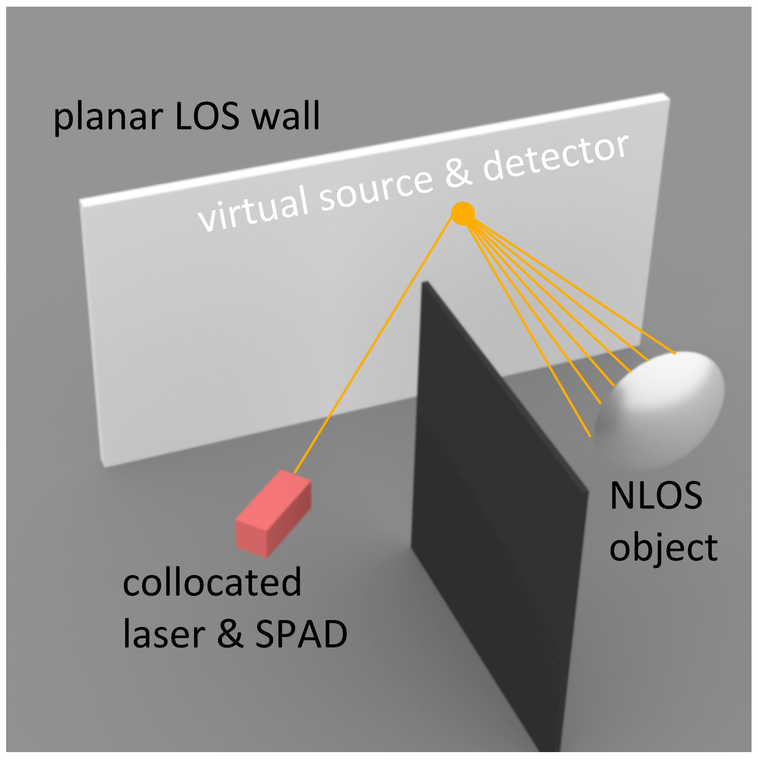
A Theory of Fermat Paths for Non-Line-of-Sight Shape Reconstruction

Shumian Xin[1], Sotiris Nousias[2,3], Kiriakos N. Kutulakos[2], Aswin C. Sankaranarayanan[1], Srinivasa G. Narasimhan[1], and Ioannis Gkioulekas[1]
[1]Carnegie Mellon University [2]University of Toronto [3]University College London

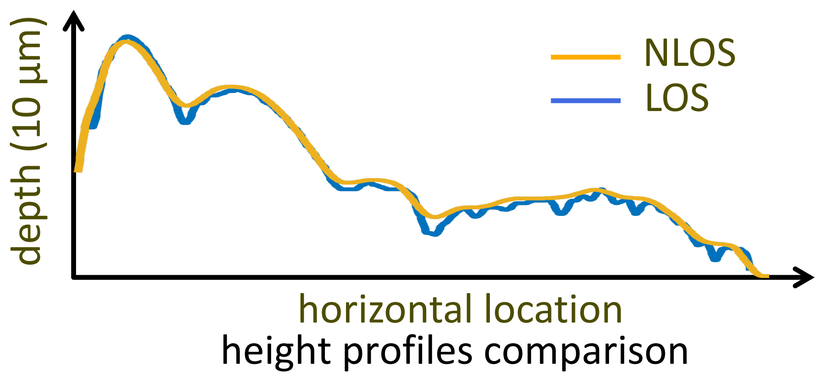
Non-line-of-sight (NLOS) setup



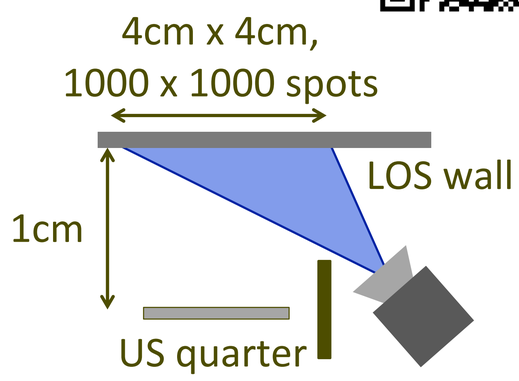
Femtosecond-scale reconstructions

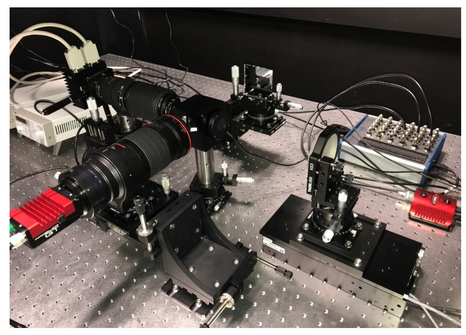

NLOS reconstruction


ground truth (direct scan)



