

Dealing with indirect illumination



15-463, 15-663, 15-862
Computational Photography
Fall 2025, Lecture 18

Course announcements

- Homework assignment 6 is due on Friday.
 - Do not leave for last minute, you won't have time to complete it.
 - You can use any leftover late days.

Overview of today's lecture

- Direct and indirect illumination.
- Direct-indirect separation using high-frequency illumination.
- Back to structured light.
- Back to time-of-flight imaging.
- Direct-indirect separation using epipolar constraints.

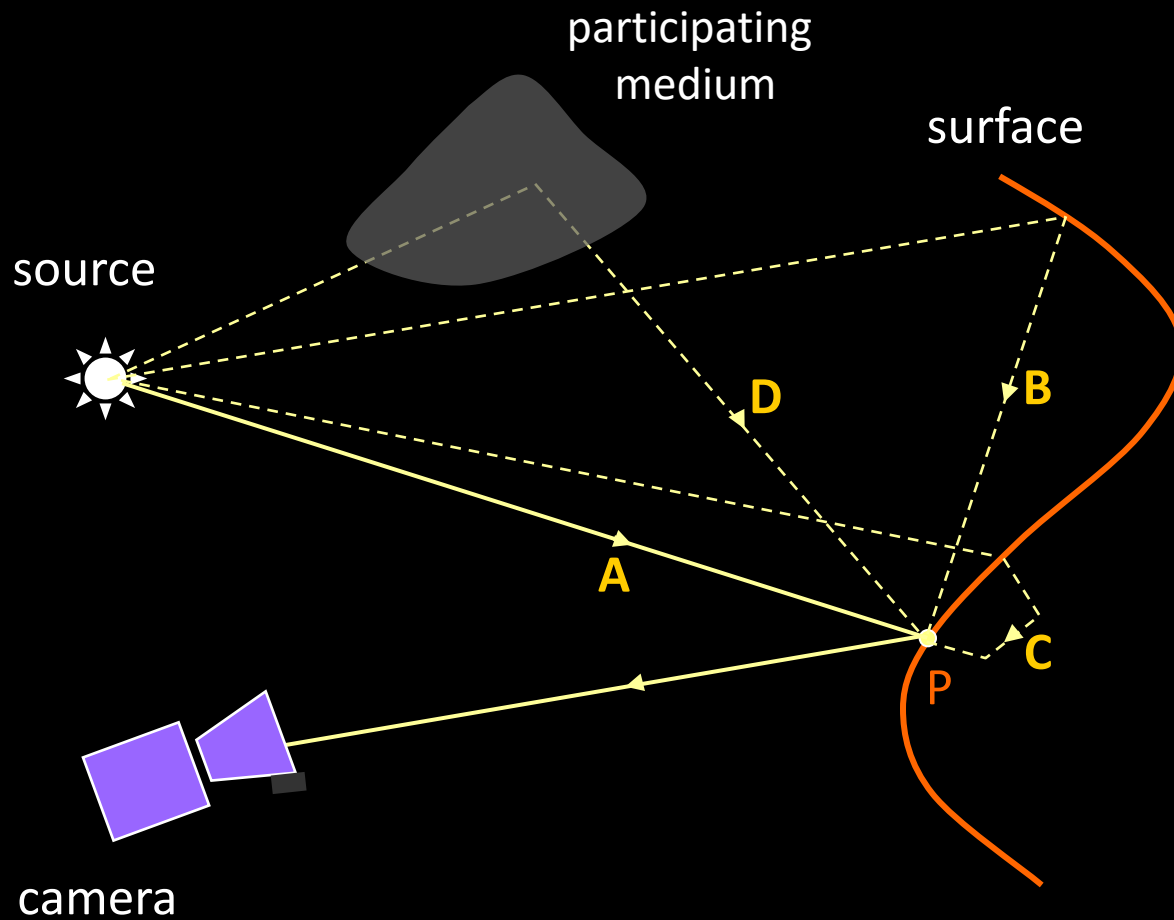
Slide credits

These slides were directly adapted from:

- Shree Nayar (Columbia).
- Matthew O'Toole (CMU).
- Supreeth Achar (Google, formerly CMU).
- Mohit Gupta (Wisconsin).

Direct and indirect illumination

Direct and Indirect Illumination



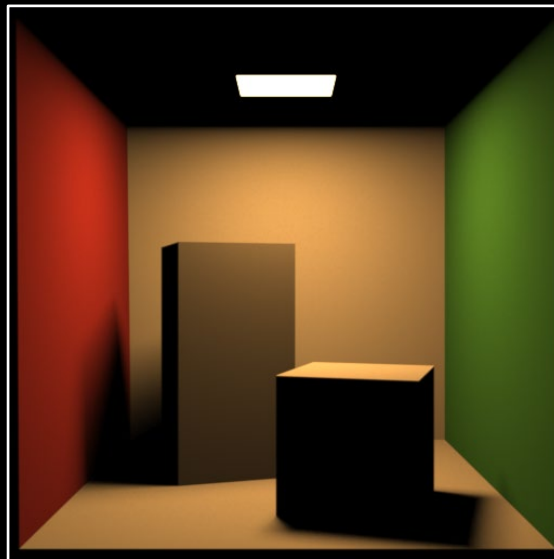
- A : Direct
- B : Interreflection
- C : Subsurface
- D : Volumetric

Easy to separate in a renderer



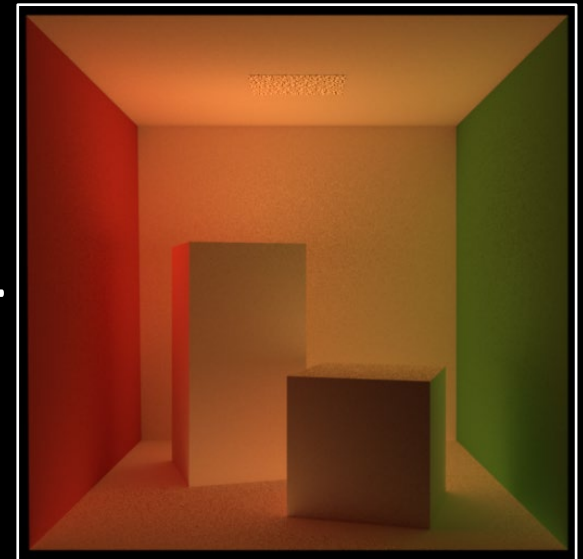
full image

=



direct

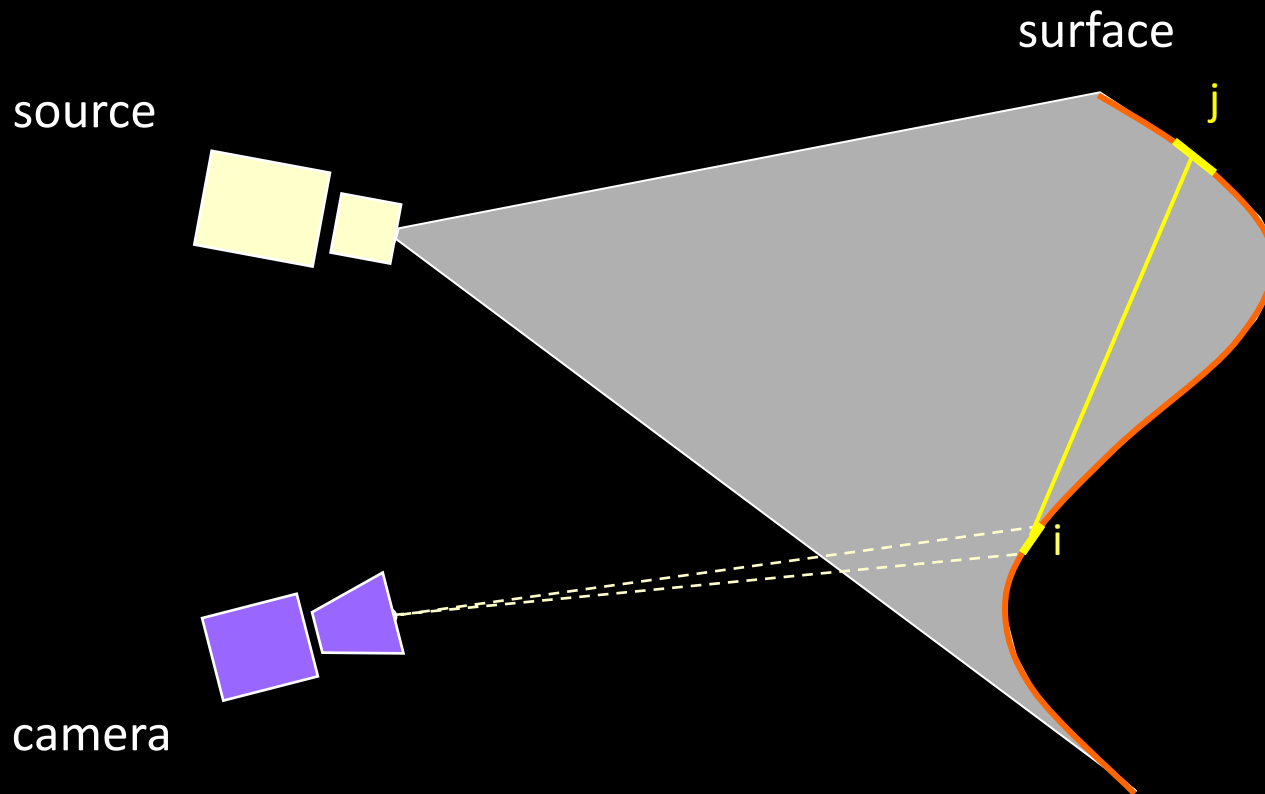
+



indirect

Direct-indirect separation using high-frequency illumination

Direct and indirect components: interreflections



$$L[c, i] = L_d[c, i] + L_g[c, i]$$

radiance

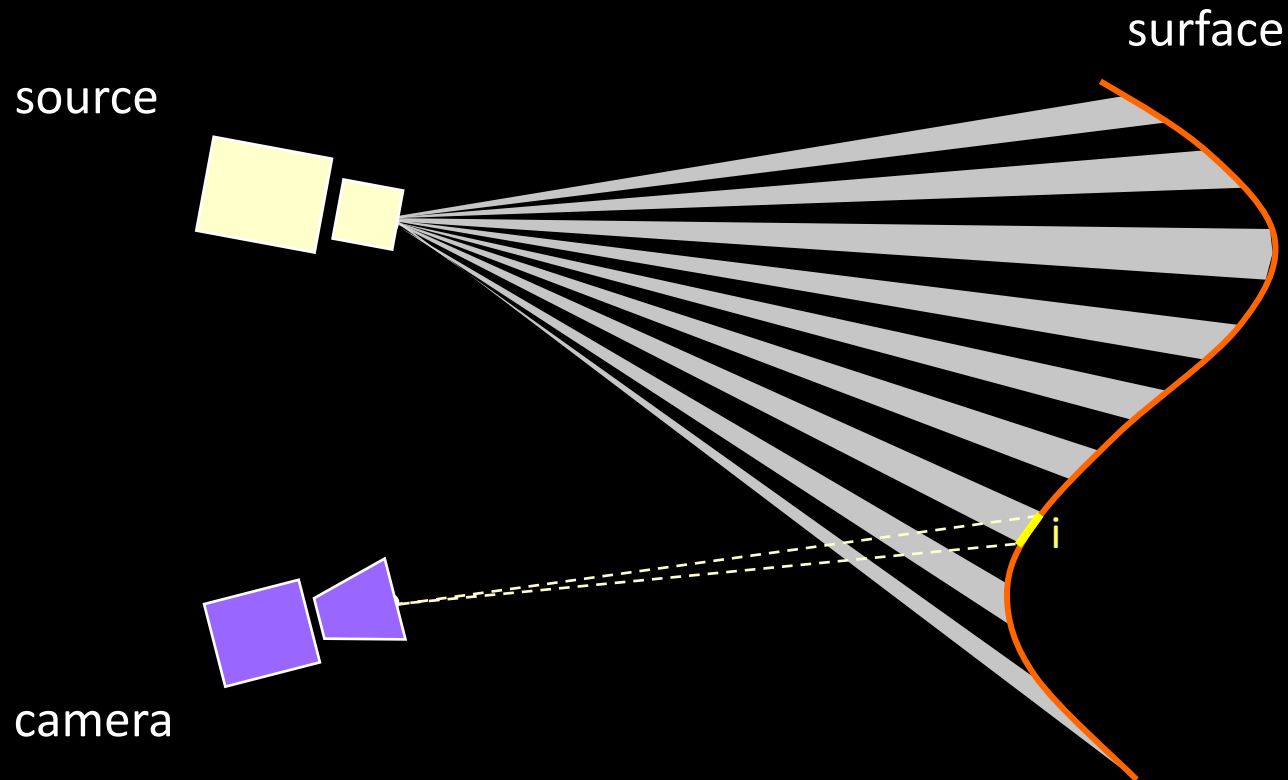
direct

indirect

$$L_g[c, i] = \sum_P A[i, j] L[i, j]$$

BRDF and geometry

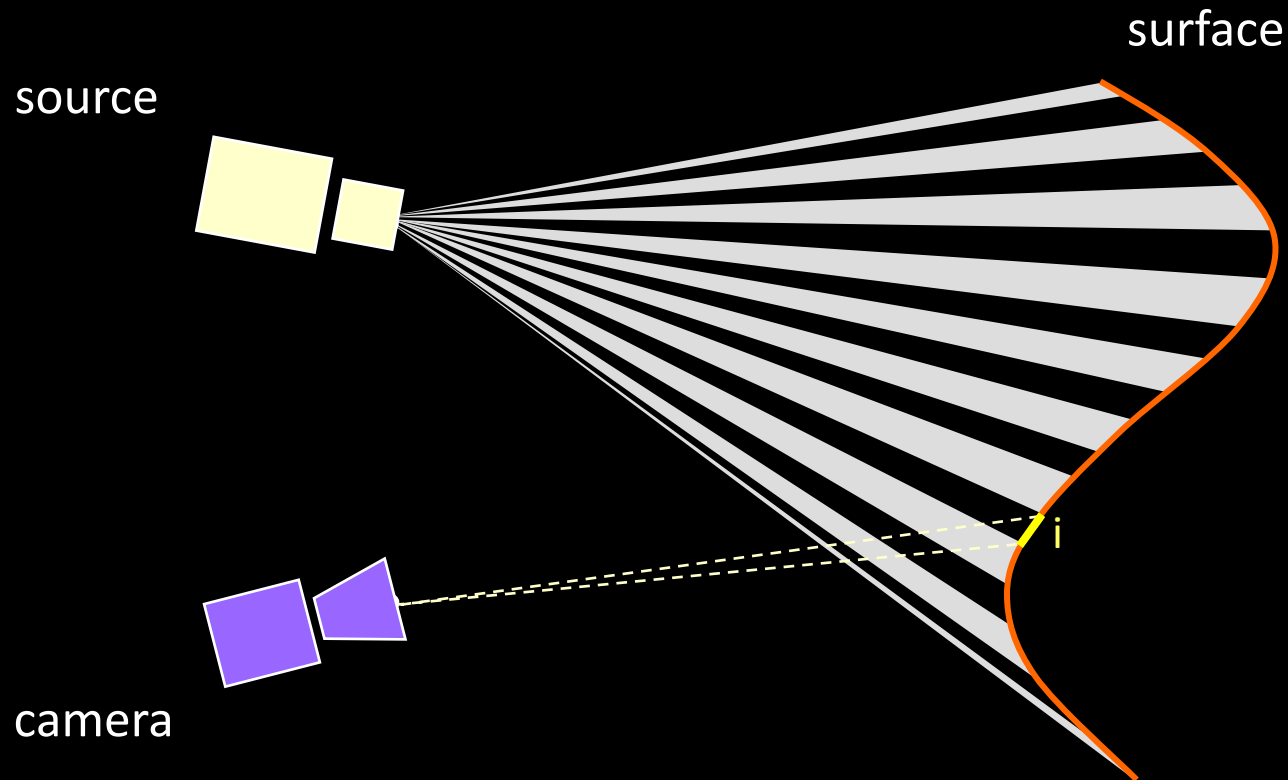
High Frequency Illumination Pattern



$$L^+[c,i] = L_d[c,i] + \alpha L_g[c,i]$$

fraction of activated source elements

High Frequency Illumination Pattern



$$L^+[c, i] = L_d[c, i] + \alpha L_g[c, i]$$

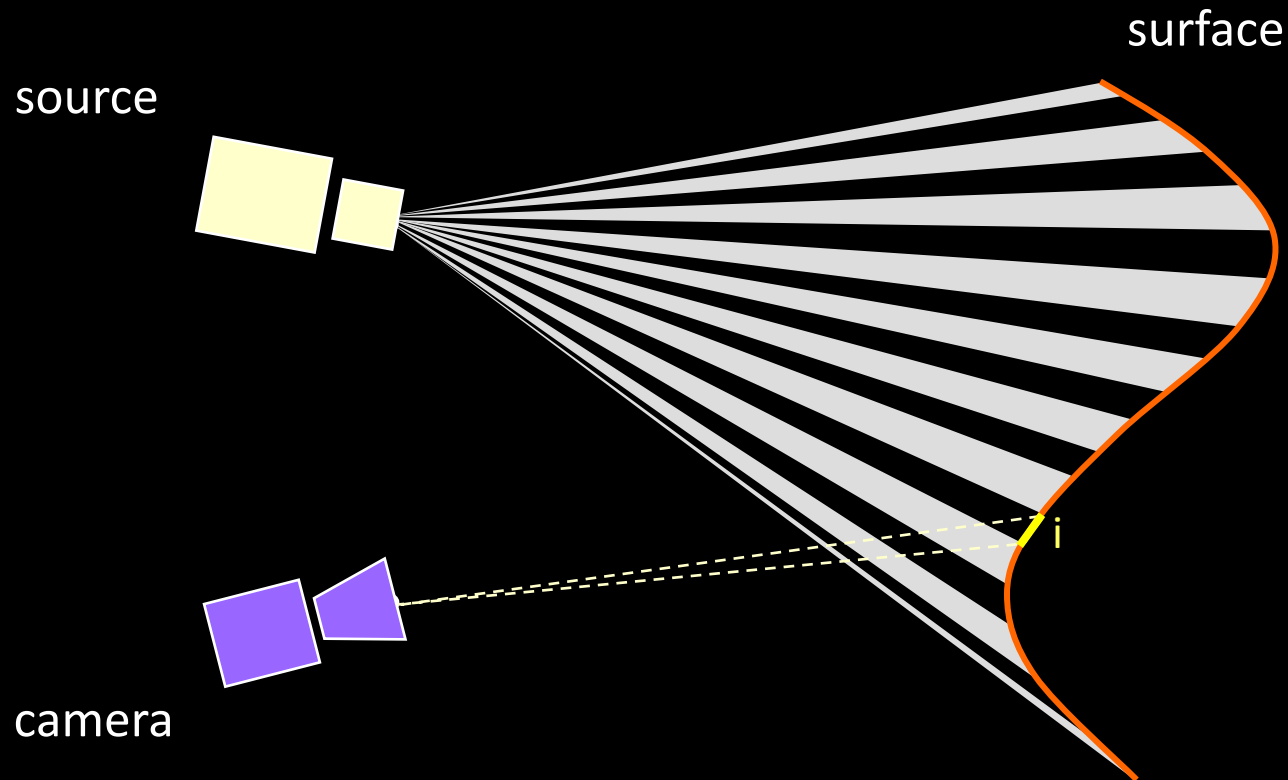
$$L^-[c, i] = (1 - \alpha) L_g[c, i]$$

fraction of activated source elements

Important insight:

- Indirect illumination is approximately invariant to high-frequency lighting.
- You can think of indirect illumination effects as a low-pass filter.

High Frequency Illumination Pattern



$$L^+[c, i] = L_d[c, i] + \alpha L_g[c, i]$$

$$\bar{L}^-[c, i] = (1 - \alpha) L_g[c, i]$$

What does approximate invariance mean in this case?

Separation from Two Images

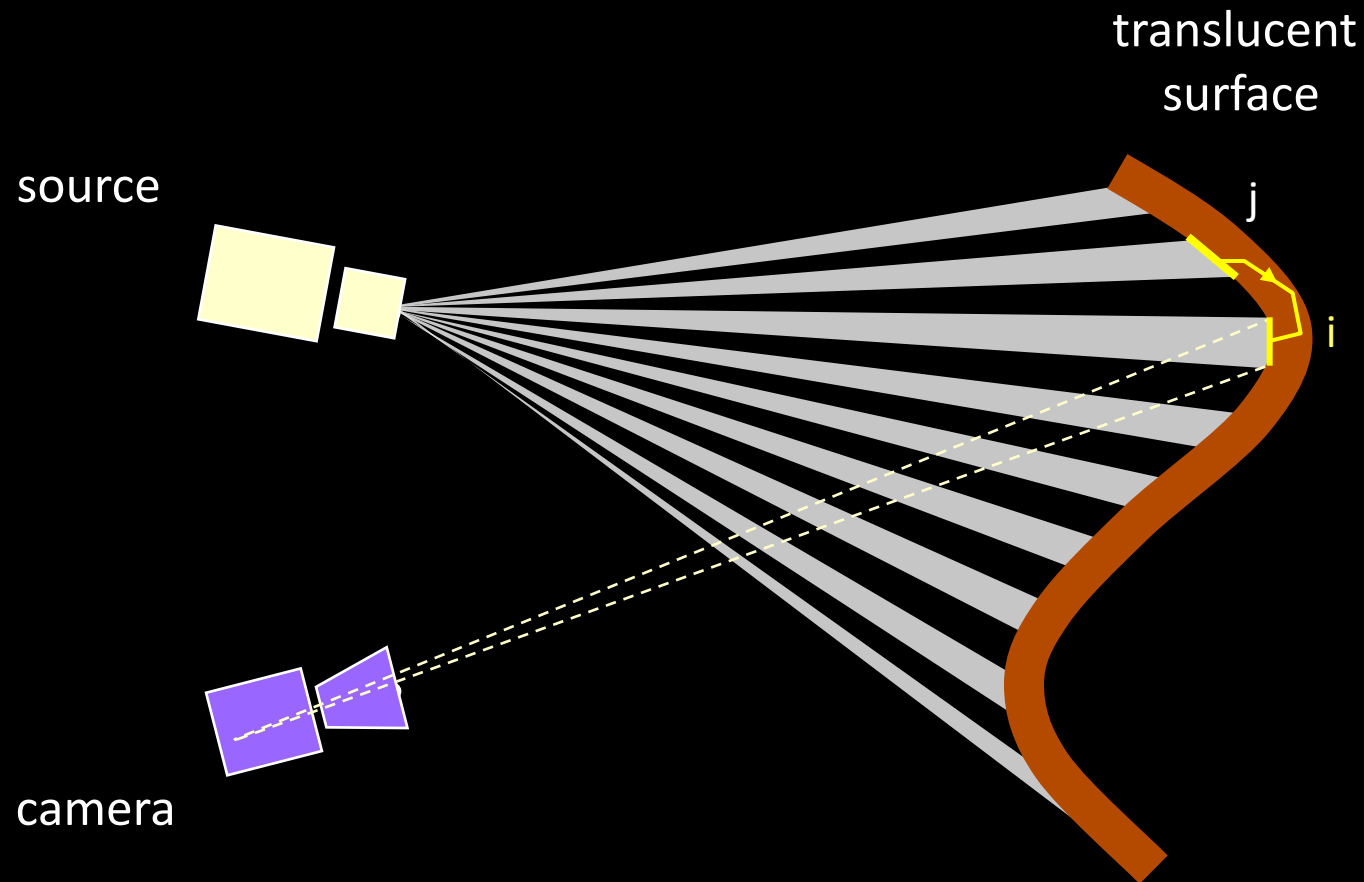
$$\alpha = \frac{1}{2}:$$

$$L_d = L_{\max} - L_{\min}, \quad L_g = 2L_{\min}$$

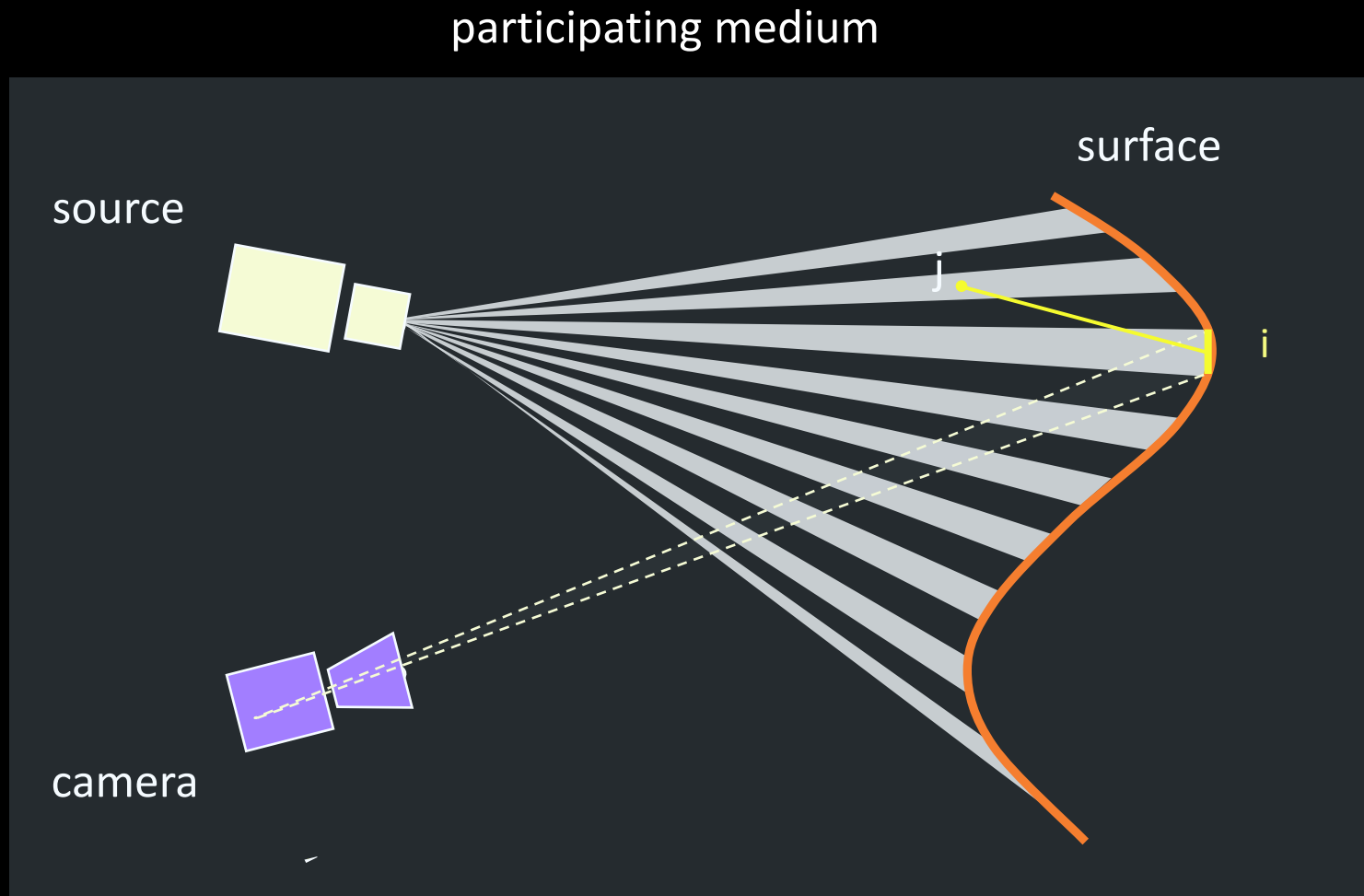
direct

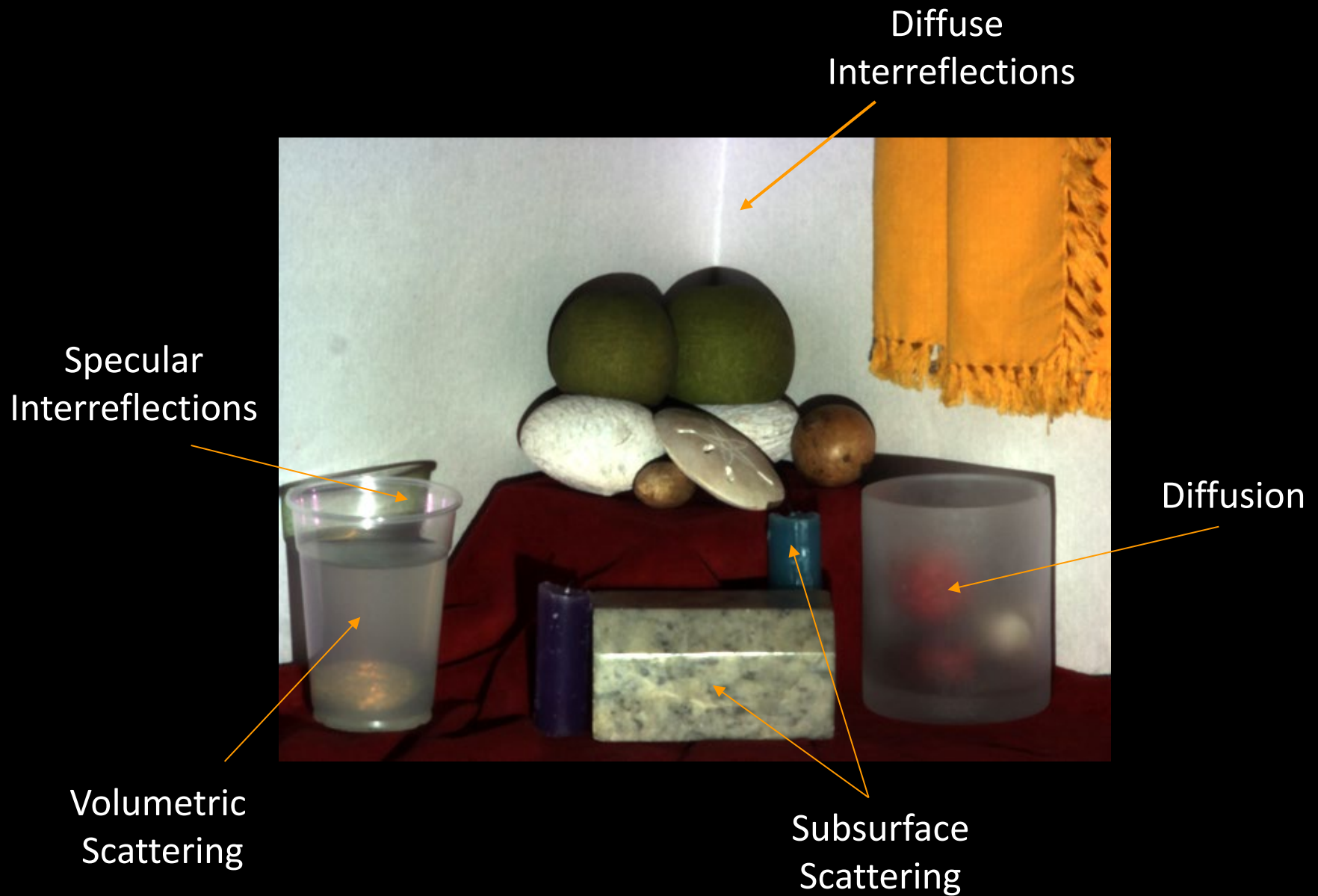
indirect

Other Indirect Effects: Subsurface Scattering

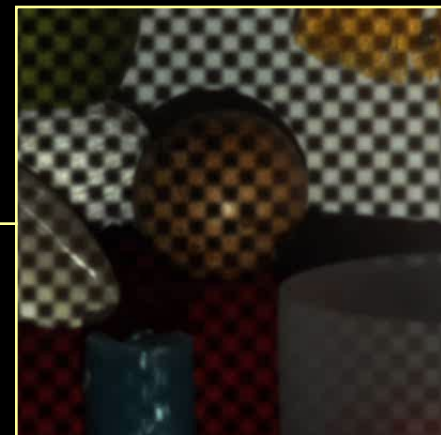


Other Indirect Effects: Volumetric Scattering





Scene



Direct

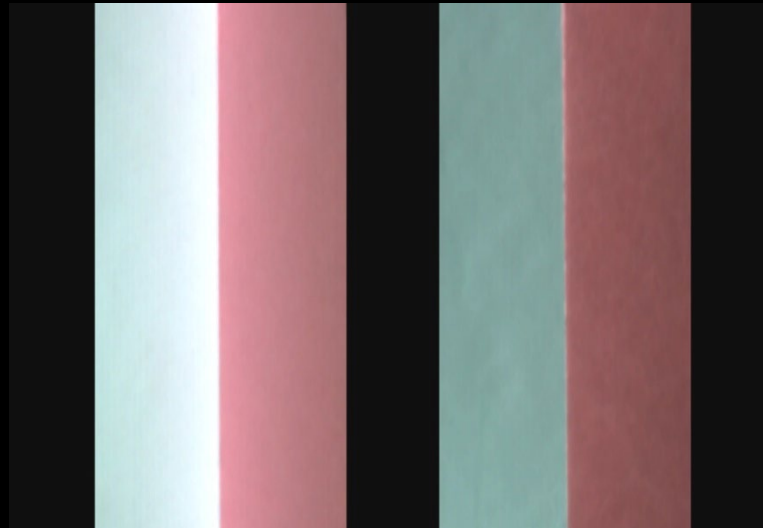


Indirect

V-Grooves: Diffuse Interreflections

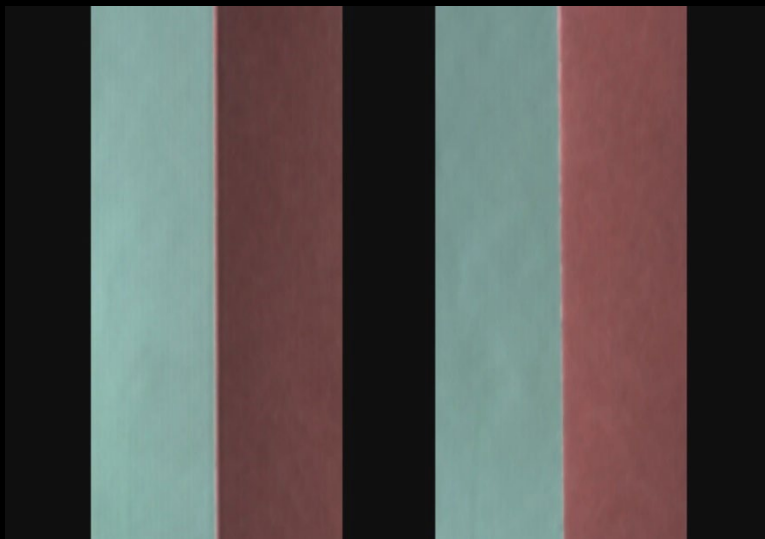
concave

convex

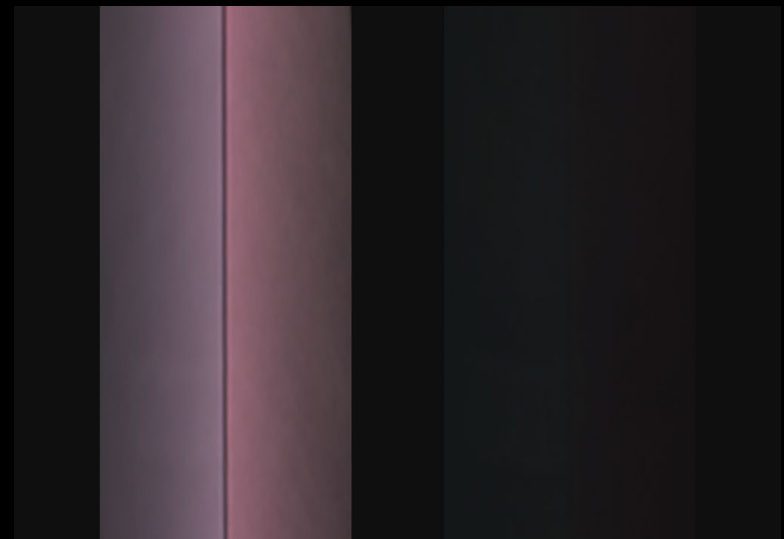


Psychophysics:

Gilchrist 79, Bloj et al. 04



Direct



Indirect

Real World Examples:

Can You Guess the Images?

