Boundary Value Caching for Walk on Spheres

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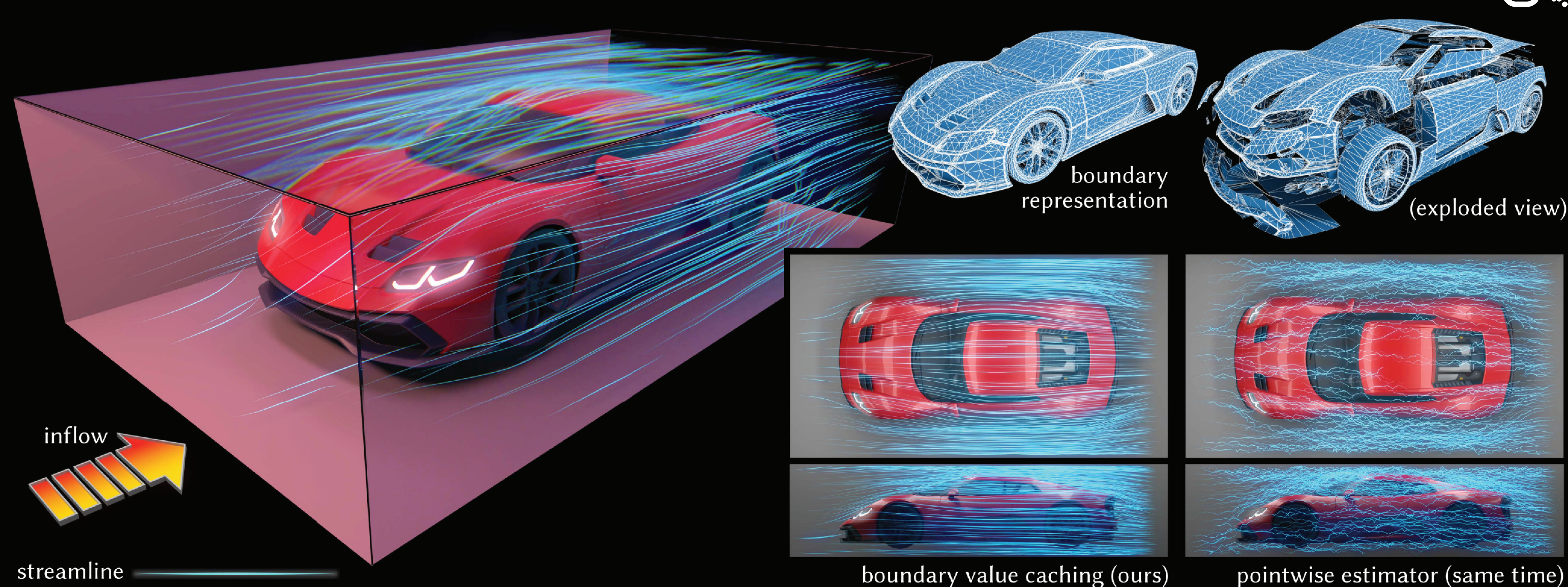
