

# Physics-based rendering and its applications in computational photography and imaging

Adithya Pediredla

Dartmouth College

[adithya.k.pediredla@dartmouth.com](mailto:adithya.k.pediredla@dartmouth.com)

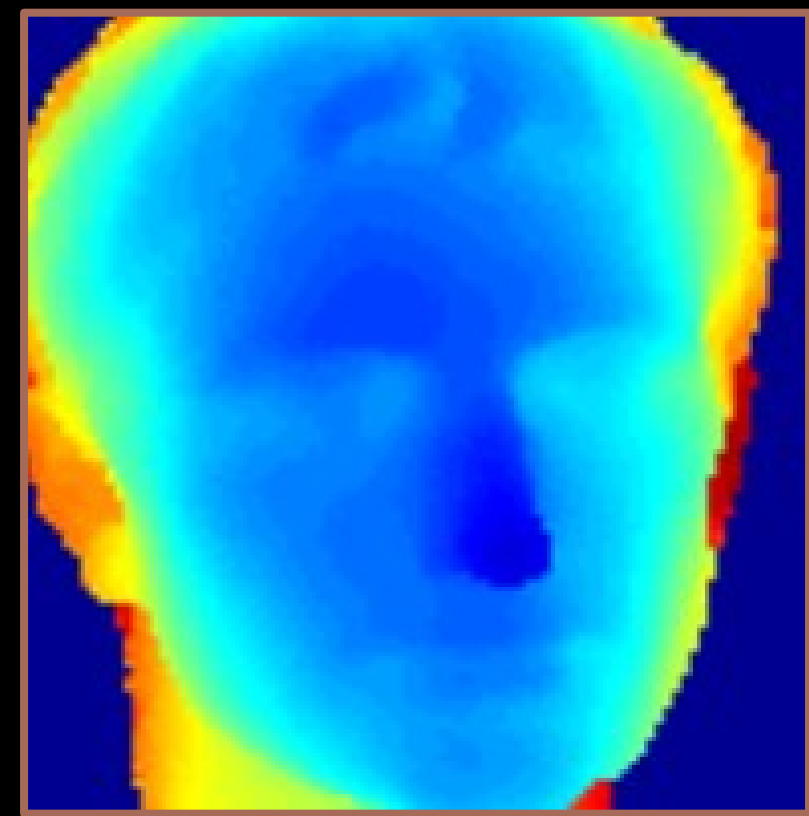
Ioannis Gkioulekas

Carnegie Mellon University

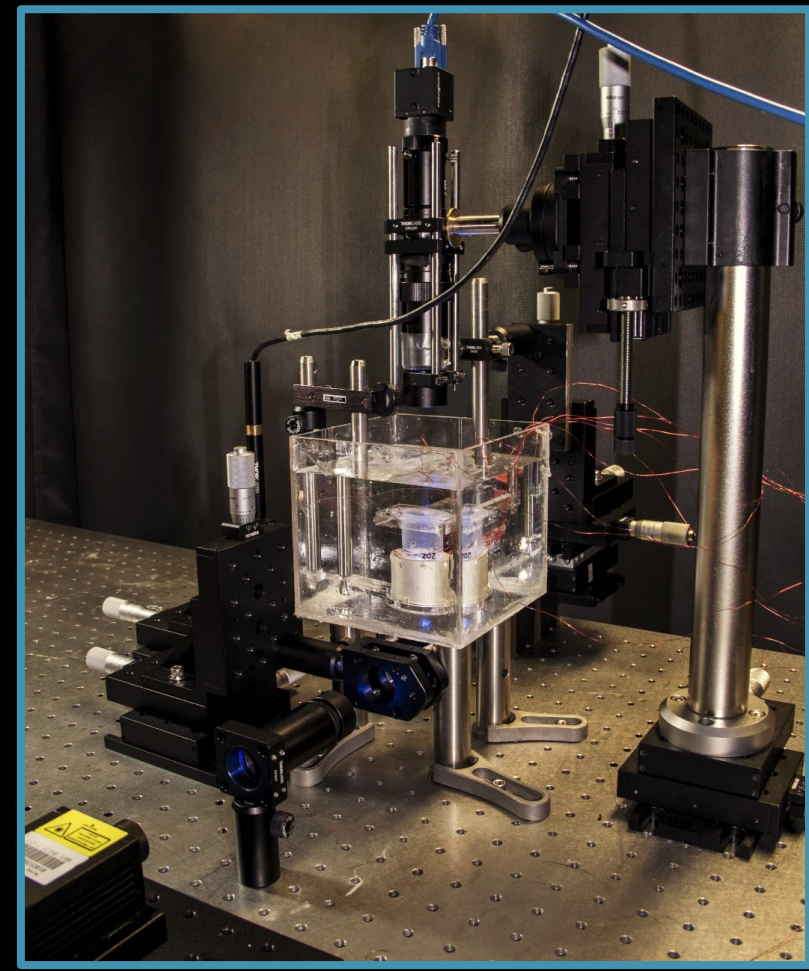
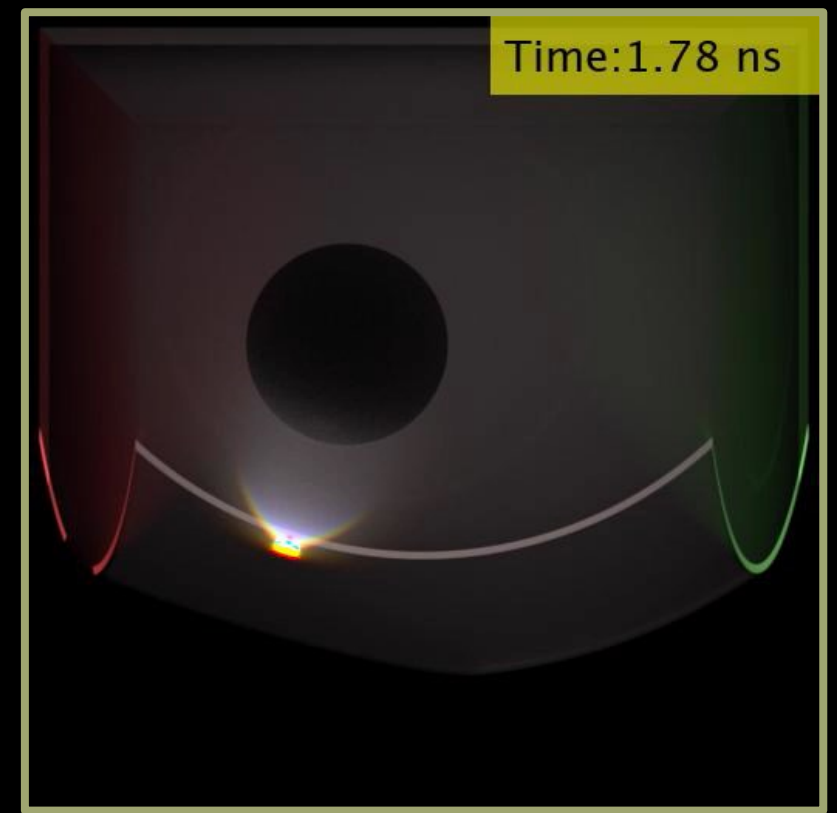
[igkioule@andrew.cmu.edu](mailto:igkioule@andrew.cmu.edu)



computer  
vision



computer  
graphics



optics

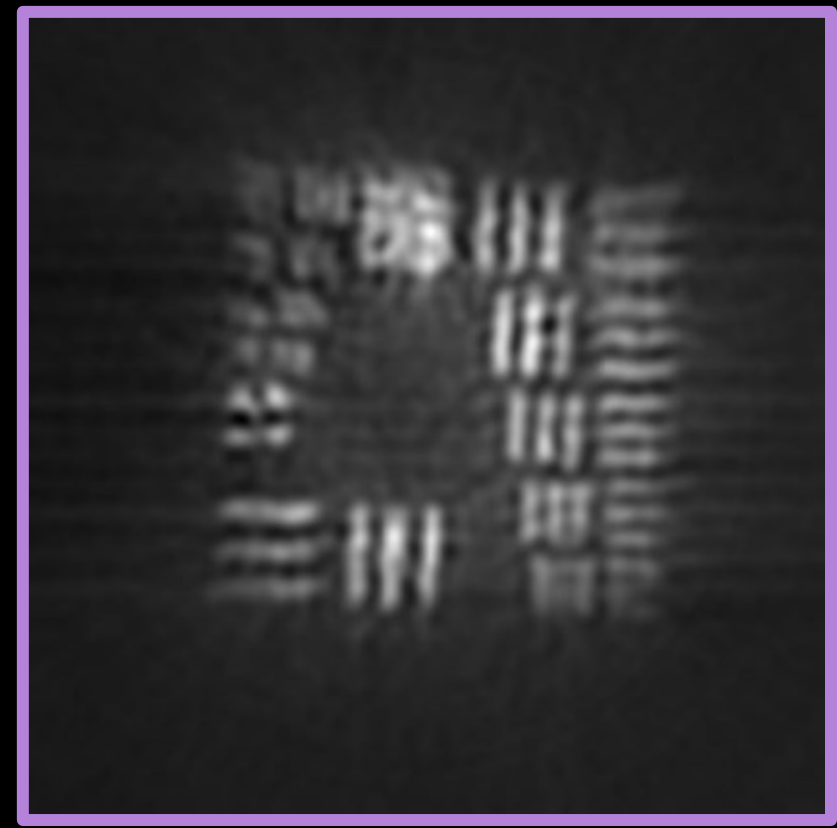
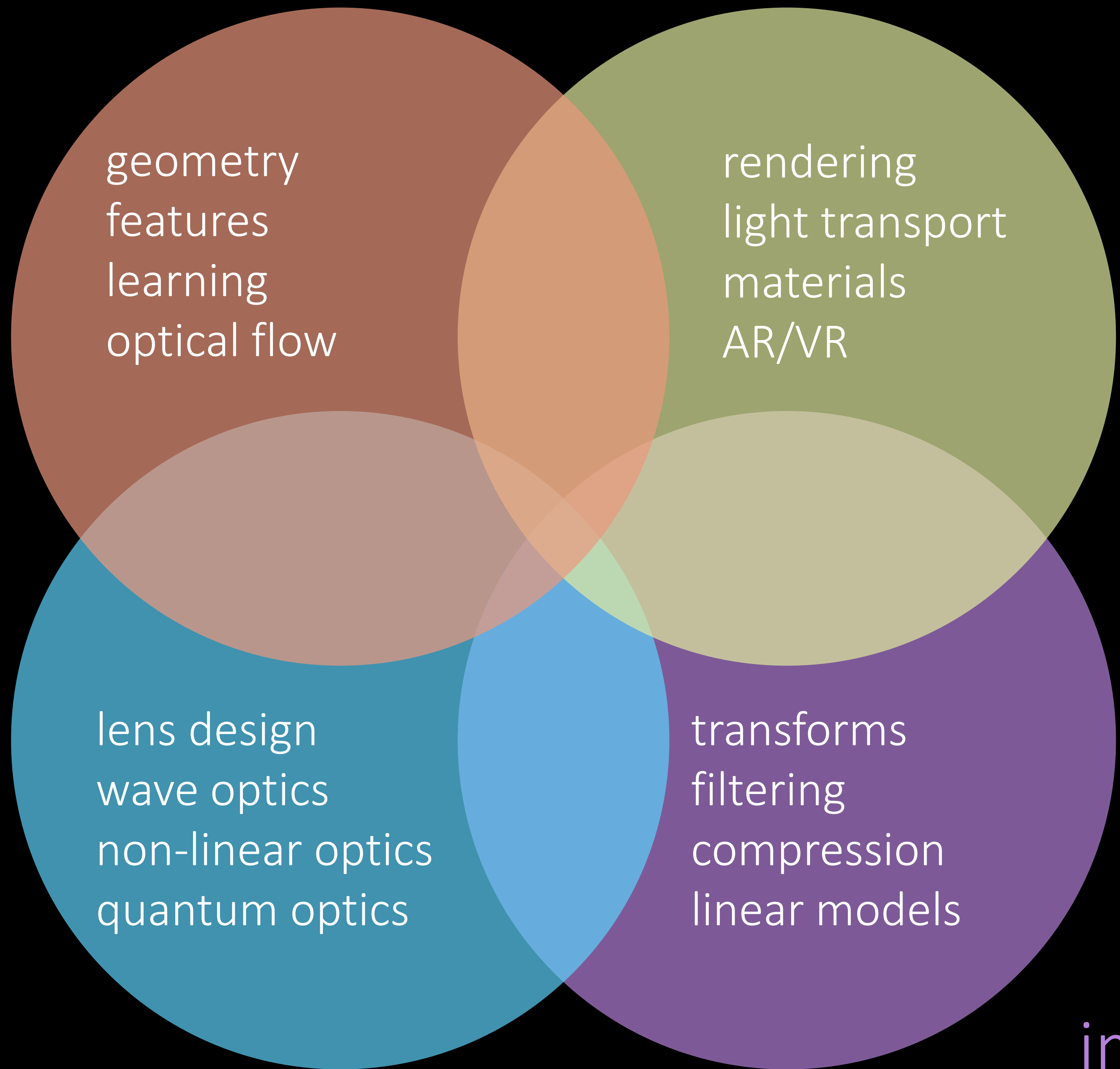
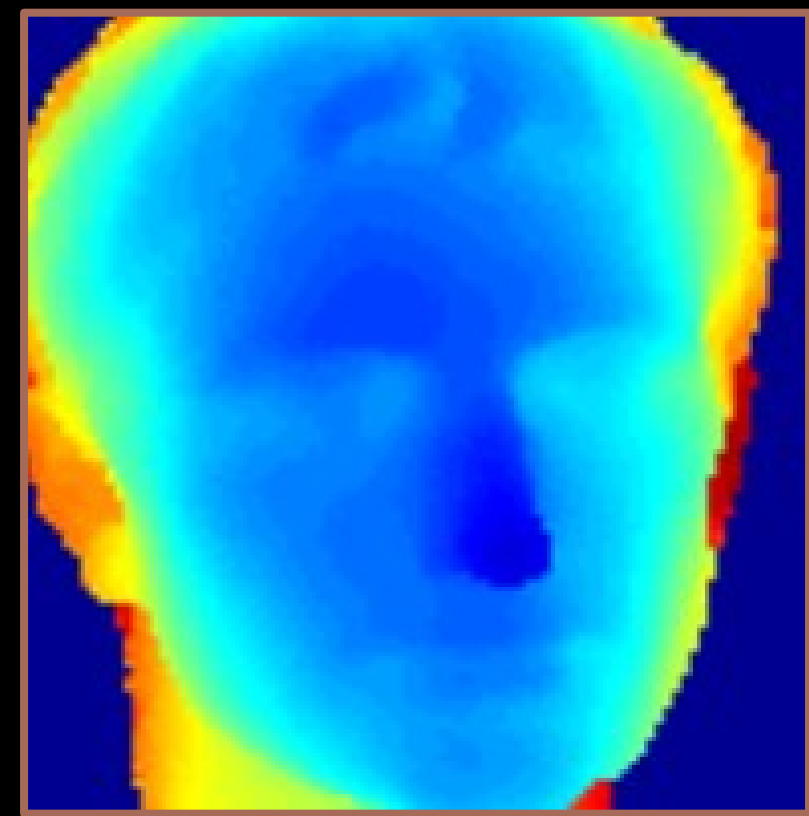


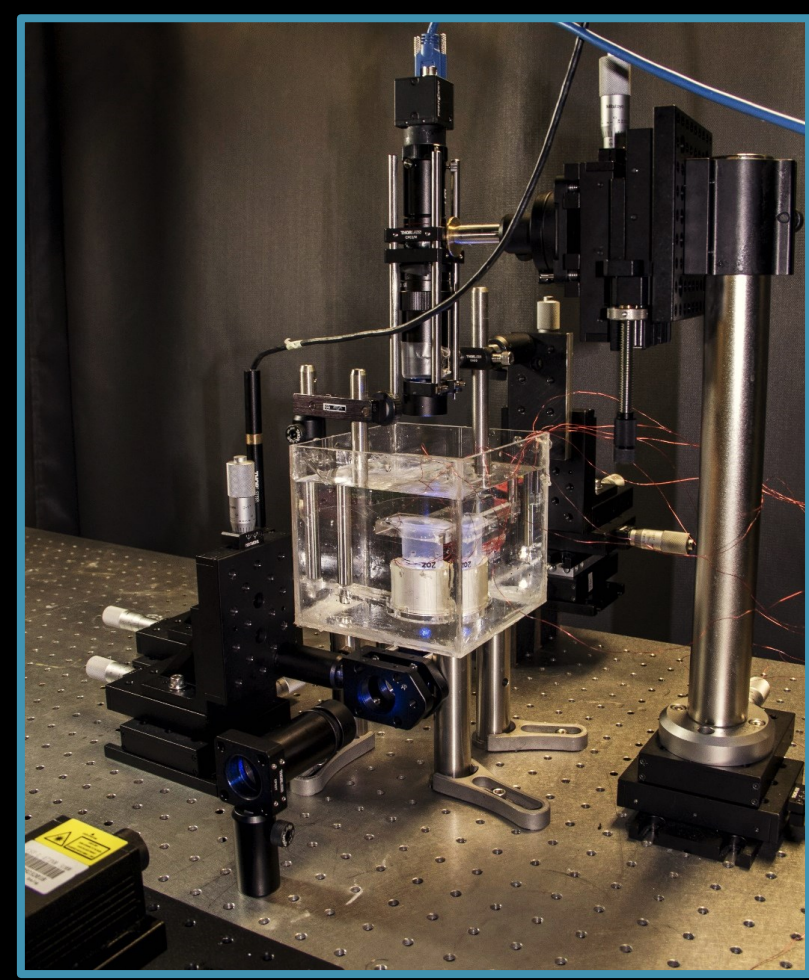
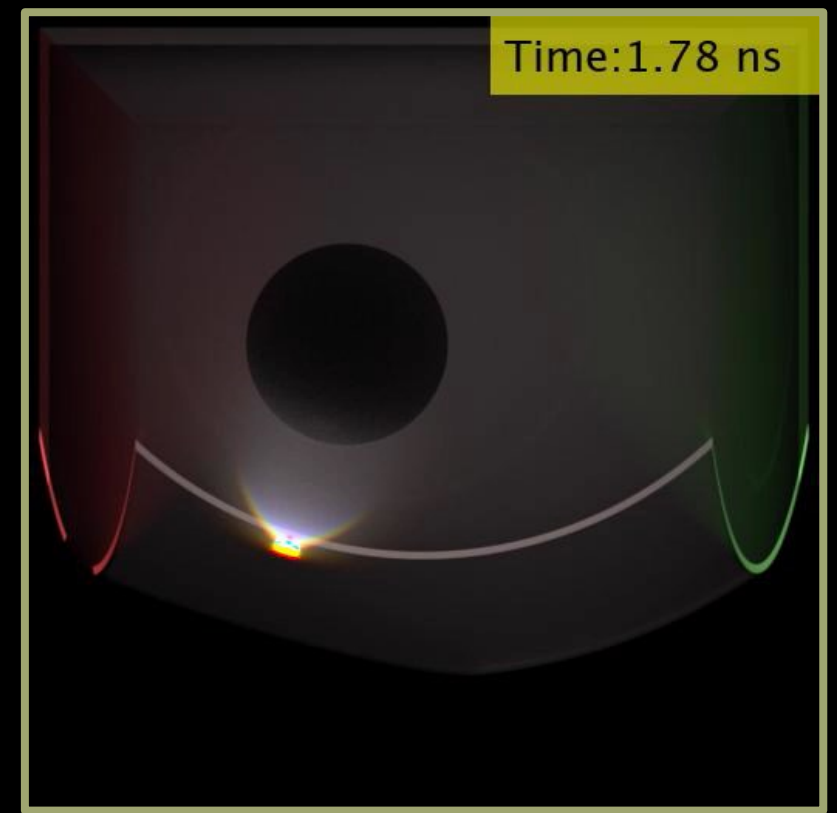
image processing



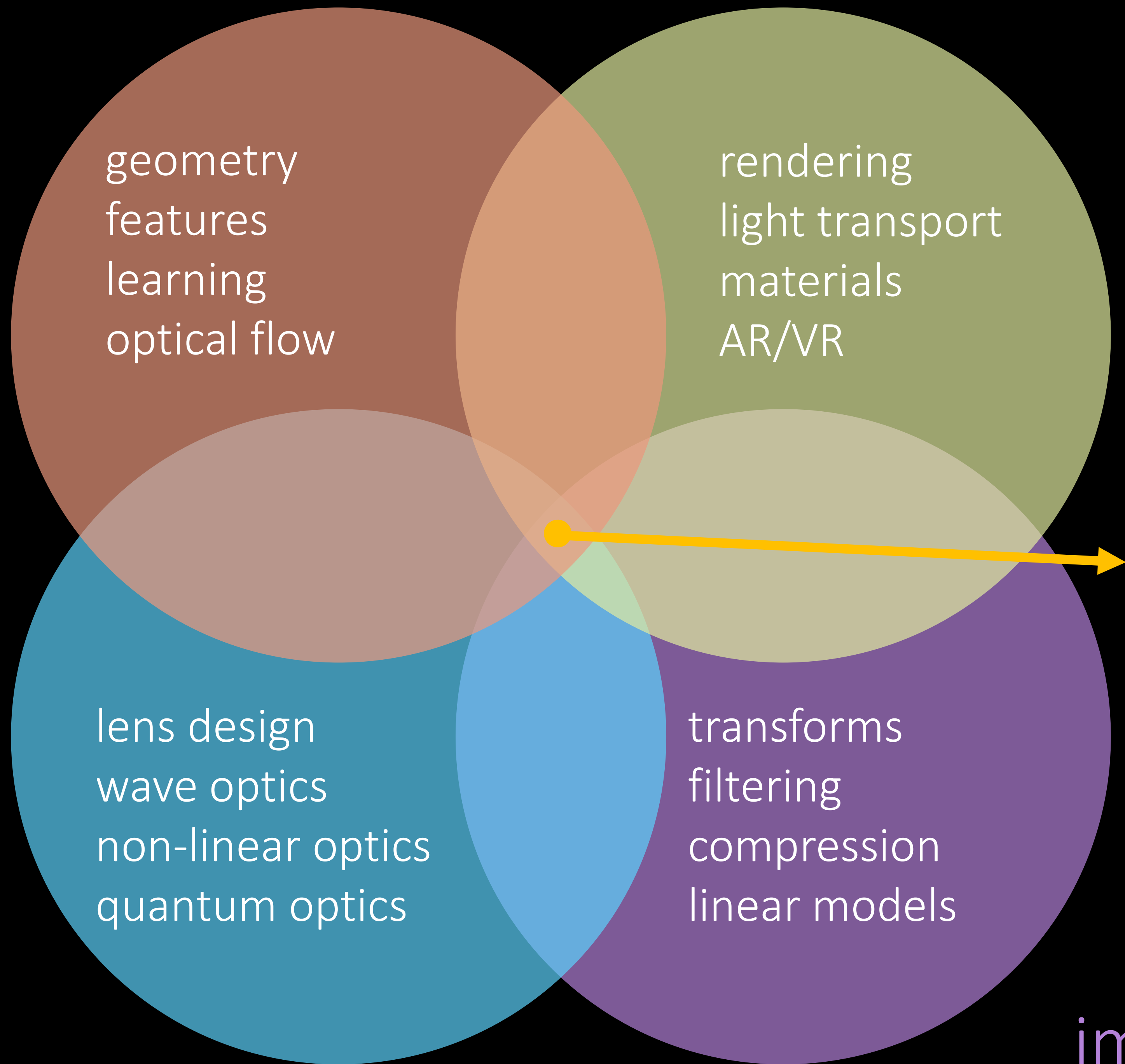
computer  
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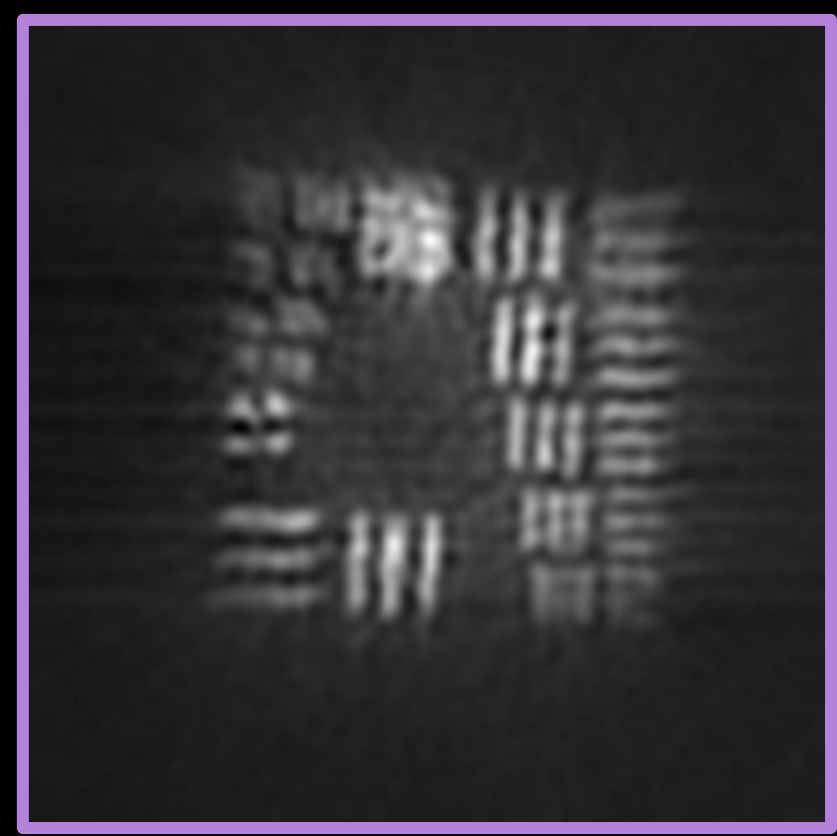
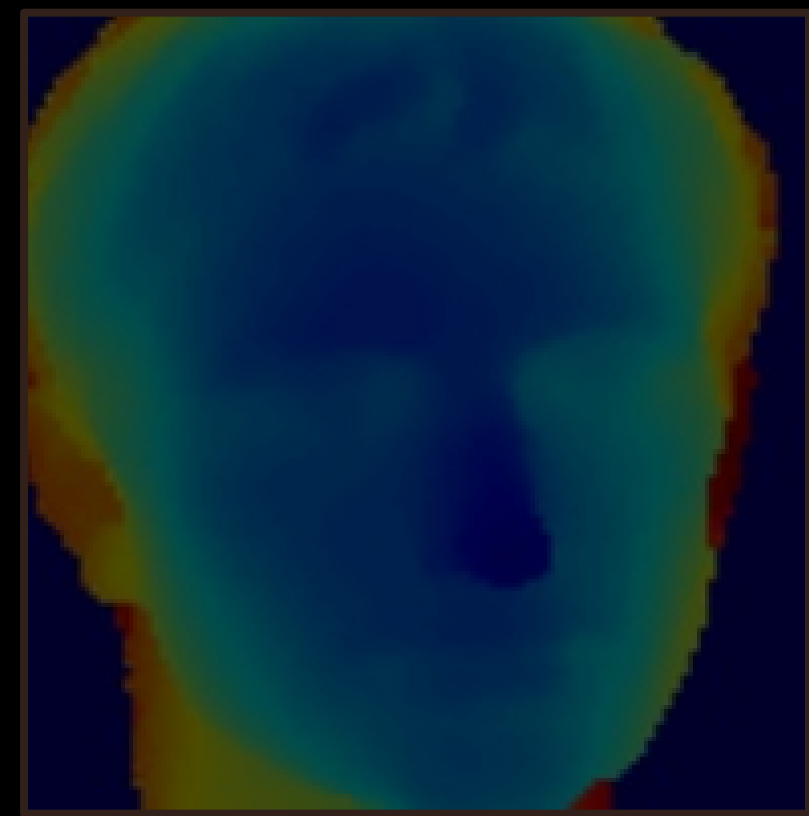


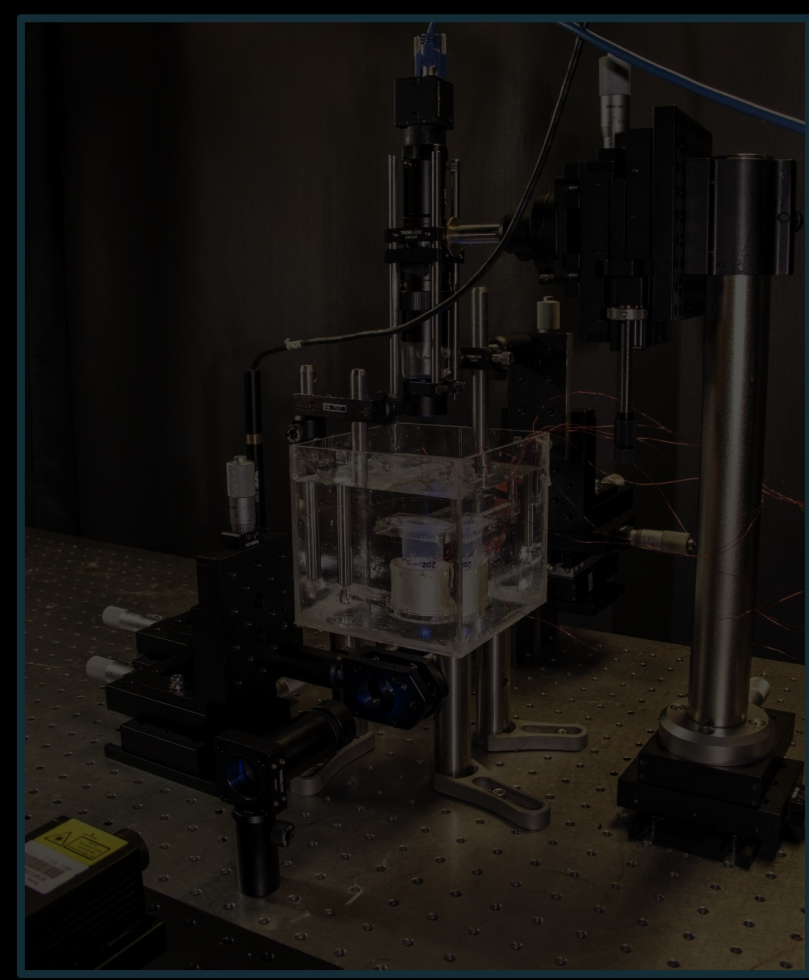
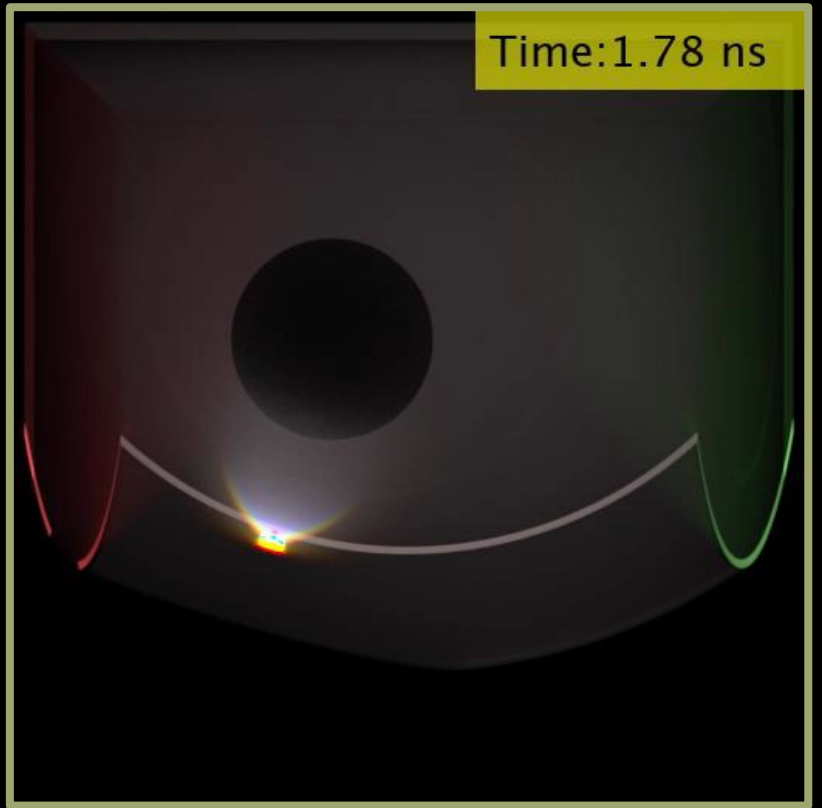
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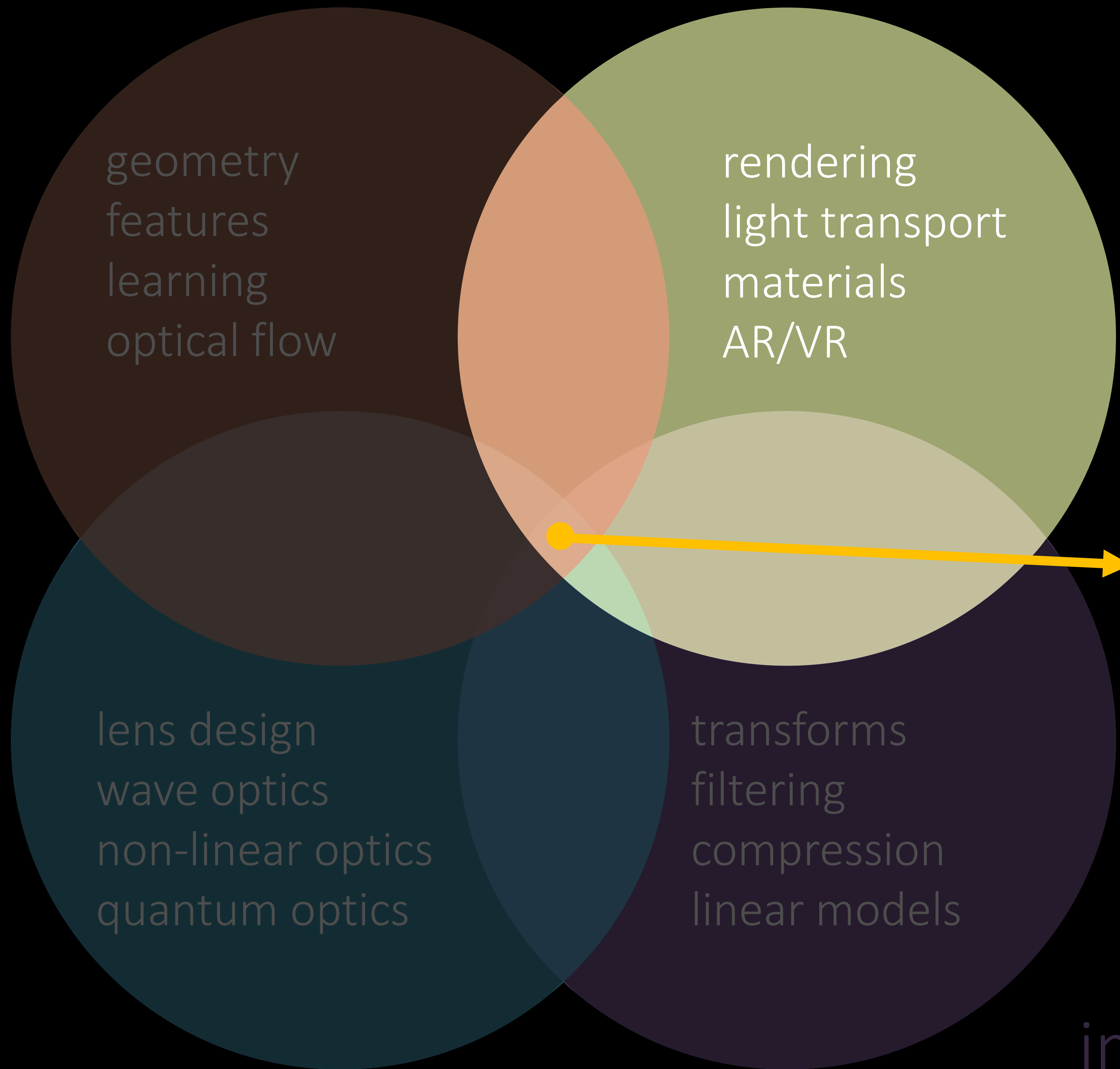
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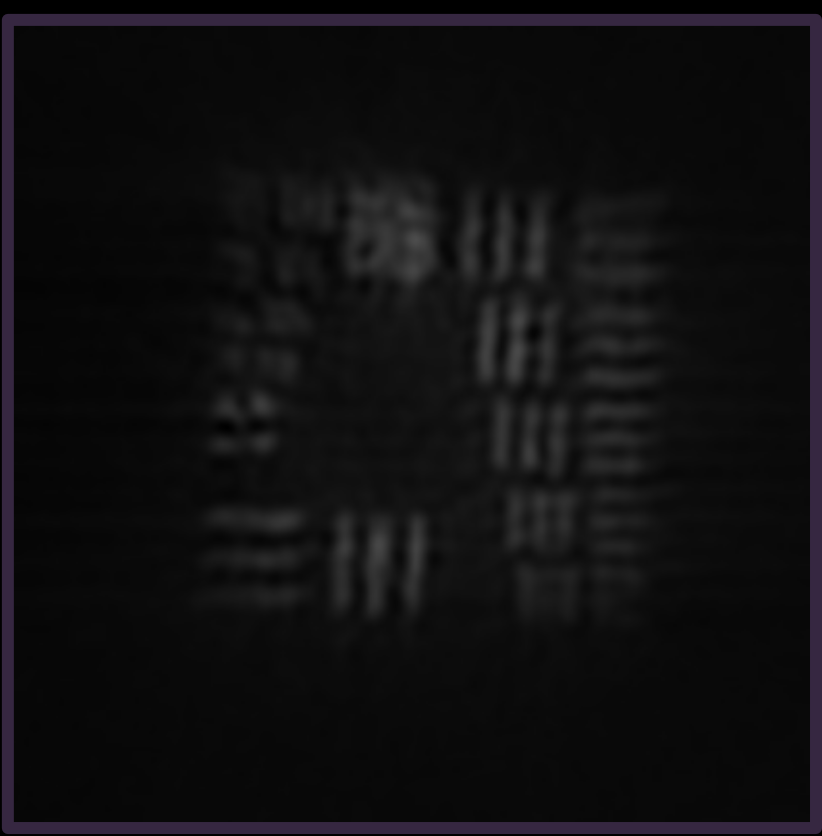
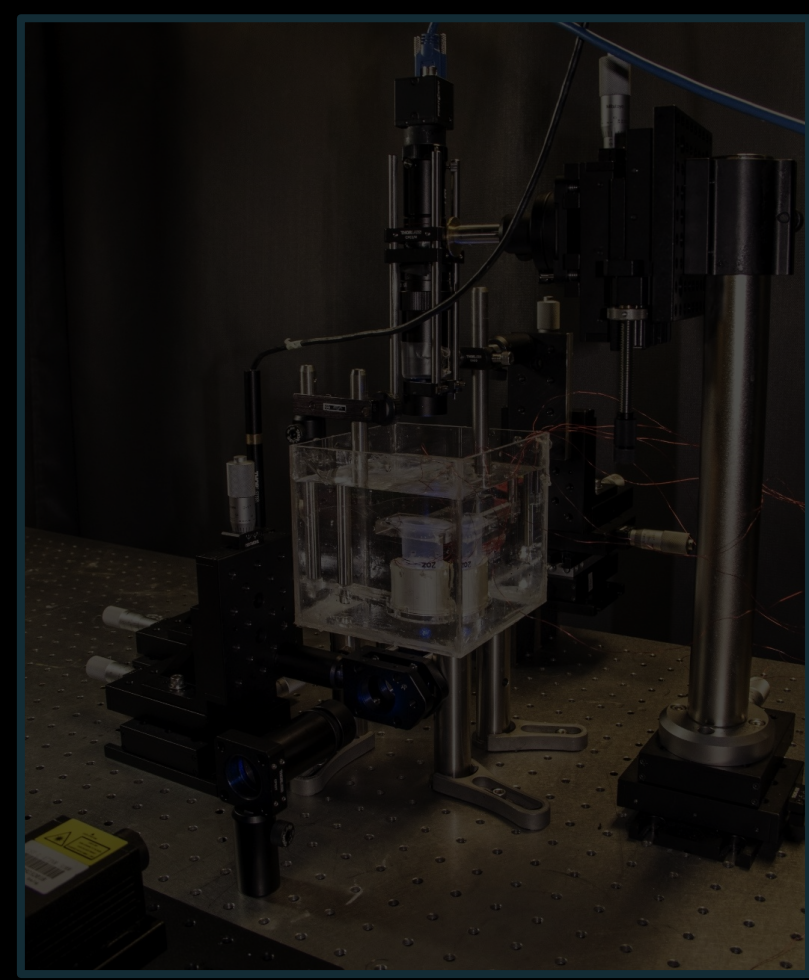
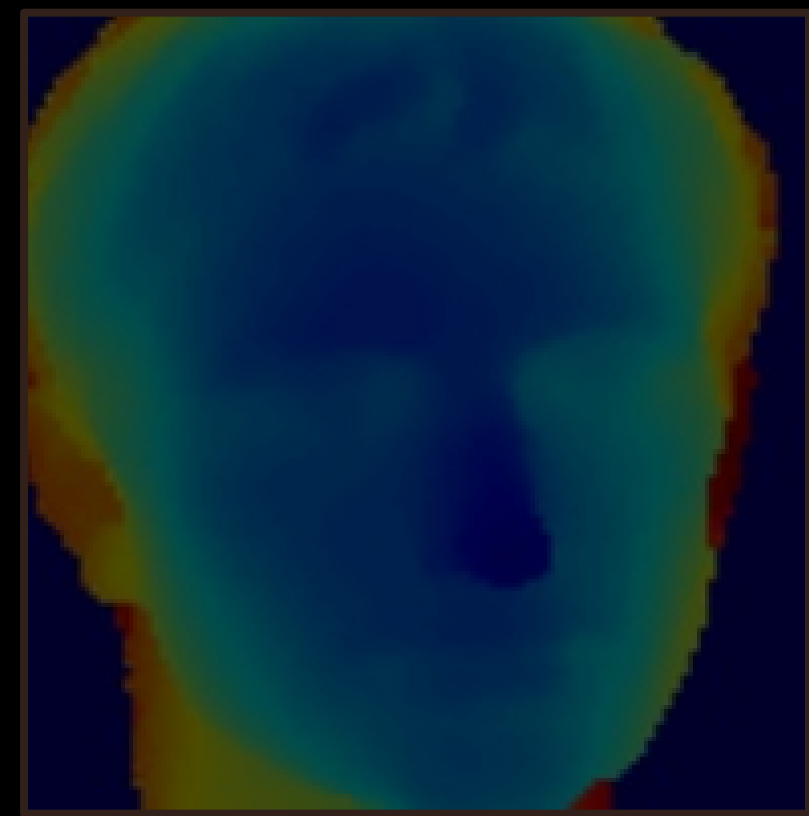


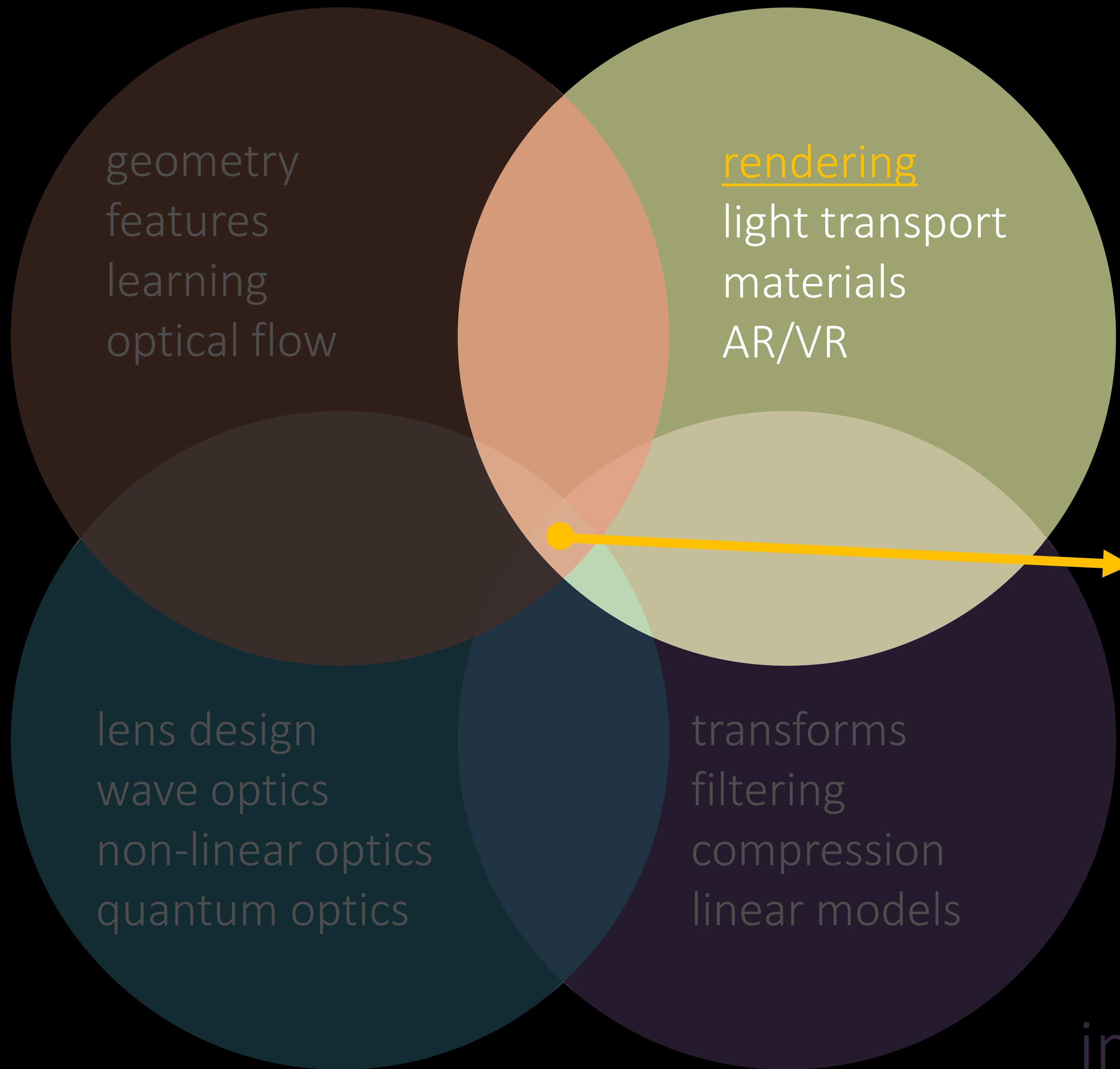
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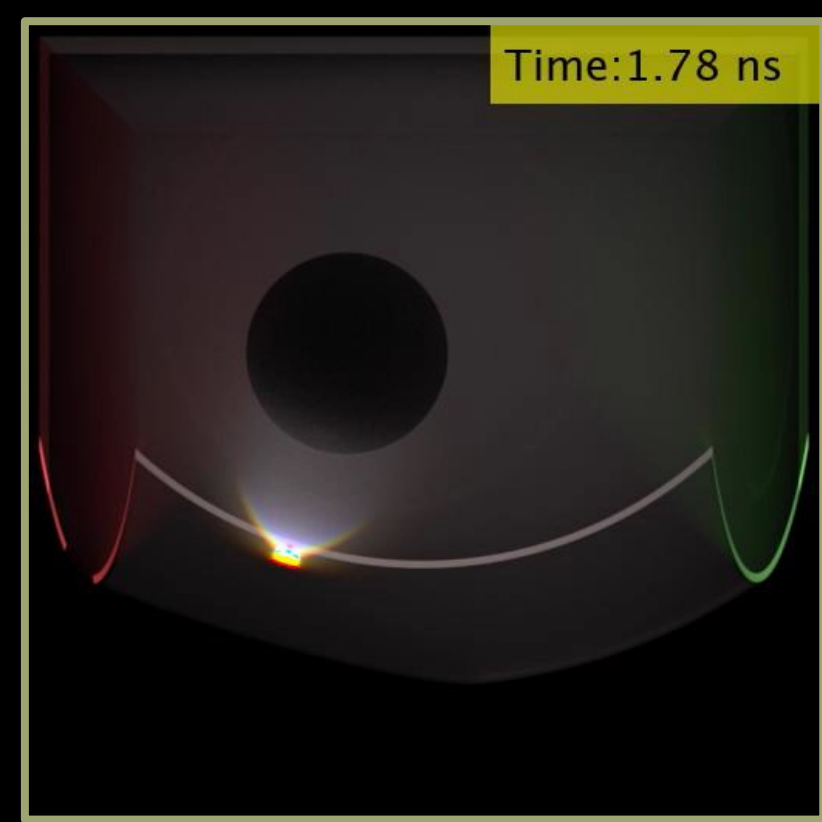
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optics



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graphics



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imaging



image processing

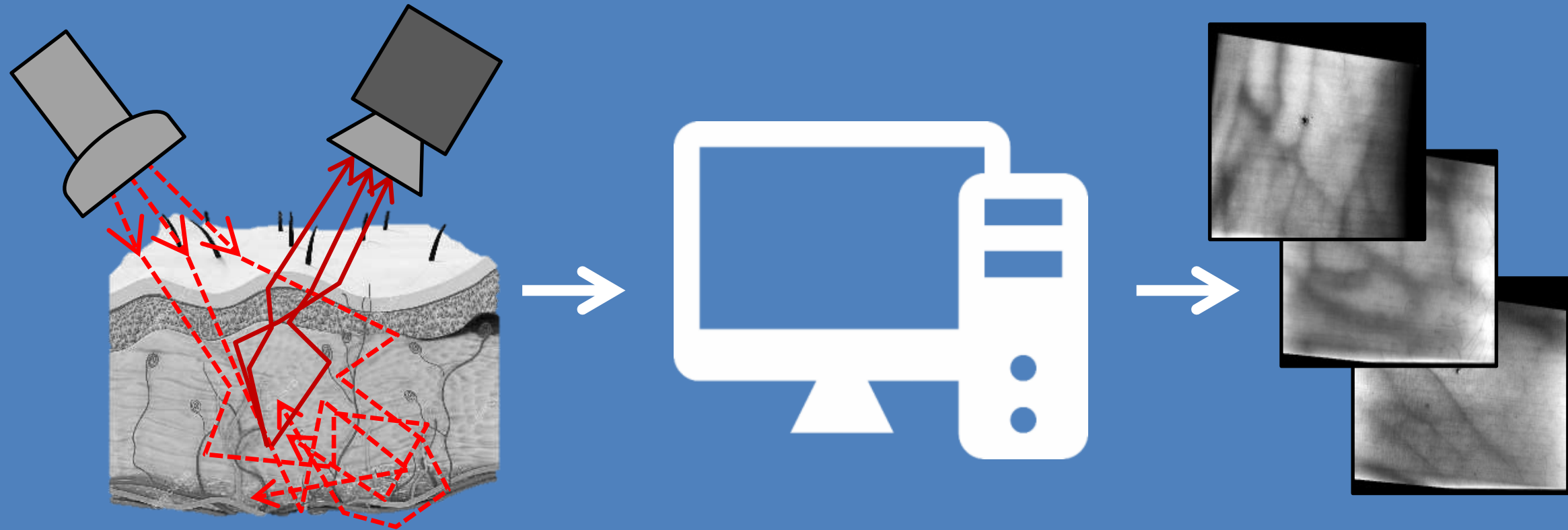


# Physics-based rendering and its applications to computational imaging



# Physics-based rendering and its applications to computational imaging

forward rendering

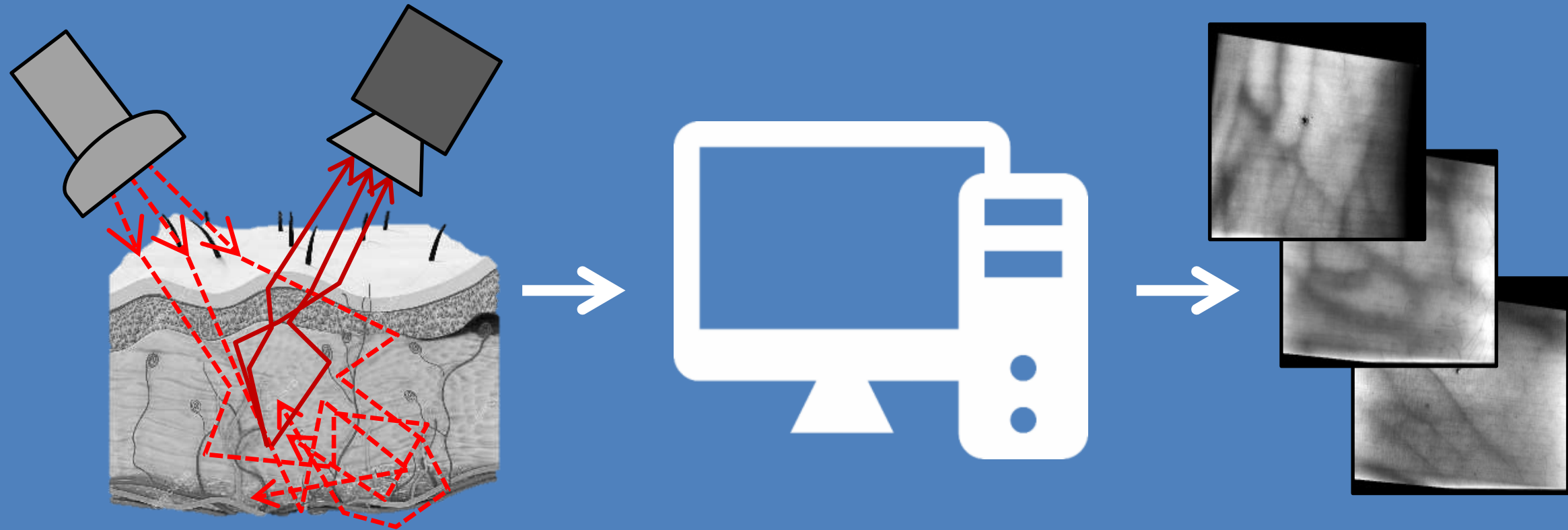


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms



# Physics-based rendering and its applications to computational imaging

## forward rendering



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## inverse rendering

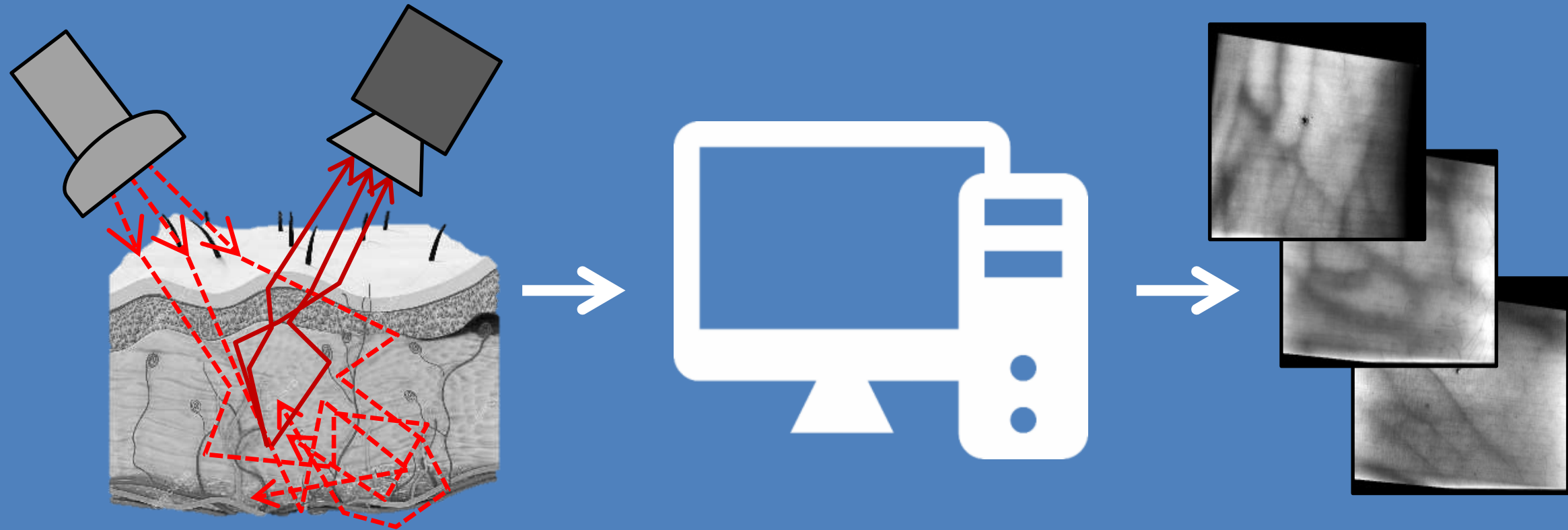


- accurate and efficient differentiable simulation
- tractably solve general inverse problems