Eggs: Diffuse Interreflections

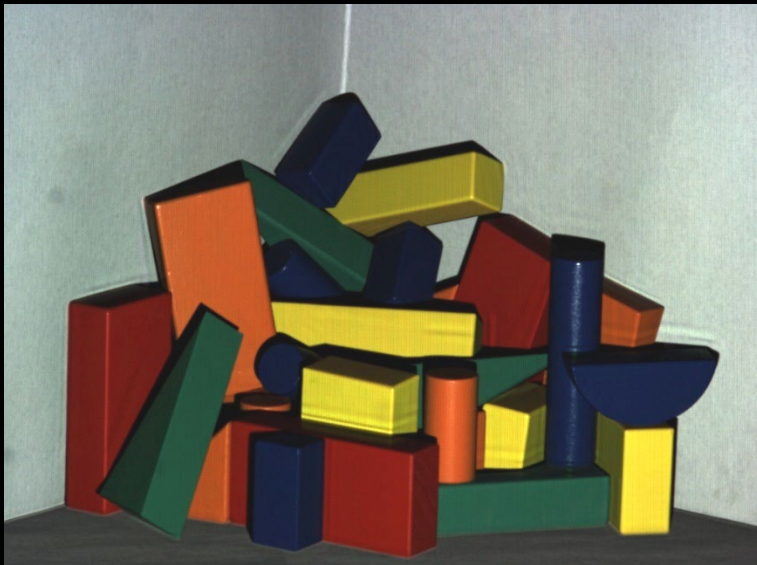
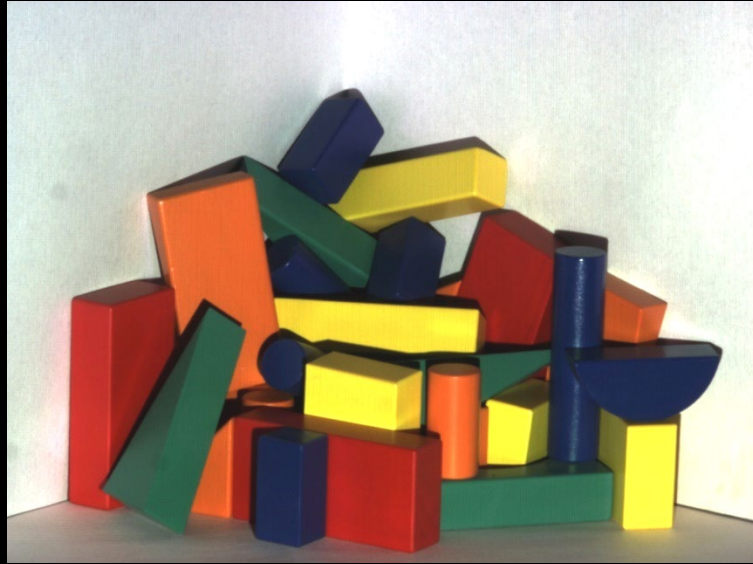


Direct



Indirect

Wooden Blocks: Specular Interreflections

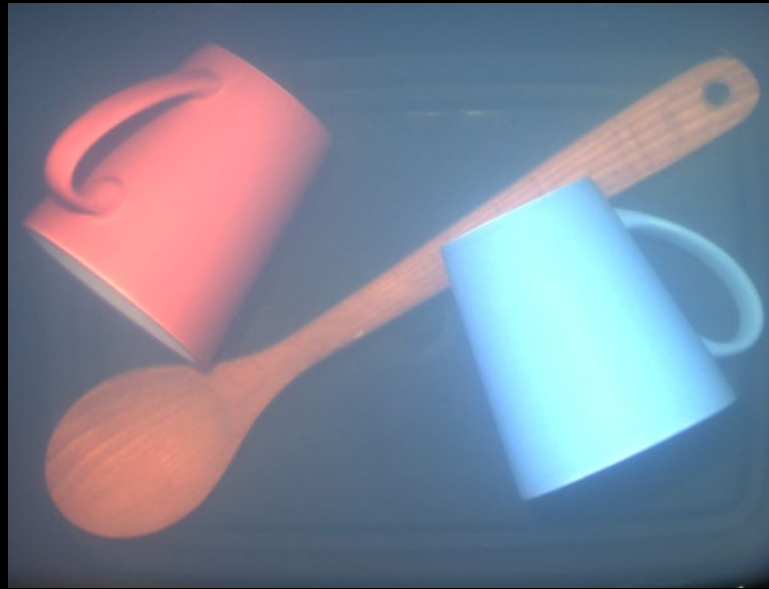


Direct



Indirect

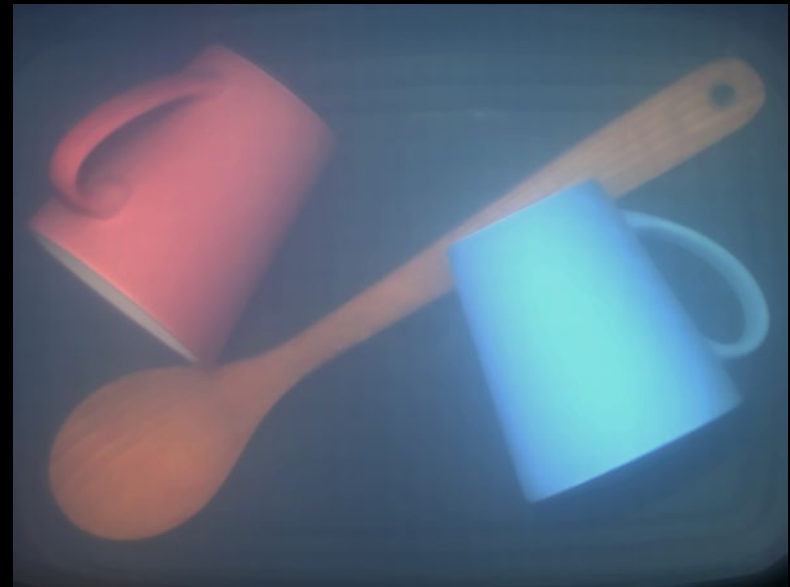
Kitchen Sink: Volumetric Scattering



Volumetric Scattering:
Chandrasekar 50, Ishimaru 78



Direct

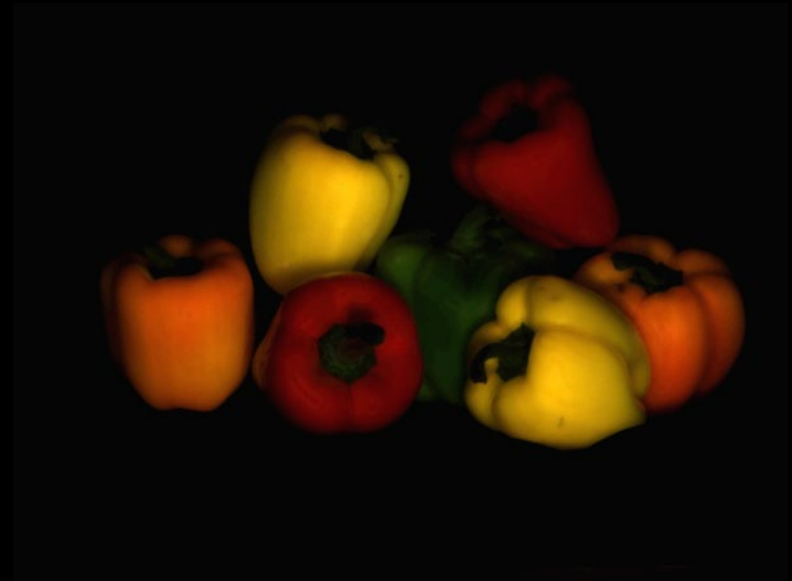


Indirect

Peppers: Subsurface Scattering



Direct

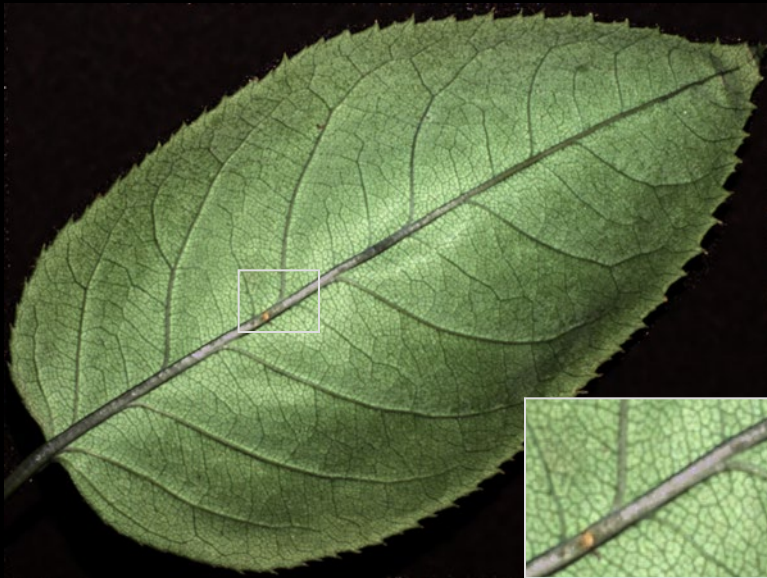


Indirect

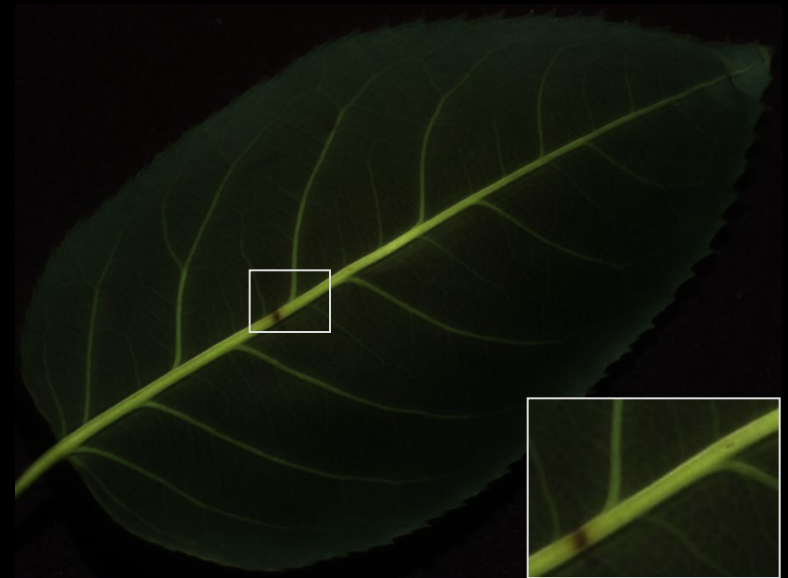
Tea Rose Leaf



Leaf Anatomy: Purves et al. 03



Direct

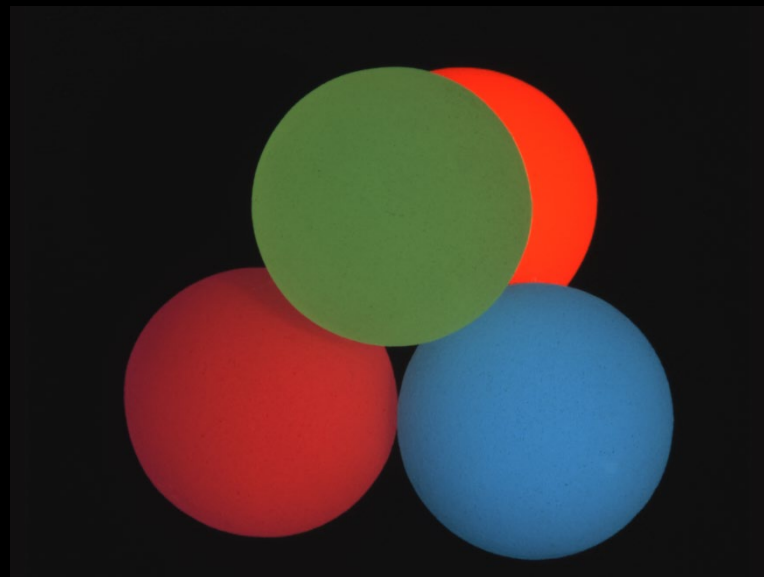


Indirect

Translucent Rubber Balls



Direct

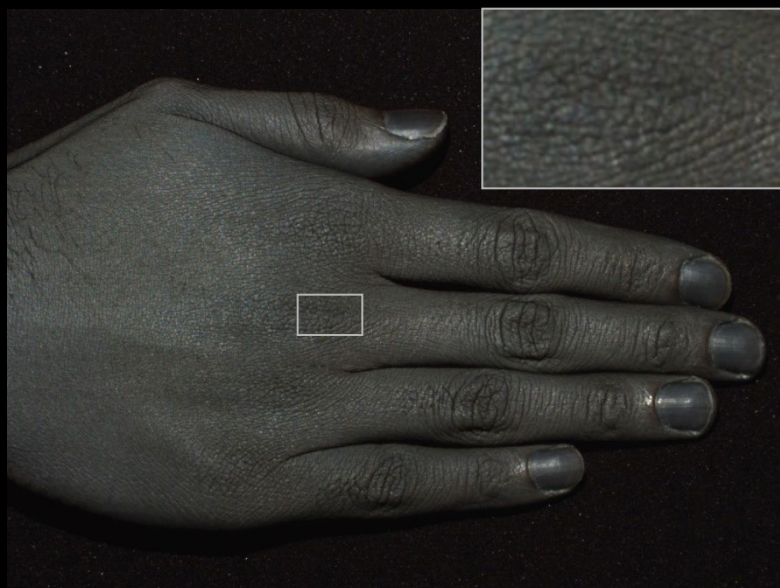


Indirect

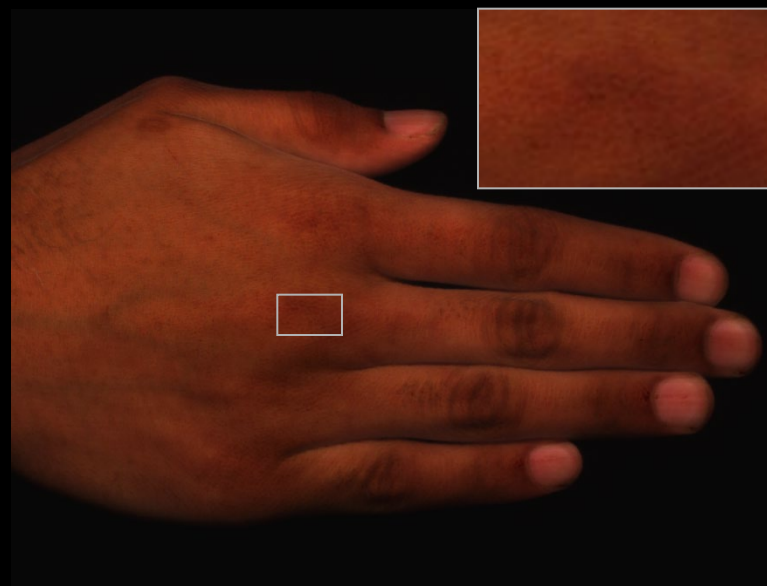
Hand



Skin: Hanrahan and Krueger 93,
Uchida 96, Haro 01, Jensen et al. 01,
Igarashi et al. 05, Weyrich et al. 05



Direct

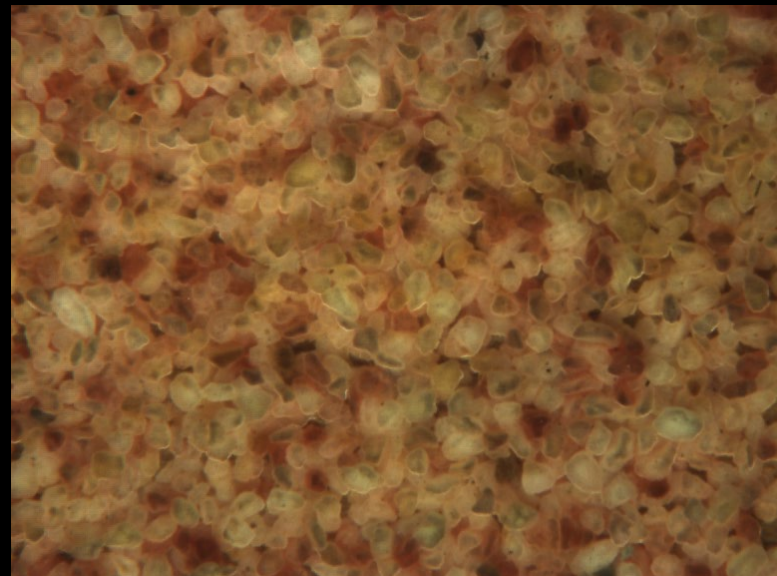


Indirect

Pebbles: 3D Texture



Direct



Indirect

Pink Carnation



Spectral Bleeding: Funt et al. 91

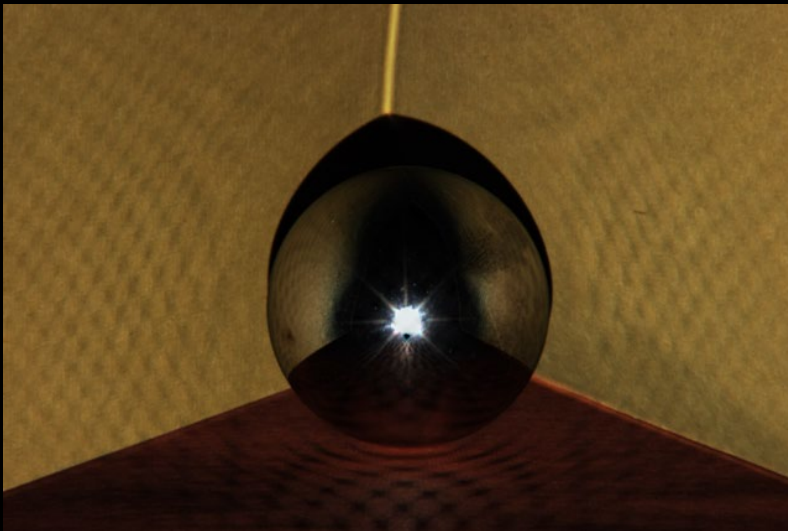
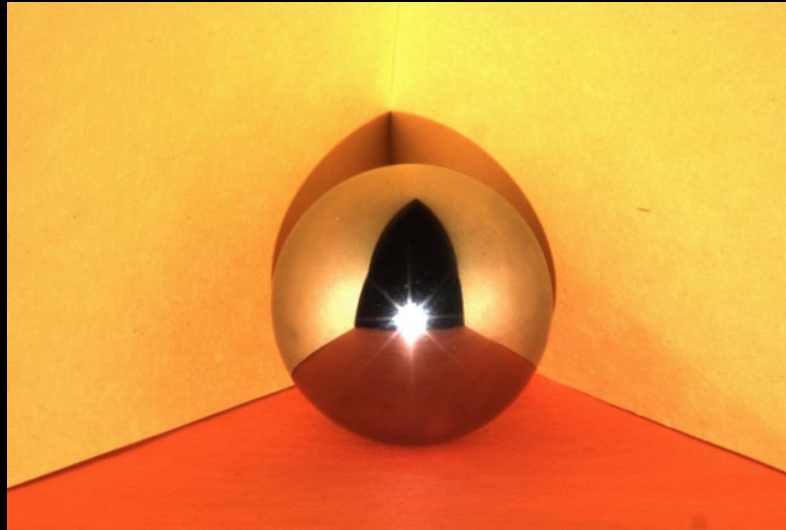


Direct

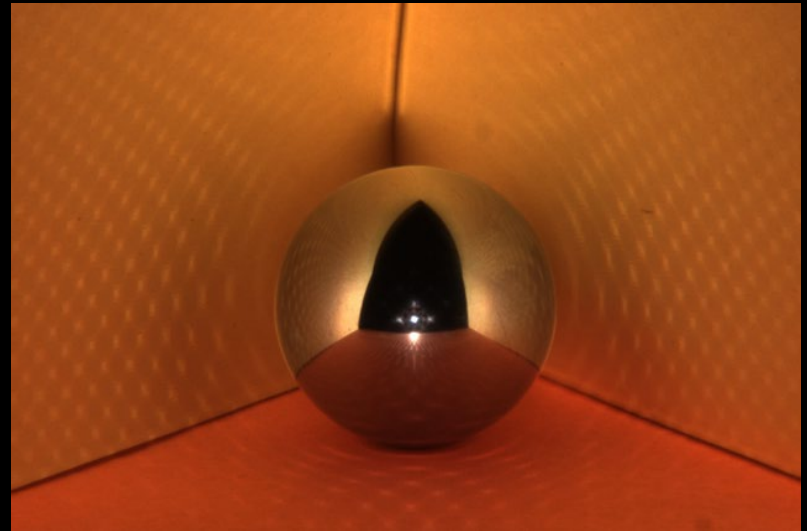


Indirect

Mirror Ball: Failure Case



Direct



Indirect

Building Corner



Stick



Shadow

3D from Shadows:
Bouguet and Perona 99

$$L_d = L_{\max} - L_{\min} , \quad L_g = L_{\min}$$

direct

indirect

Building Corner

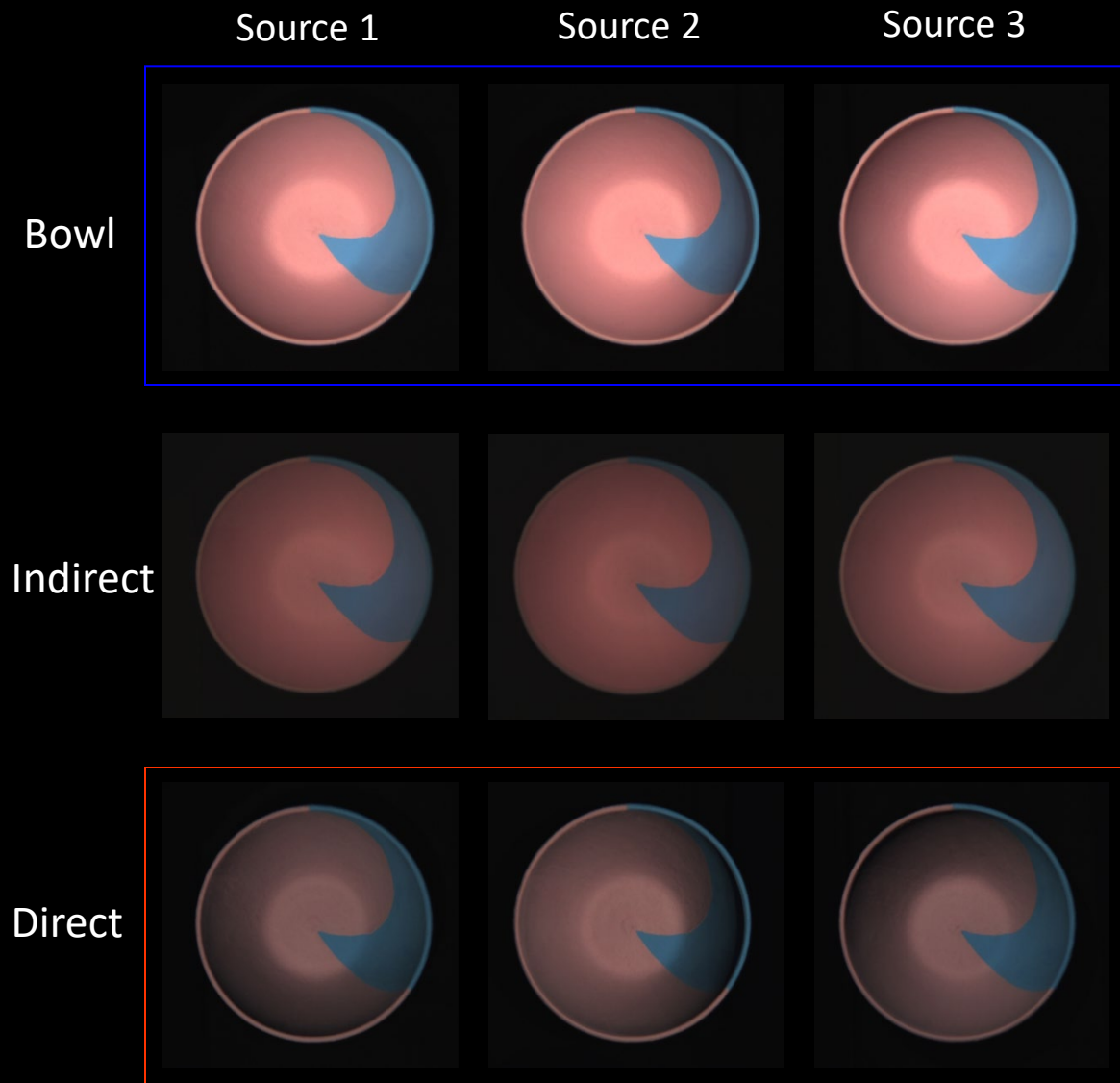


Direct

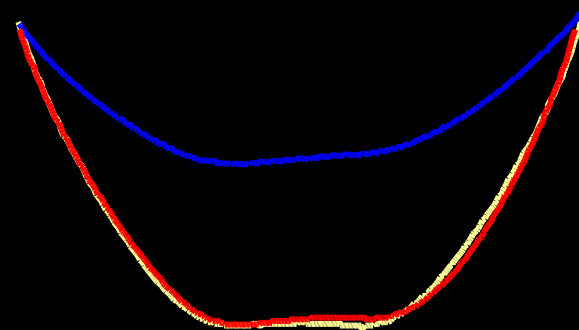


Indirect

Photometric Stereo using Direct Images



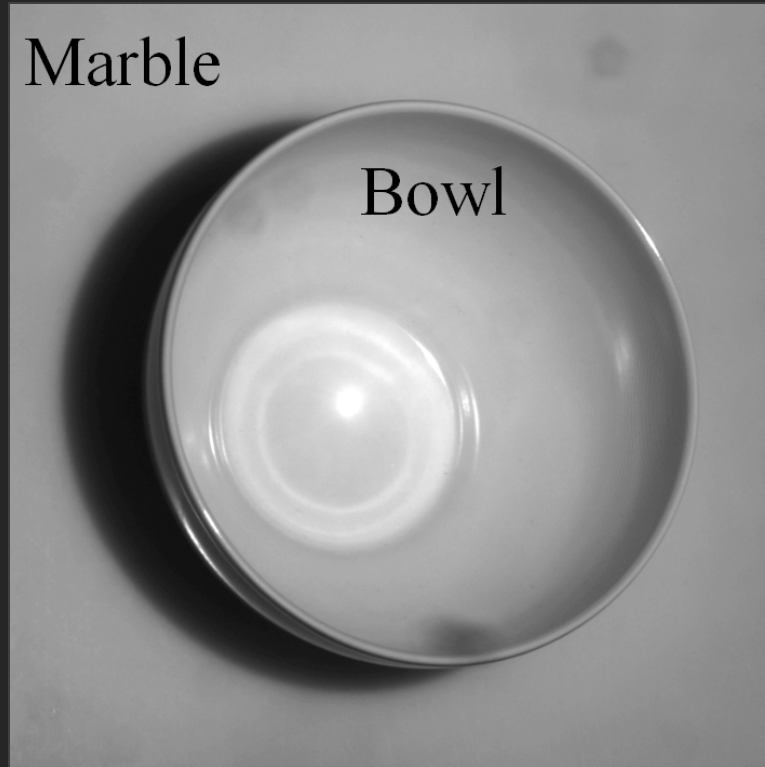
Shape



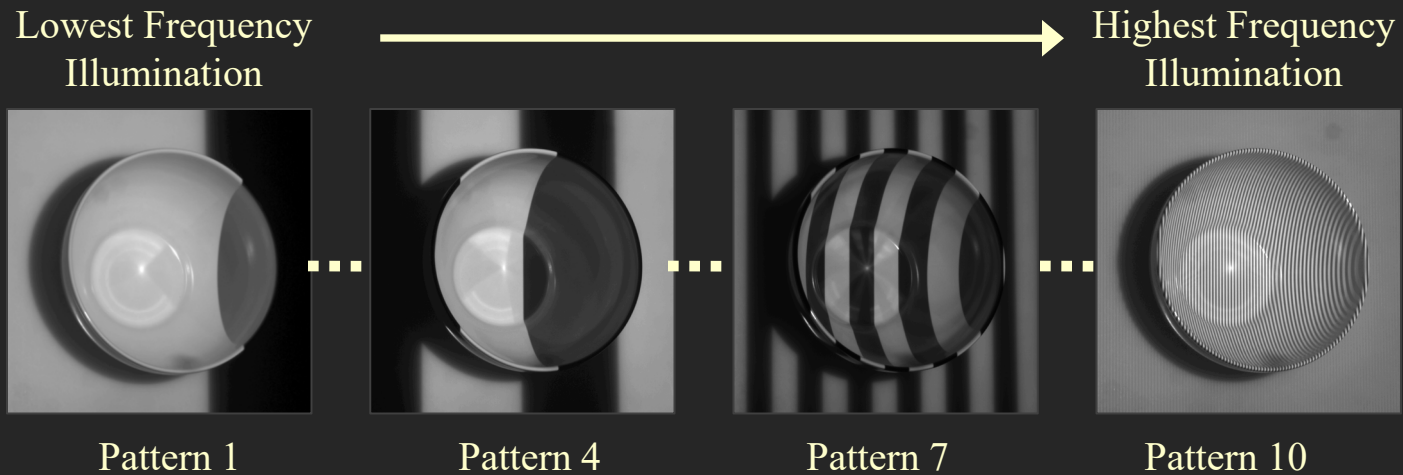
Application to structured light

Why is indirect illumination a problem?

