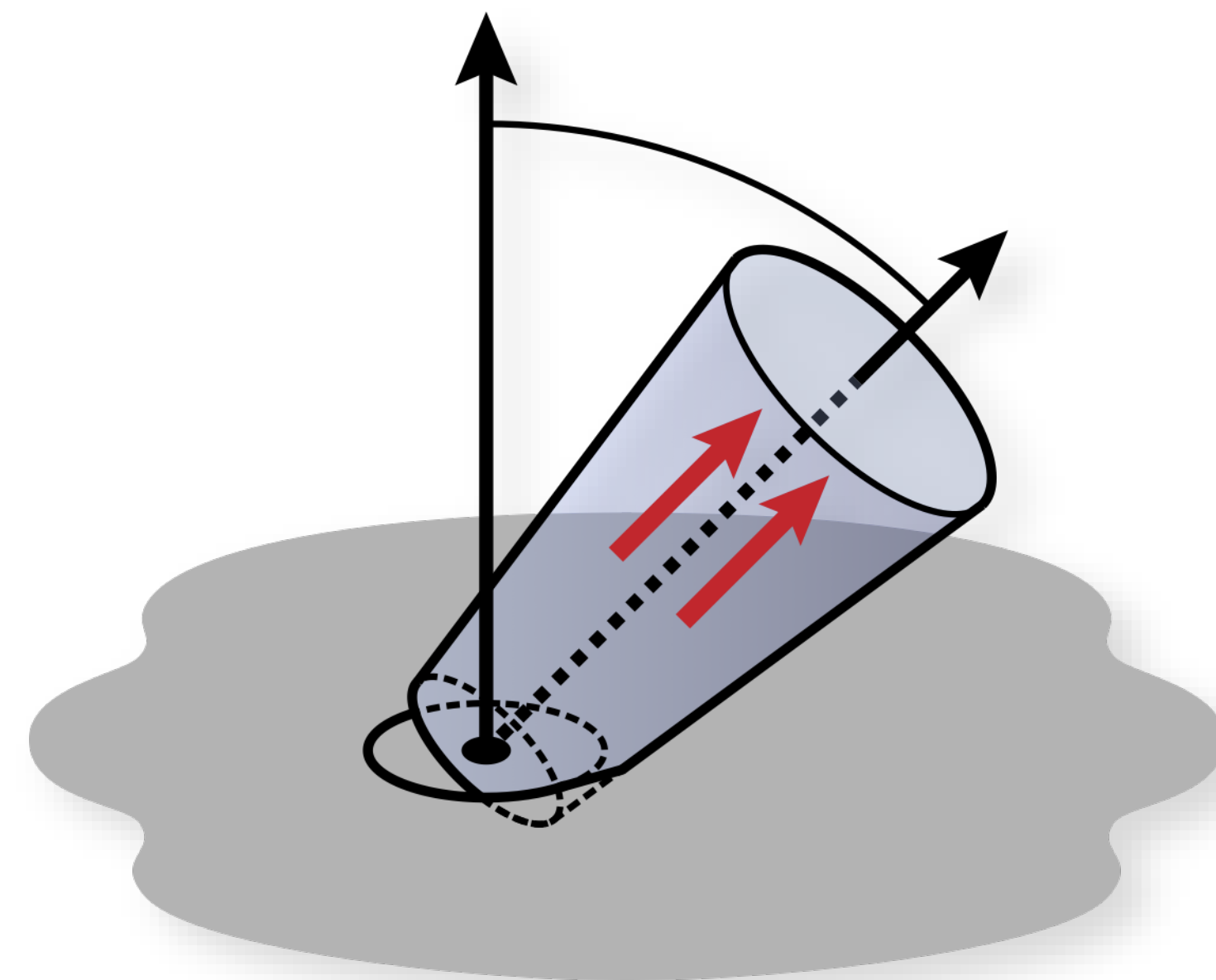
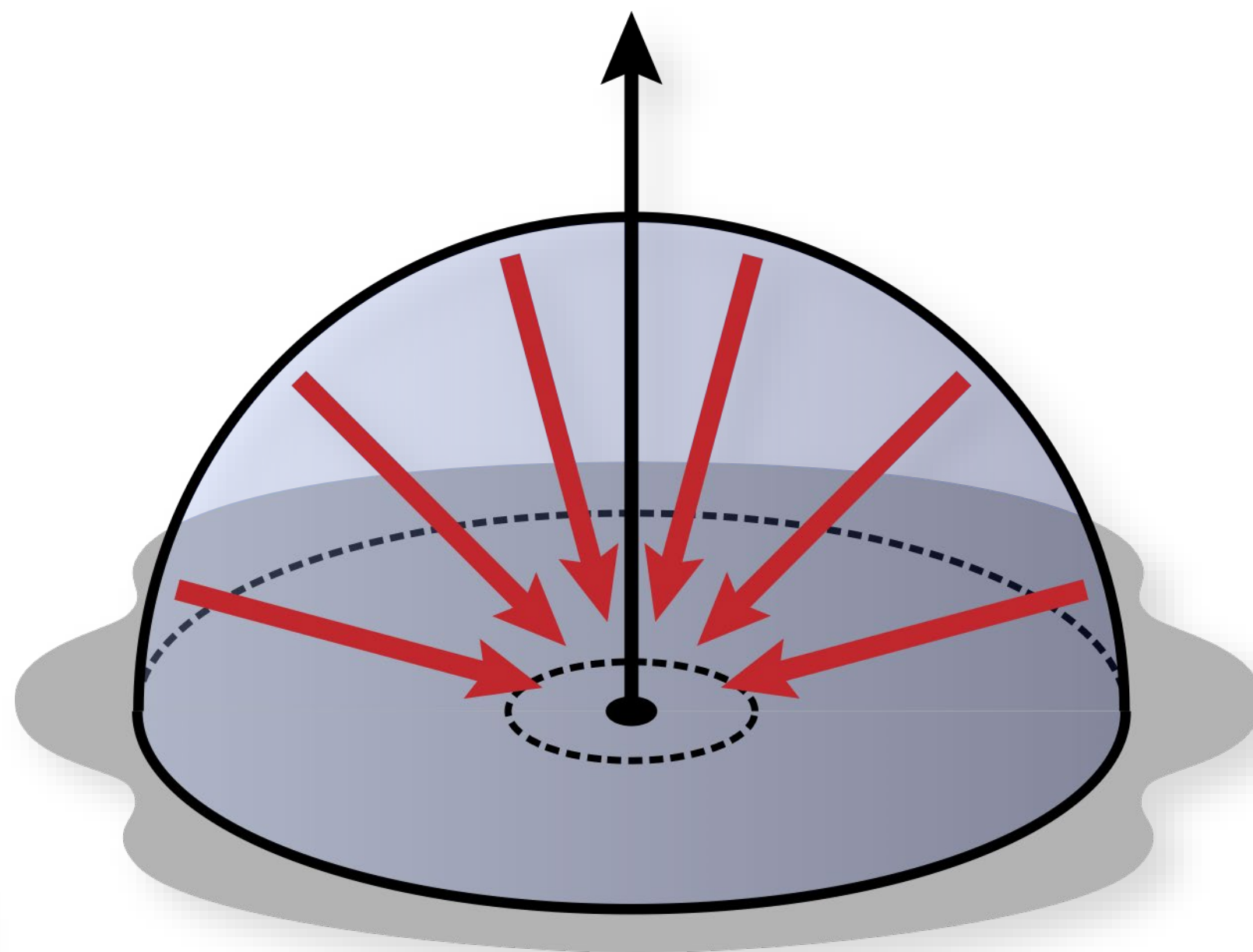
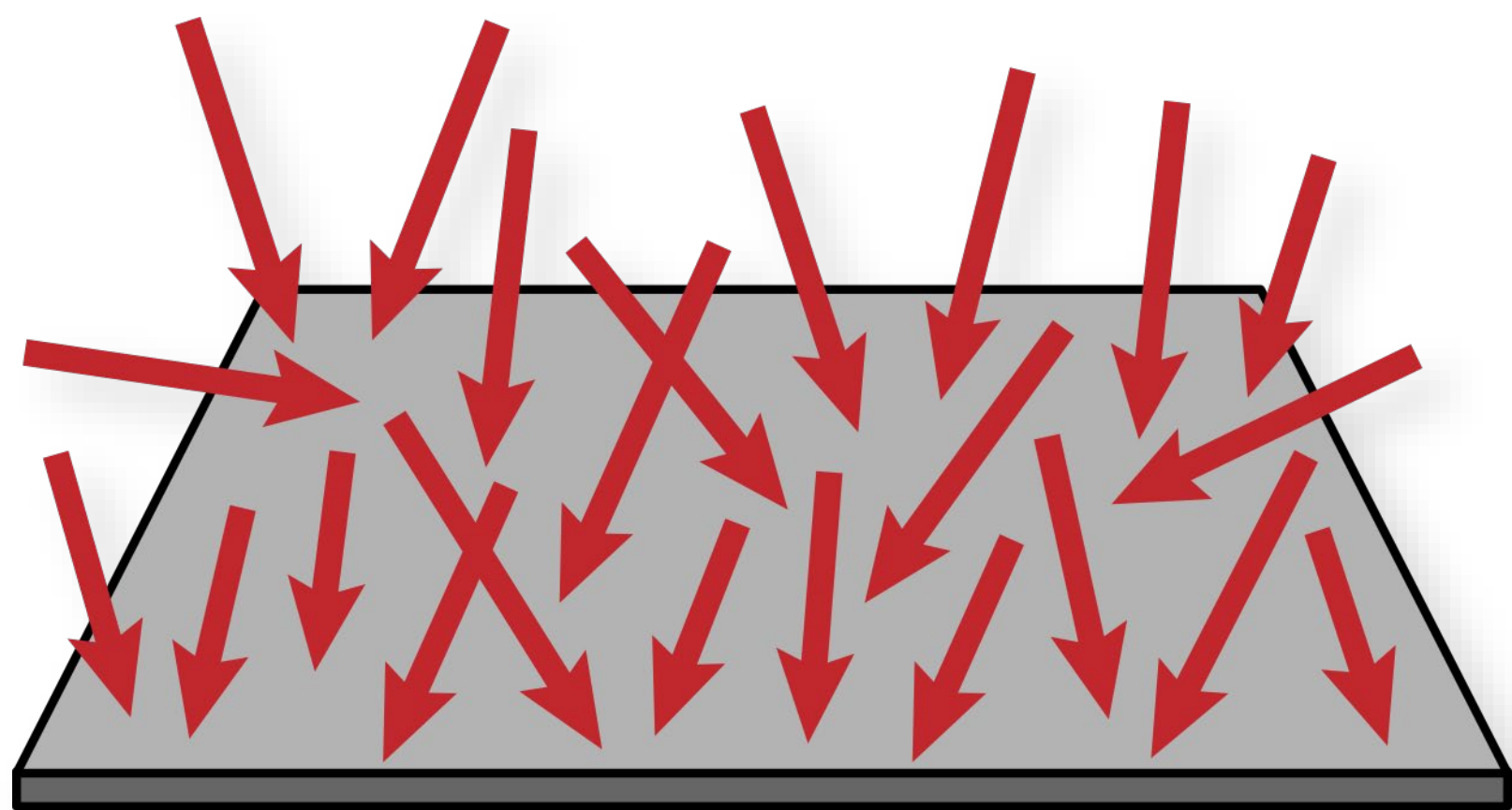


Radiometry



Course announcements

- Take-home quiz 2 posted, due tonight.
 - How many of you have looked at/started/finished it?
 - Any questions?
- Programming assignment 1 posted, this Friday.
 - How many of you have looked at/started/finished it?
 - Any questions?
- First reading group took place on Friday.
 - Any feedback?

Overview of today's lecture

- Radiometric quantities.
- A little bit about color.
- Reflectance equation.
- Standard reflectance functions revisited.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

Quantifying Light

Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, ...

Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.



Radiometry

Assume light consists of photons with:

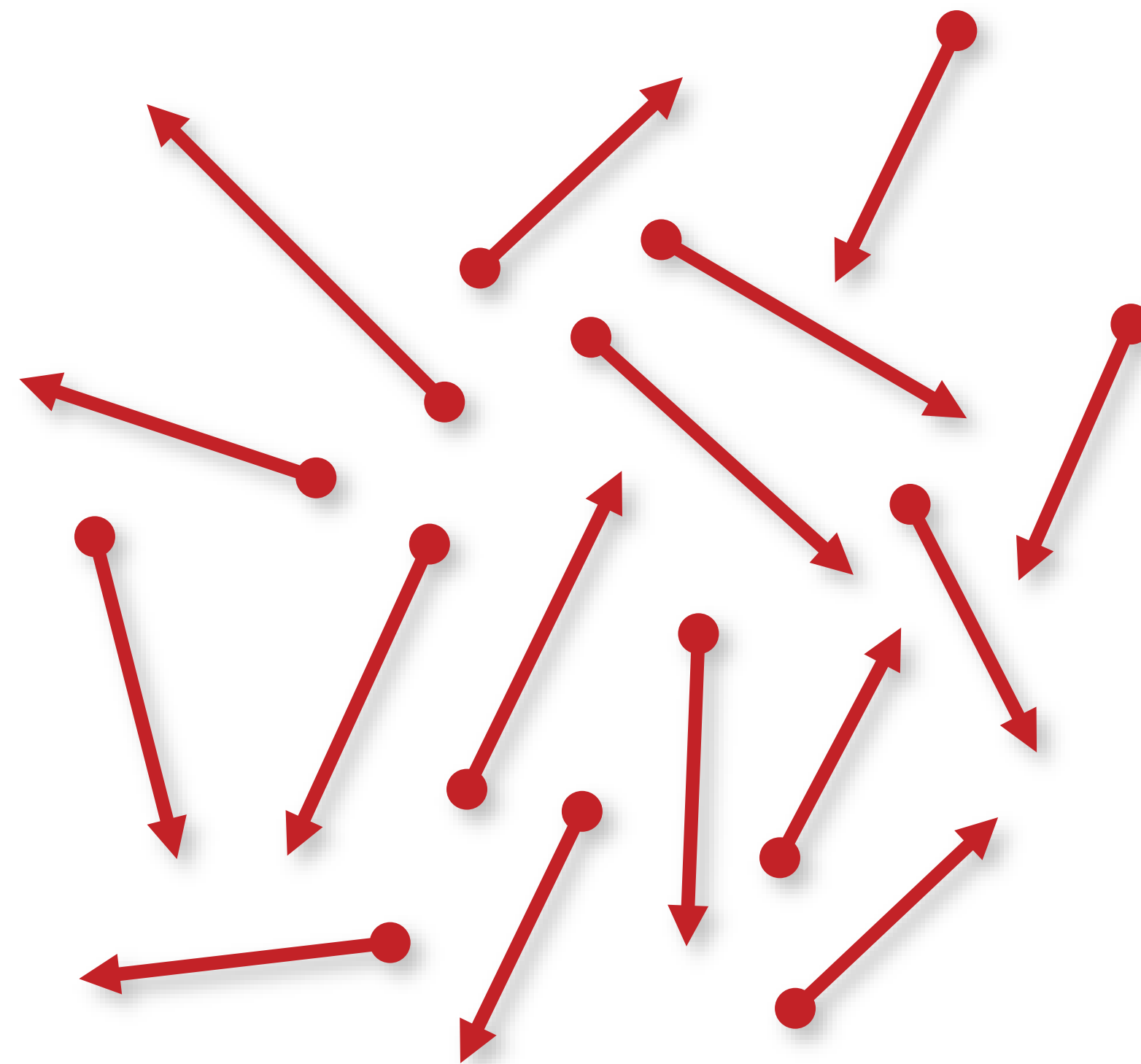
- $\mathbf{x}$: Position
- $\vec{\omega}$: Direction of travel
- λ : Wavelength

Each photon has an energy of: $\frac{h c}{\lambda}$

- $h \approx 6.63 \times 10^{-34} \text{ m}^2 \text{ kg/s}$: Planck's constant
- $c = 299,792,458 \text{ m/s}$: speed of light in vacuum
- Unit of energy, Joule: $\left[\text{J} = \text{kg m}^2 / \text{s}^2 \right]$

Radiometry

How do we measure the energy flow?



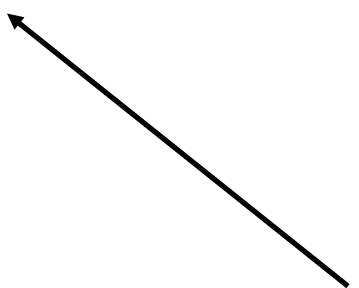
Measuring energy means “counting photons”

Radiometry

Basic quantities (depend on wavelength)

- flux Φ
- irradiance E
- radiosity B
- intensity I
- radiance L

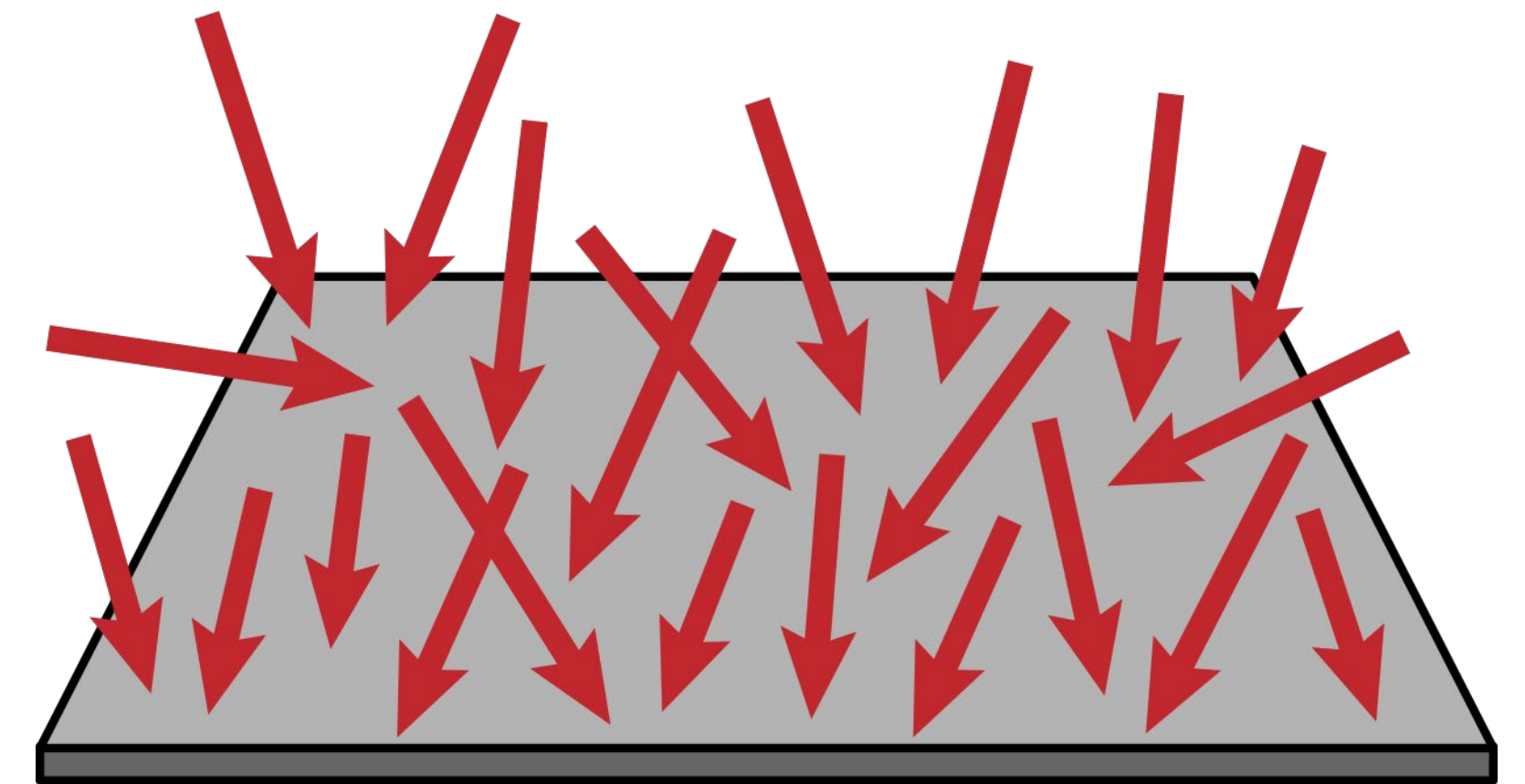
will be the most important quantity for us



Flux (Radiant Flux, Power)

total amount of radiant energy passing through surface or space
per unit time

$$\Phi(A) \quad \left[\frac{\text{J}}{\text{s}} = \text{W} \right]$$



examples:

- number of photons hitting a wall per second
- number of photons leaving a lightbulb per second (how do we quantify this exactly?)