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## forward rendering



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## inverse rendering

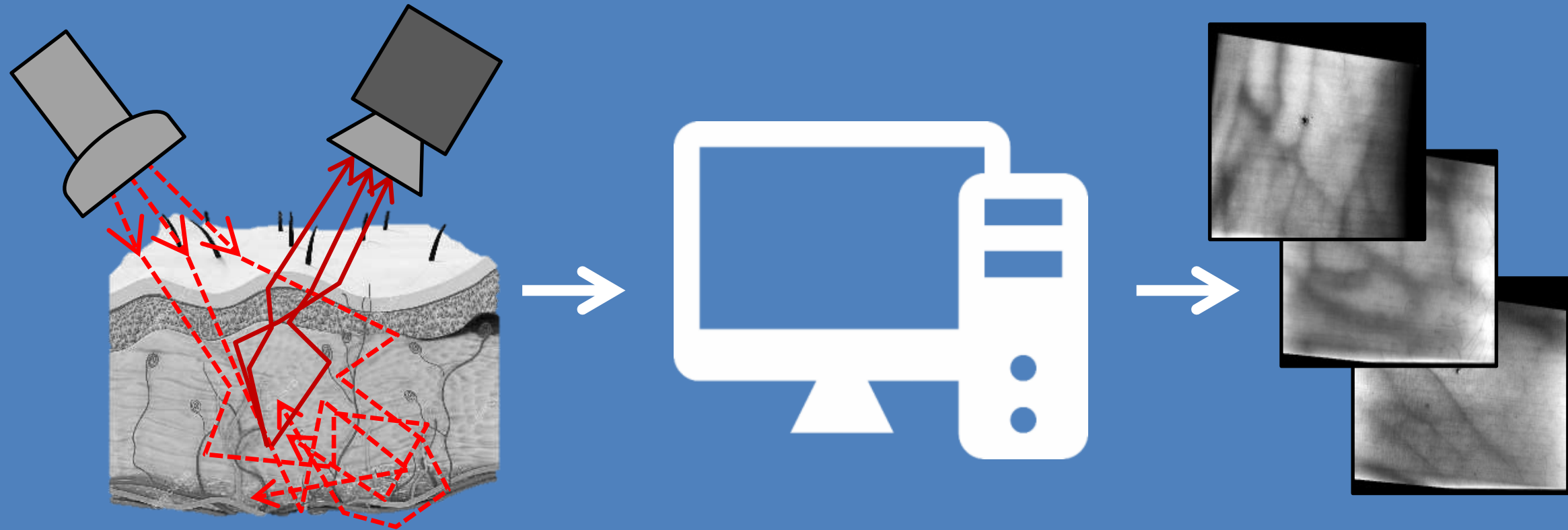


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## forward rendering

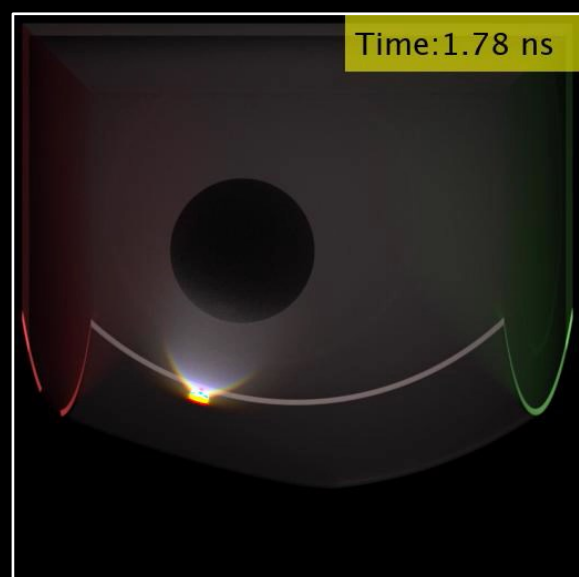


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## inverse rendering



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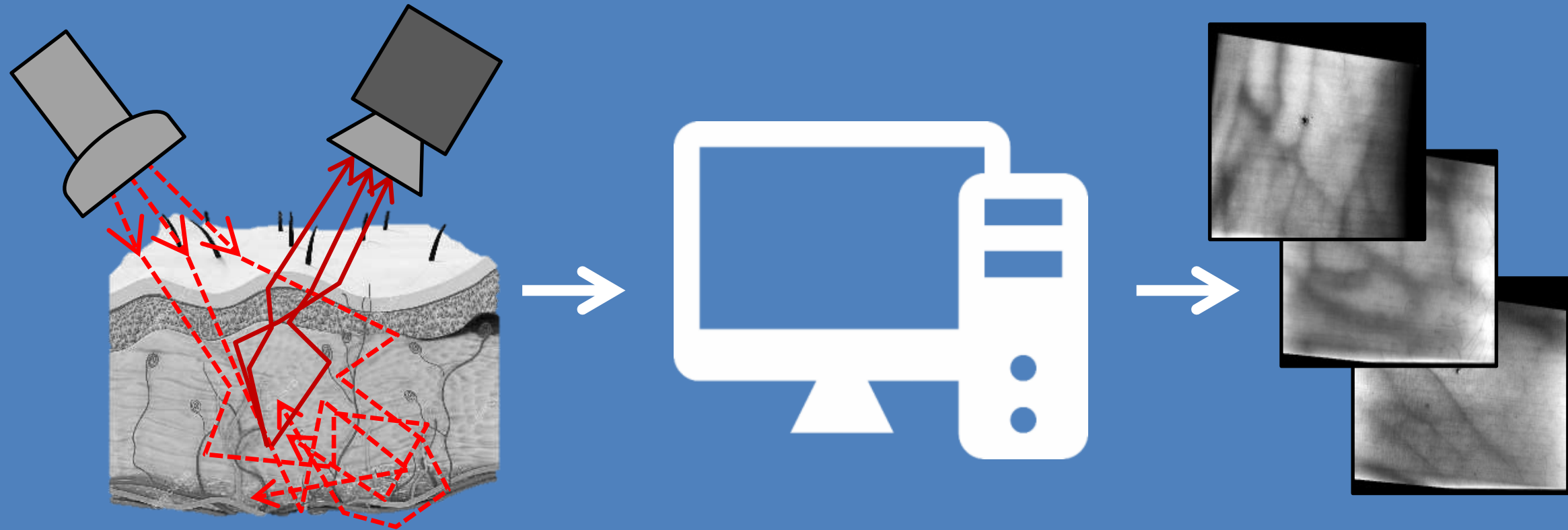


time-of-flight  
imaging



# Physics-based rendering and its applications to computational imaging

## forward rendering

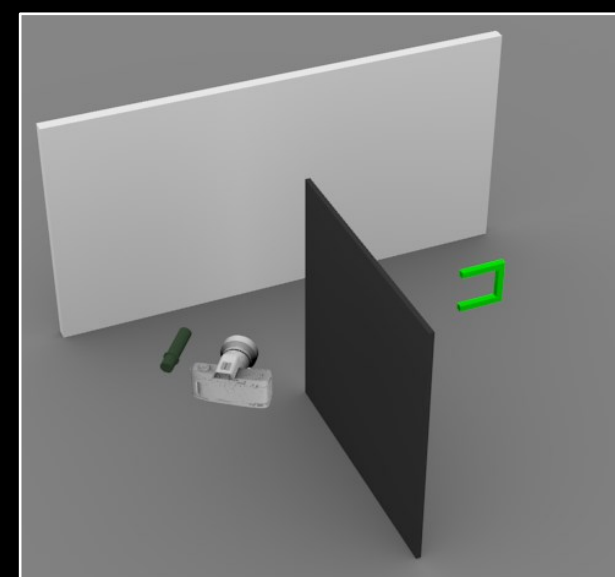
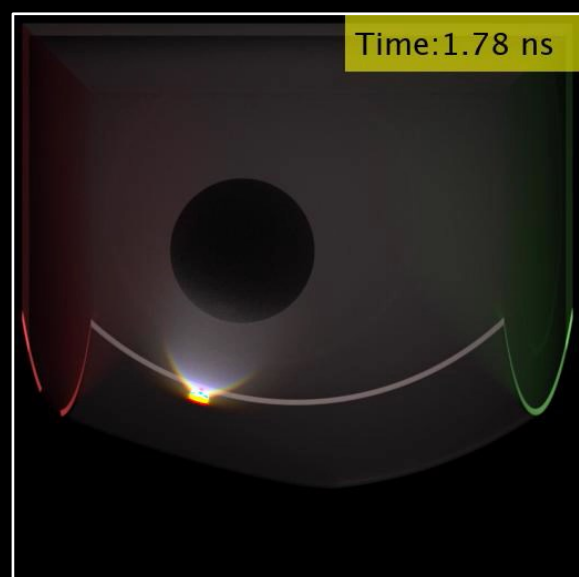


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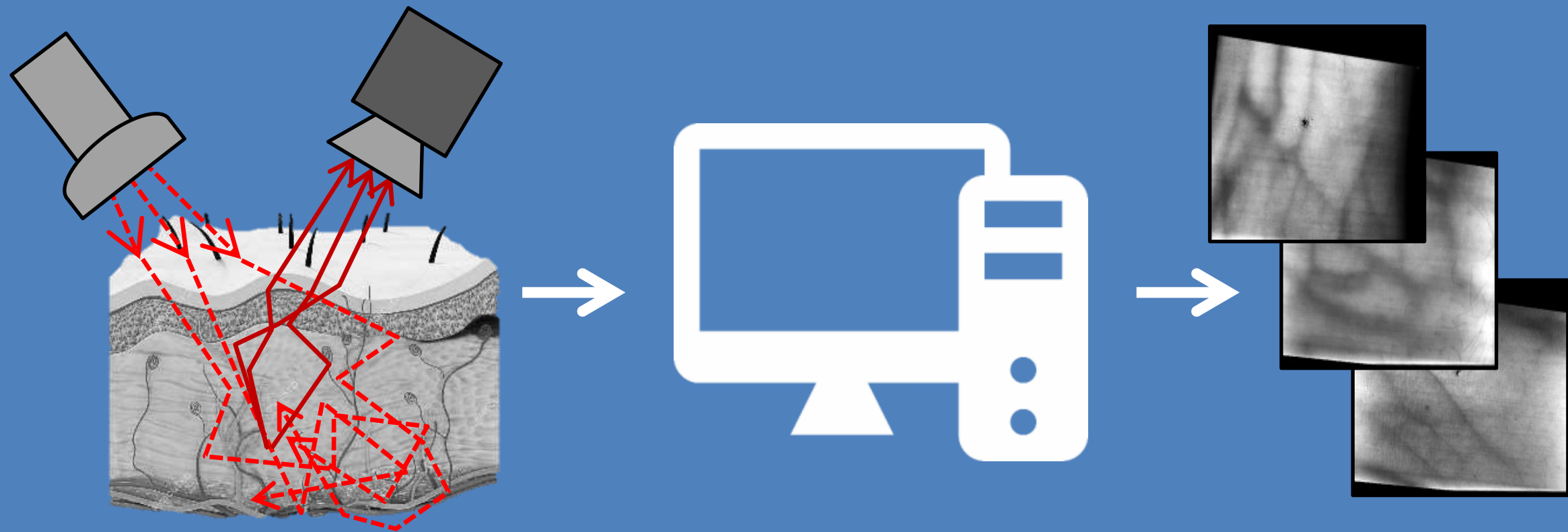


time-of-flight imaging    non-line-of-sight imaging



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## forward rendering

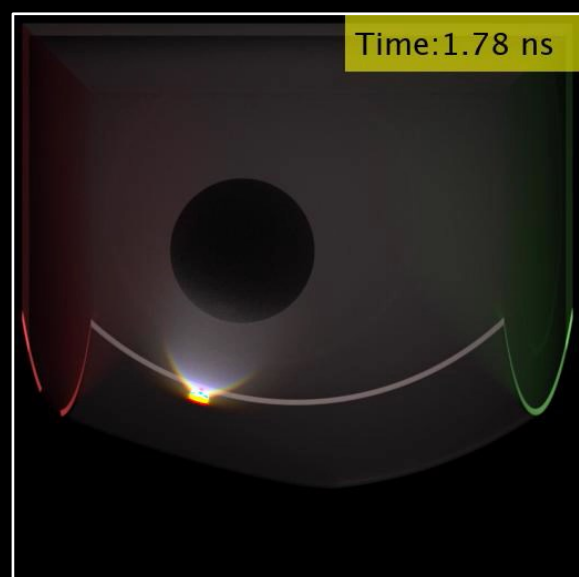


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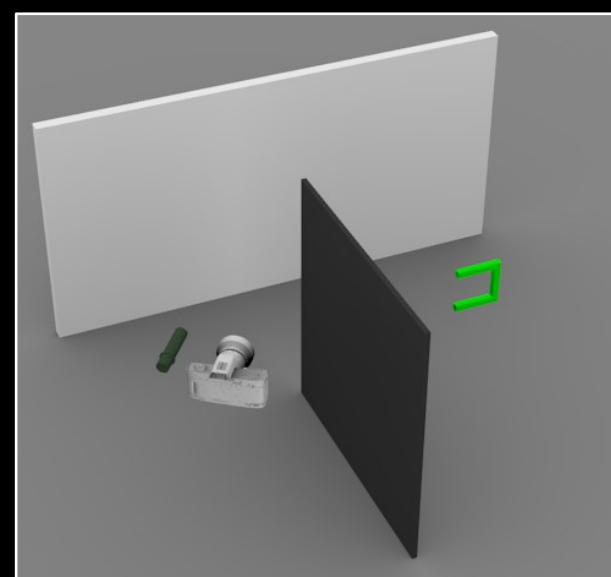
## inverse rendering



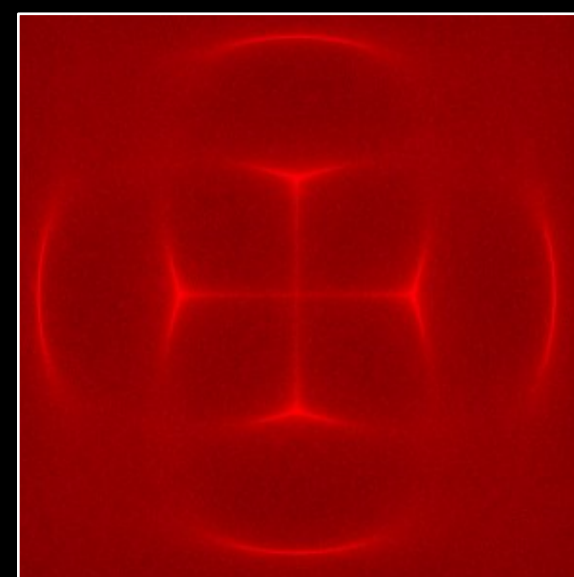
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time-of-flight  
imaging



non-line-of-sight  
imaging

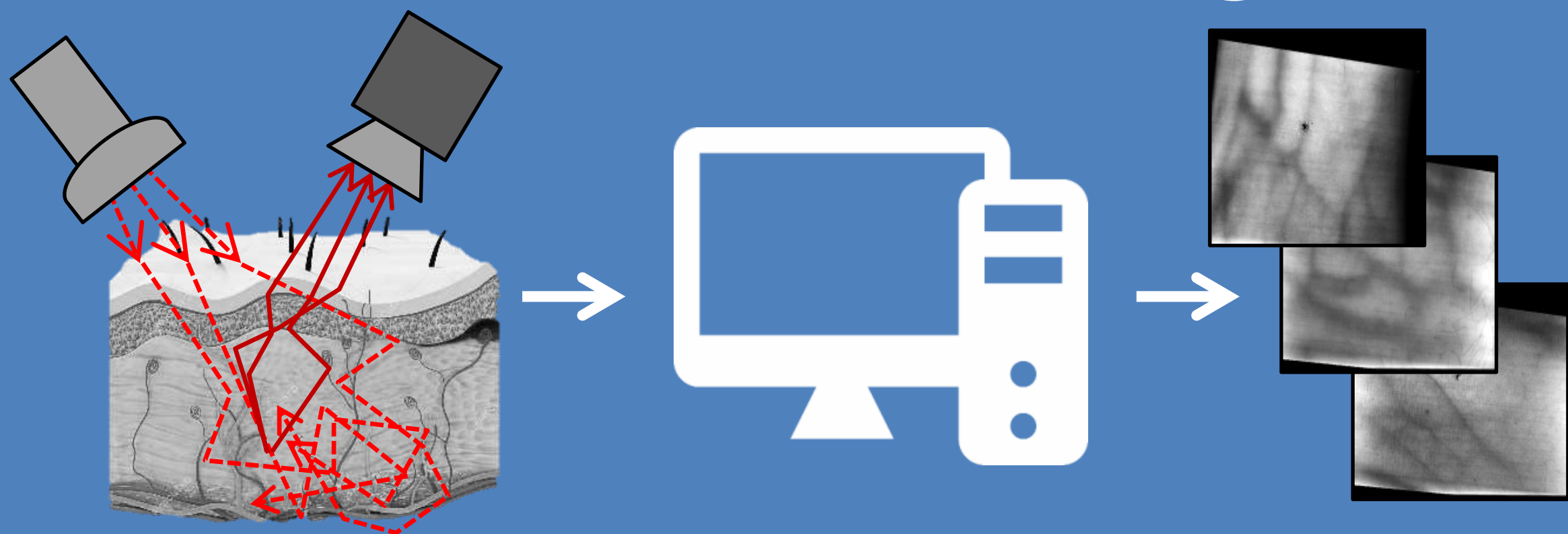


acousto-optic  
lensing



# Physics-based rendering and its applications to computational imaging

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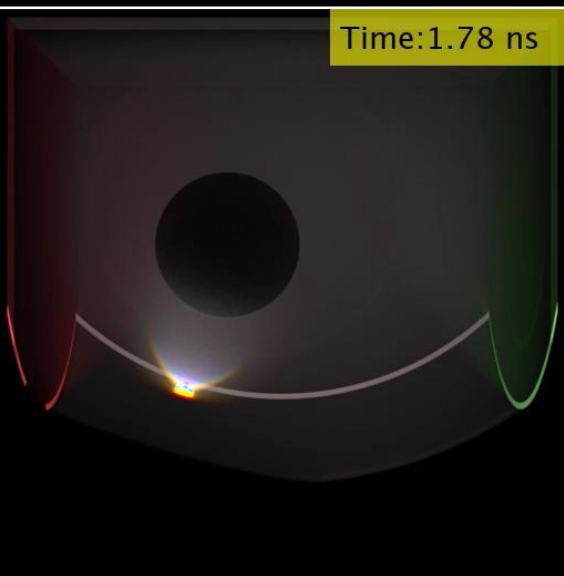


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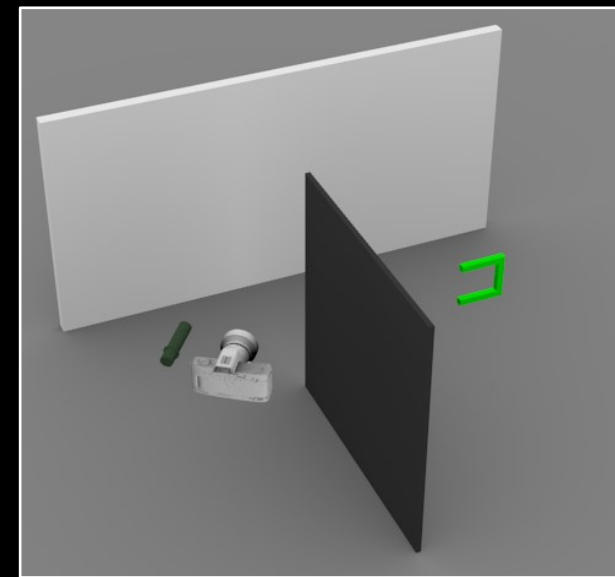
## inverse rendering



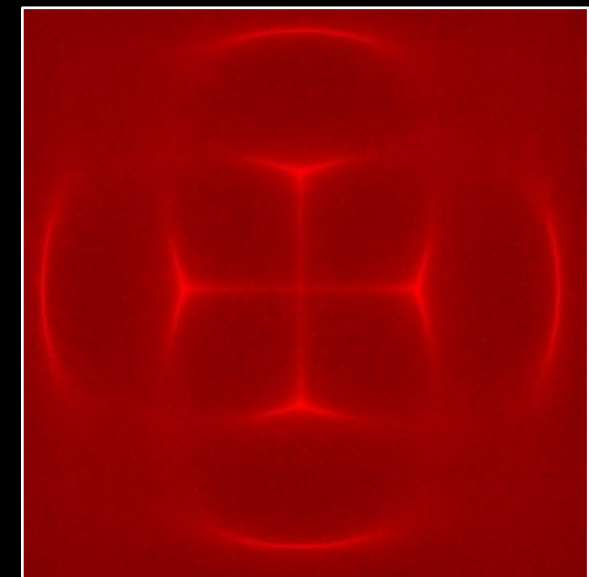
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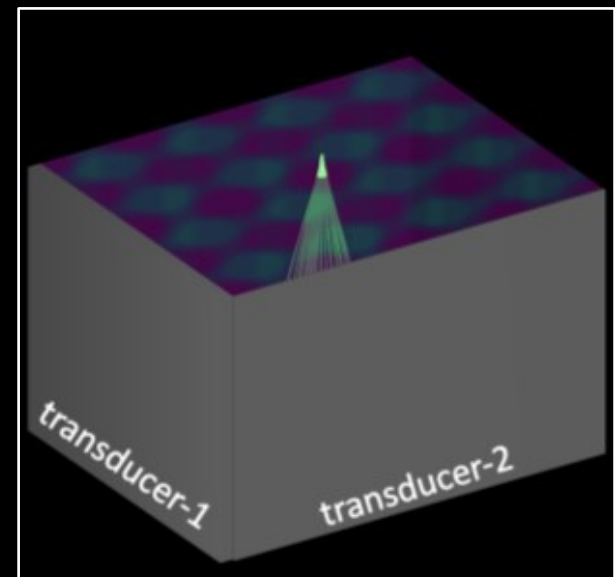
time-of-flight  
imaging



non-line-of-sight  
imaging



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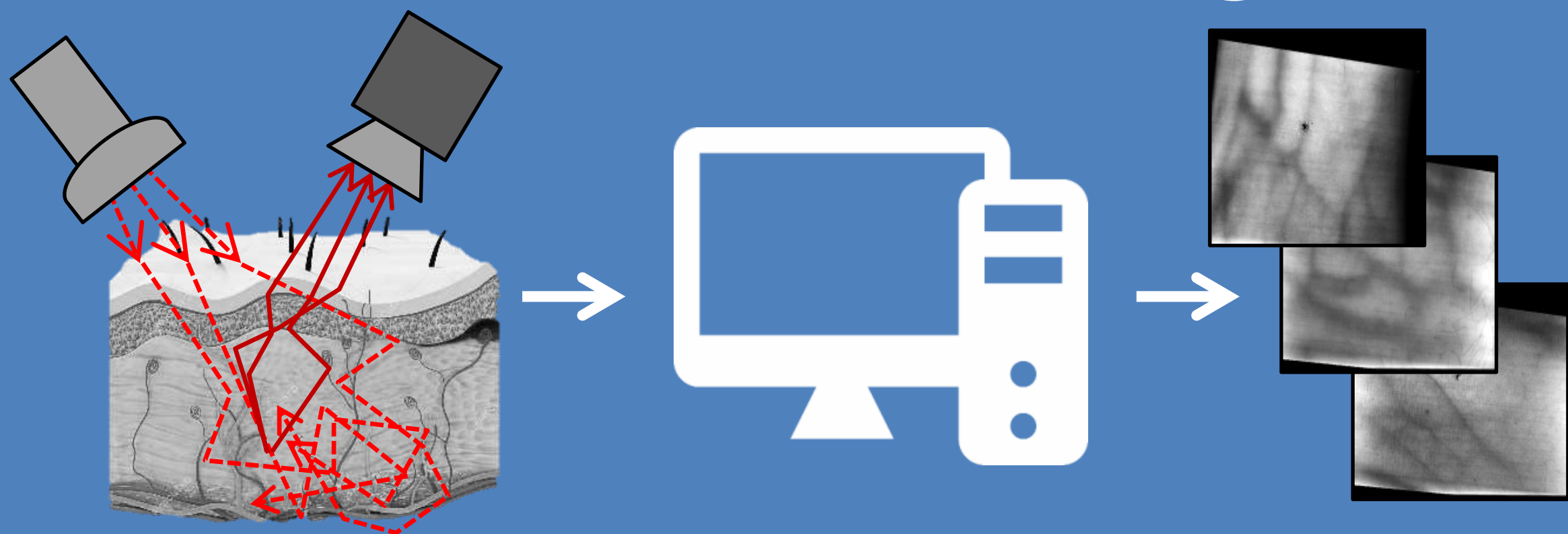


ultrafast light  
scanning



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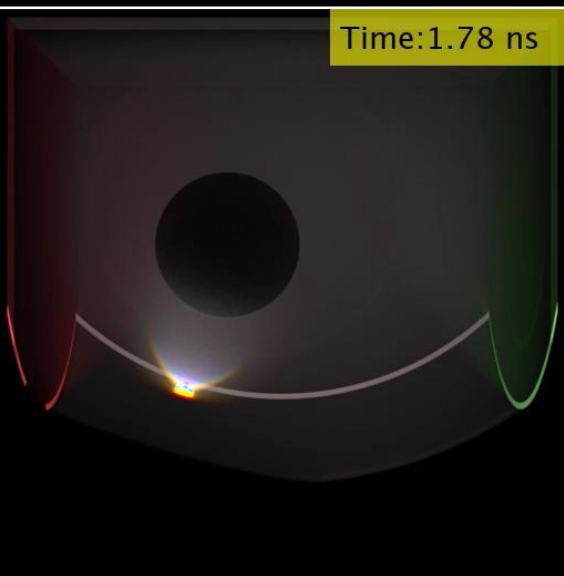


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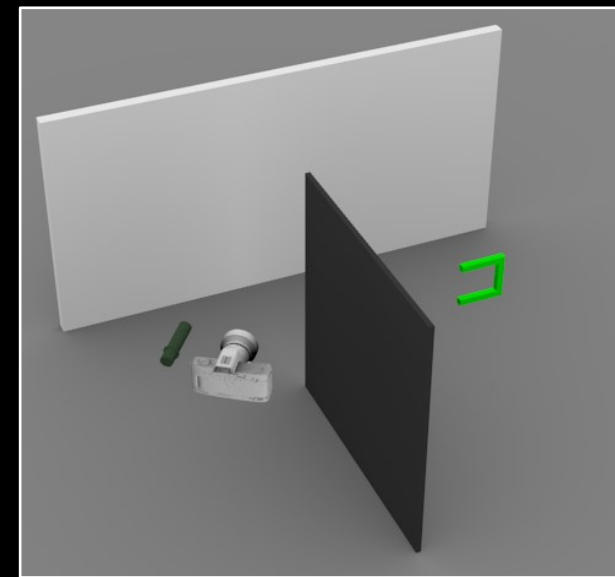
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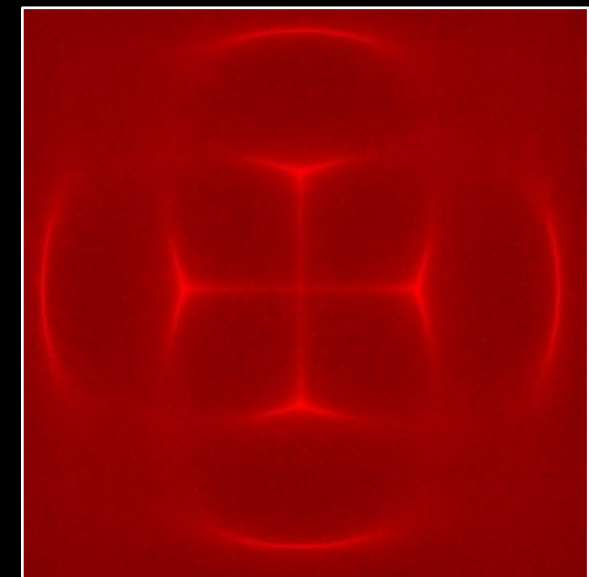
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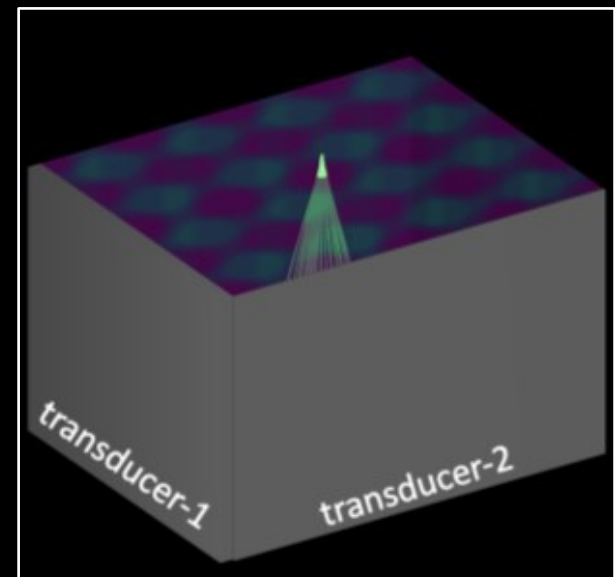
time-of-flight  
imaging



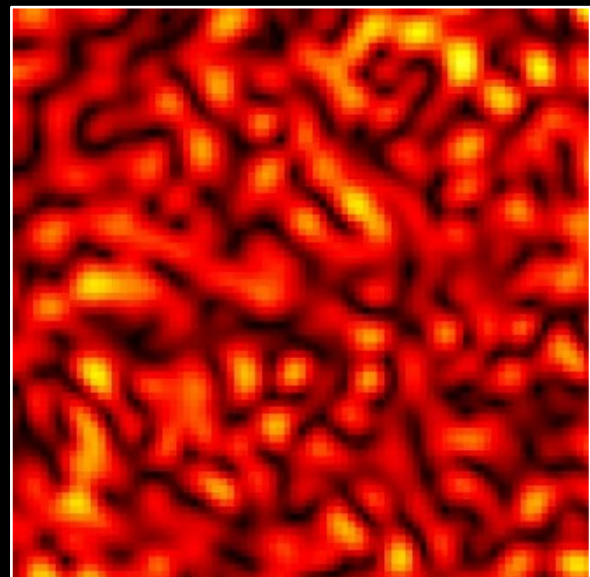
non-line-of-sight  
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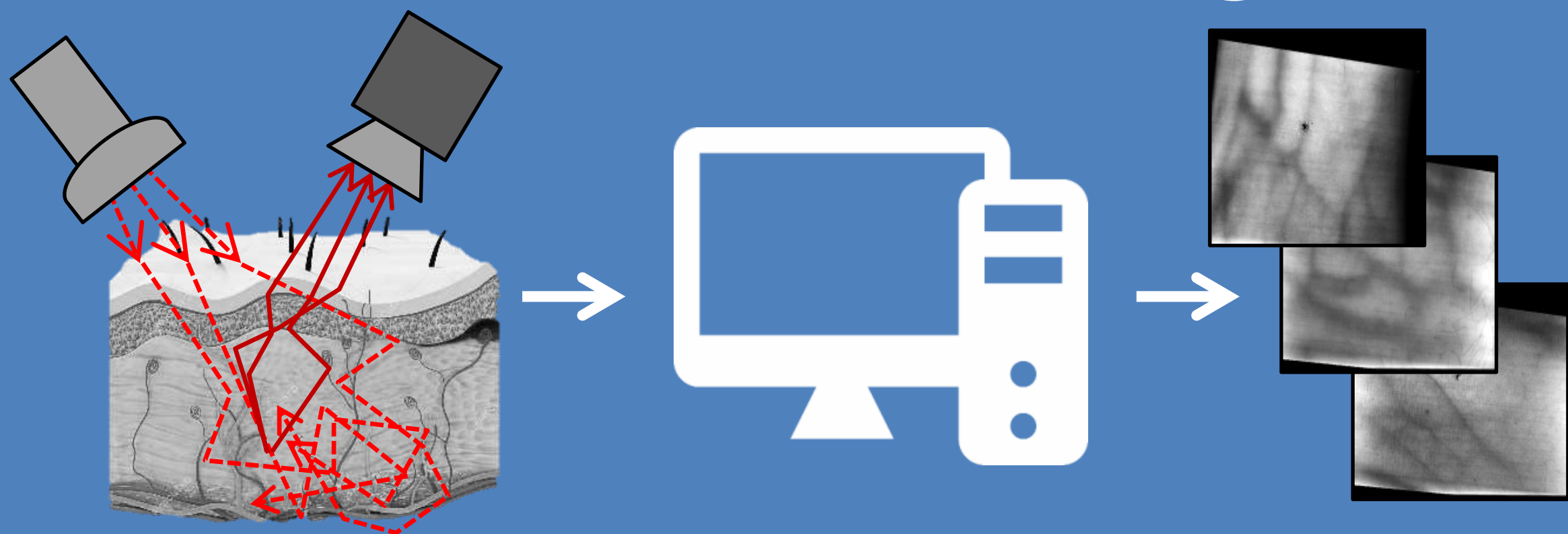


speckle  
imaging



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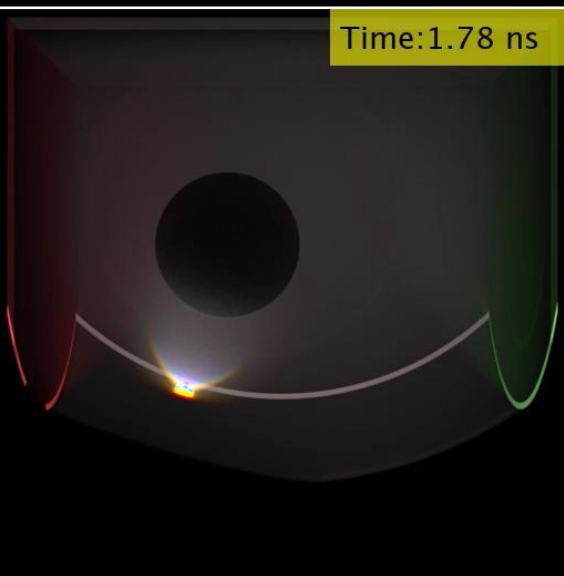


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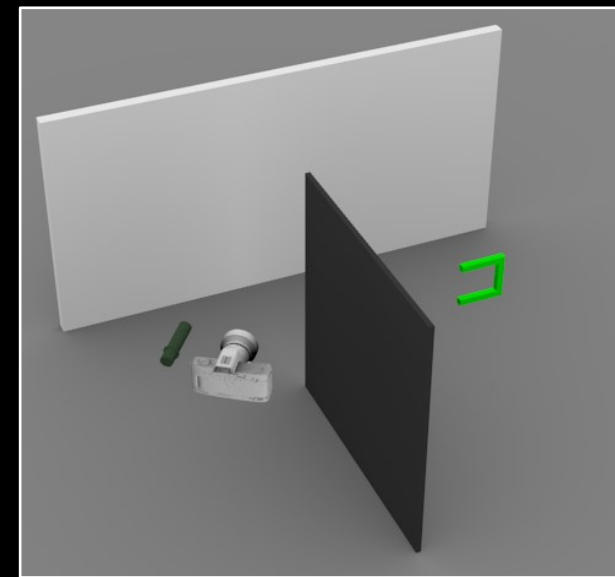
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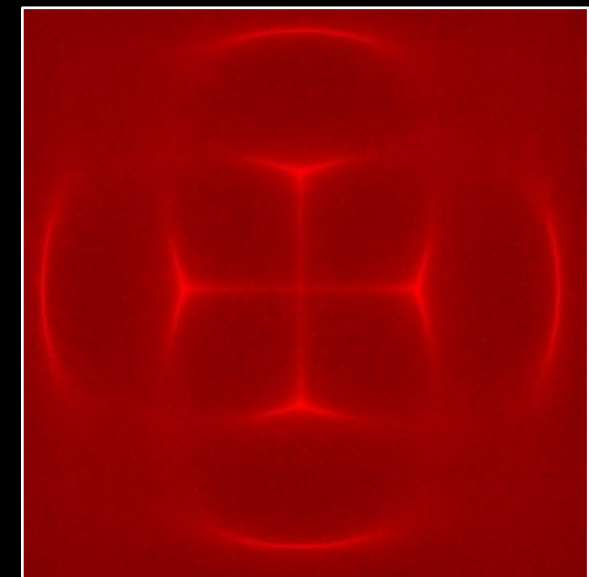
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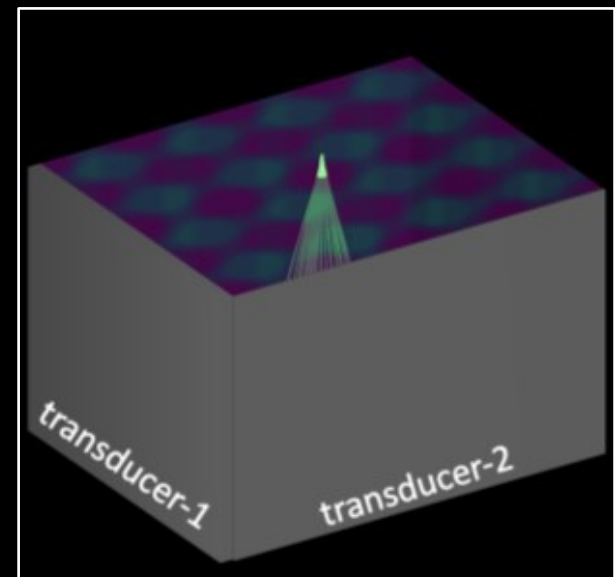
time-of-flight  
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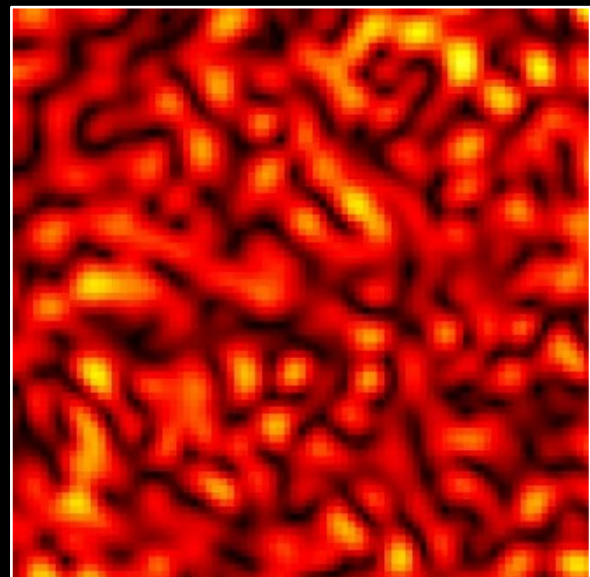
non-line-of-sight  
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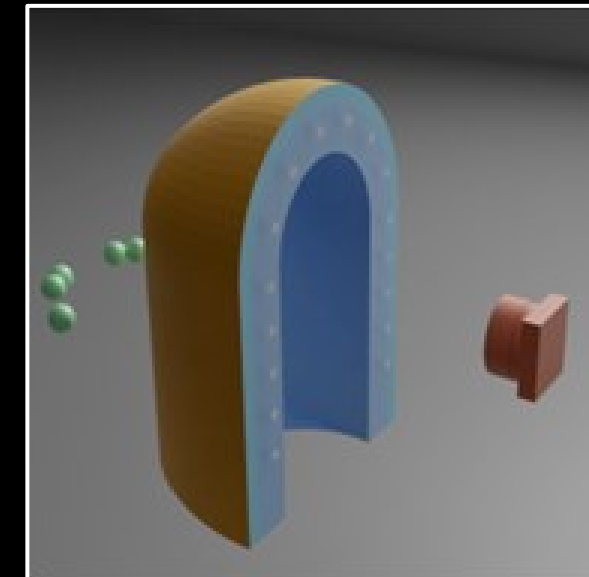
acousto-optic  
lensing



ultrafast light  
scanning



speckle  
imaging

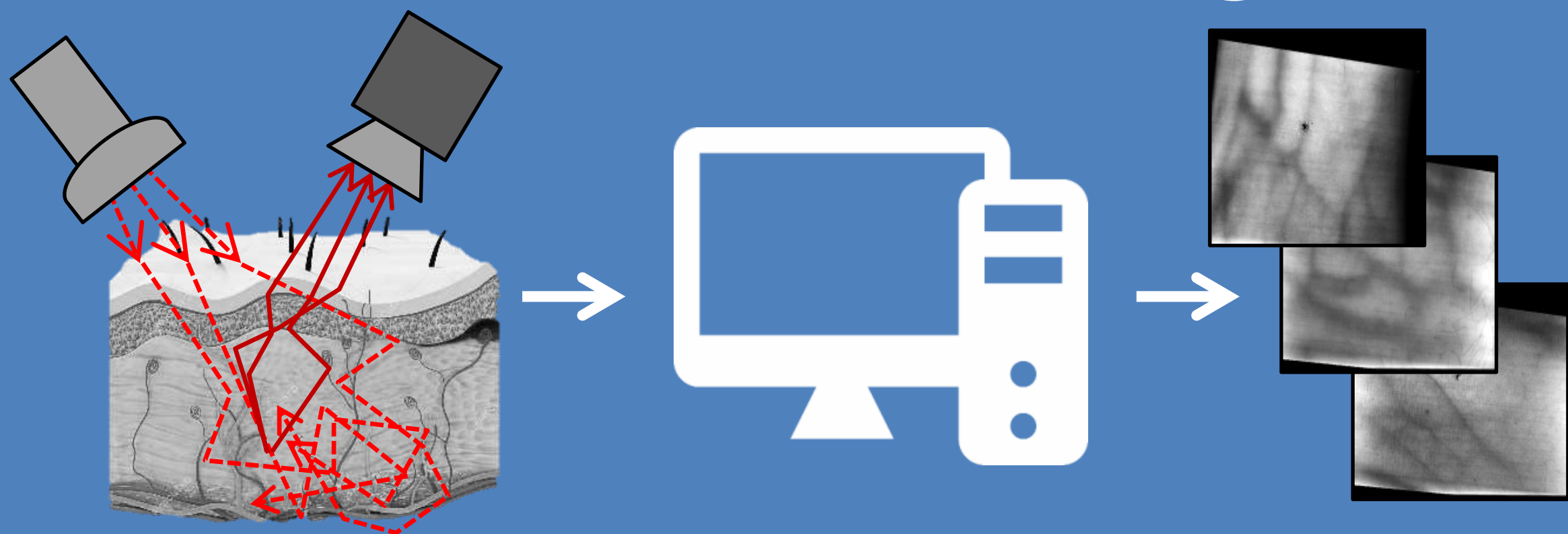


tactile sensor  
design



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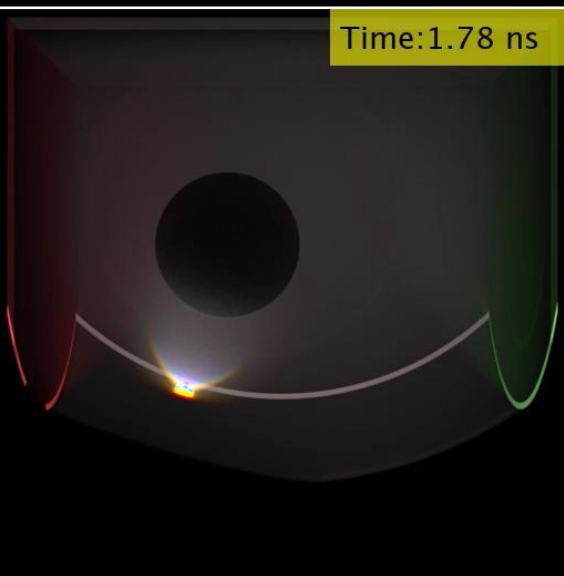


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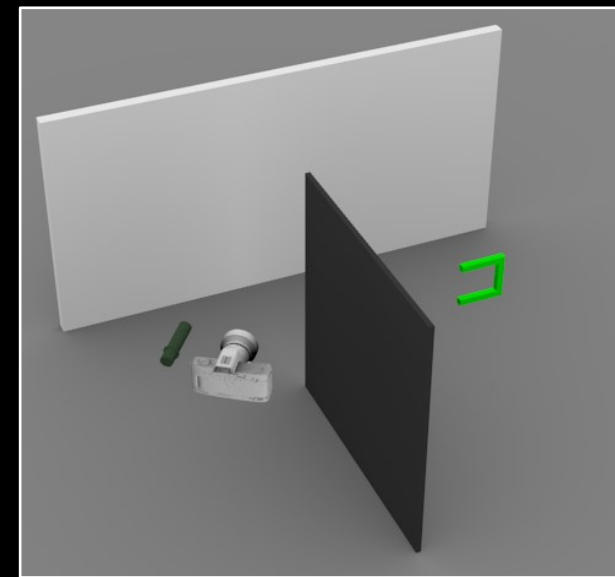
## inverse rendering



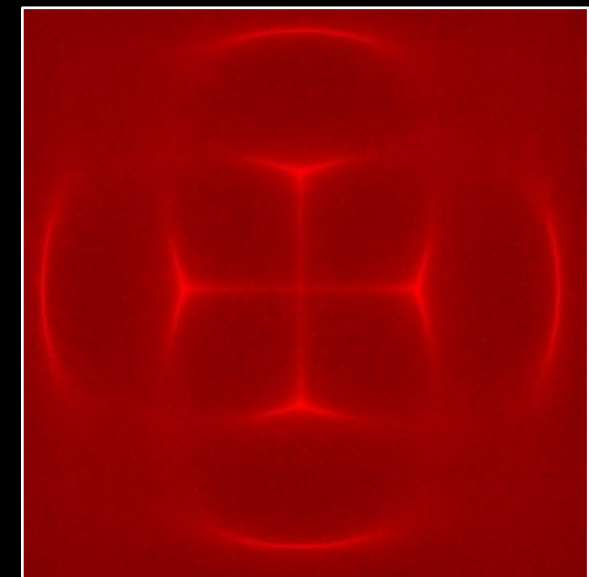
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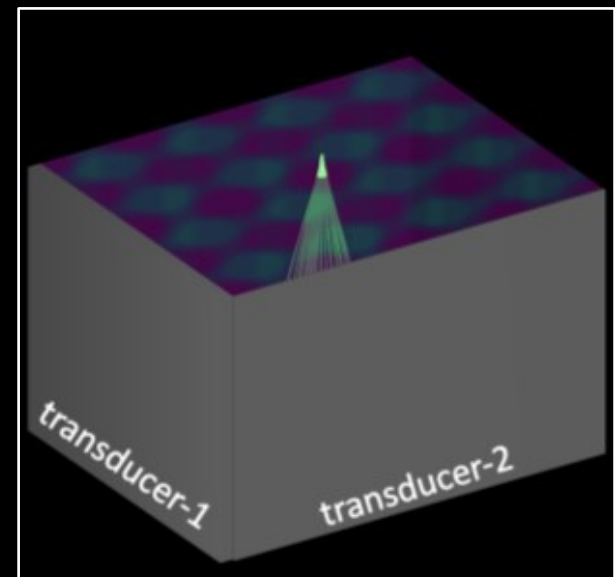
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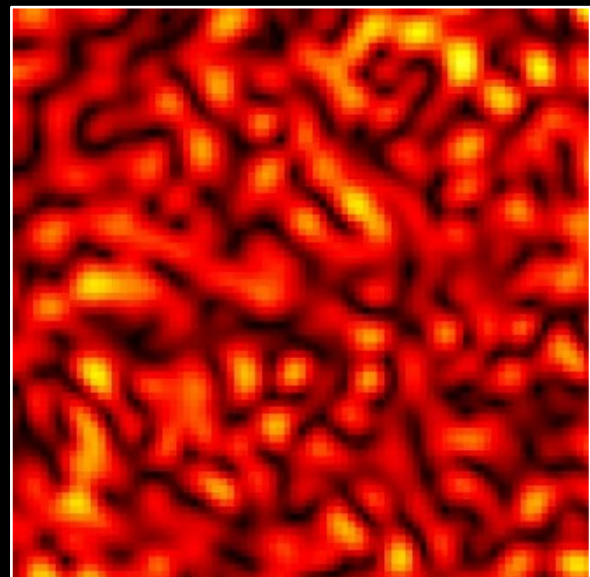
non-line-of-sight  
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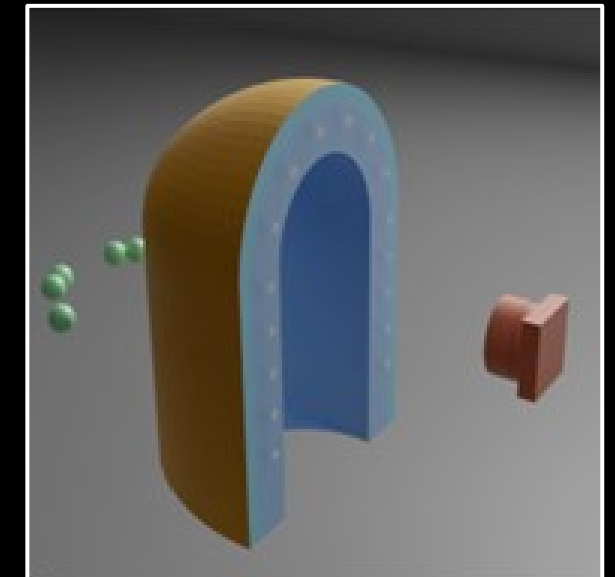
acousto-optic  
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speckle  
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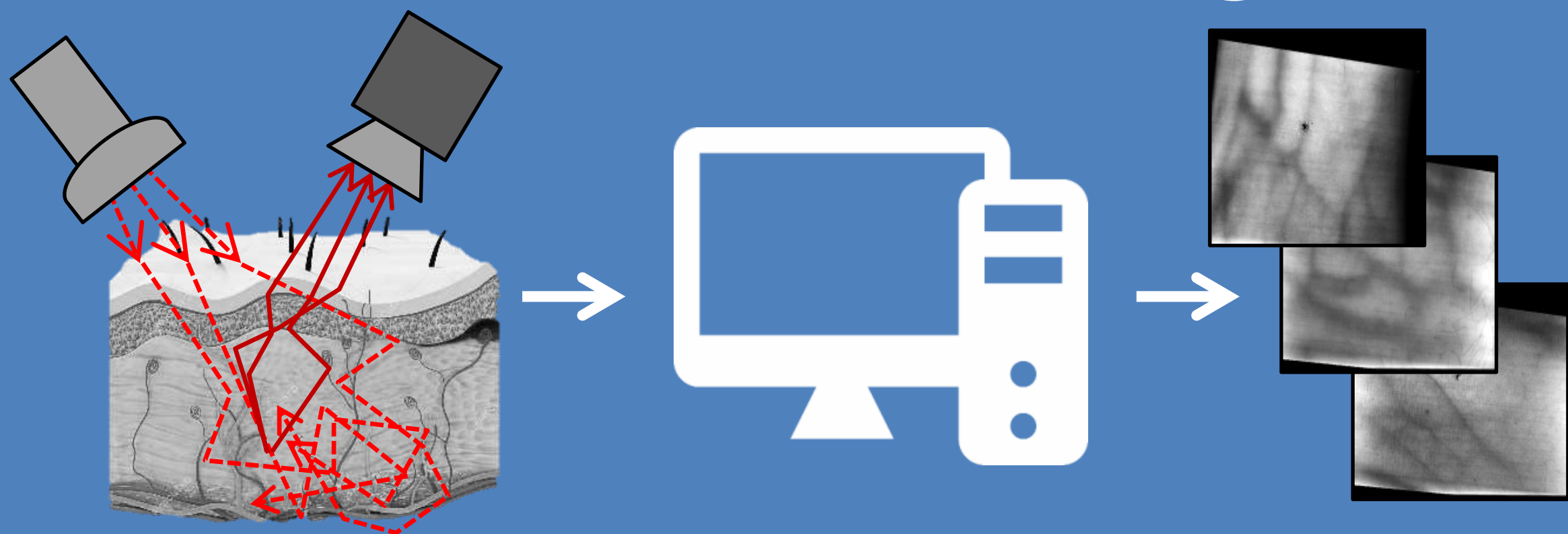


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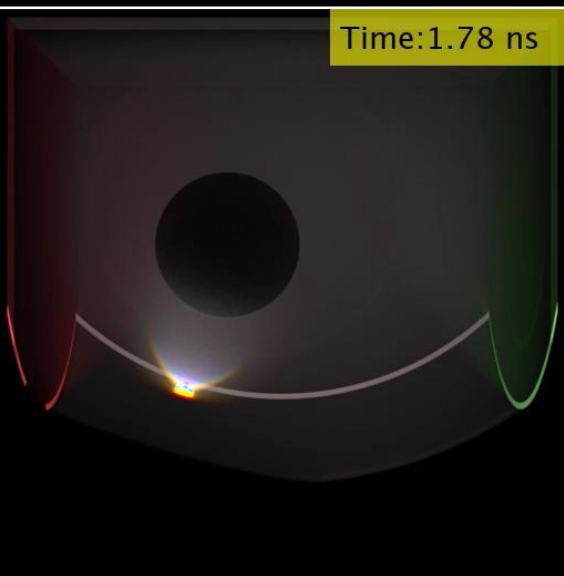