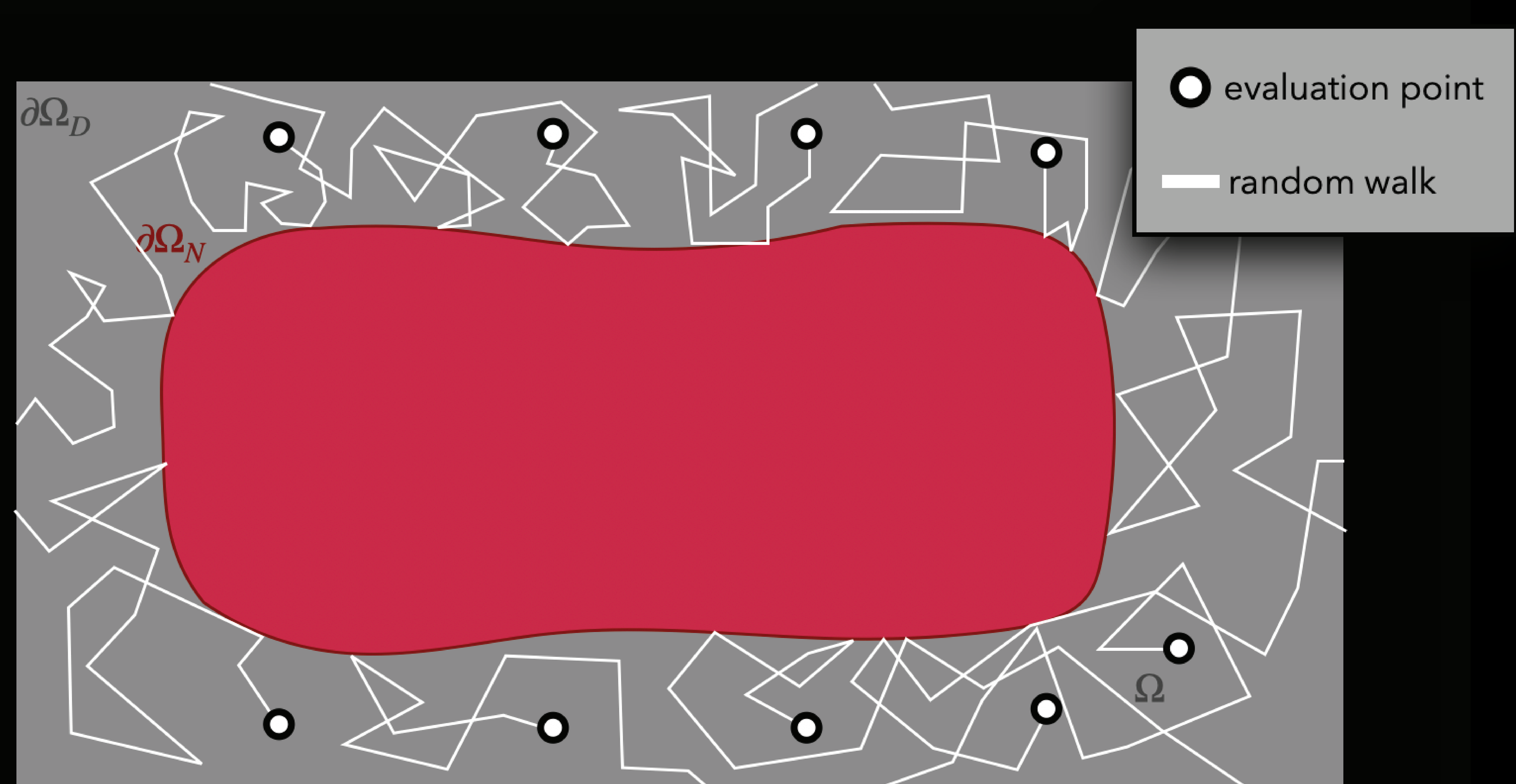
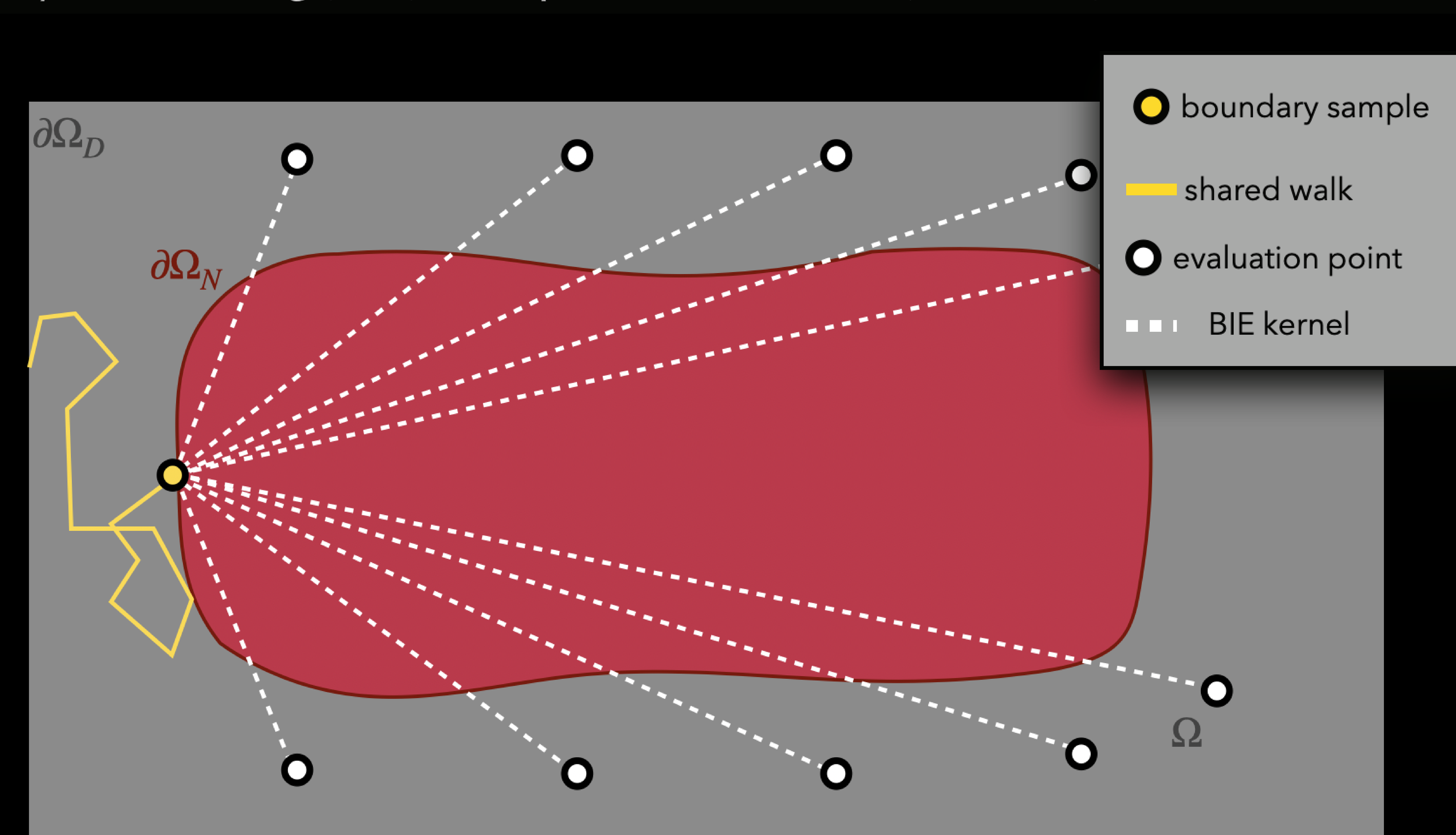
paper →