Irradiance

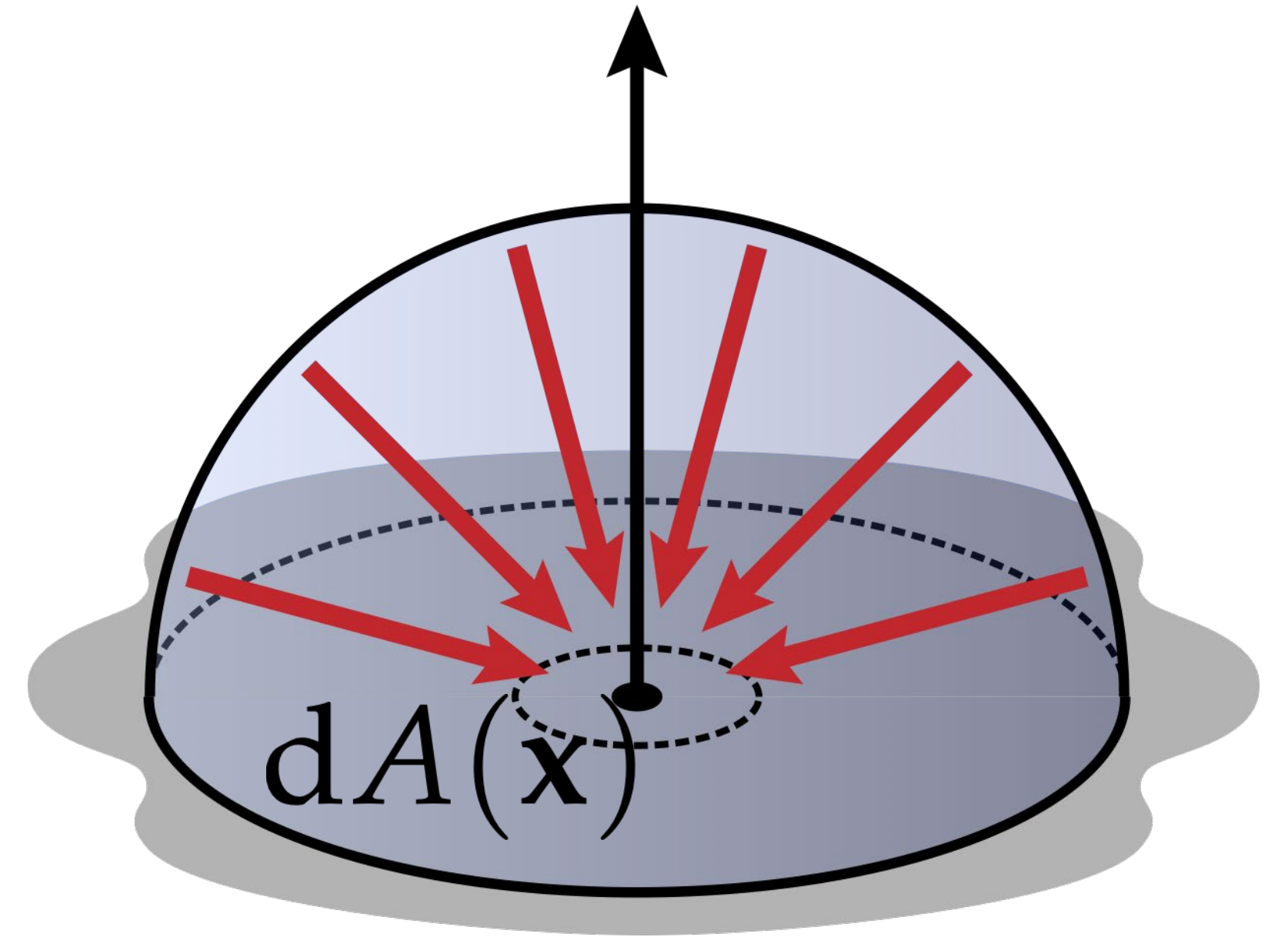
area density of flux

flux per unit area **arriving** at a surface

$$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \quad \left[\frac{\text{W}}{\text{m}^2} \right]$$

example:

- number of photons **hitting** a small patch of a wall per second,
divided by size of patch



Radiosity (Radiant Exitance)

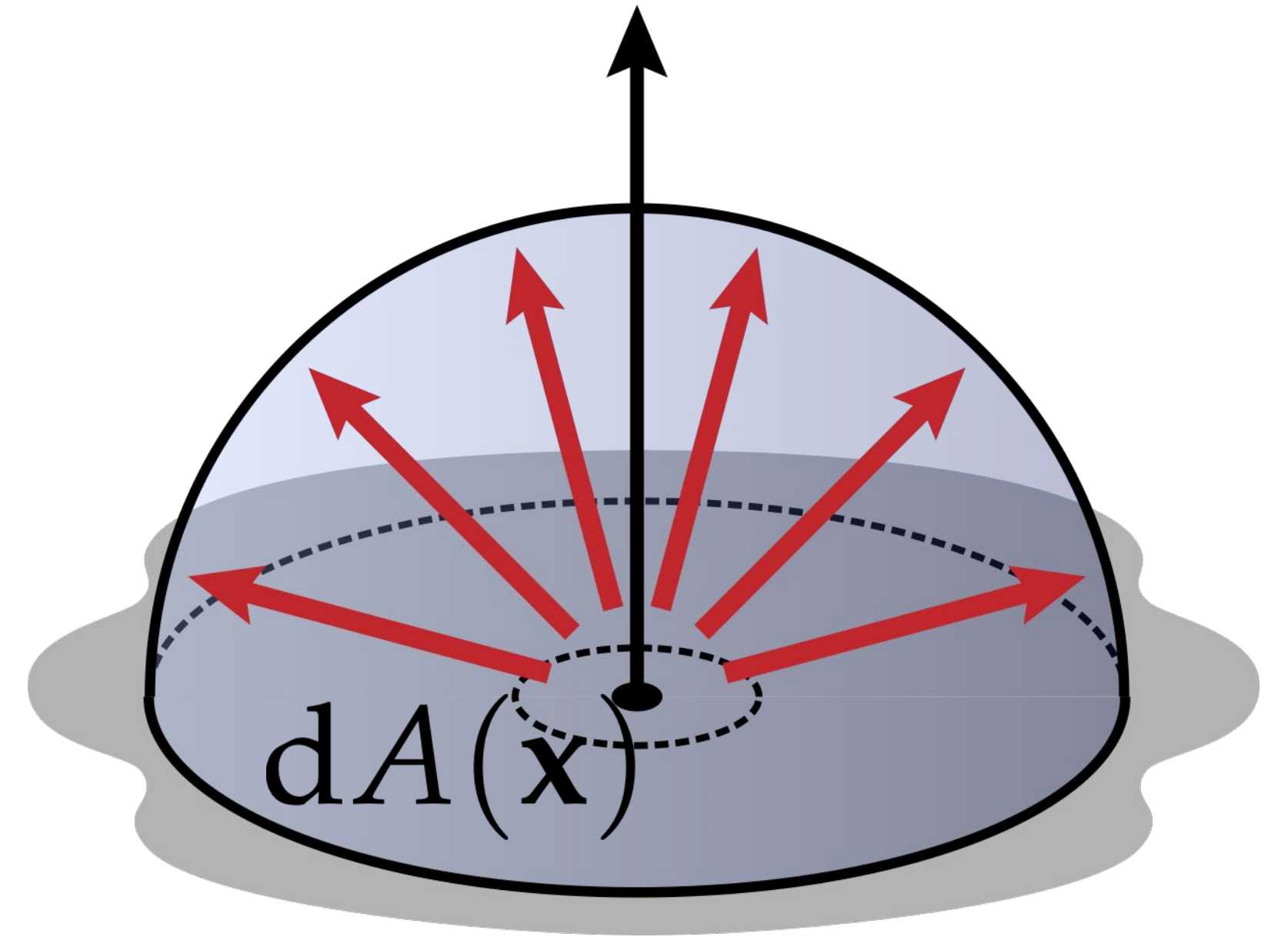
area density of flux

flux per unit area **leaving** a surface

$$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \quad \left[\frac{\text{W}}{\text{m}^2} \right]$$

example:

- number of photons **reflecting off** a small patch of a wall per second,
divided by size of patch

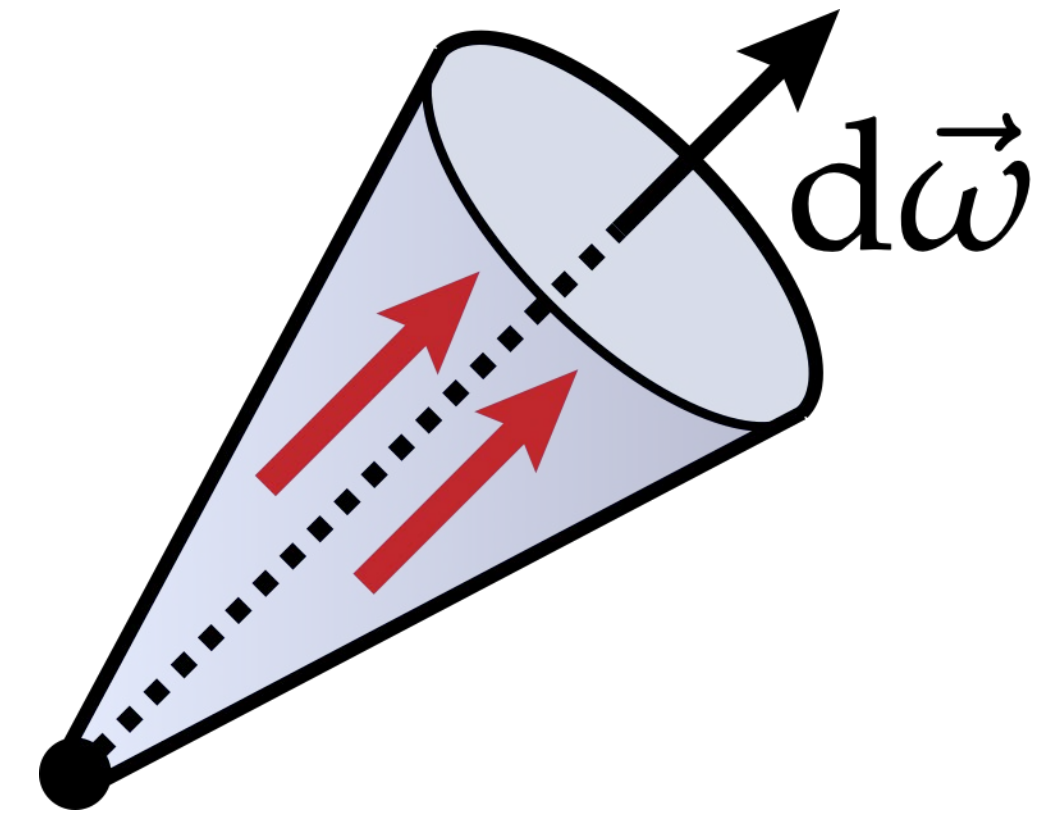


Radiant Intensity

directional density of flux

power (flux) per solid angle

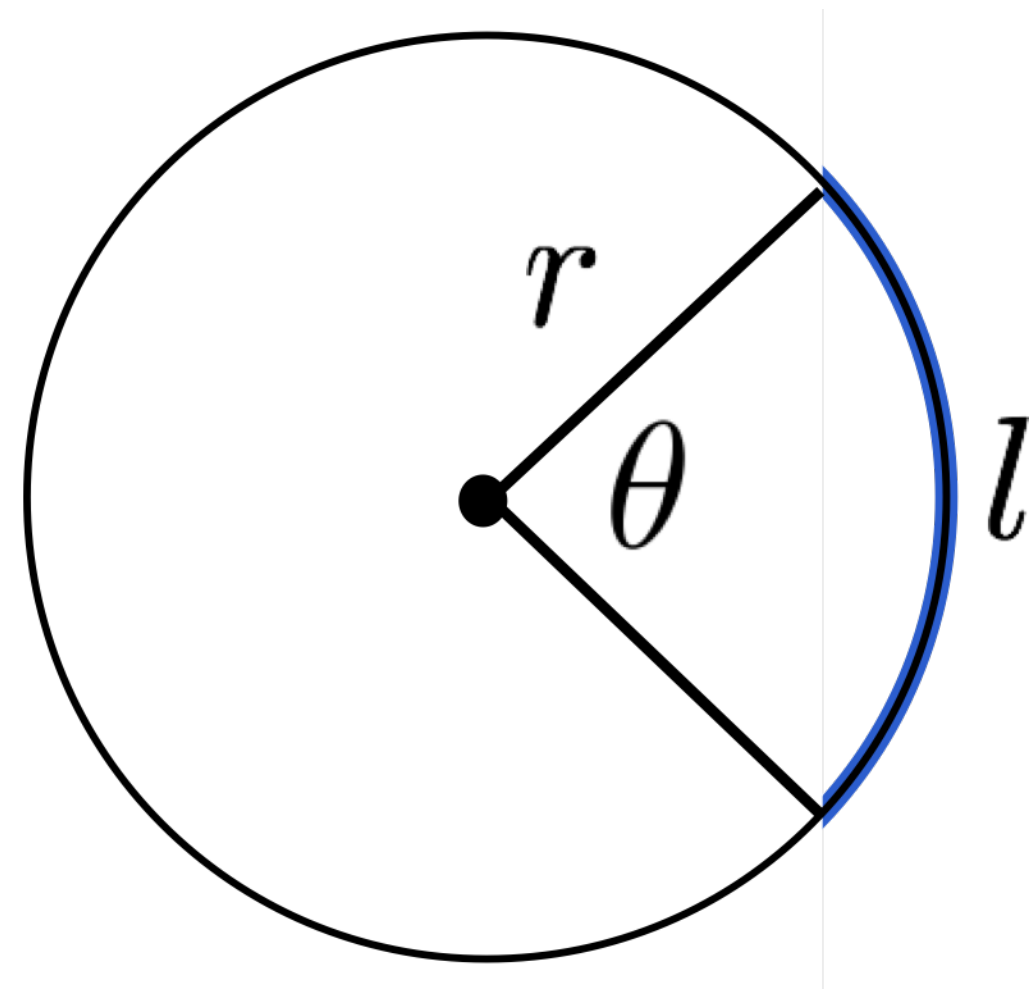
$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \quad \left[\frac{\text{W}}{\text{sr}} \right]$$



Solid Angle

Angle

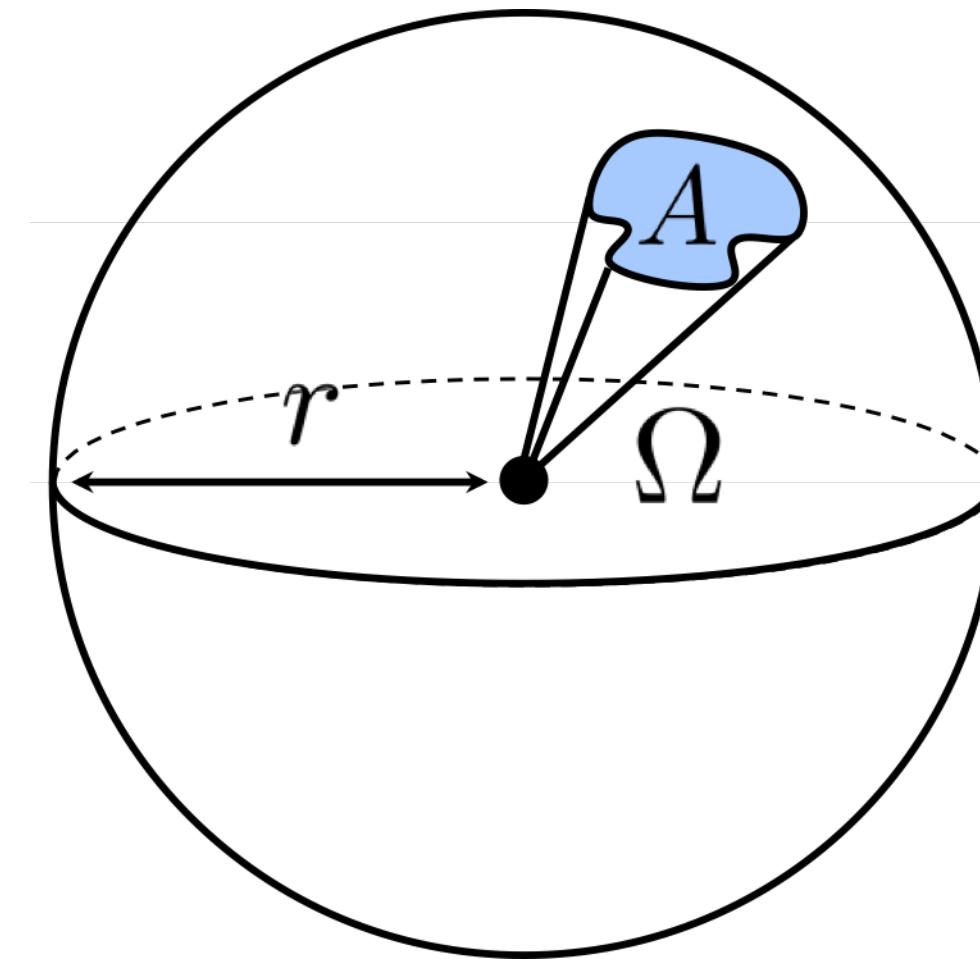
- circle: 2π radians



$$\theta = \frac{l}{r}$$

Solid angle

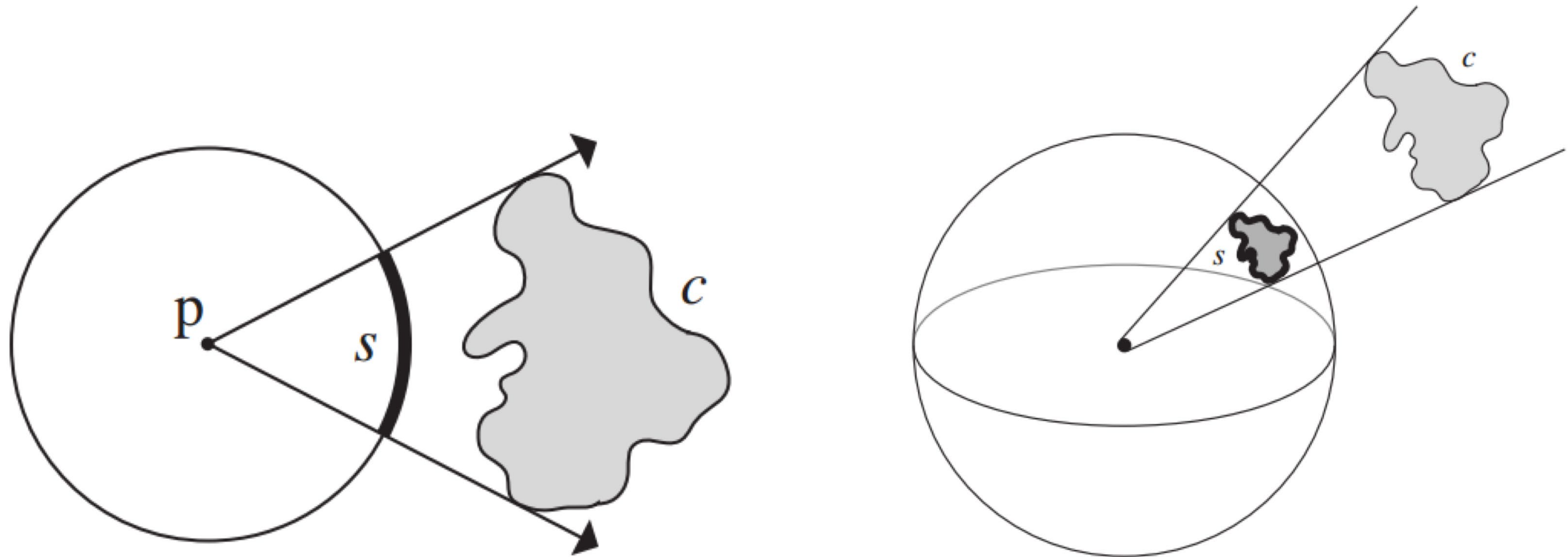
- sphere: 4π steradians



$$\Omega = \frac{A}{r^2}$$

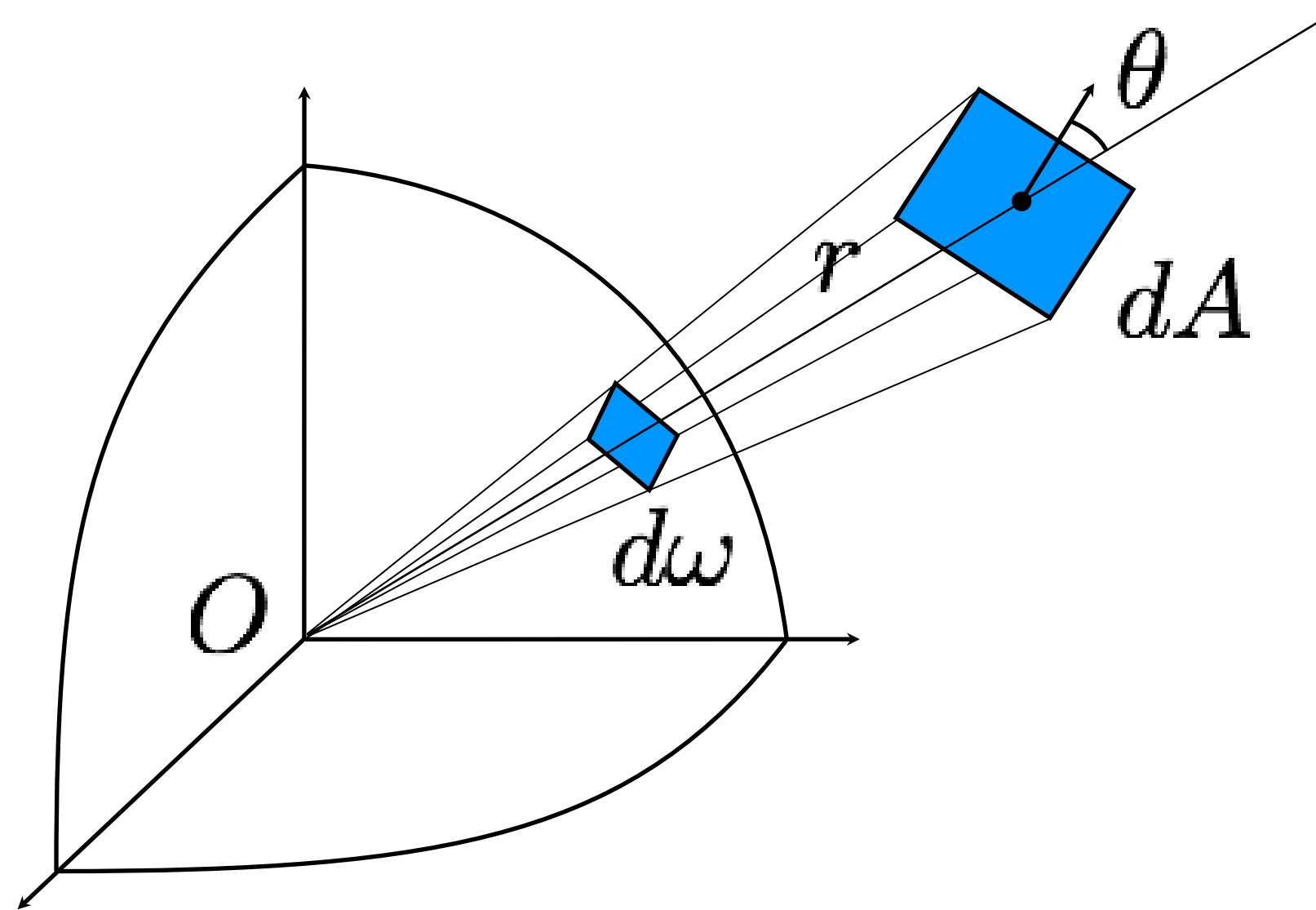
Subtended (Solid) Angle

Length/area of object's *projection* onto a unit circle/sphere



Solid angle

The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

One can show:

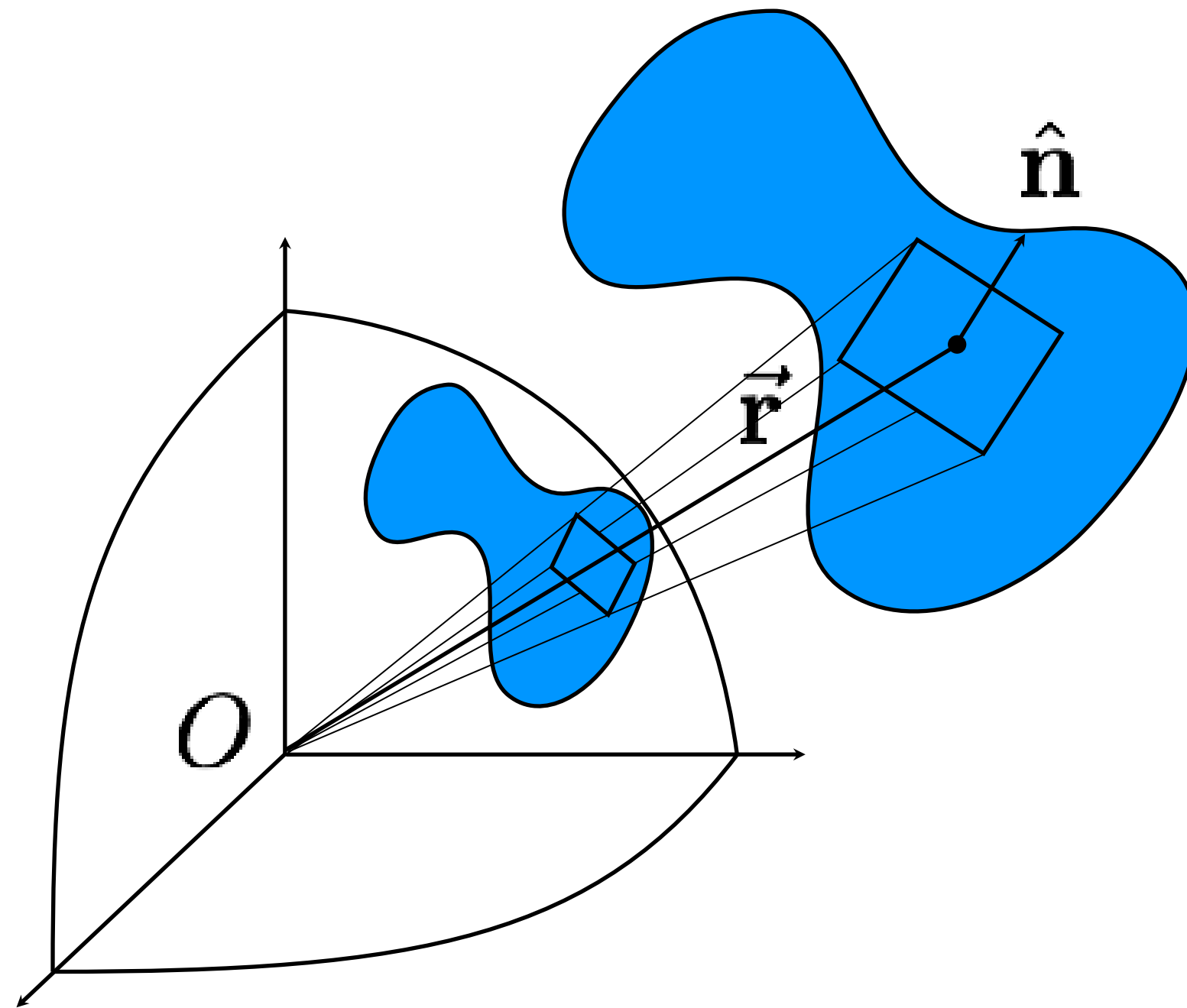
$$d\omega = \frac{dA \cos \theta}{r^2}$$

“surface foreshortening”

Units: steradians [sr]

Solid angle

To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{r} \cdot \hat{n} dS}{|\vec{r}|^3}$$

One can show:

$$d\omega = \frac{dA \cos \theta}{r^2} \quad \text{"surface foreshortening"}$$

Units: steradians [sr]

Radiant Intensity

directional density of flux

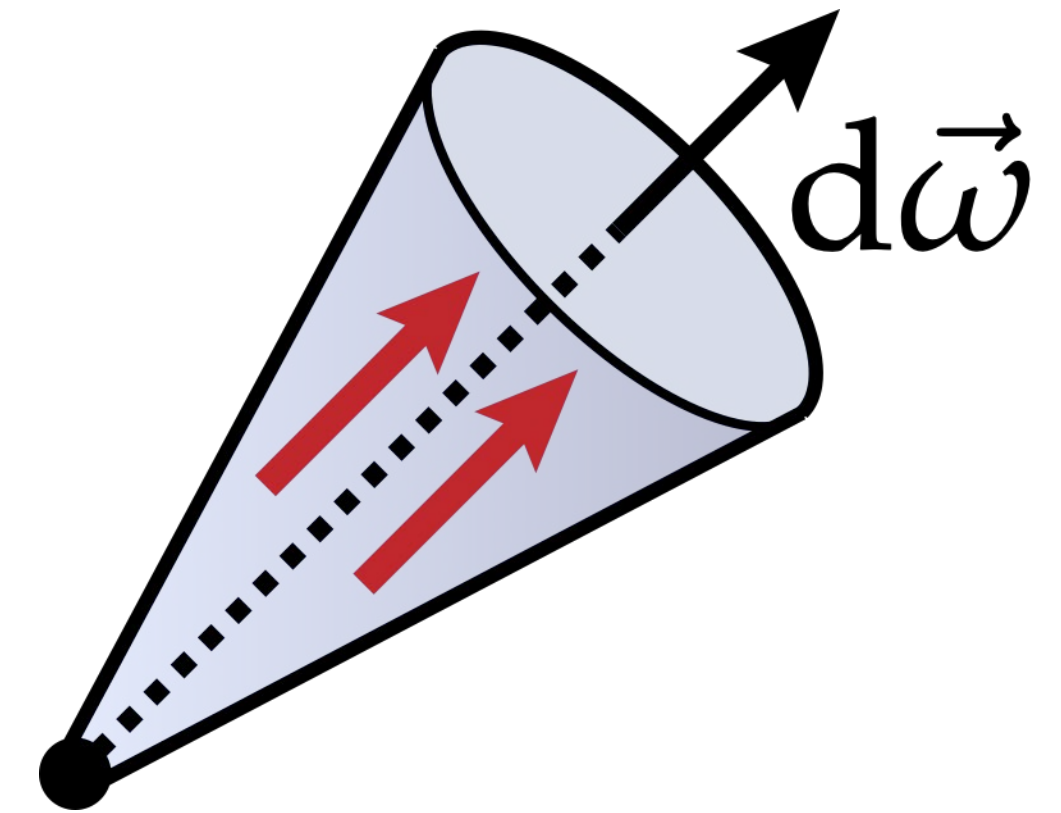
power (flux) per solid angle

$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \quad \left[\frac{\text{W}}{\text{sr}} \right]$$

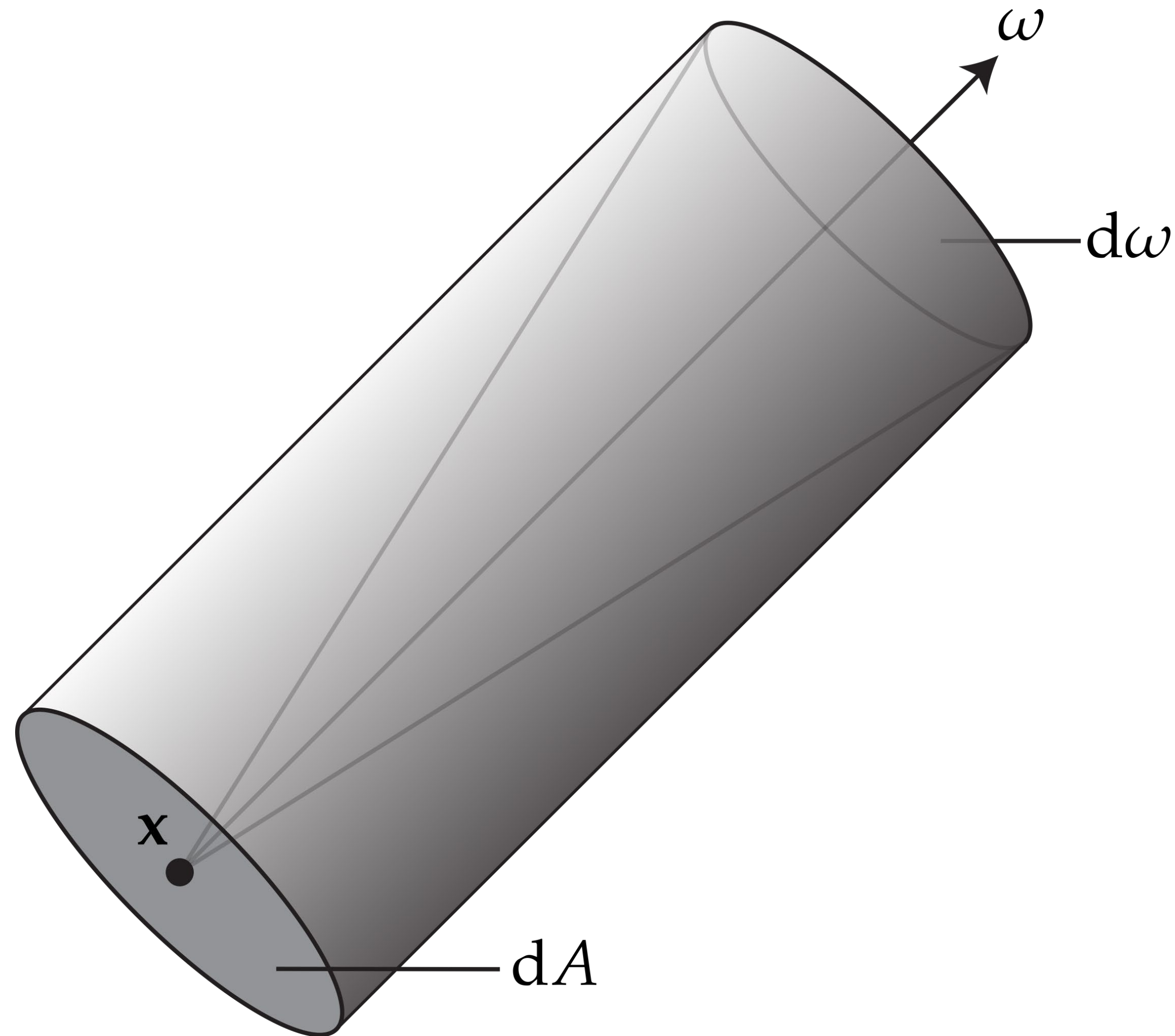
$$\Phi = \int_{S^2} I(\vec{\omega}) d\vec{\omega}$$

example: $\Phi = 4\pi I$ (for an isotropic point source)

- power per unit solid angle emanating from a point source



A hypothetical measurement device

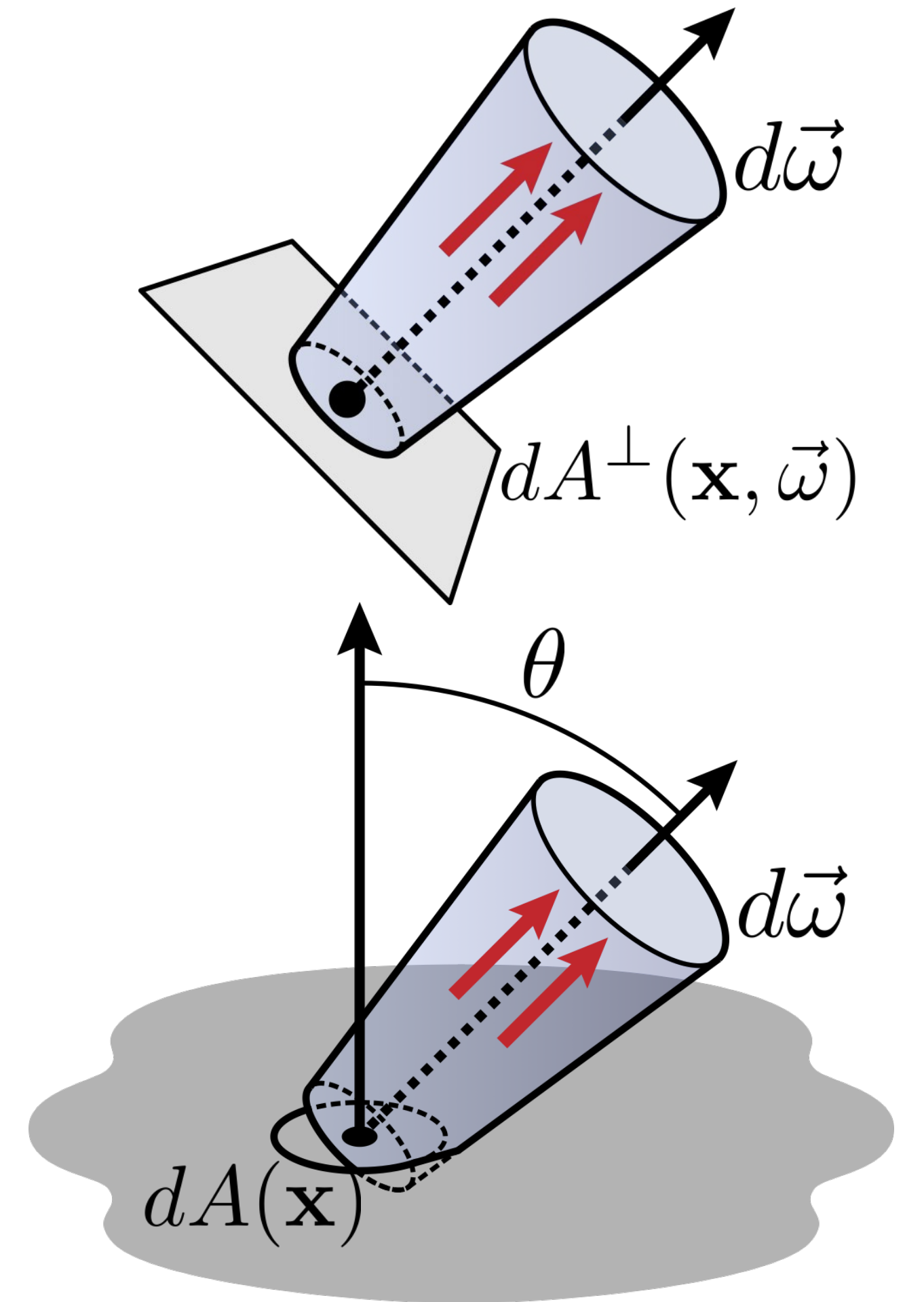


Radiance

flux density per unit solid angle, per *perpendicular* unit area

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2 \Phi(A)}{d\vec{\omega} dA^\perp(\mathbf{x}, \vec{\omega})} \left[\frac{W}{m^2 sr} \right]$$

$$= \frac{d^2 \Phi(A)}{d\vec{\omega} dA(\mathbf{x}) \cos \theta}$$

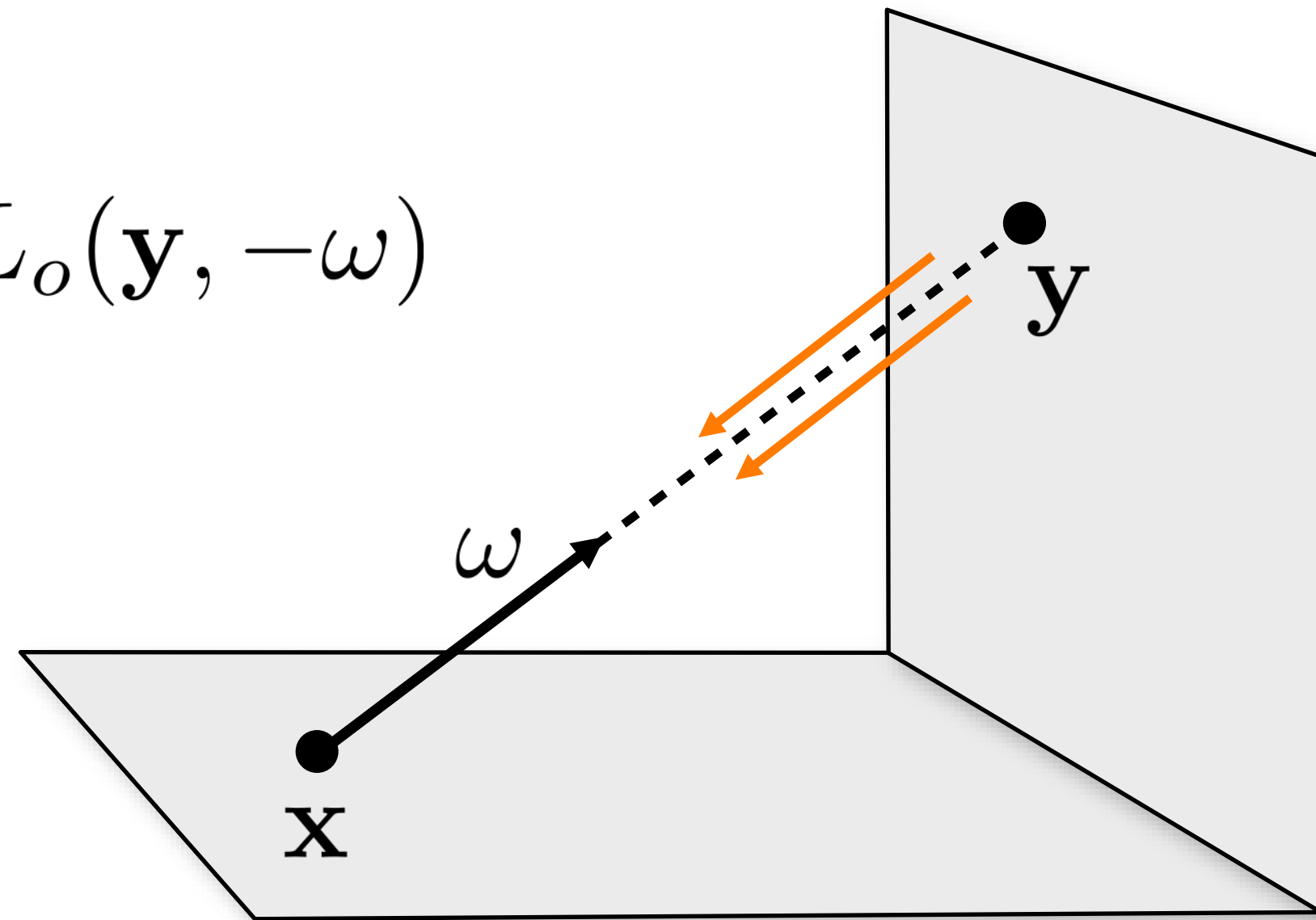


Radiance

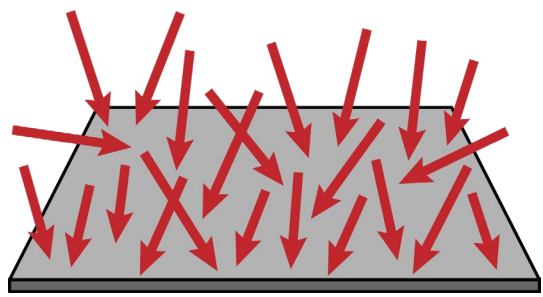
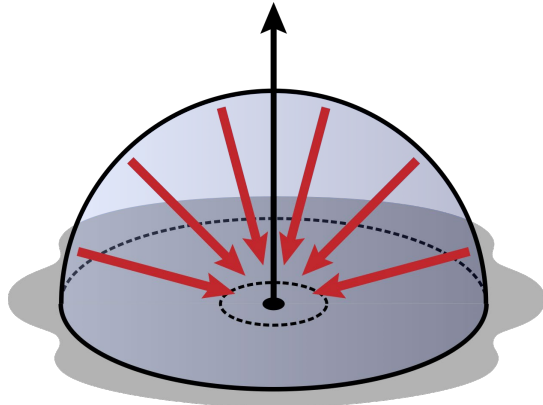
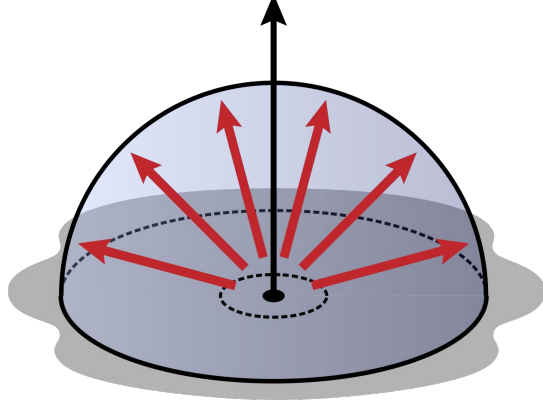
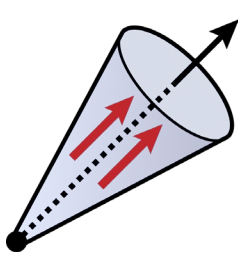
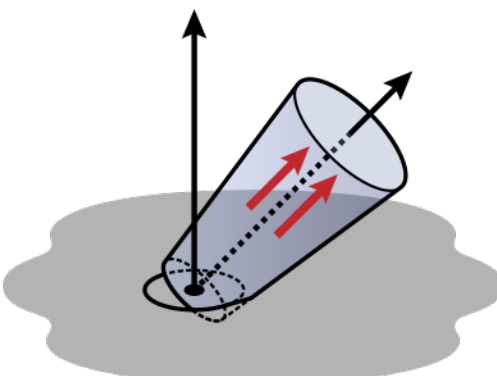
fundamental quantity for ray tracing and physics-based rendering
remains constant along a ray (*in vacuum only!*)

incident radiance L_i at one point can be
expressed as outgoing radiance L_o at another point

$$L_i(\mathbf{x}, \omega) = L_o(\mathbf{y}, -\omega)$$



Overview of Quantities

• flux:	$\Phi(A)$	$\left[\frac{J}{s} = W \right]$	
• irradiance:	$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• radiosity:	$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• intensity:	$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$	$\left[\frac{W}{sr} \right]$	
• radiance:	$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}}$	$\left[\frac{W}{m^2 sr} \right]$	

Radiance

expressing *irradiance* in terms of radiance:

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x})d\vec{\omega}} \quad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$L(\mathbf{x}, \vec{\omega}) = \frac{dE(\mathbf{x})}{\cos\theta d\vec{\omega}}$$

$$L(\mathbf{x}, \vec{\omega}) \cos\theta d\vec{\omega} = dE(\mathbf{x})$$

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta d\vec{\omega} = E(\mathbf{x})$$

Integrate cosine-weighted
radiance over hemisphere

Radiance

expressing *irradiance* in terms of radiance:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta \, d\vec{\omega} = E(\mathbf{x})$$

expressing *flux* in terms of radiance:

$$\int_A E(\mathbf{x}) \, dA(\mathbf{x}) = \Phi(A) \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$\int_A \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta \, d\vec{\omega} dA(\mathbf{x}) = \Phi(A)$$

Integrate cosine-weighted radiance
over hemisphere and area

Radiance

Allows computing the radiant flux measured by *any* sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA$$

Cameras measure integrals of radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to (integrals of) radiance.