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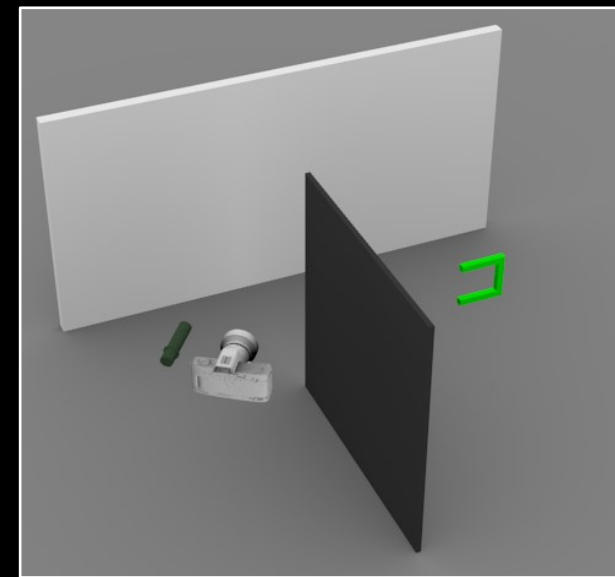
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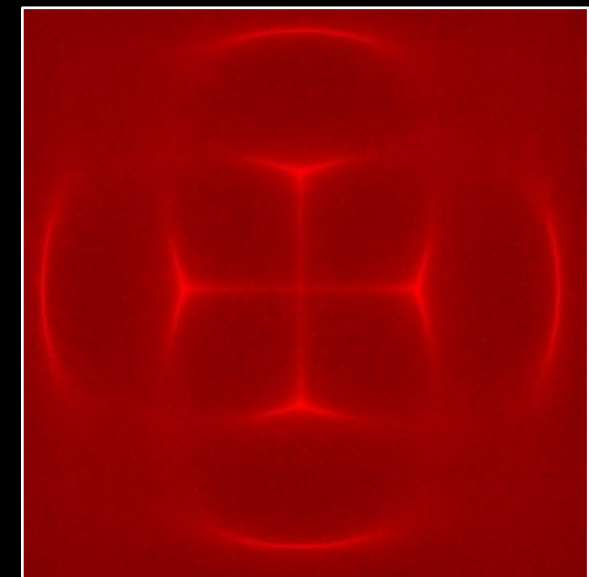
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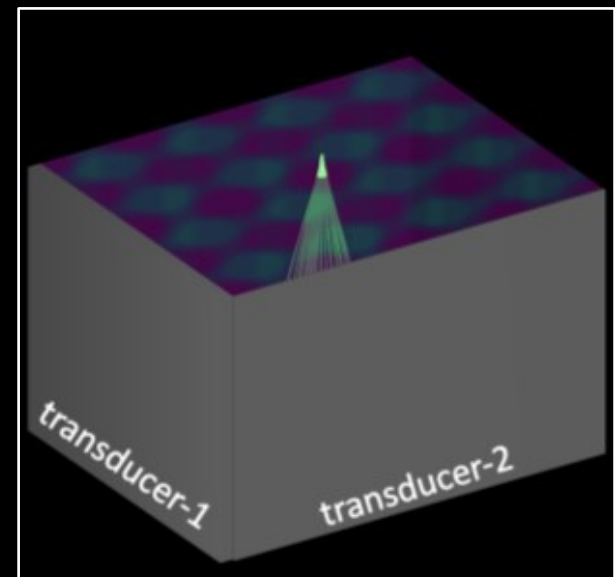
time-of-flight  
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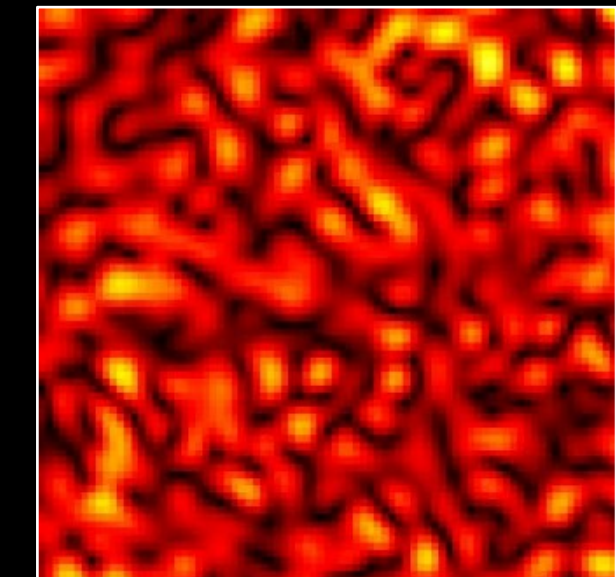
non-line-of-sight  
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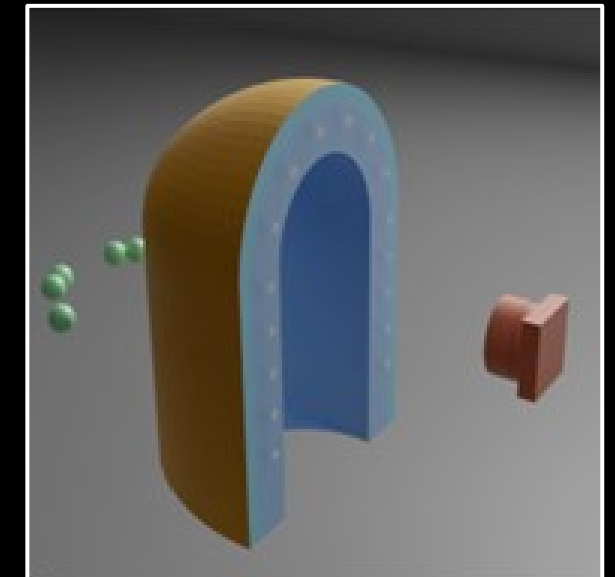
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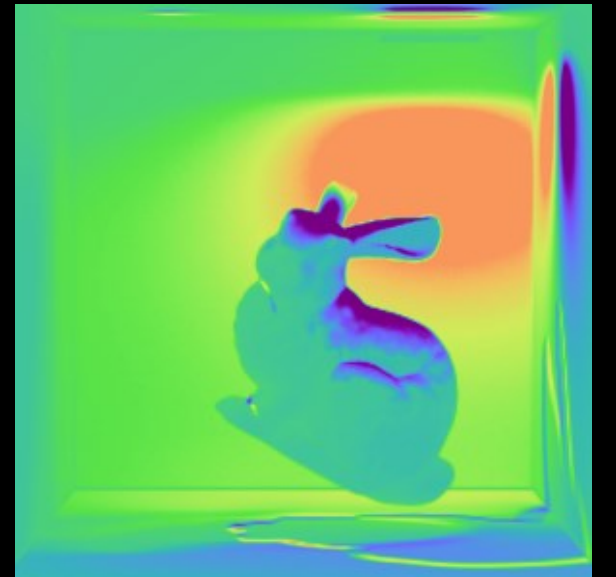
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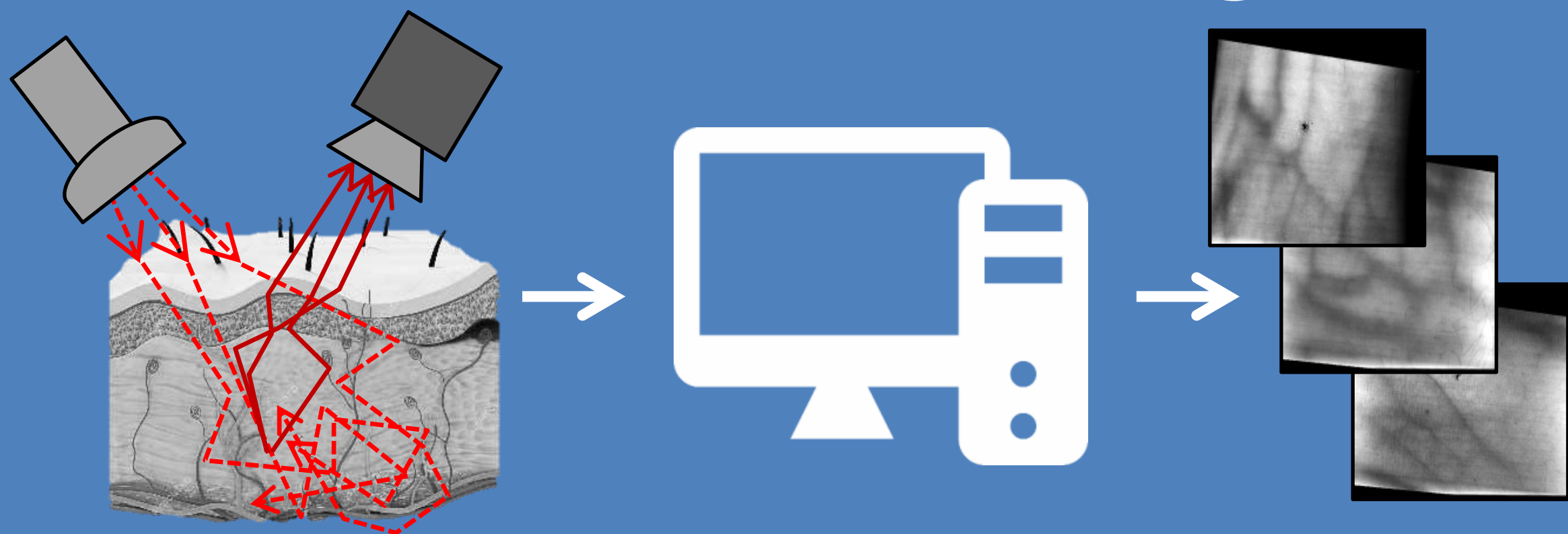


differentiable  
rendering



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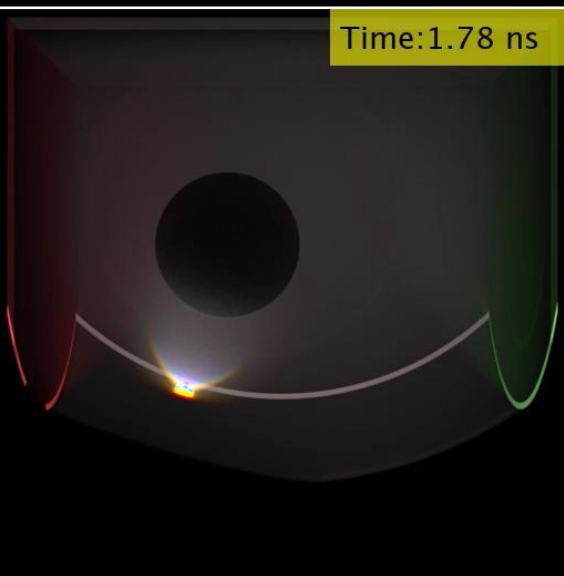


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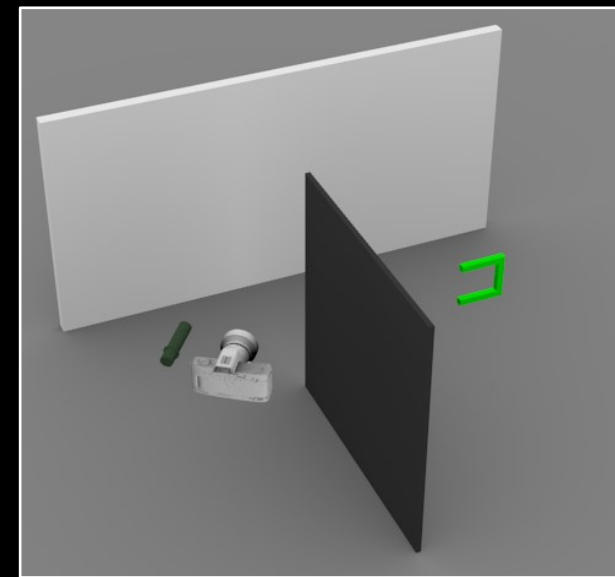
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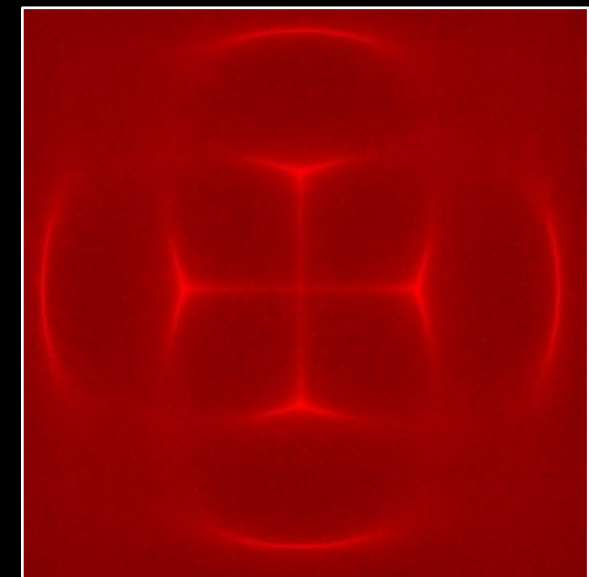
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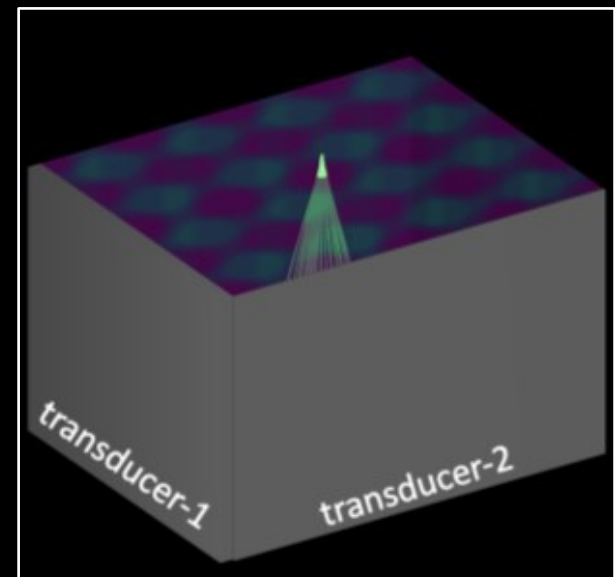
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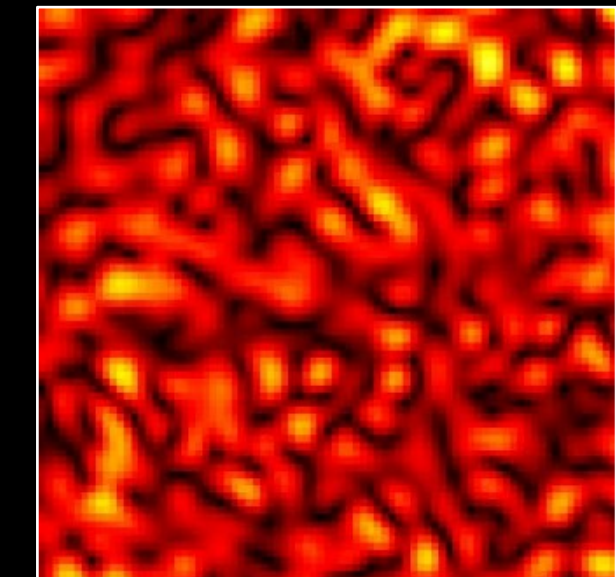
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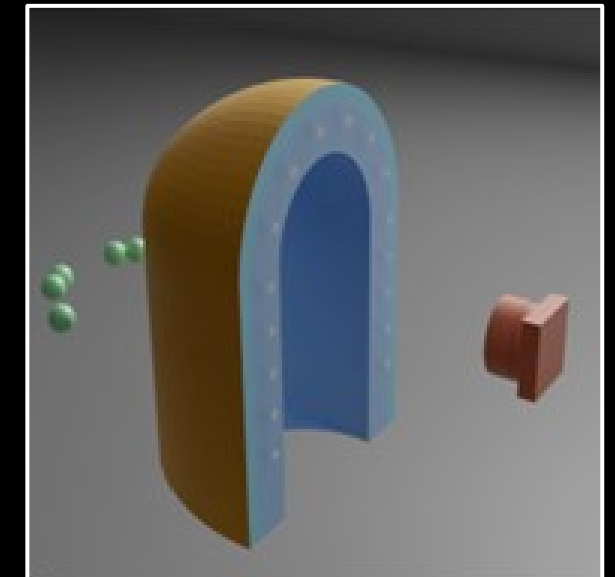
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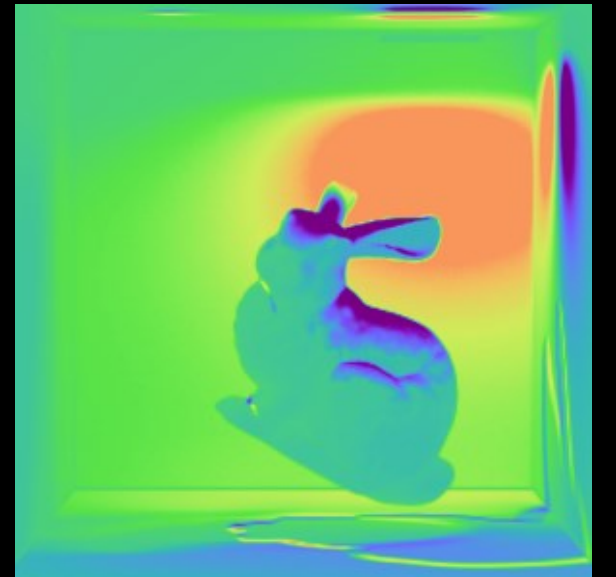
ultrafast light  
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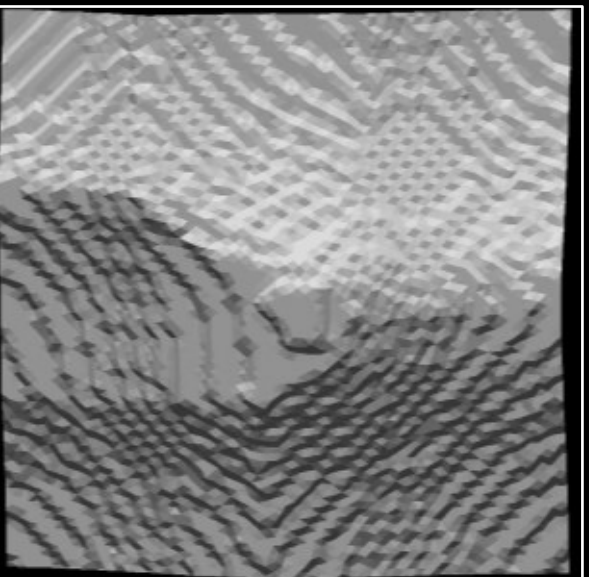
speckle  
imaging



tactile sensor  
design



differentiable  
rendering



inverse  
problems



# Complex light transport



After [Ritschel et al 2011]



# Complex light transport



After [Ritschel et al 2011]



# Complex light transport





# Complex light transport





# Complex light transport



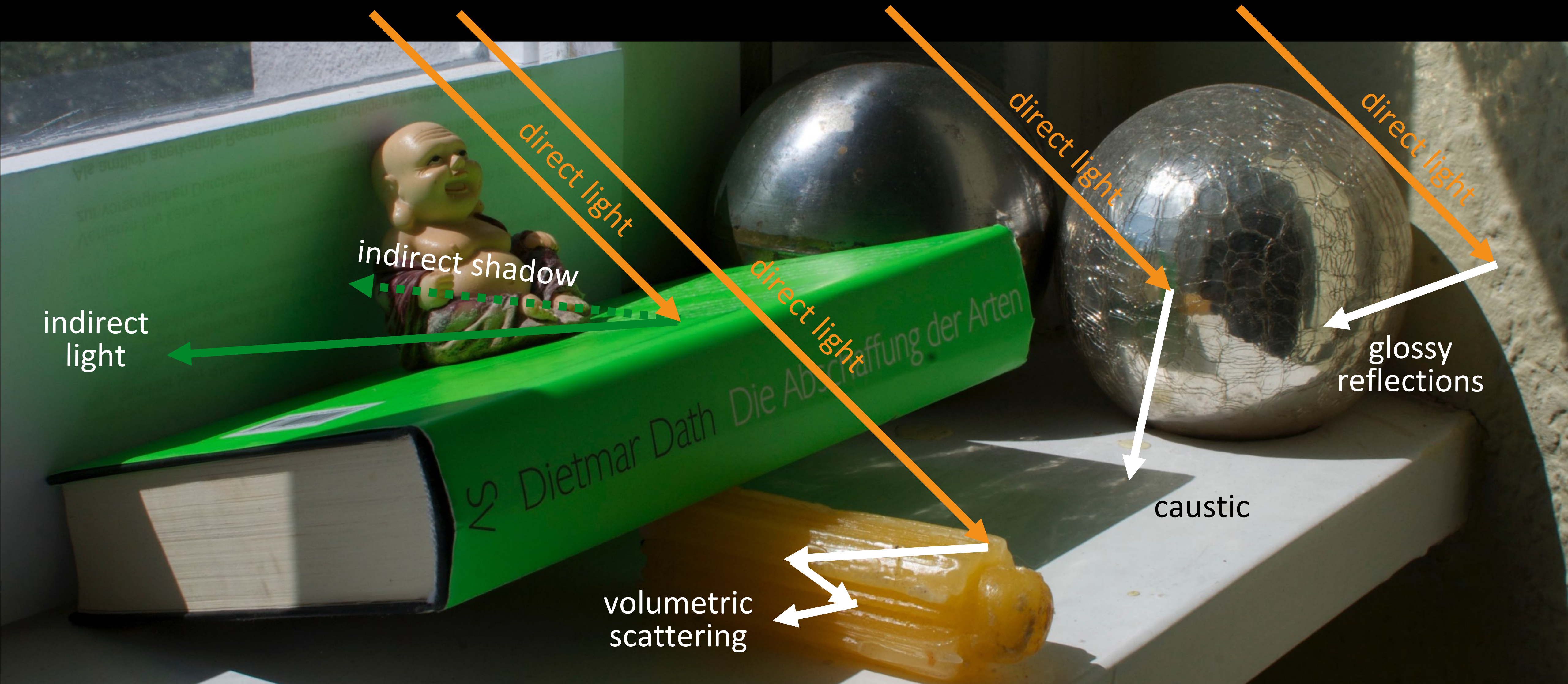


# Complex light transport





# Complex light transport





# Path integral form of light transport

image  $I = \int_{\mathcal{P}} W_e(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$

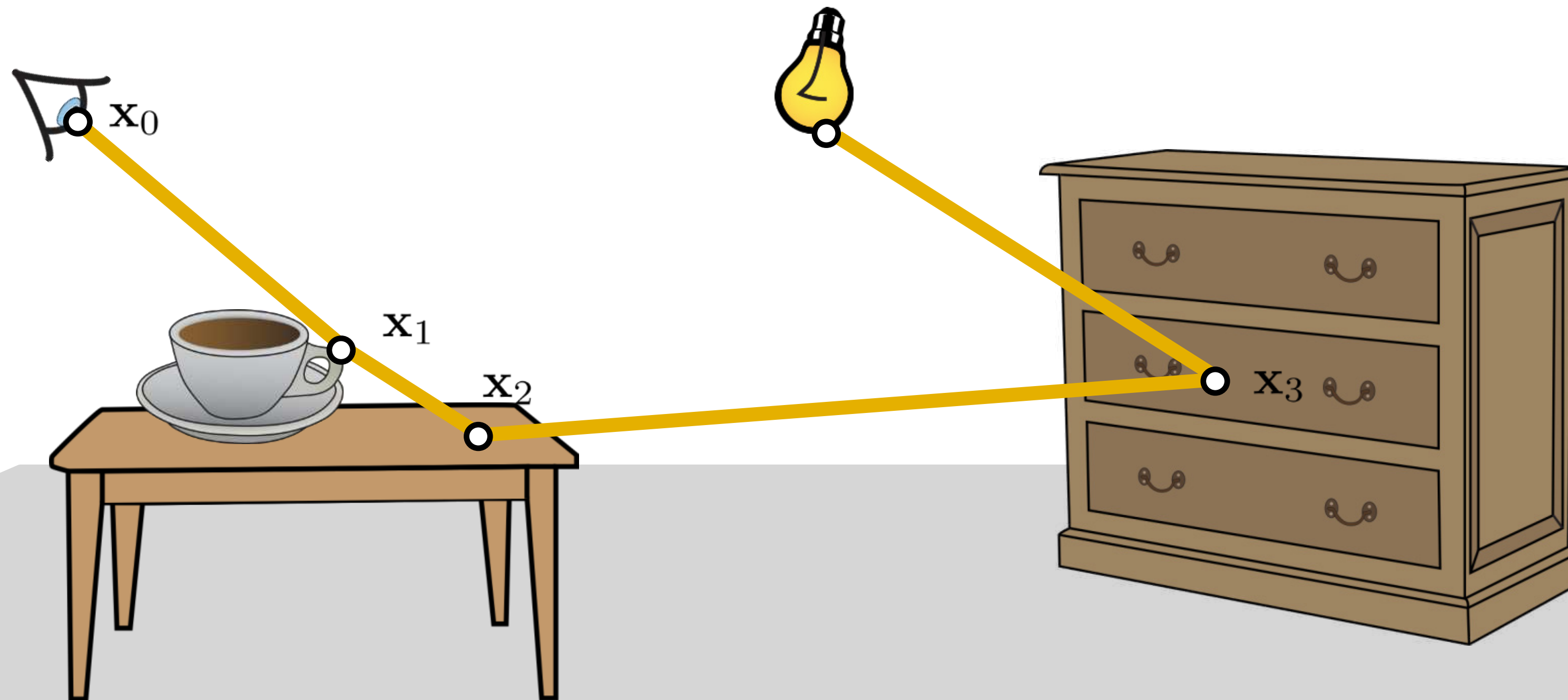




# Path integral form of light transport

image  $I = \int_{\mathcal{P}} W_e(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$  light path

space of all light paths



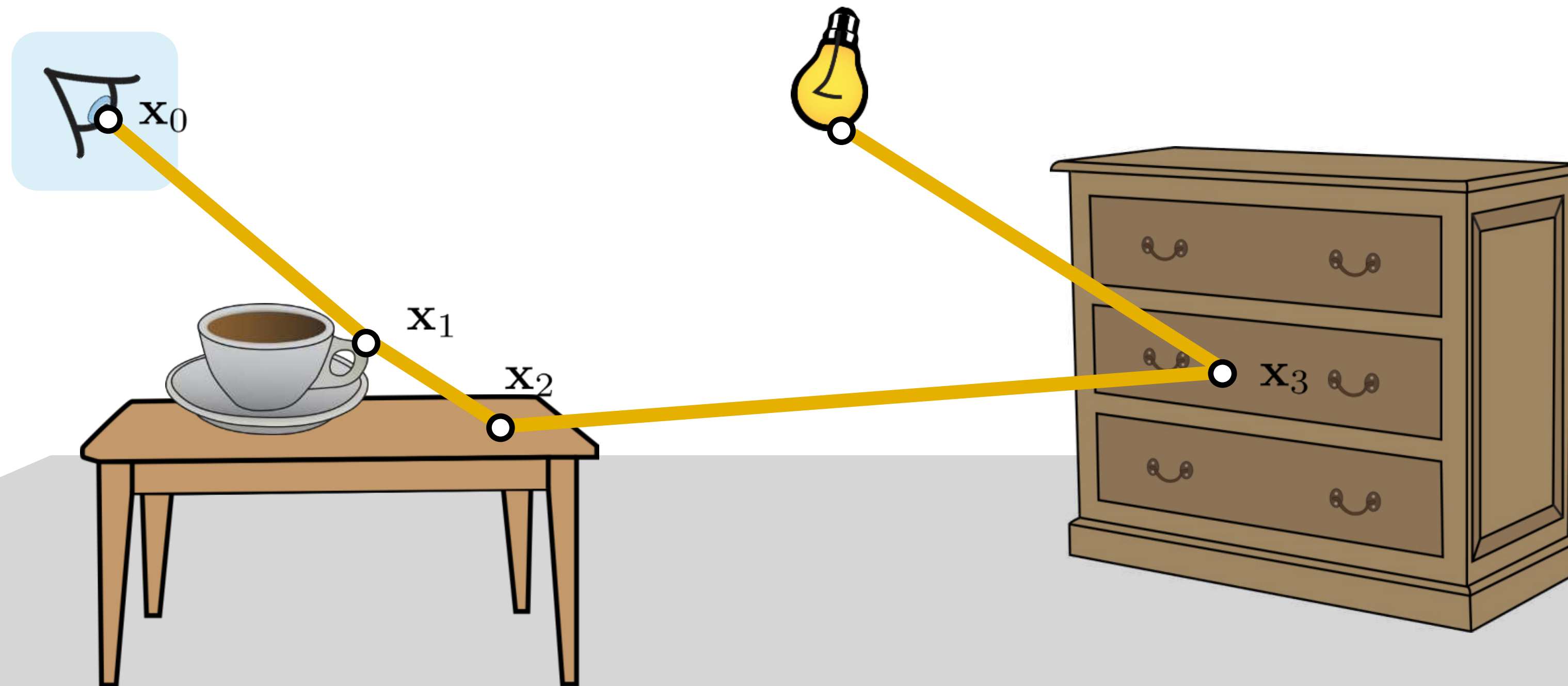


# Path integral form of light transport

image

$$I = \int_{\mathcal{P}} \overset{\text{sensor weight}}{W_e(\mathbf{x}_0, \mathbf{x}_1)} L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}}) d\bar{\mathbf{x}} \quad \text{light path}$$

space of all light paths





# Path integral form of light transport

image

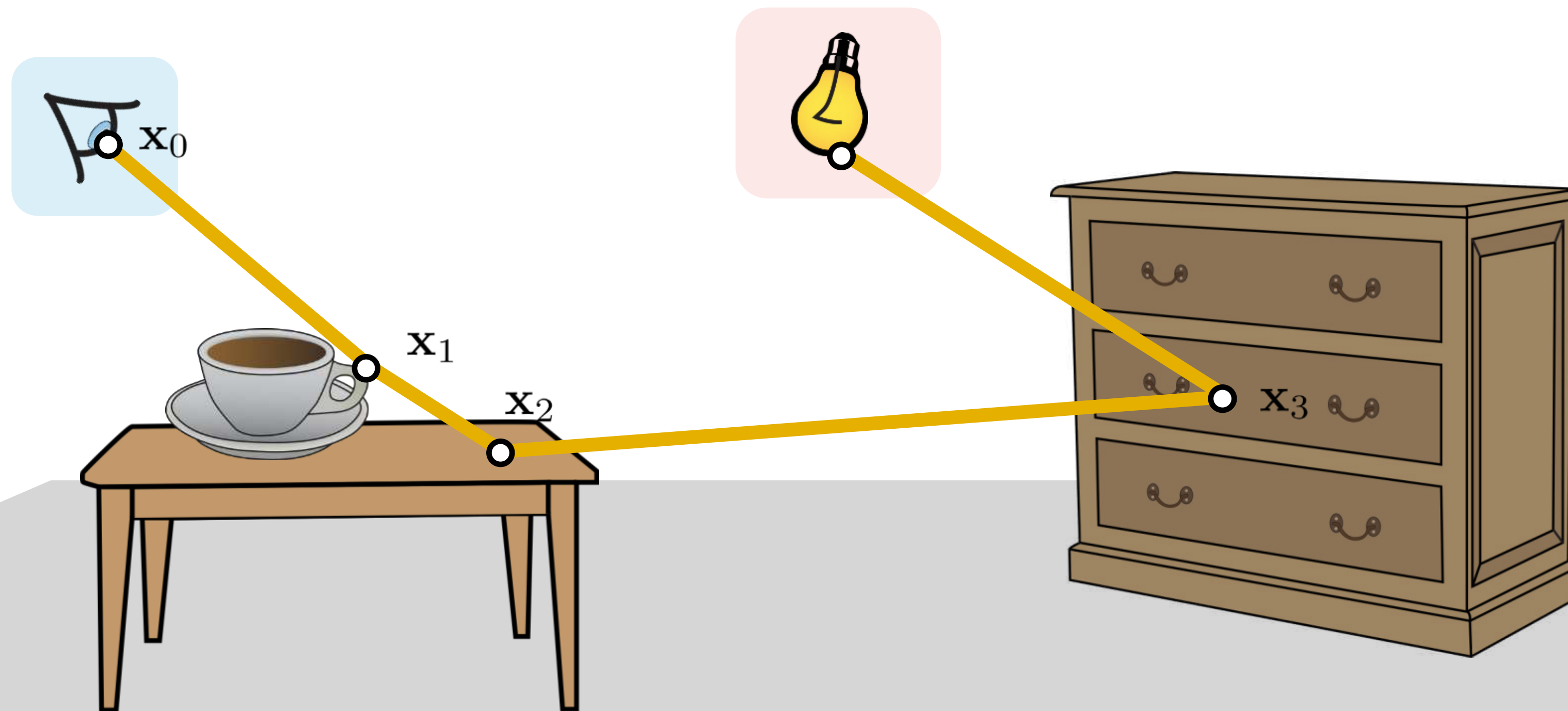
$$I = \int_{\mathcal{P}} W_e(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

space of all light paths

sensor weight

source weight

light path





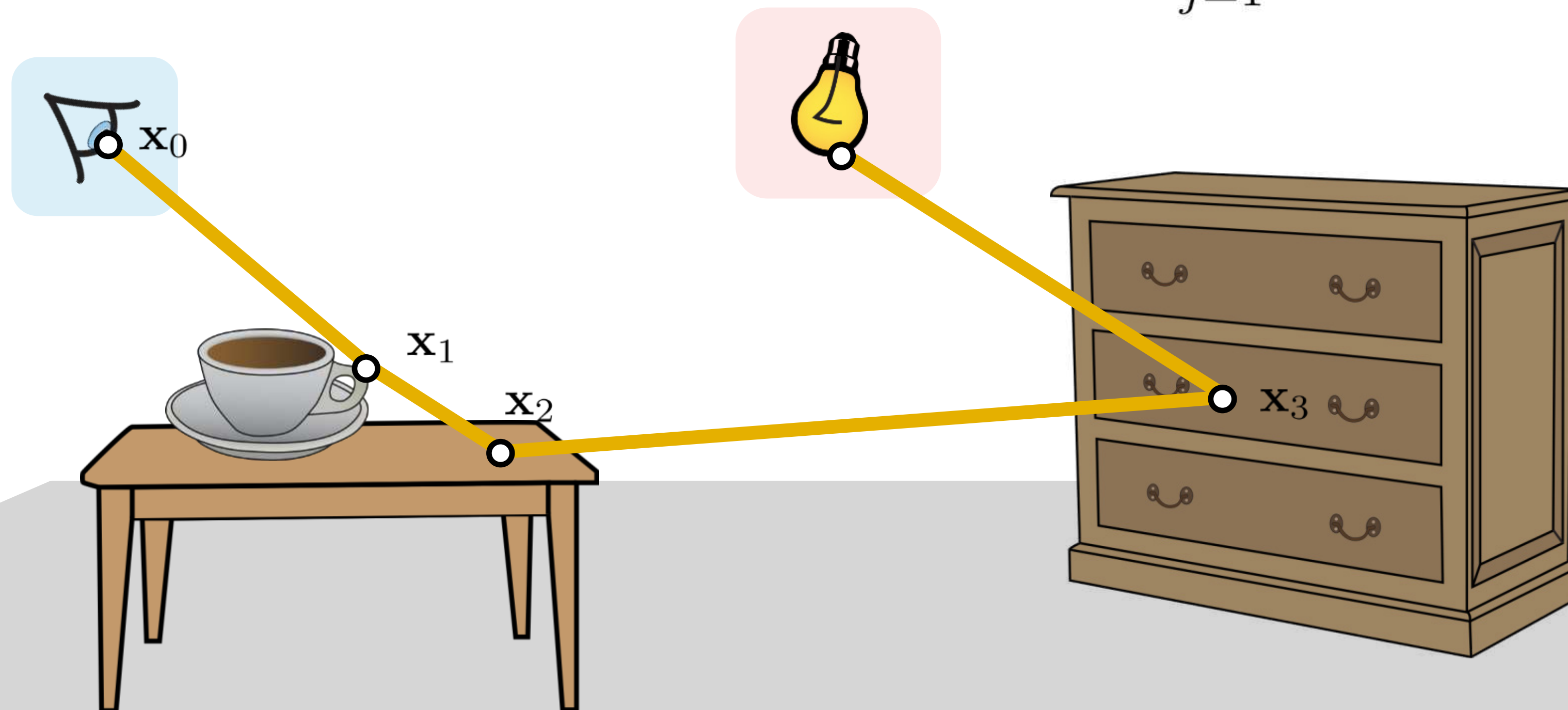
# Path integral form of light transport

image

$$I = \int_{\mathcal{P}} \overset{\text{sensor weight}}{W_e(\mathbf{x}_0, \mathbf{x}_1)} \overset{\text{source weight}}{L_e(\mathbf{x}_k, \mathbf{x}_{k-1})} \overset{\text{light path}}{T(\bar{\mathbf{x}})} d\bar{\mathbf{x}} \overset{\text{path throughput}}$$

space of all light paths

$$T(\bar{\mathbf{x}}) = G(\mathbf{x}_0, \mathbf{x}_1) \prod_{j=1}^{k-1} f(\mathbf{x}_j, \mathbf{x}_{j+1}, \mathbf{x}_{j-1}) G(\mathbf{x}_j, \mathbf{x}_{j+1})$$





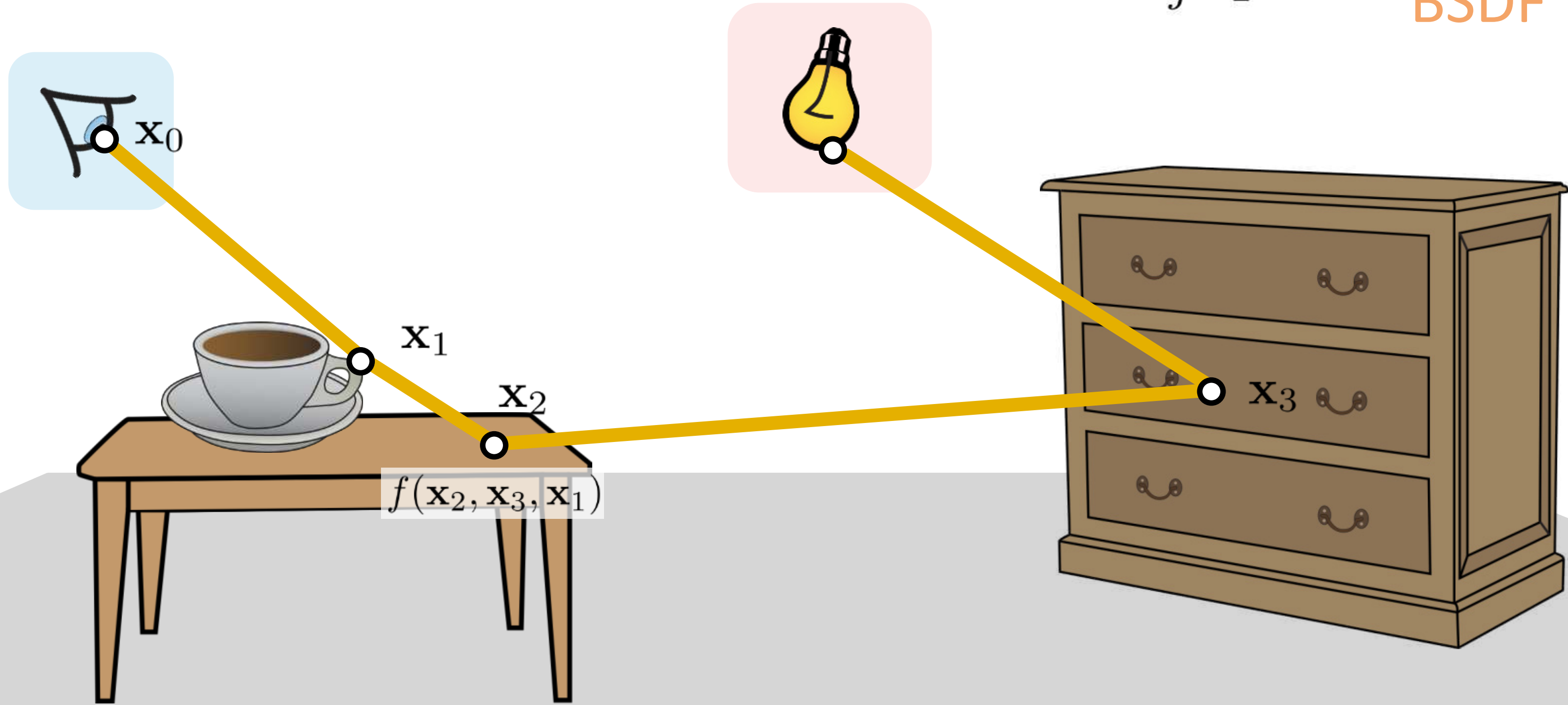
# Path integral form of light transport

image  $I = \int_{\mathcal{P}} \overset{\text{sensor weight}}{W_e(\mathbf{x}_0, \mathbf{x}_1)} \overset{\text{source weight}}{L_e(\mathbf{x}_k, \mathbf{x}_{k-1})} \overset{\text{light path}}{T(\bar{\mathbf{x}})} d\bar{\mathbf{x}}$

$\mathcal{P}$  space of all light paths

$T(\bar{\mathbf{x}}) = G(\mathbf{x}_0, \mathbf{x}_1) \prod_{j=1}^{k-1} \overset{\text{BSDF}}{f(\mathbf{x}_j, \mathbf{x}_{j+1}, \mathbf{x}_{j-1})} G(\mathbf{x}_j, \mathbf{x}_{j+1})$

path throughput





# Path integral form of light transport

image

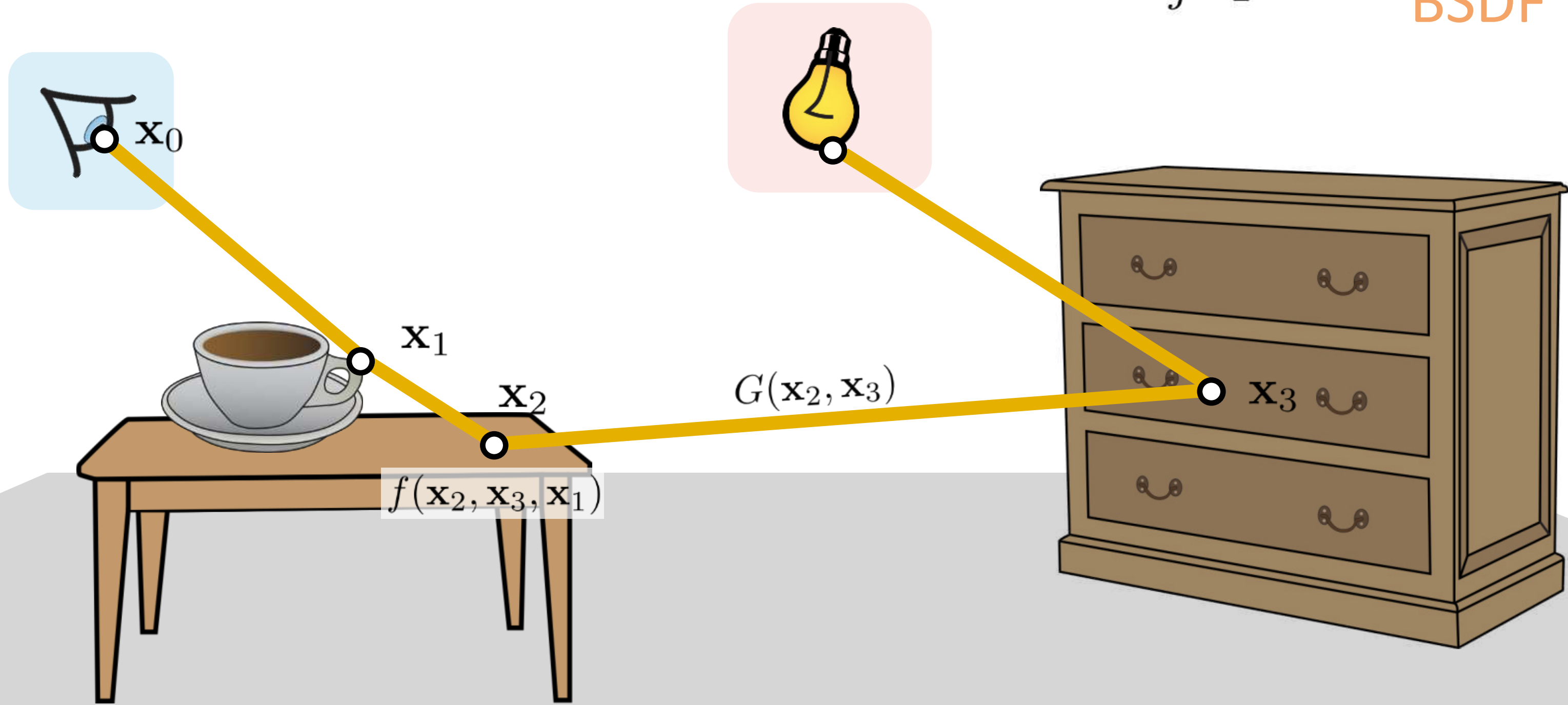
$$I = \int_{\mathcal{P}} W_e(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

sensor weight      source weight      light path

space of all light paths      path throughput

$$T(\bar{\mathbf{x}}) = G(\mathbf{x}_0, \mathbf{x}_1) \prod_{j=1}^{k-1} f(\mathbf{x}_j, \mathbf{x}_{j+1}, \mathbf{x}_{j-1}) G(\mathbf{x}_j, \mathbf{x}_{j+1})$$

BSDF      geometry





# Monte Carlo rendering

approximate image as 
$$I \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

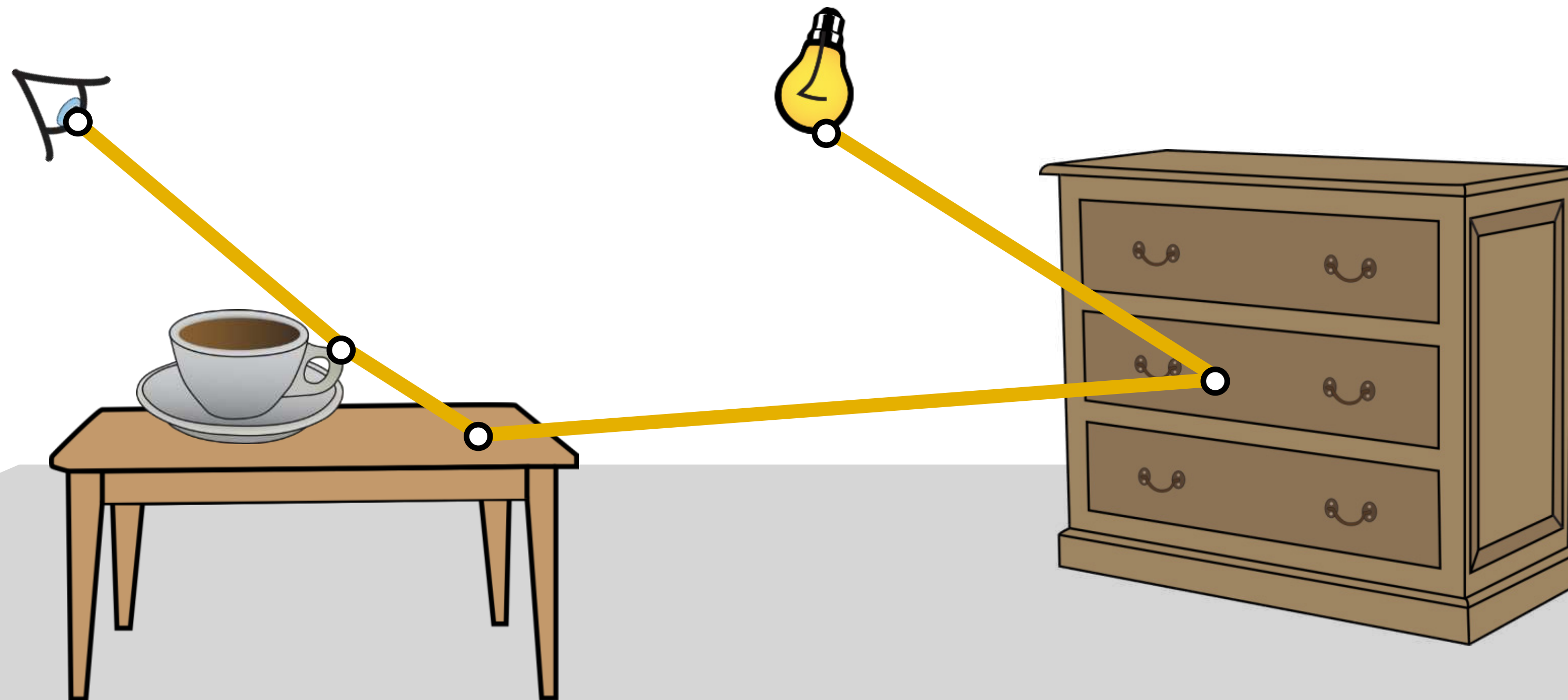




# Monte Carlo rendering

approximate image as  $I \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

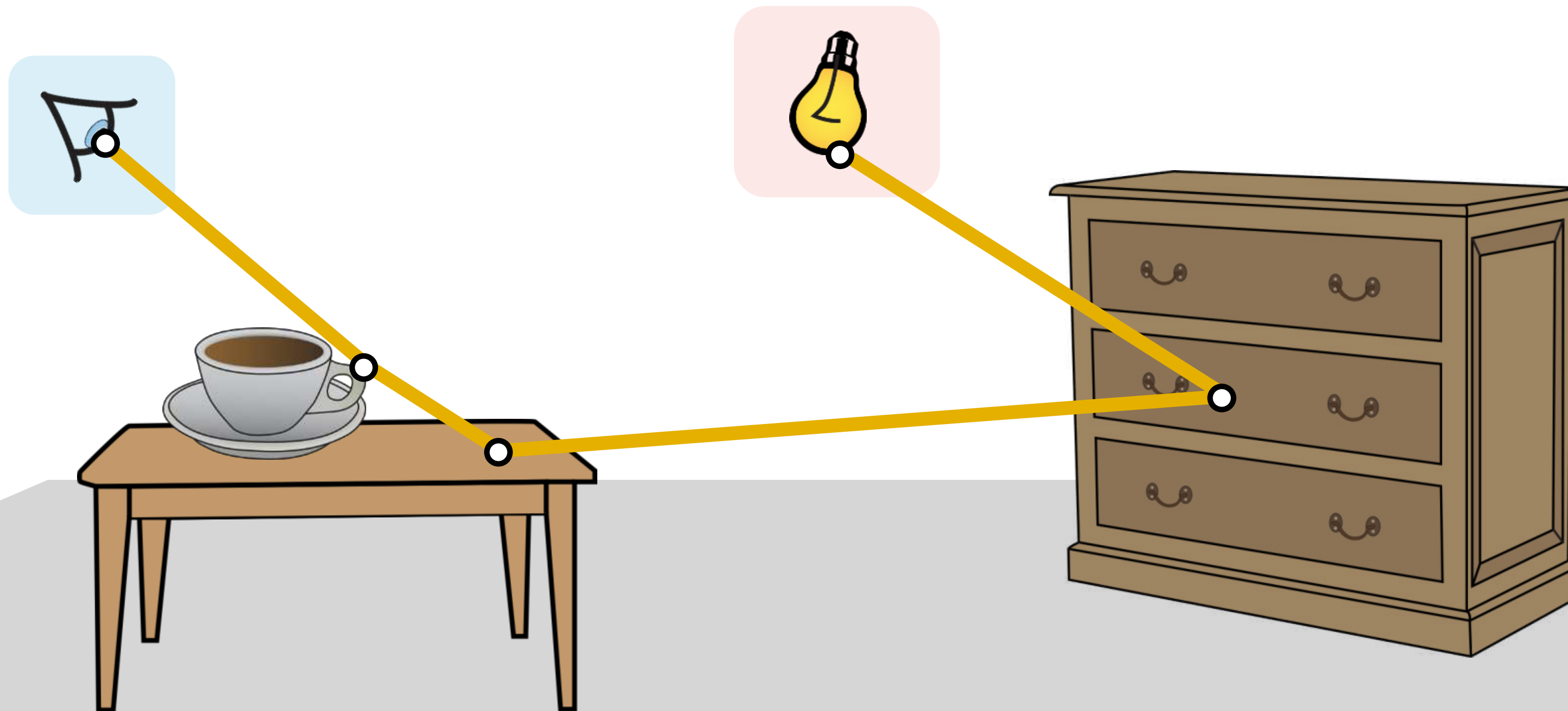




# Monte Carlo rendering

approximate image as  $I \approx \frac{1}{N} \sum_{i=1}^N \frac{\overset{\text{sensor weight}}{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1})} \overset{\text{source weight}}{L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1})} \overset{\text{path throughput}}{T(\bar{\mathbf{x}}_i)}}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