Pointwise Estimator (independent walks)



Boundary Value Caching (shared walks)

BVC Algorithm

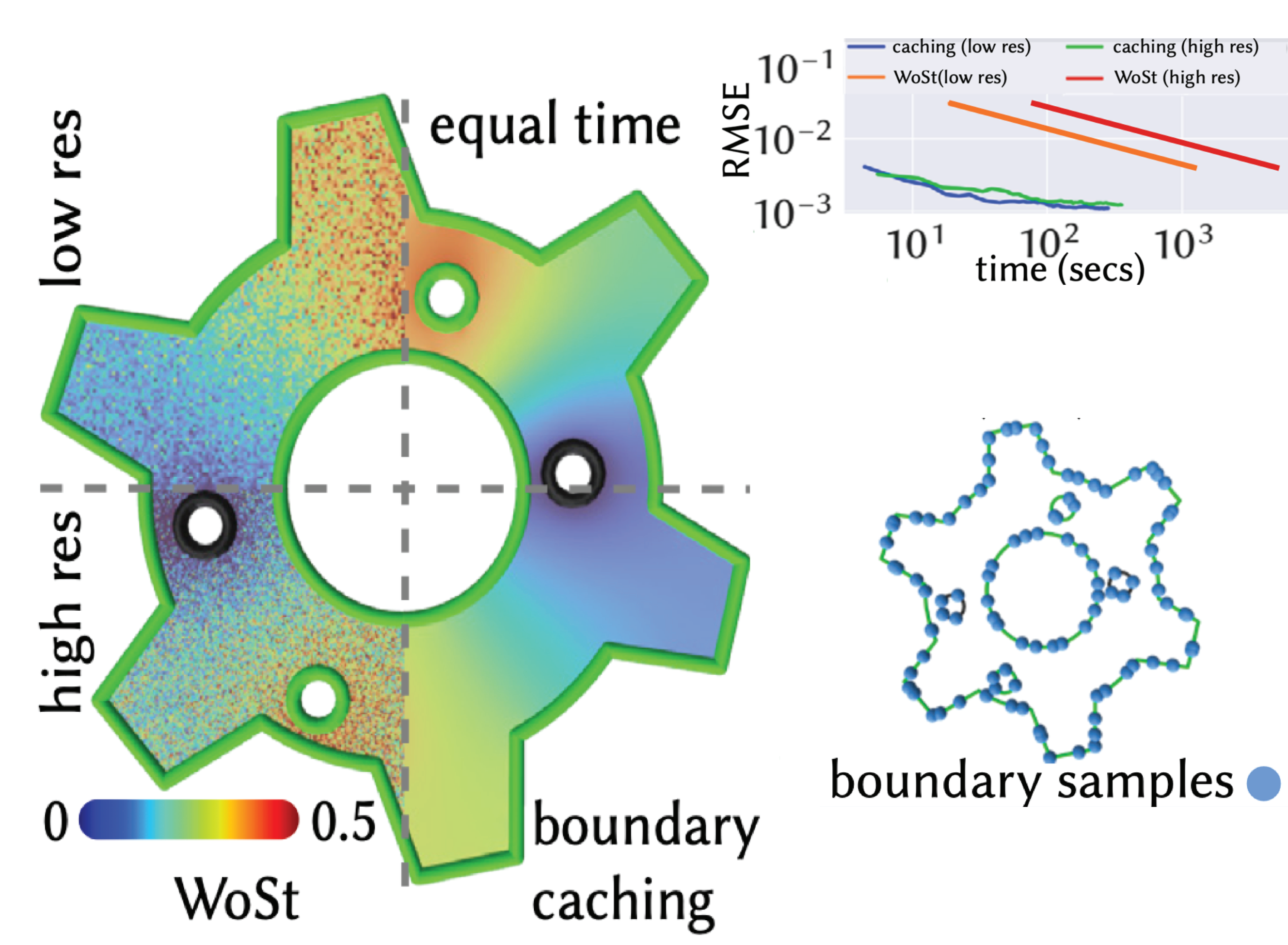
1. generate samples on boundary $\partial\Omega$
2. use WoSt to estimate u & $\frac{du}{dn}$
3. evaluate BIE inside domain Ω

Boundary Integral Equation

$$u(x) = \int_{\partial\Omega} \underbrace{\frac{\partial G(x,y)}{\partial n}}_{\text{free-space Poisson kernel}} \underbrace{u(y)}_{\text{Dirichlet values}} - \underbrace{G(x,y)}_{\text{free-space Green kernel}} \underbrace{\frac{\partial u(y)}{\partial n}}_{\text{Neumann values}} \, dy$$

$$\Delta u = 0 \quad \text{on } \Omega \qquad u = g \quad \text{on } \partial\Omega_D$$

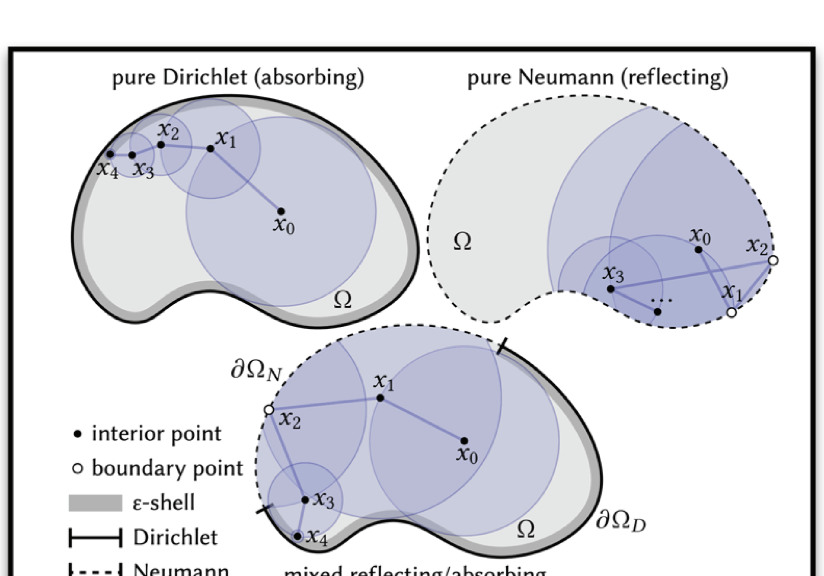
$$\frac{\partial u}{\partial n} = h \quad \text{on } \partial\Omega_N$$