- “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Computing spherical integrals

Express function using spherical coordinates:

$$\int_0^{2\pi} \int_0^\pi f(\theta, \phi) \, d\theta \, d\phi \quad ?$$

Warning: this is not correct!

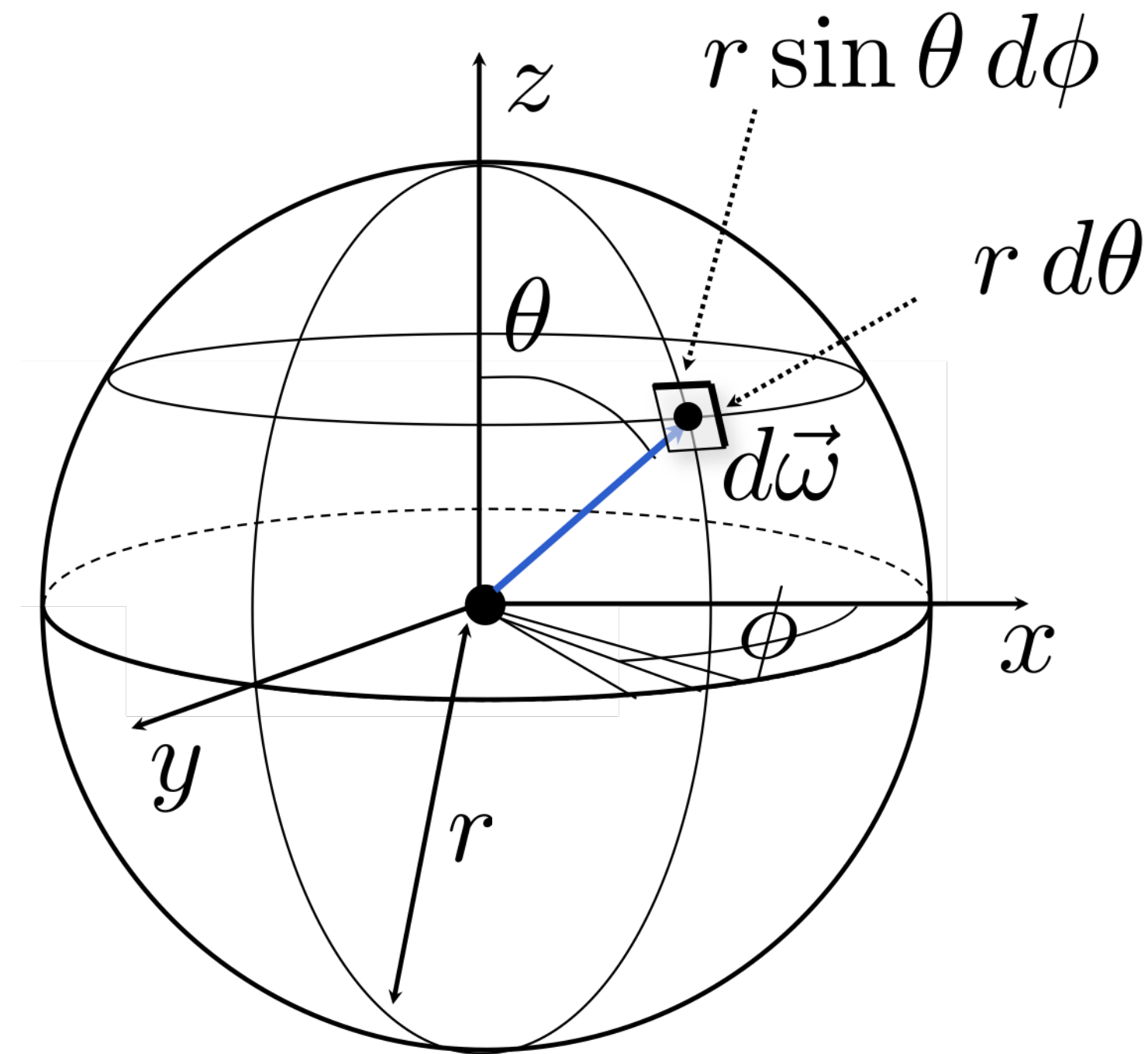
Differential Solid Angle

Differential area on the unit sphere around direction

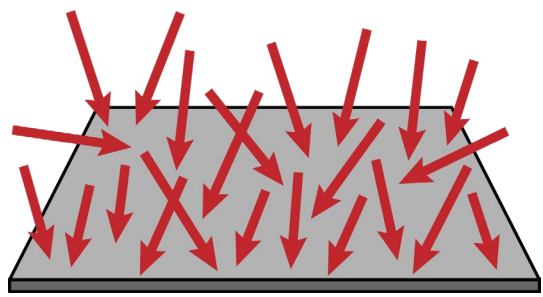
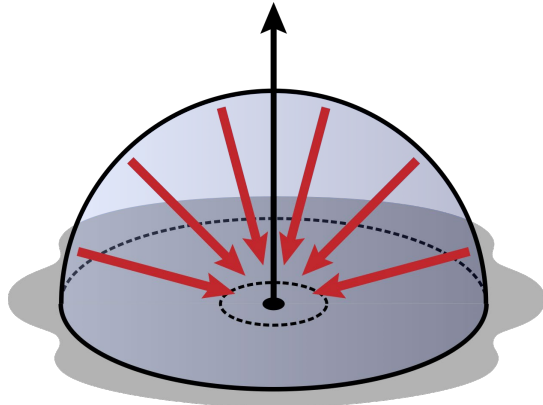
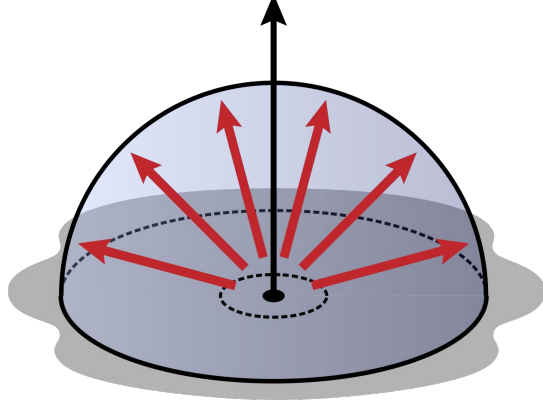
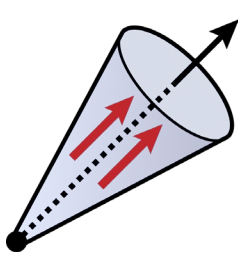
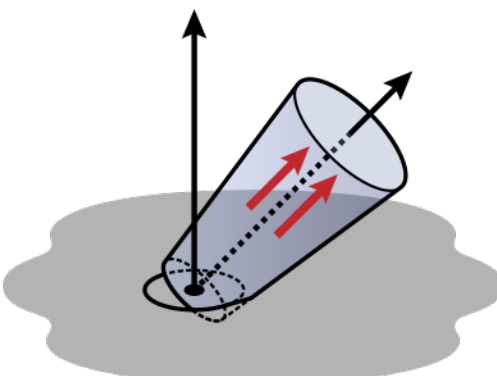
$$dA = (r d\theta)(r \sin \theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi = 4\pi$$



Overview of Quantities

• flux:	$\Phi(A)$	$\left[\frac{J}{s} = W \right]$	
• irradiance:	$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• radiosity:	$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• intensity:	$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$	$\left[\frac{W}{sr} \right]$	
• radiance:	$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}}$	$\left[\frac{W}{m^2 sr} \right]$	

All of these quantities can be a function of wavelength!

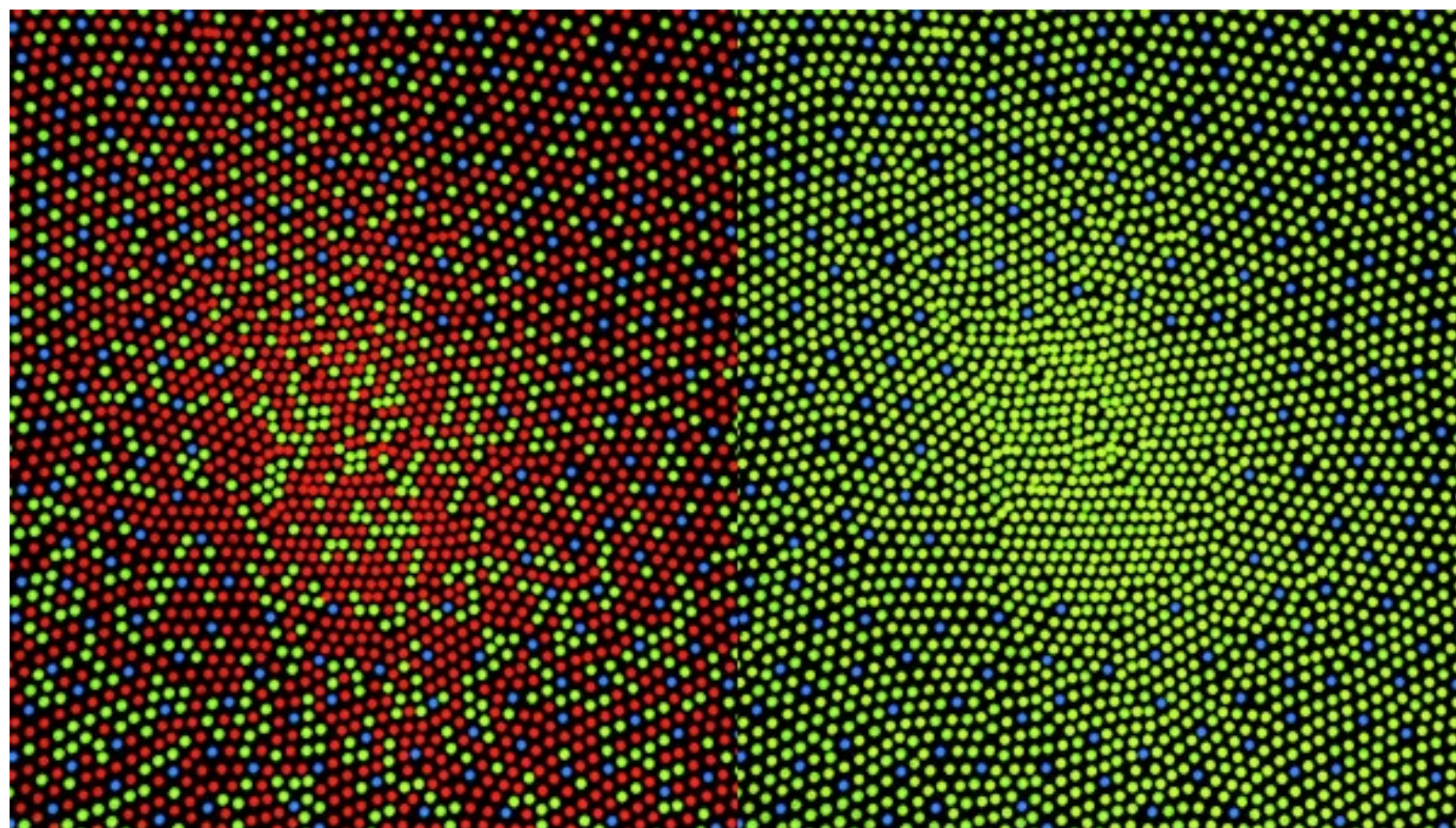
Handling color

- *Any* light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor's *spectral sensitivity function* (SSF).
- When measuring some incident *spectral* flux, the sensor produces a *scalar color* response:

$$\begin{array}{c} \text{sensor} \\ \text{response} \end{array} \longrightarrow R = \int_{\lambda} \begin{array}{c} \text{spectral flux} \\ \downarrow \\ \Phi(\lambda) \end{array} \begin{array}{c} \text{sensor SSF} \\ \downarrow \\ f(\lambda) \end{array} d\lambda$$

Handling color – the human eye

- The human eye is a collection of light sensors called cone cells.
- There are three types of cells with different spectral sensitivity functions.
- Human color perception is three-dimensional (*tristimulus color*).

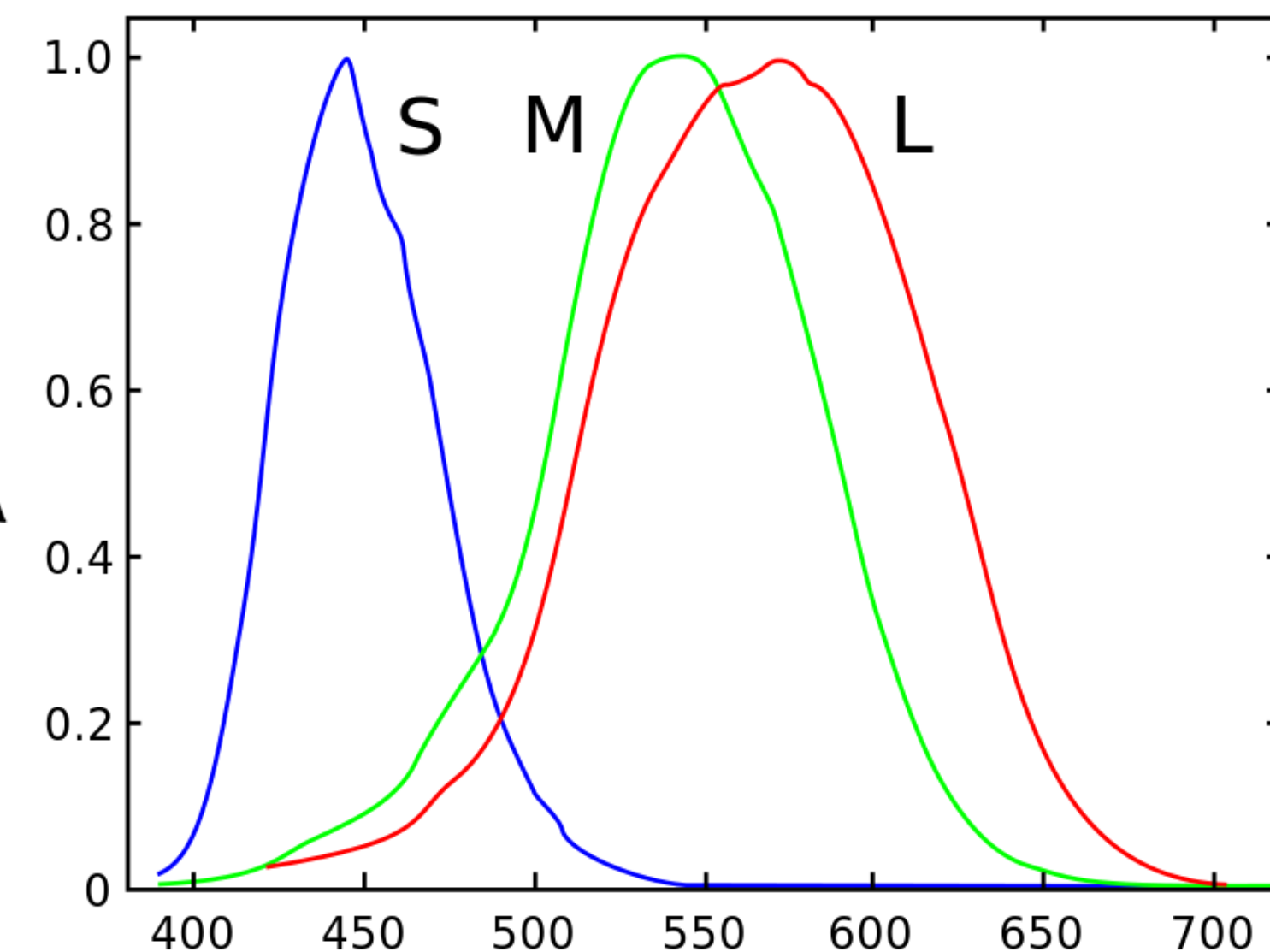


cone distribution
for normal vision
(64% L, 32% M)

“short” $S = \int_{\lambda} \Phi(\lambda) S(\lambda) d\lambda$

“medium” $M = \int_{\lambda} \Phi(\lambda) M(\lambda) d\lambda$

“long” $L = \int_{\lambda} \Phi(\lambda) L(\lambda) d\lambda$

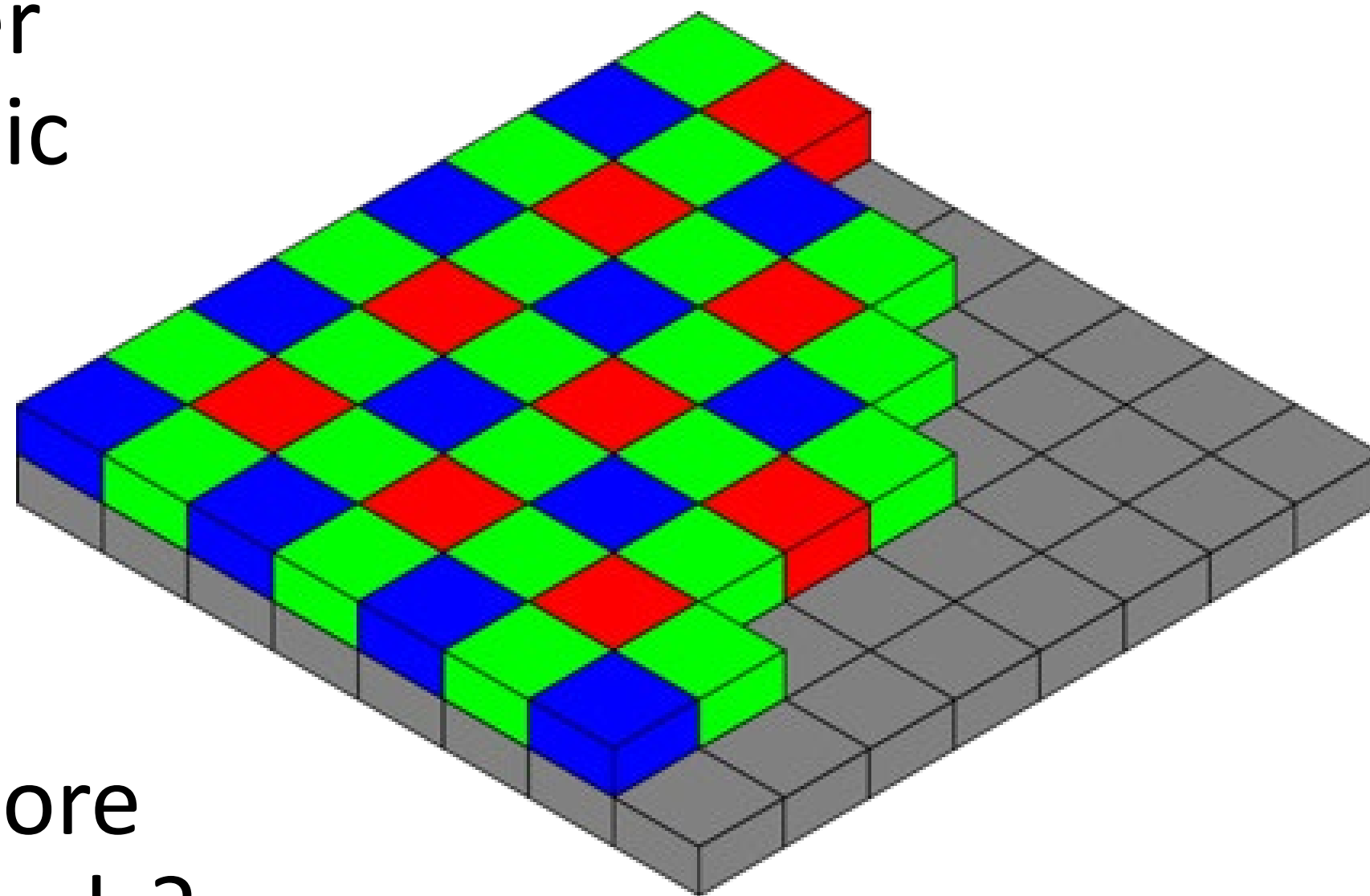


Handling color – photography

Two design choices:

- What spectral sensitivity functions $f(\lambda)$ to use for each color filter?
- How to spatially arrange (“mosaic”) different color filters

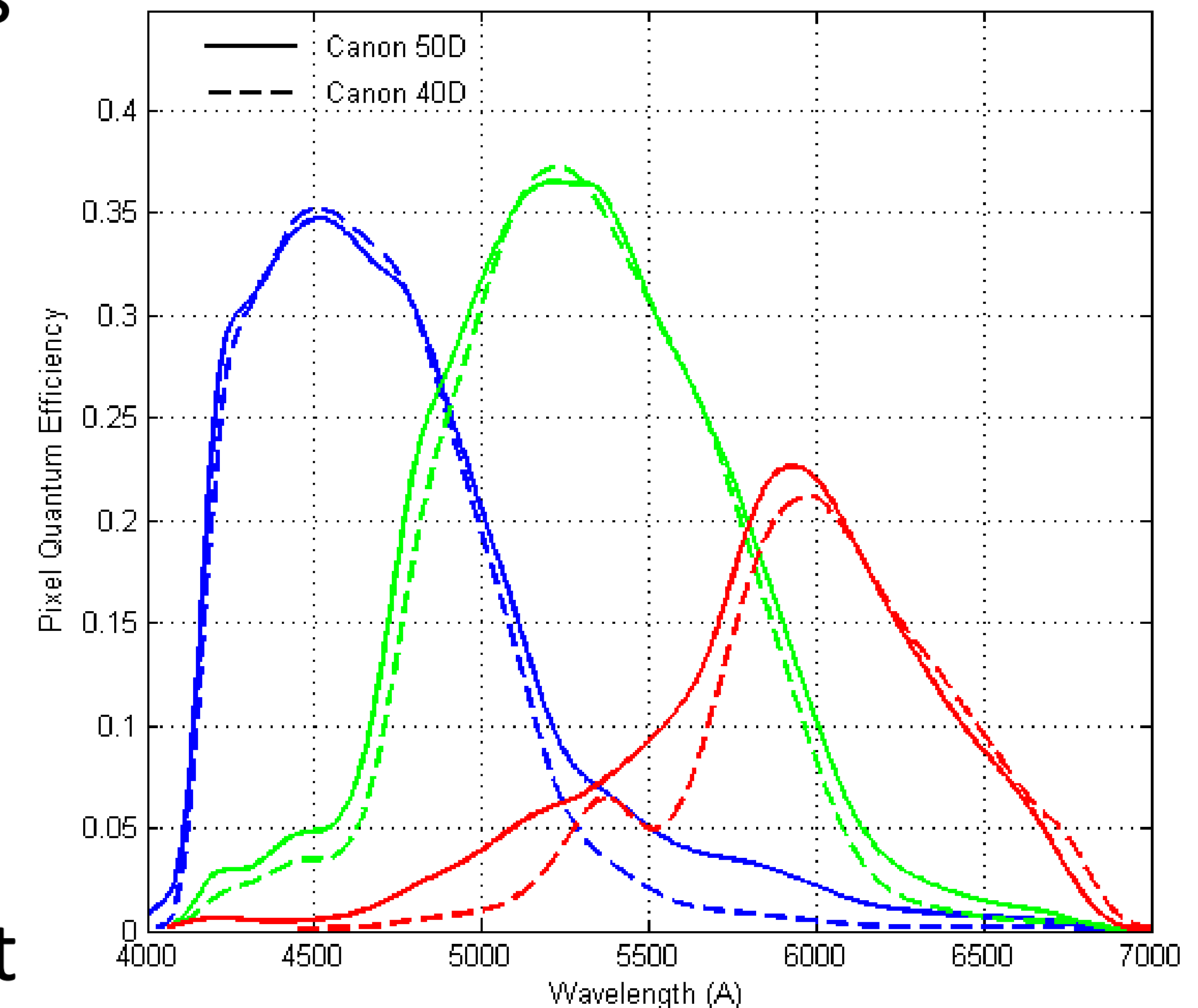
Bayer
mosaic



Why more
green pixels?