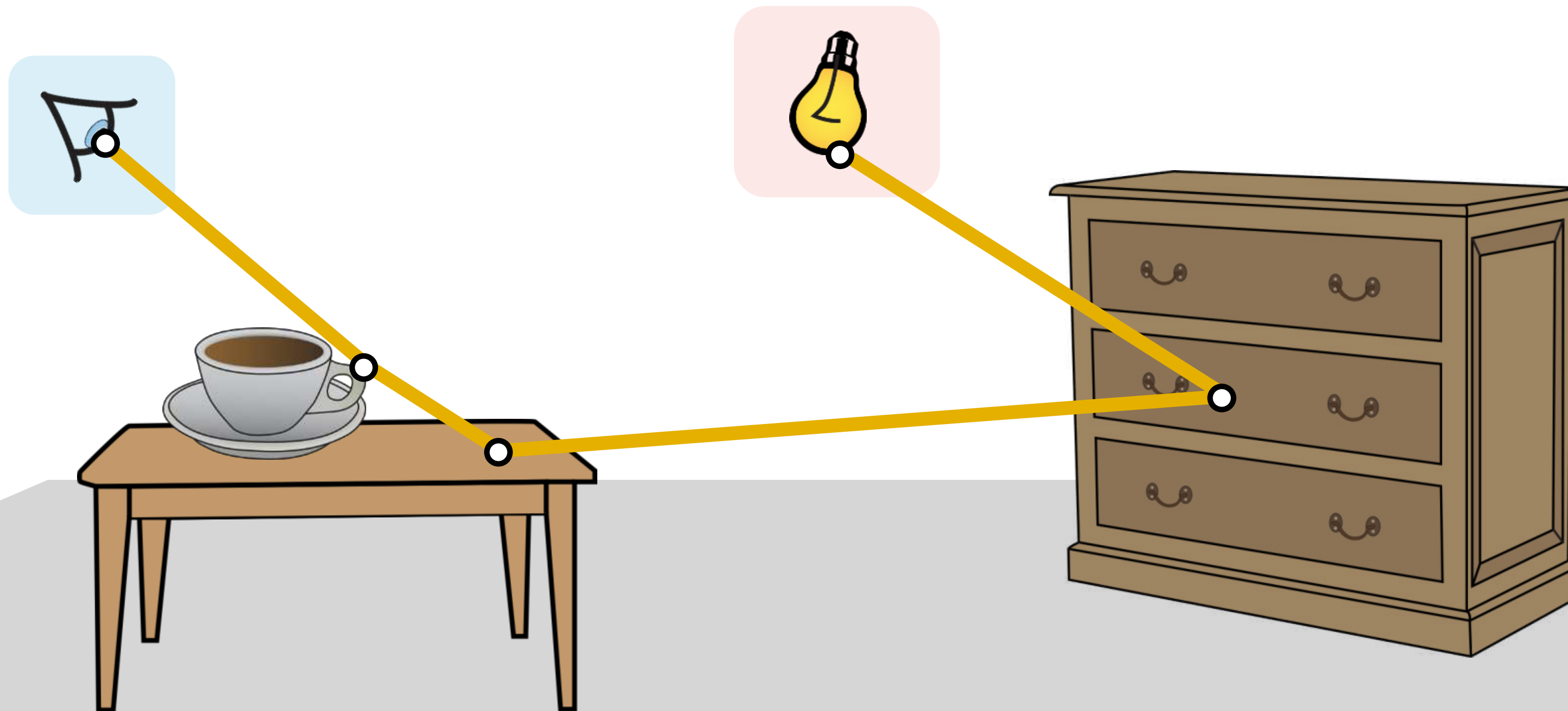




# Monte Carlo rendering

approximate image as  $I \approx \frac{1}{N} \sum_{i=1}^N \frac{\overset{\text{sensor weight}}{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1})} \overset{\text{source weight}}{L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1})} \overset{\text{path throughput}}{T(\bar{\mathbf{x}}_i)}}{\underset{\text{PDF of random path}}{p(\bar{\mathbf{x}}_i)}}$

sum over *randomly* sampled paths





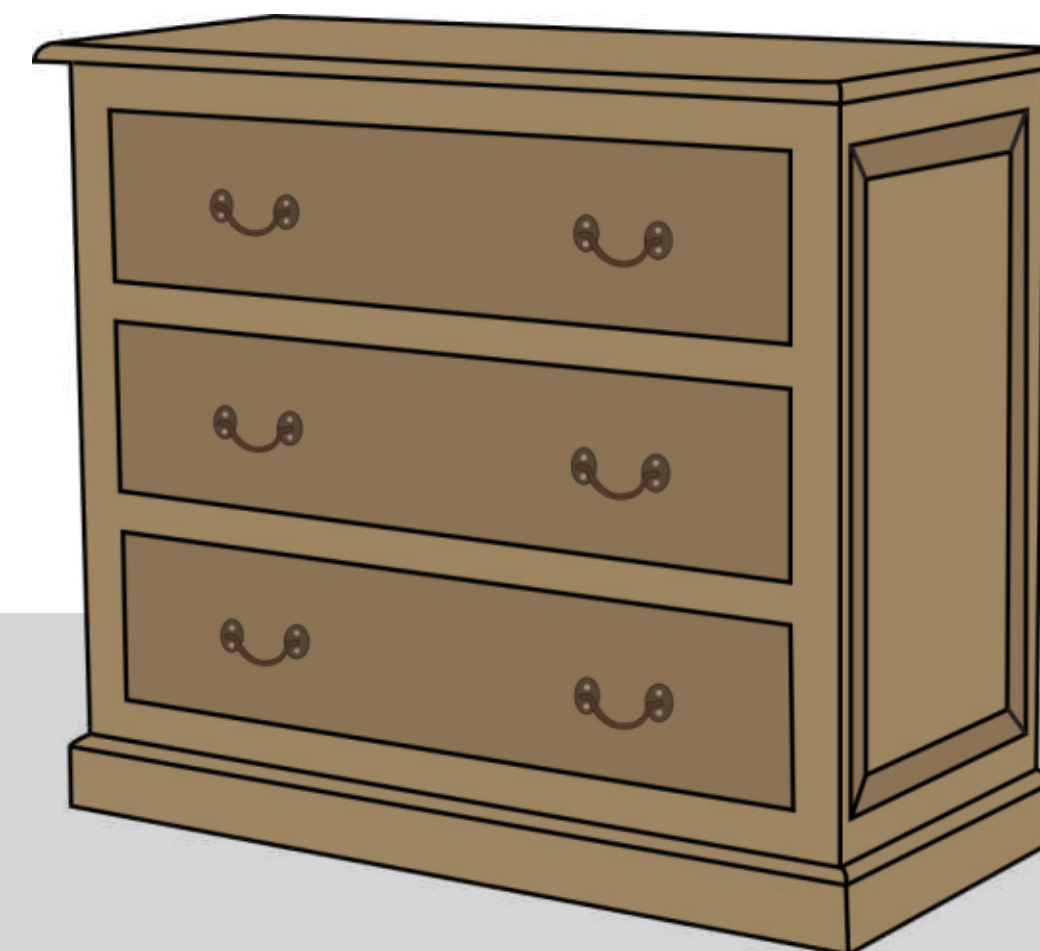
# Monte Carlo rendering

approximate image as  $I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

PDF of random path

*Path tracing*: sample path  
starting from sensor





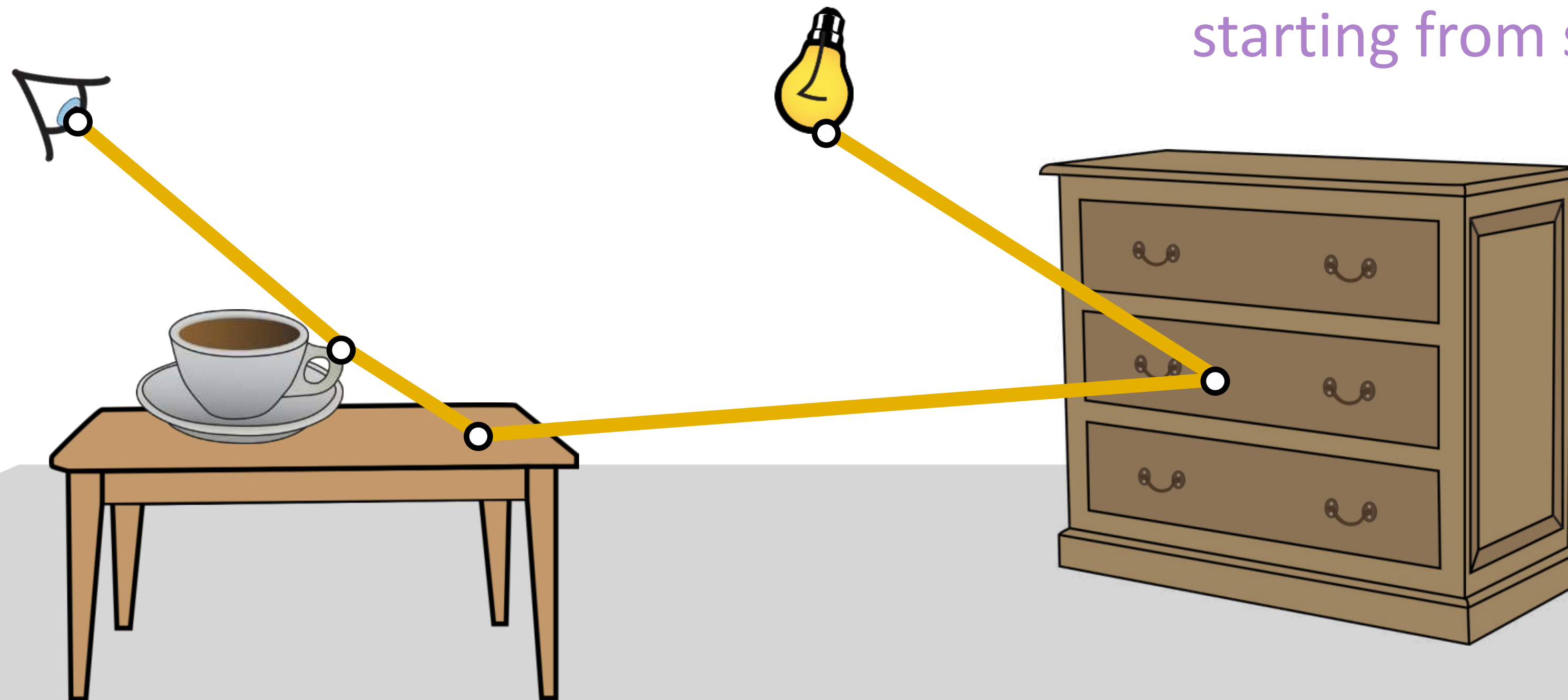
# Monte Carlo rendering

approximate image as  $I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

PDF of random path

*Path tracing: sample path starting from sensor*





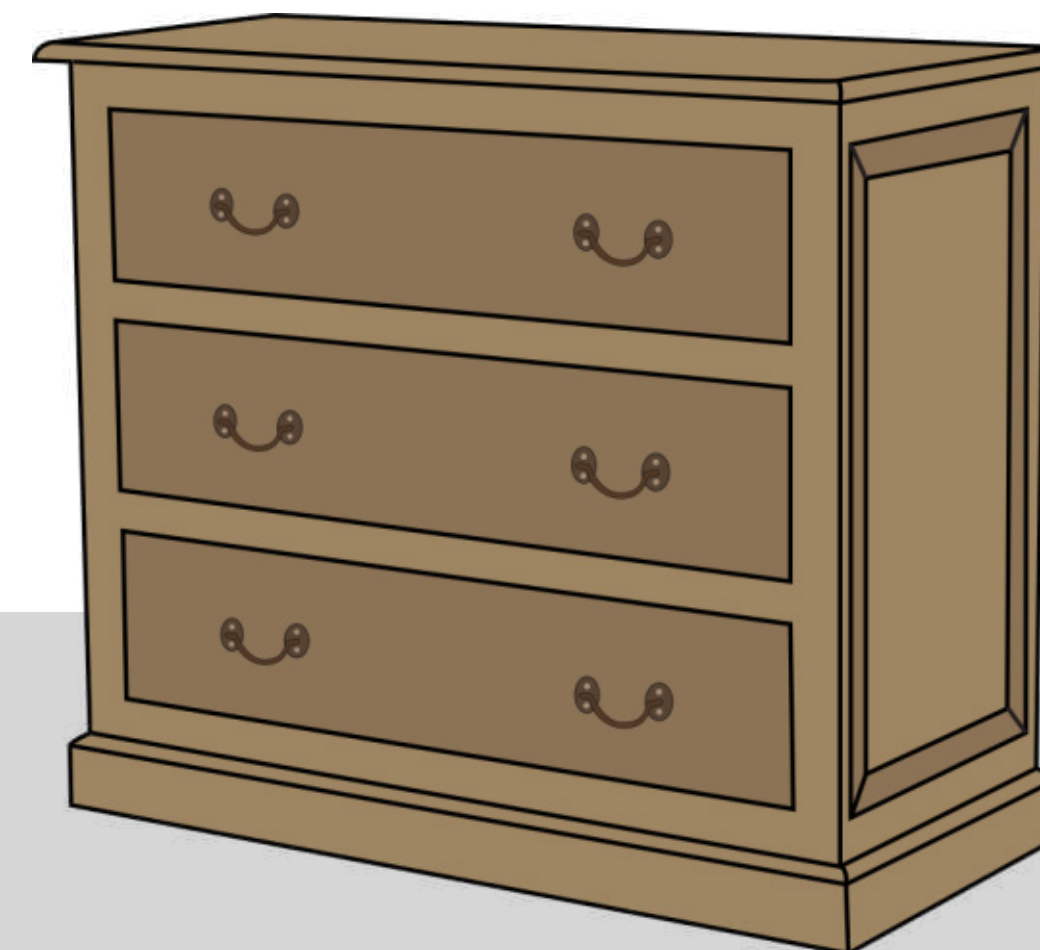
# Monte Carlo rendering

approximate image as  $I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

PDF of random path

*Light tracing*: sample path  
starting from source





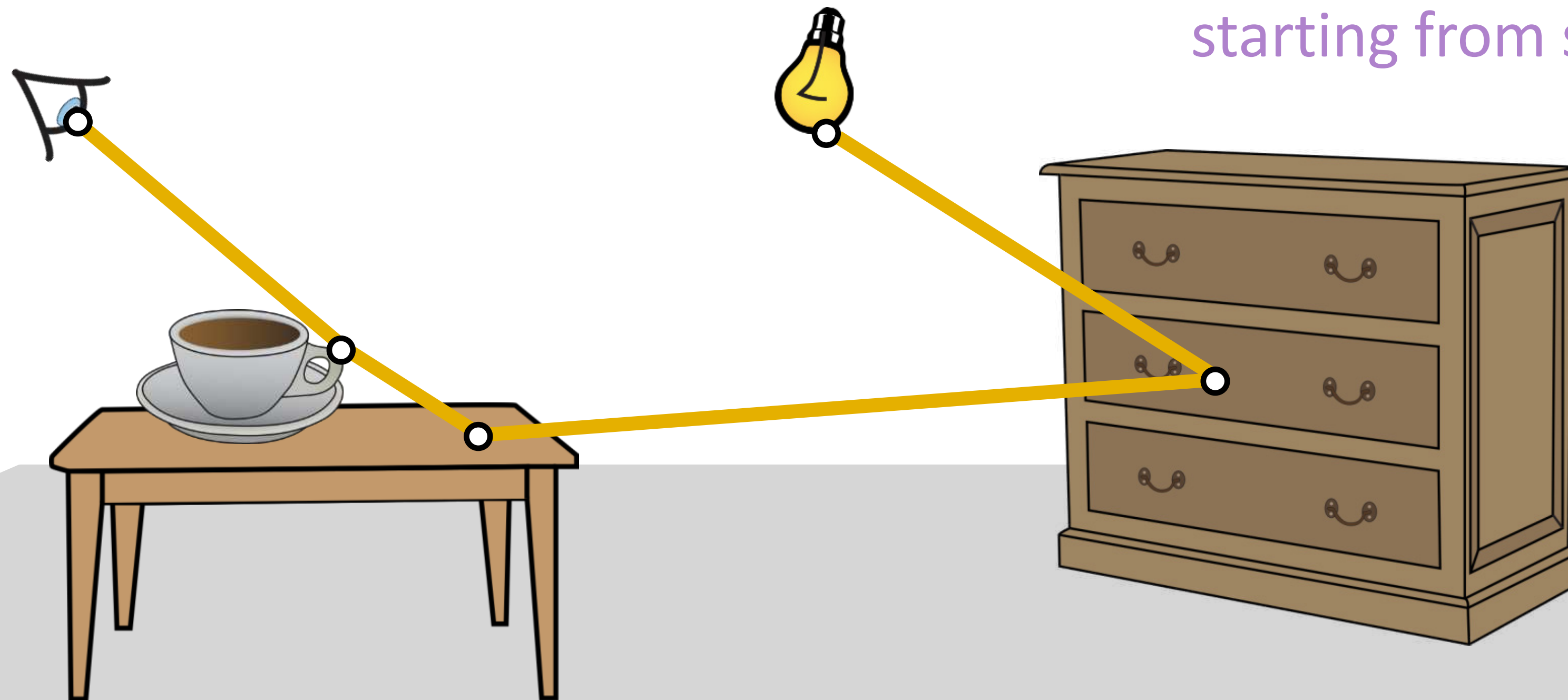
# Monte Carlo rendering

approximate image as 
$$I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

sum over *randomly* sampled paths

PDF of random path

*Light tracing: sample path starting from source*





# Monte Carlo rendering

approximate image as  $I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

PDF of random path

*Bidirectional path tracing*: sample path starting from both source and sensor





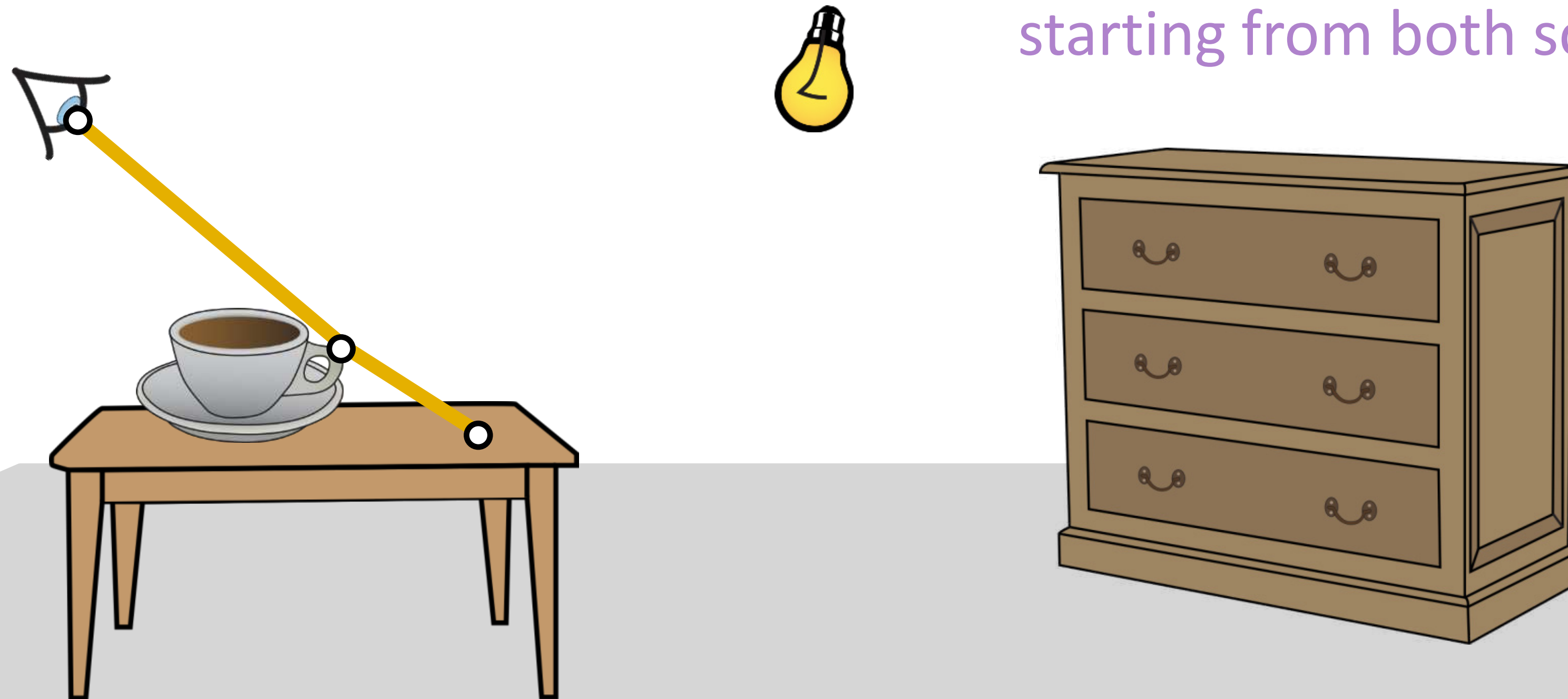
# Monte Carlo rendering

approximate image as  $I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

PDF of random path

*Bidirectional path tracing*: sample path starting from both source and sensor





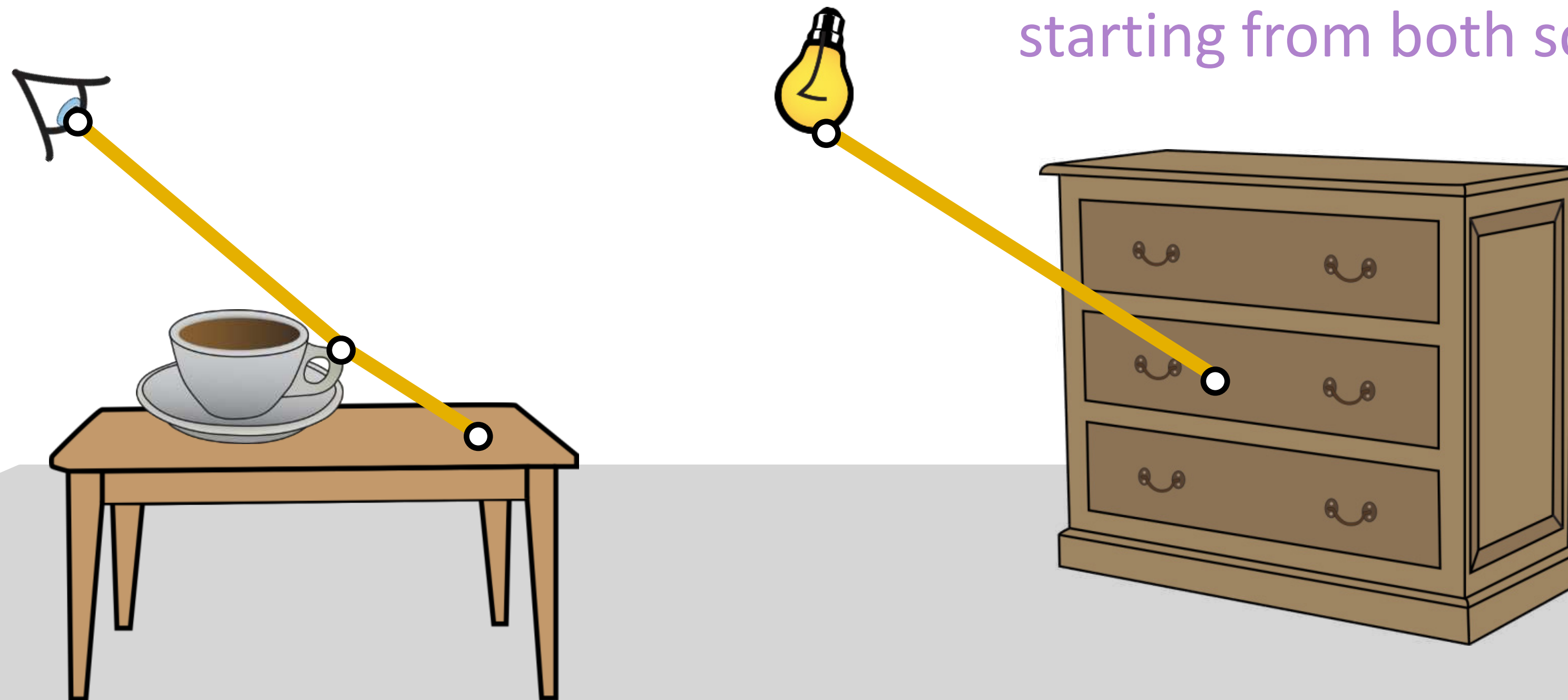
# Monte Carlo rendering

approximate image as 
$$I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

sum over *randomly* sampled paths

PDF of random path

*Bidirectional path tracing*: sample path starting from both source and sensor





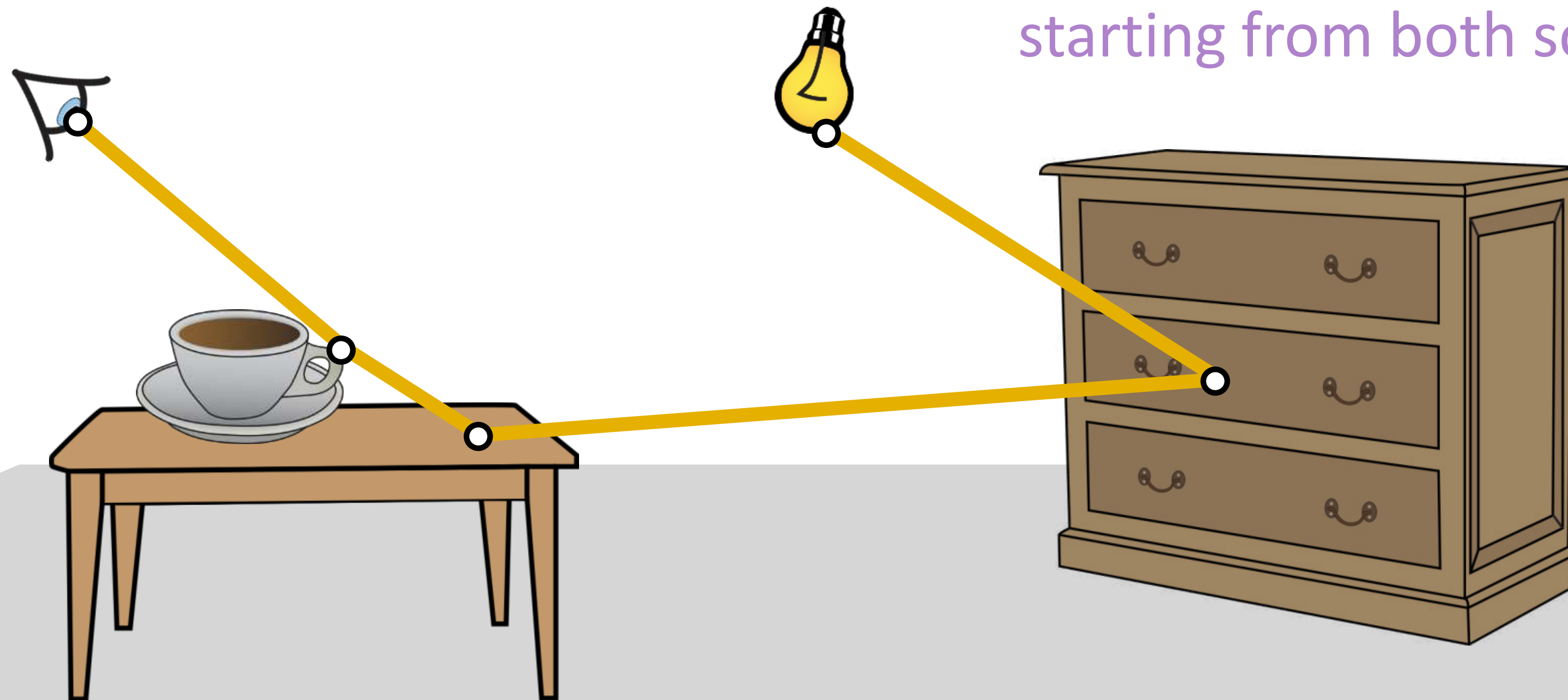
# Monte Carlo rendering

approximate image as  $I_j \approx \frac{1}{N} \sum_{i=1}^N \frac{W_e(\mathbf{x}_{i,0}, \mathbf{x}_{i,1}) L_e(\mathbf{x}_{i,k}, \mathbf{x}_{i,k-1}) T(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$

sum over *randomly* sampled paths

PDF of random path

*Bidirectional path tracing*: sample path starting from both source and sensor





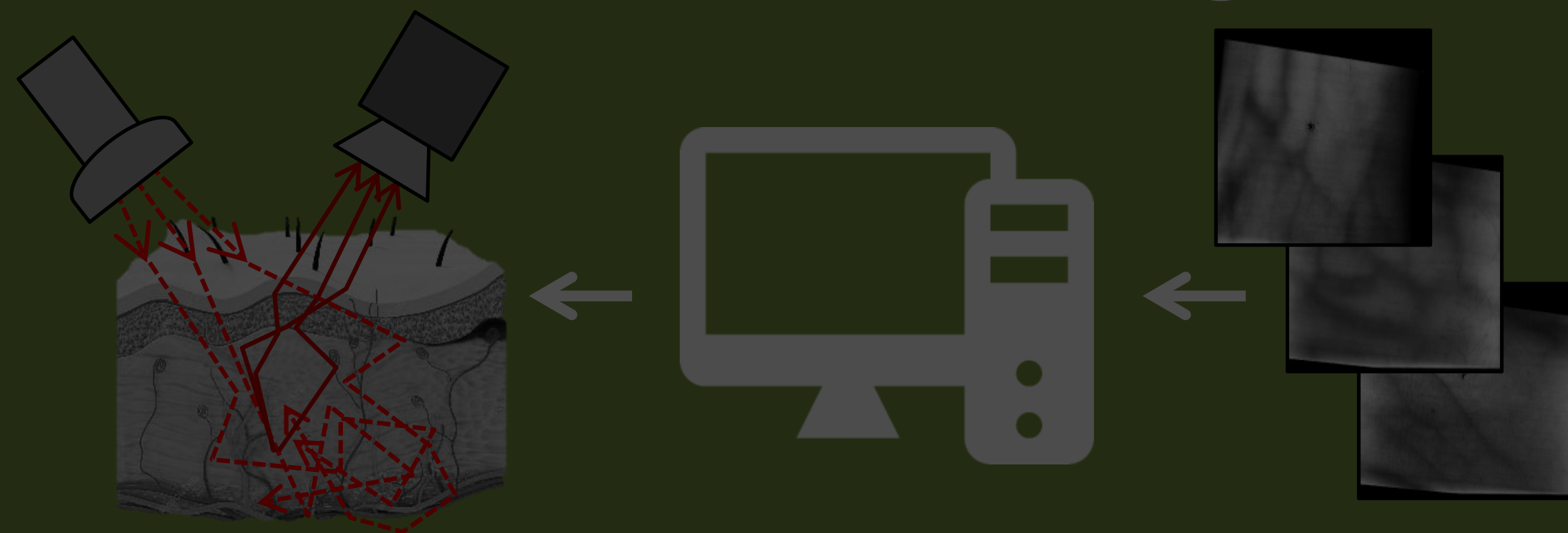
# Physics-based rendering and its applications to computational imaging

## forward rendering

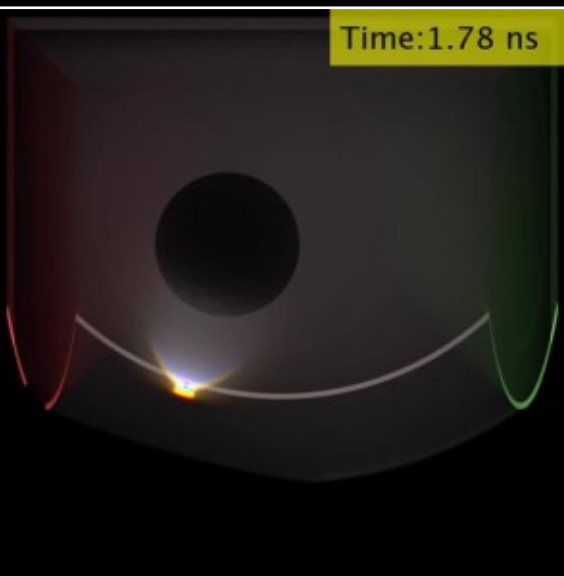


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

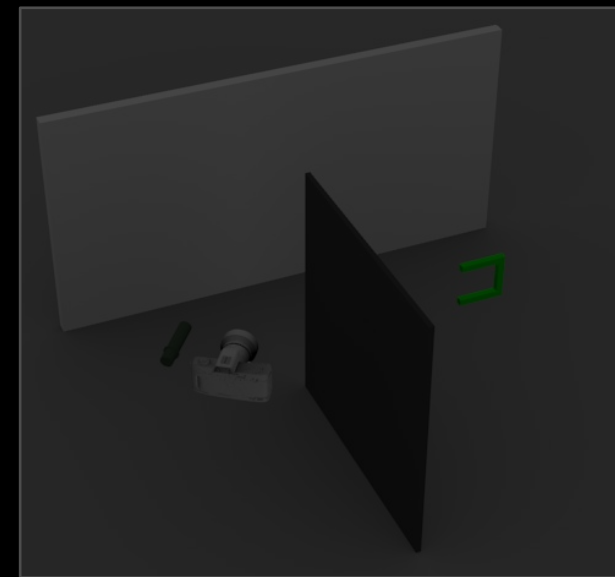
## inverse rendering



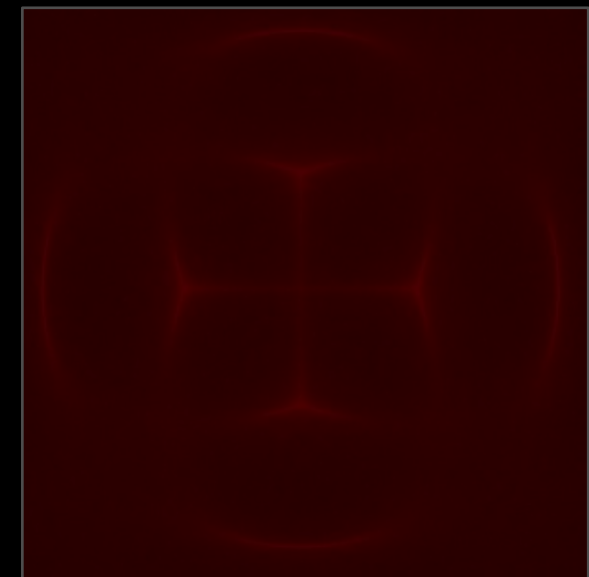
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



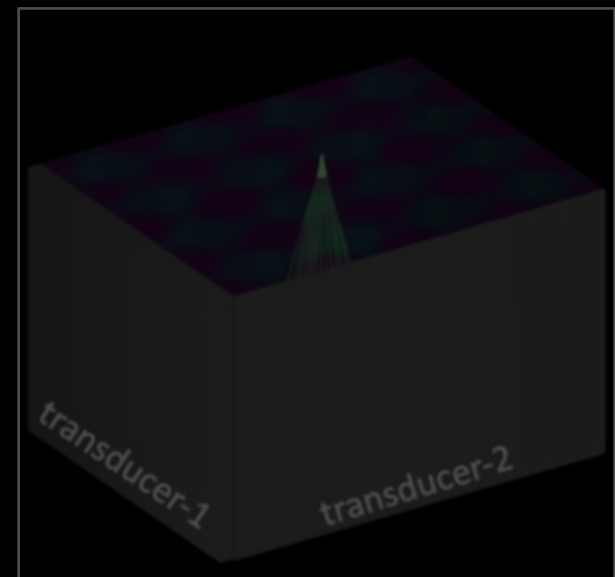
time-of-flight  
imaging



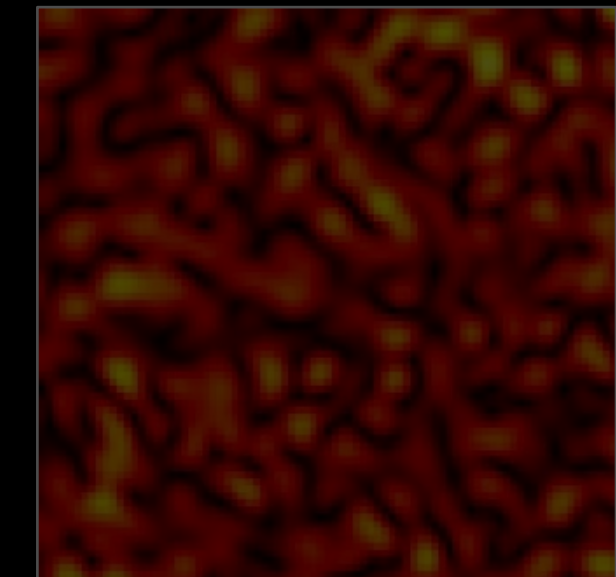
non-line-of-sight  
imaging



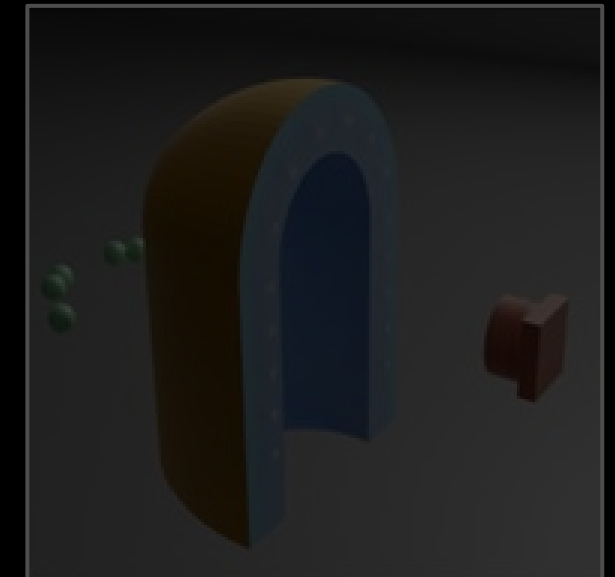
acousto-optic  
lensing



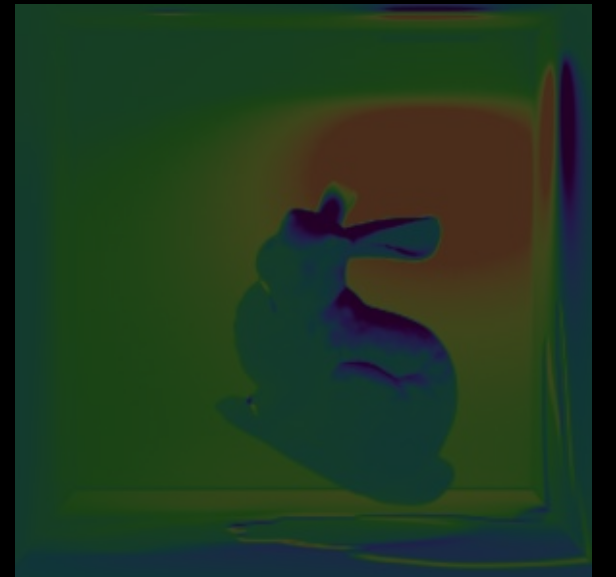
ultrafast light  
scanning



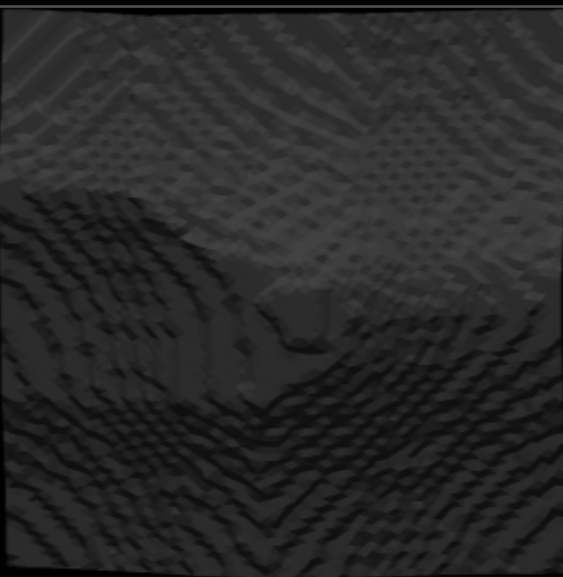
speckle  
imaging



tactile sensor  
design



differentiable  
renderer



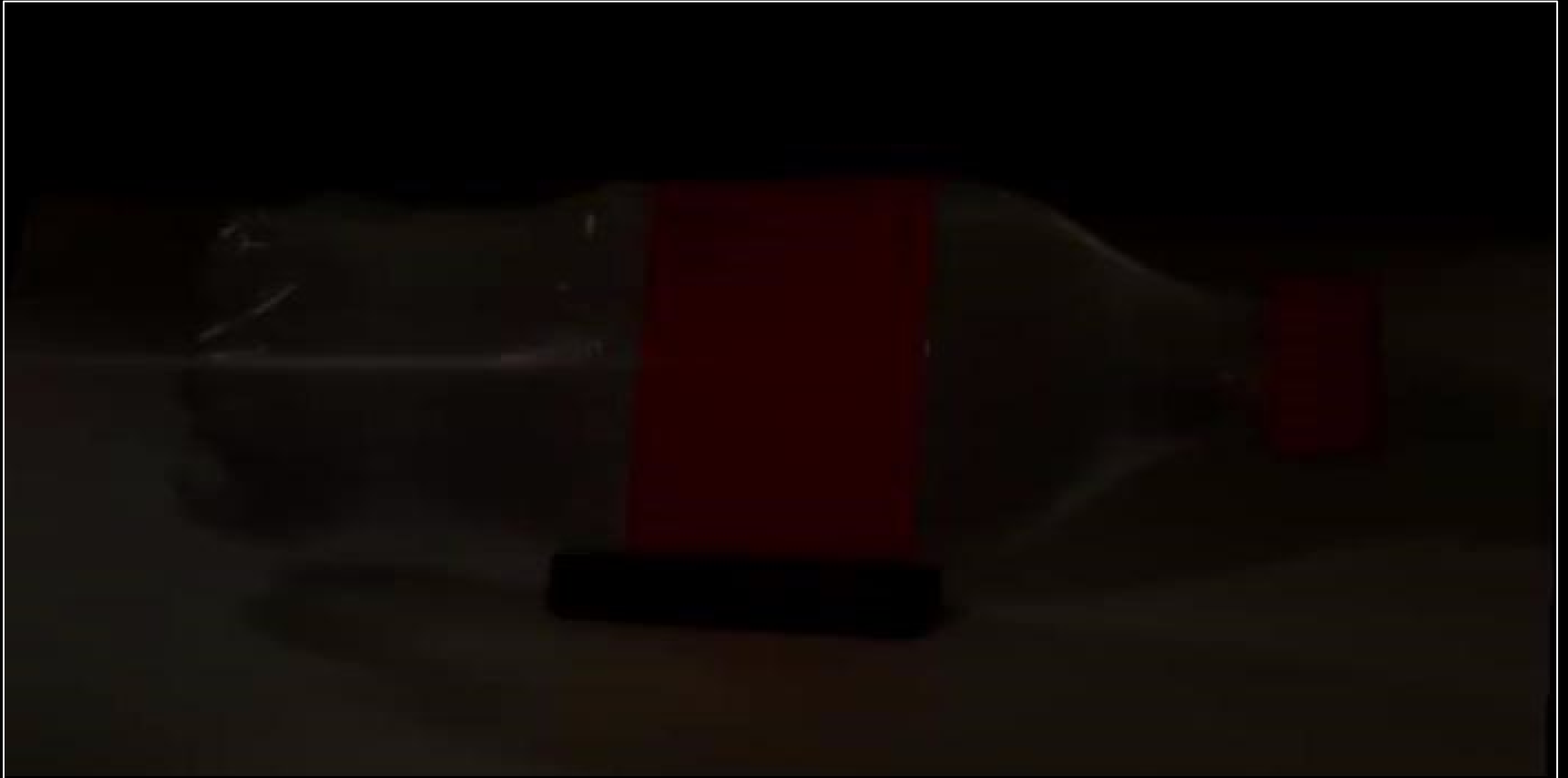
inverse  
problems



# Time-of-flight cameras

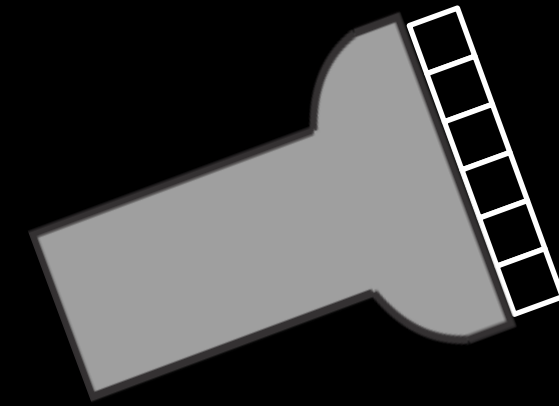
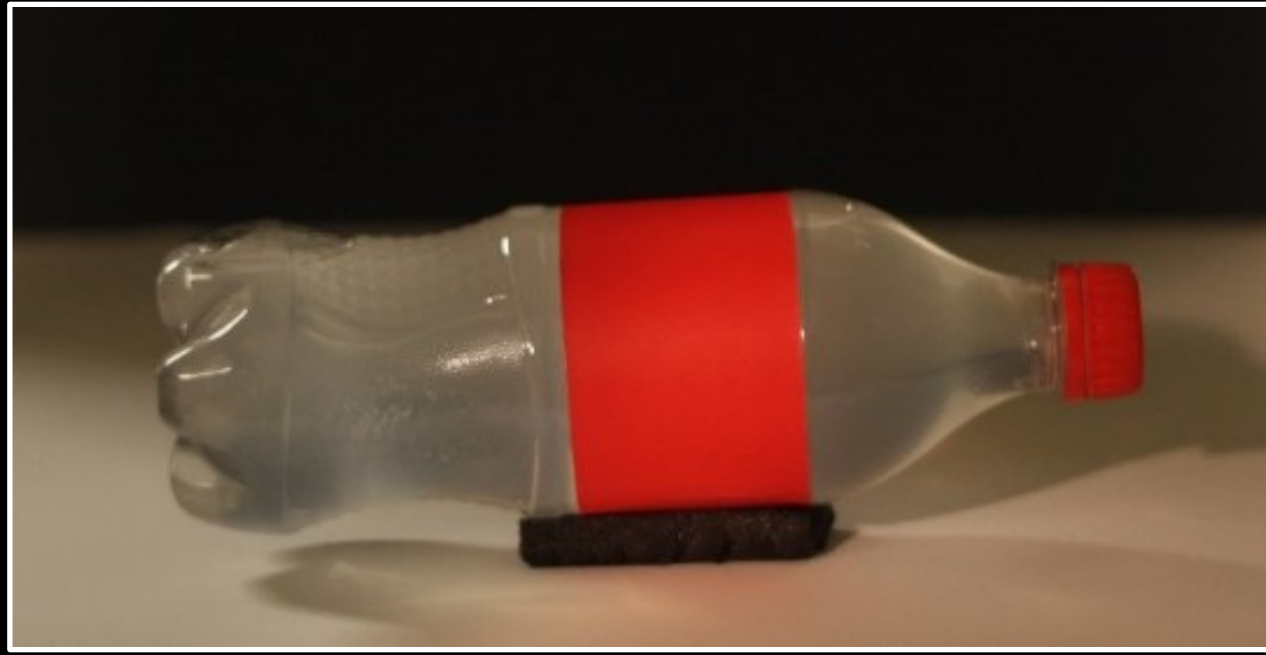


# Time-of-flight cameras

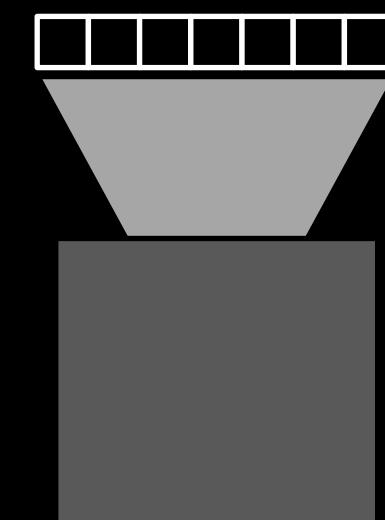
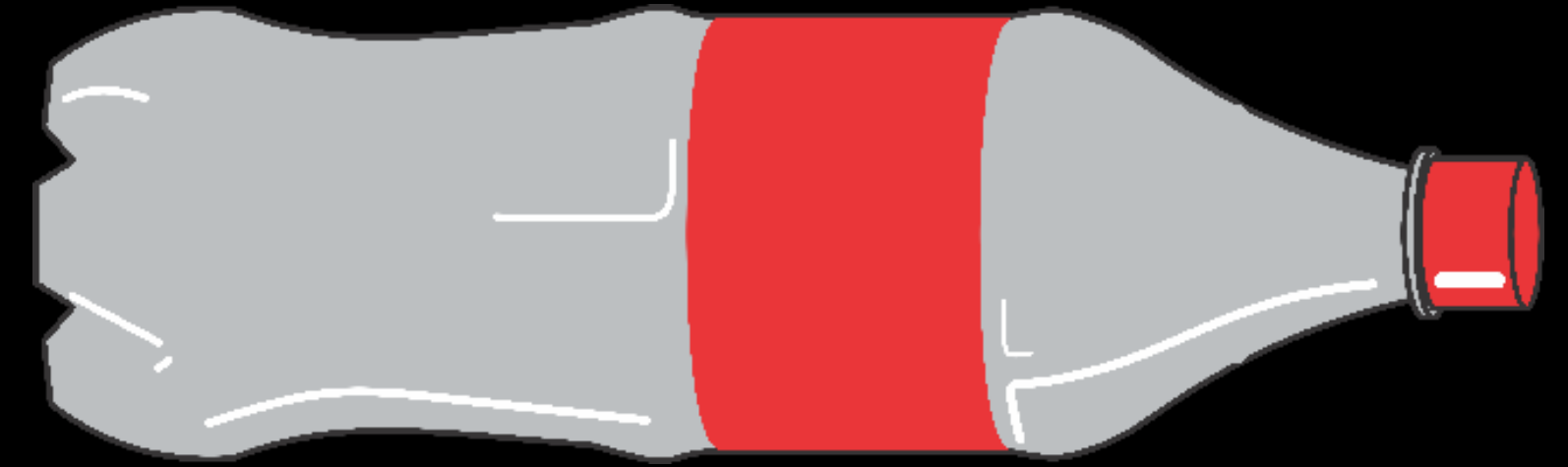