Bowl on a Marble Slab

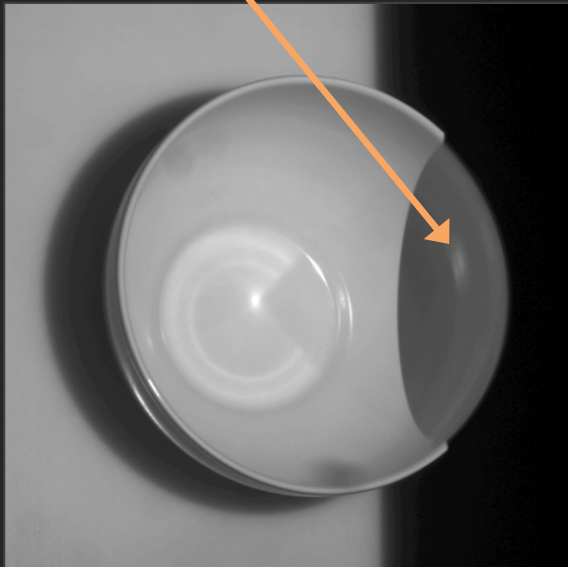


Captured images under conventional Gray codes



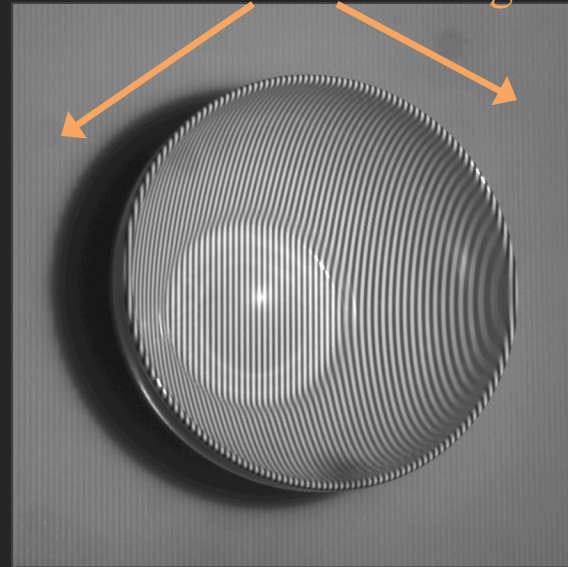
Issues due to indirect illumination effects

Strong Inter-reflections



Low-frequency pattern

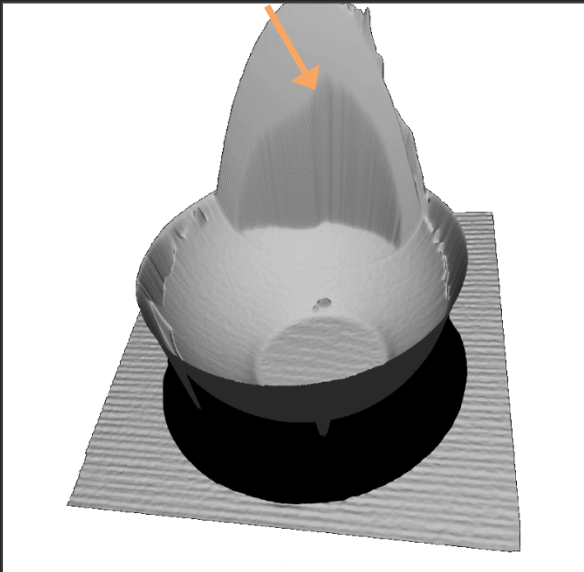
Blurring due to
Sub-surface Scattering



High-frequency pattern

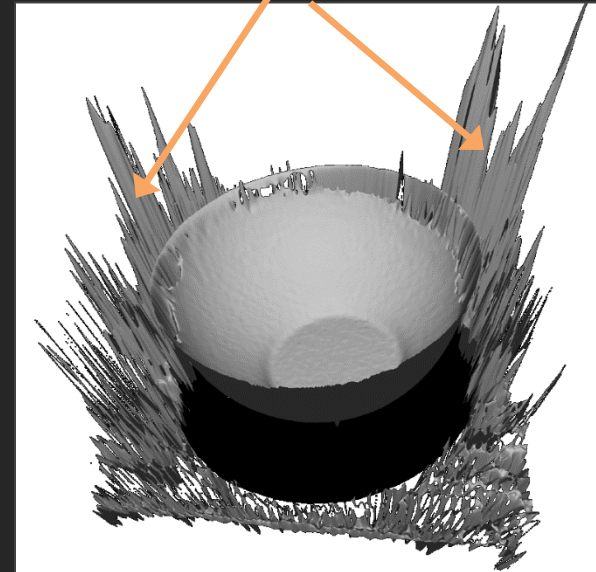
3D Visualizations: State of the Art

Errors due to
interreflections



Conventional Gray
(11 images)

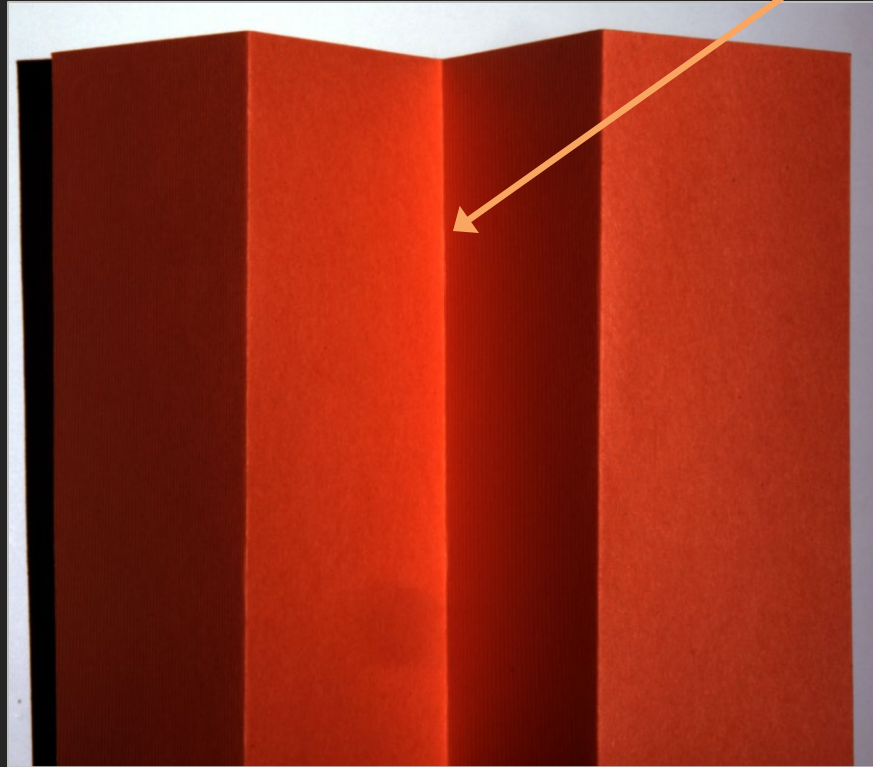
Errors due to
sub-surface scattering



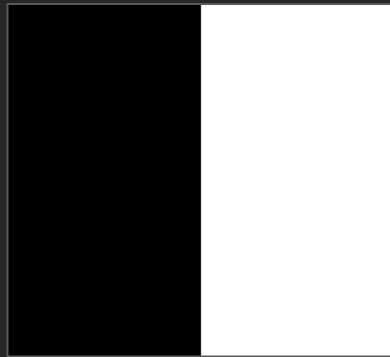
Modulated Phase-Shifting
(162 images)

V-Groove Scene

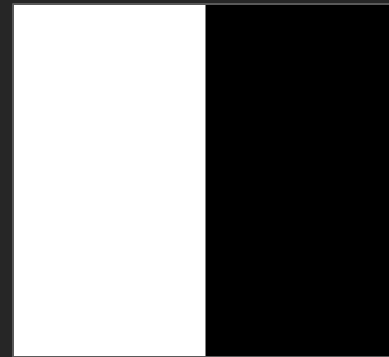
Inter-reflections



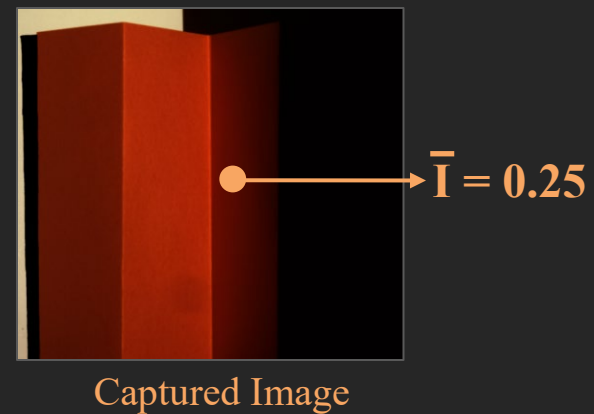
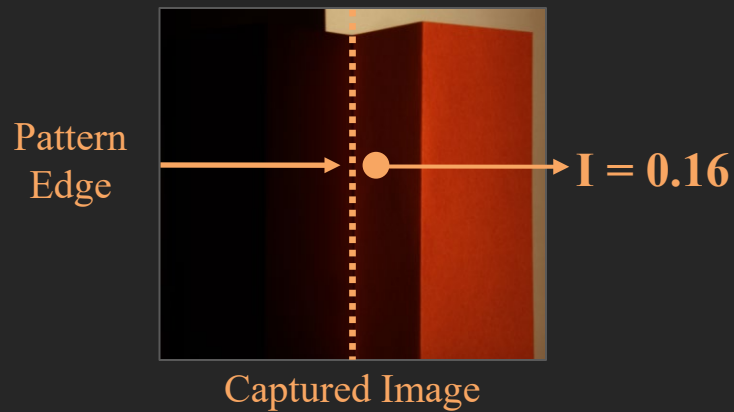
Conventional Gray codes



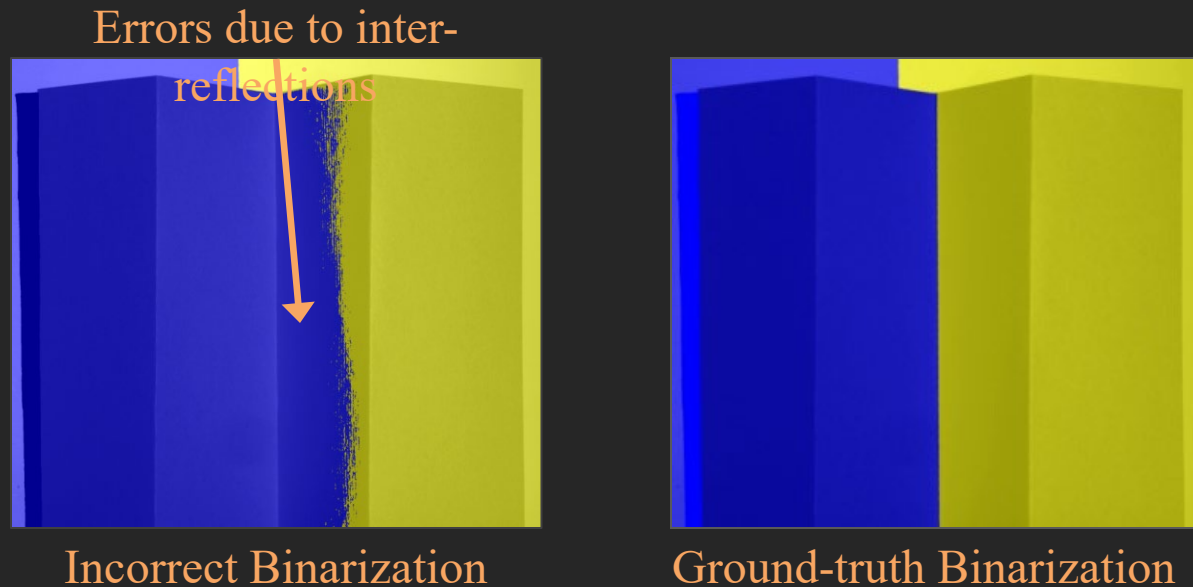
Low frequency pattern



Inverse Pattern

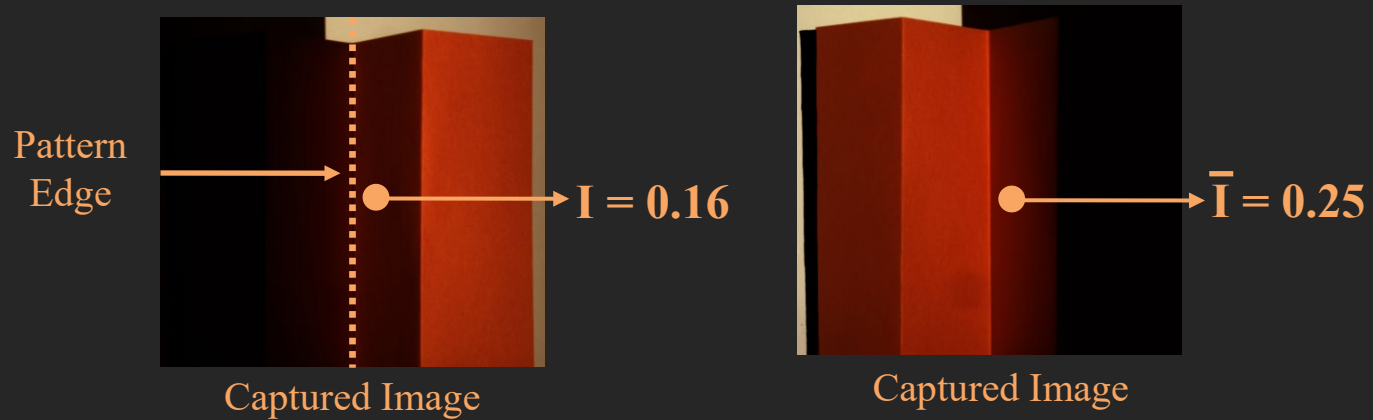


Binarization error



■ One (illuminated) ■ Zero (not-illuminated)

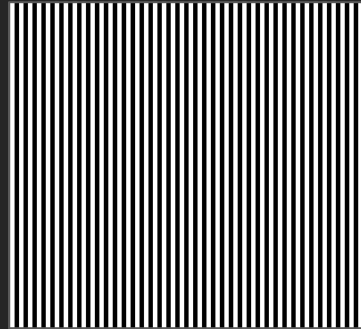
Low-frequency patterns



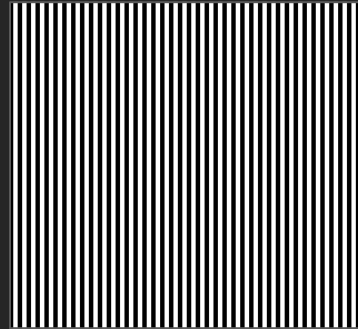
$$I = \text{Direct} + \alpha \cdot \text{Indirect} \quad \bar{I} = (1 - \alpha) \cdot \text{Indirect}$$

$$\alpha \approx 0, \text{ Direct} < \text{Indirect} \Rightarrow I < \bar{I}$$

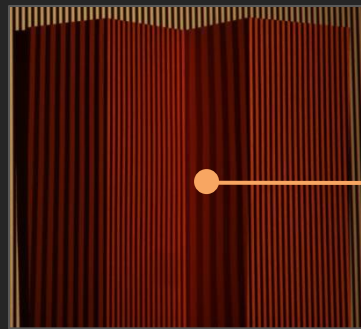
High-frequency patterns



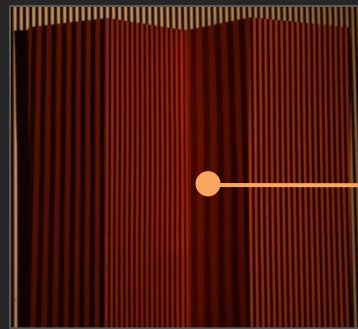
Pattern



Inverse Pattern



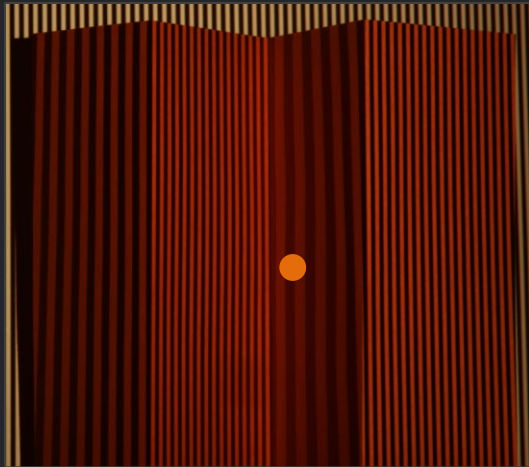
Captured Image



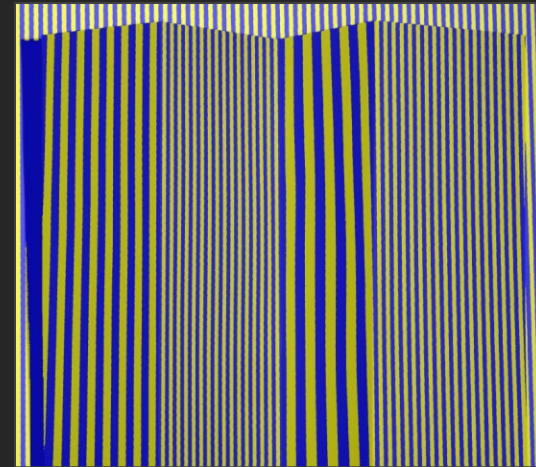
Captured Image

$$I = \text{Direct} + 0.5 \text{ Indirect} > \bar{I} = 0.5 \text{ Indirect}$$

High-frequency Patterns are Decoded Correctly



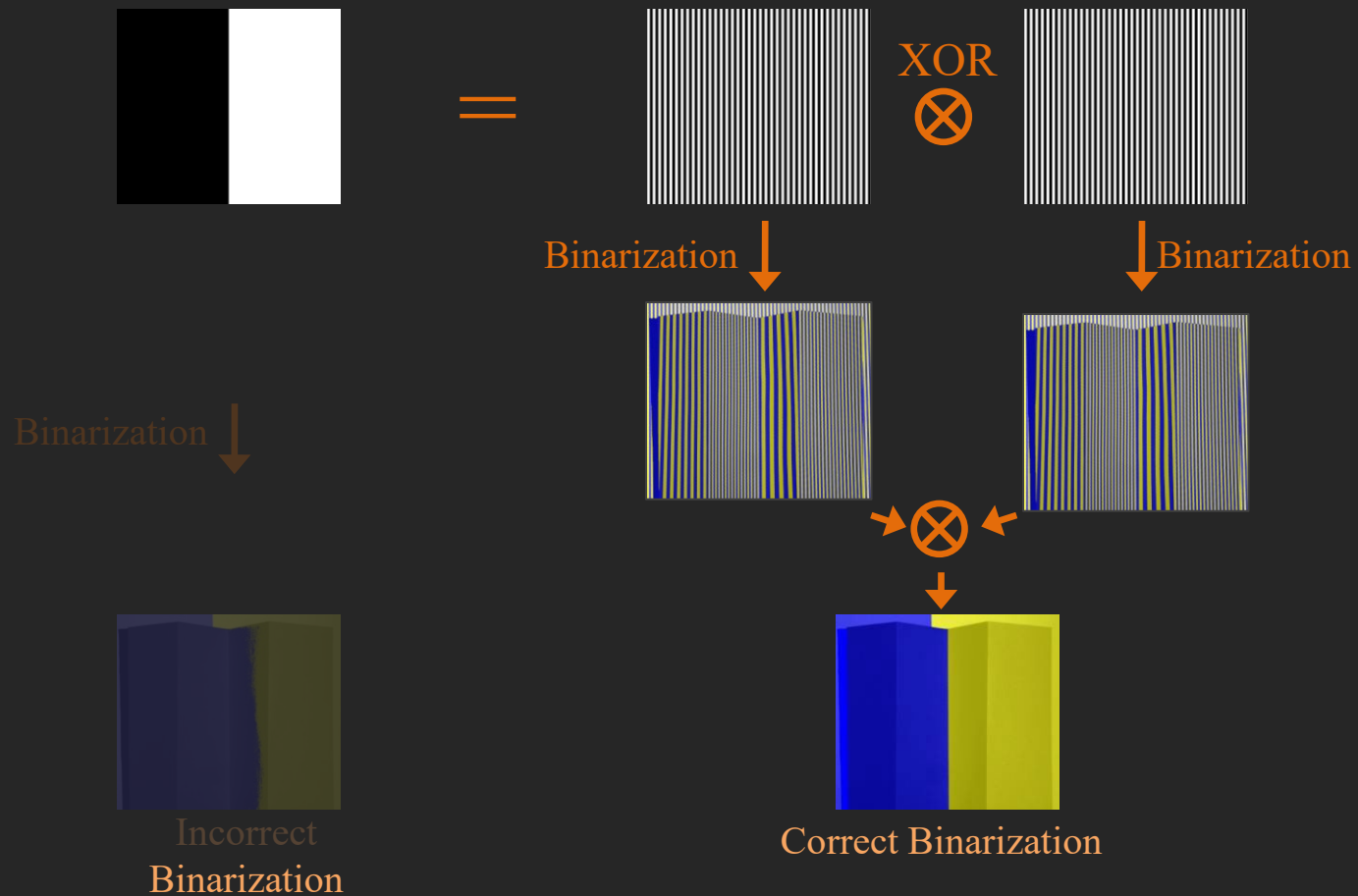
Captured Image



Binary Decoding

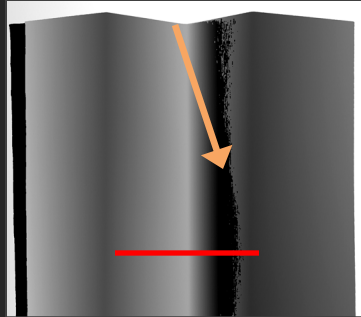
Logical Coding and Decoding

Logical Coding and Decoding

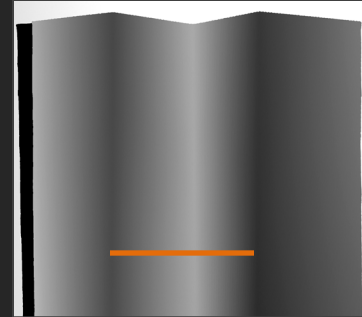


Depth Map Comparison

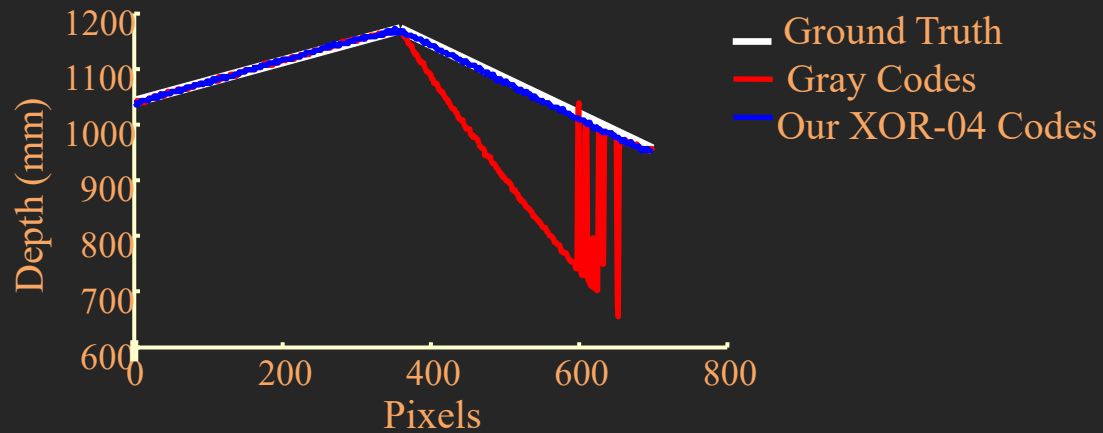
Errors due to Inter-reflections



Conventional Gray Codes (11 images)



Our XOR-04 Codes (11 images)



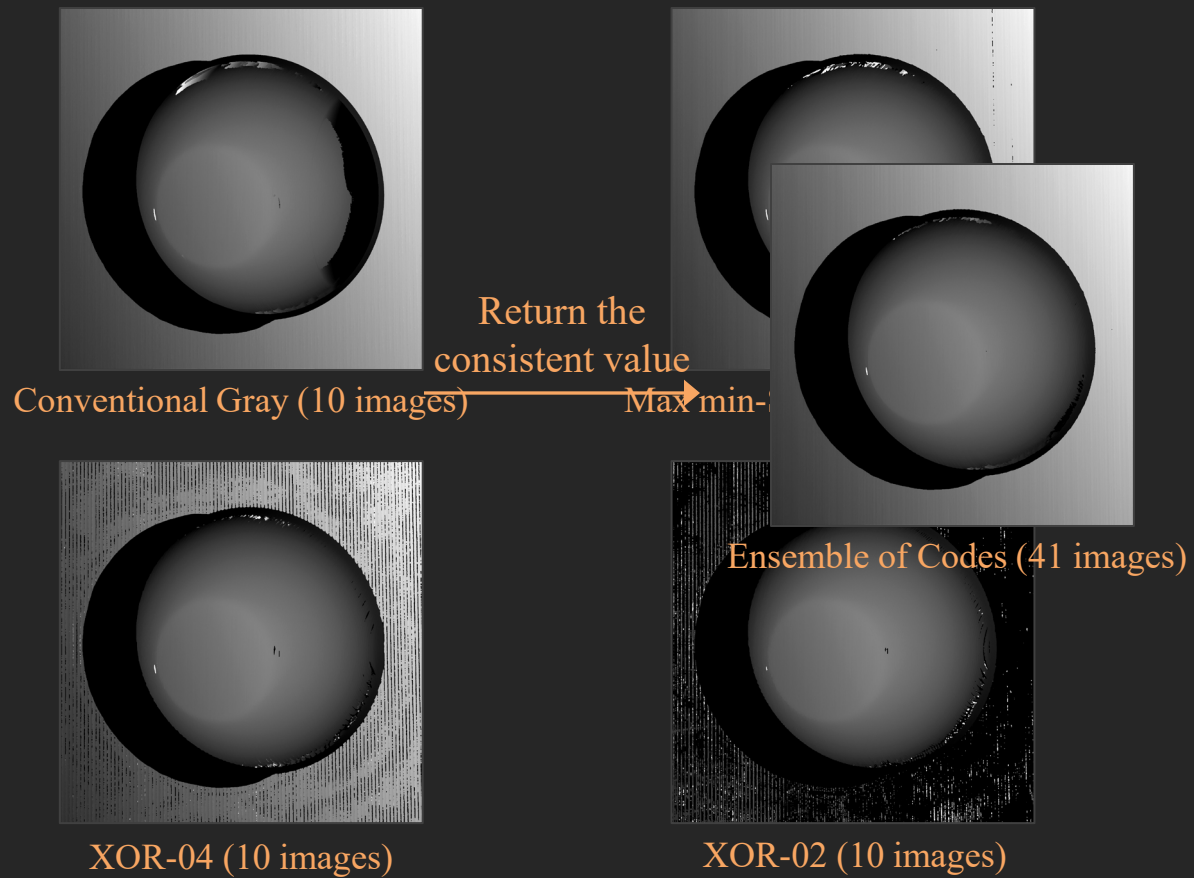
Making the Logical XOR Codes



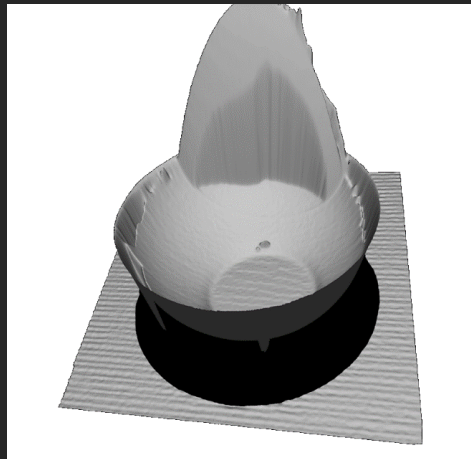
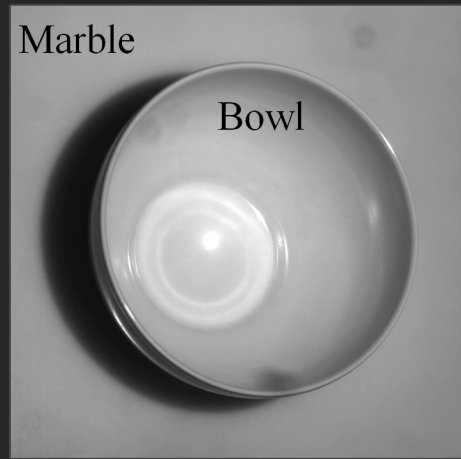
XOR of Last Pattern with Patterns 1-9 → XOR-02 Codes (10 patterns)

XOR of Second-last Pattern with Patterns 1-8 → XOR-04 Codes (10 patterns)

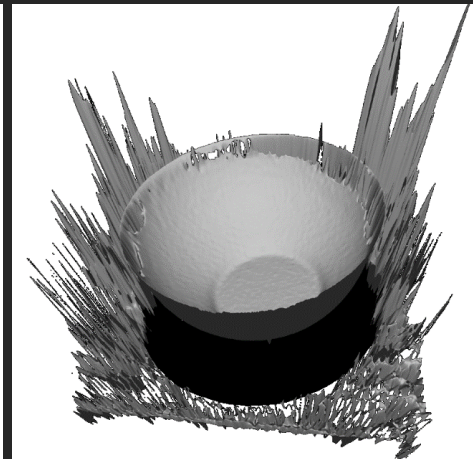
Ensemble of Codes for General Scenes



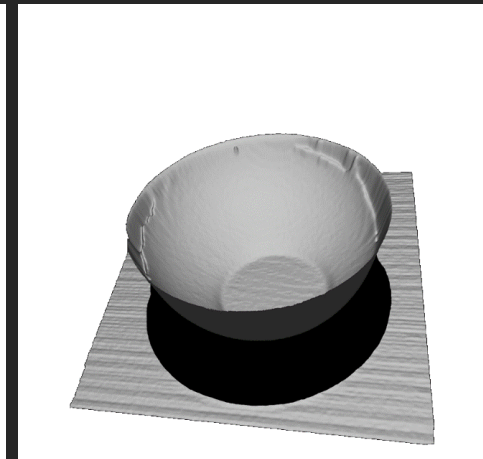
Shape Comparison



Conventional Gray
(11 images)



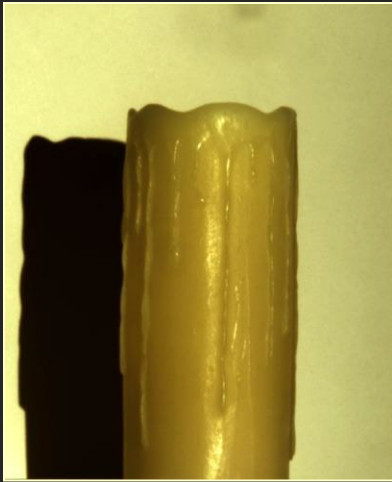
Modulated Phase-Shifting
(162 images)



Our Technique
(41 images)

Translucent Wax Candle

Errors due to strong
sub-surface scattering



Scene



Modulated Phase-
Shifting (162 images)



Our Ensemble Codes
(41 images)

Translucent Wax Object

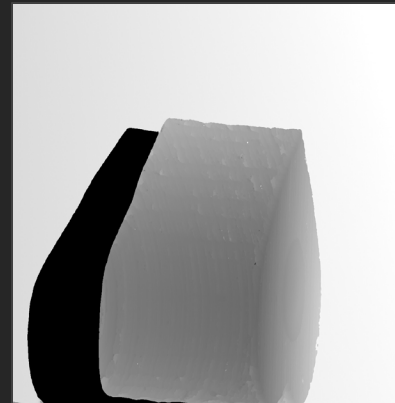
Errors due to strong
sub-surface scattering



Scene



Modulated Phase-
Shifting (162 images)



Our Ensemble Codes
(41 images)

Depth-Map Comparison



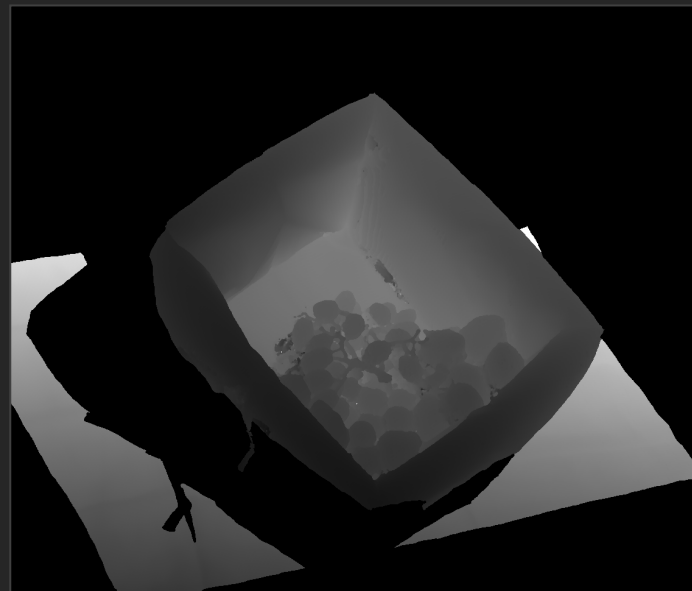
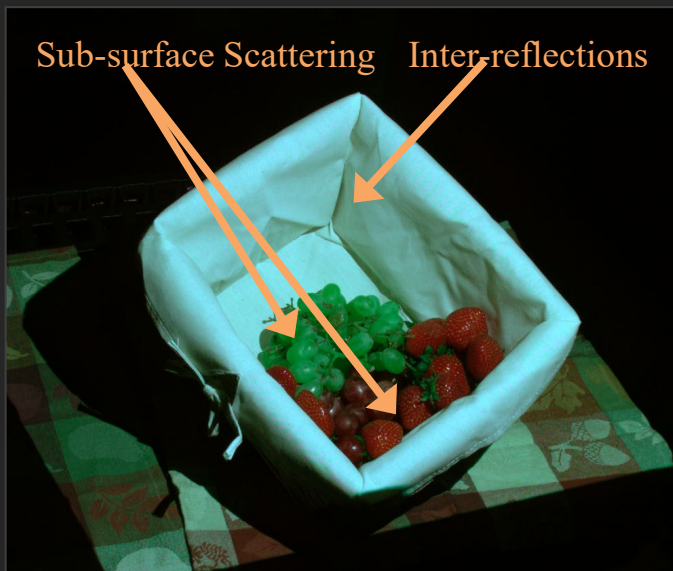
Regular Gray Codes (11 images)



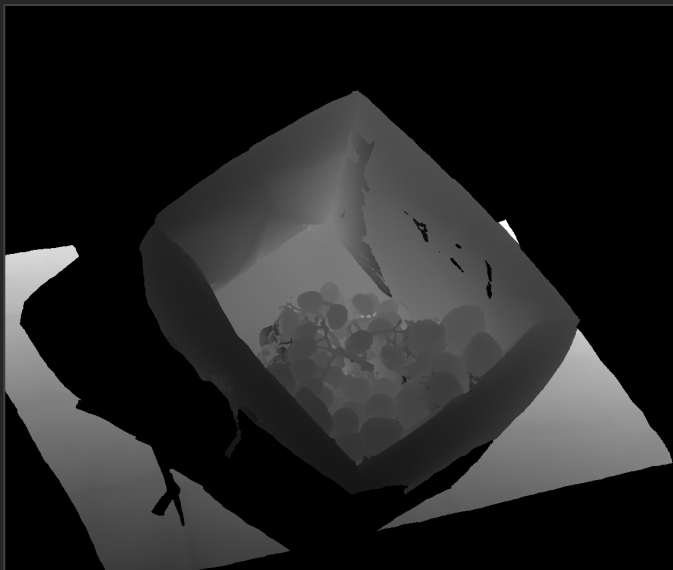
Our Ensemble Codes (41 images)