SSF for
Canon 50D

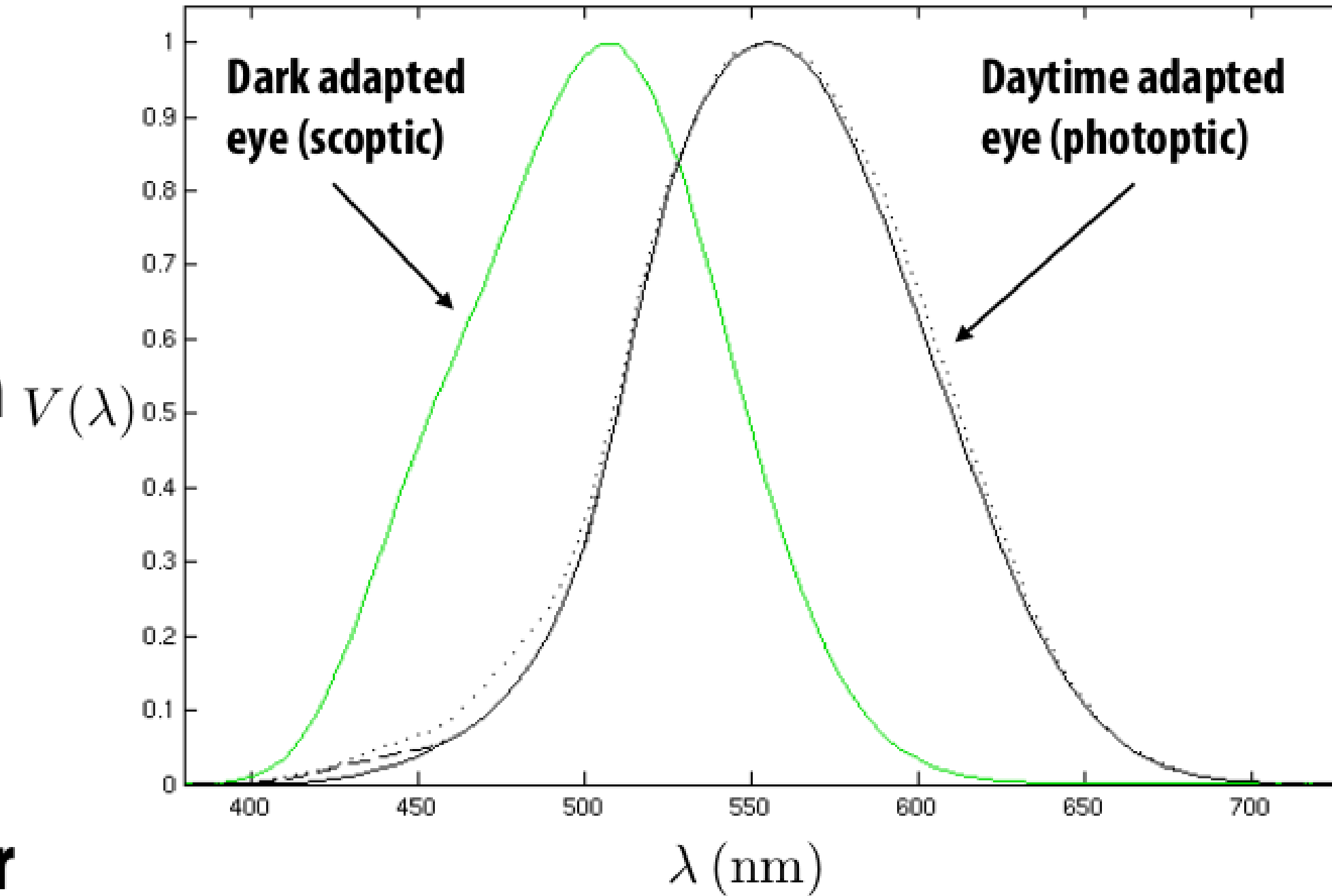
Generally do not
match human LMS.



$f(\lambda)$

Radiometry versus photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance (Y) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye's luminous efficacy curve, e.g.:



$$Y(p, \omega) = \int_0^\infty L(p, \omega, \lambda) V(\lambda) d\lambda$$

Radiometry versus photometry

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Radiometry versus photometry

Photometry	MKS	CGS	British
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

Modern LED light

Input power: 11 W

**Output: 815 lumens
(~ 80 lumens / Watt)**

**Incandescent bulbs:
~15 lumens / Watt)**



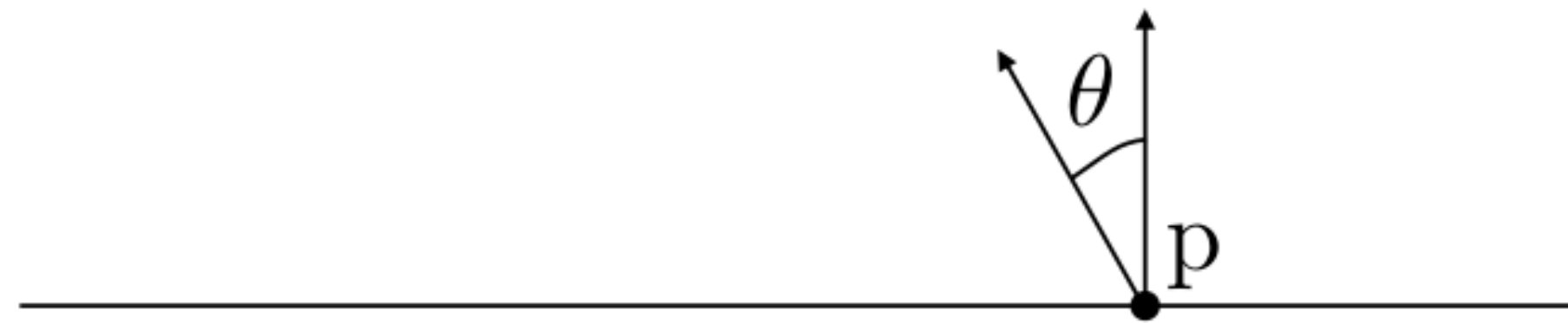
A simple derivation

Measurement of a sensor using a thin lens

Lens aperture



Sensor plane



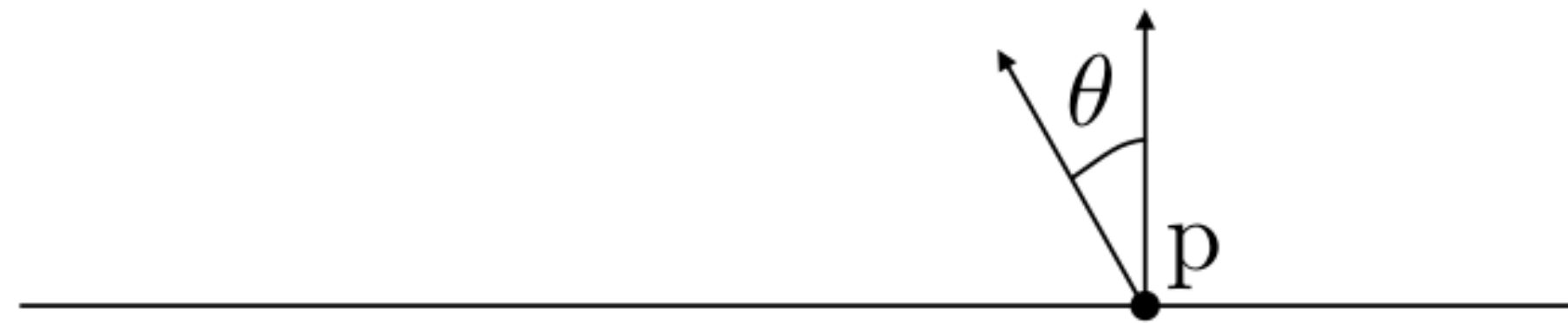
What integral should we write for the power measured by infinitesimal pixel p ?

Measurement of a sensor using a thin lens

Lens aperture



Sensor plane



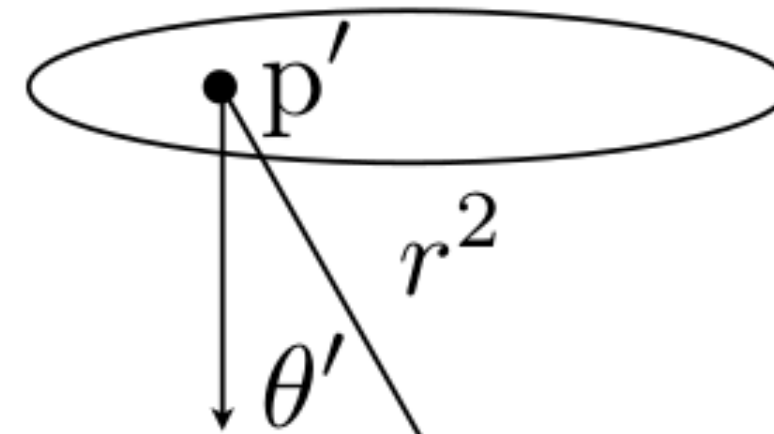
What integral should we write for the power measured by infinitesimal pixel p?

$$E(p, t) = \int_{H^2} L_i(p, \omega', t) \cos \theta \, d\omega'$$

Can I transform this integral over the hemisphere to an integral over the aperture area?

Measurement of a sensor using a thin lens

Lens aperture



Sensor plane



What integral should we write for the power measured by infinitesimal pixel p ?

$$E(p, t) = \int_{H^2} L_i(p, \omega', t) \cos \theta \, d\omega'$$

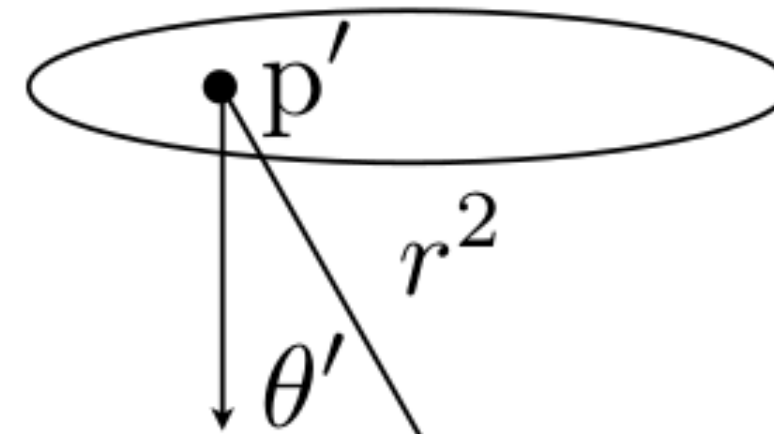
Can I transform this integral over the hemisphere to an integral over the aperture area?

$$E(p, t) = \int_A L(p' \rightarrow p, t) \frac{\cos \theta \cos \theta'}{\|p' - p\|^2} \, dA'$$

**Transform integral over solid
angle to integral over lens
aperture**

Measurement of a sensor using a thin lens

Lens aperture



Sensor plane

$$E(p, t) = \int_A L(p' \rightarrow p, t) \frac{\cos \theta \cos \theta'}{\|p' - p\|^2} dA'$$

Transform integral over solid angle to integral over lens aperture

$$= \int_A L(p' \rightarrow p, t) \frac{\cos^2 \theta}{\|p' - p\|^2} dA'$$

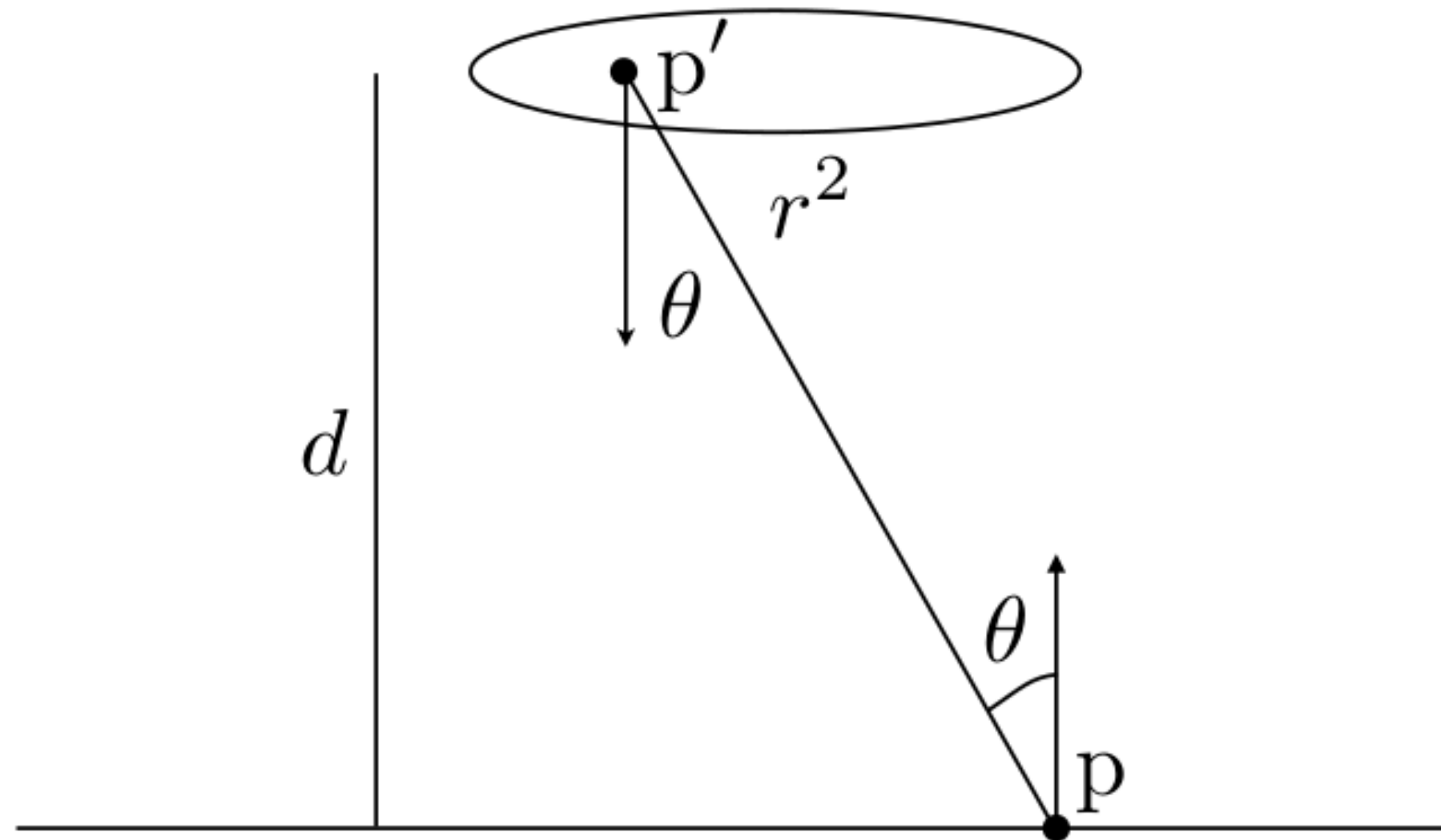
Assume aperture and film plane are parallel: $\theta = \theta'$

Can I write the denominator in a more convenient form?

Measurement of a sensor using a thin lens

Lens aperture

$$||p' - p|| = \frac{d}{\cos \theta}$$



Sensor plane

$$\begin{aligned} E(p, t) &= \int_A L(p' \rightarrow p, t) \frac{\cos^2 \theta}{||p' - p||^2} dA' \\ &= \frac{1}{d^2} \int_A L(p' \rightarrow p, t) \cos^4 \theta dA' \end{aligned}$$

What does this say about the image I am capturing?

Vignetting

Fancy word for: pixels far off the center receive less light



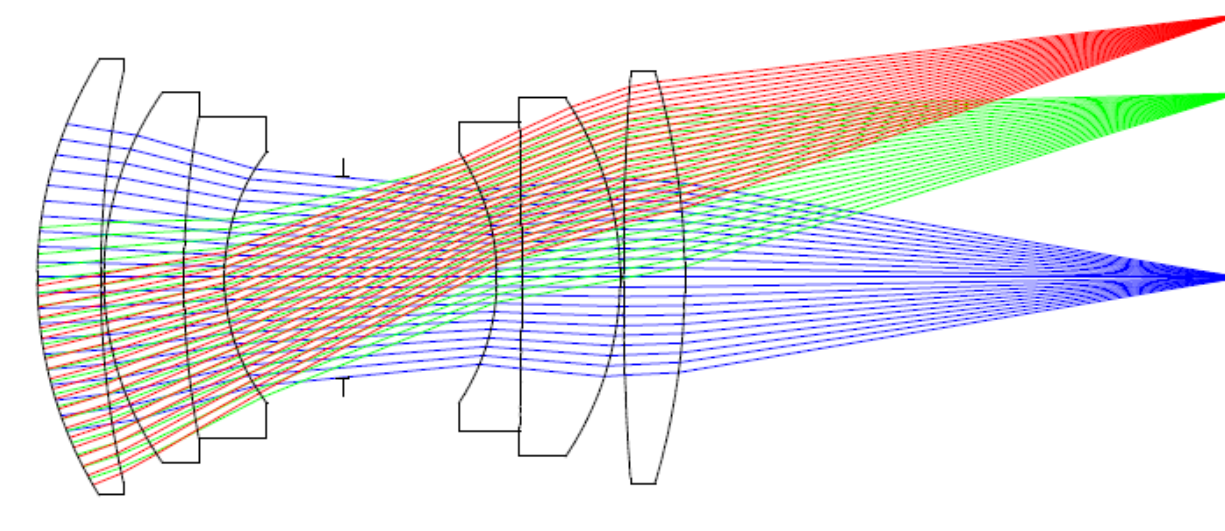
white wall under uniform light



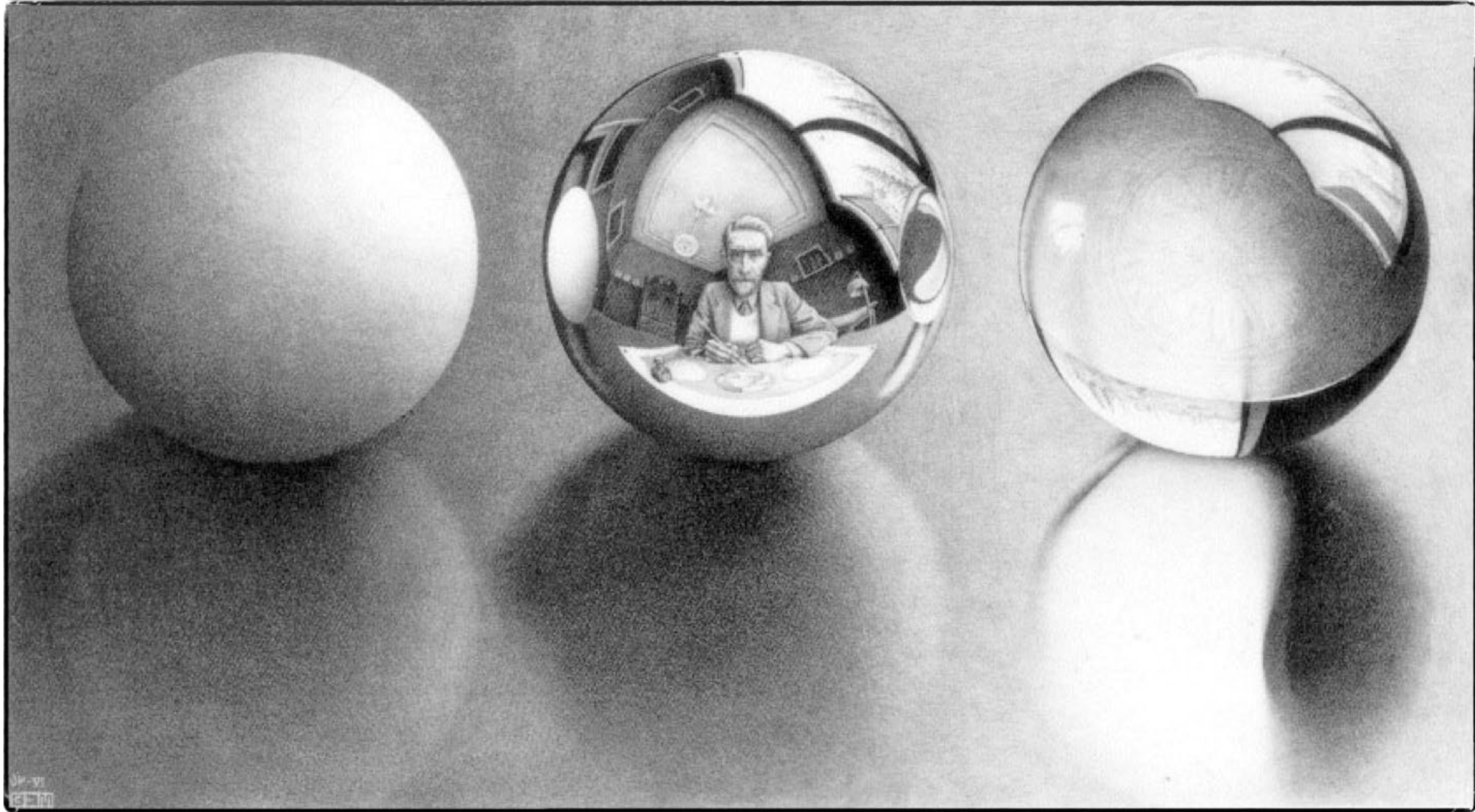
more interesting example of vignetting

Four types of vignetting:

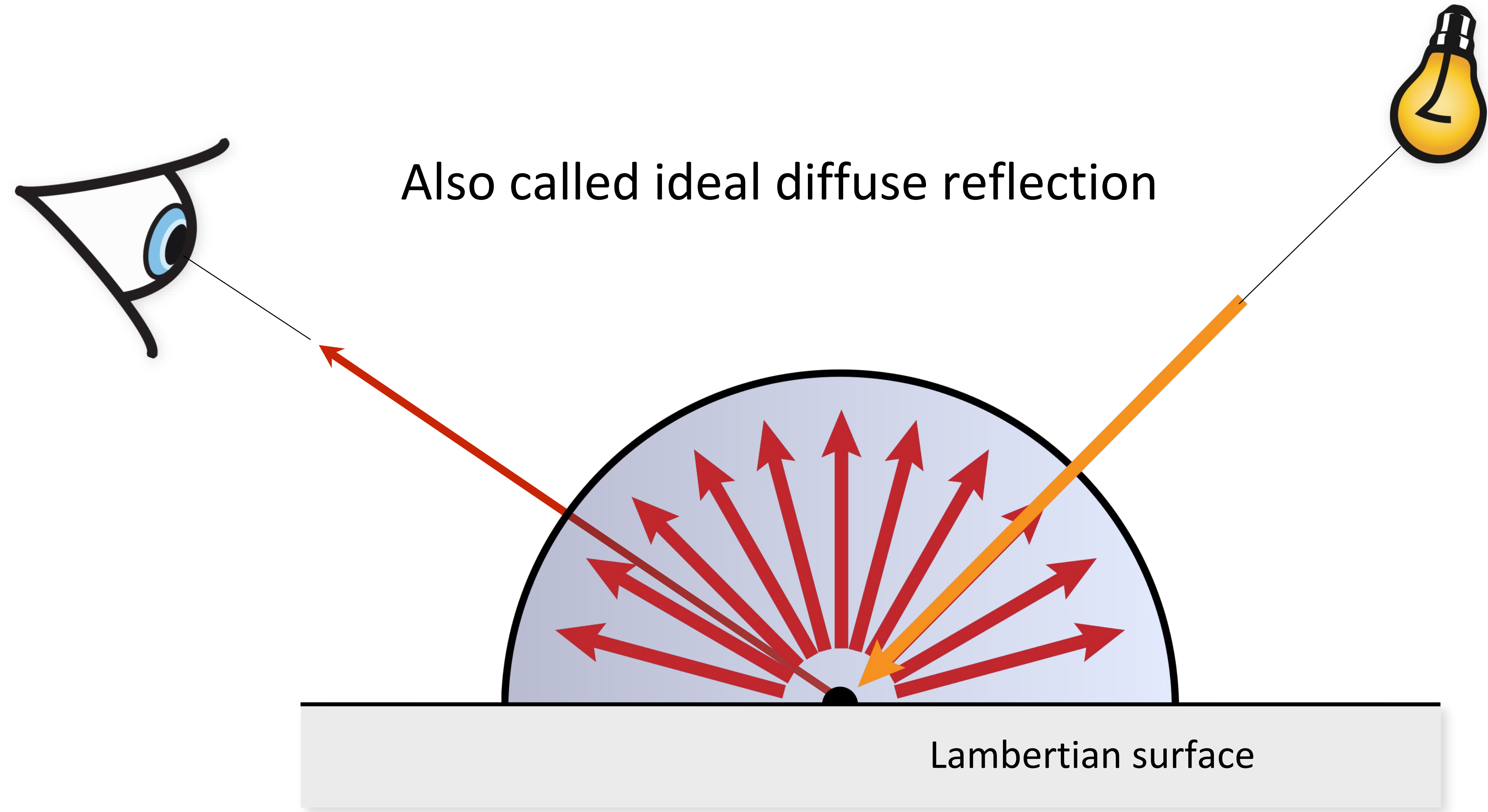
- Mechanical: light rays blocked by hoods, filters, and other objects.
- Lens: similar, but light rays blocked by lens elements.
- Natural: due to radiometric laws (“cosine fourth falloff”).
- Pixel: angle-dependent sensitivity of photodiodes.



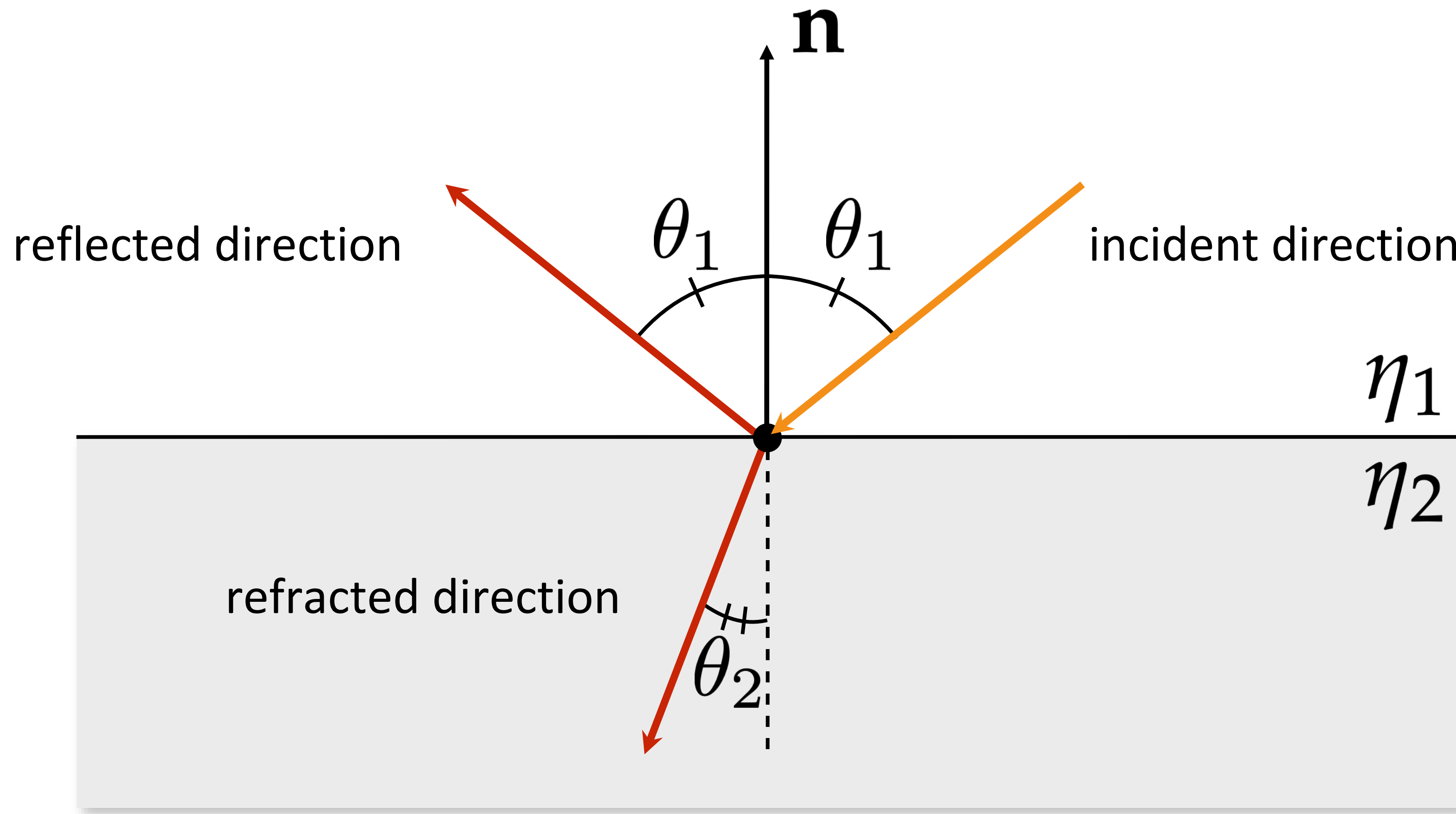
Reflection equation



Lambertian reflection

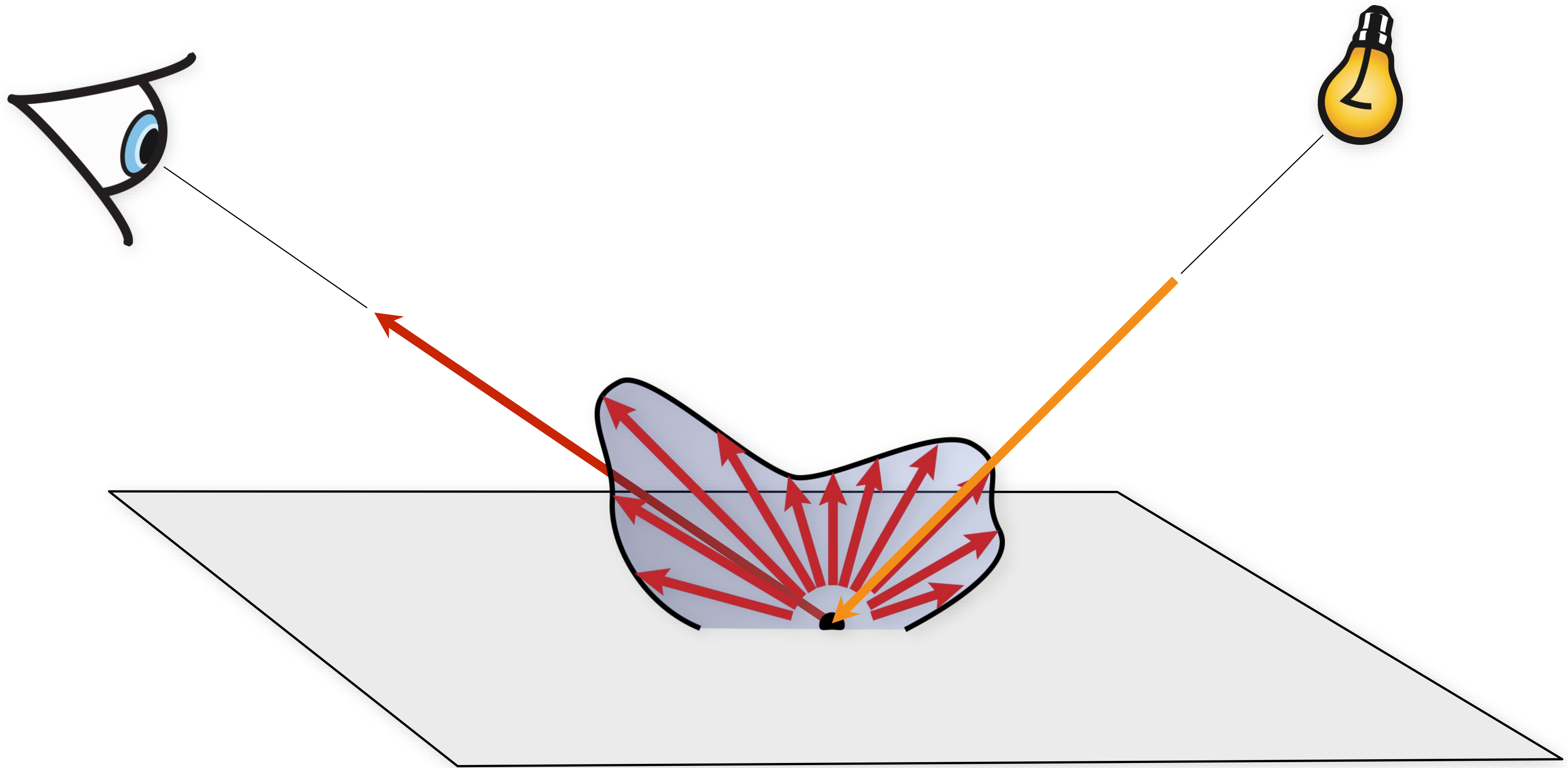


Ideal specular reflection/refraction



$$\eta_1 \sin \theta_1 = \eta_2 \sin \theta_2$$

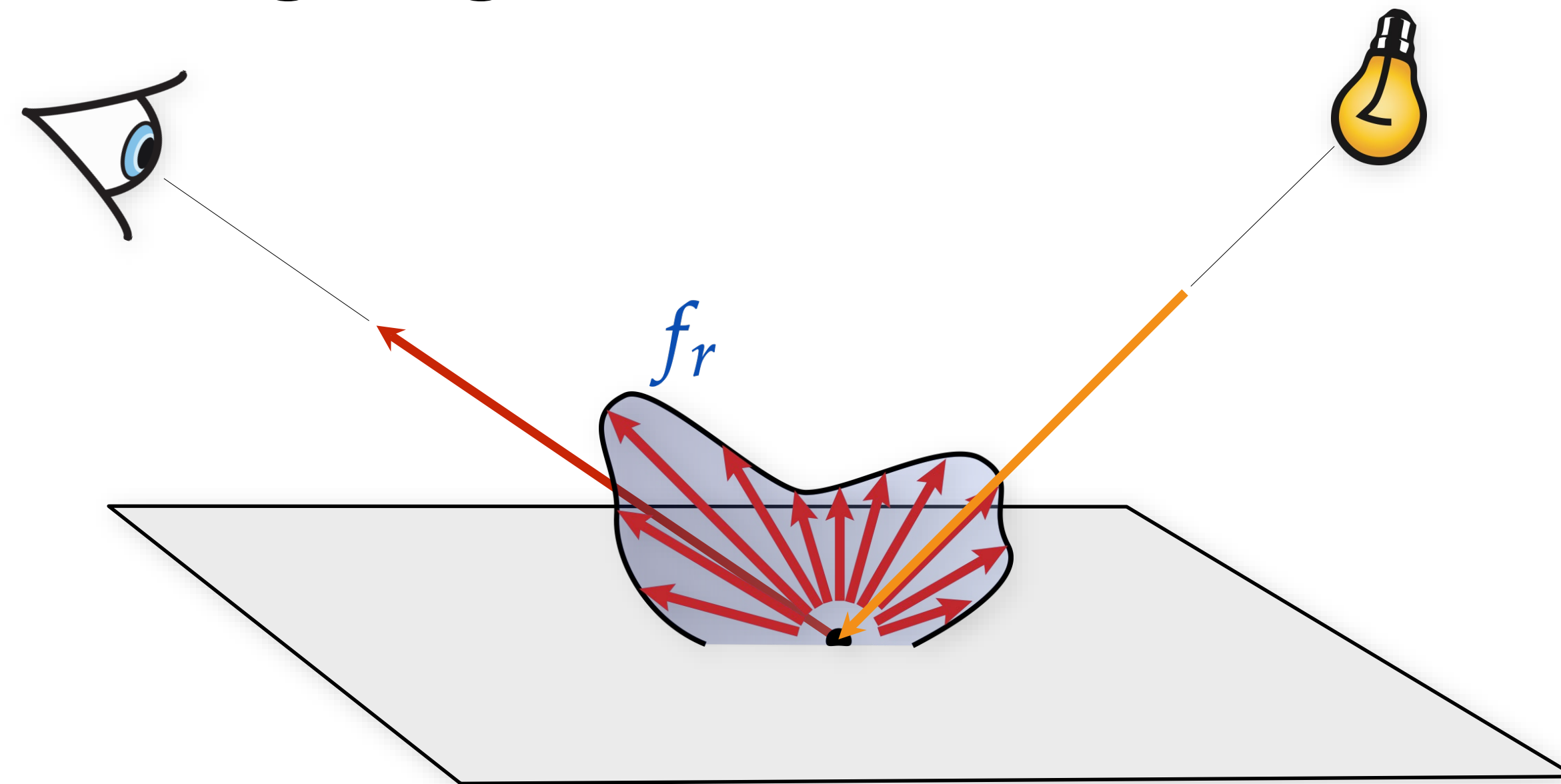
Light-Material Interactions



The BRDF

Bidirectional Reflectance Distribution Function

- how much light gets scattered from **one direction** into **each other direction**
- formally: ratio of outgoing *radiance* to incident *irradiance*

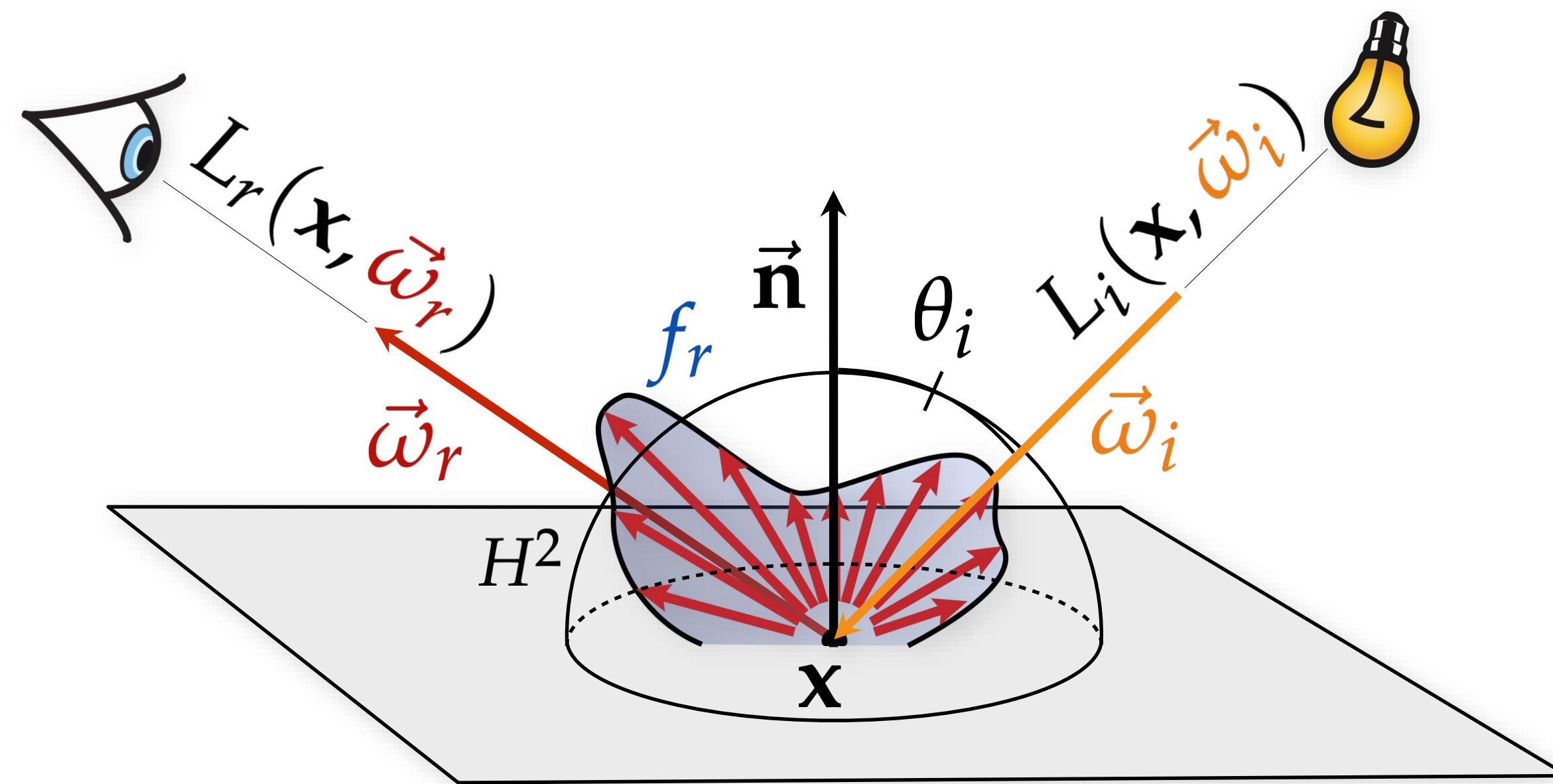


The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Where does the cosine come from?