# Time-of-flight cameras



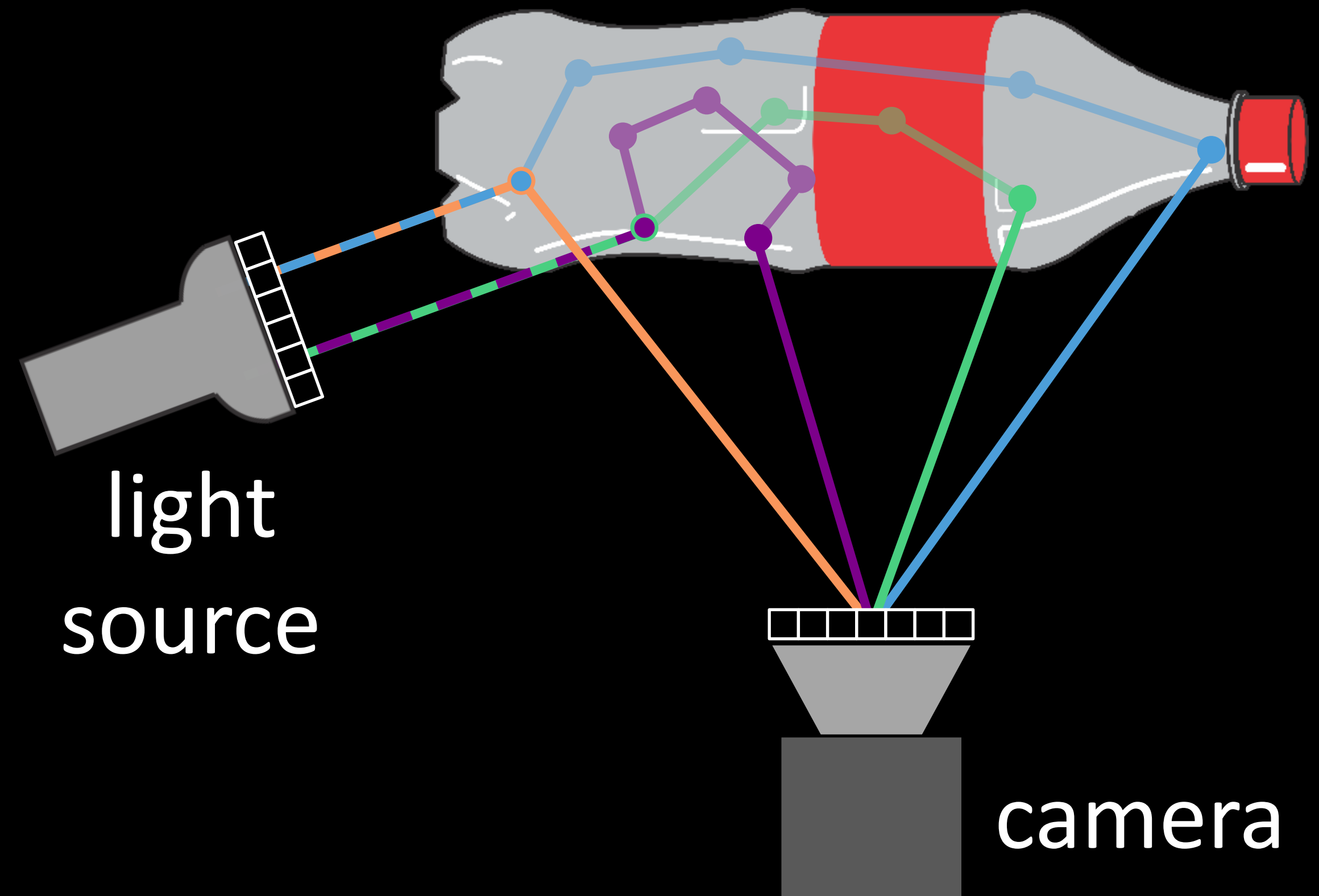
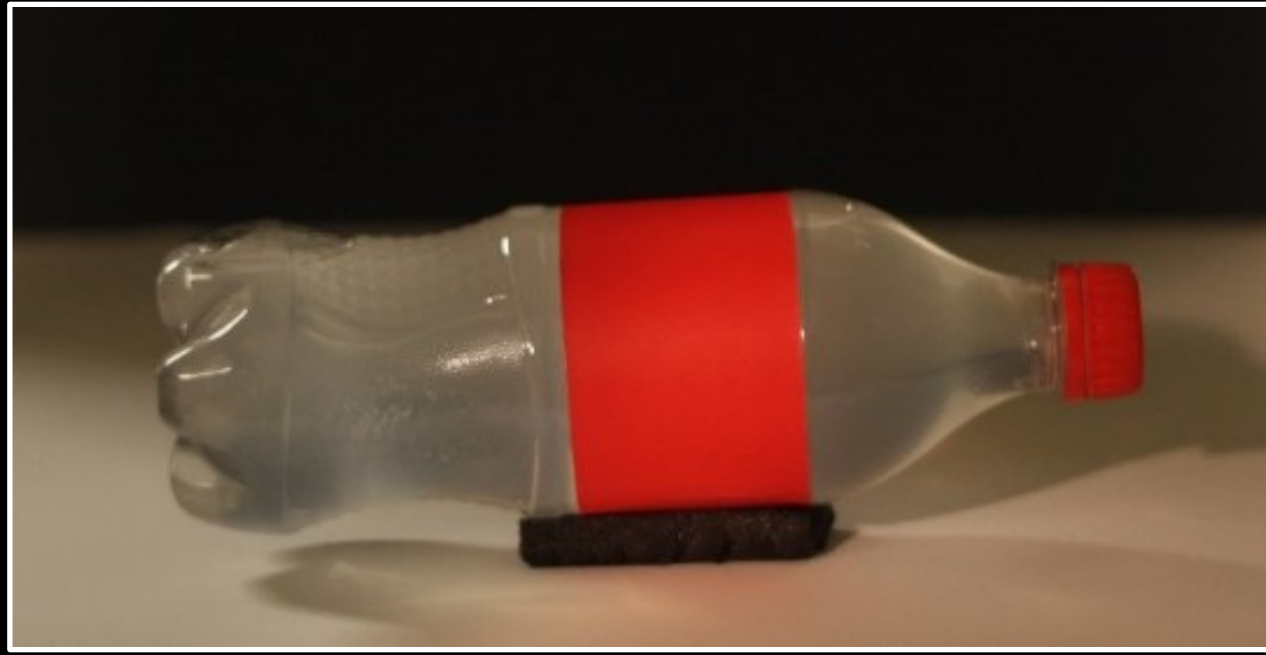
light  
source



camera

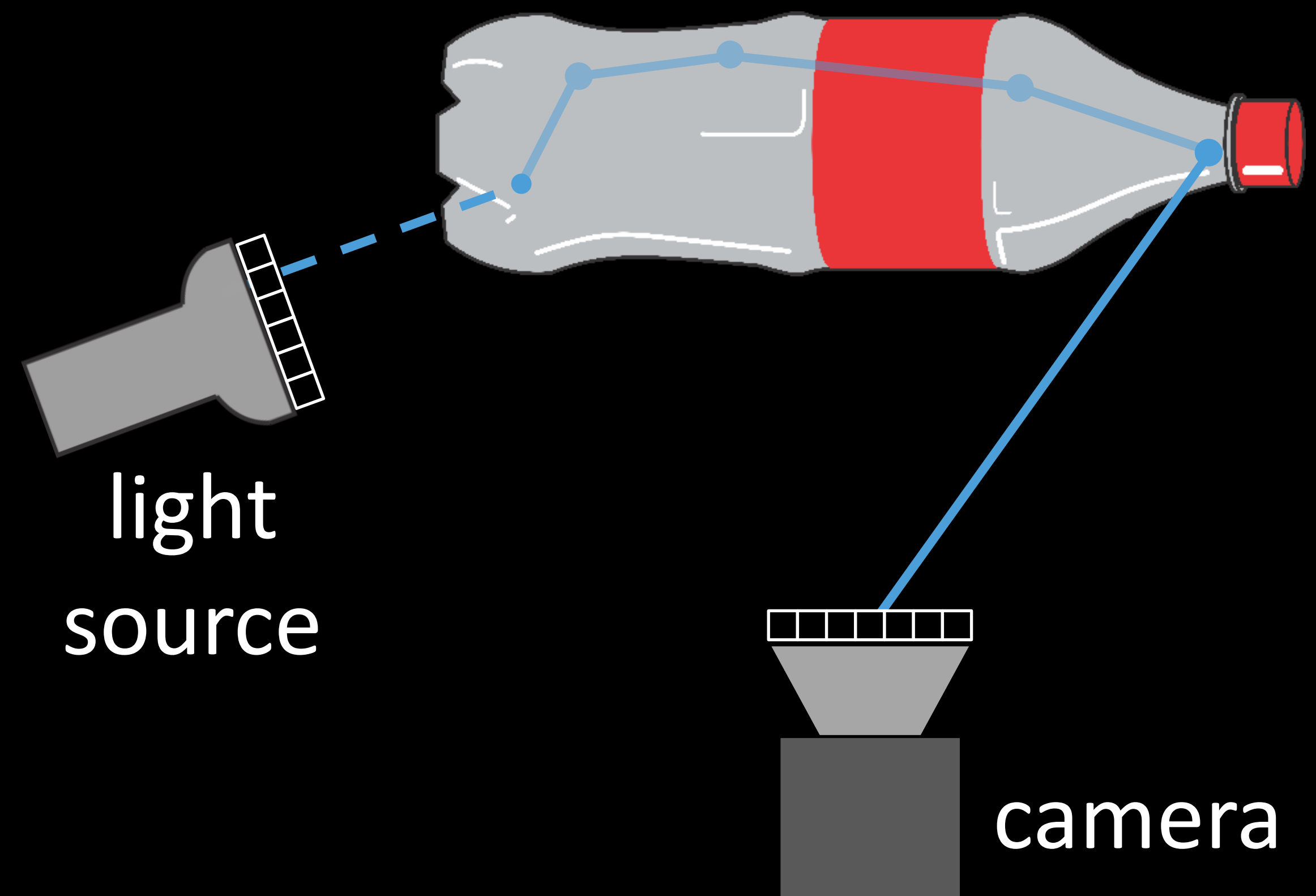
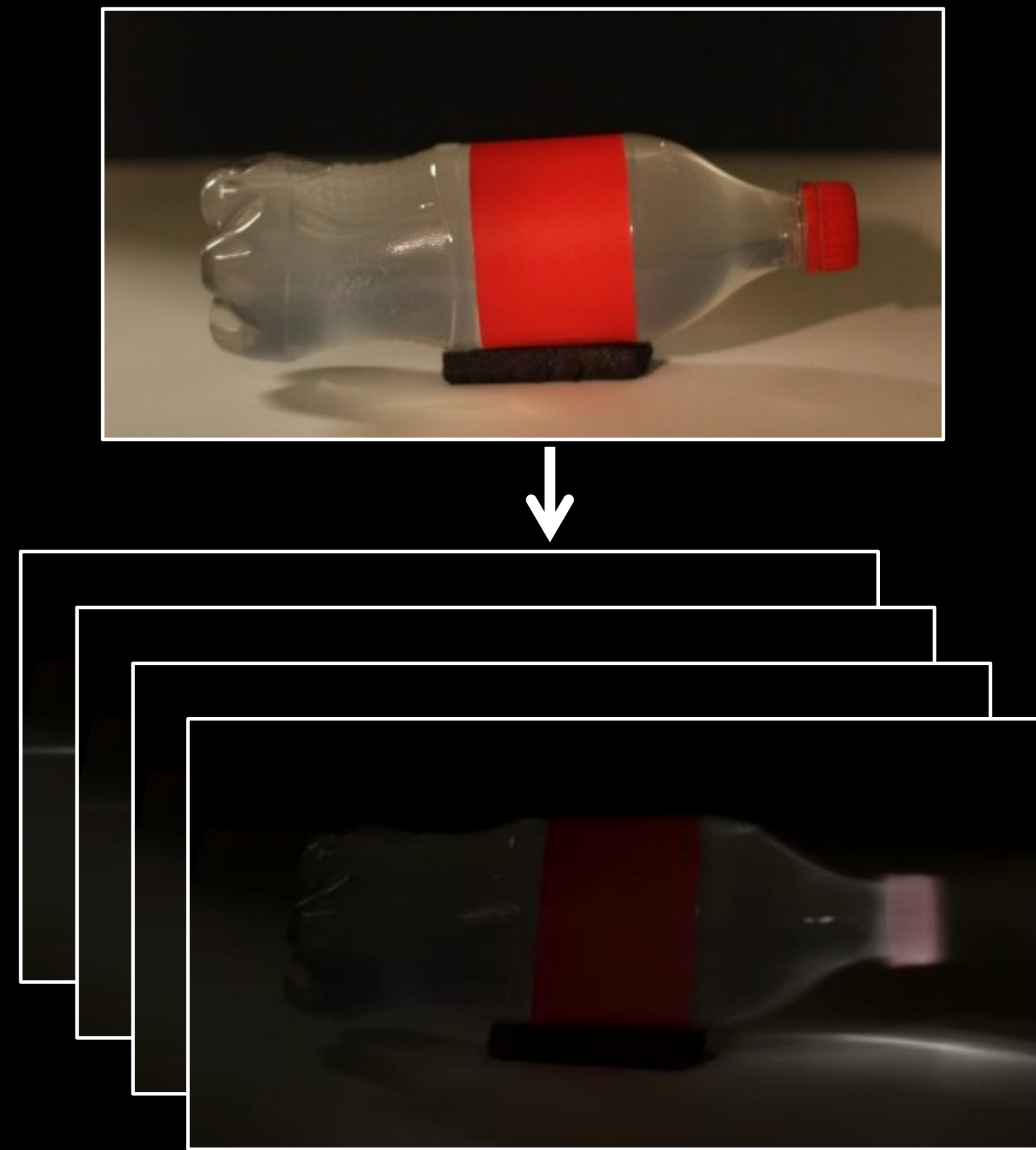


# Time-of-flight cameras





# Time-of-flight cameras





# Time-of-flight cameras



SwissRanger SR4000



Hamamatsu streak



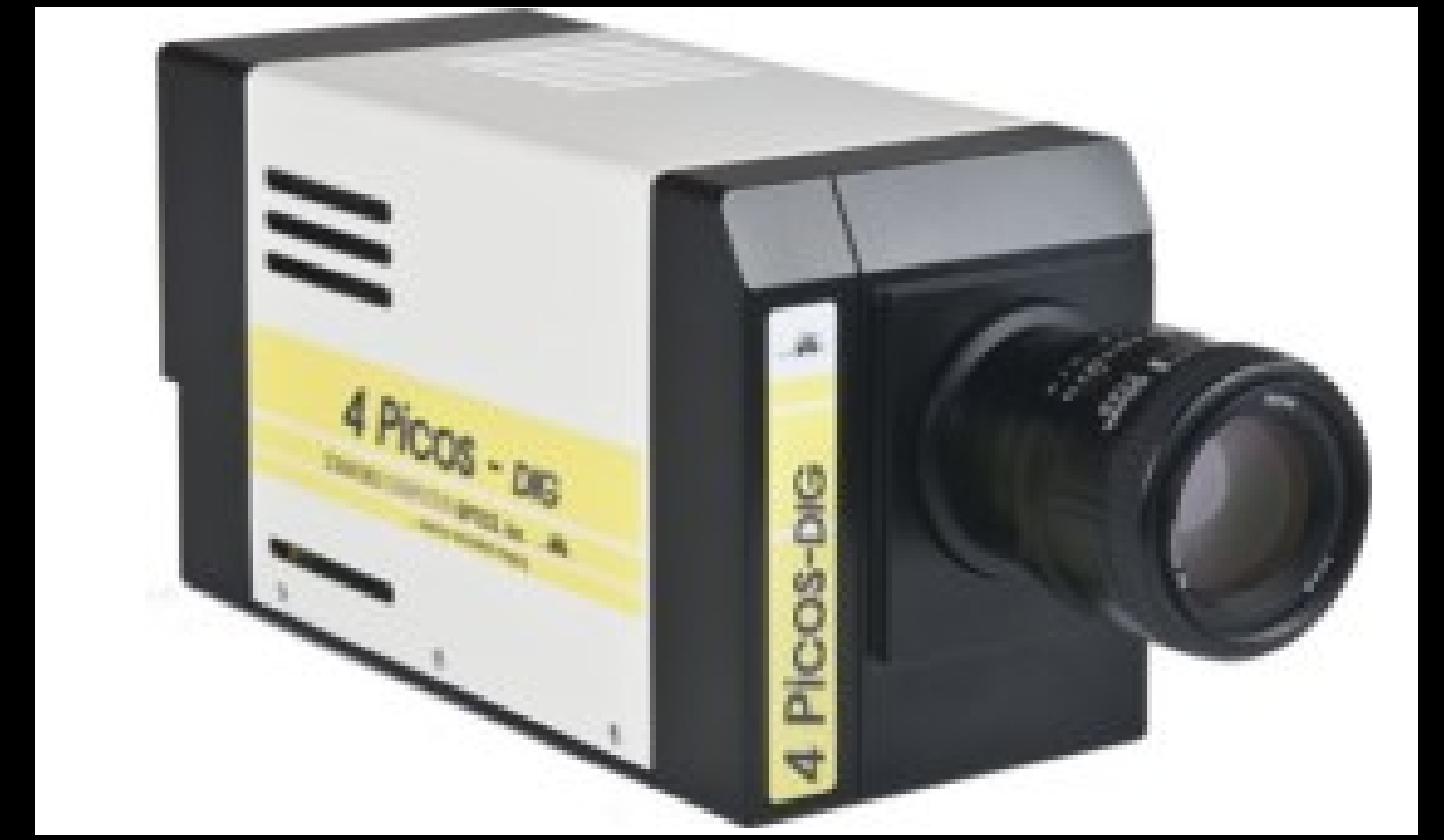
Brightway VISDOM



Microsoft Kinect



MPD SPAD



Stanford ICCD



# Time-of-flight cameras



SwissRanger SR4000



Hamamatsu streak



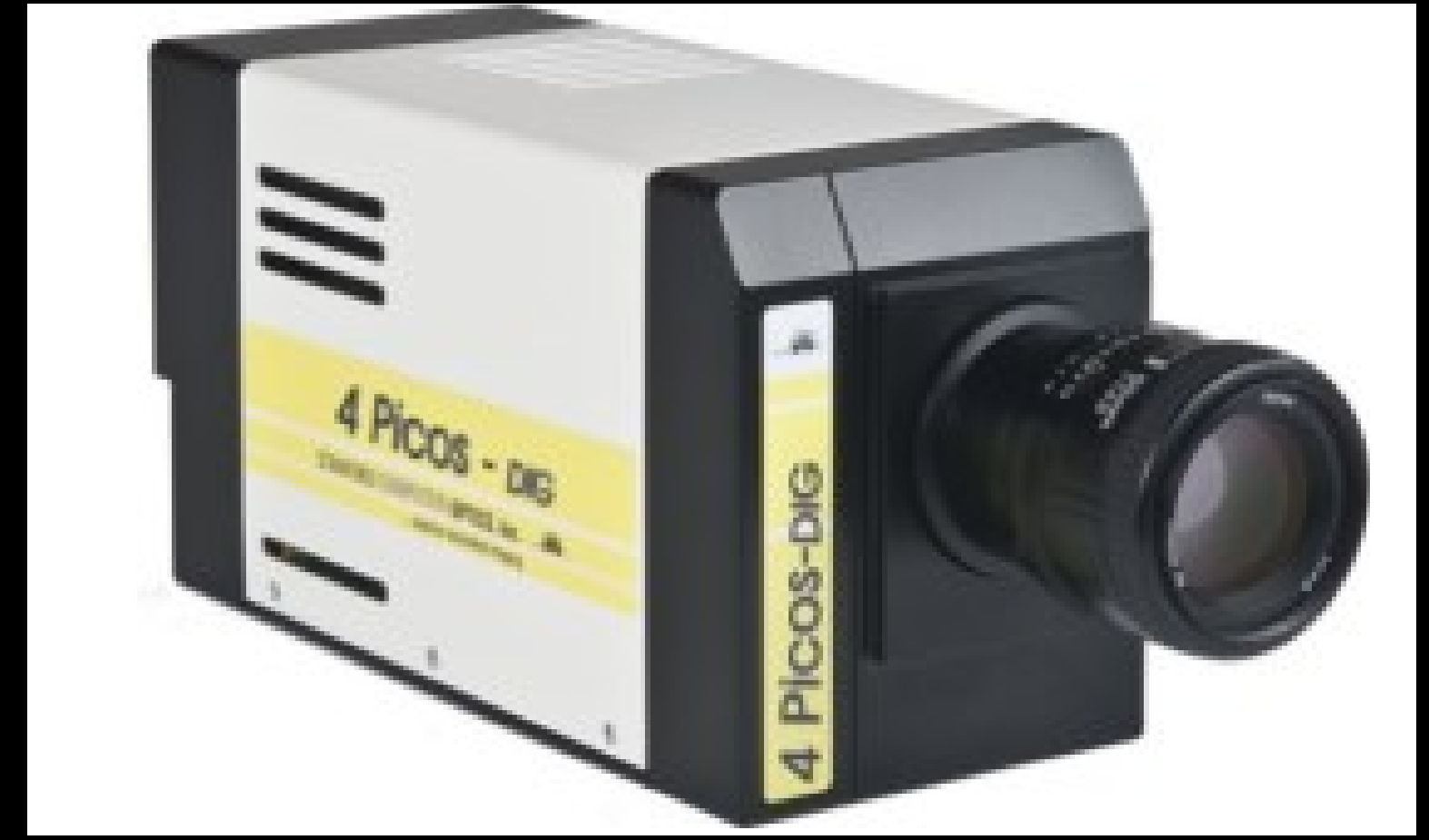
Brightway VISDOM



Microsoft Kinect  
continuous-wave



MPD SPAD  
transient

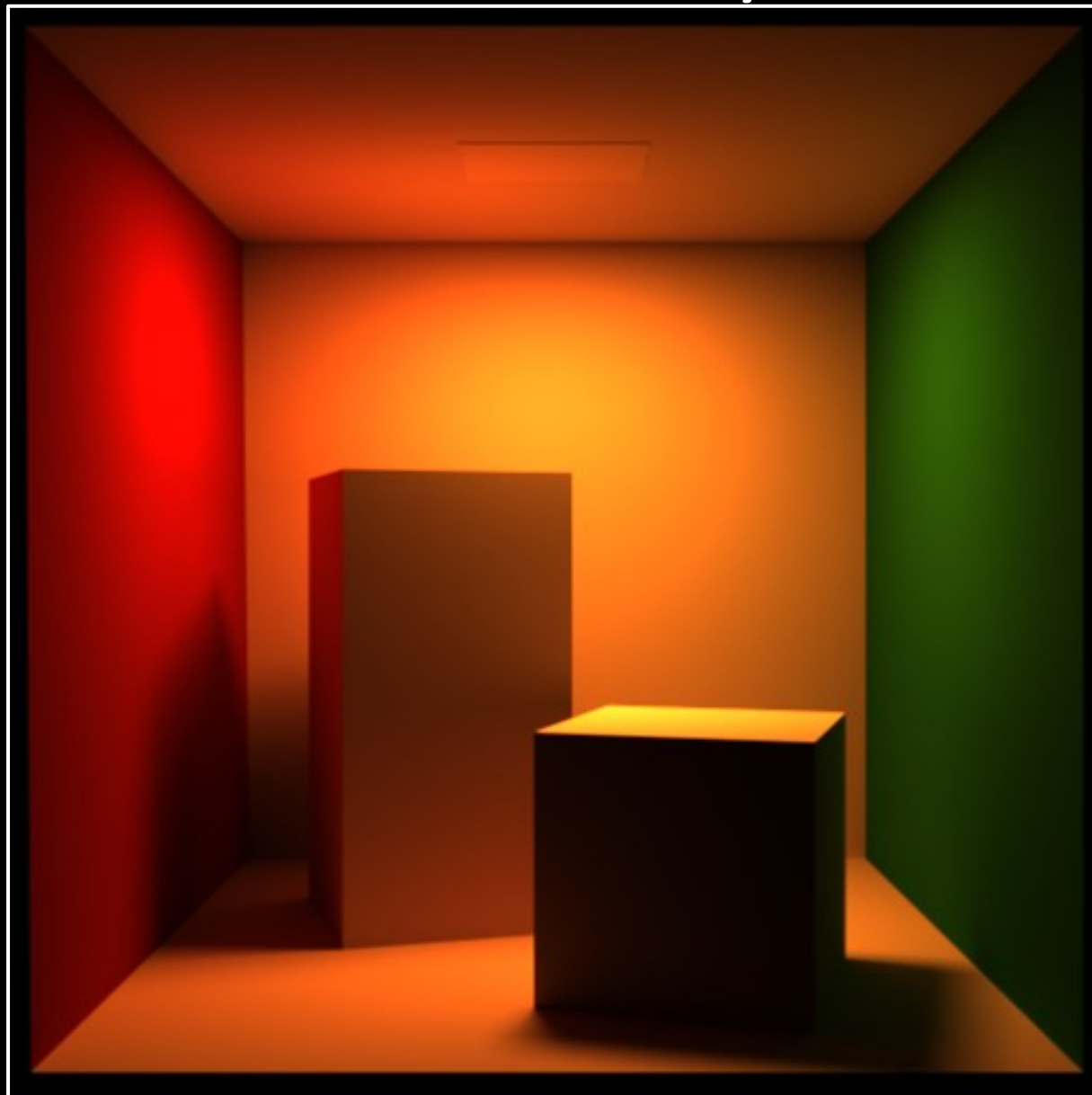


Stanford ICCD  
time-gated

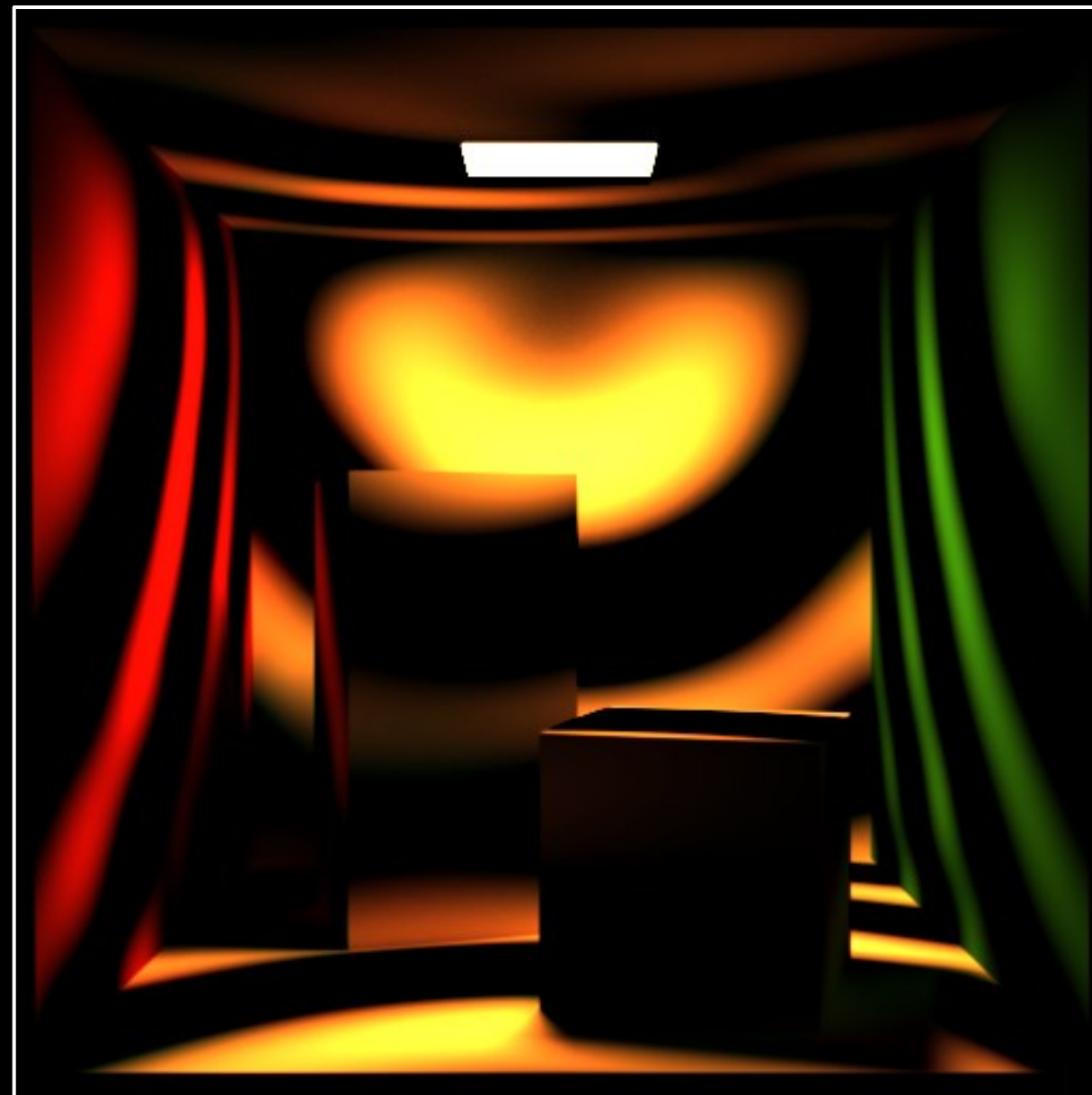


# Time-of-flight cameras

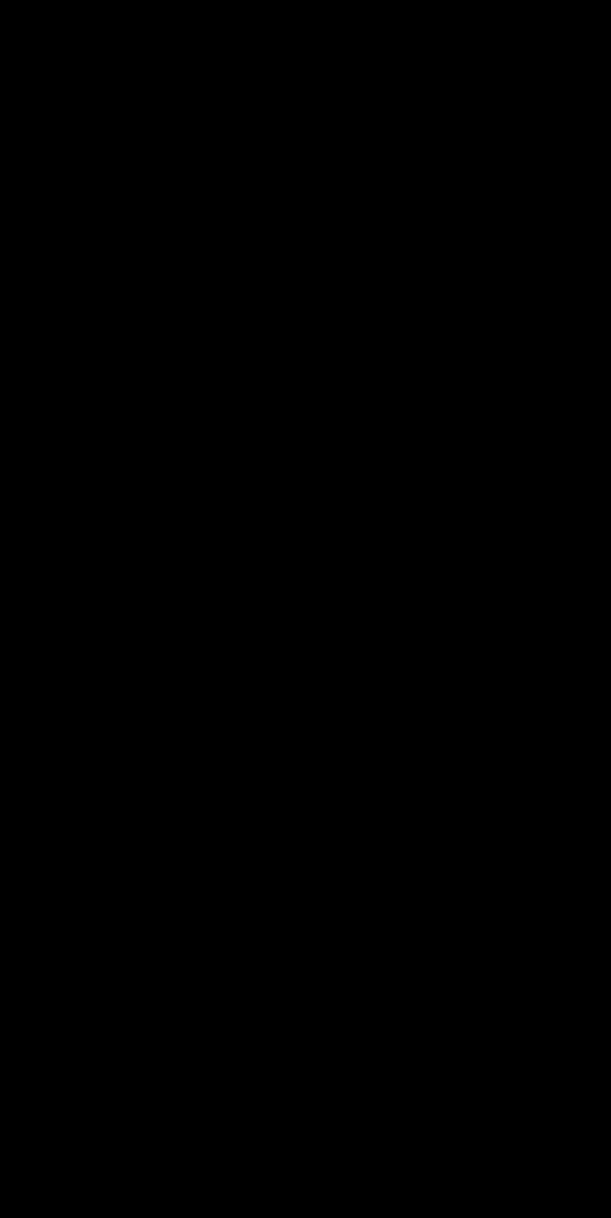
intensity



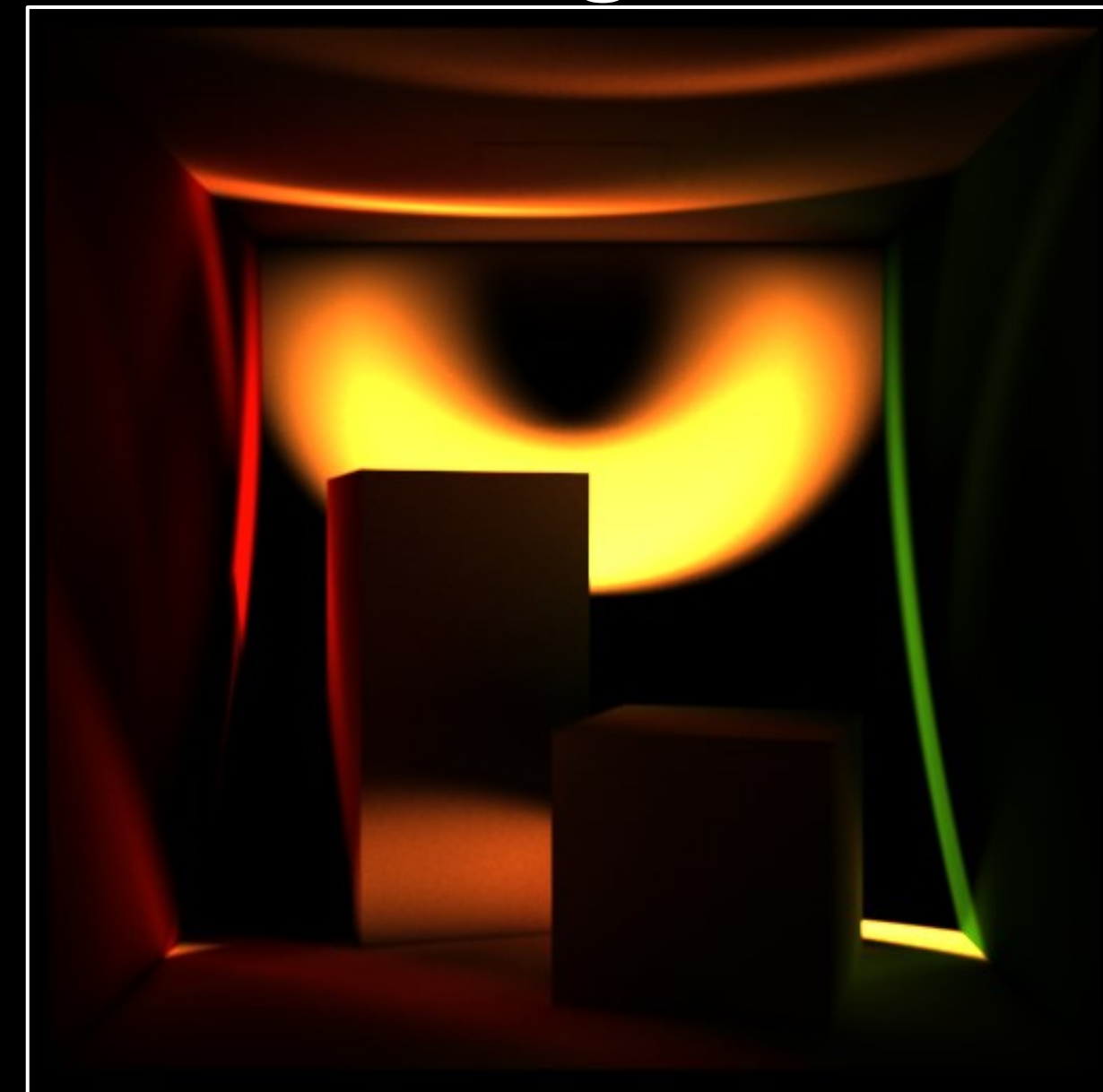
continuous wave



transient



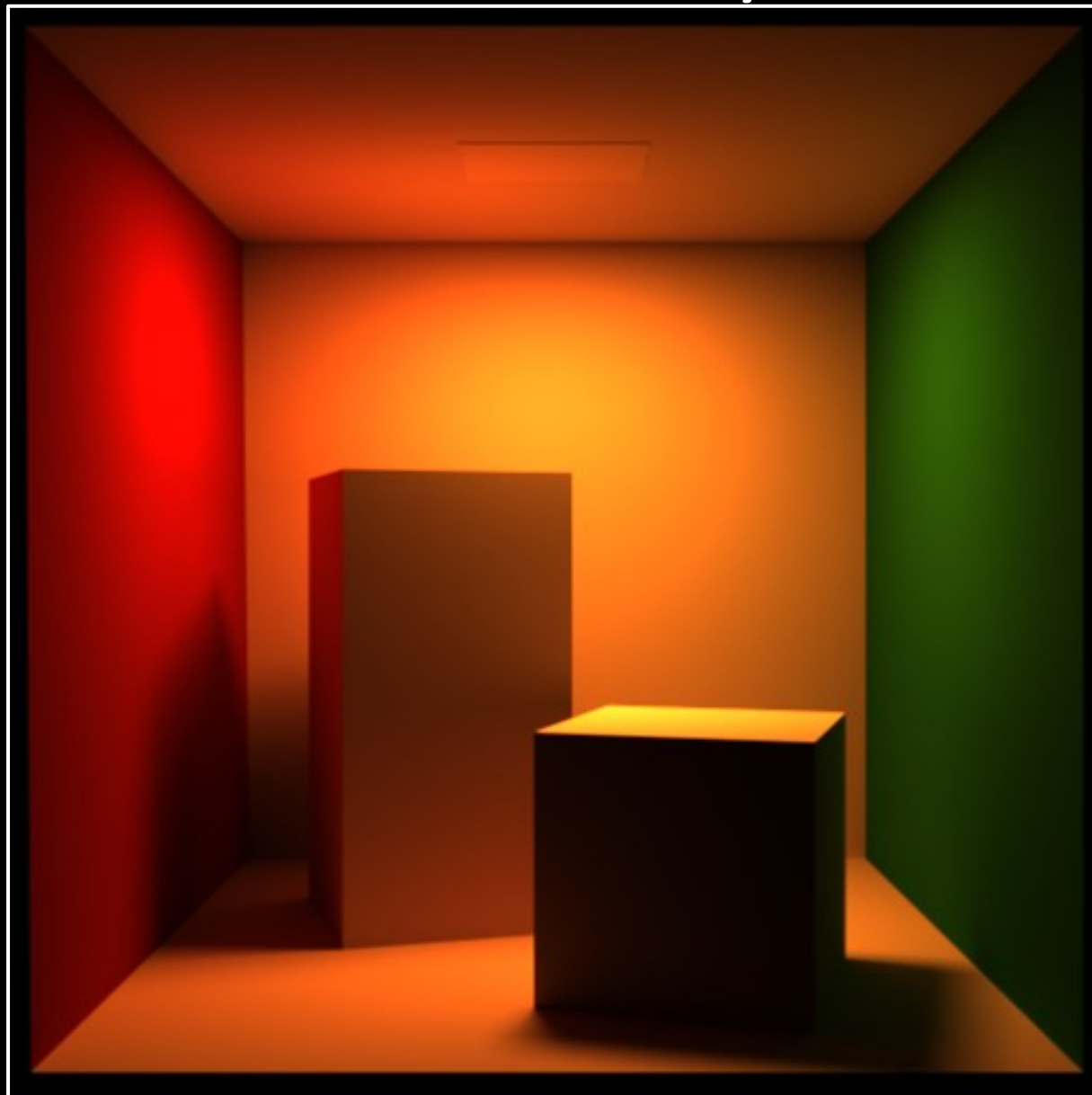
time-gated



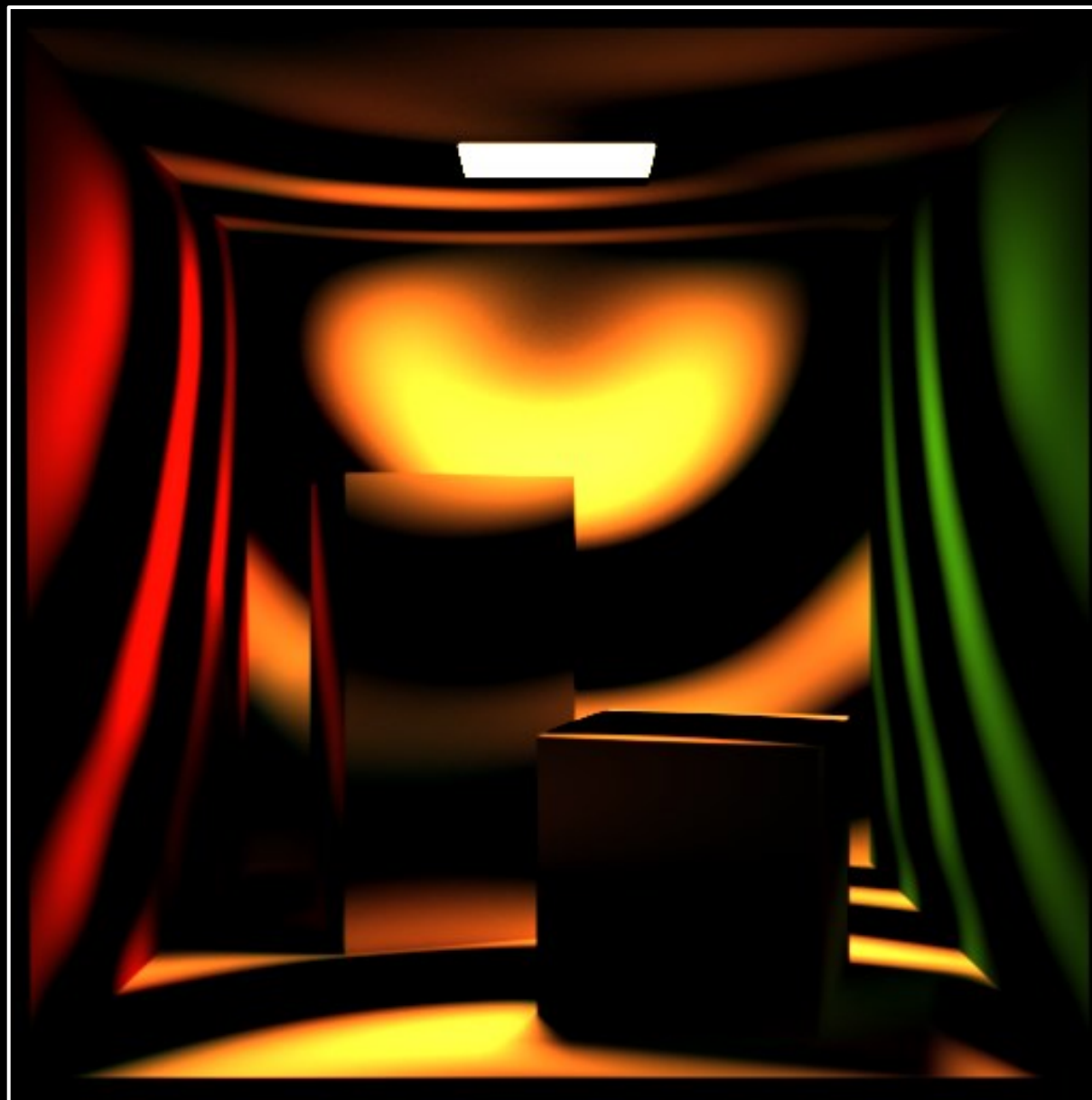


# Time-of-flight cameras

intensity



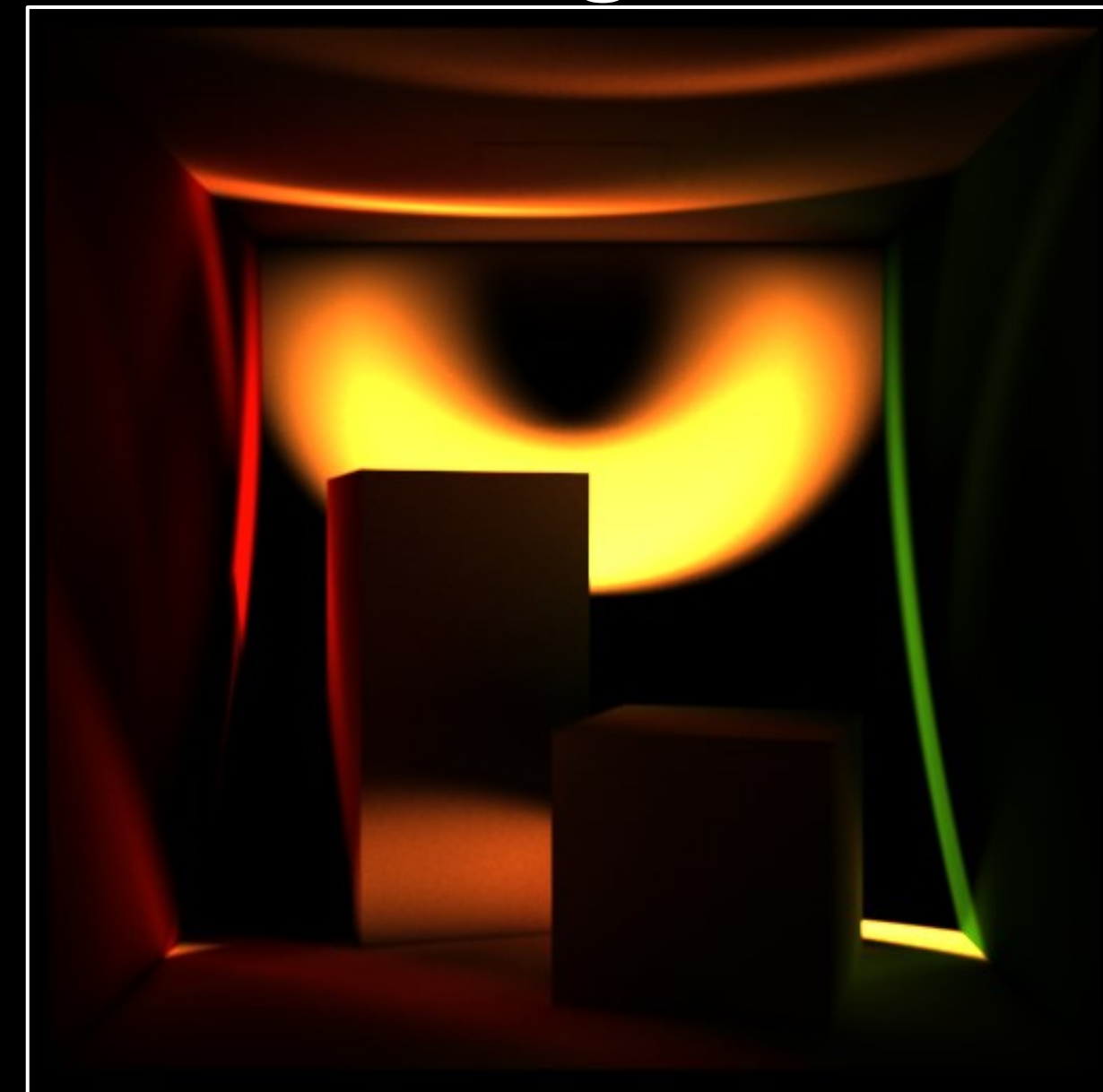
continuous wave



transient



time-gated





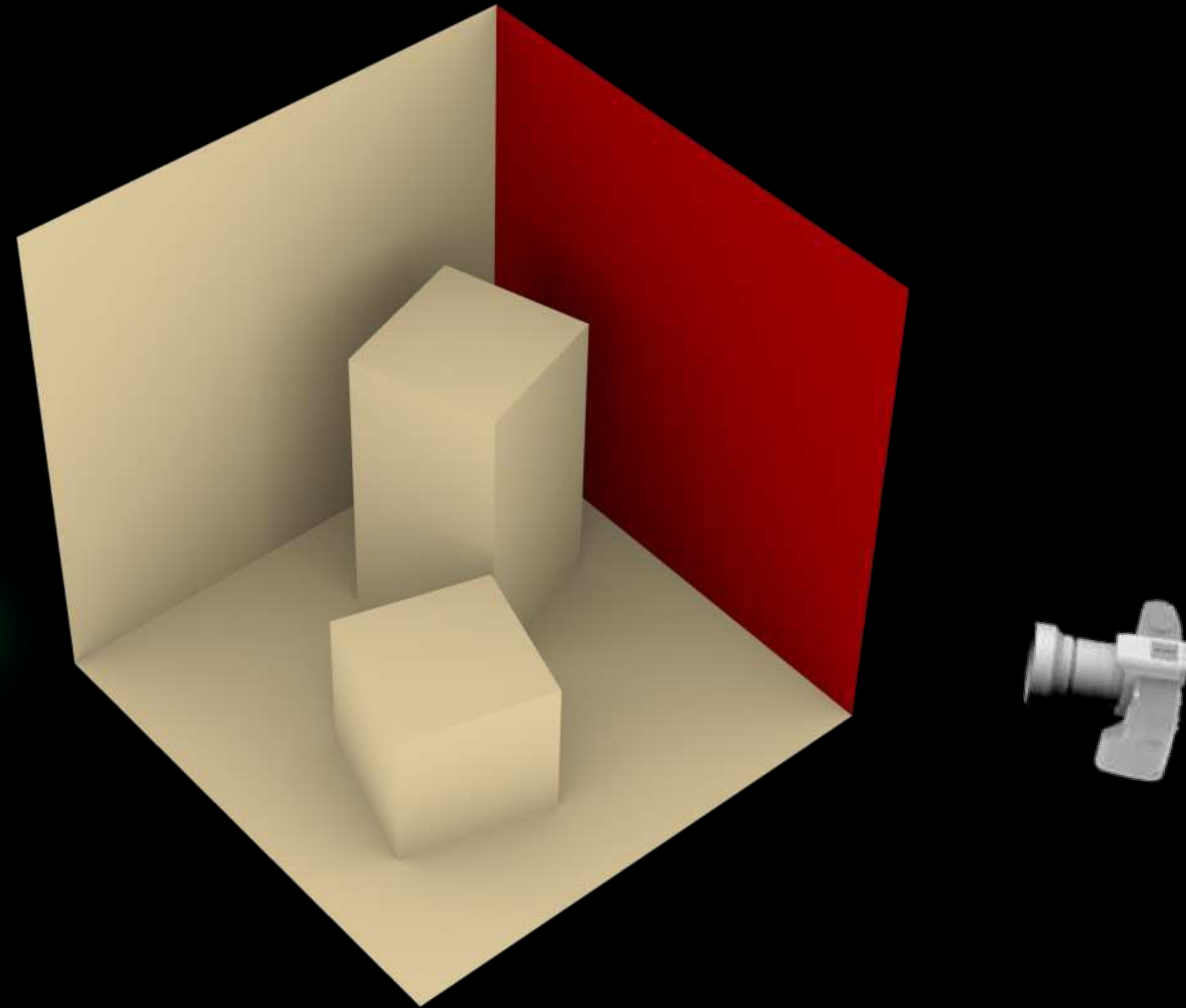
# Rendering time-of-flight cameras: Path space integral

$$\text{image} = \int_{\text{light paths}} f(\text{path})$$

light  
paths

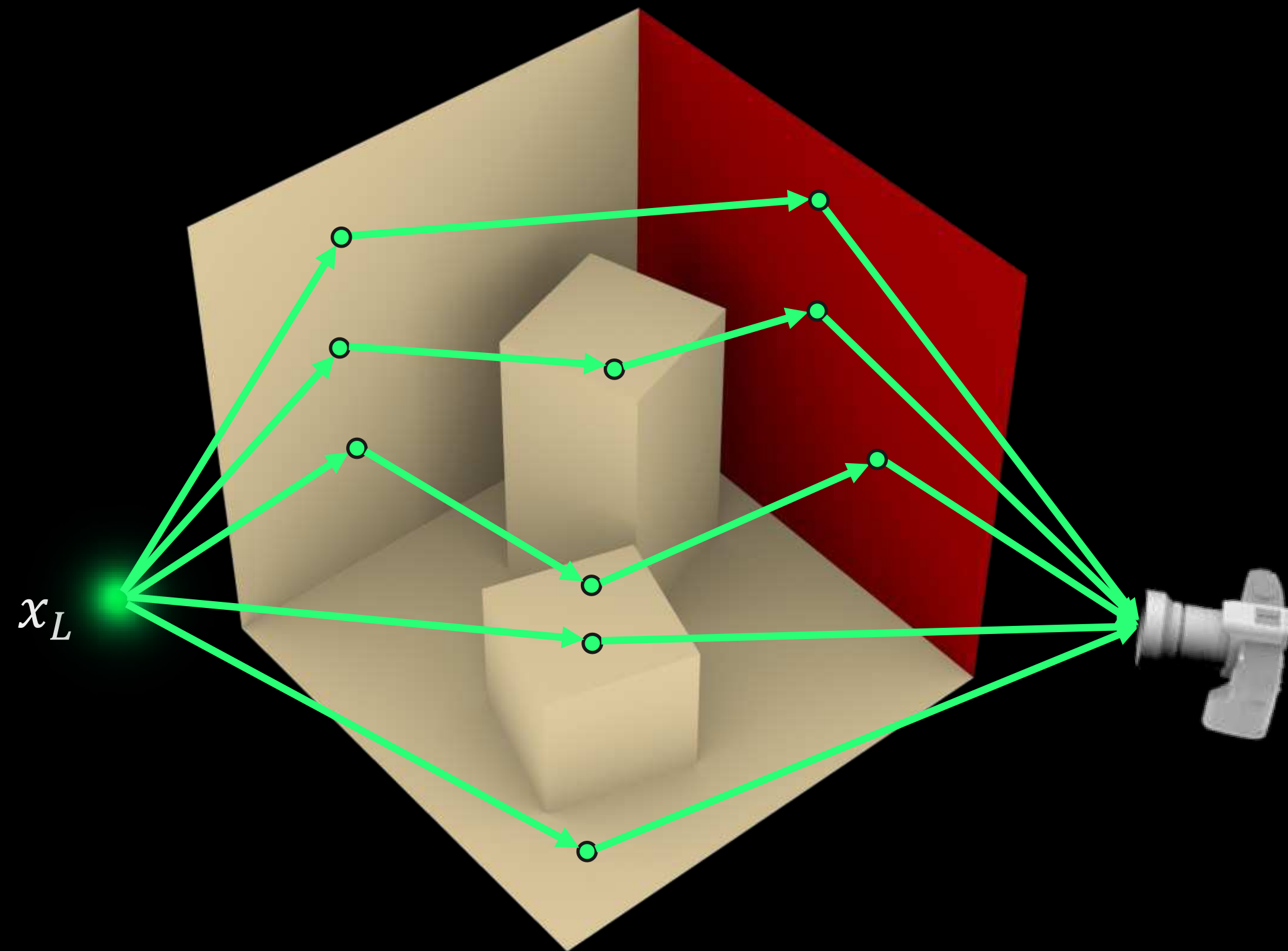
path contribution, depends  
on scene properties, light  
source, and sensor

$x_L$





# Rendering time-of-flight cameras: Path space integral



$$\text{image} = \int \text{f(path)}$$

light  
paths

path contribution, depends  
on scene properties, light  
source, and sensor

Monte Carlo rendering:

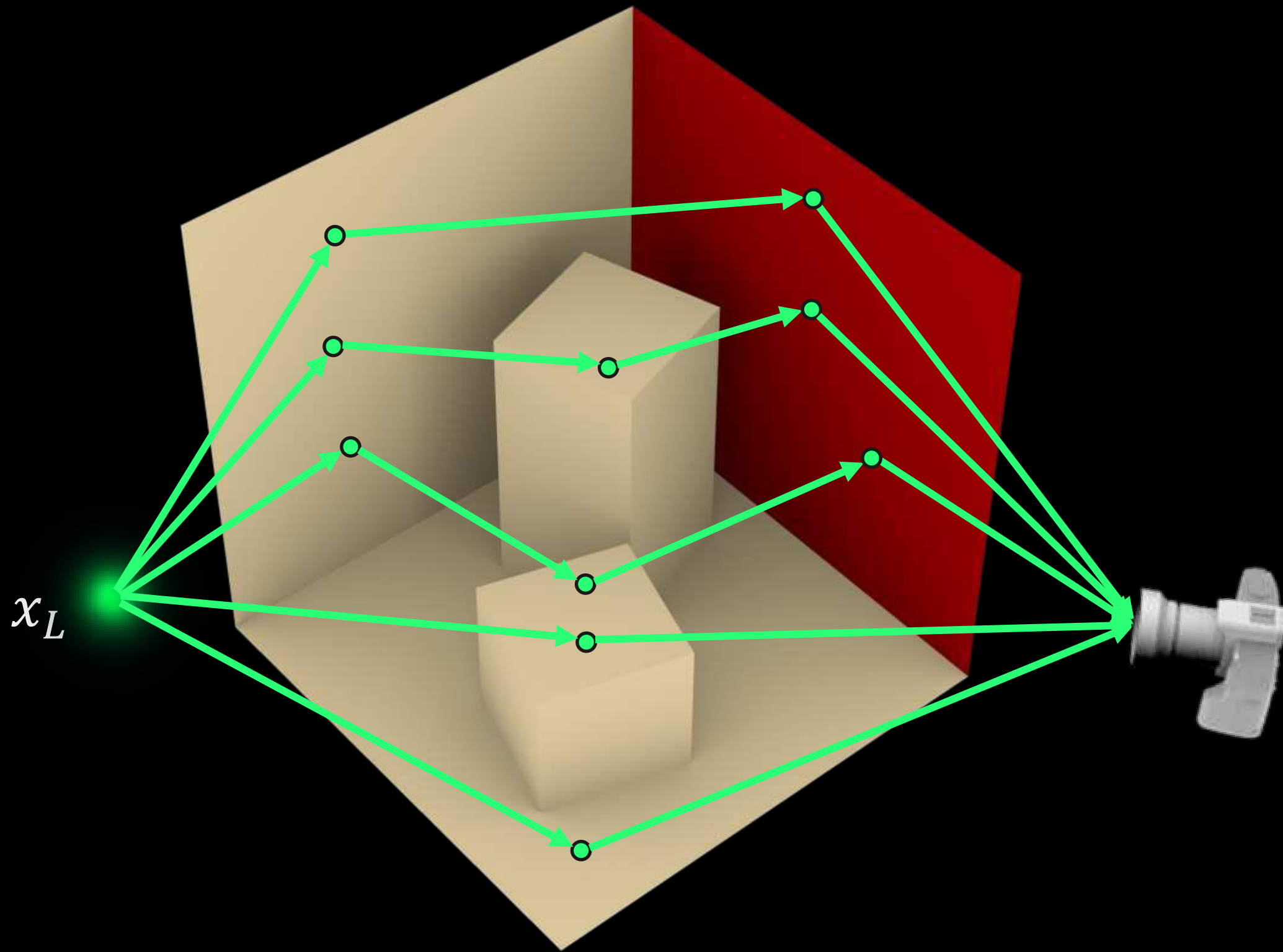
- randomly sample paths:  $\text{path}_1, \text{path}_2, \dots, \text{path}_N$
- approximate image as:

$$\text{image} \approx \sum_n \frac{\text{f(path}_n\text{)}}{\text{prob}(\text{path}_n)}$$

# Rendering time-of-flight cameras: Path space integral

$$\text{image} = \int_{\text{light paths}} f(\text{path}) W(|\text{path}|)$$

path length weight depends only on sensor



Monte Carlo rendering:

- randomly sample paths:  $\text{path}_1, \text{path}_2, \dots, \text{path}_N$
- approximate image as:

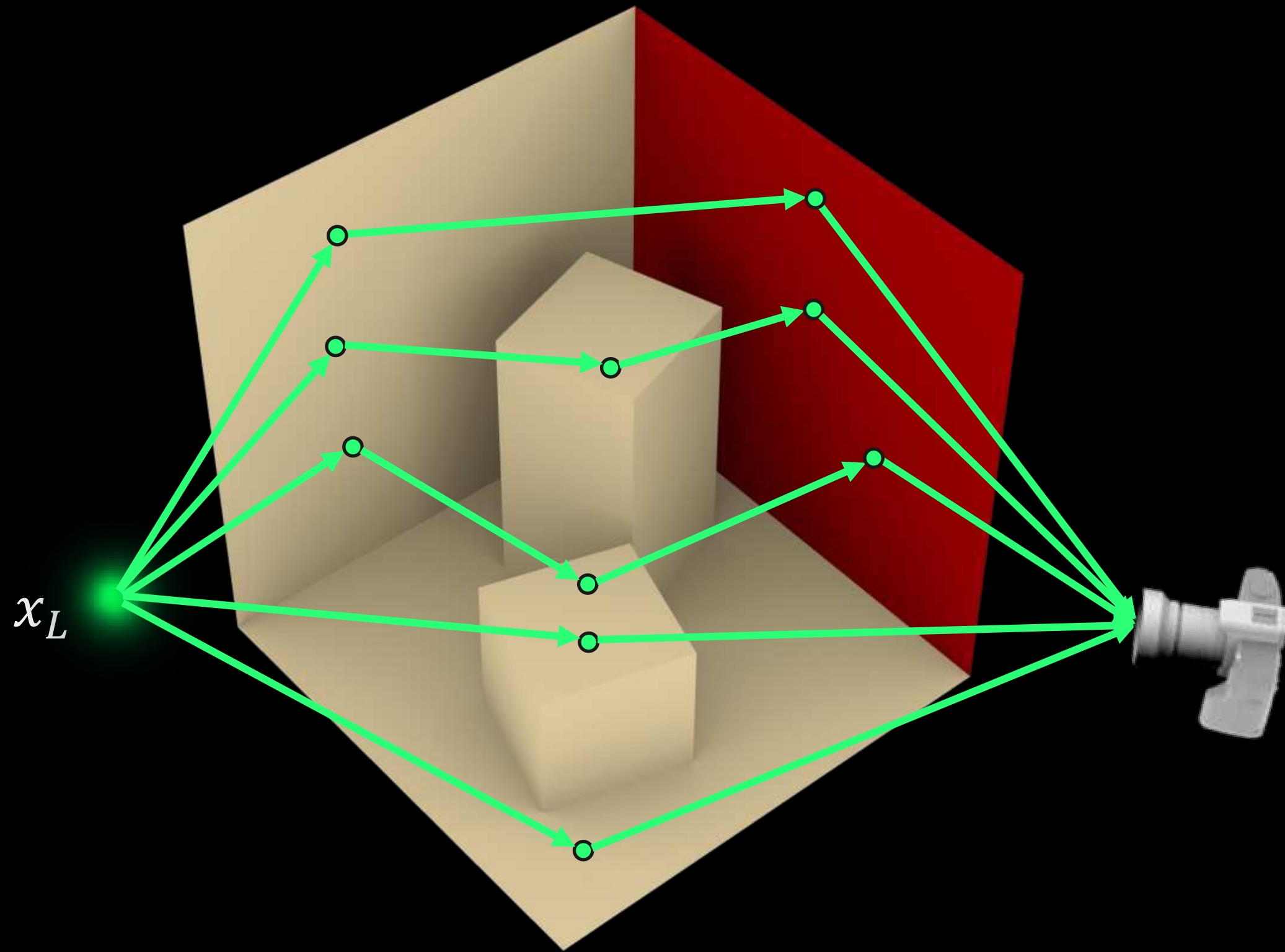
$$\text{image} \approx \sum_n \frac{f(\text{path}_n)}{\text{prob}(\text{path}_n)}$$