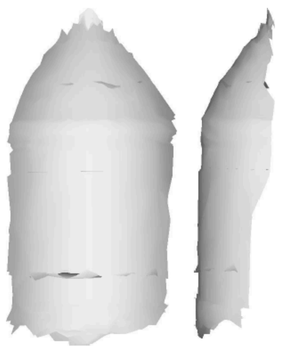
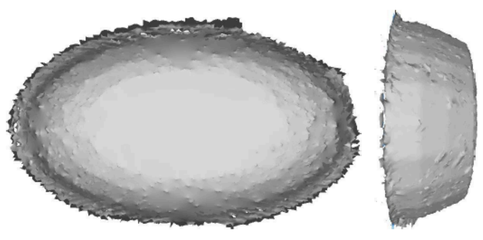
height profiles comparison

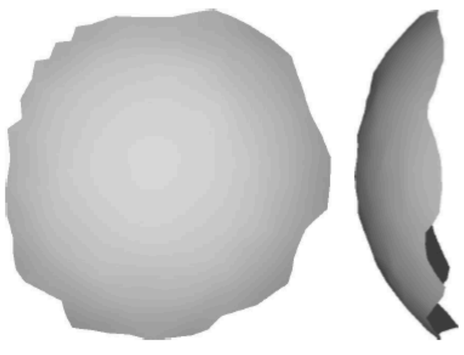


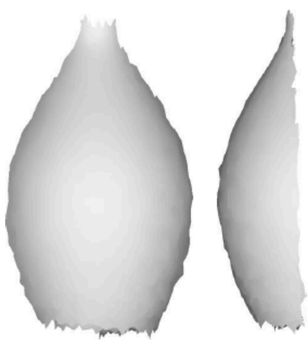

transient imaging using optical coherence tomography

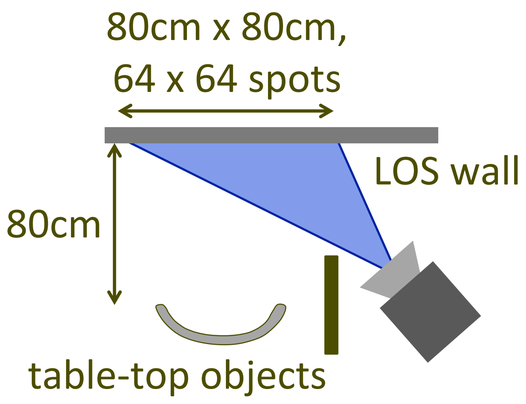
Picosecond-scale reconstructions

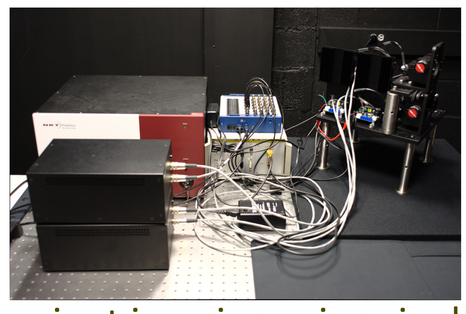




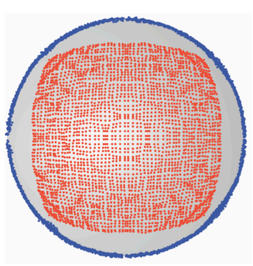
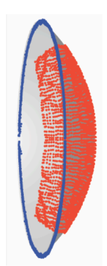
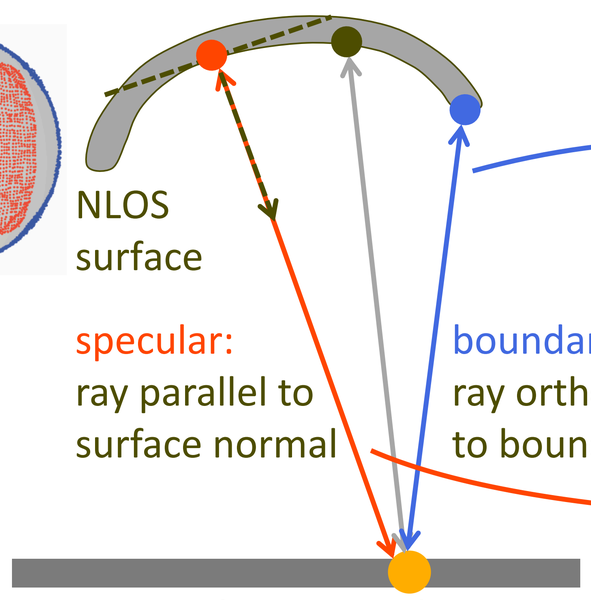





transient imaging using single-photon avalanche diode (SPAD)