Fruit Basket: Multiple Effects

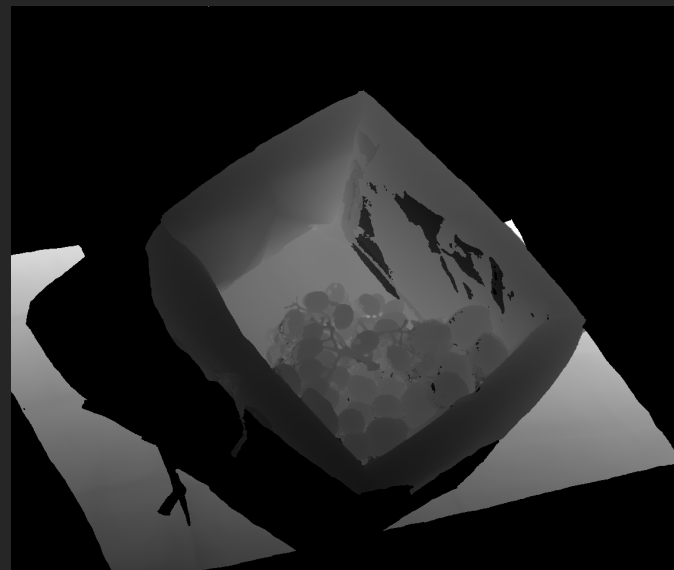
Depth-maps with previous state of the art



Our Ensemble Codes (41 images)

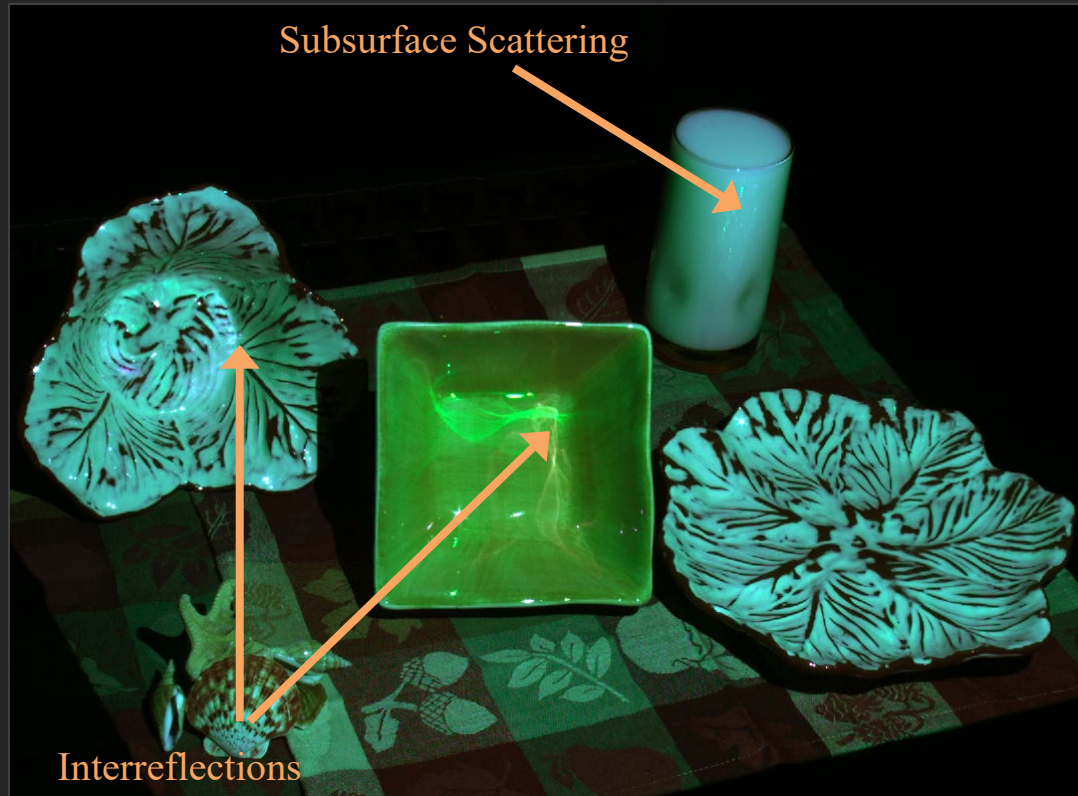


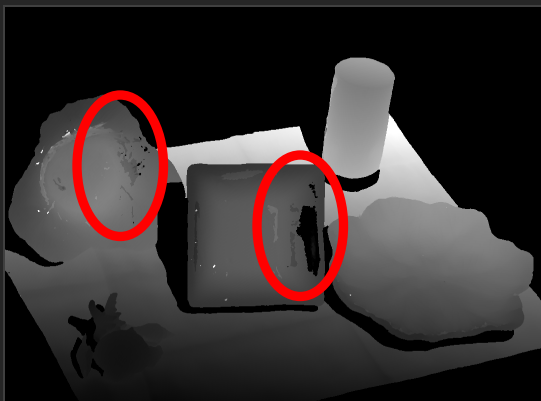
Regular Gray (11 images)



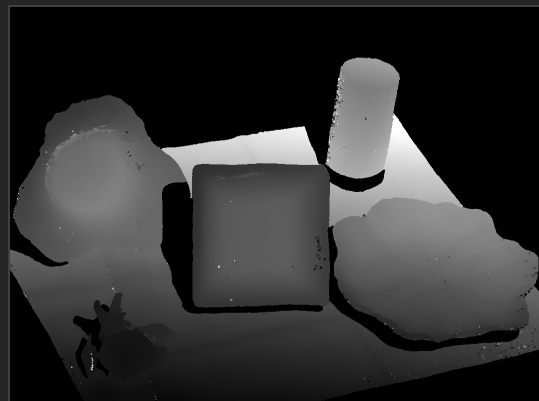
Phase-Shifting (18 images)

Bowls and Milk: Multiple Effects

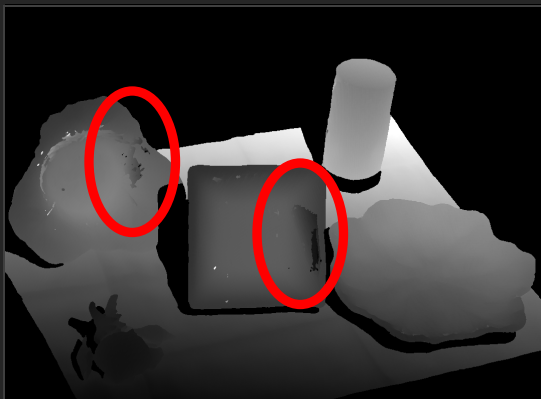




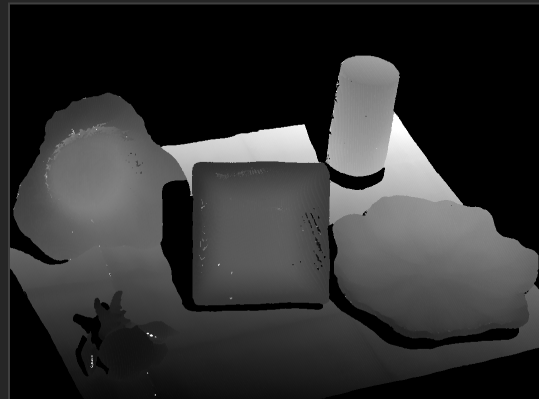
Phase-Shifting (18 images)



Modulated Phase-Shifting (162 images)

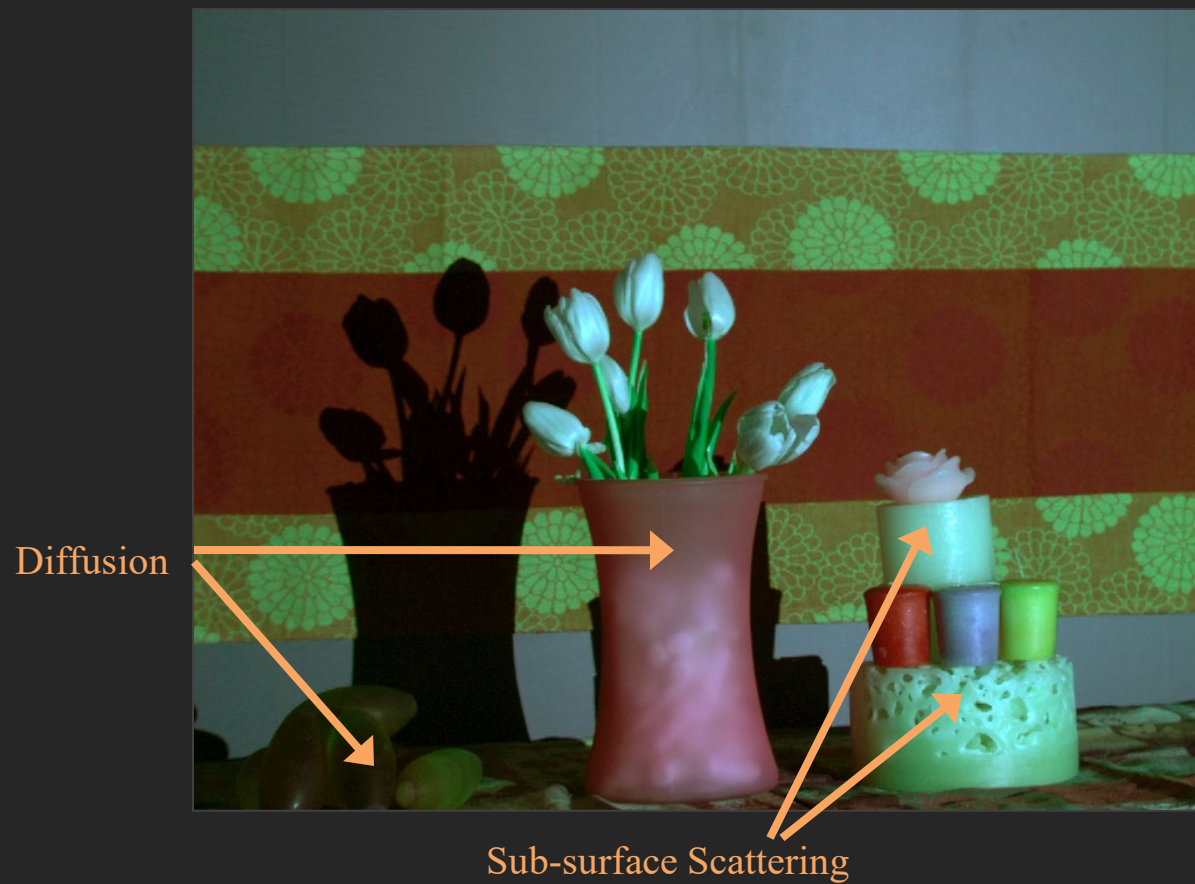


Regular Gray Codes (11 images)



Our XOR Codes (11 images)

Flower-Vase



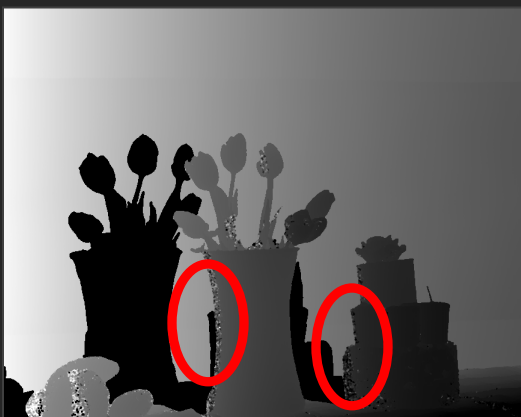
Comparison



Phase-Shifting (18 images)



Regular Gray Code (11 images)



Modulated Phase-Shifting (162 images)

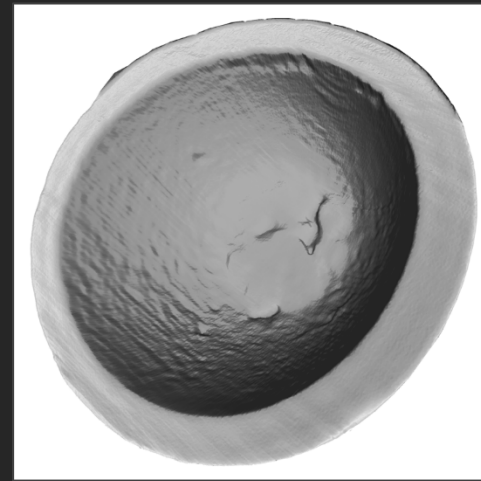


Our Ensemble Codes (41 images)

Multiple Indirect Illumination Effects



Wax Bowl

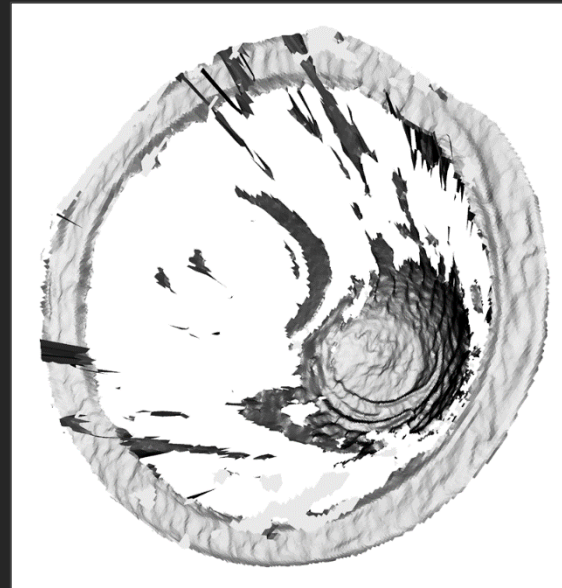


Shape Using Ensemble Codes

Multiple Indirect Illumination Effects



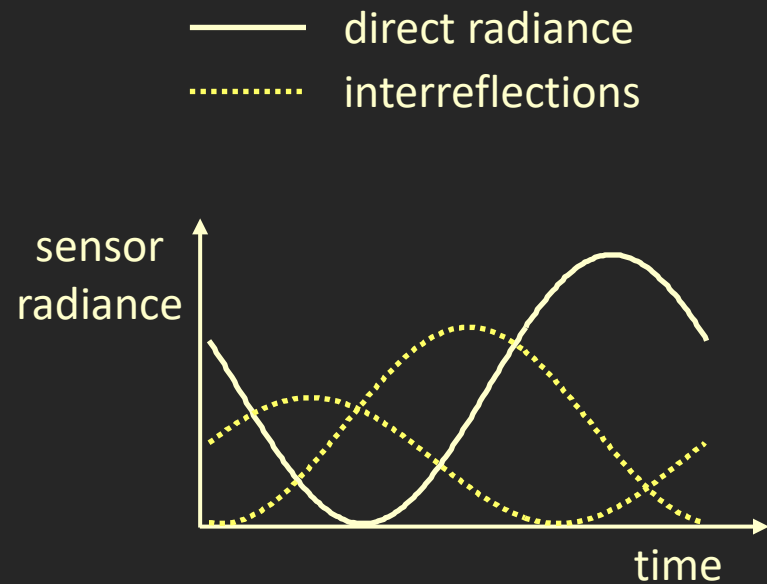
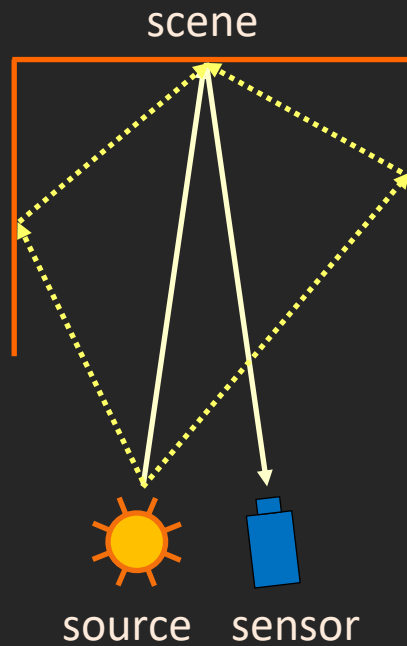
Deep Wax Container



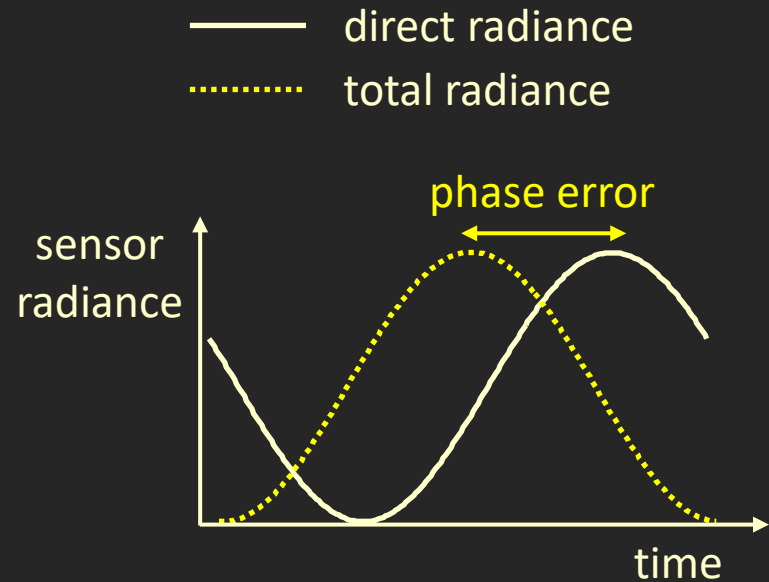
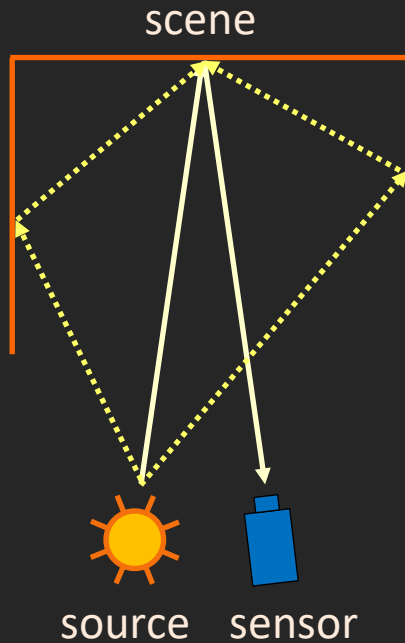
Shape Using Ensemble Codes

Application to time-of-flight imaging

Interreflections and ToF Imaging

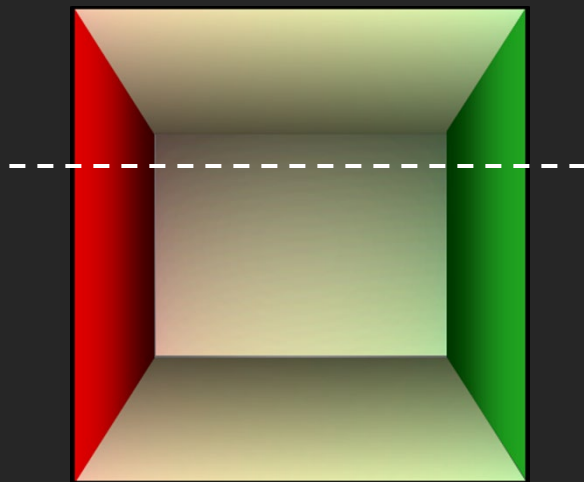


Interreflections and ToF Imaging

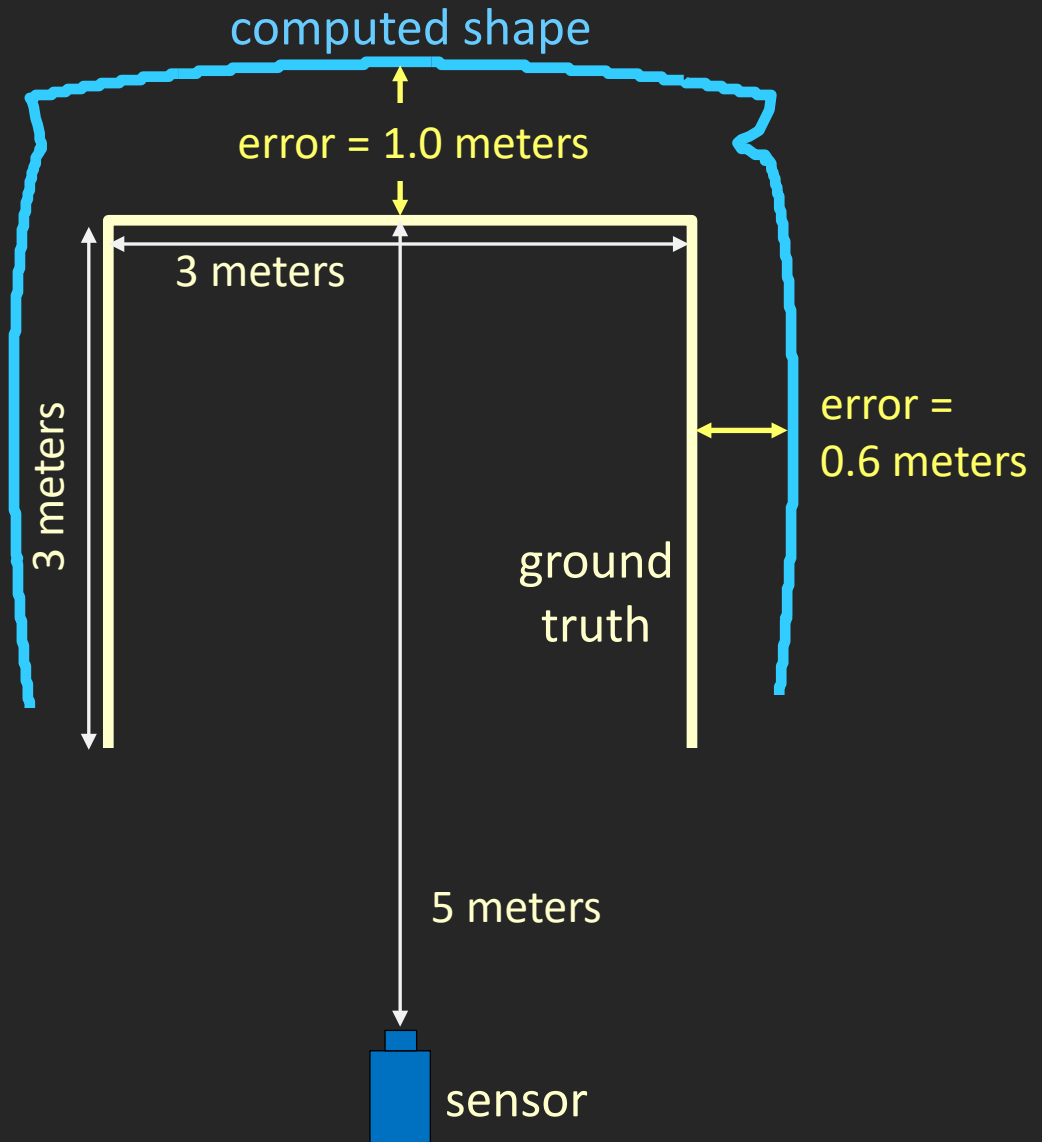


Interreflections Produce Incorrect Phase

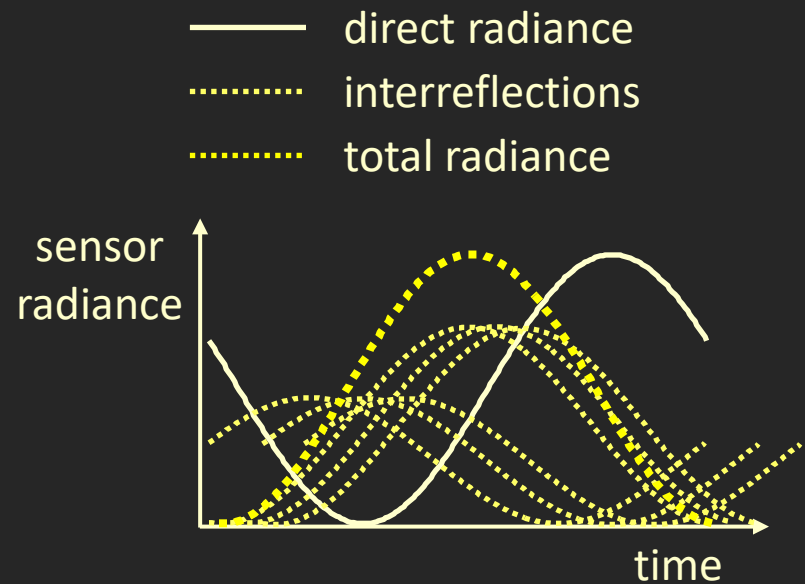
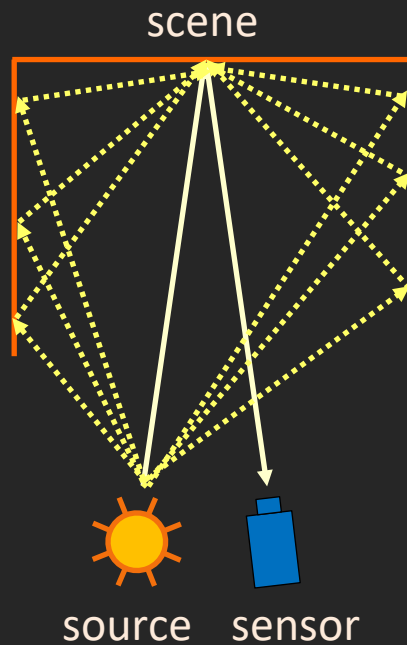
Errors in Shape Recovery



camera view

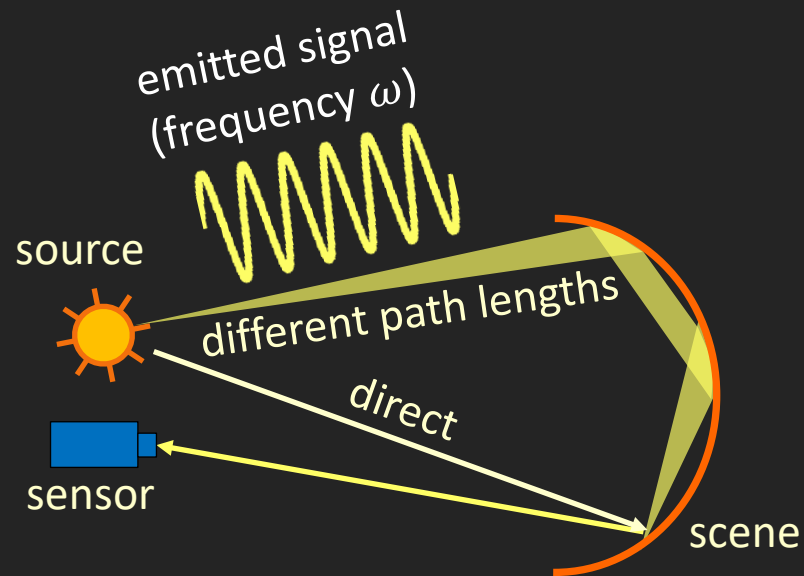


Multipath Interference: Existing Work

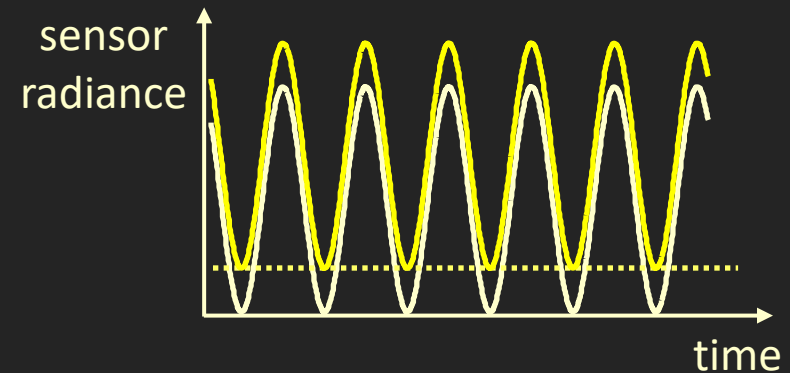


How To Separate Different Components?

Interreflections vs. Modulation Frequency

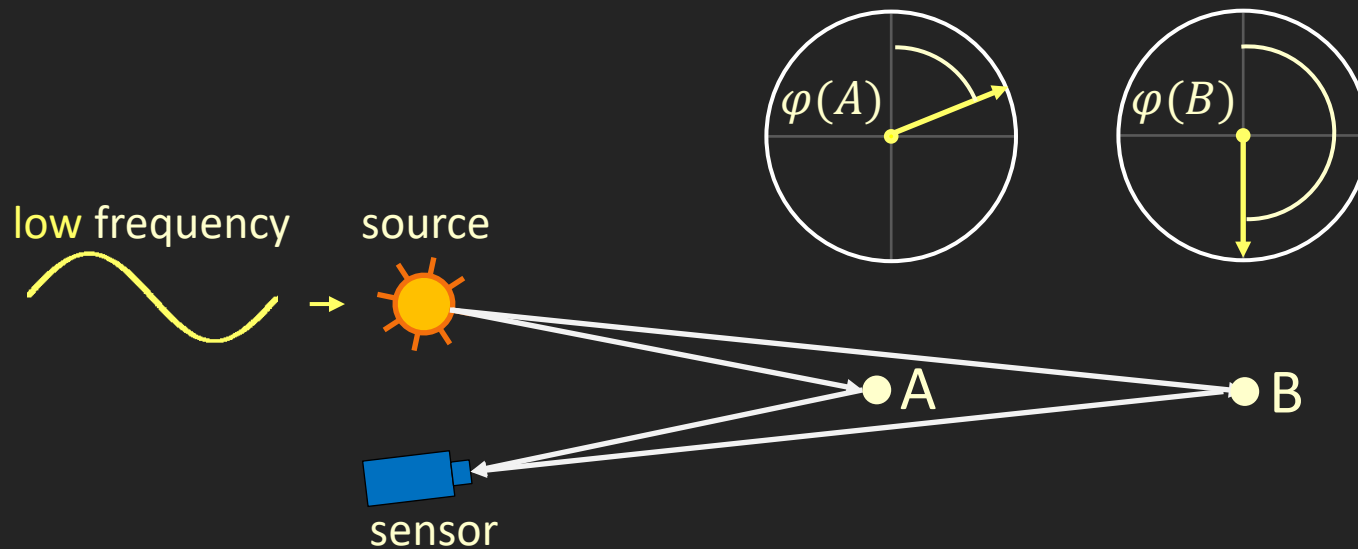


— total radiance
 — direct radiance
 interreflection



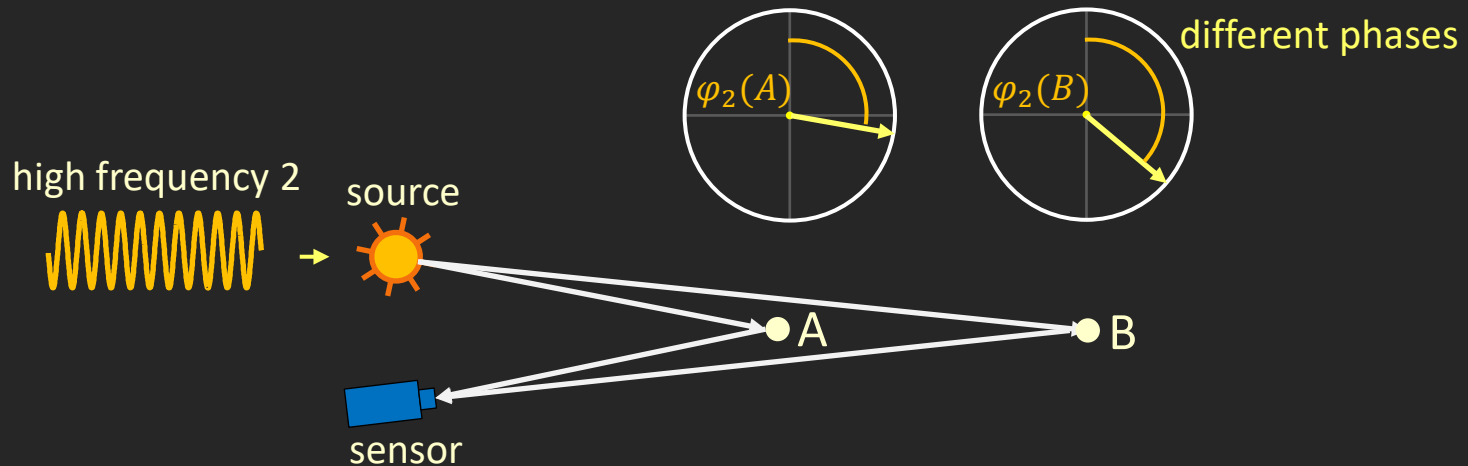
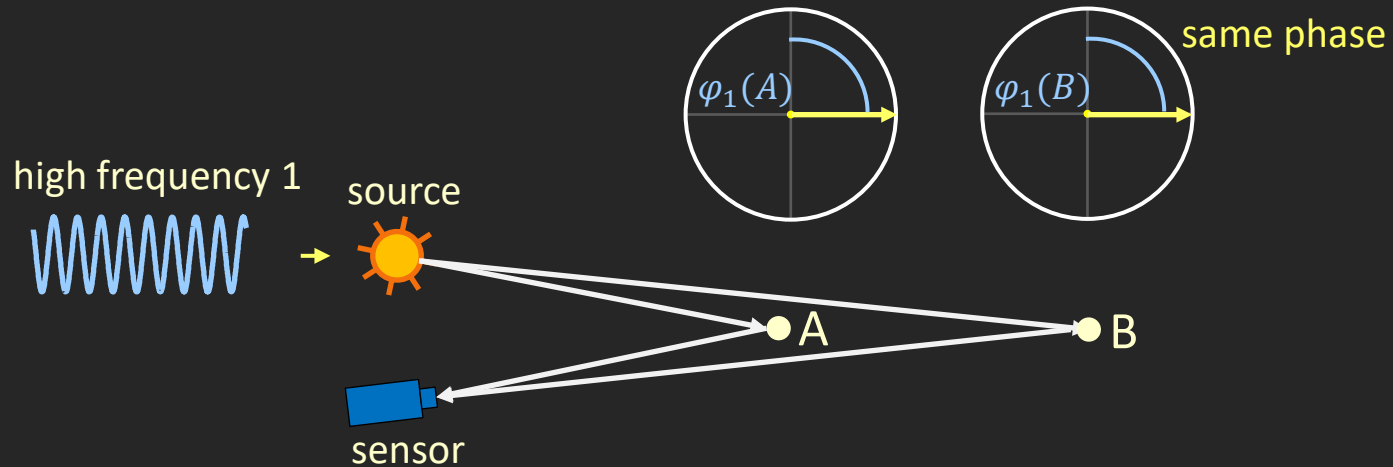
For High Temporal Frequency
 Interreflections Do Not Affect Phase