This describes a local illumination model

Motivation



Motivation



Derivation of the Reflectance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) = \frac{E^{surface}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r)}{}$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r, \phi_r) = \frac{L^{src}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i}{}$$

Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r, \phi_r) = \int_{2\pi} L^{src}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_0^{\pi/2} L^{src}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \underline{\sin \theta_i d\theta_i d\phi_i}$$

BRDF Properties

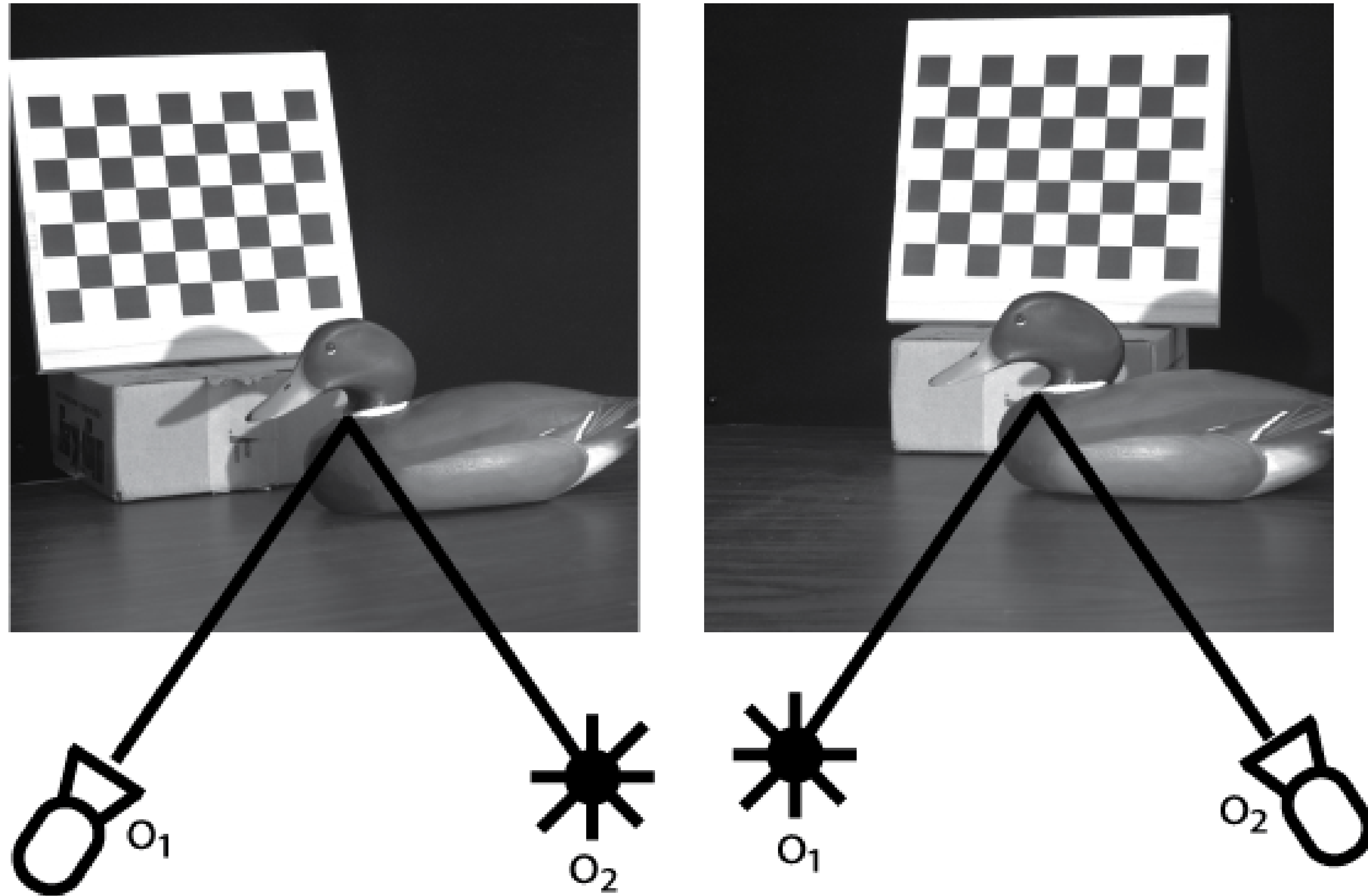
Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_r d\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i$$

Where does the
cosine come from?

Helmholtz Reciprocity



BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_r d\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i$$

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$
$$f_r(\mathbf{x}, \vec{\omega}_i \leftrightarrow \vec{\omega}_r)$$

BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_r \, d\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i$$

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$

- Together:

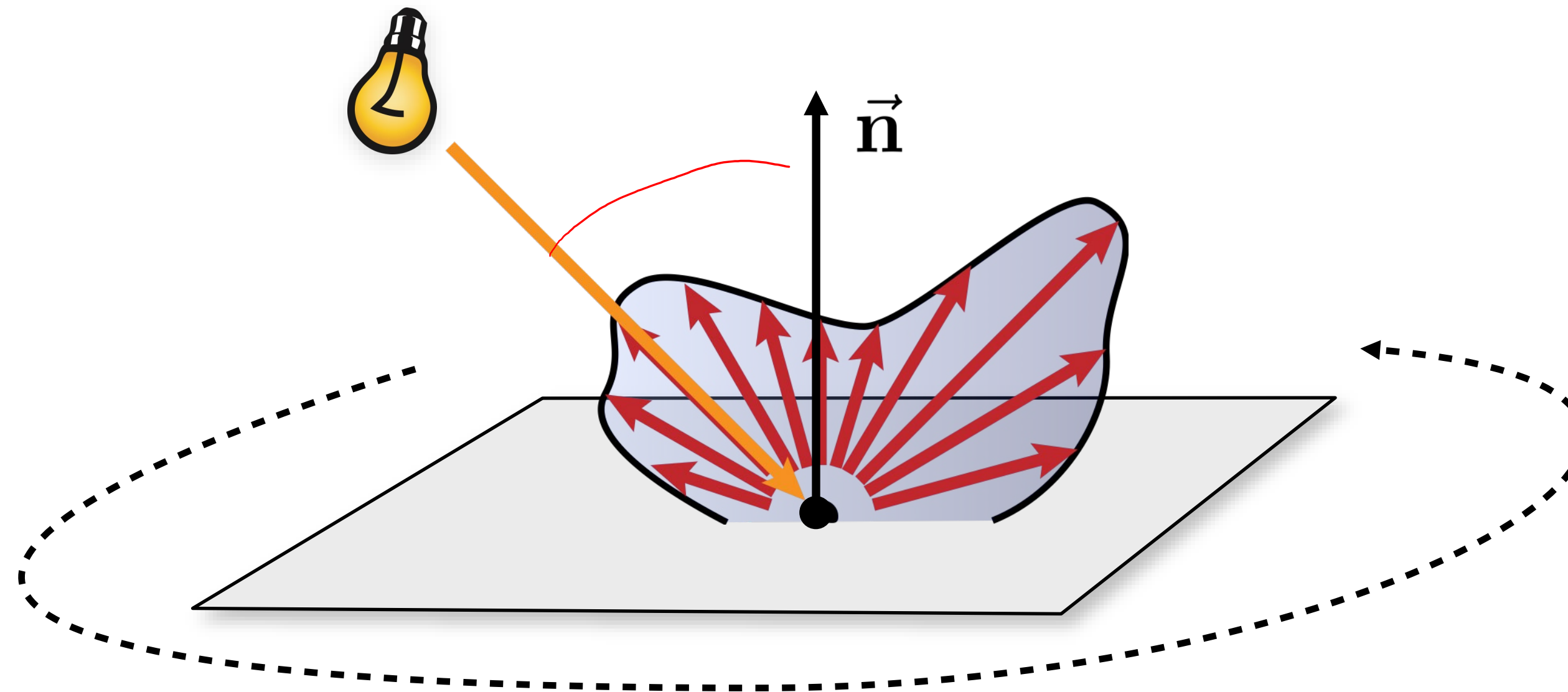
$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_i \, d\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r$$

BRDFs Properties

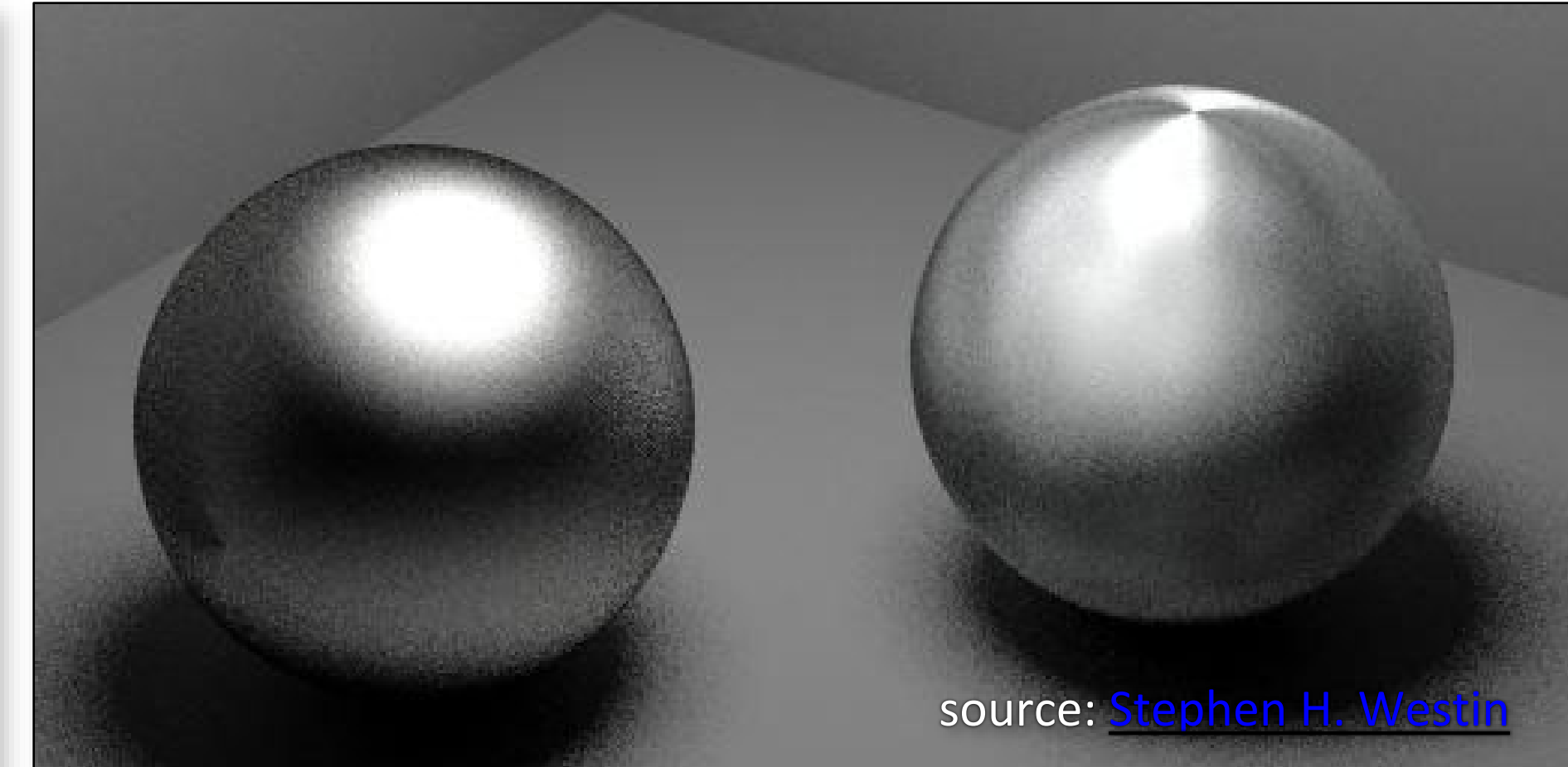
If the BRDF is unchanged as the material is rotated around the normal, then it is ***isotropic***, otherwise it is ***anisotropic***.

Isotropic BRDFs are functions of just 3 variables

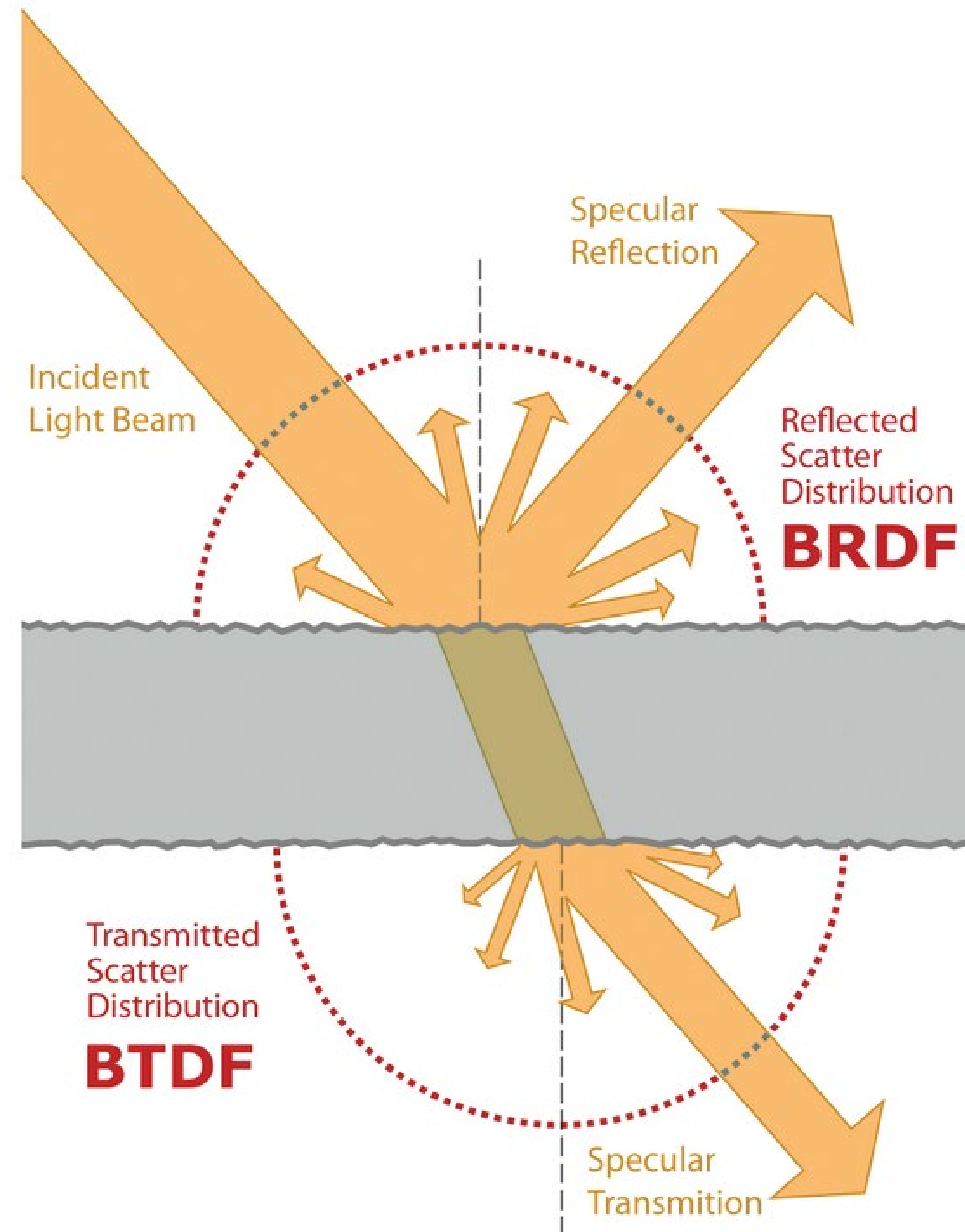
$$(\theta_i, \theta_r, \Delta\phi)$$



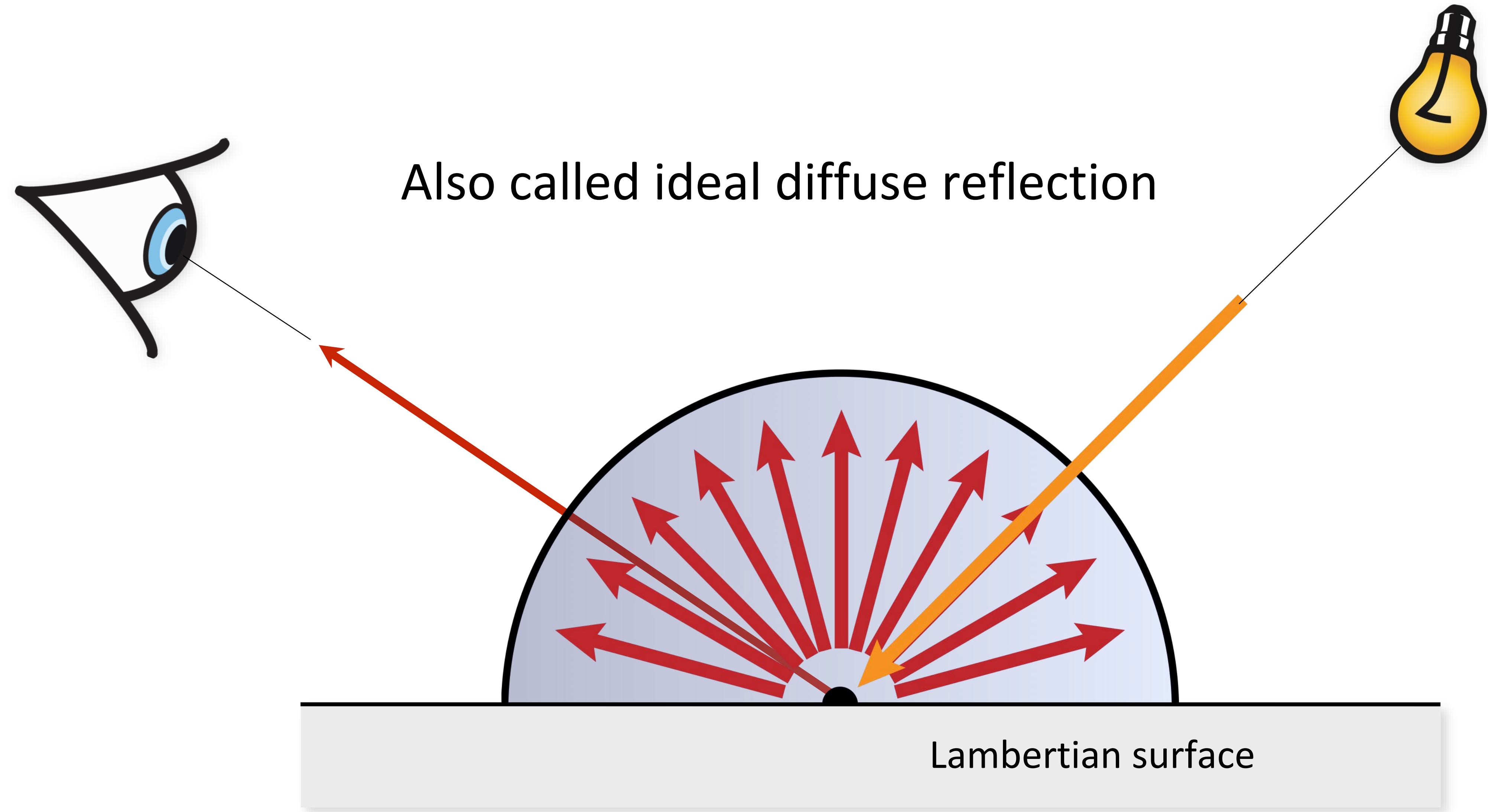
Isotropic vs Anisotropic Reflection



Reflection vs. Refraction



Lambertian reflection



BRDF for Lambertian reflection?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Scatters light equally in all directions
BRDF is a constant

Lambertian BRDF

For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Note: we can
drop ω_r

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r E(\mathbf{x})$$

If *all* incoming light is reflected:

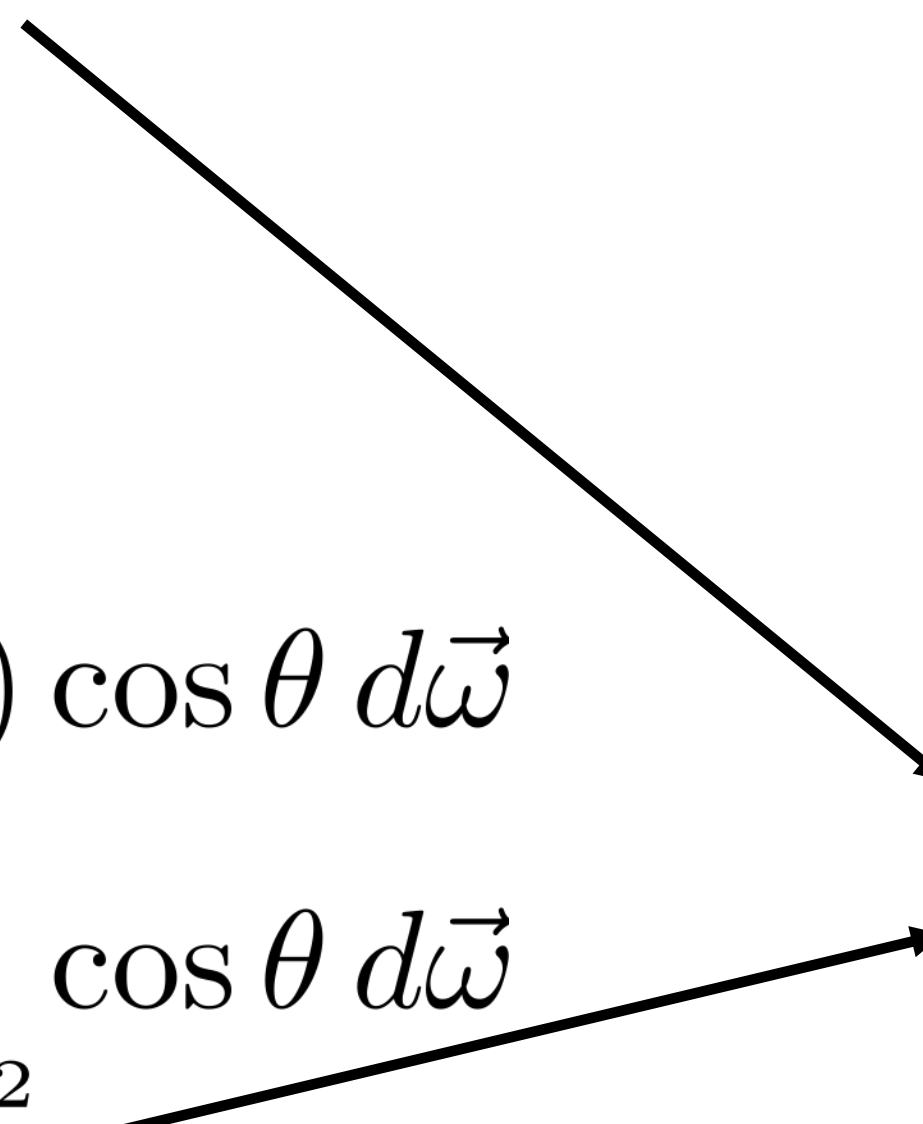
$$E(\mathbf{x}) = B(\mathbf{x})$$

$$E(\mathbf{x}) = \int_{H^2} L_r(\mathbf{x}) \cos \theta d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \int_{H^2} \cos \theta d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \pi$$