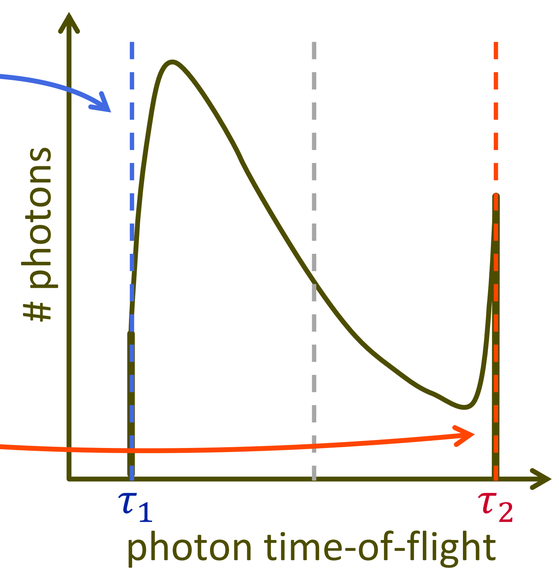
What are Fermat paths

Fermat paths: specular or boundary

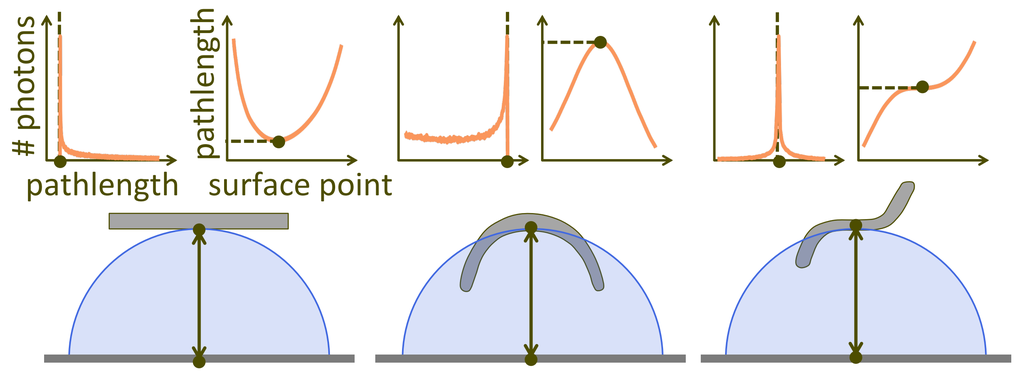
How to find Fermat paths

transient discontinuous at Fermat pathlengths

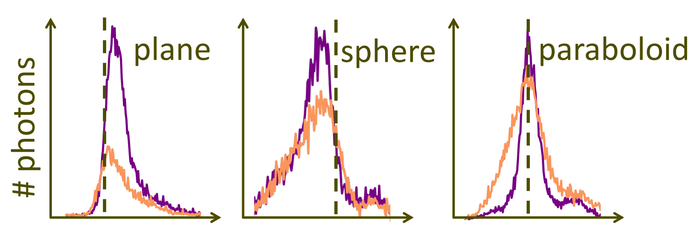


Why Fermat

Fermat's principle: paths of stationary length




aluminum foil
diffuse paint

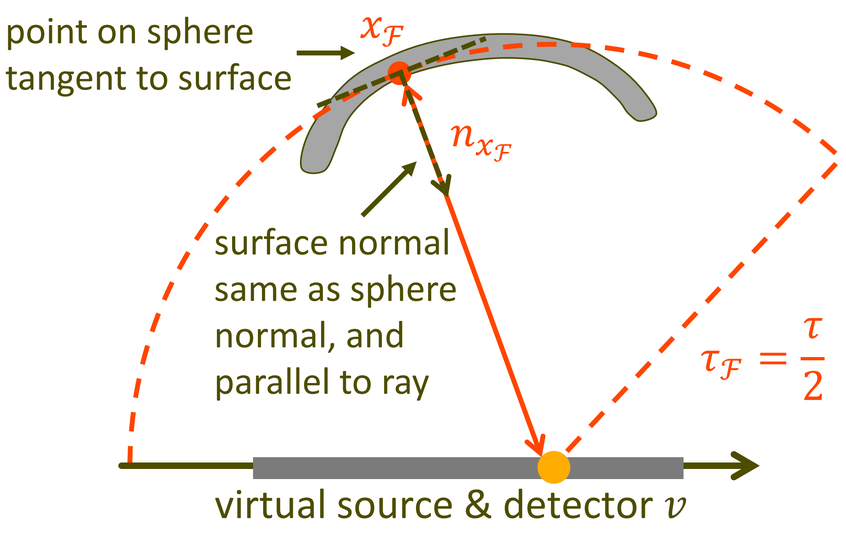


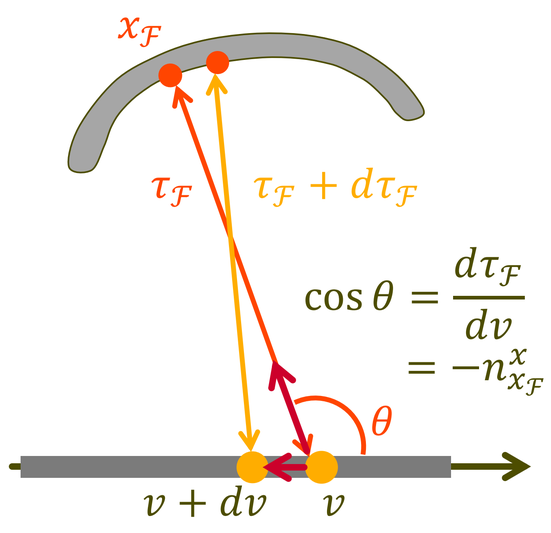
BRDF invariance and curvature

How to reconstruct a point and its normal