Phase Ambiguity



Unambiguous Depth Range: $R_{unambiguous} = \frac{1}{2\omega}$

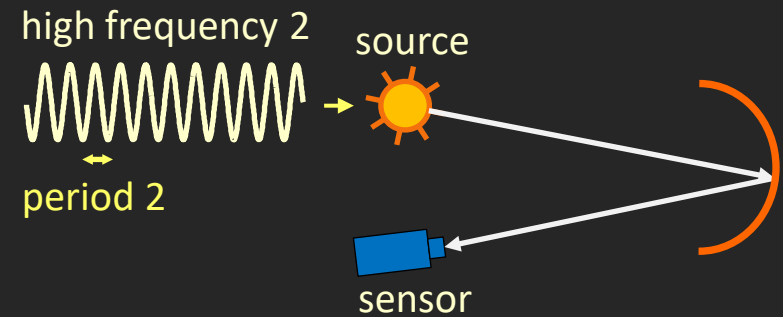
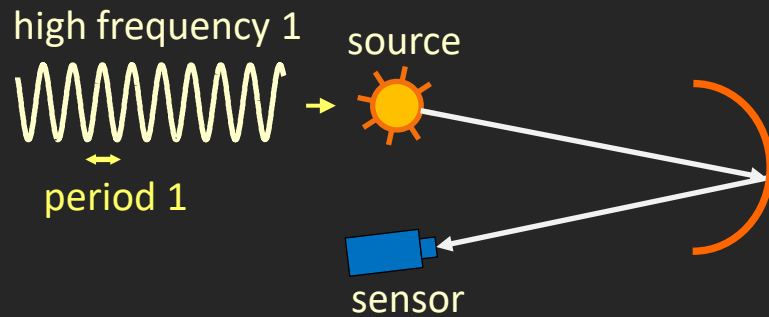
Disambiguating Phase



Compute Phases at Two High Frequencies

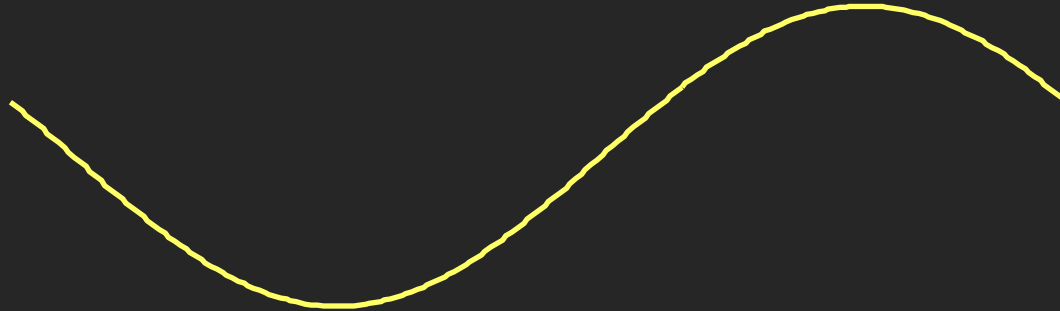
[Jongenelen *et al.* 2010, 2011]

Micro Time-of-Flight Imaging

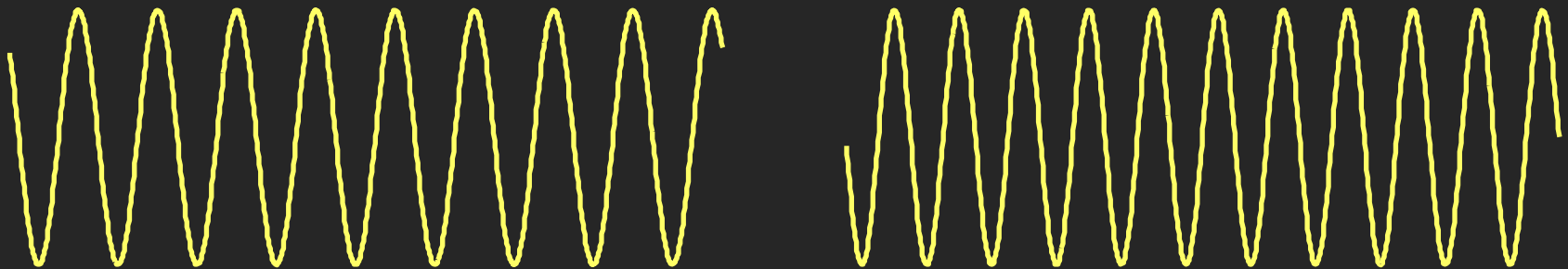


Modulation Signals With Micro (Small) Periods

Conventional vs. Micro ToF Imaging

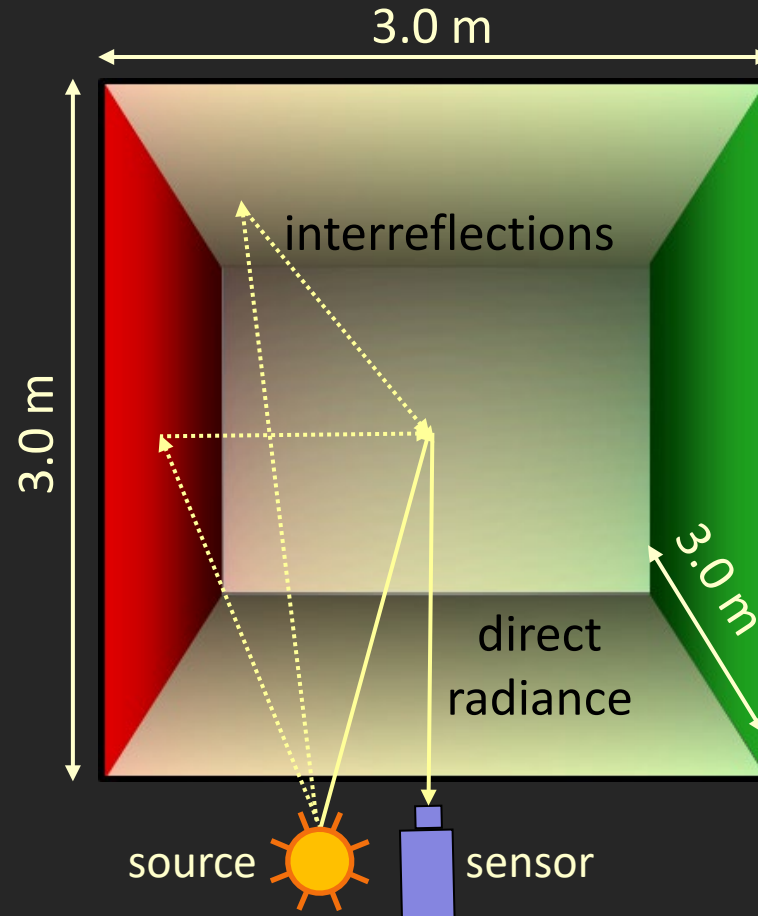


Conventional ToF Shifting: One Low Frequency
Three Measurements

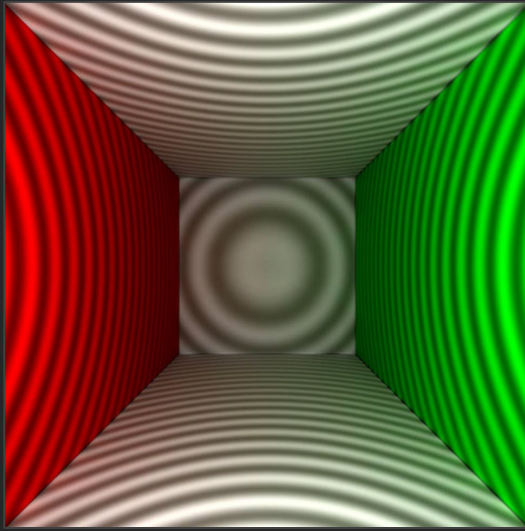


Micro ToF Shifting: Two High Frequencies
Four Measurements

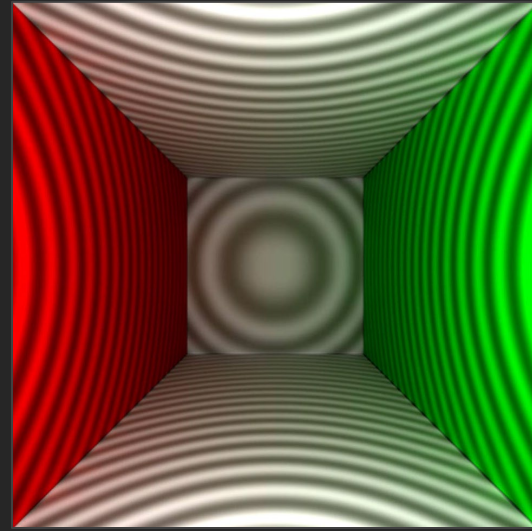
Simulations: Cornell Box



Cornell Box: Input Images

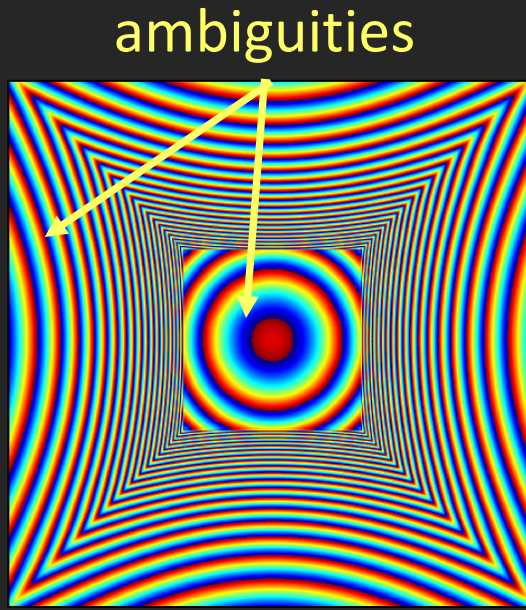


957 MHz.

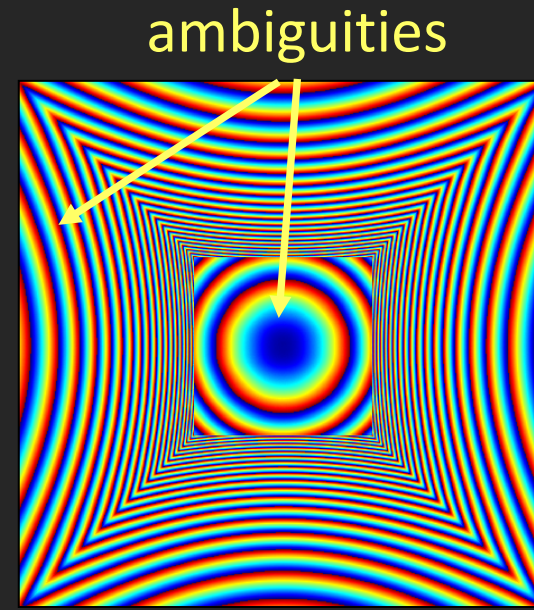


930 MHz.

Cornell Box: Phase Maps



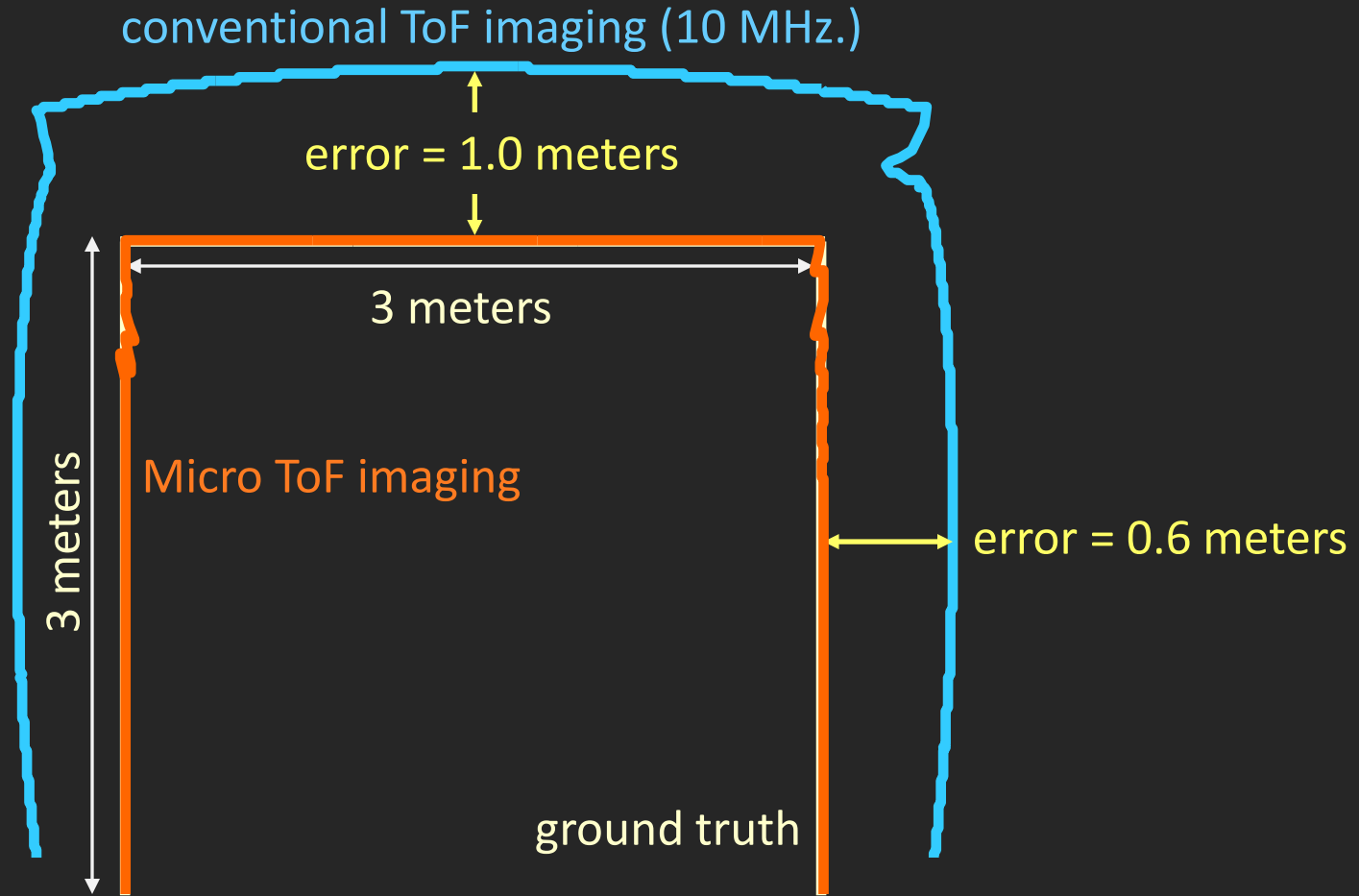
957 MHz.



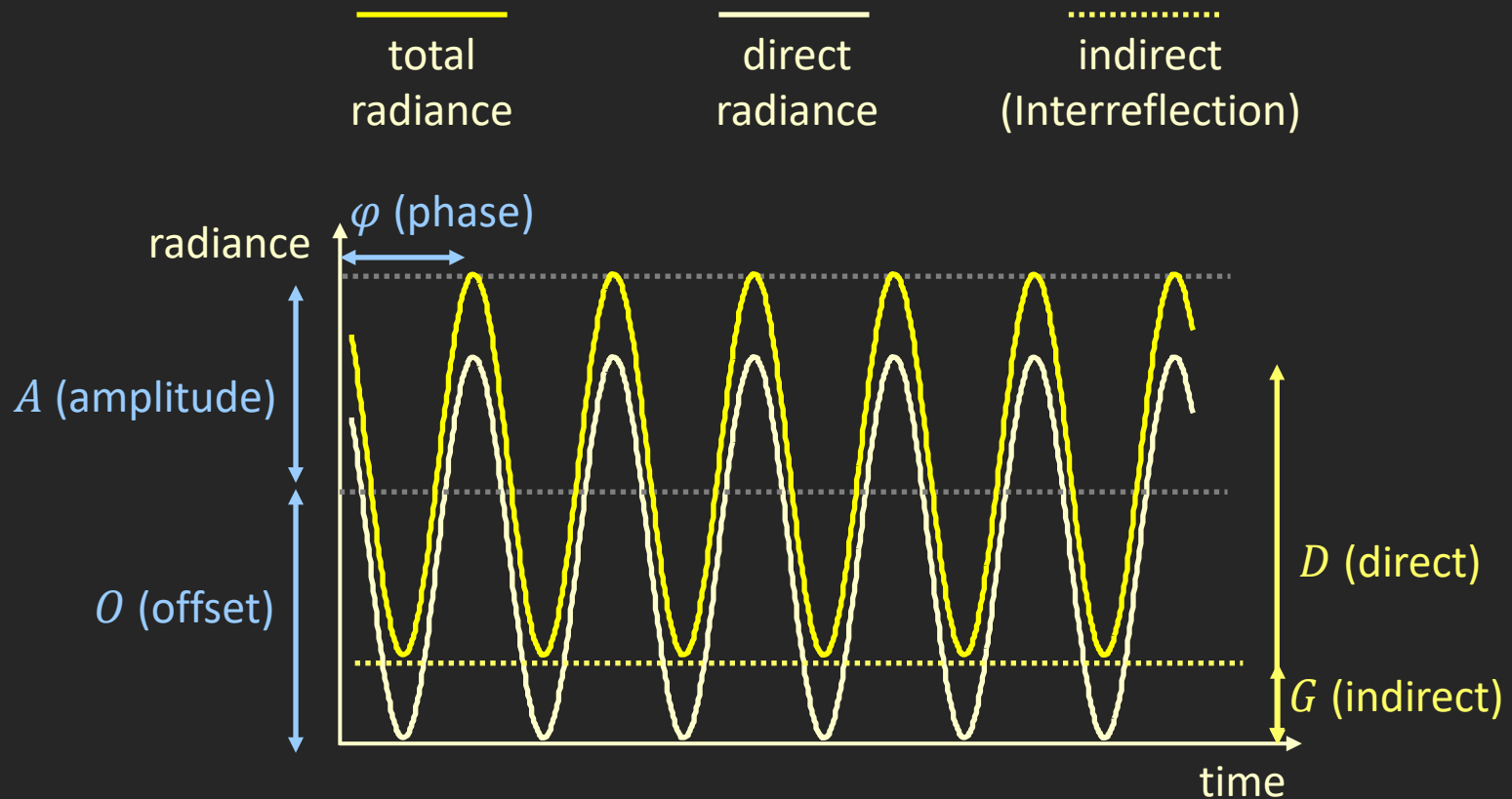
930 MHz.



Cornell Box: Shape Comparison



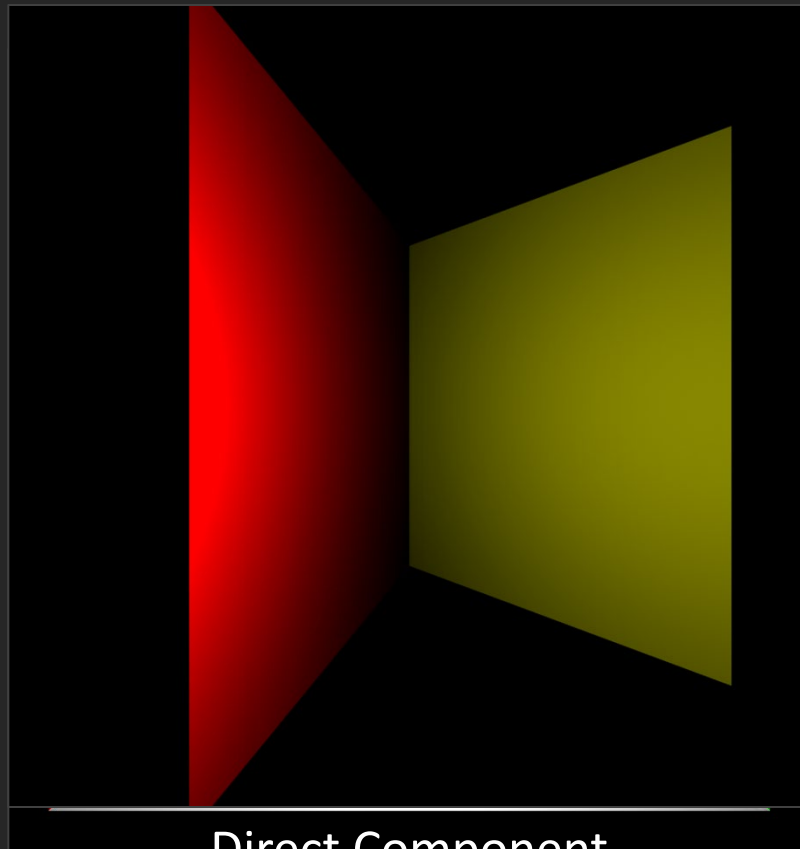
Direct-Indirect Separation



$$D = 2A \quad G = O - A$$

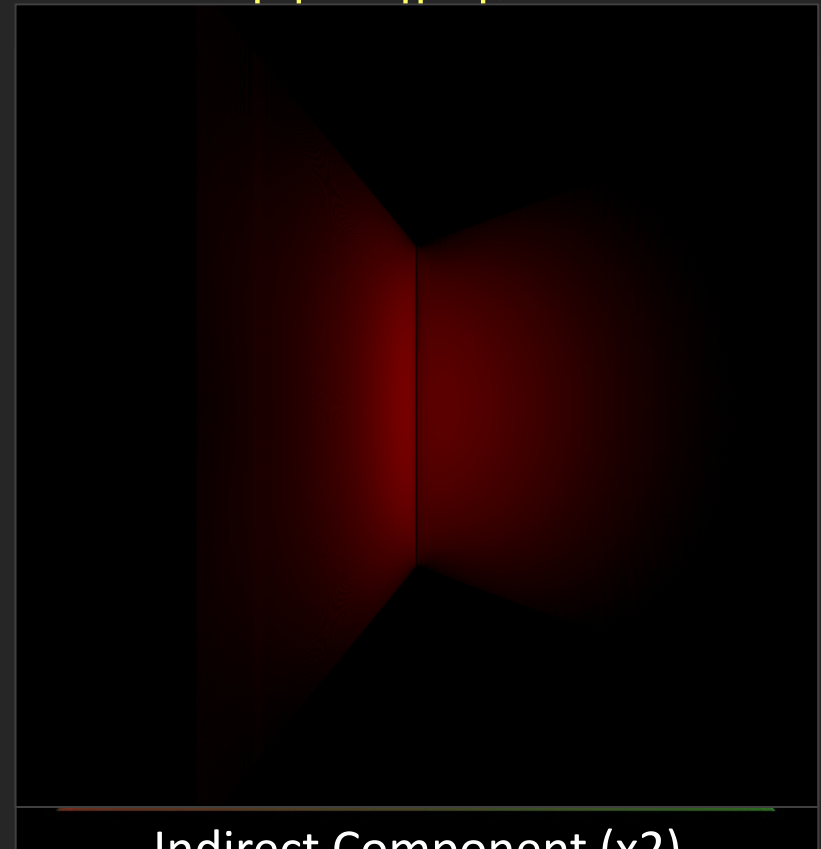
Direct-Indirect Separation Using Three Measurements

Direct-Indirect Separation



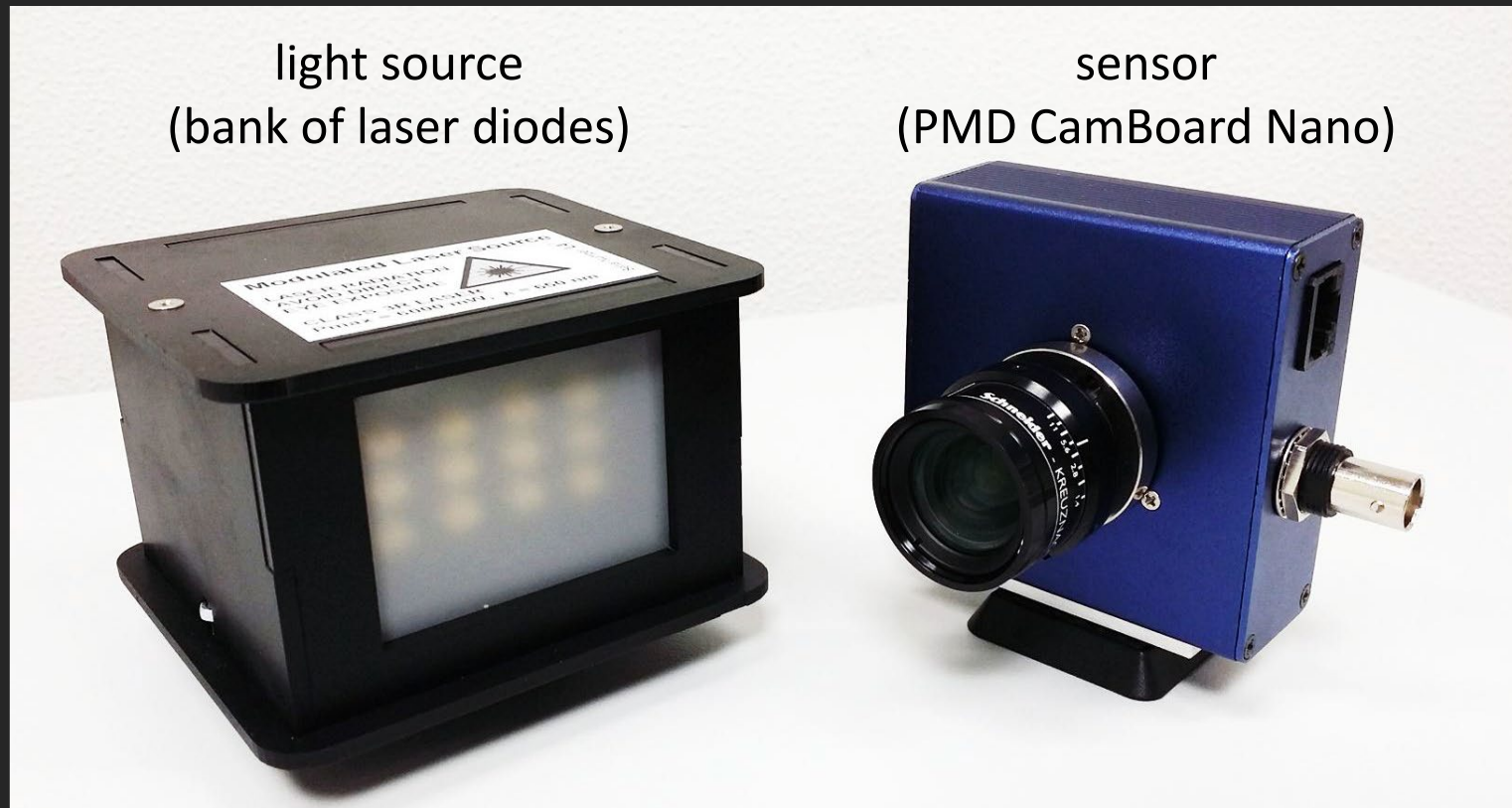
Direct Component
Direct Component

Color Bleeding due to
Indirect Component



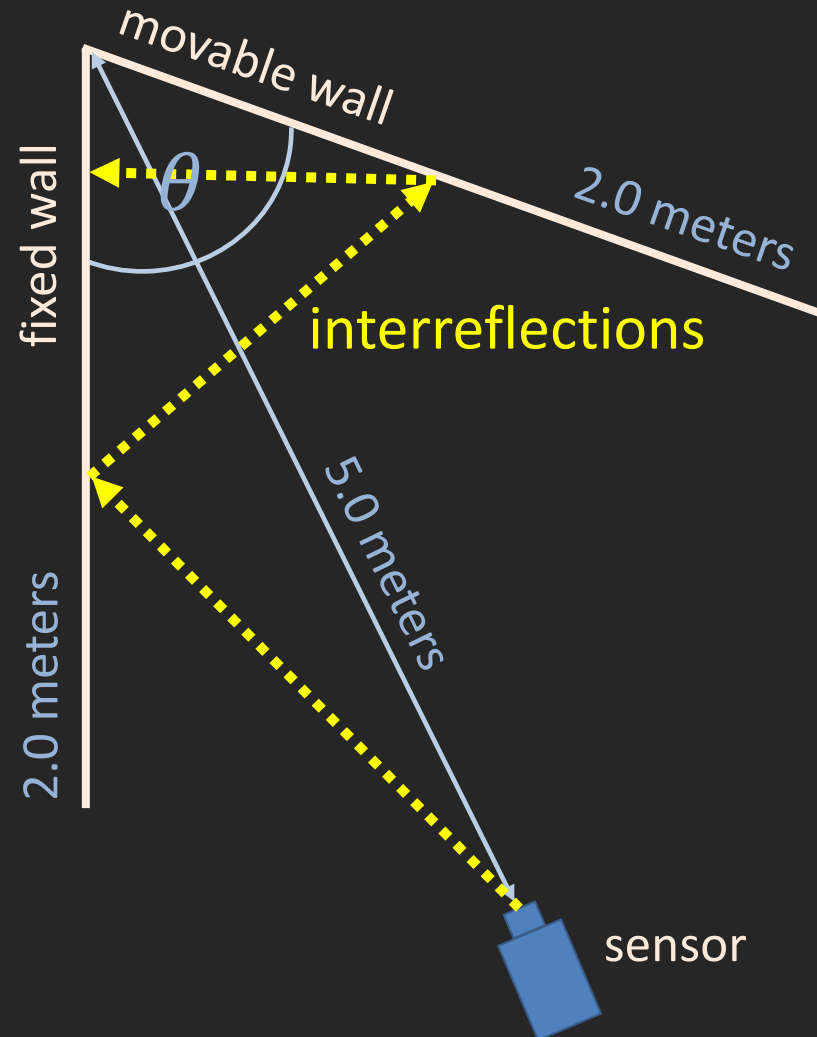
Indirect Component (x2)
Indirect Component (x2)

Experimental Setup

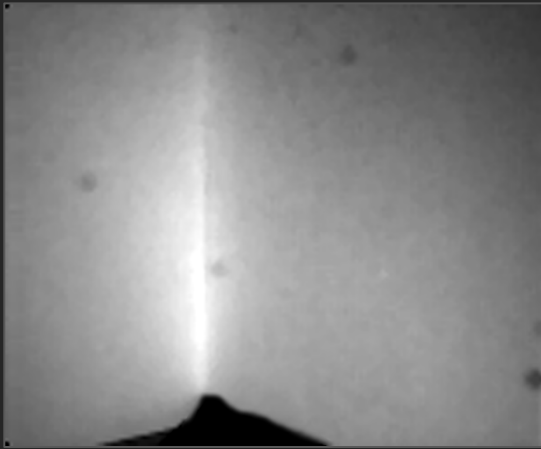


Maximum System Modulation Frequency = 125 MHz.