Note: can also be
derived from energy
conservation

$$f_r = \frac{1}{\pi}$$


Lambertian BRDF

For Lambertian reflection, the BRDF is a constant:

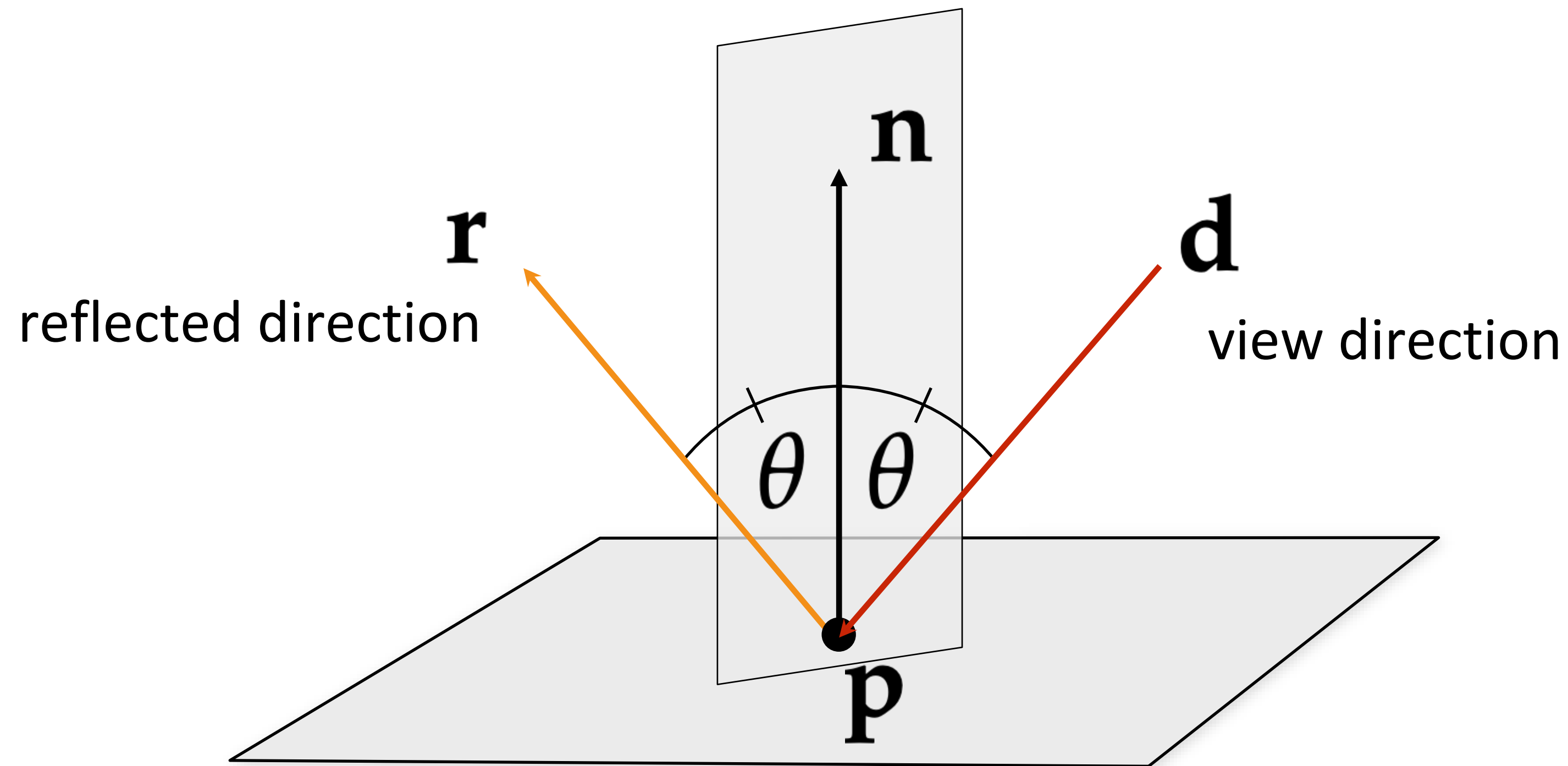
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

ρ : Diffuse reflectance (albedo) [0...1]

Specular BRDF

Assume $\mathbf{n}$ is unit length



$$\mathbf{r} = -2\mathbf{n}(\mathbf{n} \cdot \mathbf{d}) + \mathbf{d}$$

Specular BRDF?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

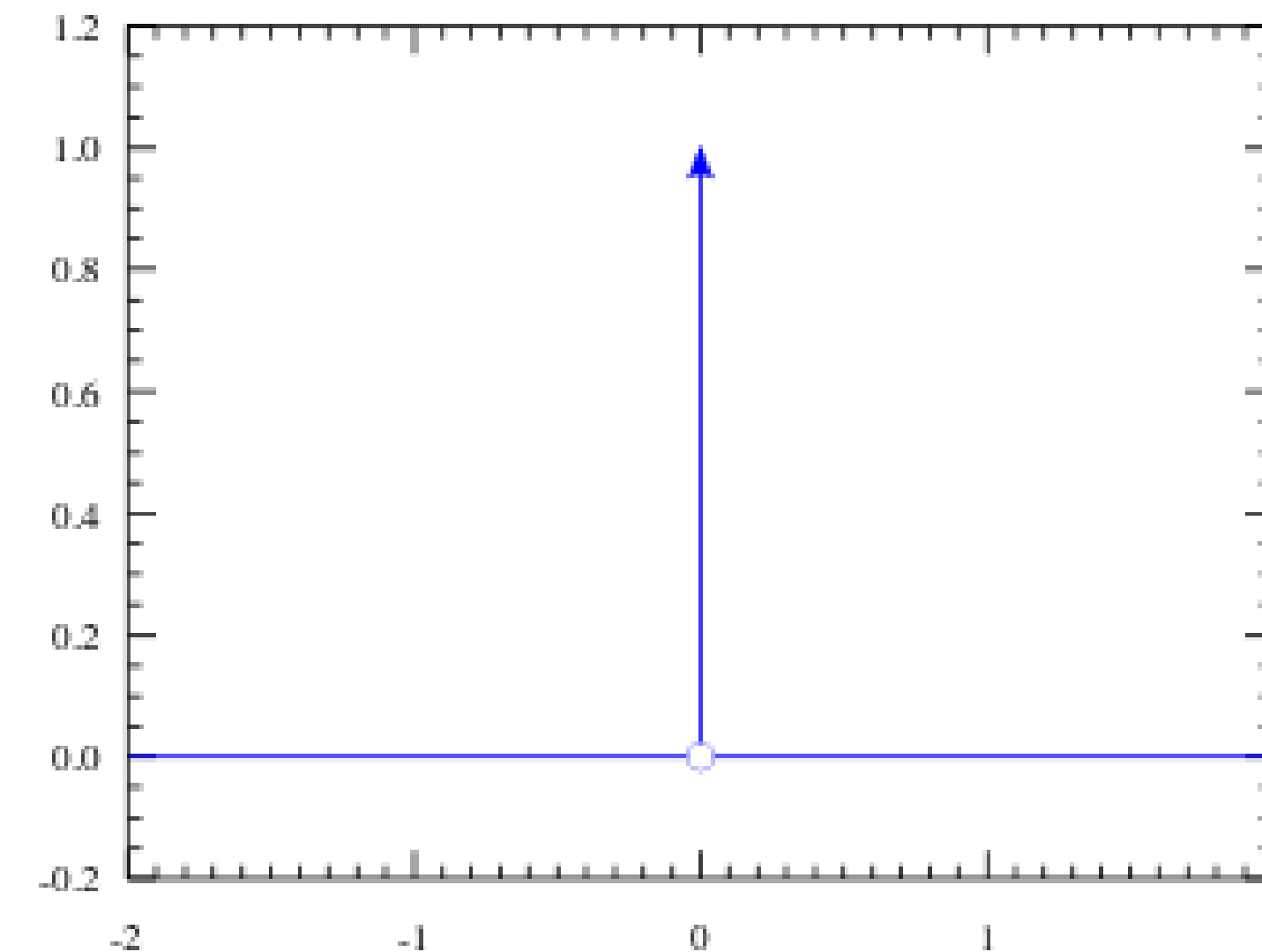
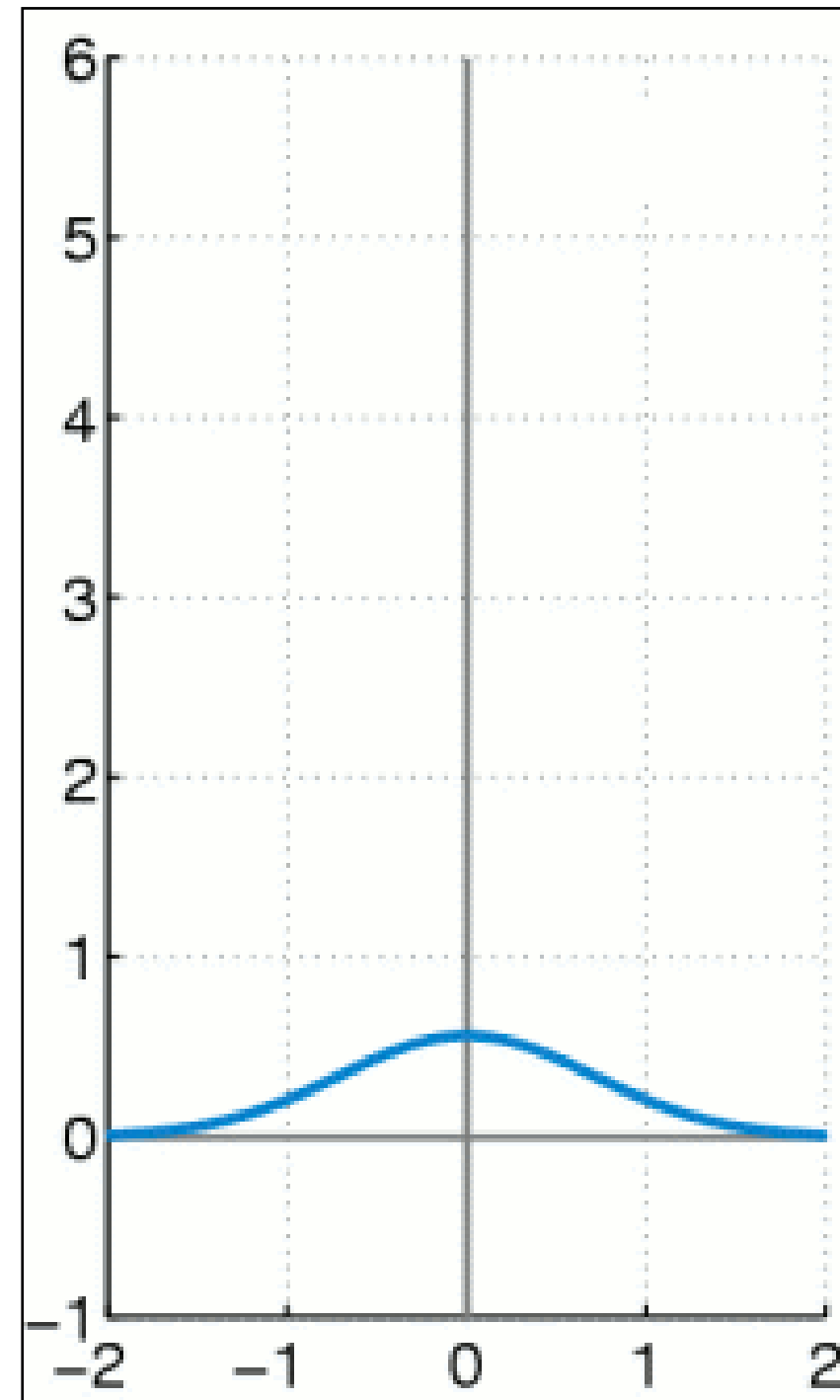
Scatters all light into one (or two) directions

Contains a Dirac delta

Integral drops out

What is the BRDF for specular reflection/refraction?

Dirac delta functions



$$\int_{-\infty}^{\infty} f(x) \delta(x - a) \, dx = f(a)$$

Note: careful when performing changes of variables in Dirac delta functions!

BRDF of Ideal Specular Reflection

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i$$

What is the BRDF for specular reflection?

Diagram illustrating the BRDF for ideal specular reflection:

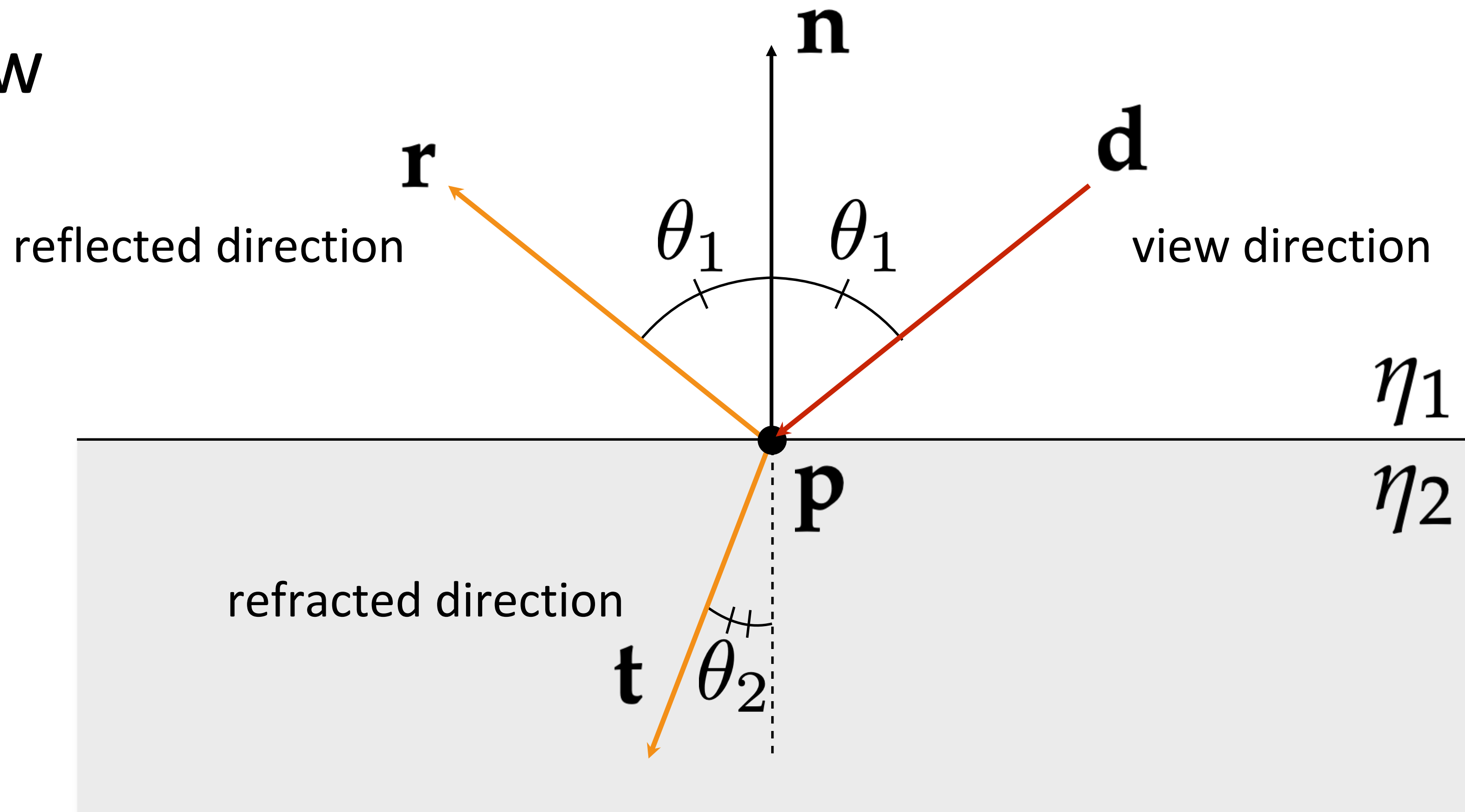
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos \theta_i}$$

Annotations:

- Fresnel reflection (points to $F_r(\vec{\omega}_i)$)
- Dirac delta (points to $\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{\mathbf{n}}))$)
- Reflection function (flips about normal) (points to $R(\vec{\omega}_r, \vec{\mathbf{n}})$)
- to cancel the cosine term in the reflection equation (Fresnel eqs. account for it) (points to $\cos \theta_i$)

Specular transmission/refraction

Snell's law



$$\mathbf{t} = \eta_1/\eta_2 (\mathbf{d} - (\mathbf{d} \cdot \mathbf{n}) \mathbf{n}) - \mathbf{n} \sqrt{1 - \eta_1^2/\eta_2^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}$$

BTDF of Ideal Specular Refraction

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i$$

What is the BTDF for specular refraction?

The diagram shows the BTDF equation for specular refraction with several annotations:

- Fresnel reflection**: An arrow points to the term $F_r(\vec{\omega}_i)$ in the equation.
- Dirac delta**: An arrow points to the $\delta(\vec{\omega}_i - T(\vec{\omega}_r, \vec{\mathbf{n}}))$ term.
- Refraction function**: An arrow points to the $T(\vec{\omega}_r, \vec{\mathbf{n}})$ term.
- to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)**: An arrow points to the $\cos \theta_i$ term in the denominator.

$$f_t(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \boxed{\frac{\eta_1^2}{\eta_2^2}} (1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_i - T(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos \theta_i}$$

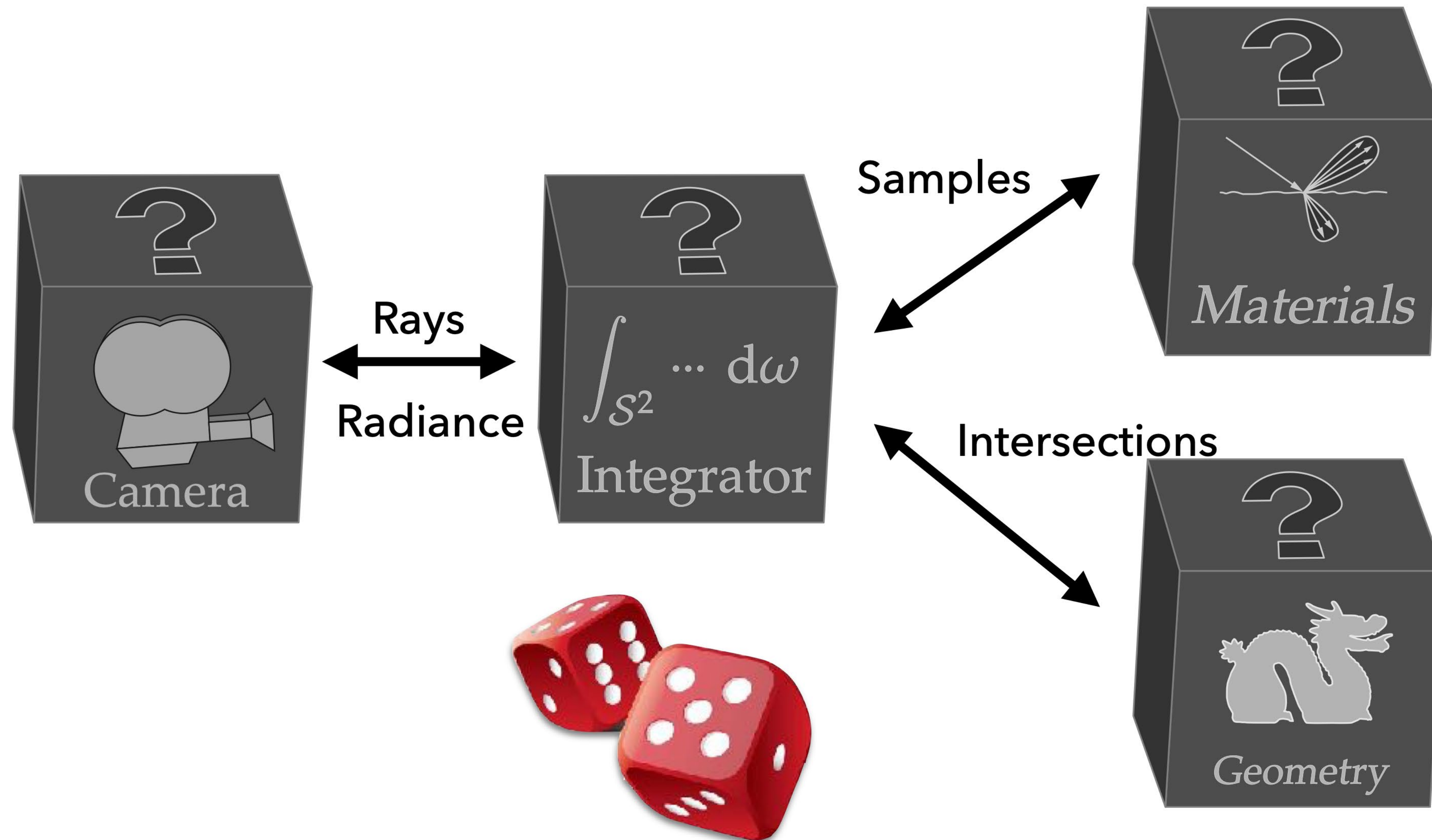
Approximating integrals with Monte Carlo

No need to be scared of math like this:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta \, d\vec{\omega} = E(\mathbf{x})$$

- integrals will just turn into **for** loops in your code
- evaluating $L(\mathbf{x}, \omega)$ will correspond to tracing a ray

Architecture of a rendering system



Architecture of a rendering system