Fermat pathlength: spherical constraint $x_{\mathcal{F}} \in \text{sphere}\ (v,\ \tau/2)$

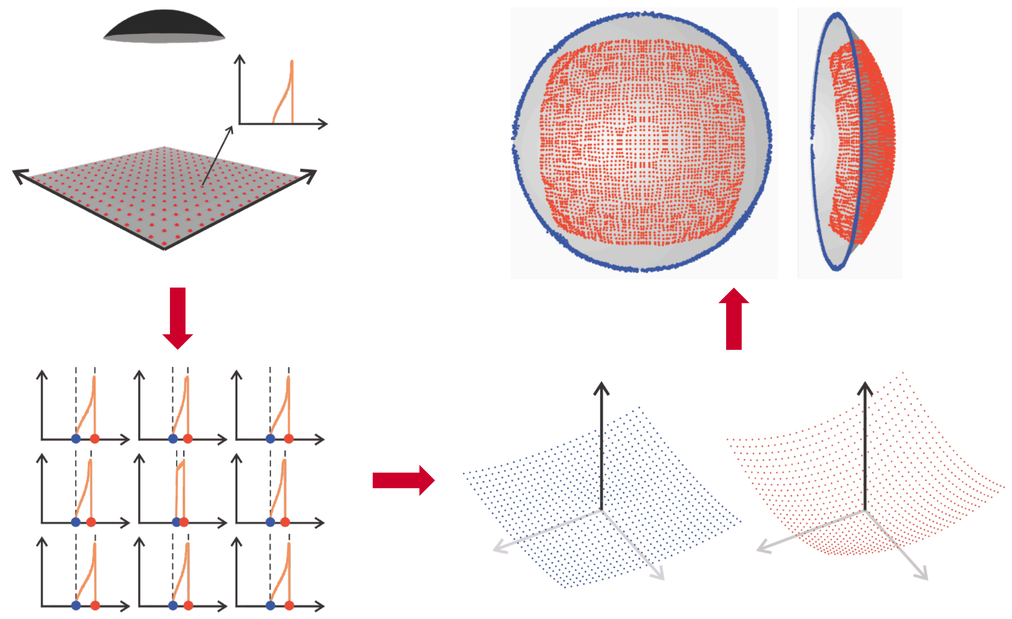
Fermat flow: ray constraint $n_{x_{\mathcal{F}}} = -\nabla_v \tau_{\mathcal{F}}(v)$





Reconstruction from sphere-ray intersection: $x_{\mathcal{F}} = v - \tau_{\mathcal{F}} \nabla_v \tau_{\mathcal{F}}(v)$

NLOS reconstruction pipeline



This work was supported by the DARPA REVEAL program under contract HR0011-16-C-0025, NSF Expeditions award CCF-1730147, the NSERC RGPIN and RTI programs, and an ONR DURIP award.

imaging.cs.cmu.edu/publications/fermat_paths/