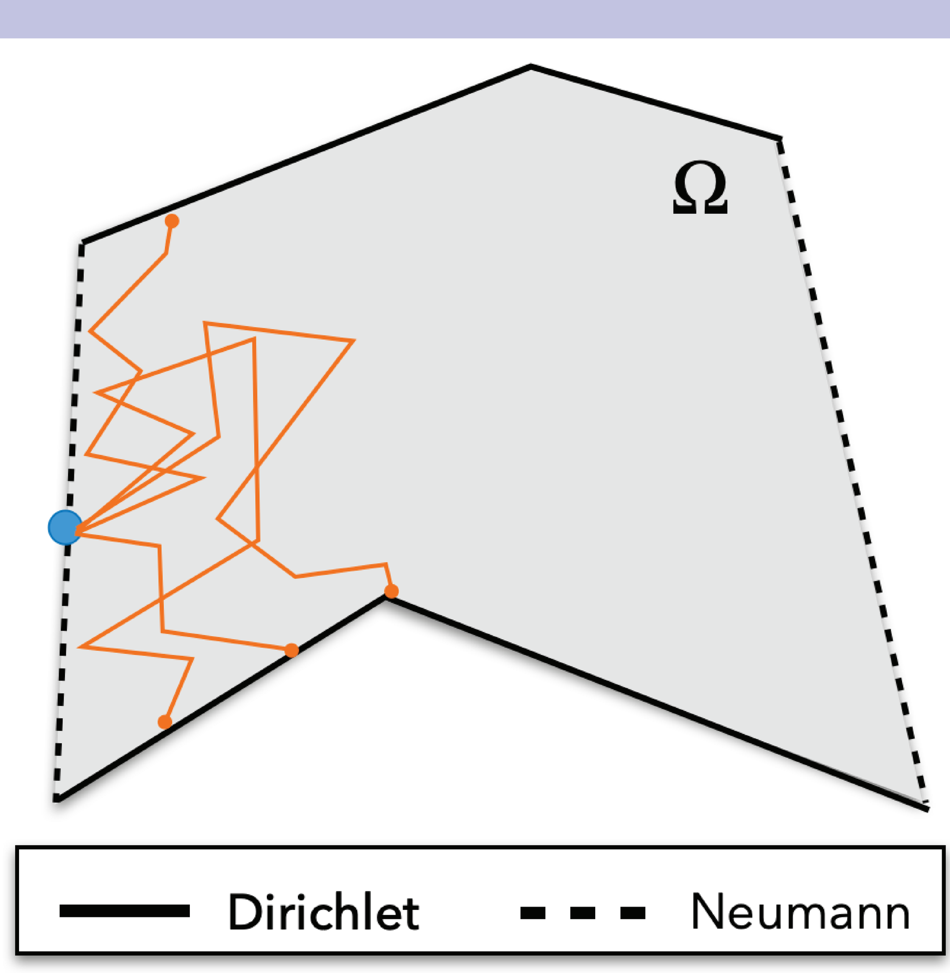


Dirichlet Value Estimates $\hat{u}$

Neumann Value Estimates $d\hat{u}/dn$

Walk on Stars [Sawhney et al. 2023]:





Spatial derivative **inside a ball** [Sawhney & Crane 2020]:

$$\nabla_x u(x) = \frac{1}{|B|} \int_{\partial B} u(y)\, \nu(y)\, dy$$

Normal derivative **on the boundary**:

$$\frac{du(x)}{dn_x} = n_x \cdot \nabla_x u(x)$$