# Rendering time-of-flight cameras: Path space integral

$$\text{image} = \int_{\text{light paths}} f(\text{path}) W(|\text{path}|)$$

path length weight depends only on sensor

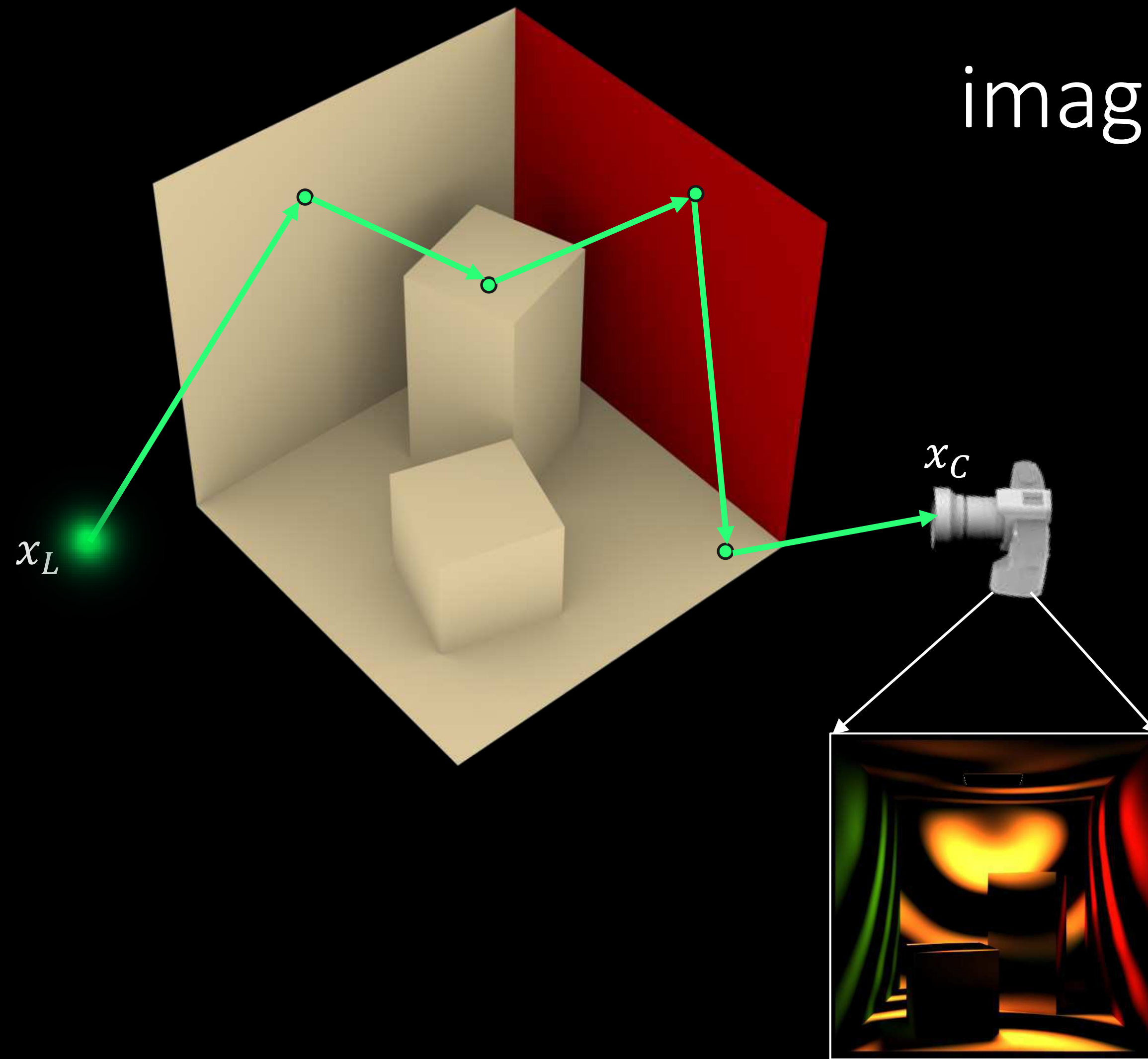


Monte Carlo rendering:

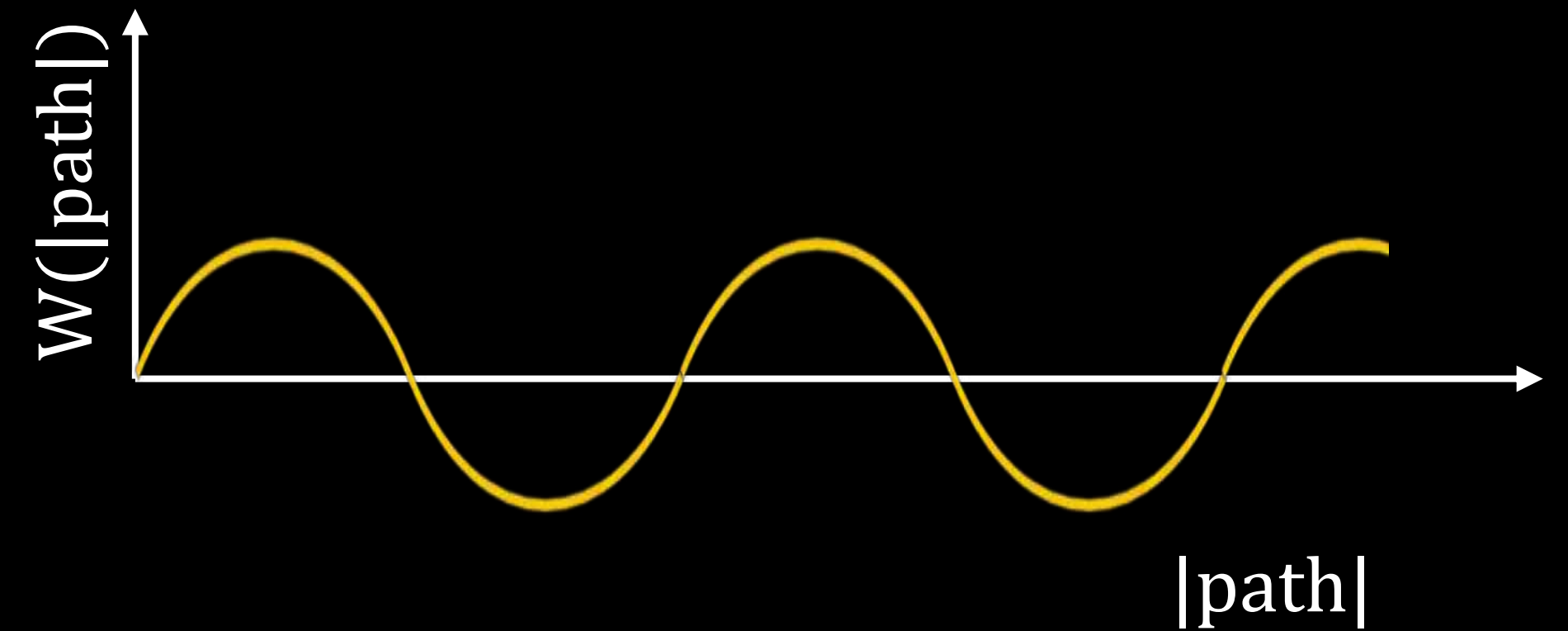
- randomly sample paths:  $\text{path}_1, \text{path}_2, \dots, \text{path}_N$
- approximate image as:

$$\text{time-of-flight image} \approx \sum_n \frac{f(\text{path}_n)}{\text{prob}(\text{path}_n)} W(|\text{path}_n|)$$

# Rendering continuous wave time-of-flight camera

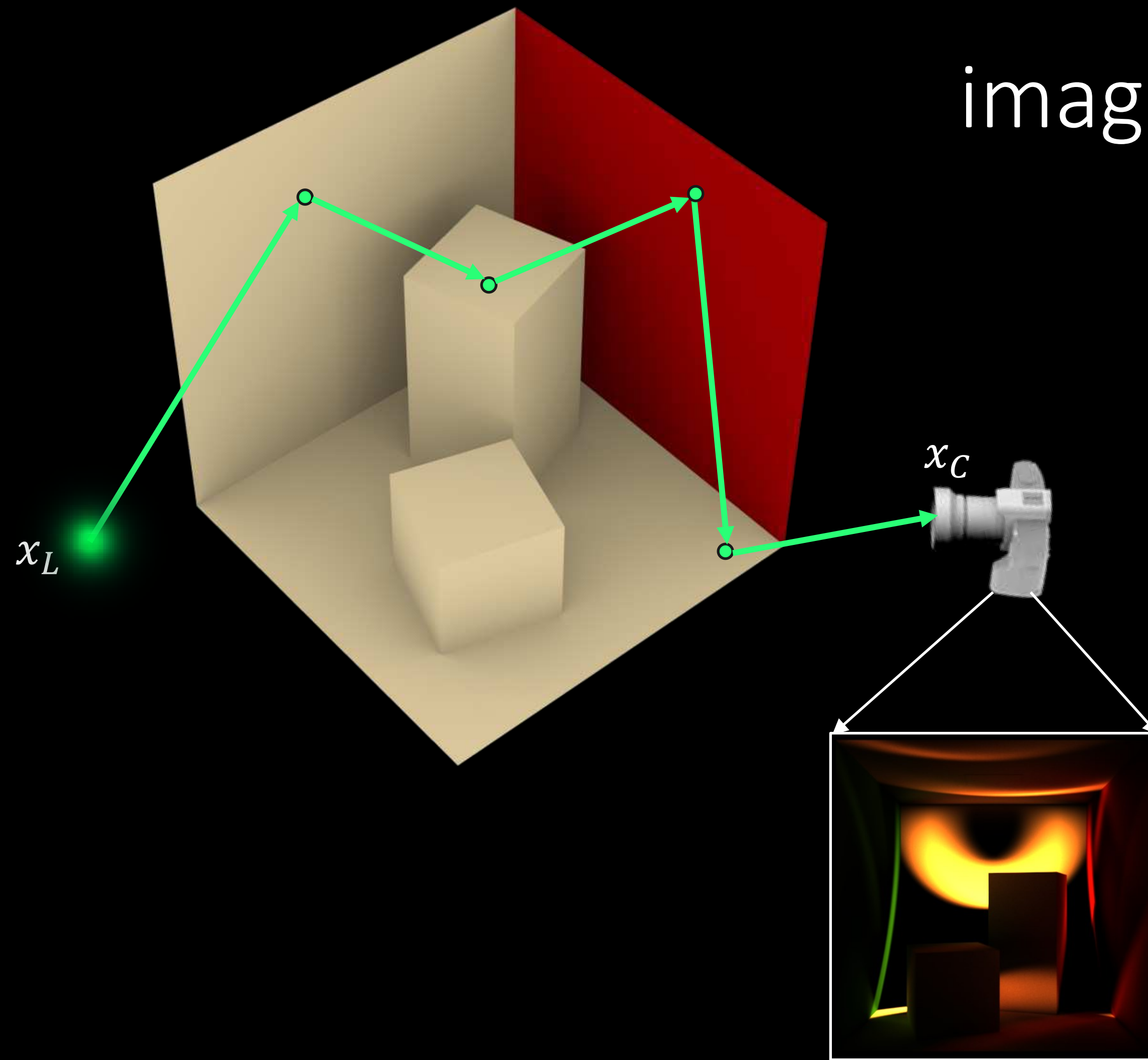


$$\text{image} = \int_{\text{light paths}} f(\text{path}) W(|\text{path}|)$$

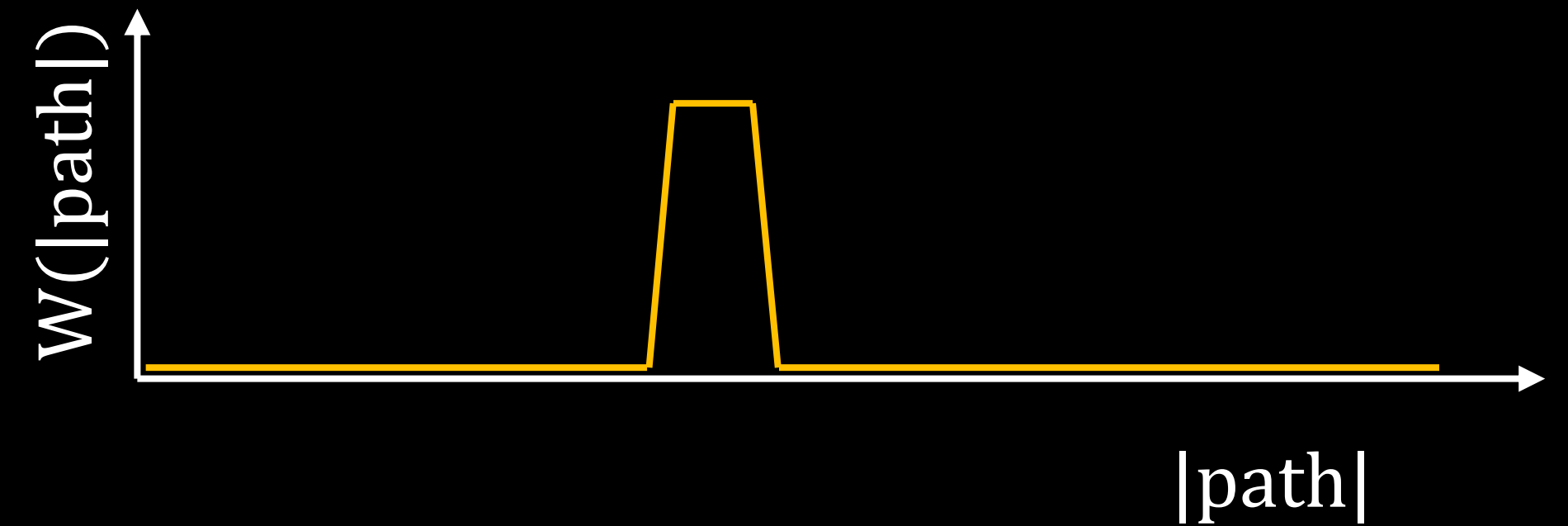




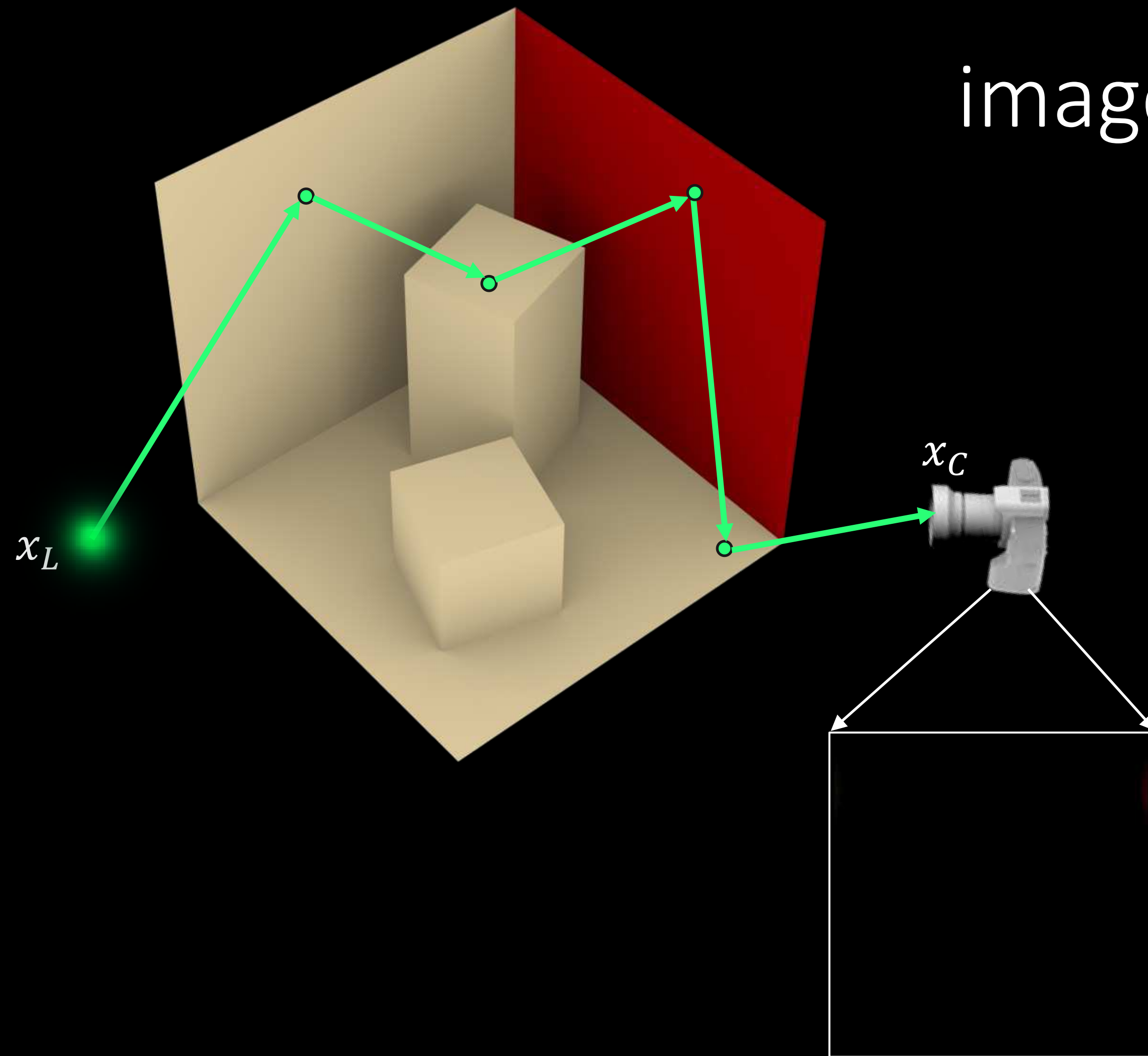
# Classification: time-gated camera



$$\text{image} = \int_{\text{light paths}} f(\text{path}) W(|\text{path}|)$$



# Rendering transient camera



$$\text{image} = \int_{\text{light paths}} f(\text{path}) W(|\text{path}|)$$

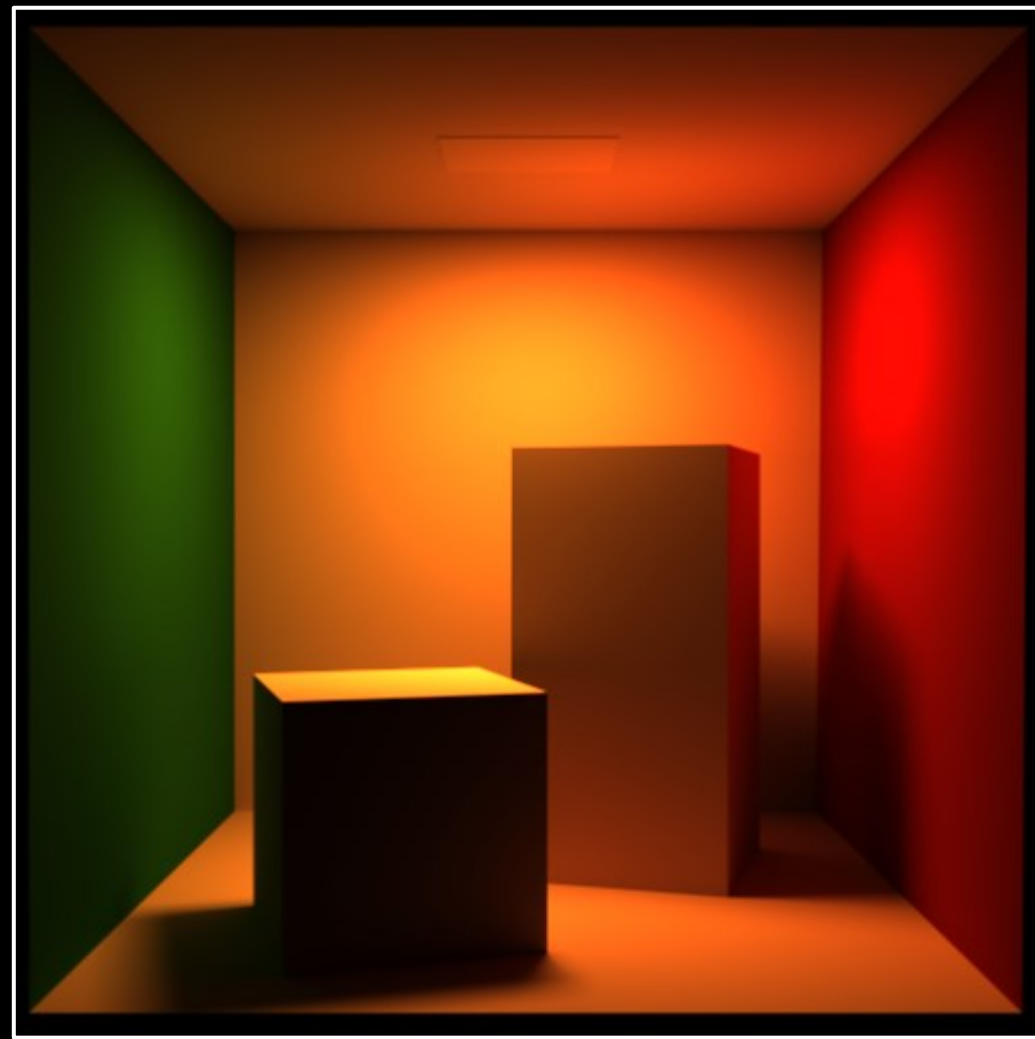




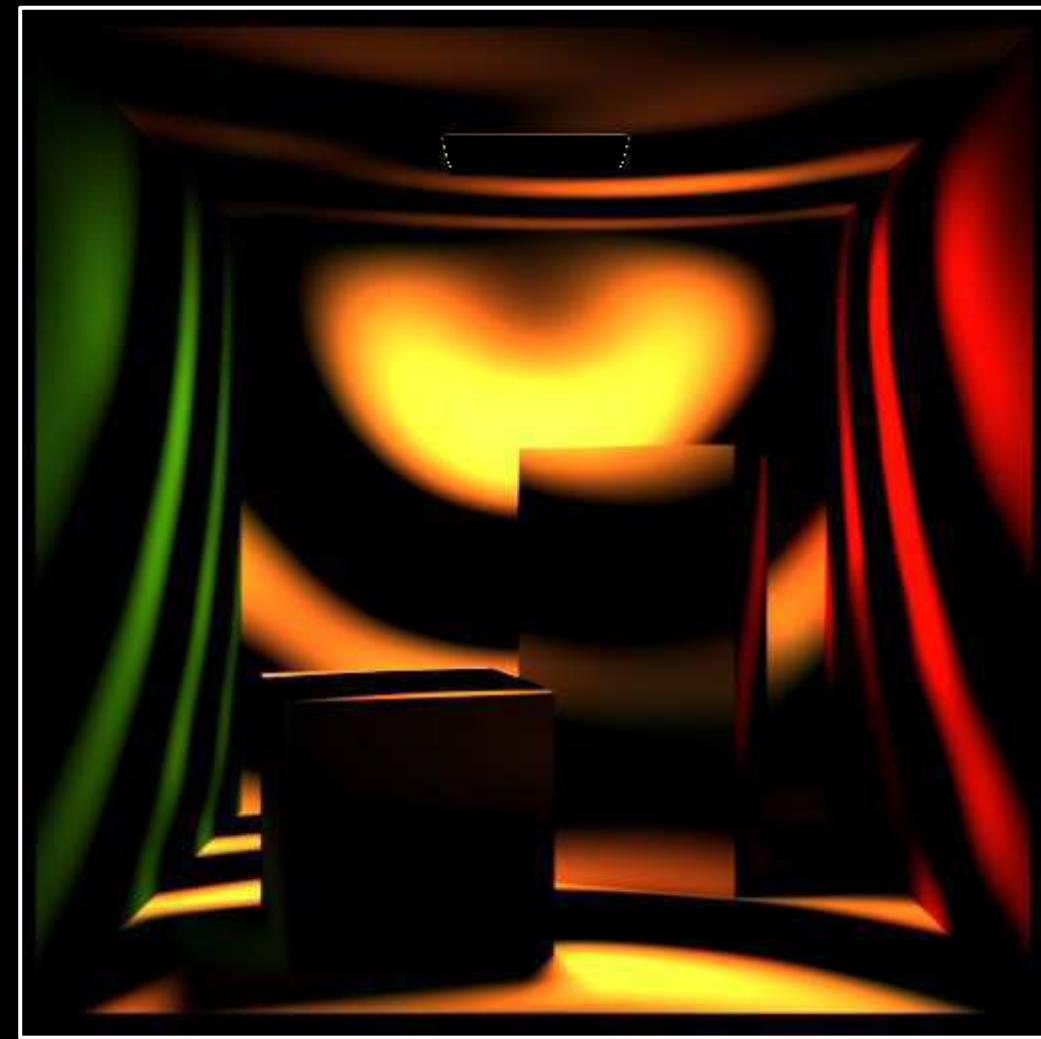
# Path space integral for time-of-flight cameras

$$\text{image} = \int_{\text{light paths}} f(\text{path}) W(|\text{path}|)$$

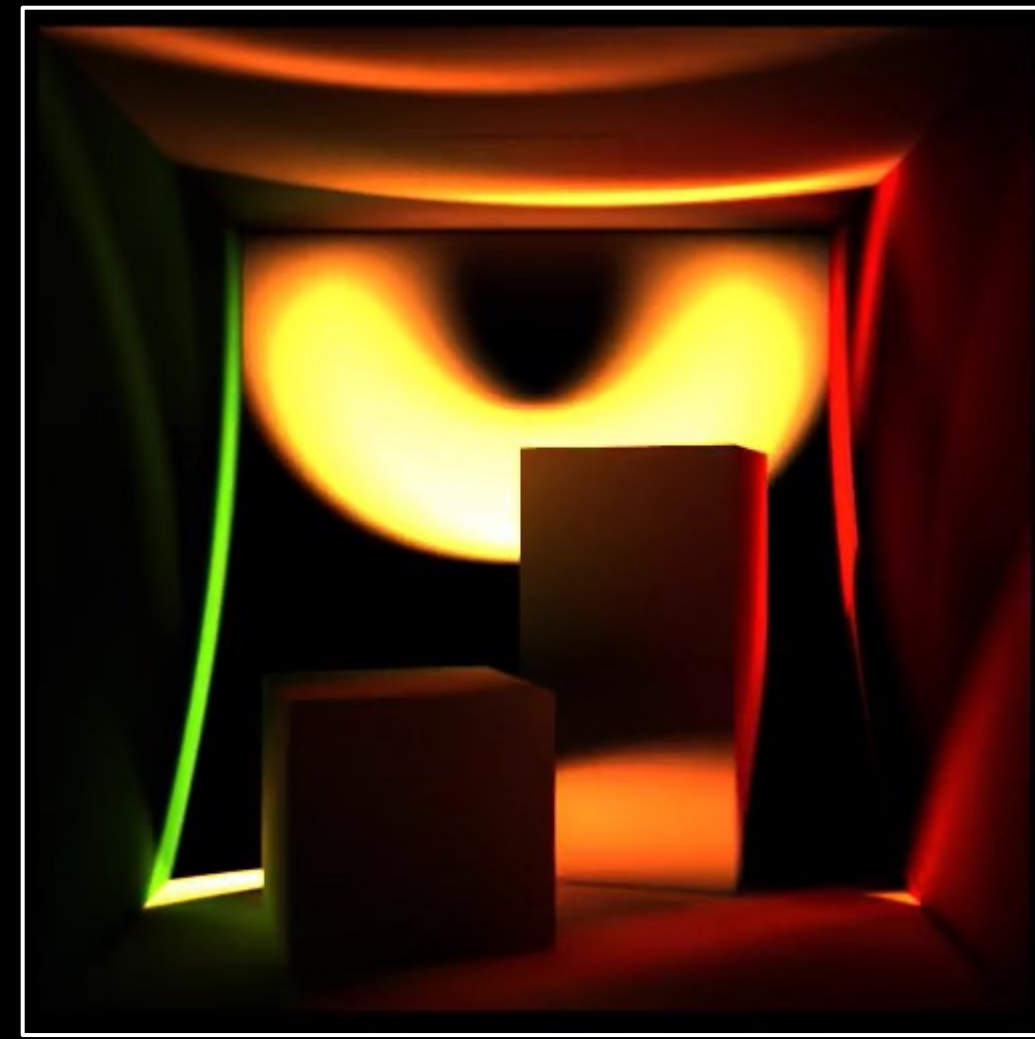
intensity



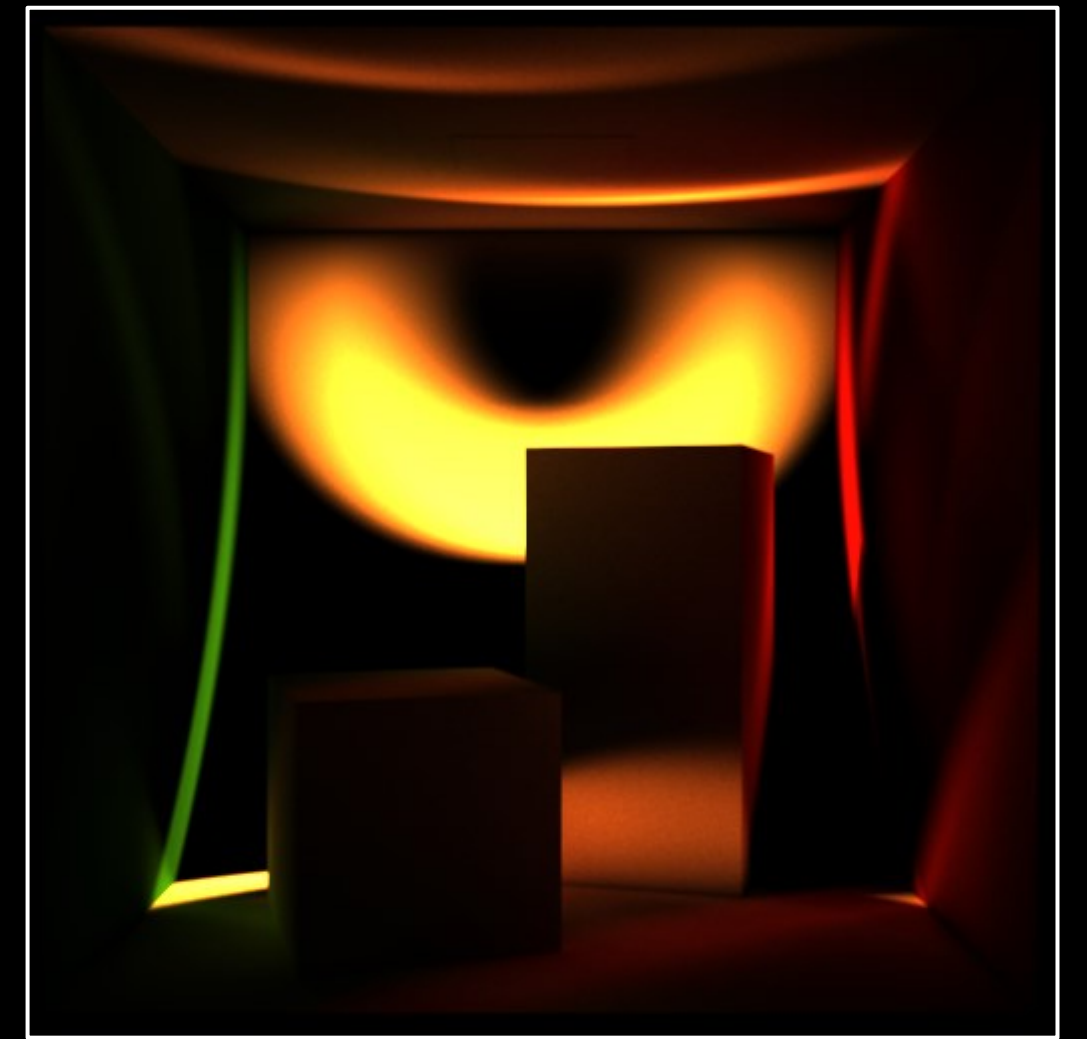
continuous-wave



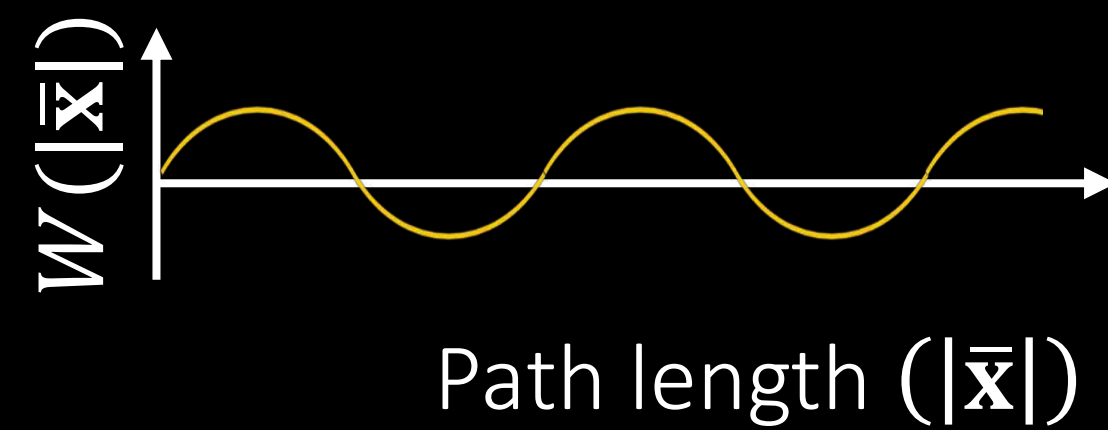
transient



time-gated



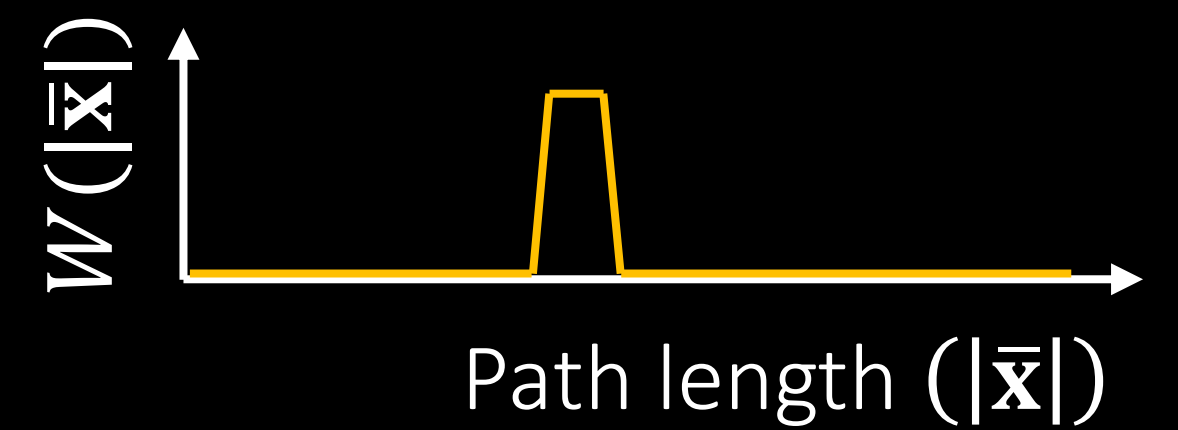
BDPT, PT, PM, KDE, etc



BDPT, PT, PM, KDE, etc

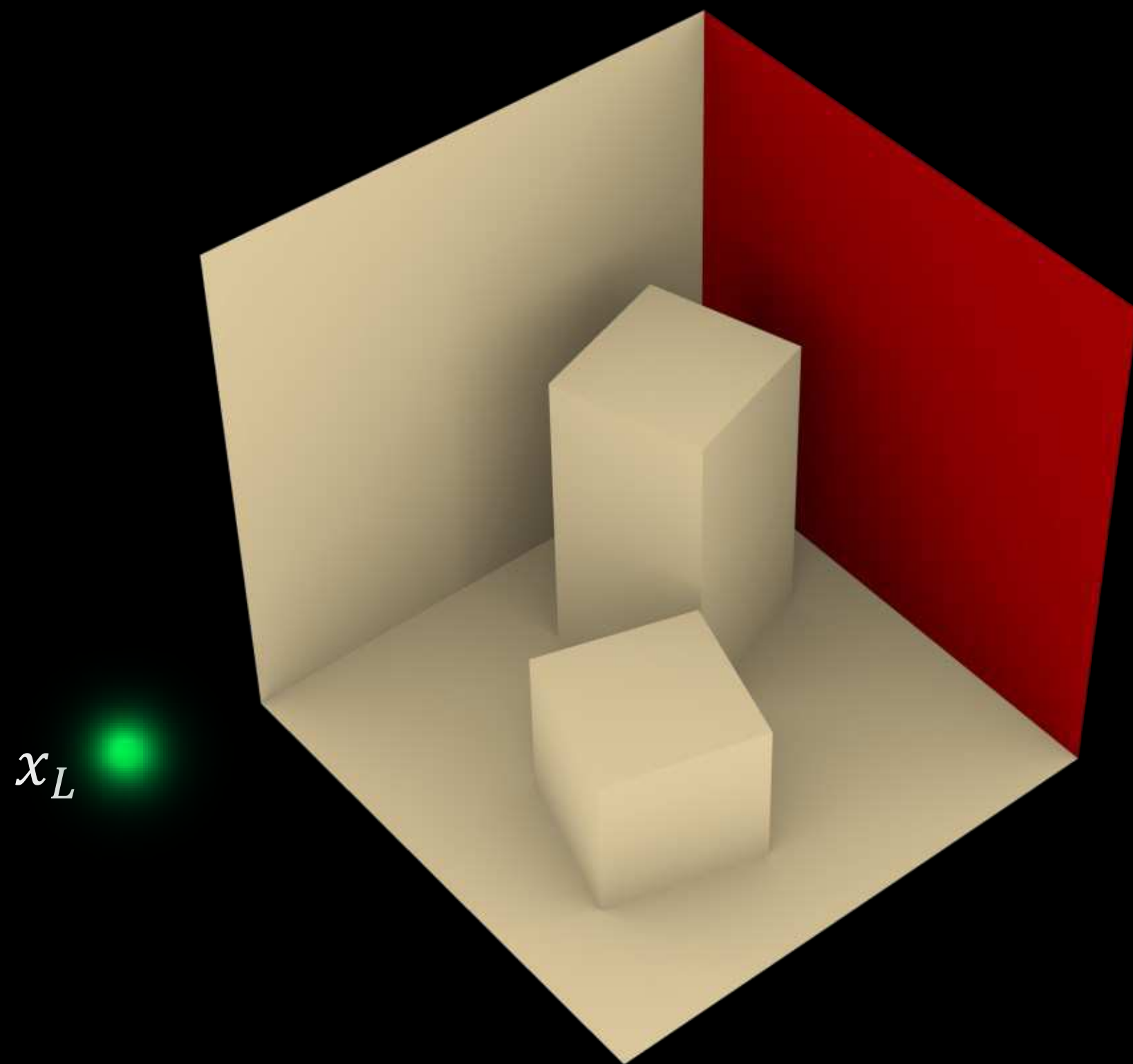


BDPT, PT, PM, KDE, etc  
[Jarabo et al., 2014, 2017]  
[Marco et al. 2017, 2018]

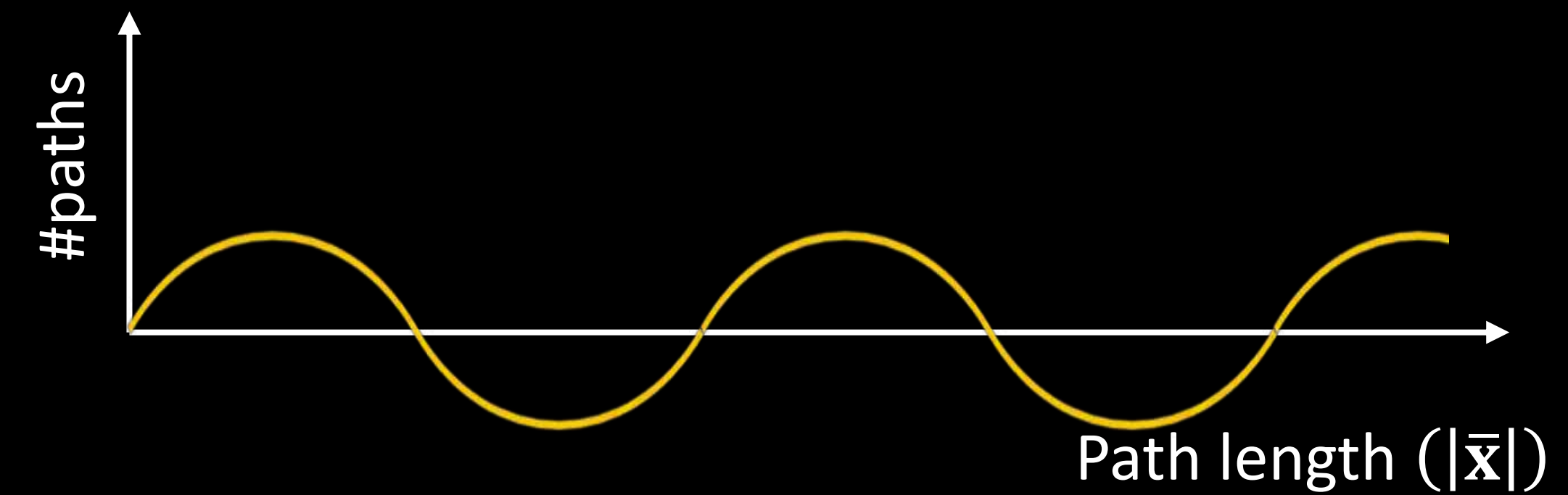


no efficient renderer

# Path sampling for time-gated rendering is challenging



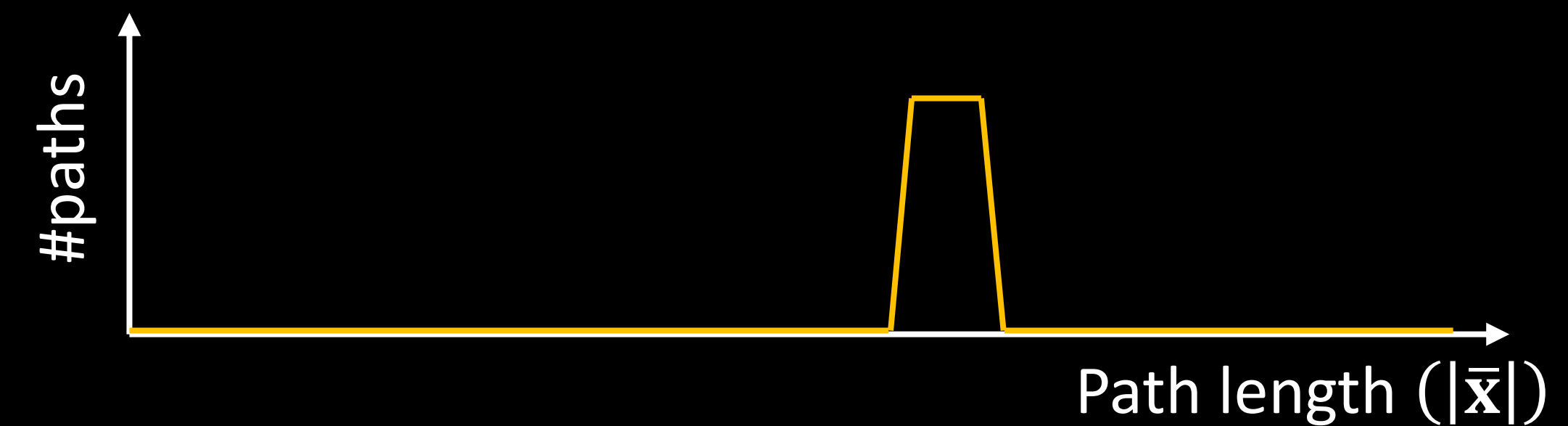
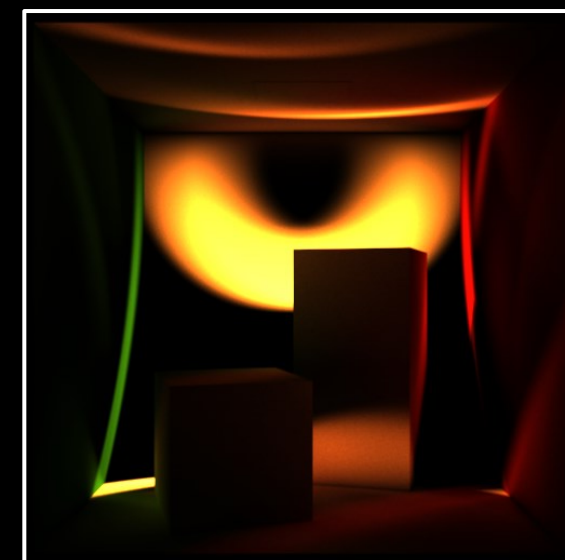
continuous-wave



transient



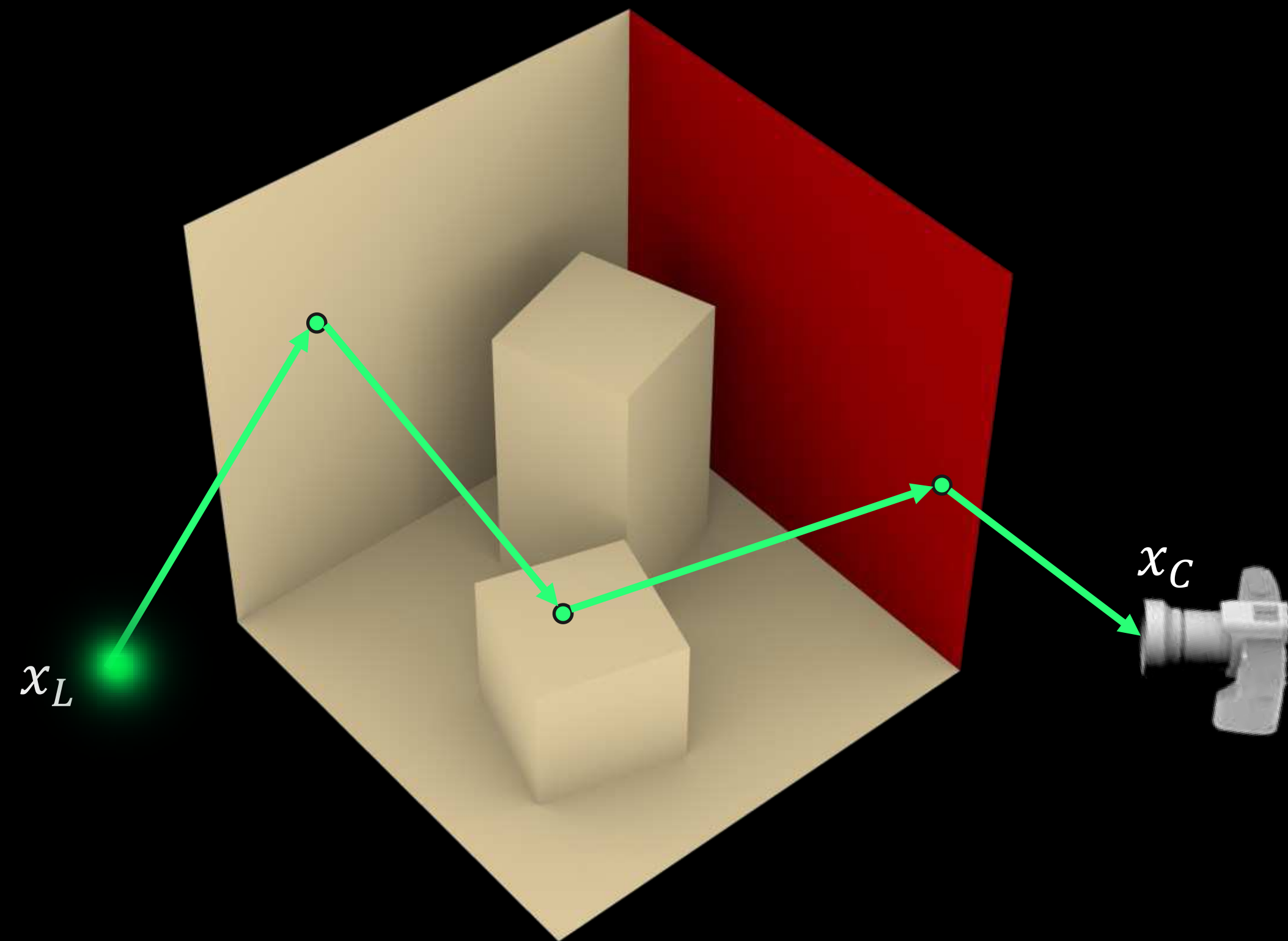
time-gated



No control over path length

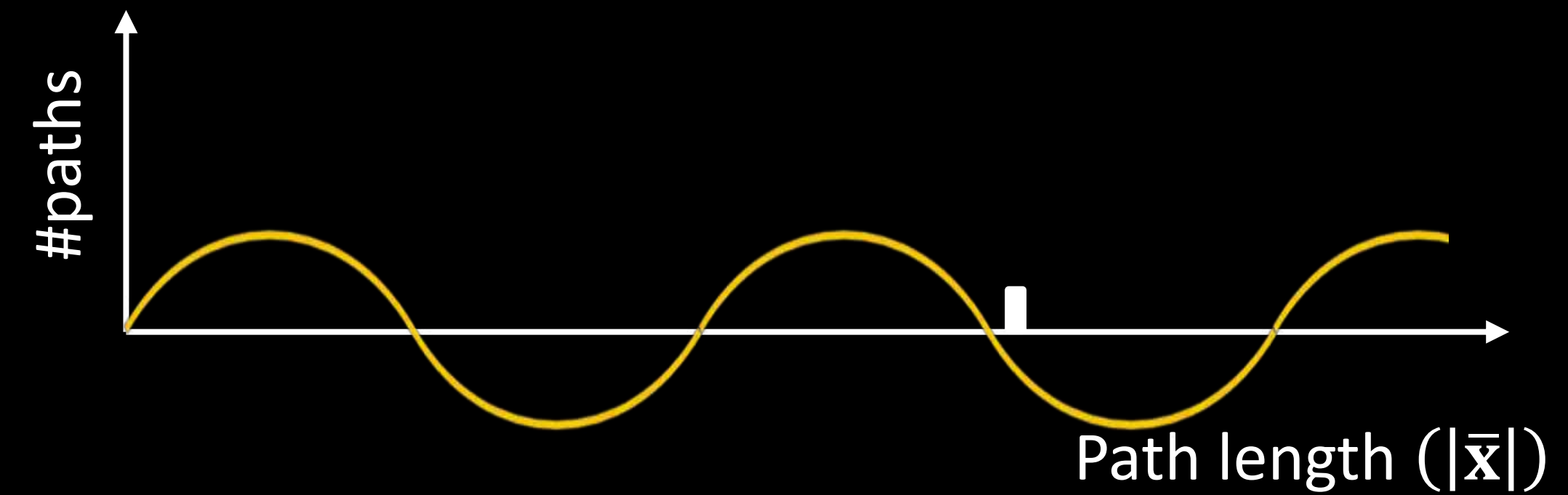
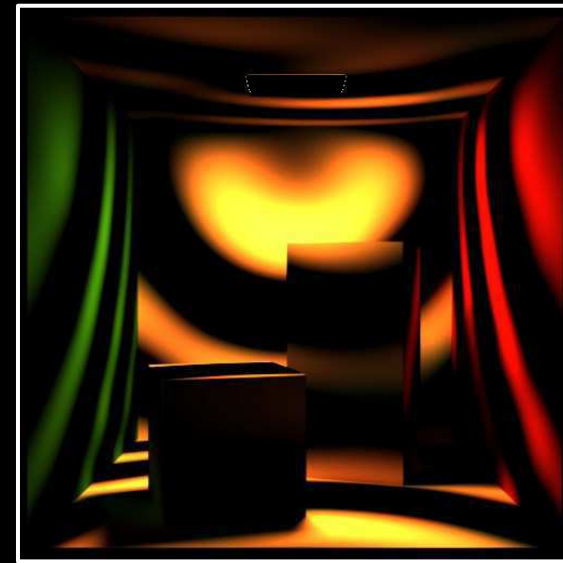