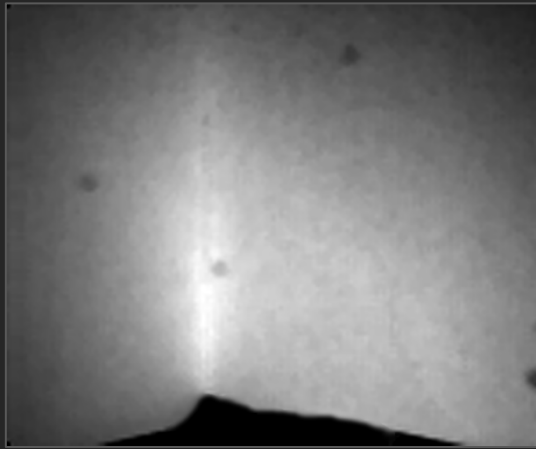
Experiments: V-Groove



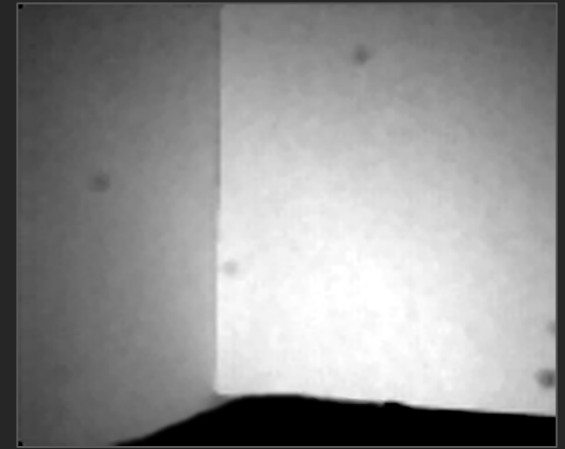
Scene Images Captured By PMD Sensor



apex angle = 45°



apex angle = 60°

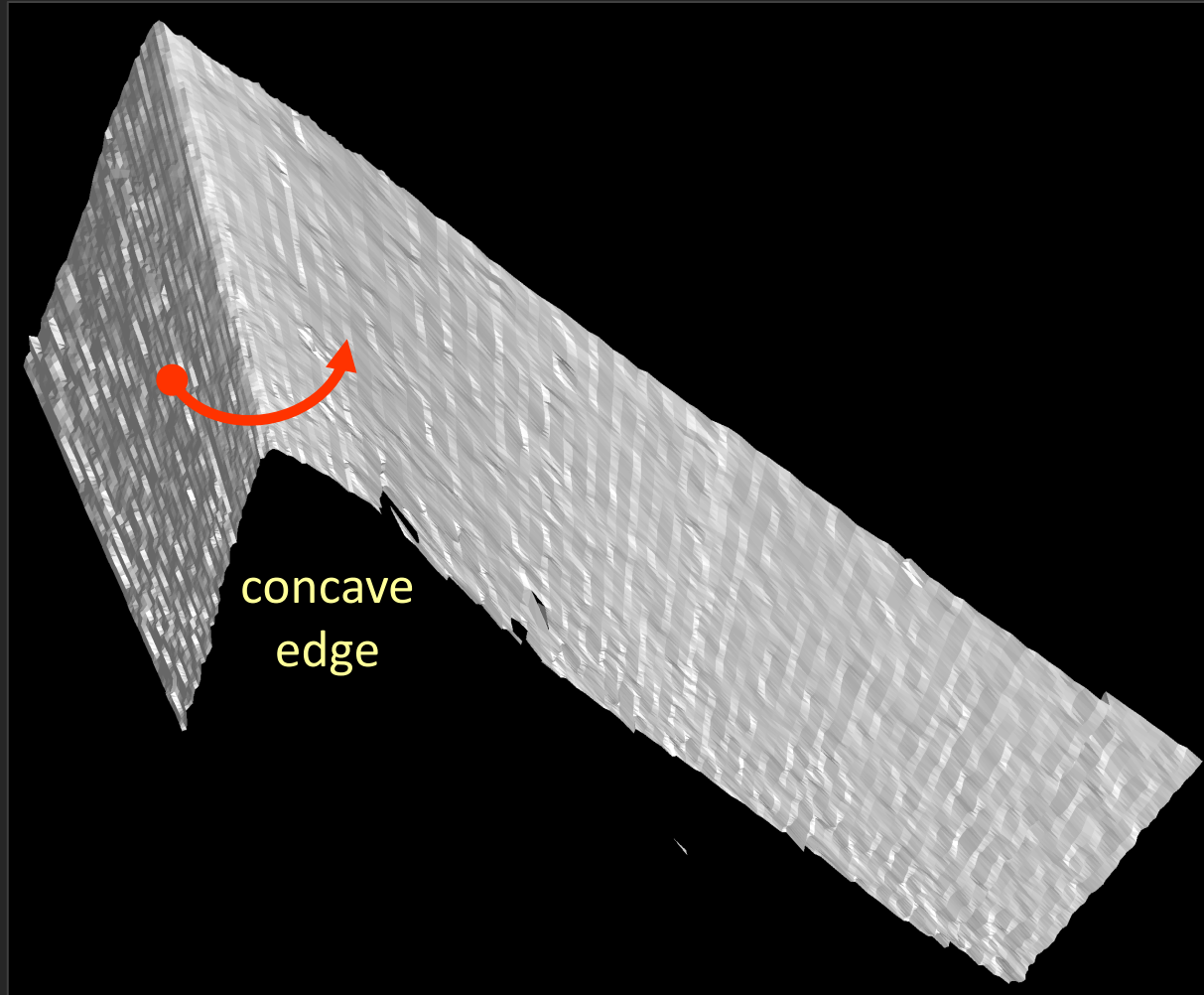


apex angle = 90°

image resolution = 120 x 165

Reconstructed Shape using Micro ToF

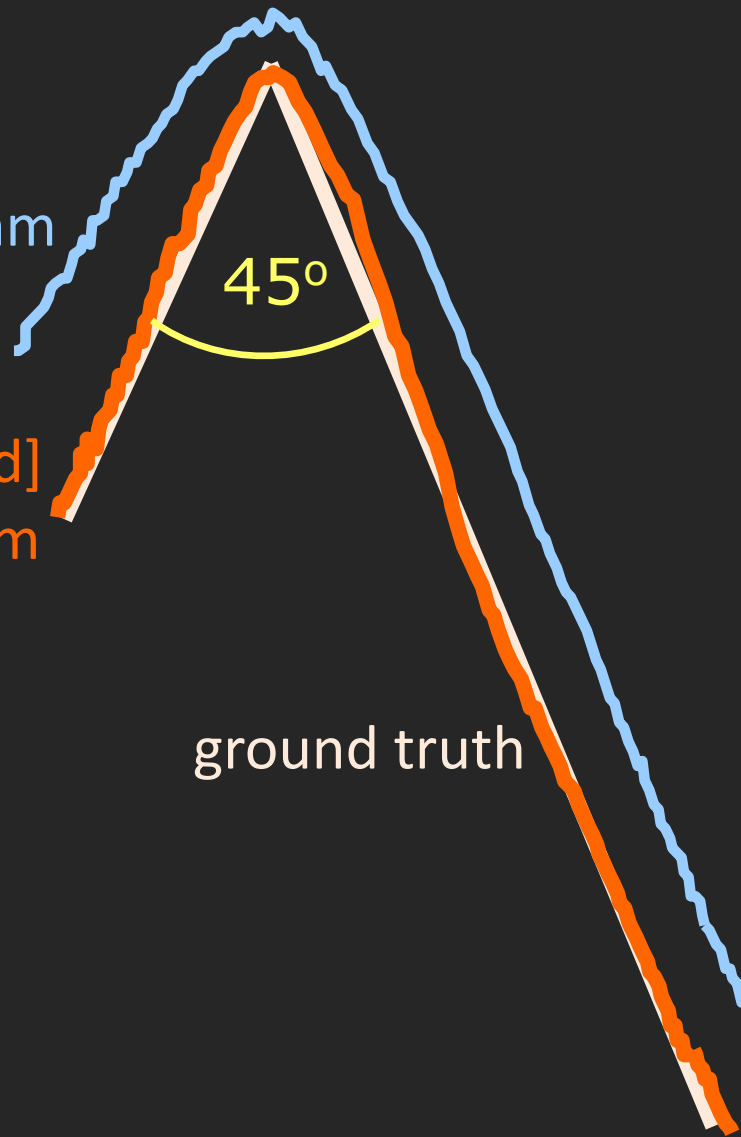
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Shape Comparisons

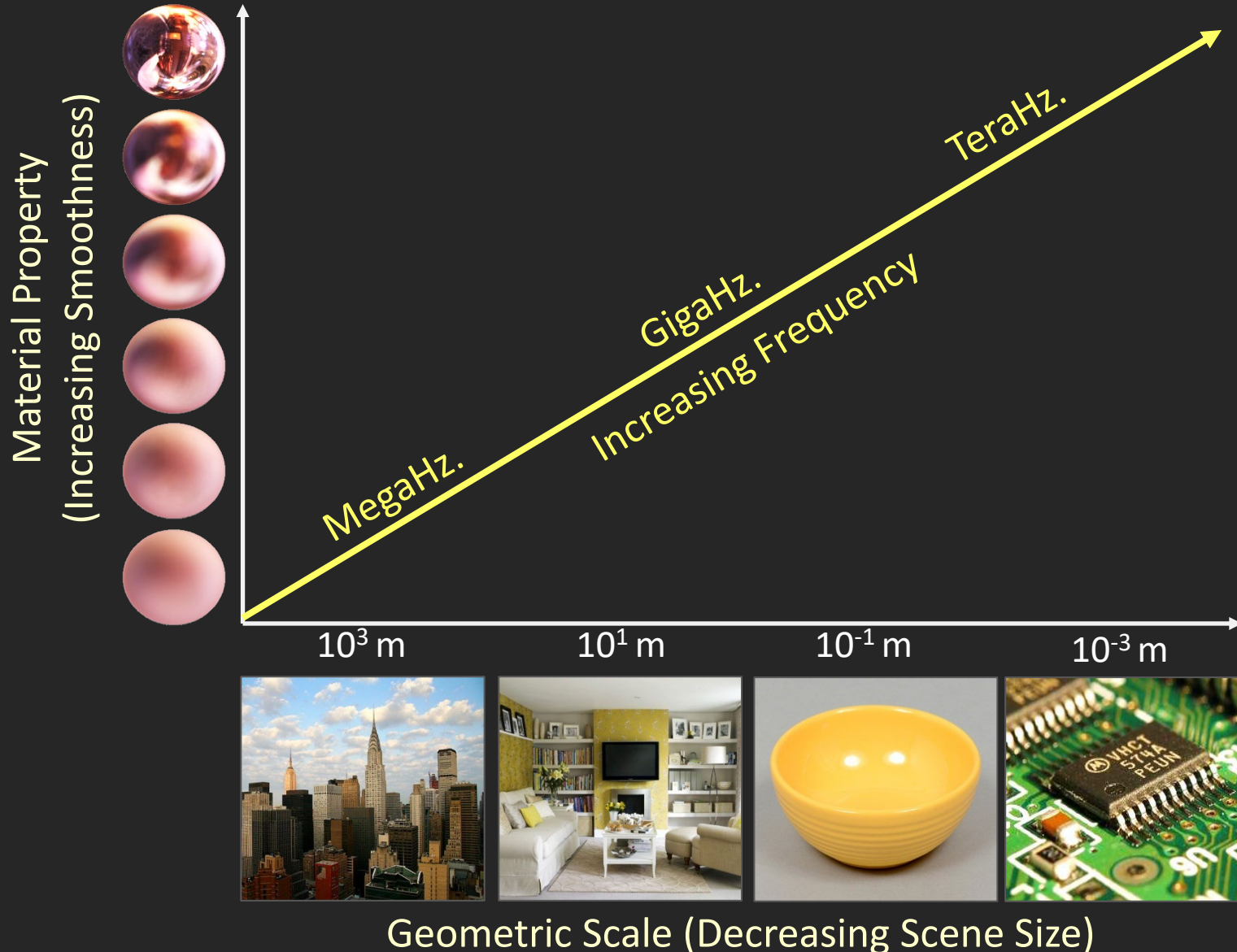
conventional ToF
mean error = 86.6 mm

Micro ToF [proposed]
mean error = 2.8 mm

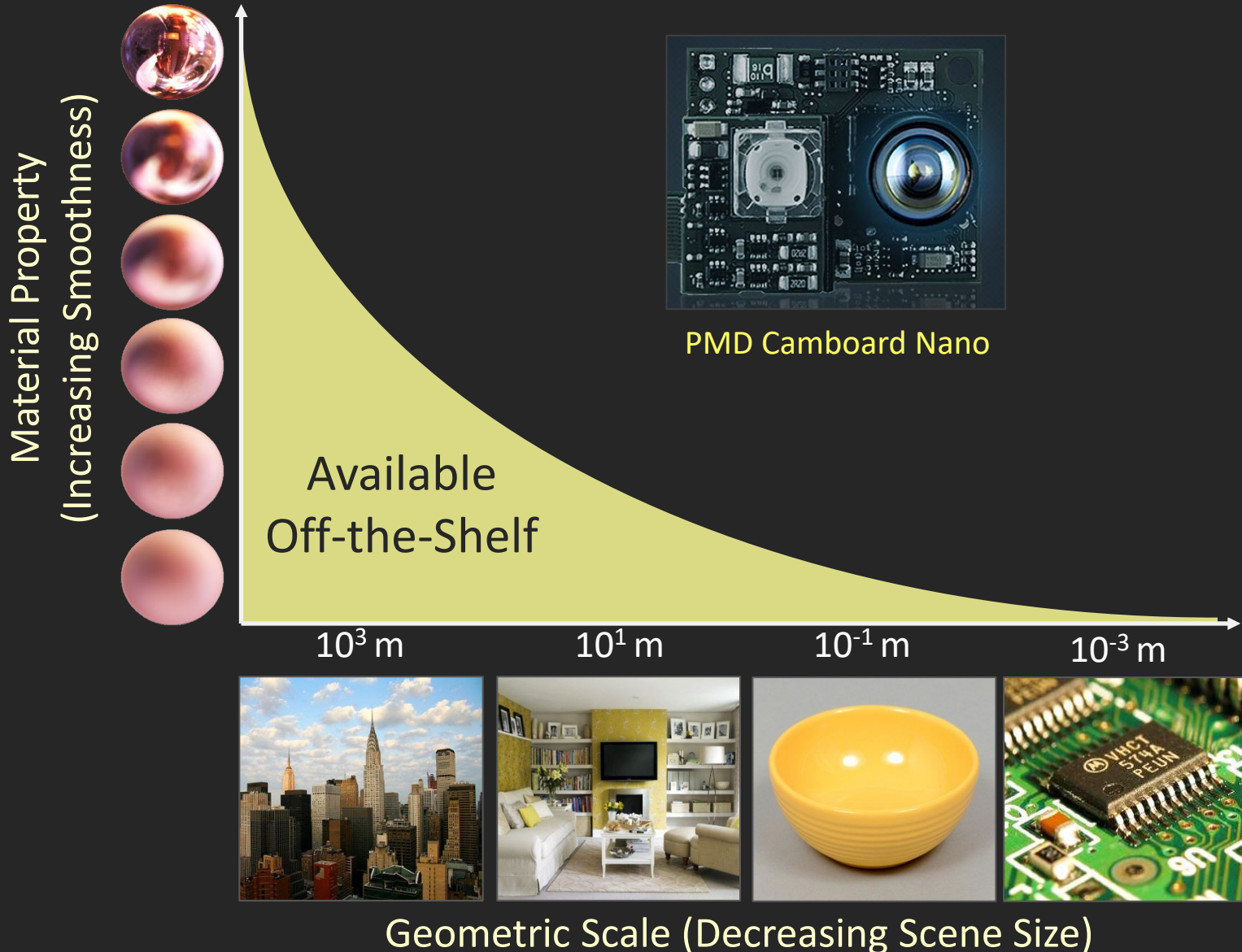


ground truth

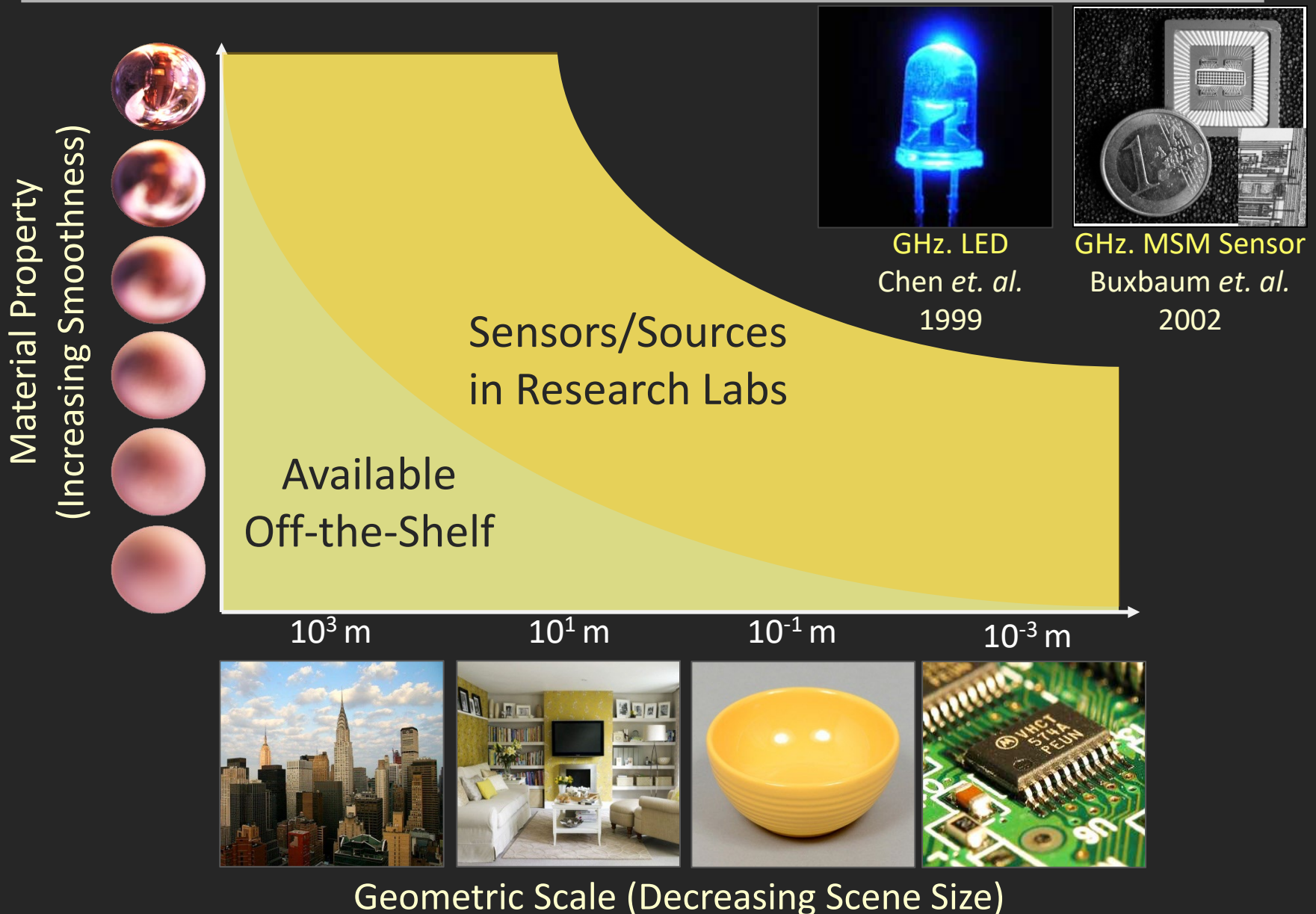
How High Should The Frequency Be?



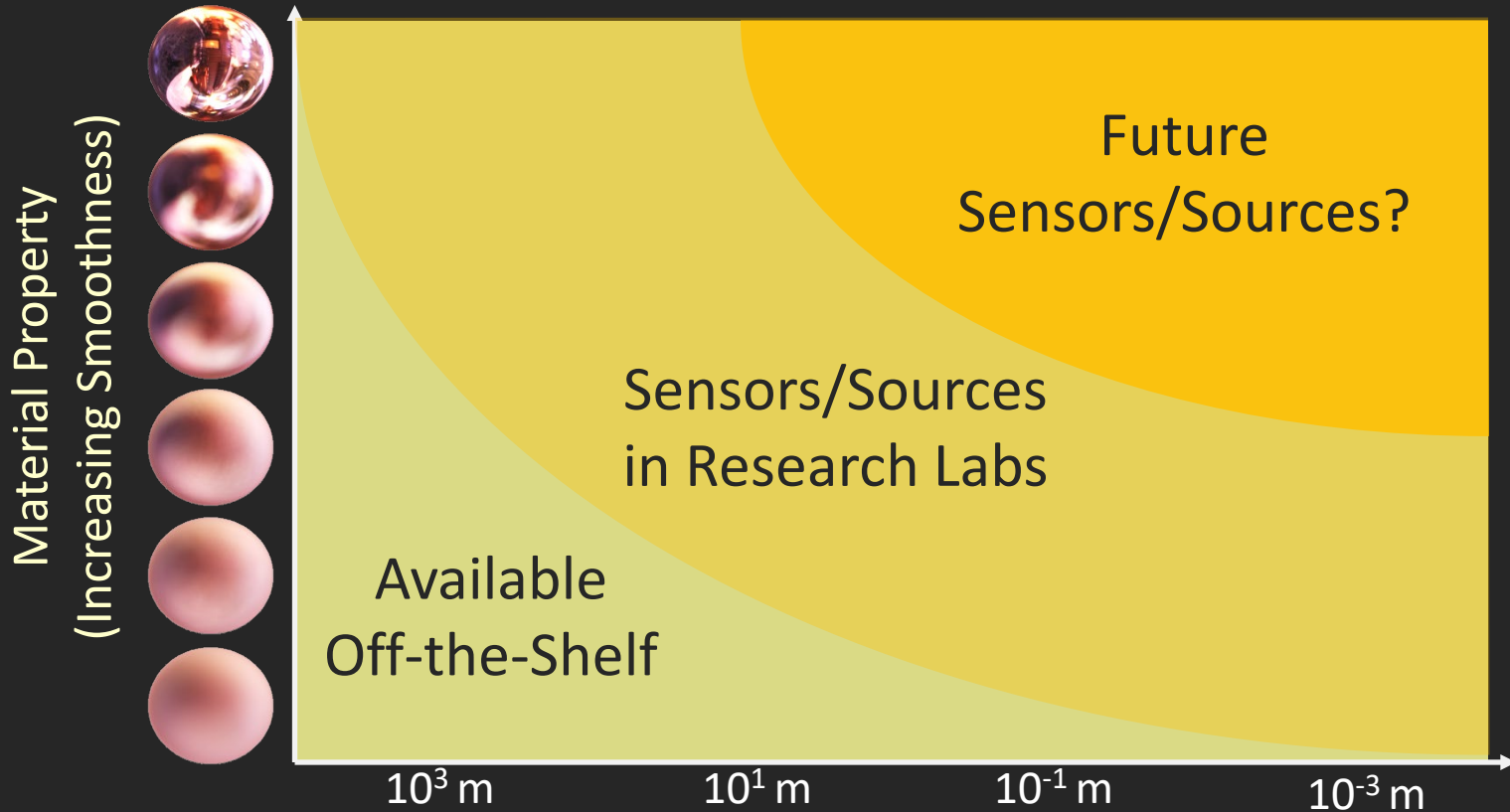
Technology (Devices) Required



Technology (Devices) Required



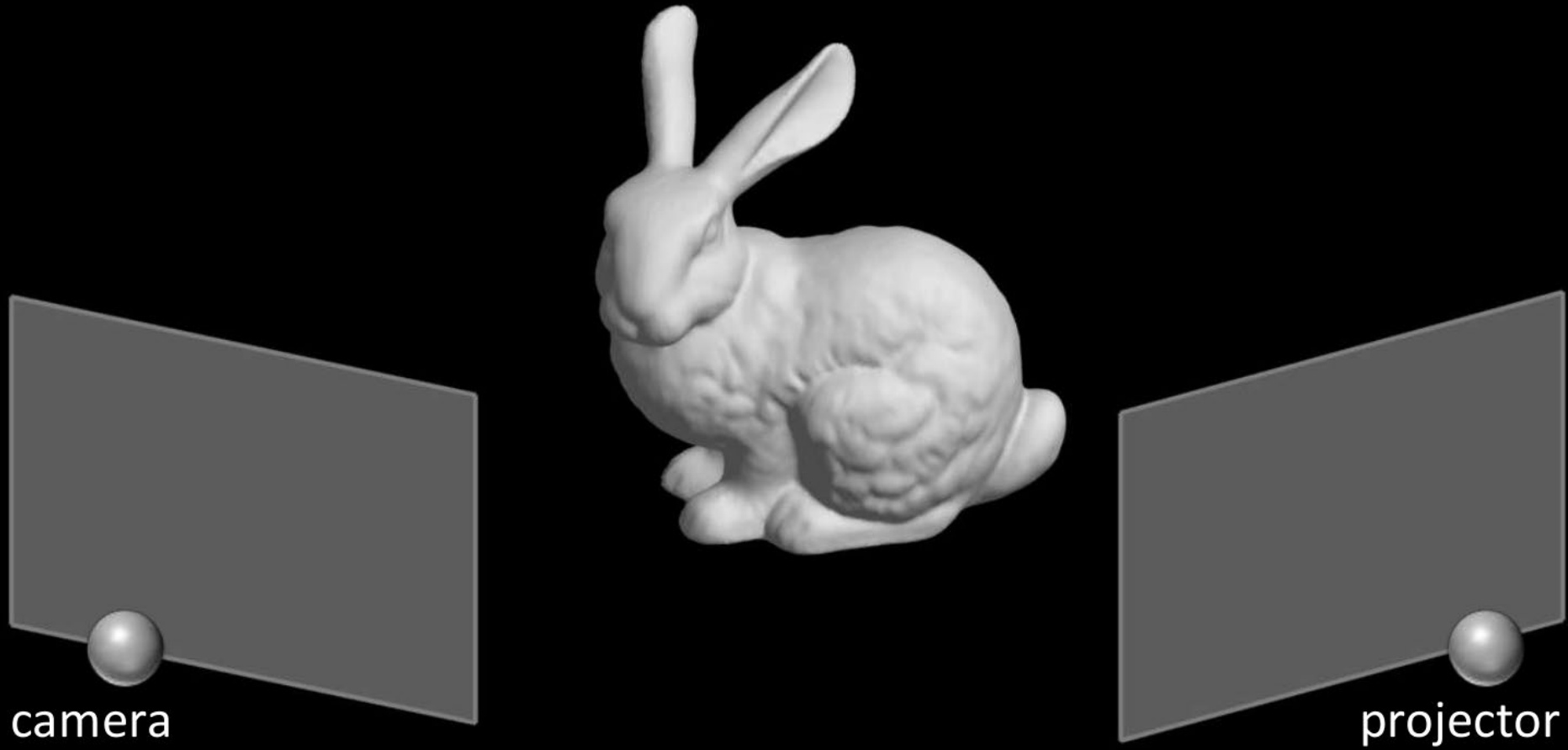
Technology (Devices) Required



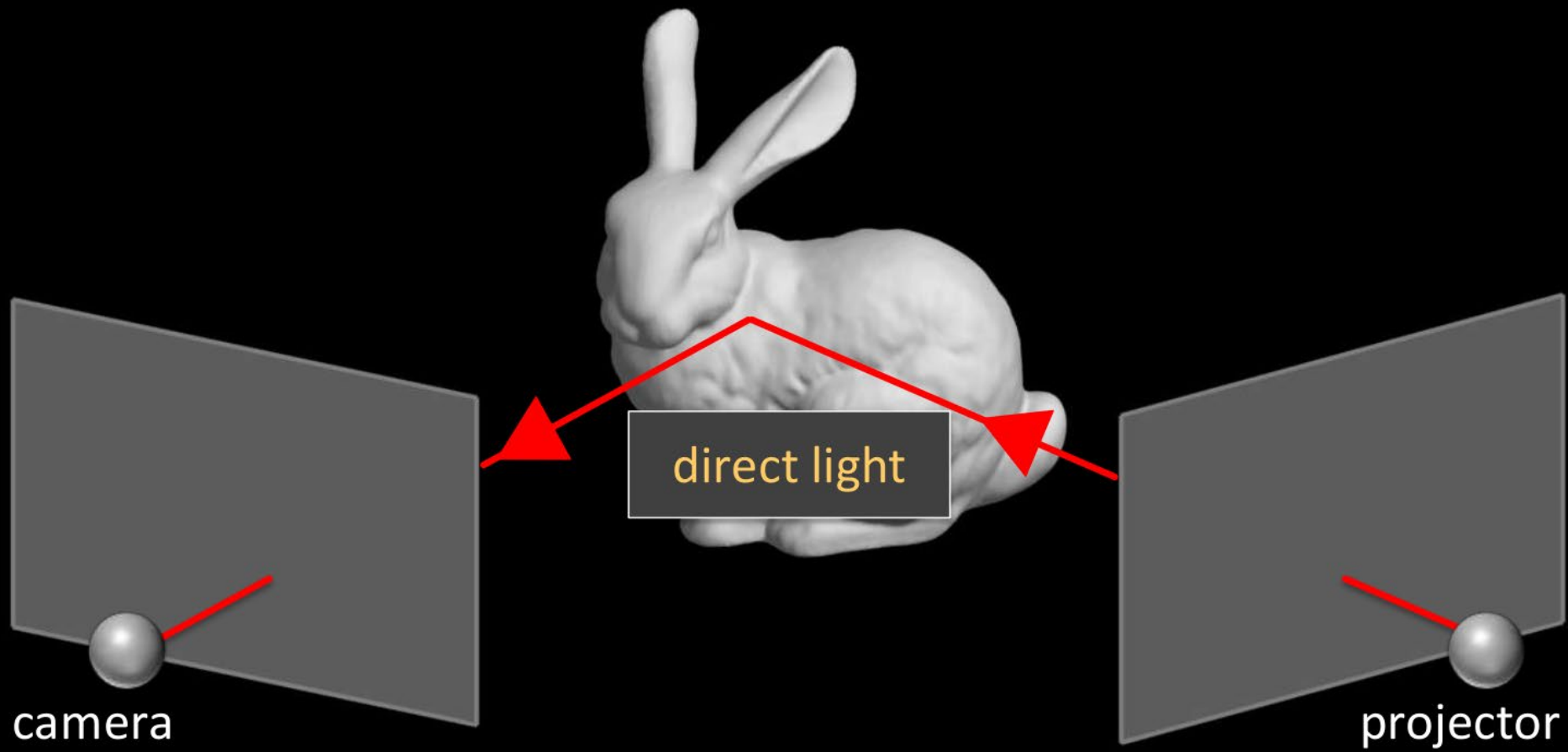
Geometric Scale (Decreasing Scene Size)

