3D Reconstruction with Fast Dipole Sums—Erratum and Addendum

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CCS Concepts: • **Computing methodologies → Point-based models**; **Ray tracing**.

Additional Key Words and Phrases: Winding number, point-based modeling, inverse rendering

1 ERRATUM

Immediately after Equation (14) on page 6 of the main paper, we state an incorrect value for $\mathrm{P}_\varepsilon(y, y)$. The correct value is $\mathrm{P}_\varepsilon(y, y) = 0$. The stated value is for Green's function, $\mathrm{G}_\varepsilon(y, y) = 3^{-1}\varepsilon^{-3}\pi^{-3/2}$.

2 ADDENDUM

We elaborate on our implementation of the volume rendering model in Sections 3.1 and 3.2 of the main paper. We use numeric labels for equations from the main paper, and labels prefixed with 'A' for equations in the addendum.

We evaluate the volume rendering equation (1) using a quadrature procedure implementing Equations (A1) and (A4) below, rather than Equation (2) by Max [1995]. Below we derive and contrast these two quadrature procedures. We use an analogous quadrature procedure to evaluate the transmittance T in Equation (28).

Both quadrature procedures start use ray samples $\tau_\mathrm{n} = \tau_0 < \cdots < \tau_J = \tau_\mathrm{f}$ to approximate the volume rendering equation (1) as

$$c(o, v) \approx \sum_{j=1}^{J} \prod_{i=1}^{j} \mathrm{V}_i \left(1 - \mathrm{V}_j\right) \mathrm{L}_j, \tag{A1}$$

where at each sample location τ_j, $\mathrm{L}_j \equiv \mathrm{L}\left(r_{o,v}(\tau_j), -v\right)$, and

$$\mathrm{V}_j \equiv \exp\left(-\int_{\tau_{j-1}}^{\tau_j} \sigma\left(r_{o,v}(t), v\right) \mathrm{d}t\right) \tag{A2}$$

is the *vacancy probability*, that is, the probability that there is no intersection with the scene geometry along the ray segment $\left[\tau_{j-1}, \tau_j\right]$. This approximation is due to Max [1995] and assumes that the radiance is constant along each ray segment, $\mathrm{L}(t) \approx \mathrm{L}_j,\ t \in \left[\tau_{j-1}, \tau_j\right]$.

Max [1995] proceeds by further assuming that the attenuation coefficient is constant along each ray segment, that is, $\sigma\left(r_{o,v}(t), v\right) \approx \sigma\left(r_{o,v}(\tau_j), v\right) \equiv \sigma_j,\ t \in \left[\tau_{j-1}, \tau_j\right]$. Under this further assumption, and using $\Delta_j \equiv \tau_j - \tau_{j-1}$, Equation (A2) becomes:

$$\mathrm{V}_j \overset{\text{[Max 1995]}}{\approx} \exp\left(-\sigma_j \Delta_j\right). \tag{A3}$$

Combining Equations (A1) and (A3) results in Equation (2).

By contrast, we approximate each vacancy probability V_j as:

$$\mathrm{V}_j \overset{\text{ours}}{\approx} \frac{\min\left\{\mathrm{v}_j, \mathrm{v}_{j-1}\right\}}{\max\left\{\mathrm{v}_j, \mathrm{v}_{j-1}\right\}}, \tag{A4}$$

where $\mathrm{v}_j \equiv \mathrm{v}\left(r_{o,v}(\tau_j)\right)$. This approximation is specific to Equation (4) for σ, which we reproduce for convenience:

$$\sigma(x, \omega) \equiv \frac{\left|\omega \cdot \nabla \mathrm{v}(x)\right|}{\mathrm{v}(x)} = \left|\omega \cdot \nabla \log \mathrm{v}(x)\right|, \tag{A5}$$

and is inspired by Wang et al. [2021, Equation (13)], who use an analogous approximation for their closely related attenuation coefficient expression [Miller et al. 2024, Section 4].

Deriving Equation (A3) assumes only that the *sign* of the argument of absolute value is constant along each ray segment $\left[\tau_{j-1}, \tau_j\right]$.

- If the sign is non-negative, by noting that the argument of the absolute value is the directional derivative of the logarithm of the attenuation coefficient along the ray direction, we have:

$$\mathrm{V}_j \approx \exp\left(-\int_{\tau_{j-1}}^{\tau_j} v \cdot \nabla \log \mathrm{v}\left(r_{o,v}(t)\right) \mathrm{d}t\right) \tag{A6}$$

$$= \exp\left(-\int_{\tau_{j-1}}^{\tau_j} \frac{\partial}{\partial t} \log \mathrm{v}(o + tv)\, \mathrm{d}t\right) \tag{A7}$$

$$= \exp\left(-\log \mathrm{v}\left(o + t_j v\right) + \log \mathrm{v}\left(o + t_{j-1} v\right)\right) \tag{A8}$$

$$= \frac{\mathrm{v}_{j-1}}{\mathrm{v}_j}. \tag{A9}$$

- If the sign is negative, we have exactly analogously,

$$\mathrm{V}_j \approx \frac{\mathrm{v}_j}{\mathrm{v}_{j-1}}. \tag{A10}$$

Then, Equation (A4) follows from Equations (A9) and (A10) by noting that a non-negative sign for σ in $\left[\tau_{j-1}, \tau_j\right]$ implies that $\mathrm{v}_j \geq \mathrm{v}_{j-1}$, whereas a negative sign implies $\mathrm{v}_{j-1} > \mathrm{v}_j$.

Equation (A4) has two advantages over Equation (A3): 1. It uses a weaker assumption on the attenuation coefficient σ—constant sign rather than constant value along each ray segment $\left[\tau_{j-1}, \tau_j\right]$. 2. It requires evaluating only the vacancy v and not its gradient ∇v. Empirically, we have found that these advantages result in greatly improved accuracy and efficiency during volume rendering.

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