# Path sampling for time-gated rendering is challenging



No control over path length

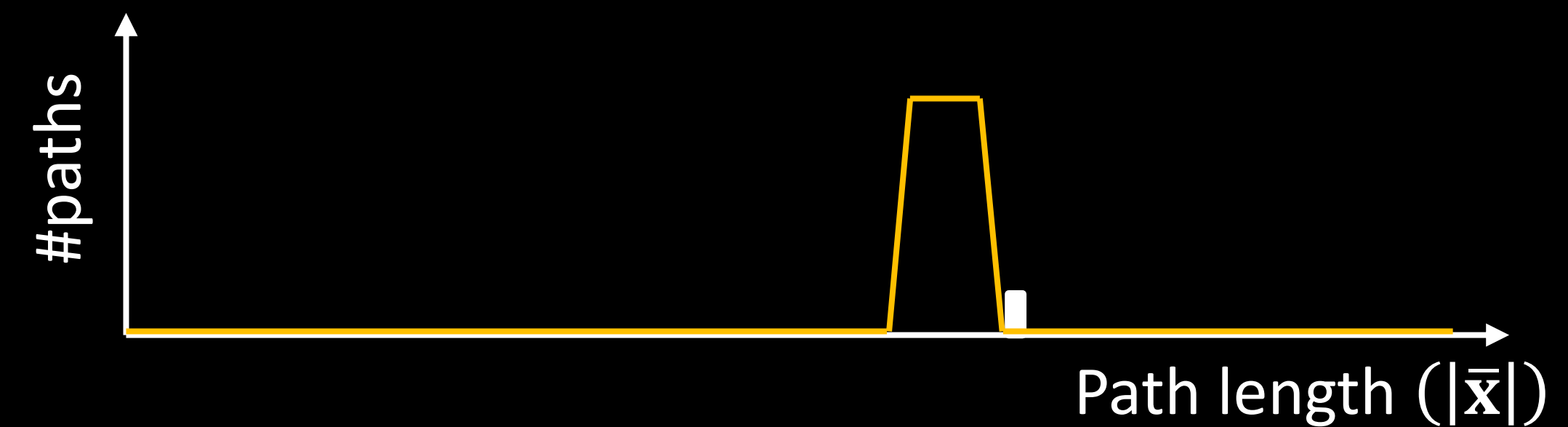
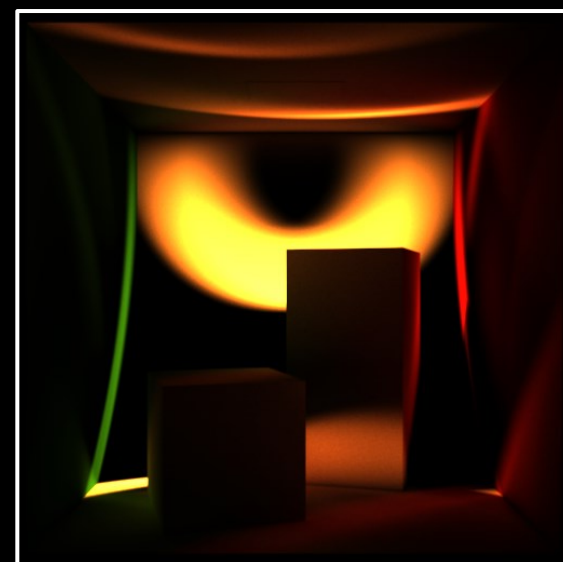
continuous-wave



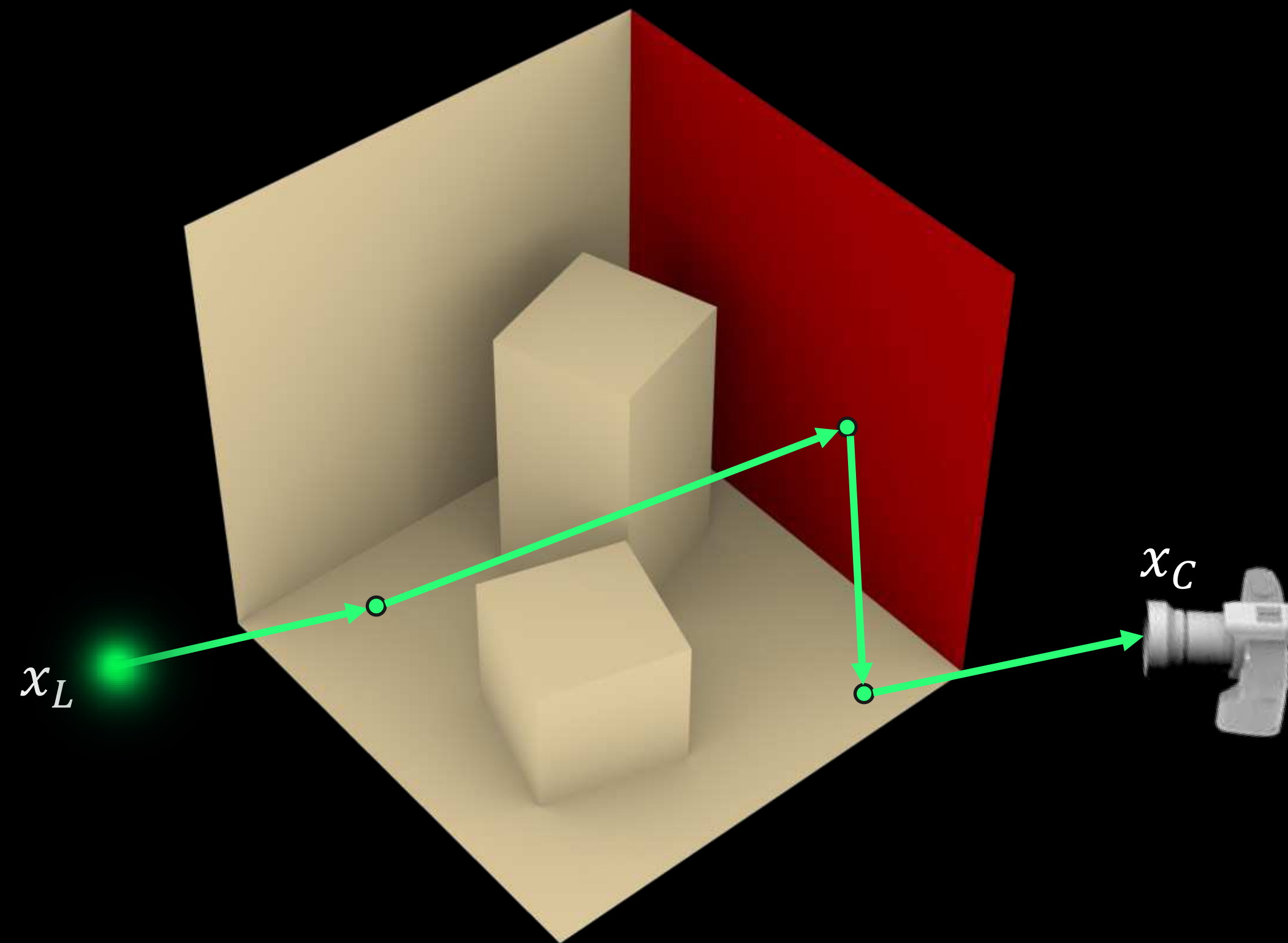
transient



time-gated

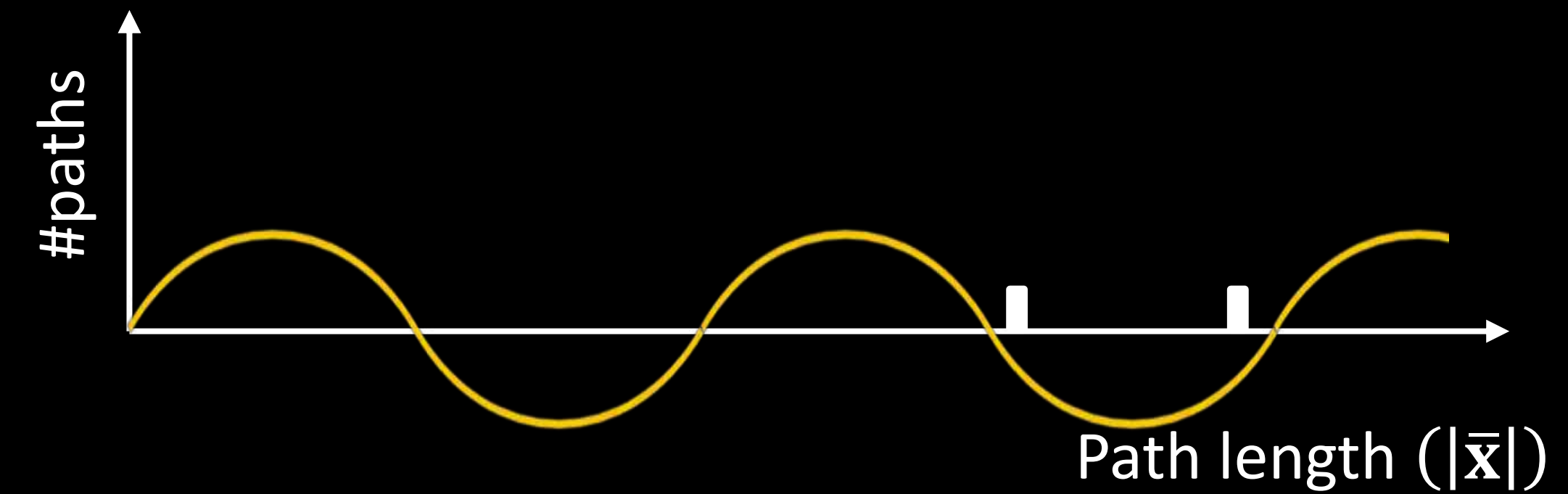


# Path sampling for time-gated rendering is challenging



No control over path length

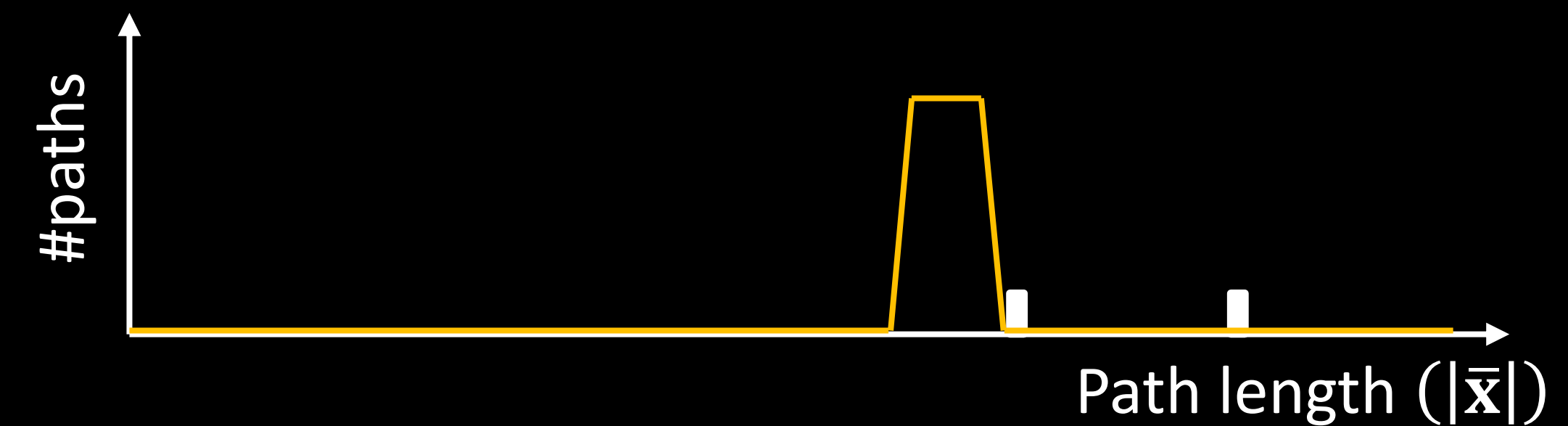
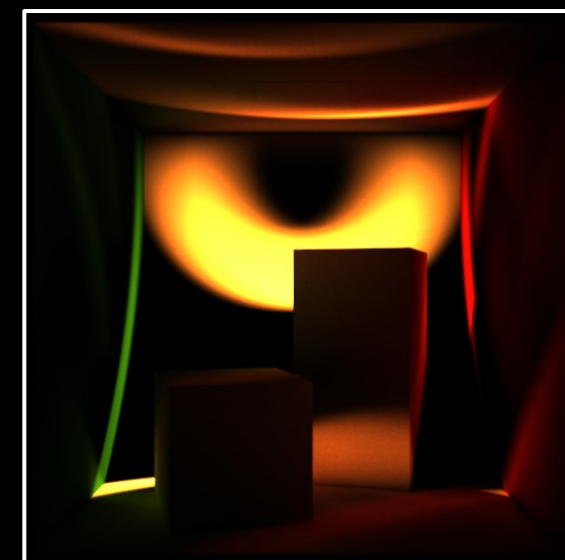
continuous-wave



transient

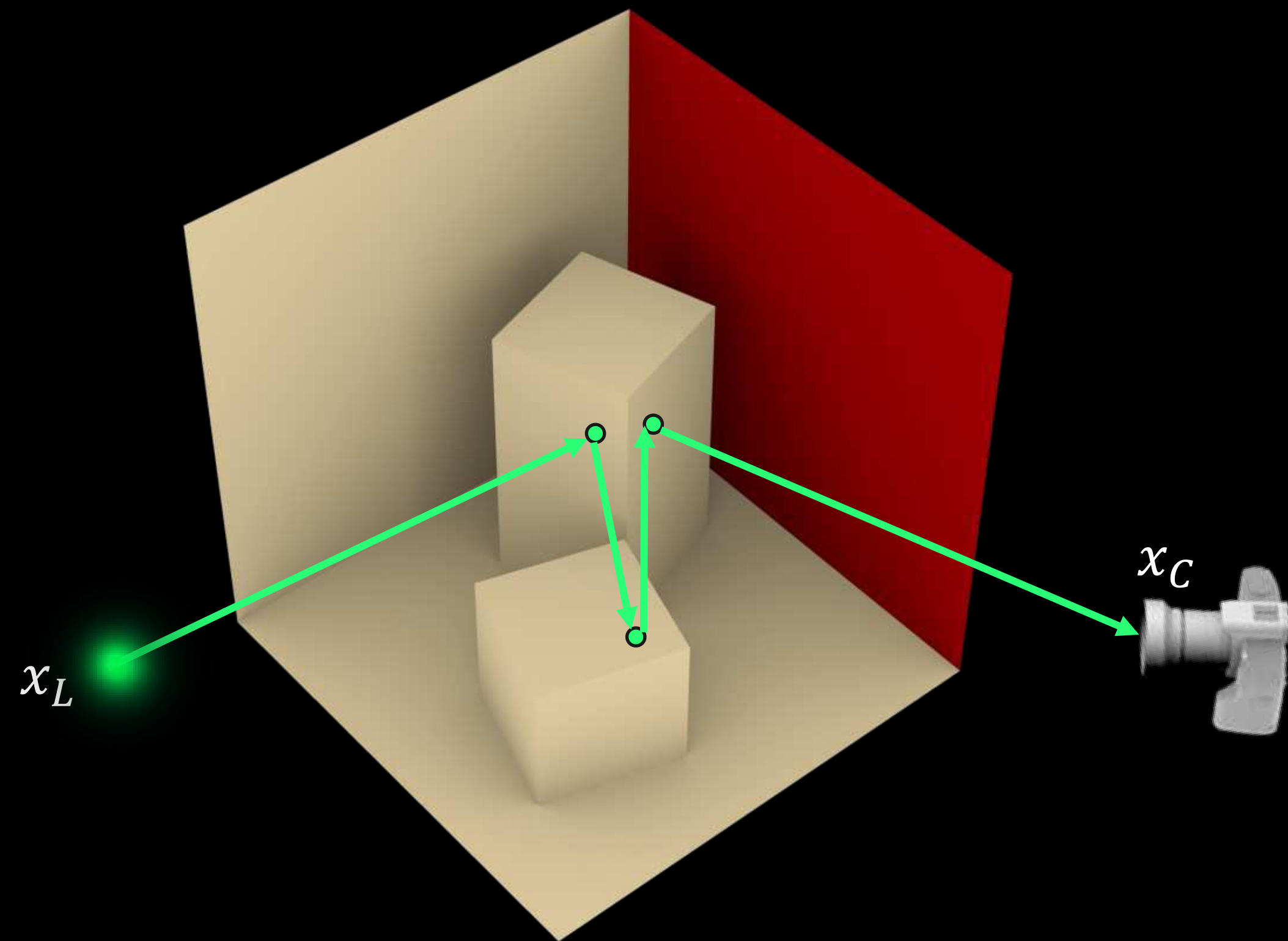


time-gated



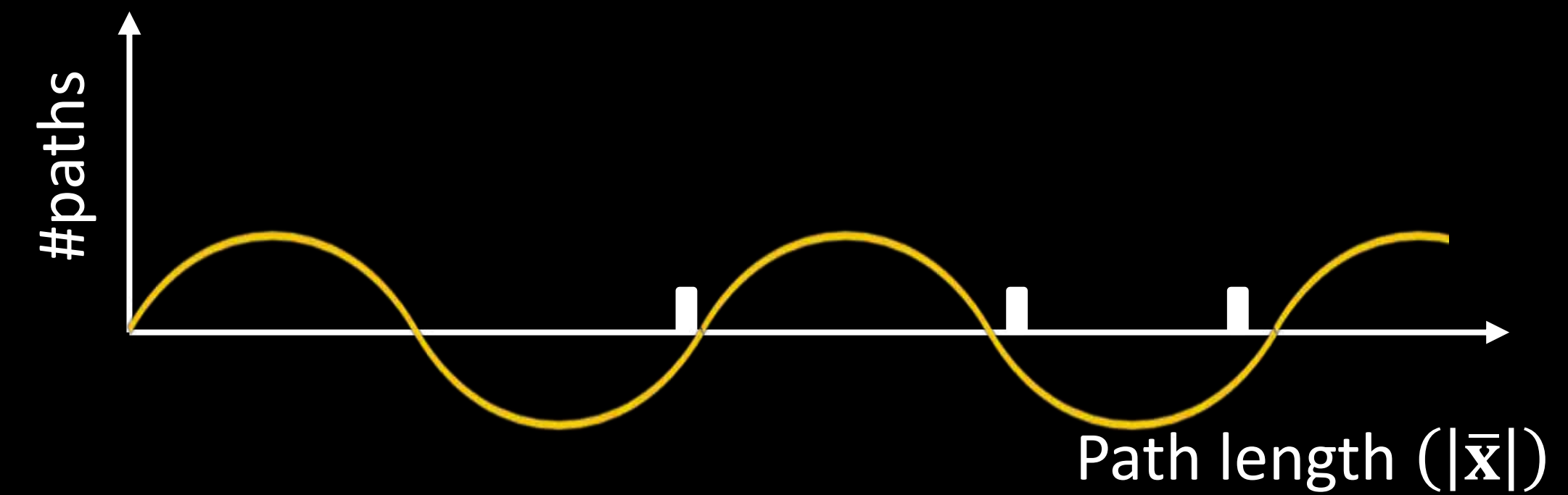
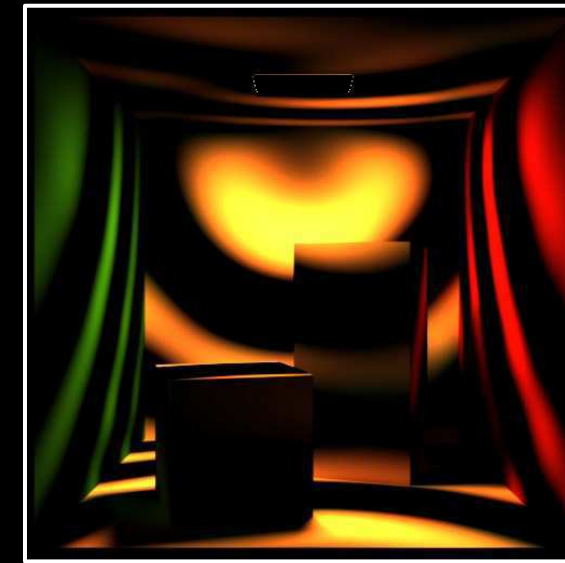


# Path sampling for time-gated rendering is challenging

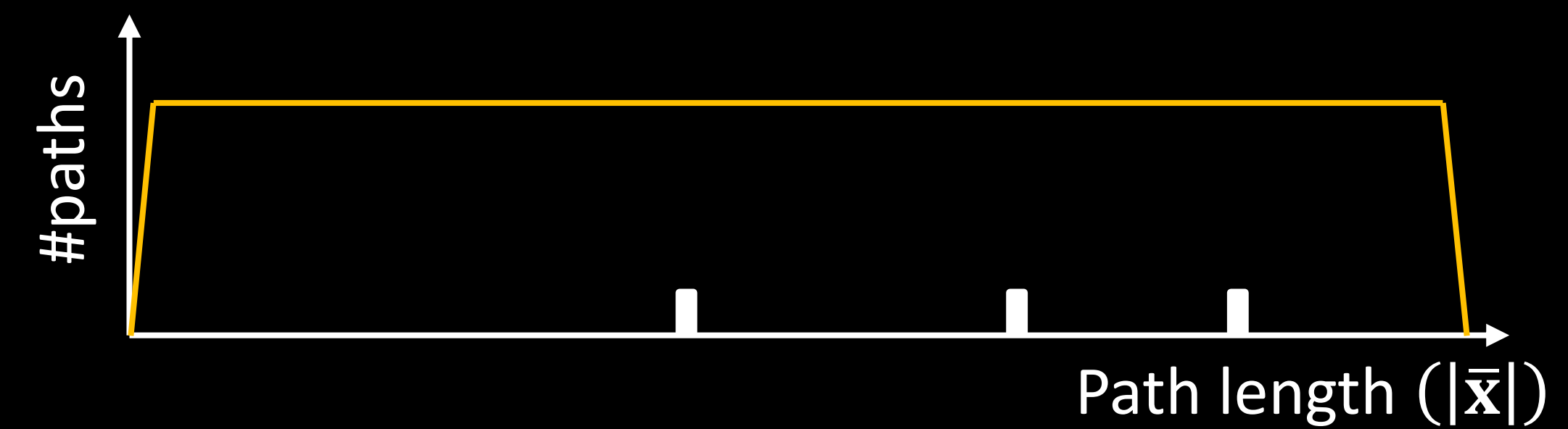


No control over path length

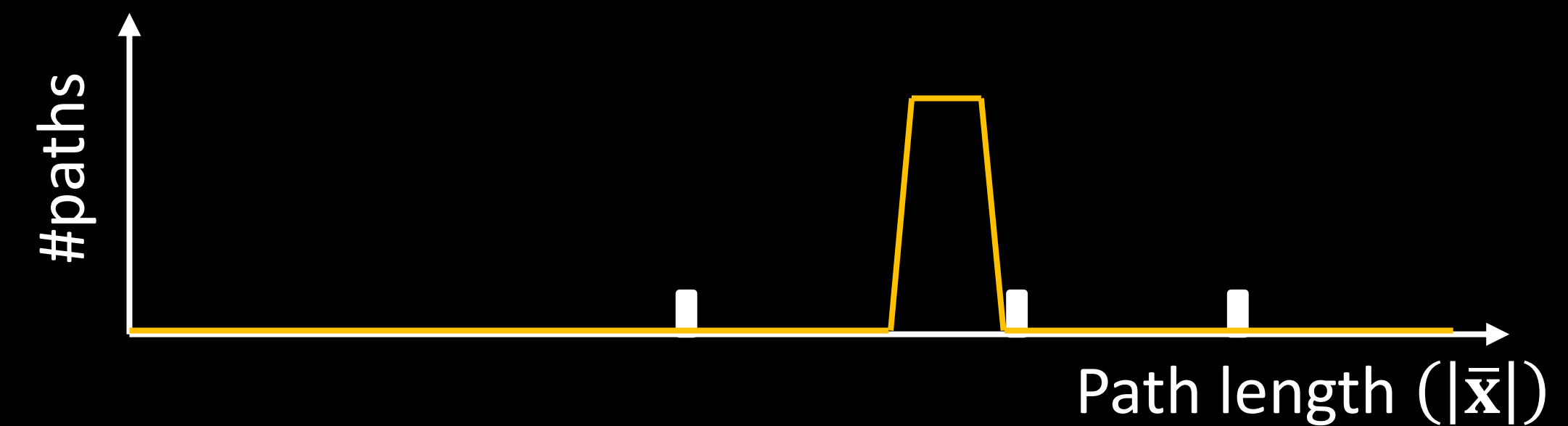
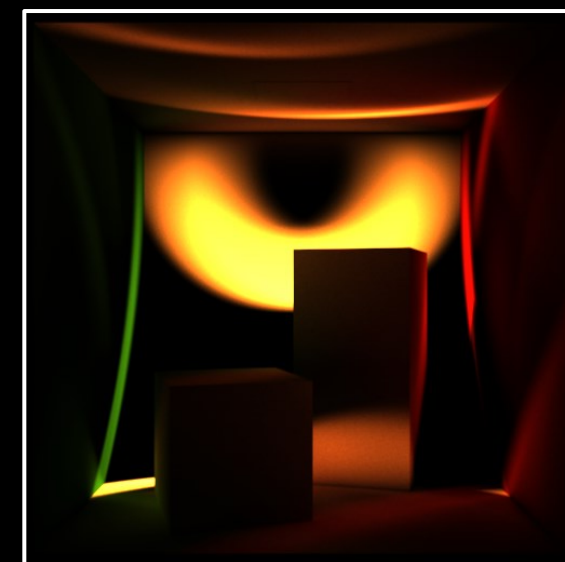
continuous-wave



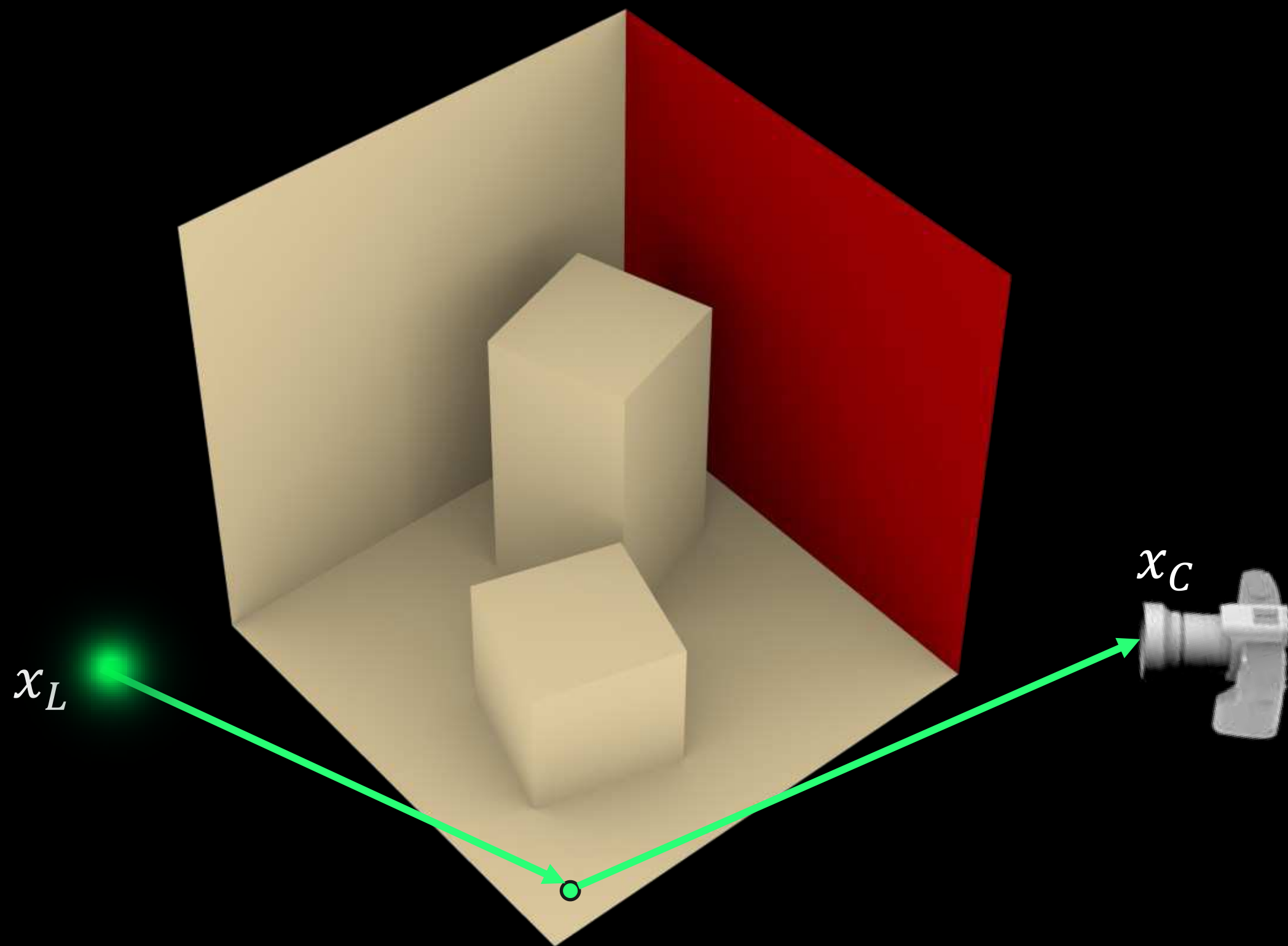
transient



time-gated

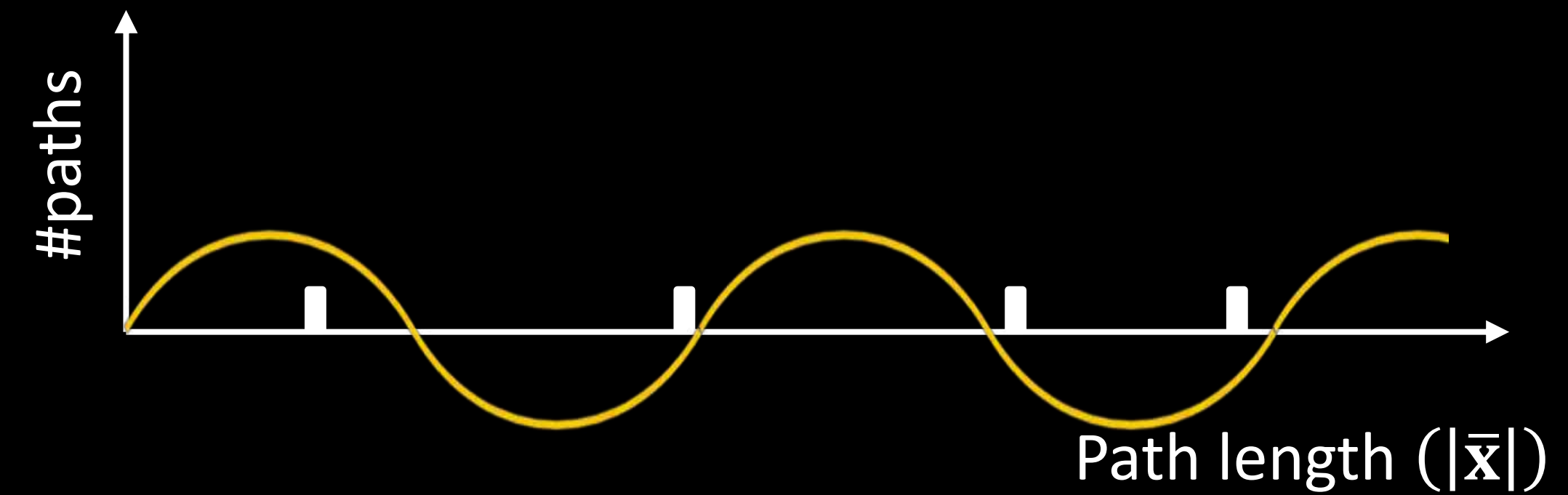
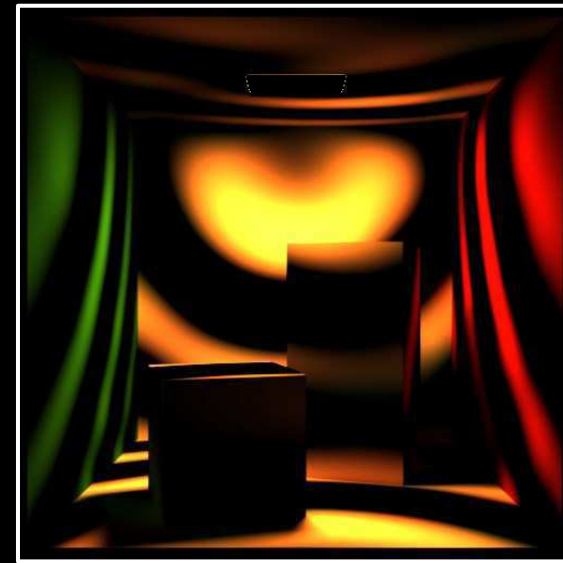


# Path sampling for time-gated rendering is challenging



No control over path length

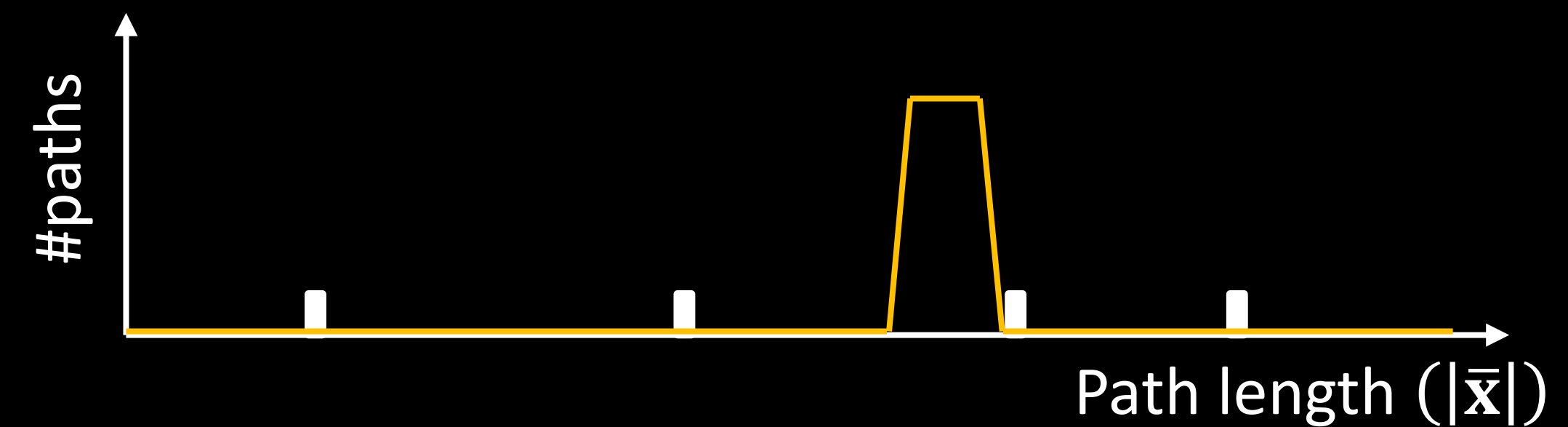
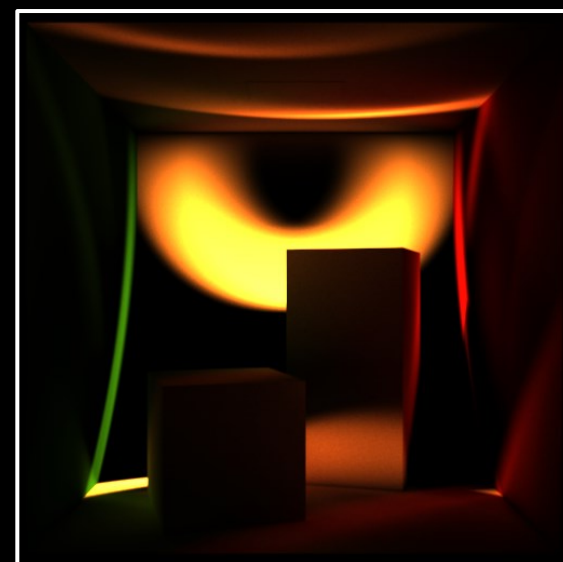
continuous-wave



transient

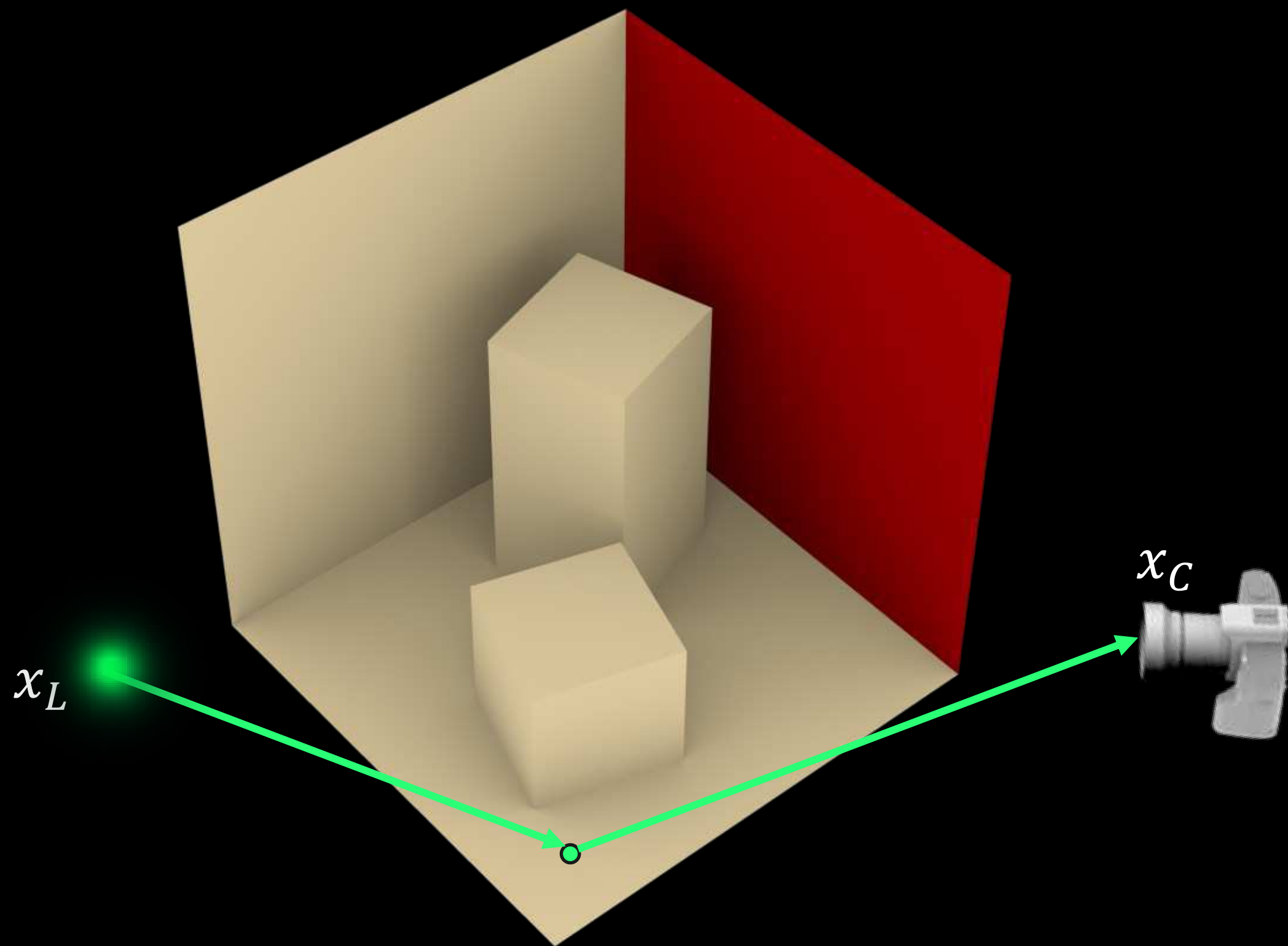


time-gated



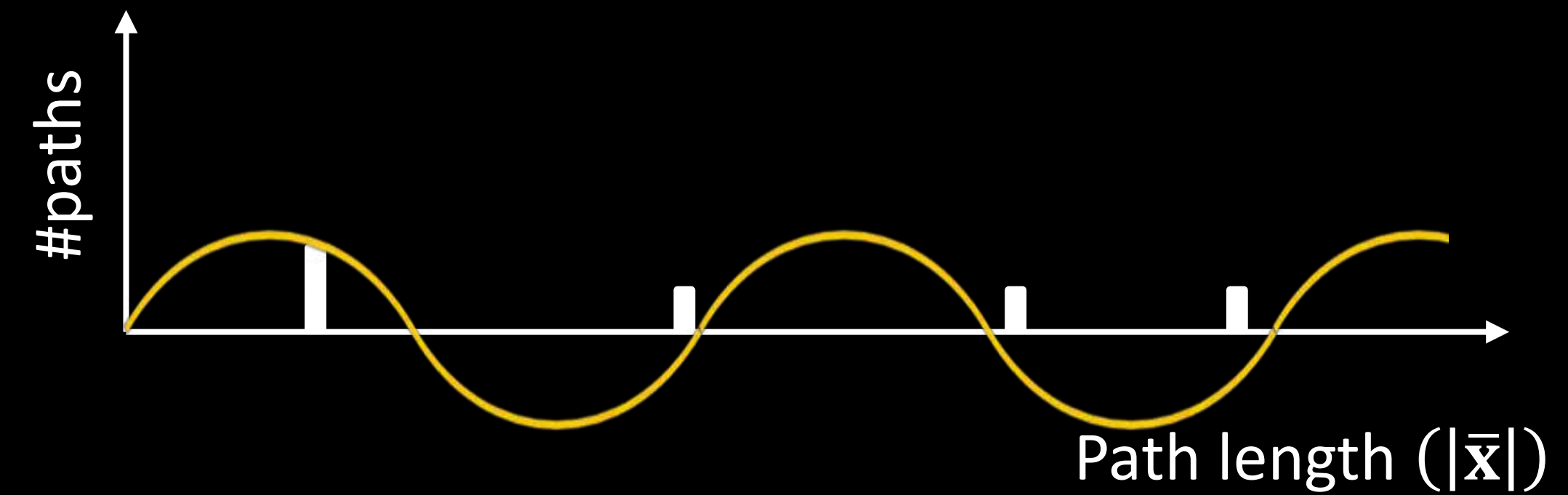
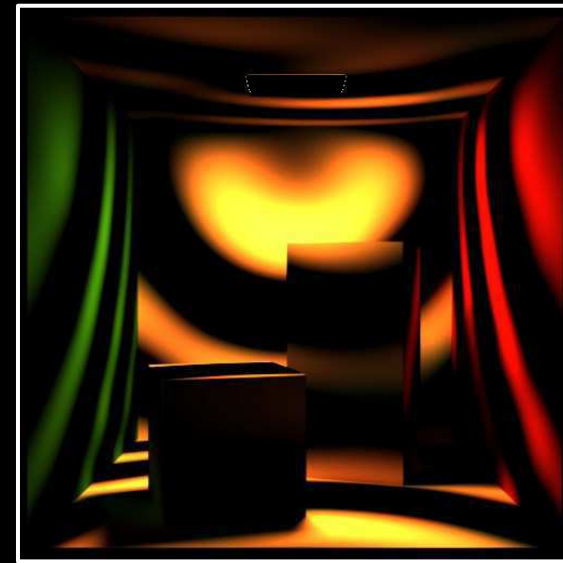


# Path sampling for time-gated rendering is challenging



No control over path length

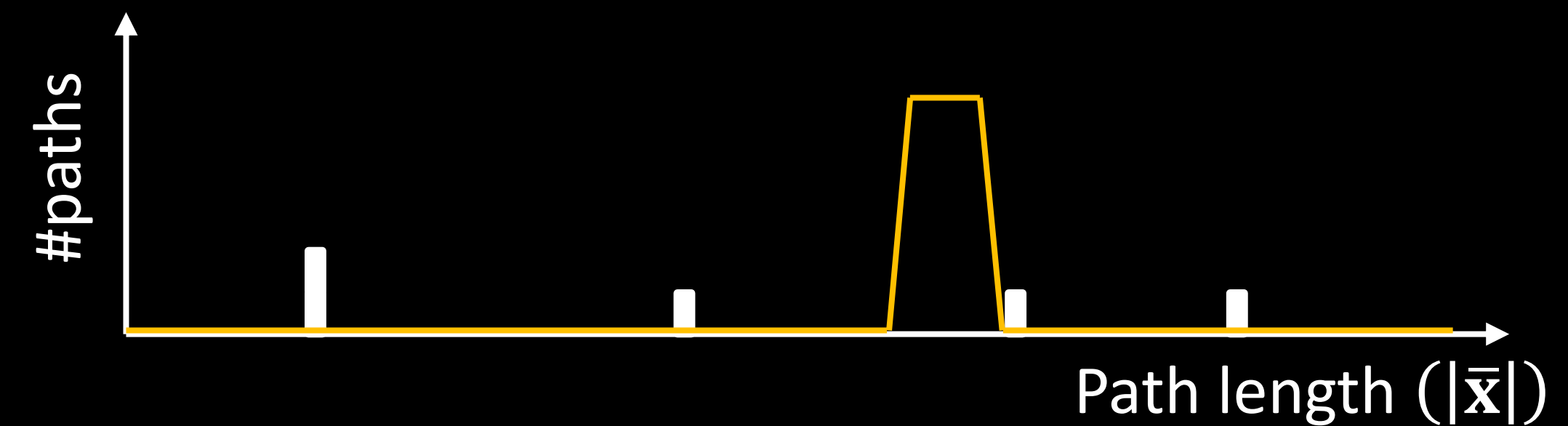
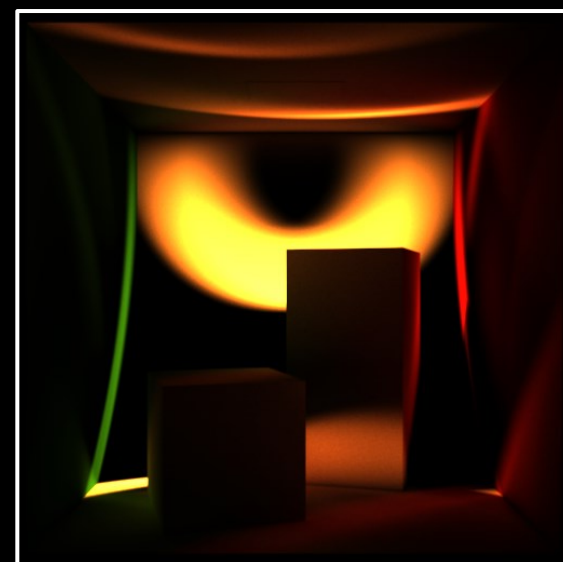
continuous-wave



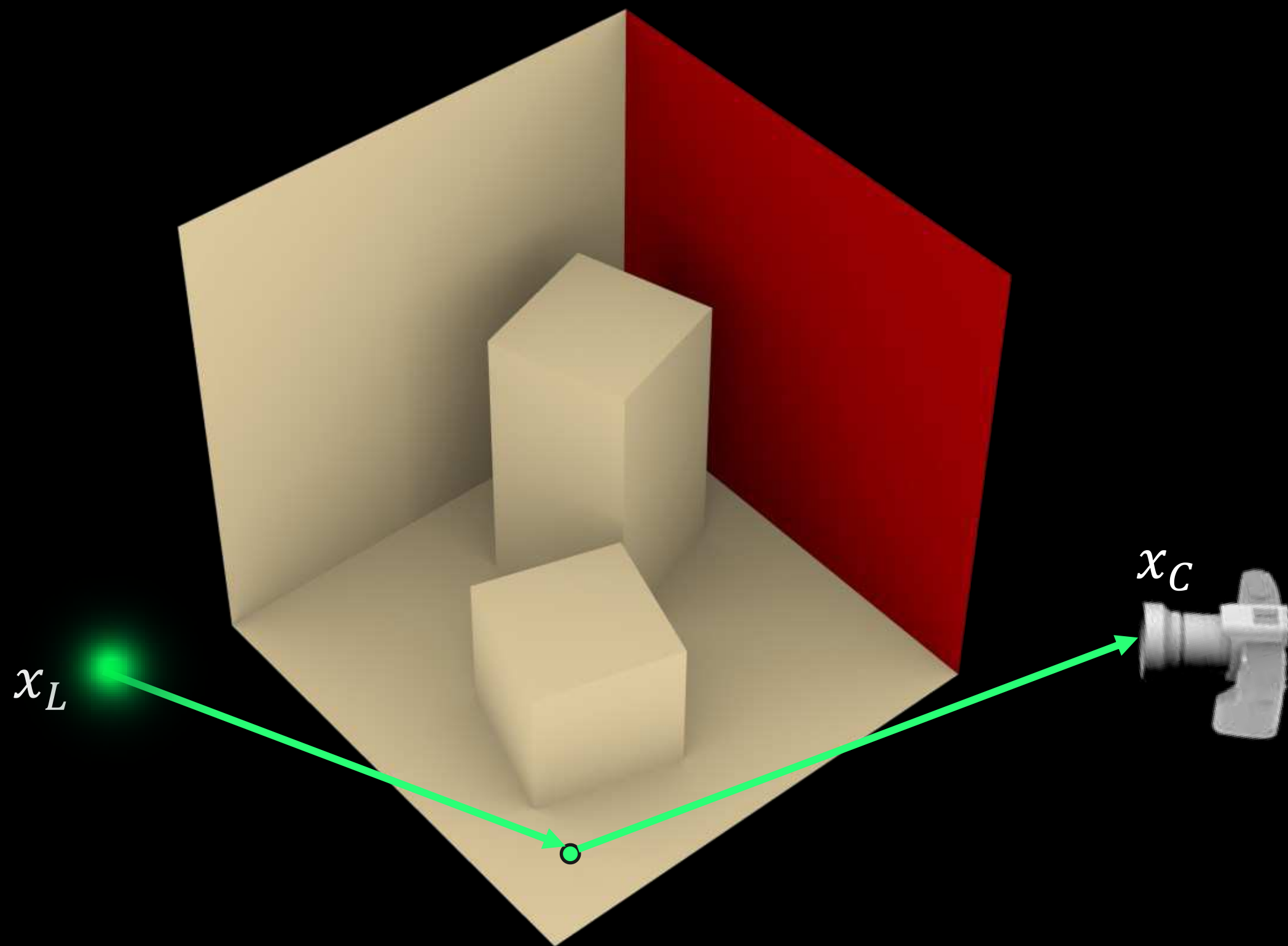
transient



time-gated

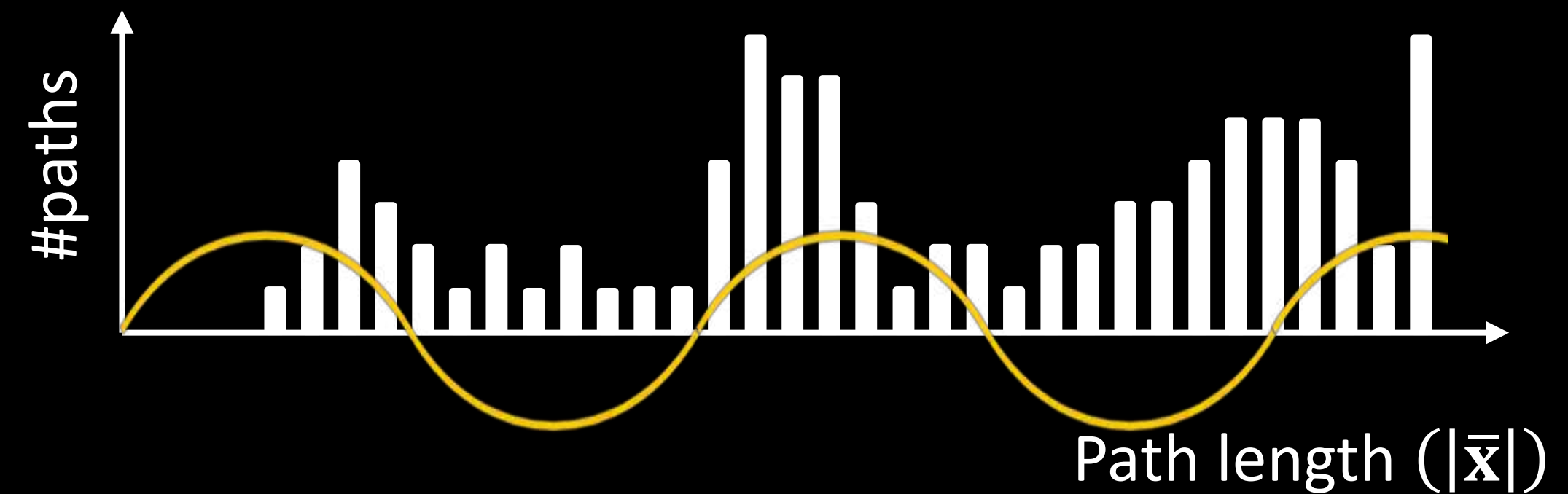
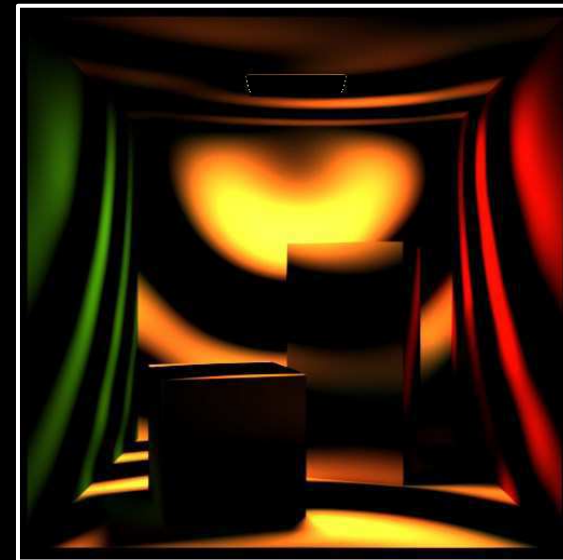