Direct-indirect separation using epipolar probing

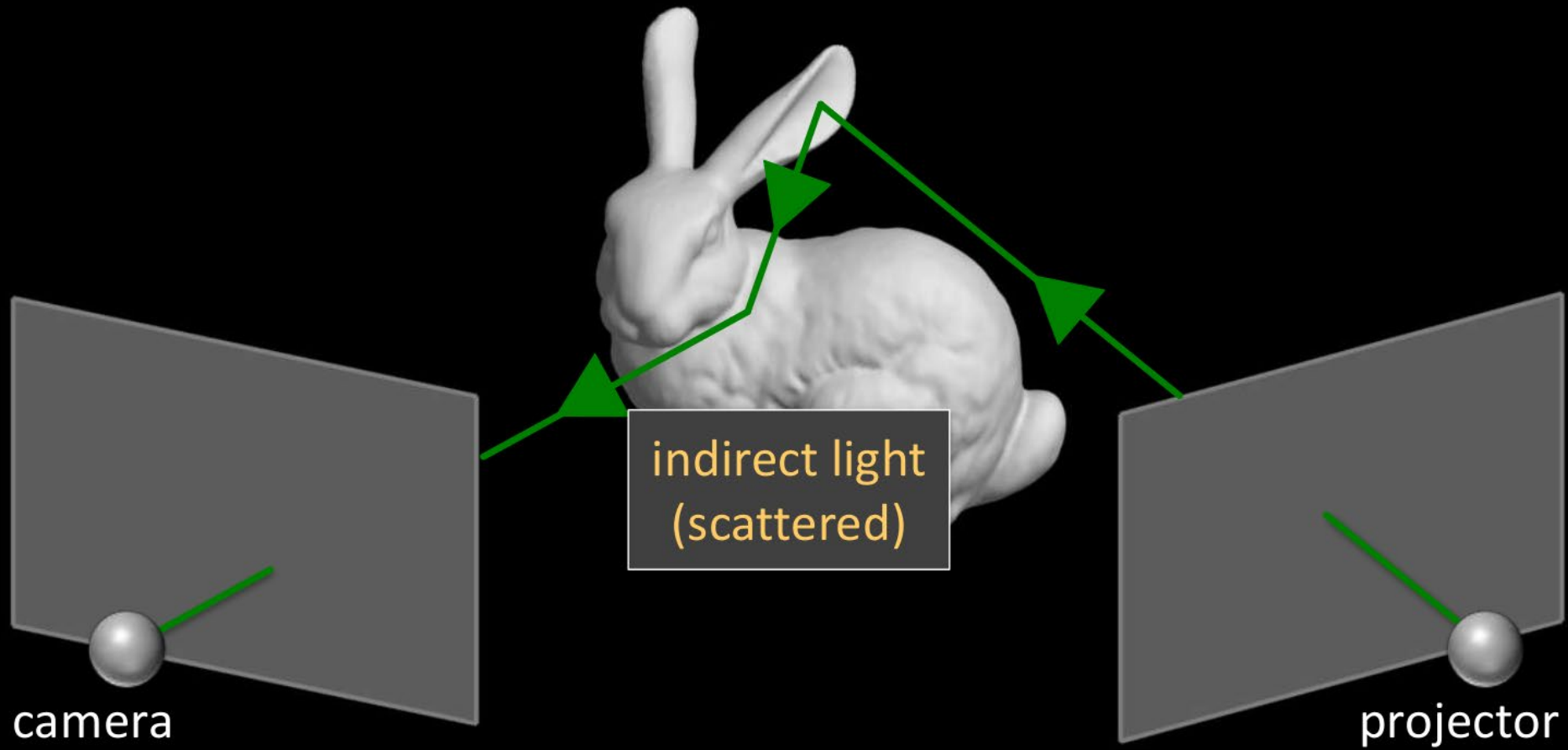
basic light paths



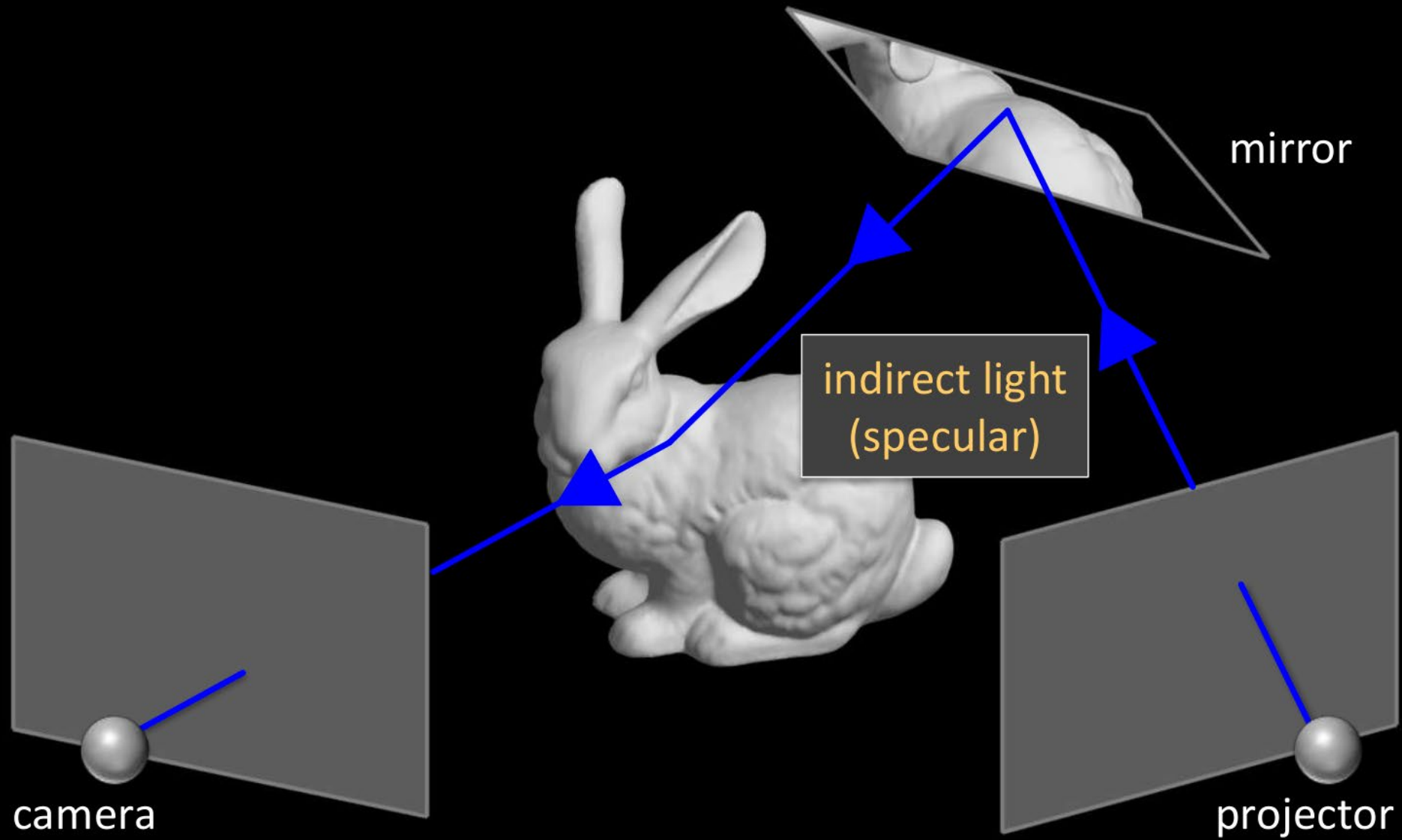
basic light paths



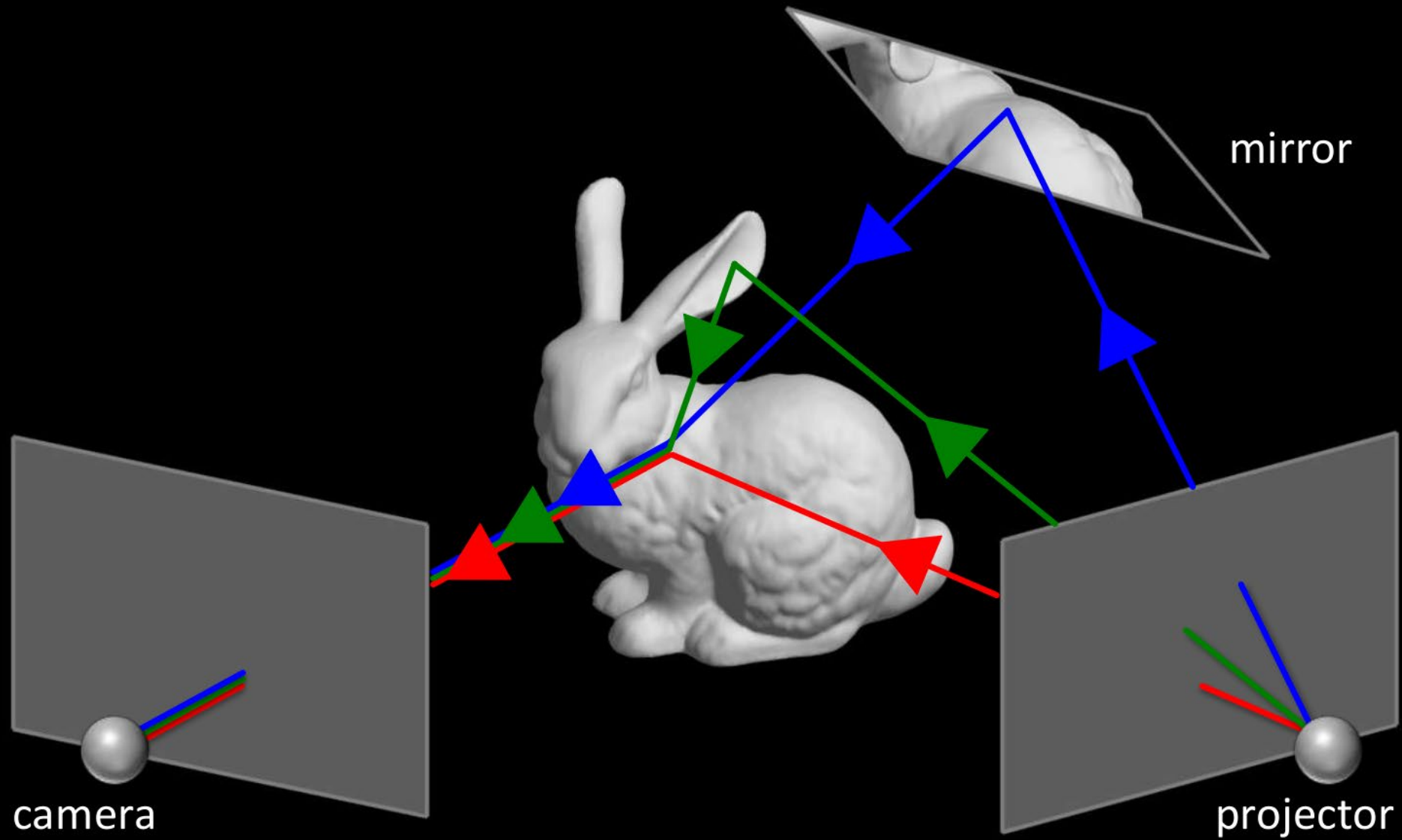
basic light paths



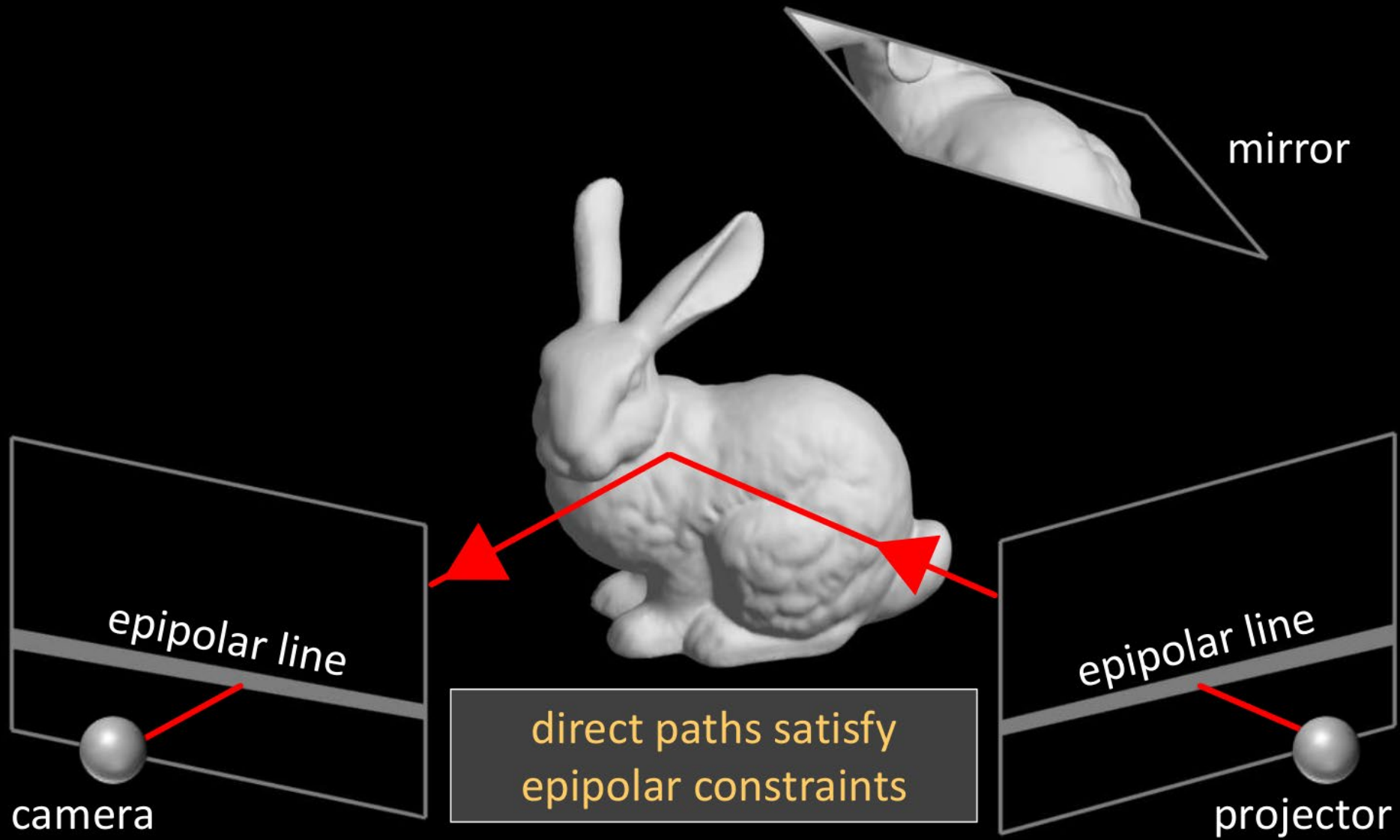
basic light paths



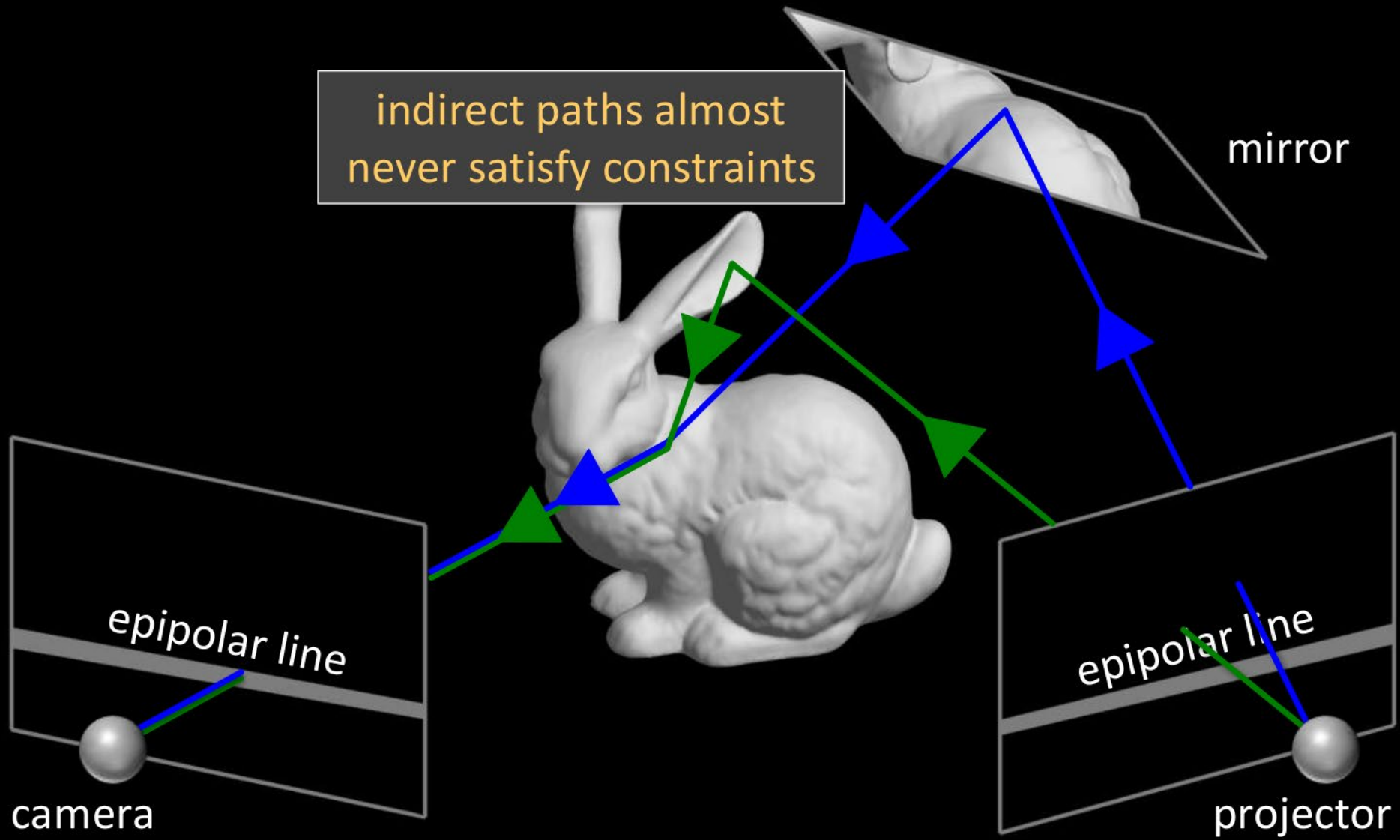
basic light paths



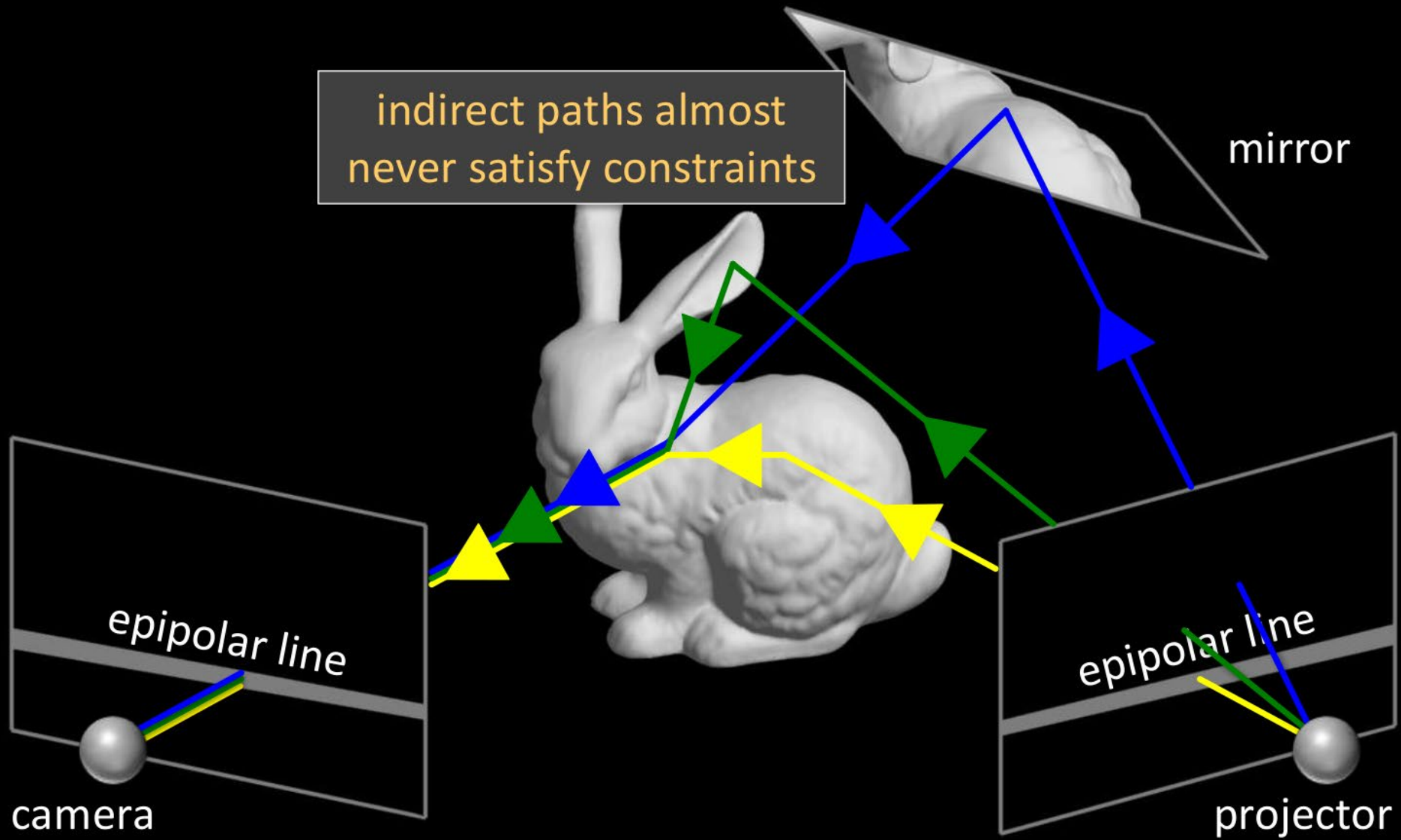
epipolar constraint & light transport



epipolar constraint & light transport

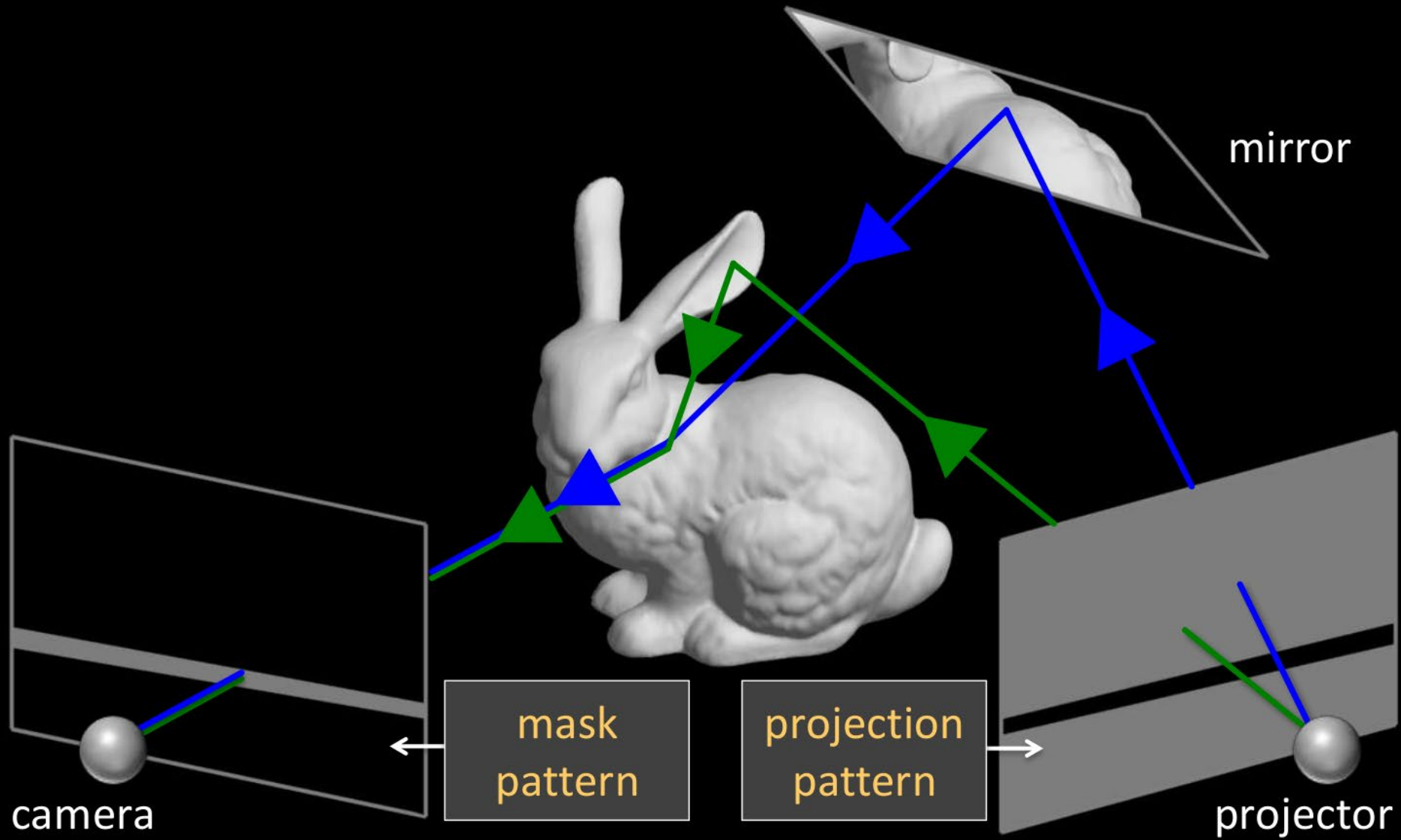


epipolar constraint & light transport



blocking epipolar paths with patterns & masks

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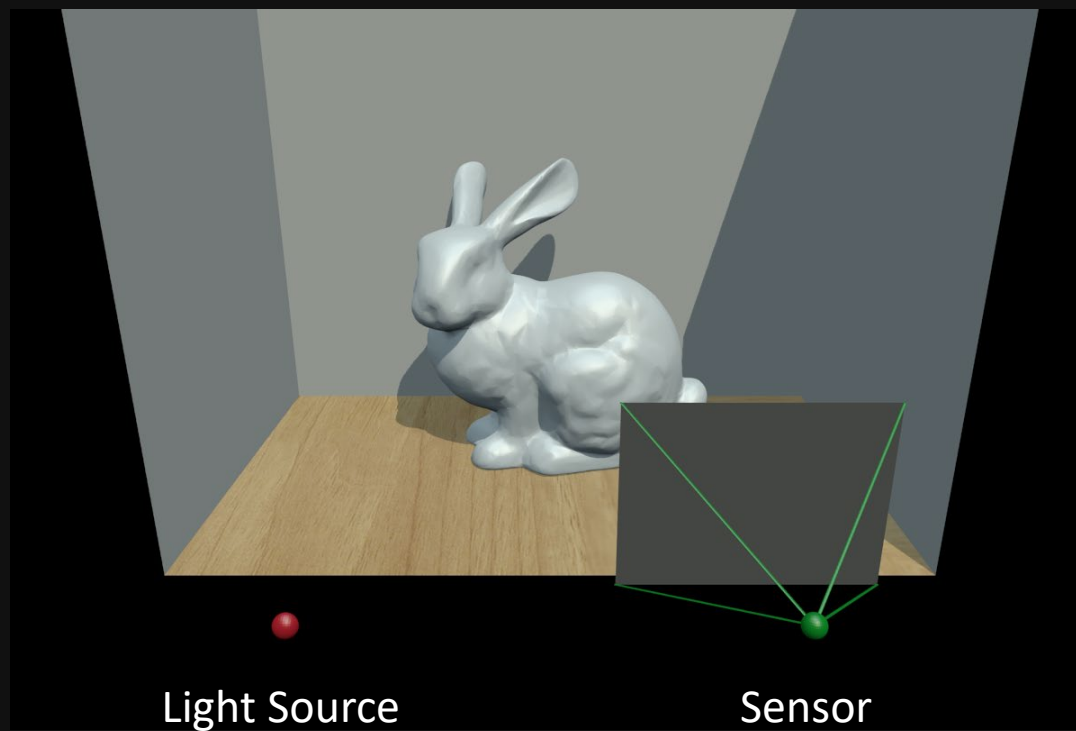


Energy-efficient epipolar imaging

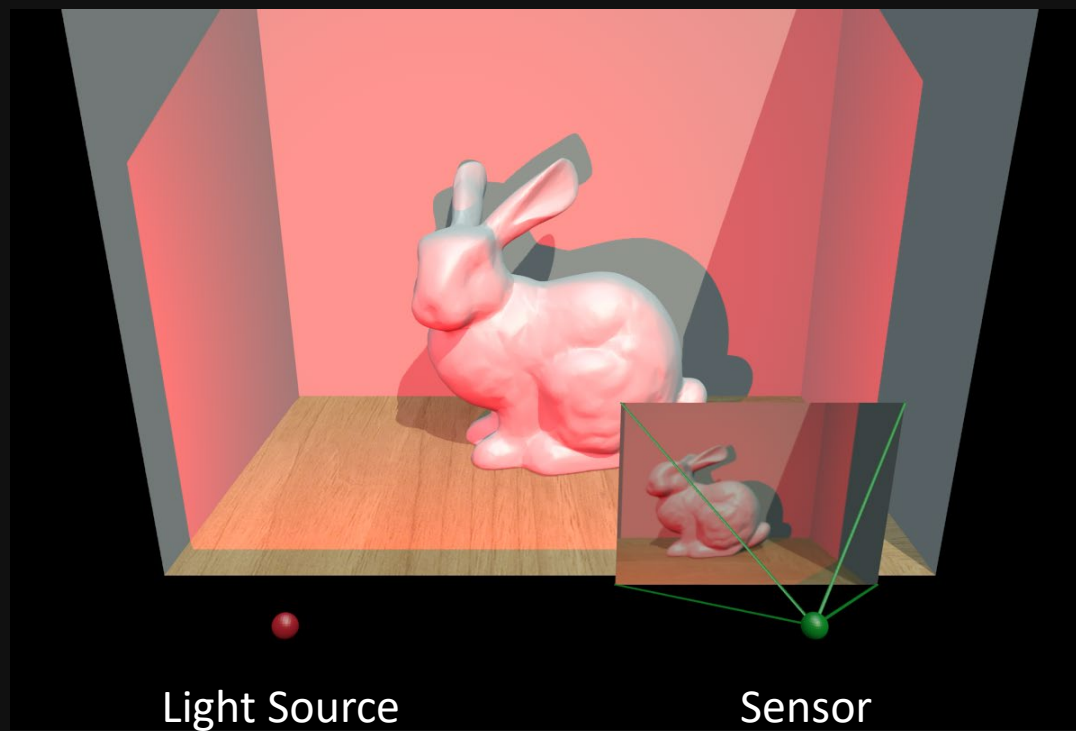
Energy-efficient transport parsing



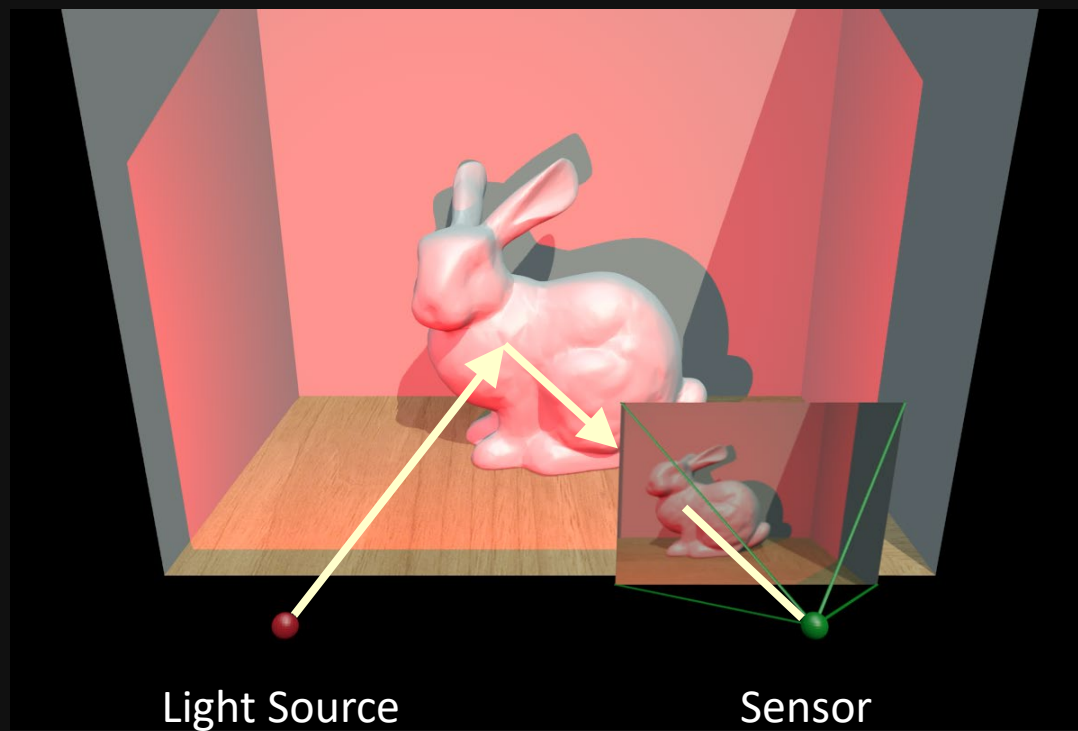
Regular Imaging



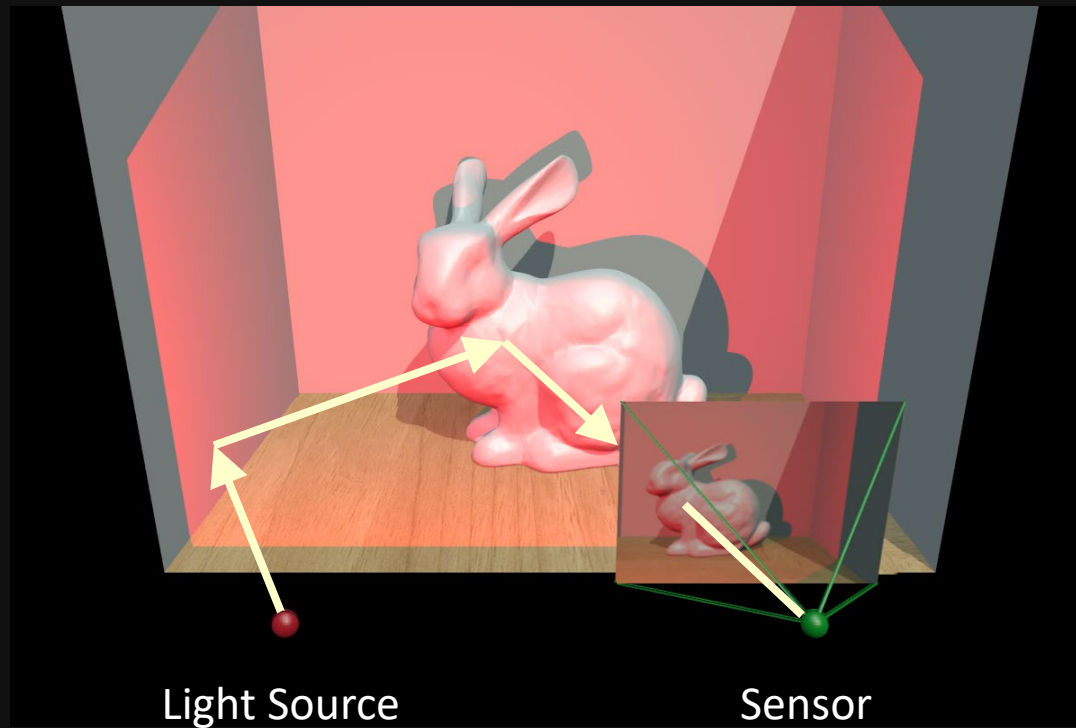
Regular Imaging



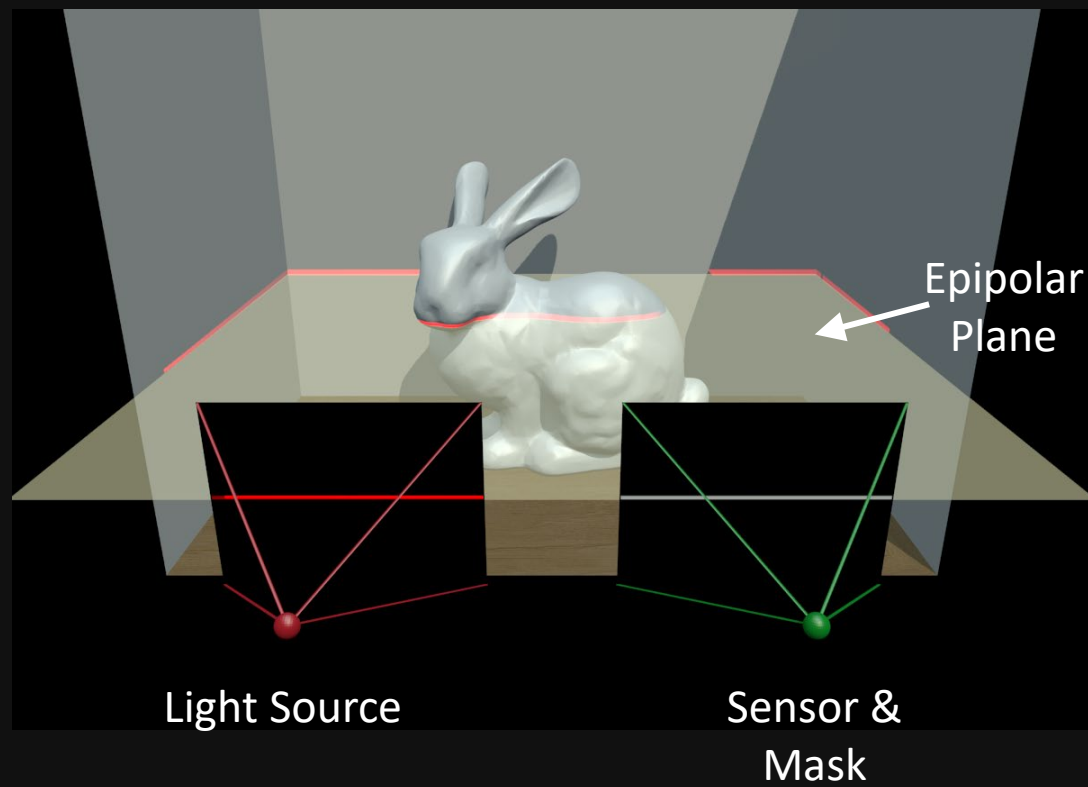
Regular Imaging



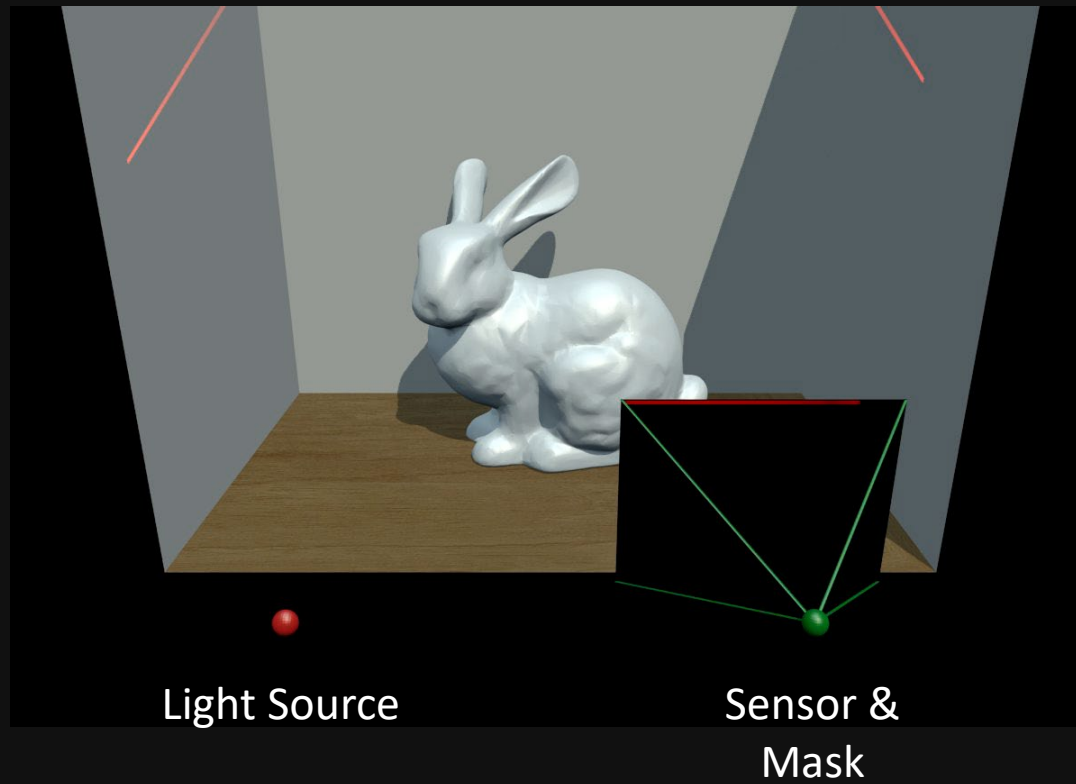
Regular Imaging



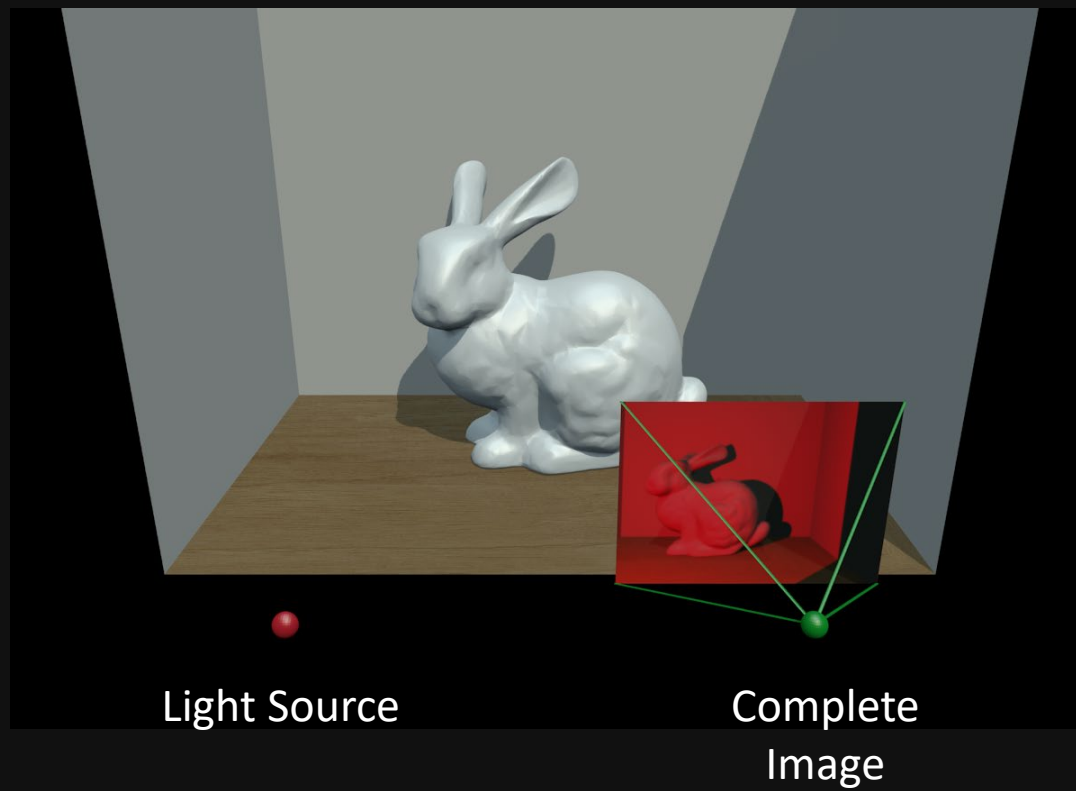
Epipolar Imaging



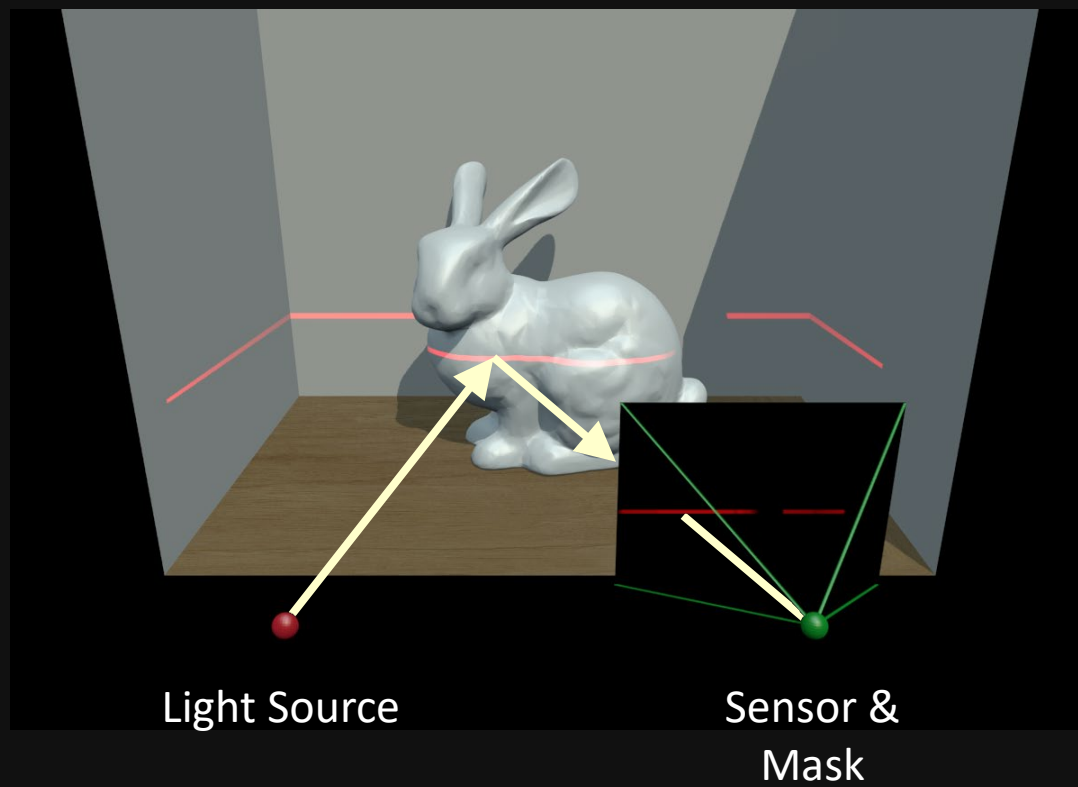
Epipolar Imaging



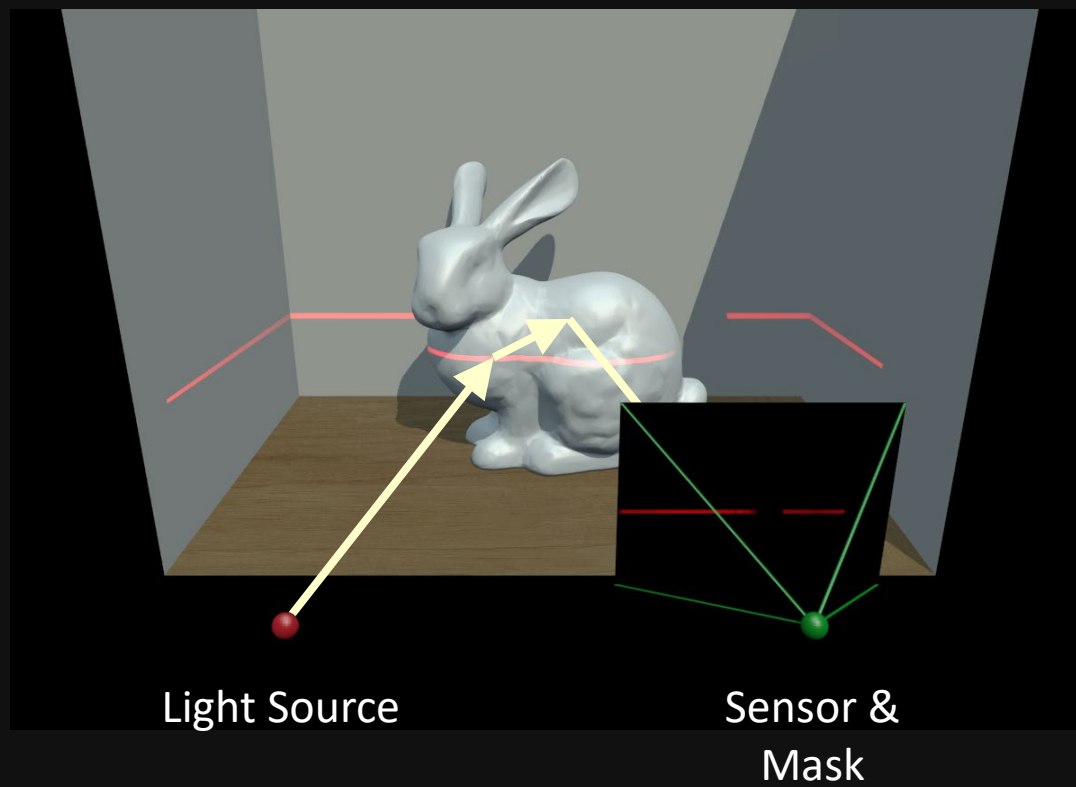
Epipolar Imaging



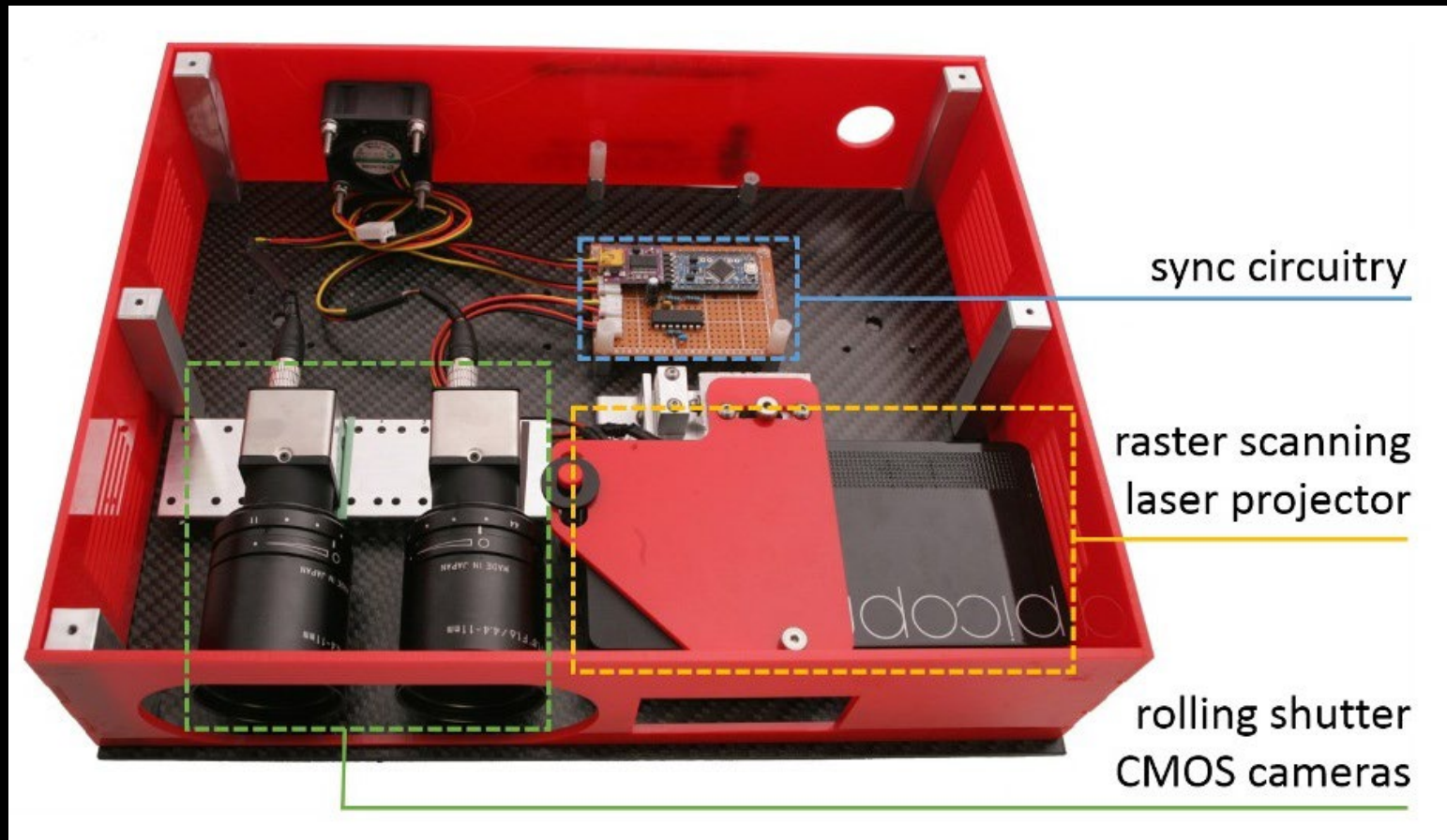
Epipolar Imaging



Epipolar Imaging



Energy-efficient transport parsing



all paths

planar (mostly direct)

non-planar (always indirect)



all paths

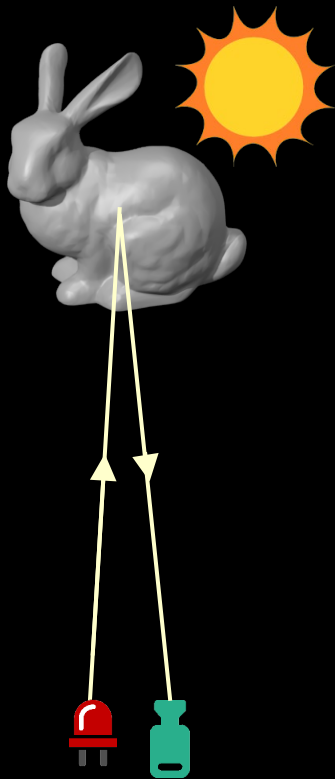
planar (mostly direct)

non-planar (always indirect)

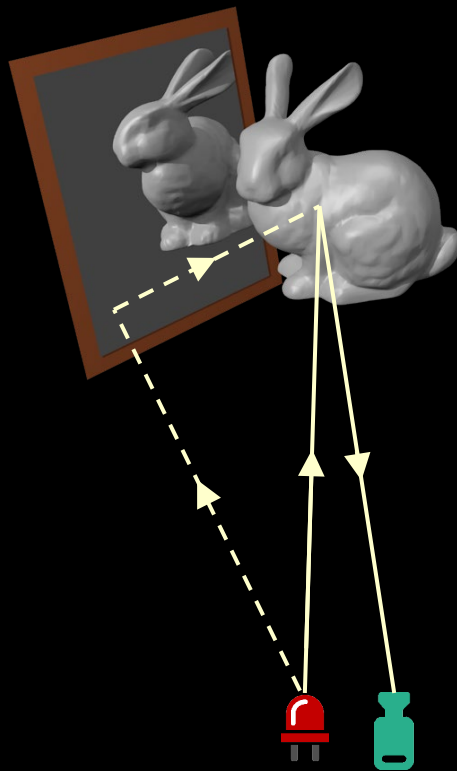


Benefits of Epipolar ToF Imaging

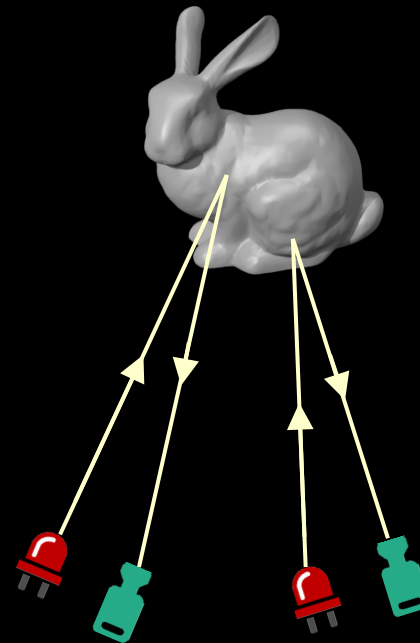
Ambient Light



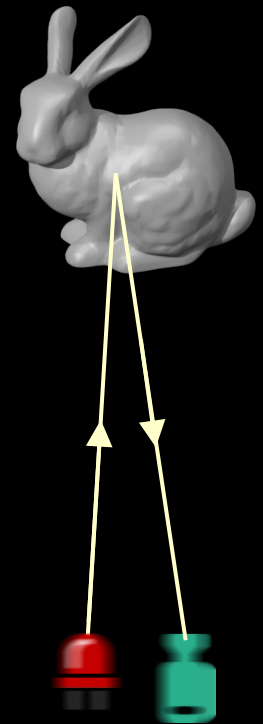
Multi-Path Interference



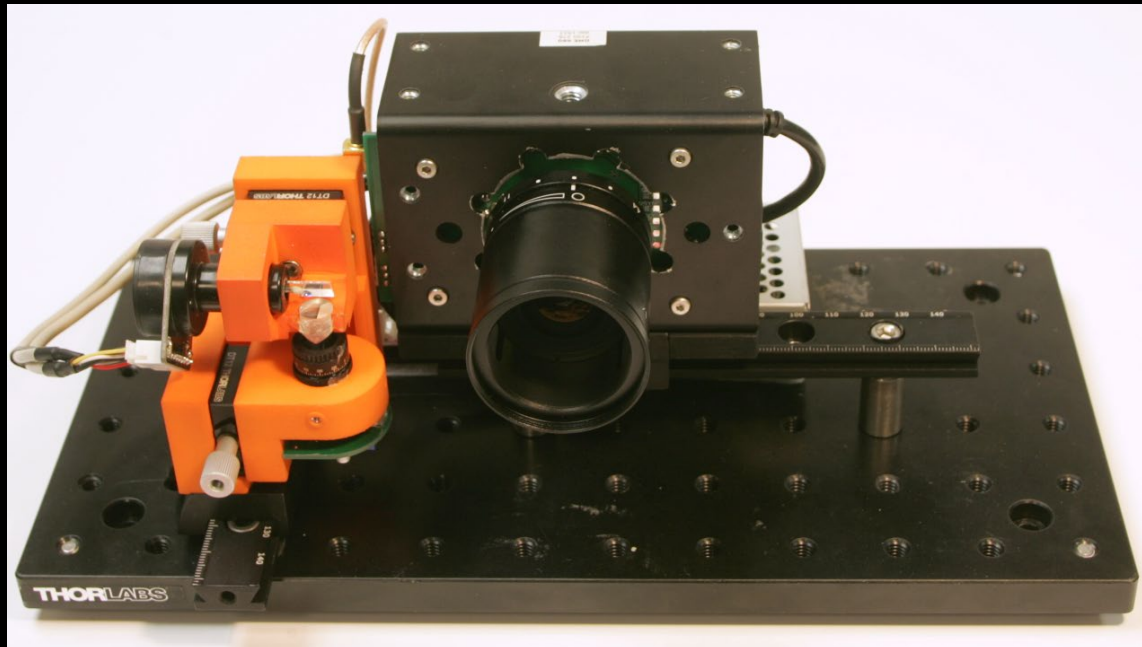
Multi-Device Interference



Camera Motion



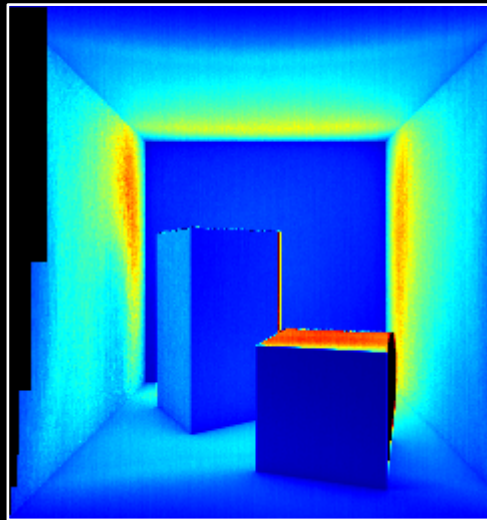
Epipolar ToF Prototype



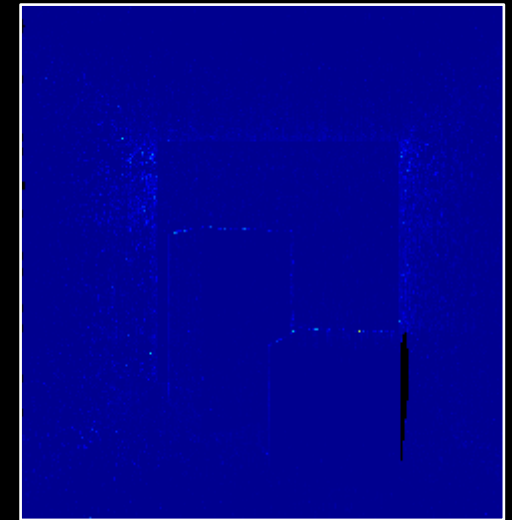
Epipolar ToF and Global Illumination



Depth Errors (in meters)



Regular ToF @ 30MHz



Epipolar ToF @ 30MHz

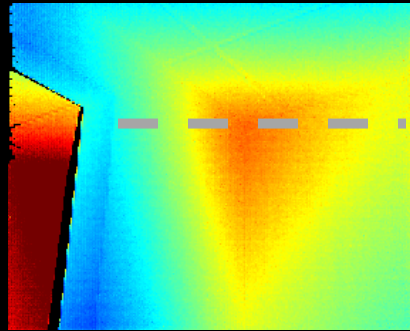


Epipolar ToF and Global Illumination

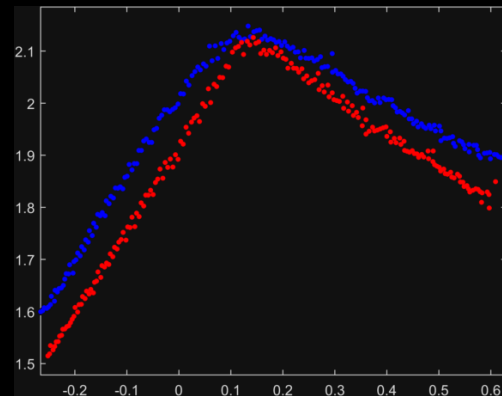
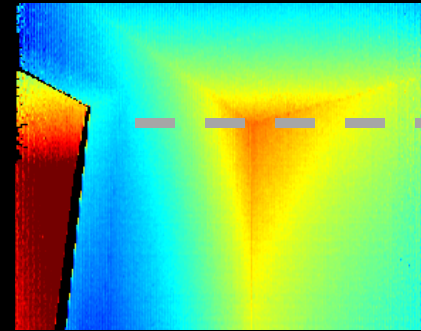


Corner of Room

Regular ToF

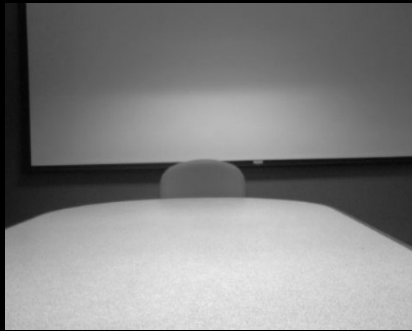


Epipolar ToF



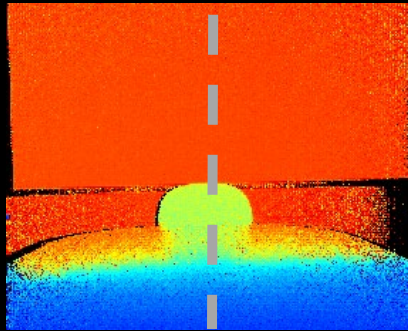
- Regular ToF
- Epipolar ToF

Epipolar ToF and Global Illumination

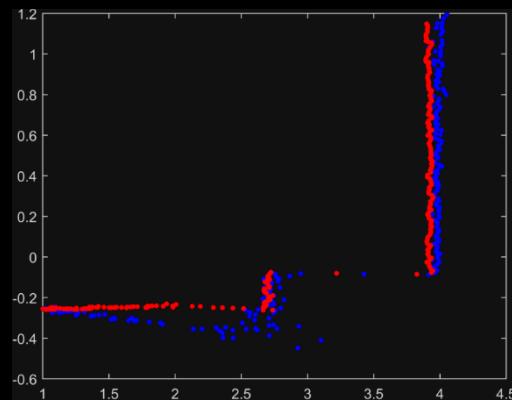
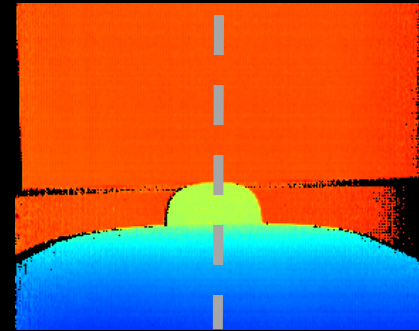


Conference Room

Regular ToF



Epipolar ToF



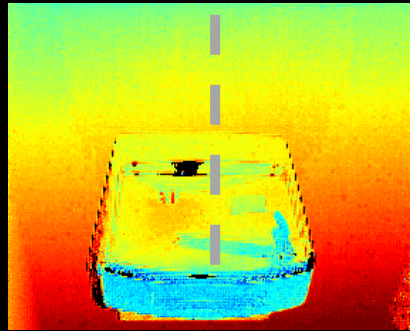
- Regular ToF
- Epipolar ToF

Epipolar ToF and Global Illumination

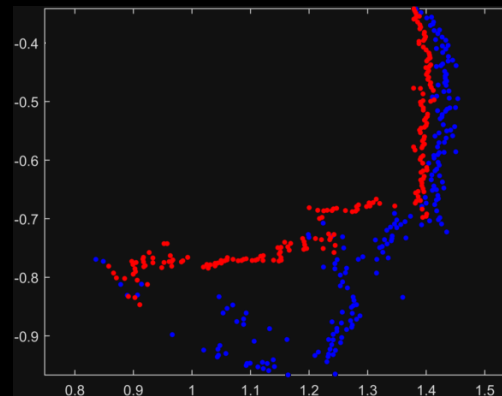
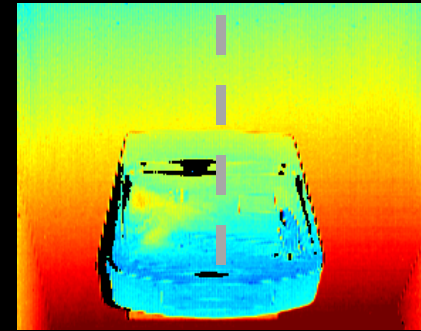


Water Fountain

Regular ToF



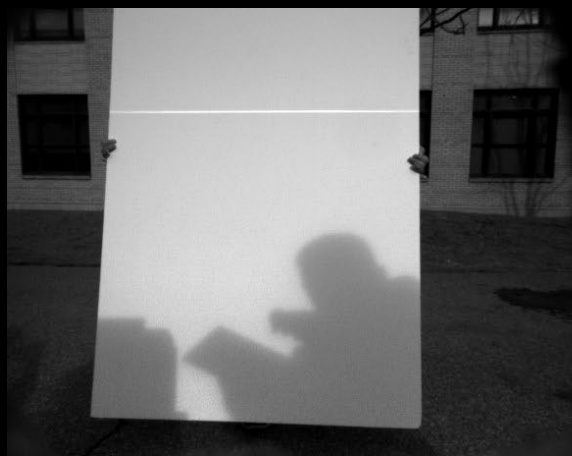
Epipolar ToF



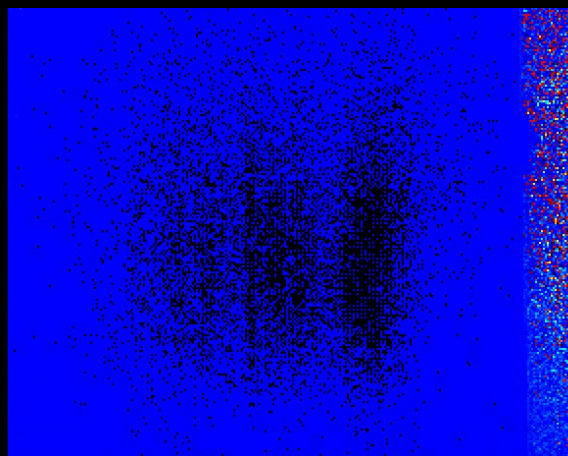
- Regular ToF
- Epipolar ToF

Outdoors (Cloudy – 10 kilolux)

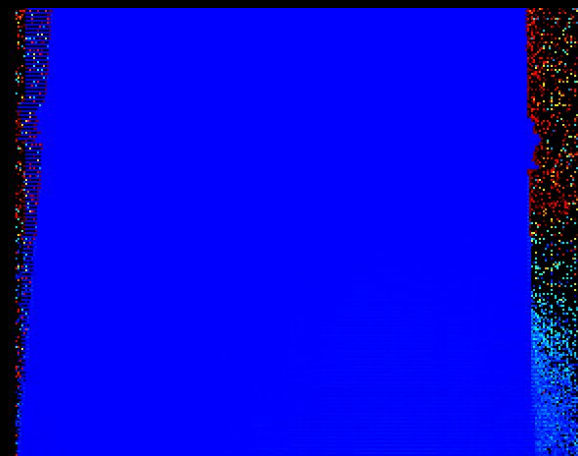
Scene



Regular ToF



Epipolar ToF

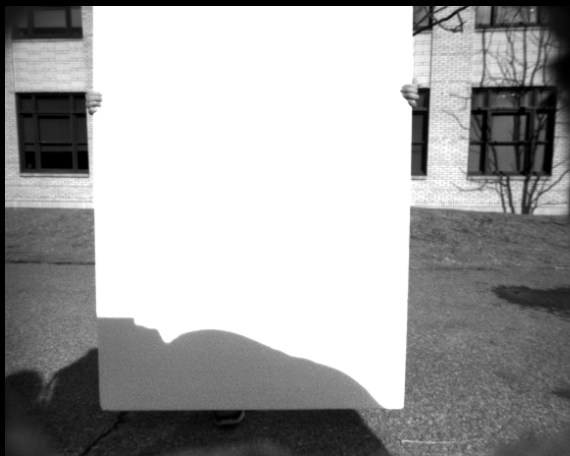


Depth (meters)

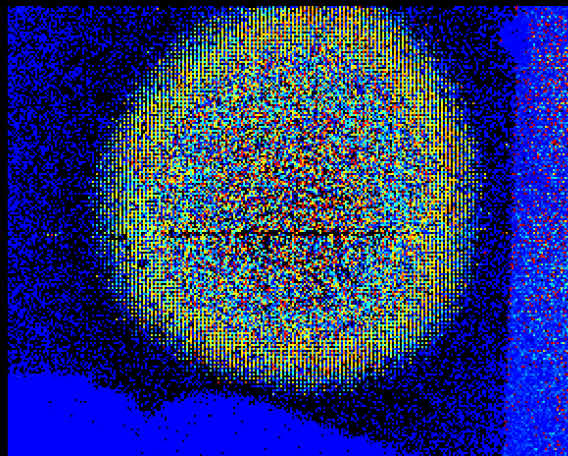


Outdoors (Sunny – 70 kilolux)

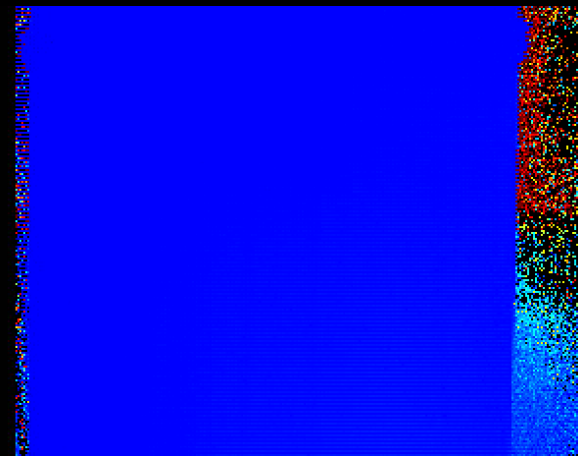
Scene



Regular ToF



Epipolar ToF



Depth (meters)



Outdoors (Sunny – 70 kilolux)



References

Basic reading:

- Nayar et al., “Fast separation of direct and indirect components of a scene using high frequency illumination,” SIGGRAPH 2004.
The paper on separation of direct and indirect illumination using high-frequency illumination.
- Gupta et al., “A Practical Approach to 3D Scanning in the Presence of Interreflections, Subsurface Scattering and Defocus,” IJCV 2013.
The paper on using XOR codes to deal with indirect illumination in structured light 3D.
- Gupta et al., “Phasor imaging: A generalization of correlation-based time-of-flight imaging,” TOG 2015.
The paper on using high-frequency modulation to deal with interreflections and MPI in CW-ToF imaging.
- O’Toole et al., “Primal-dual coding to probe light transport,” SIGGRAPH 2012.
- O’Toole et al., “3d shape and indirect appearance by structured light transport,” CVPR 2014.
These two papers introduce the concepts of light transport probing and epipolar probing, as well as explain how to use primal-dual coding to achieve them.
- O’Toole et al., “Homogeneous codes for energy-efficient illumination and imaging,” SIGGRAPH 2015.
This paper shows how to efficiently implement epipolar imaging with a simple projector and camera.
- Achar et al., “Epipolar time-of-flight imaging,” SIGGRAPH 2017.
This paper combines epipolar imaging and time-of-flight imaging.

Additional reading:

- Seitz et al., “A theory of inverse light transport,” ICCV 2005.
This early paper shows a way to *exactly* decompose light transport by number of bounces, under certain assumptions for the imaged scene.
- Chandraker et al., “On the duality of forward and inverse light transport,” PAMI 2011.
- Reddy et al., “Frequency-space decomposition and acquisition of light transport under spatially varying illumination,” ECCV 2012.
These two papers have additional analysis about the relationship between direct and indirect illumination and illumination frequency.
- Durand et al., “A frequency analysis of light transport,” SIGGRAPH 2005.
This paper more formally discusses the notion of light transport frequency, and the frequency characteristics of different light transport effects (specular versus diffuse reflections, hard versus smooth shadows).