# Path sampling for time-gated rendering is challenging

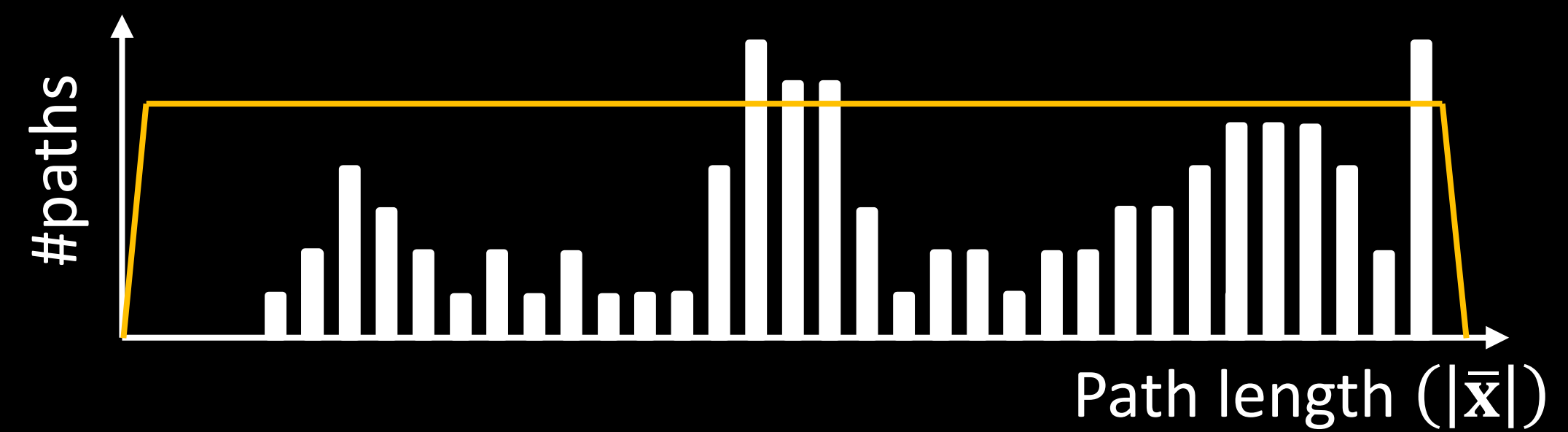


No control over path length

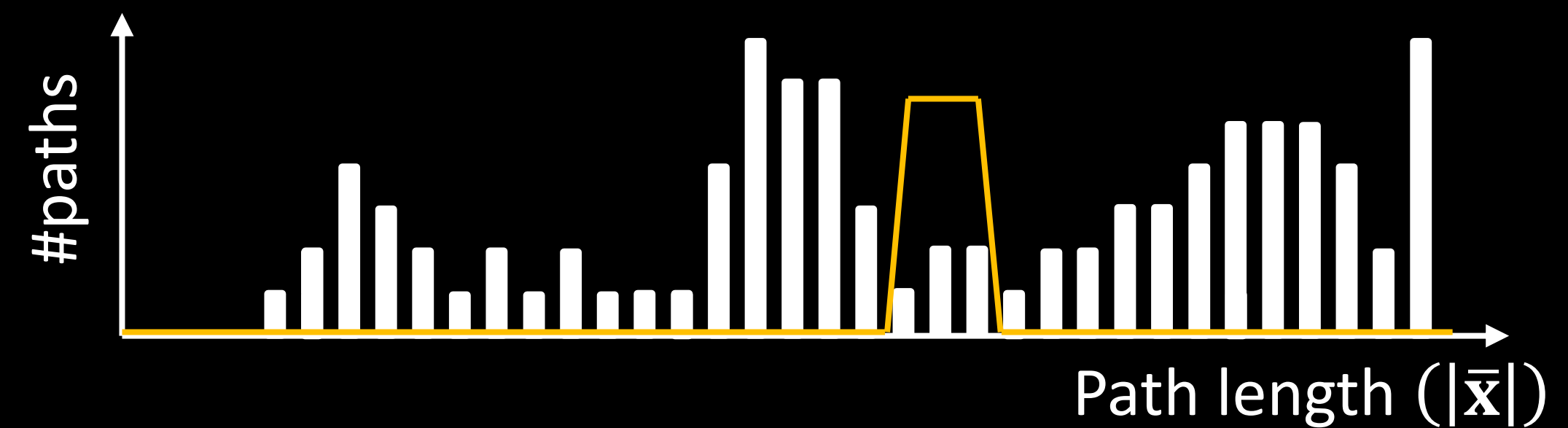
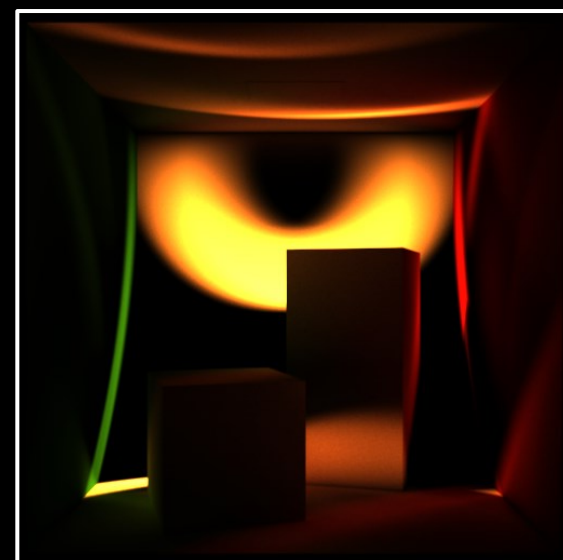
continuous-wave



transient

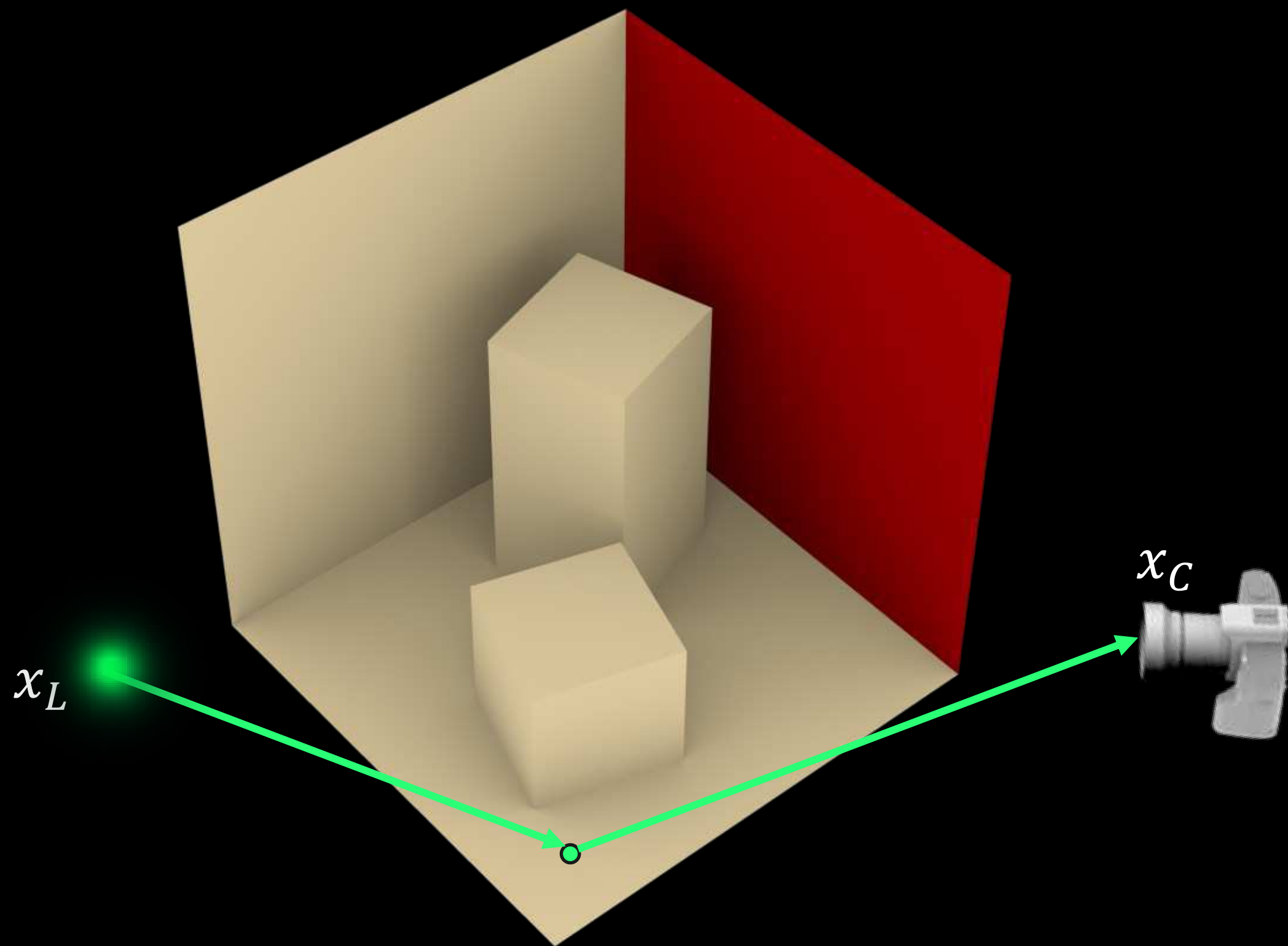


time-gated



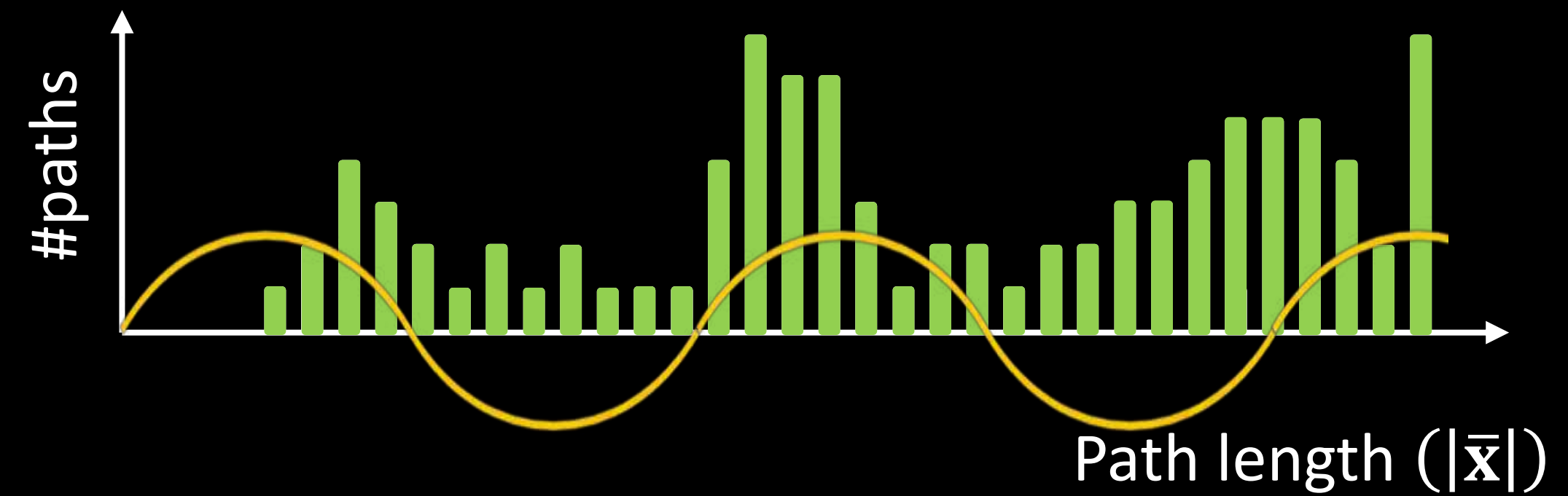
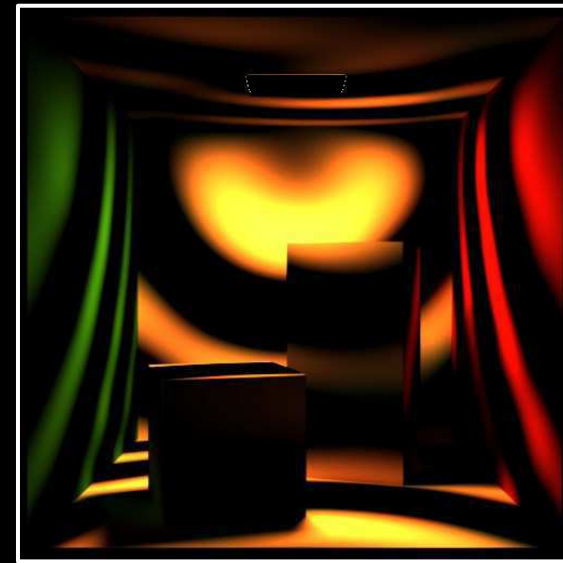


# Path sampling for time-gated rendering is challenging

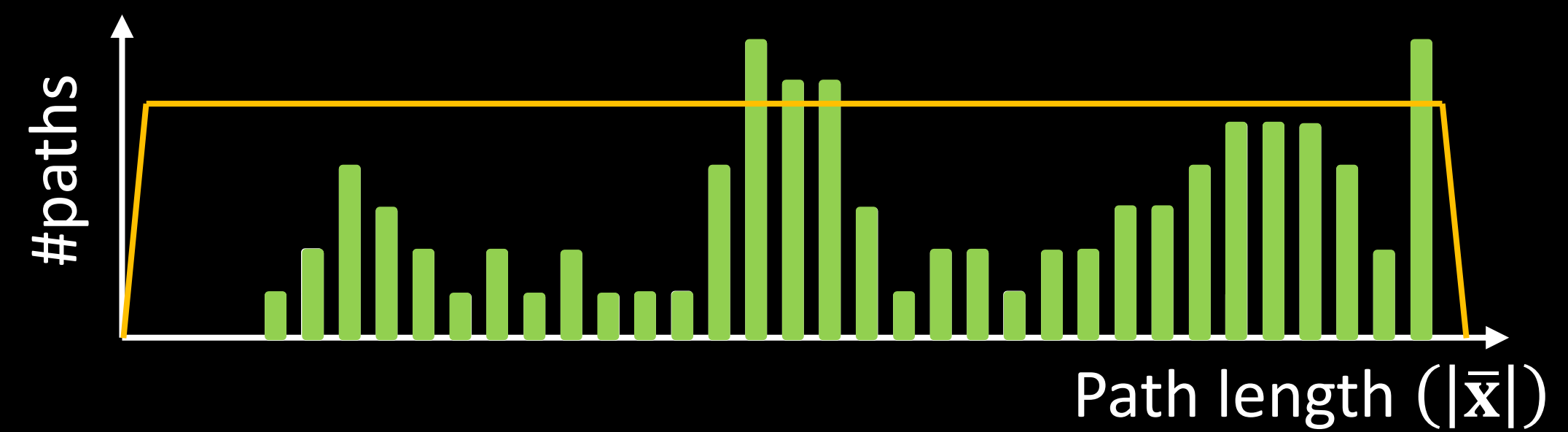


No control over path length

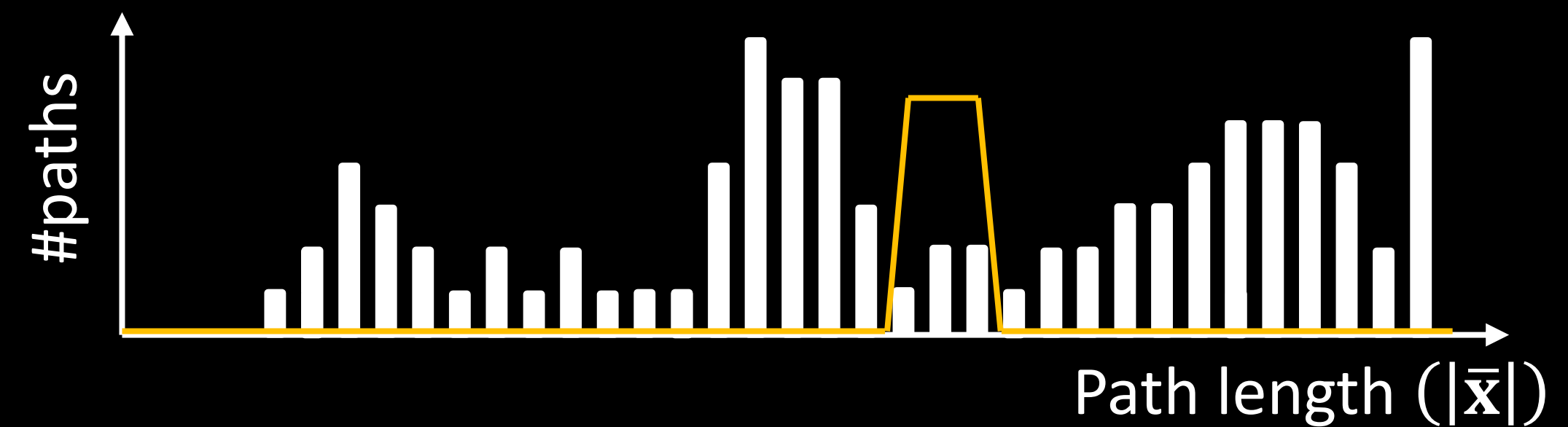
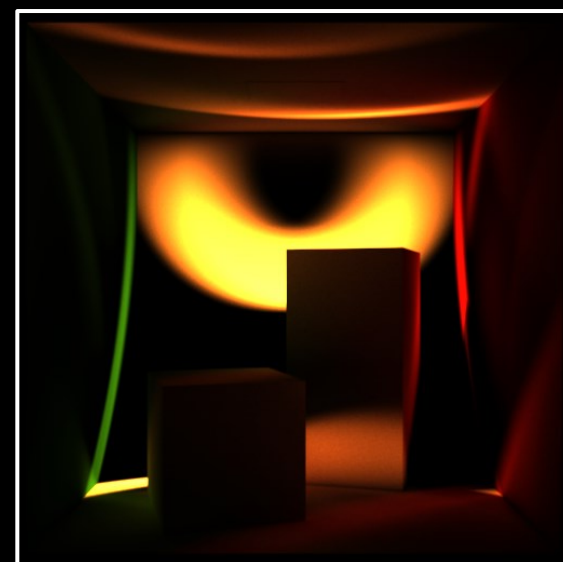
continuous-wave



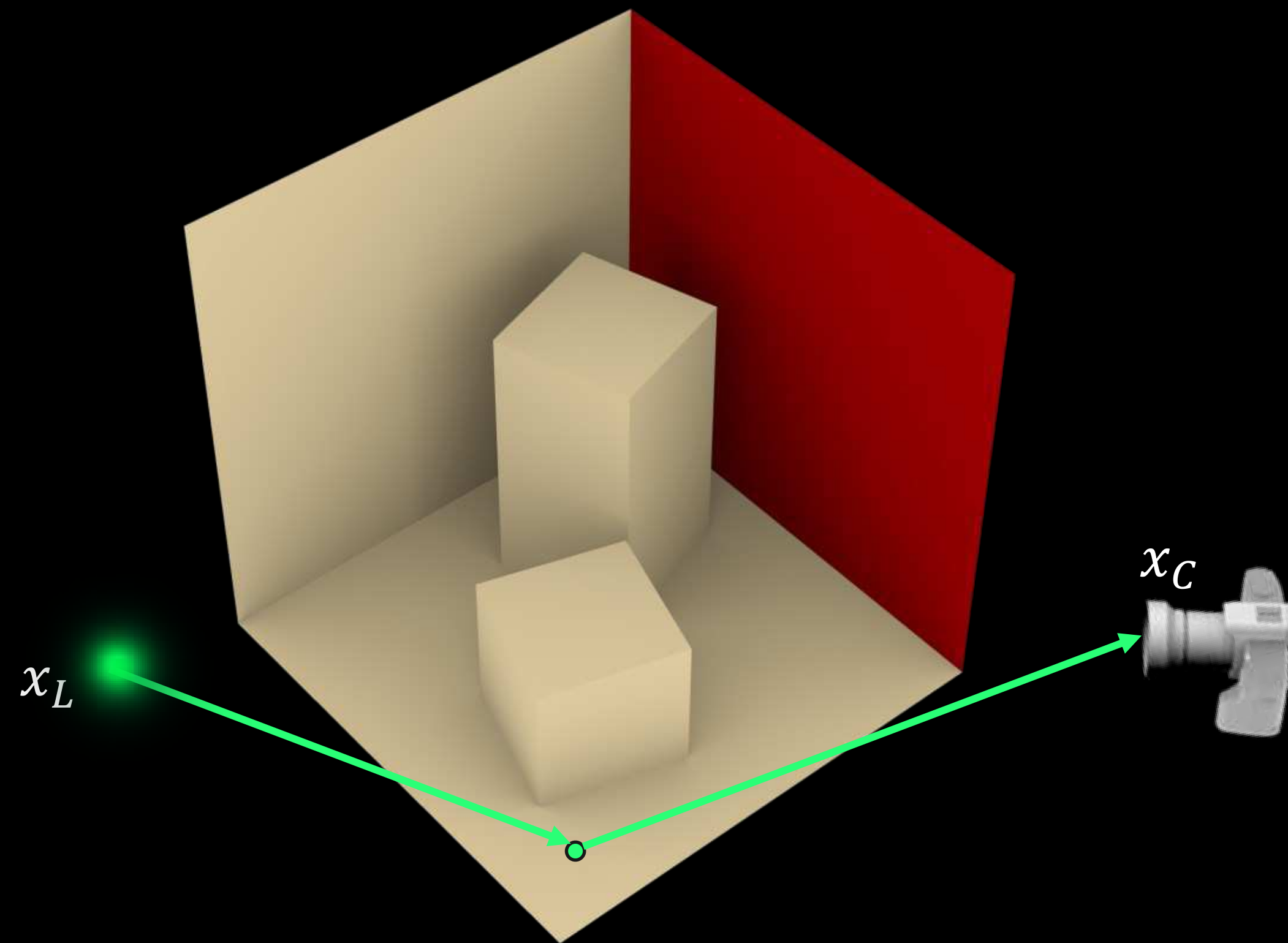
transient



time-gated

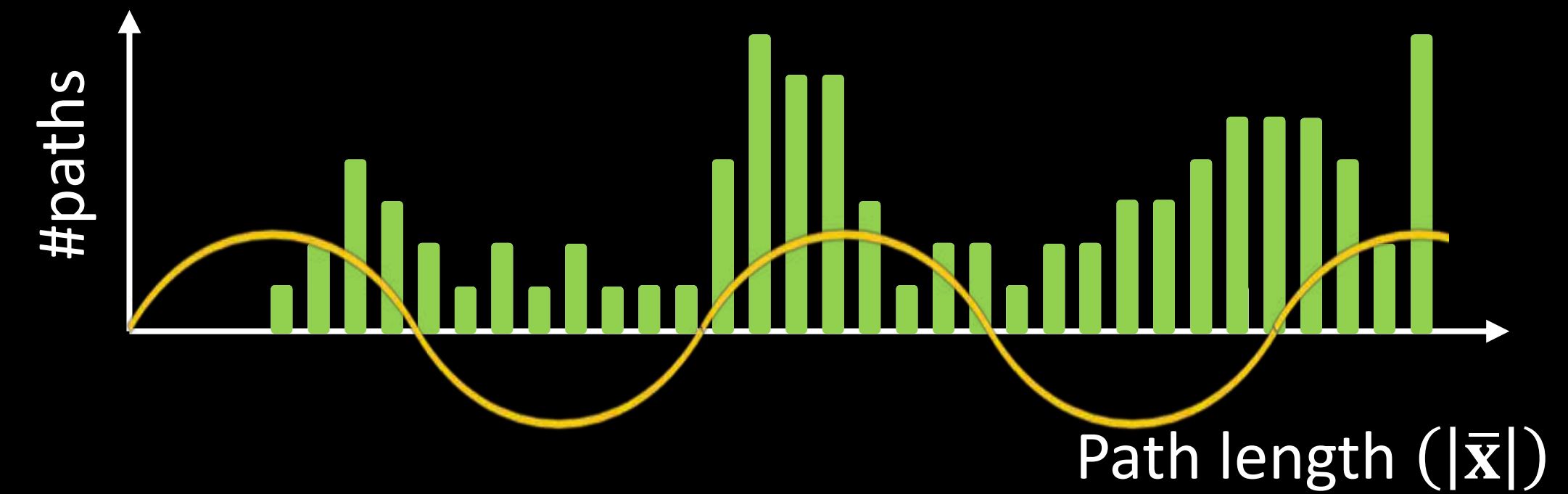


# Path sampling for time-gated rendering is challenging

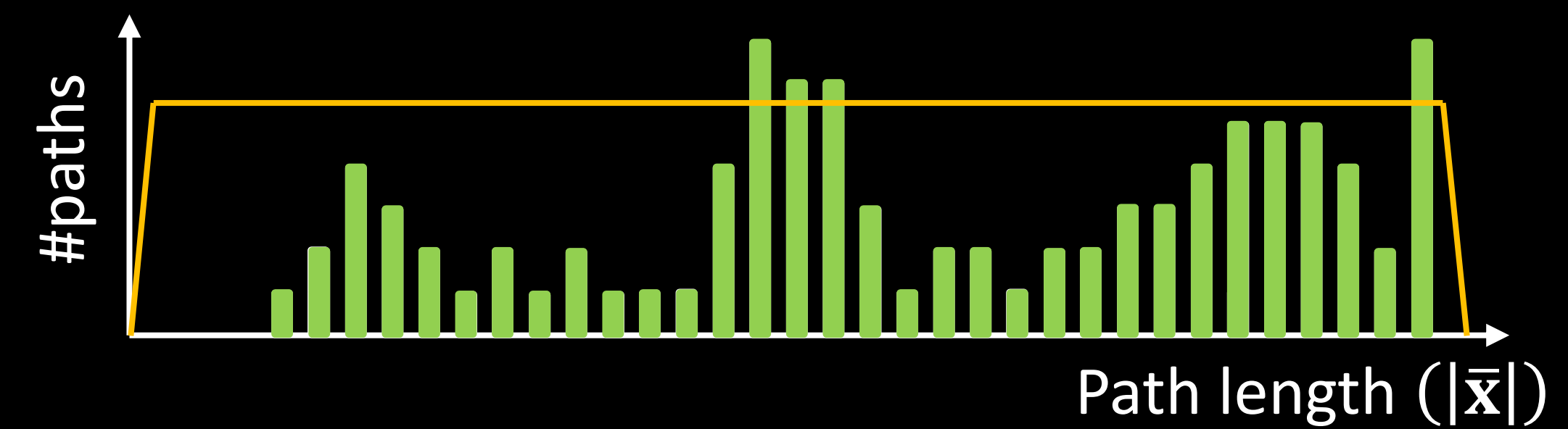


No control over path length

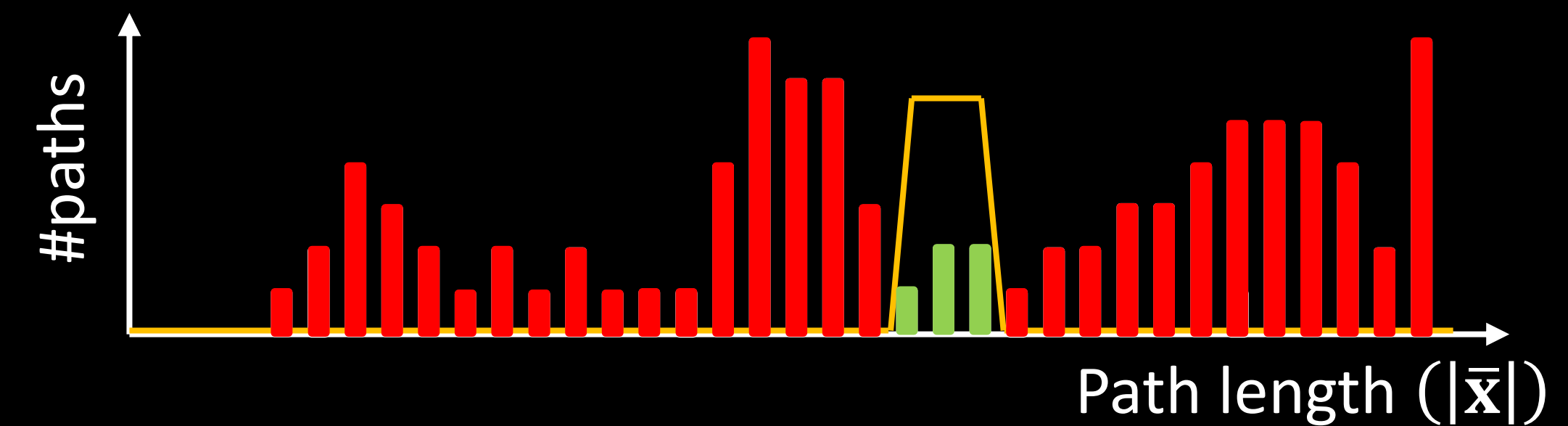
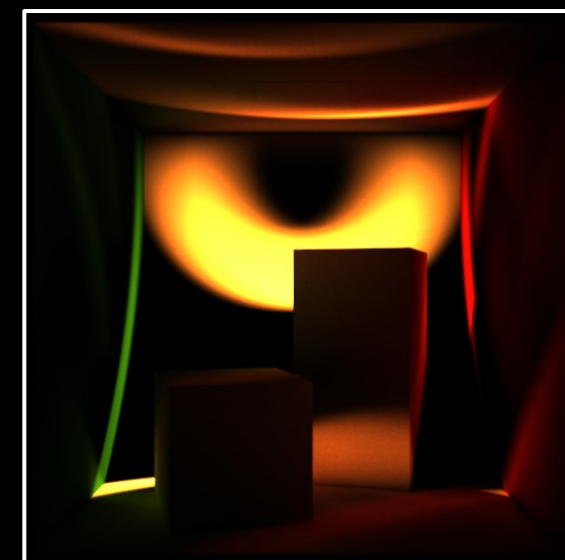
continuous-wave



transient

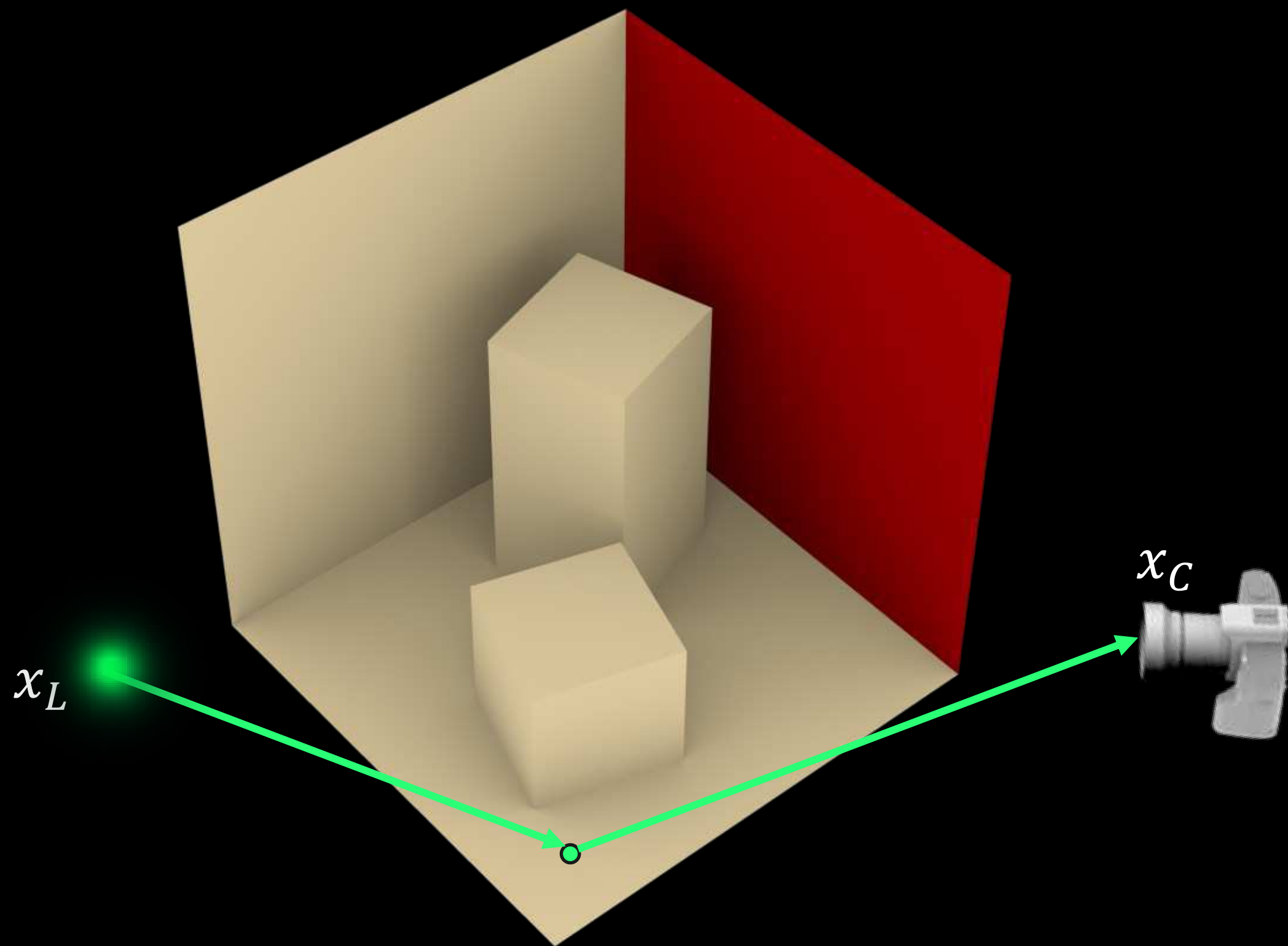


time-gated



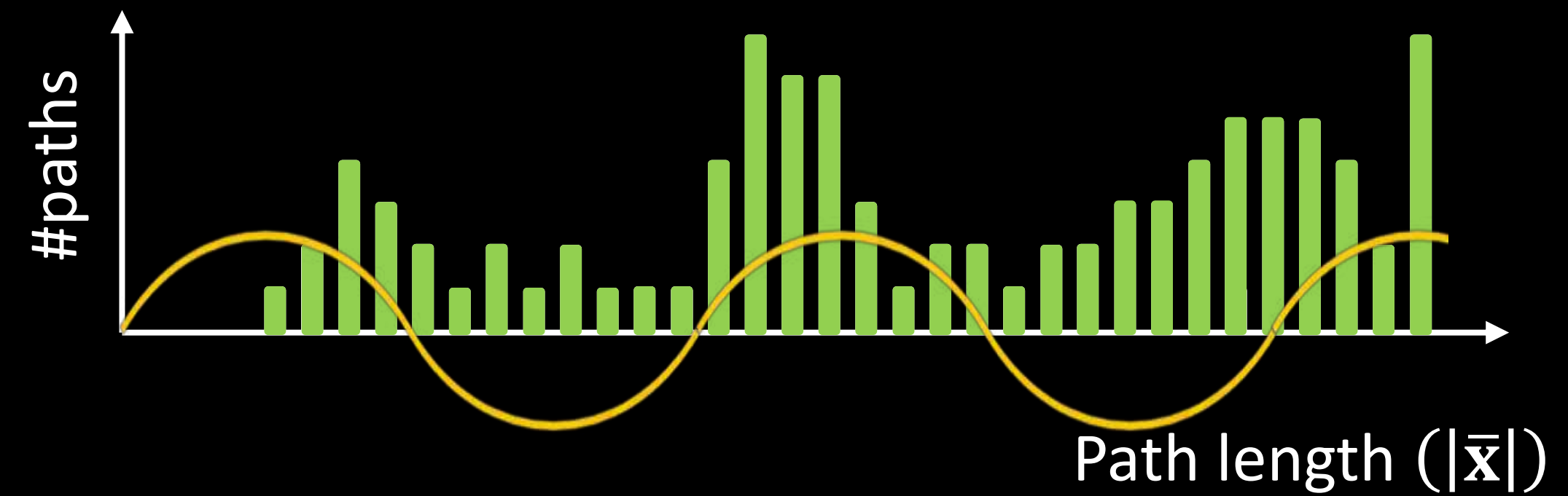
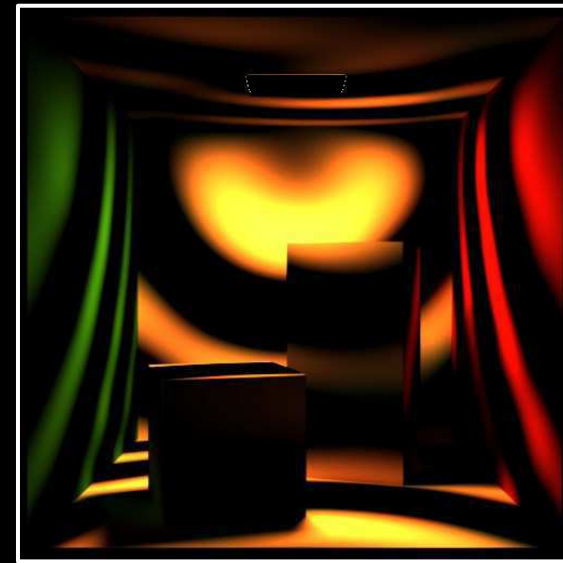


# Path sampling for time-gated rendering is challenging

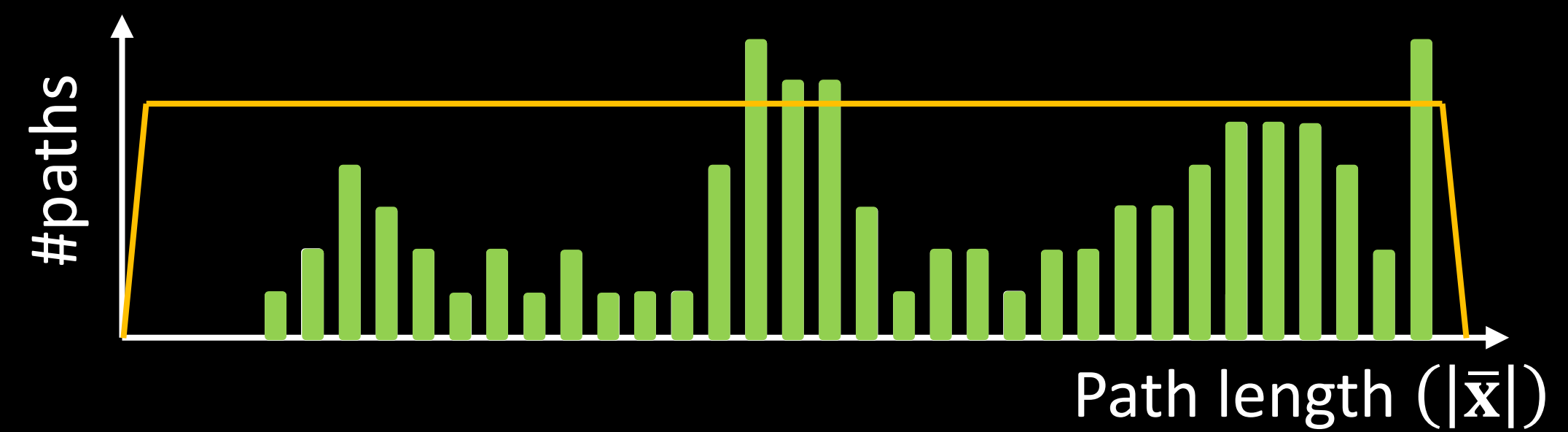


No control over path length

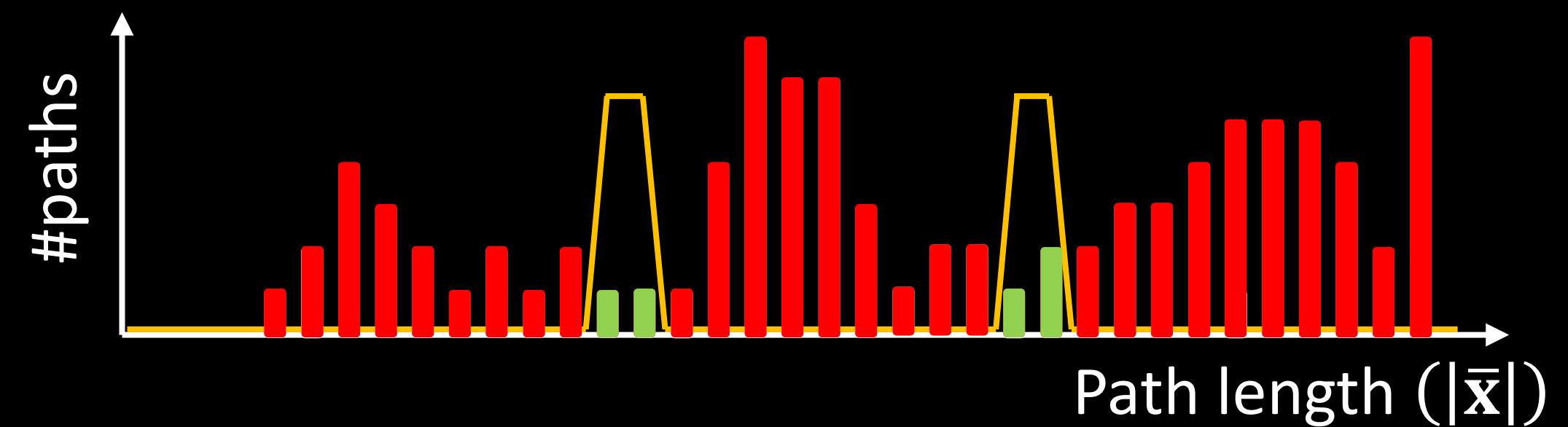
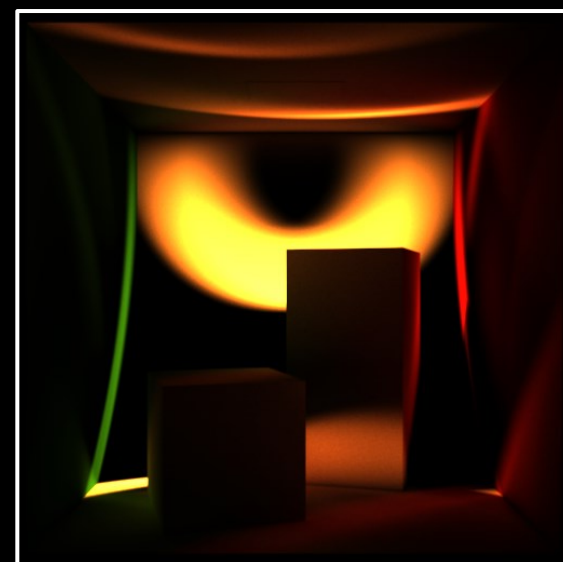
continuous-wave



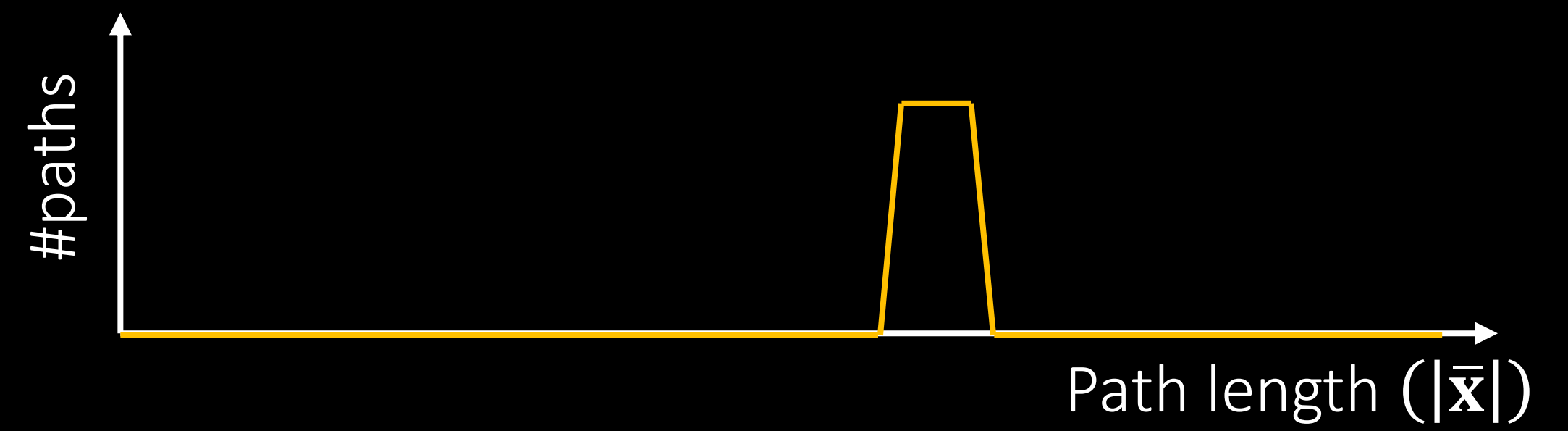
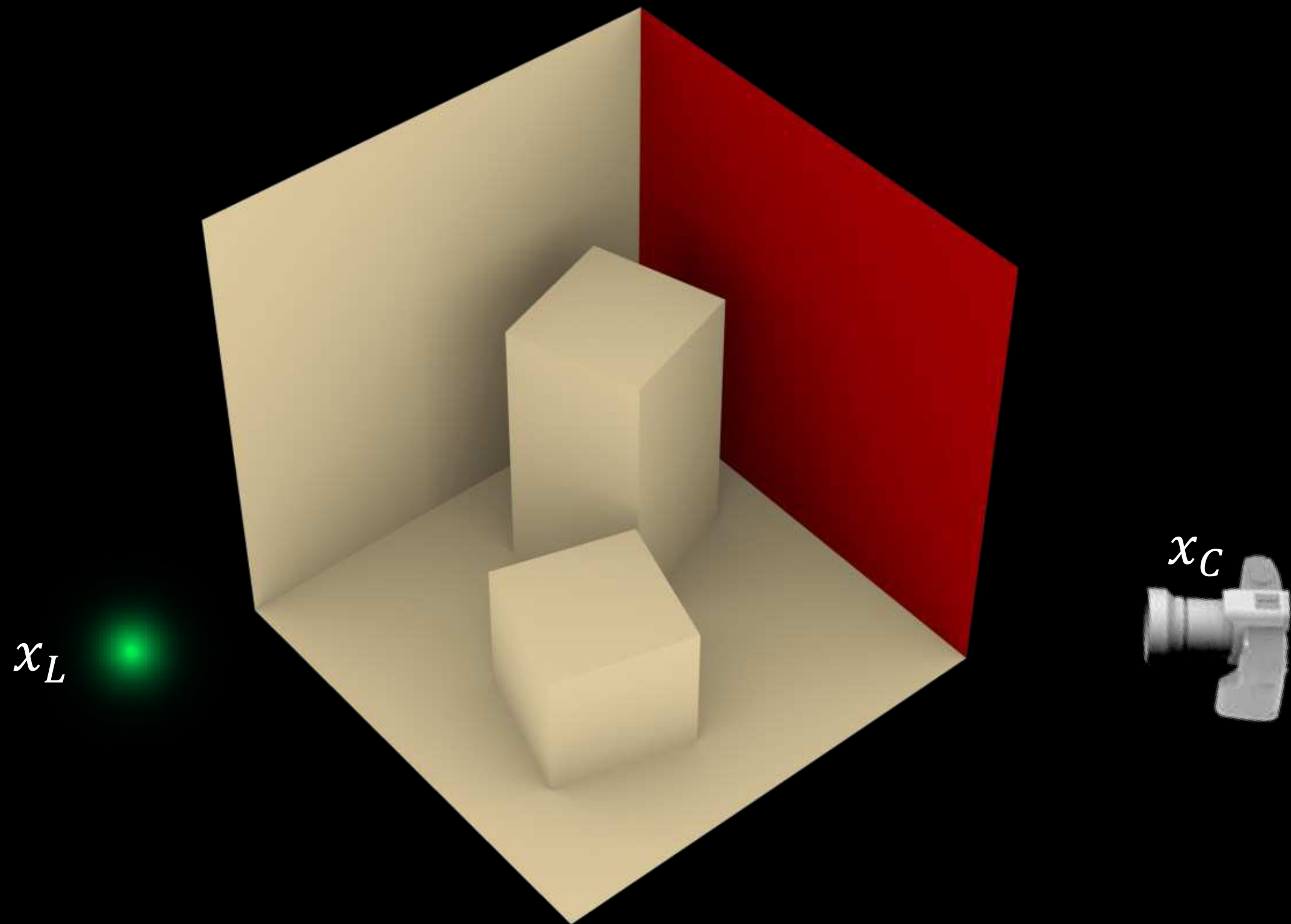
transient



time-gated

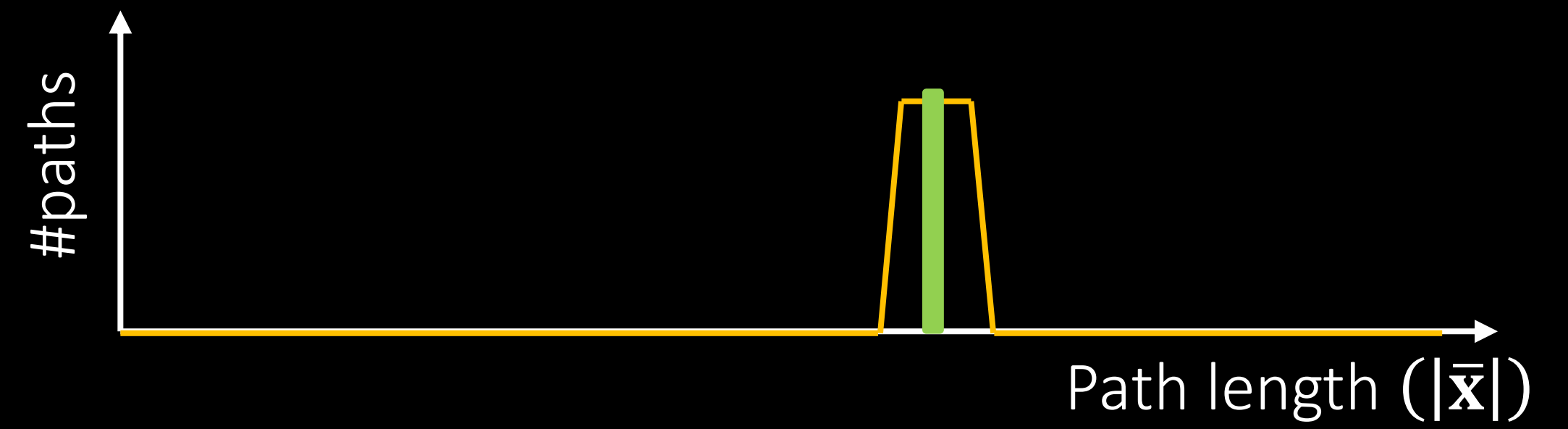
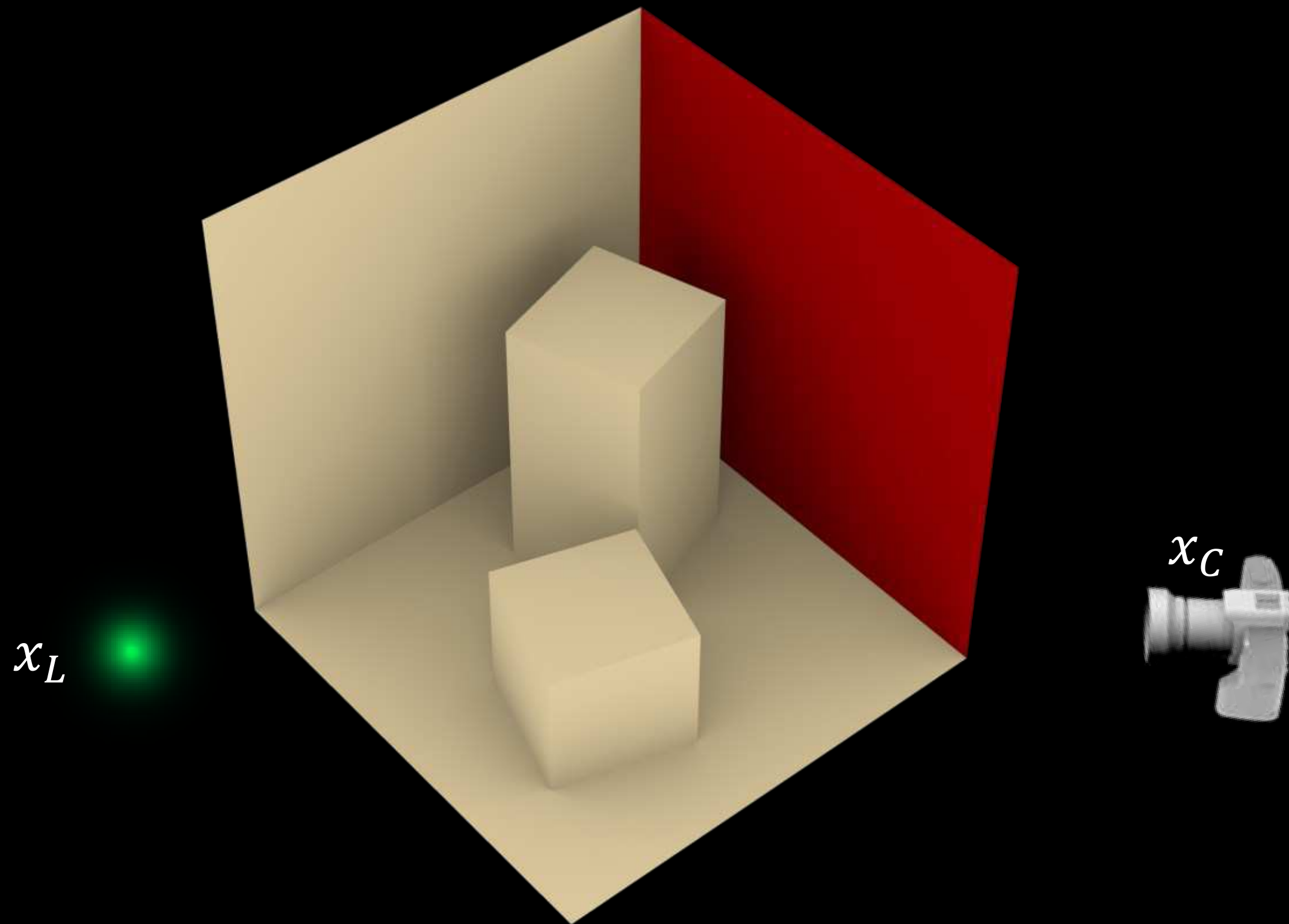


# Path sampling for time-gated case



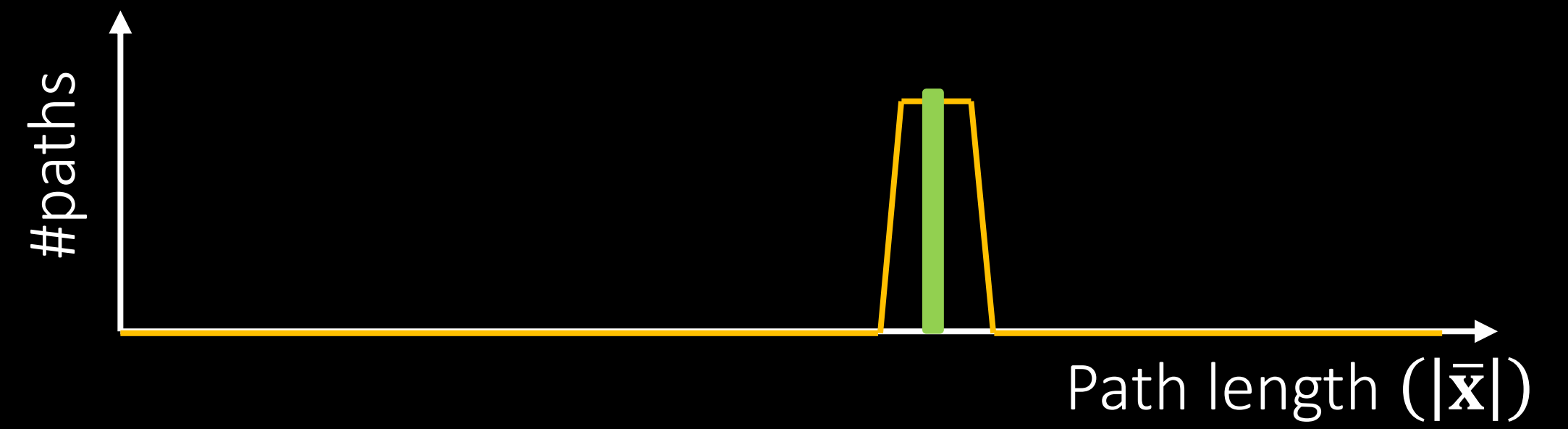
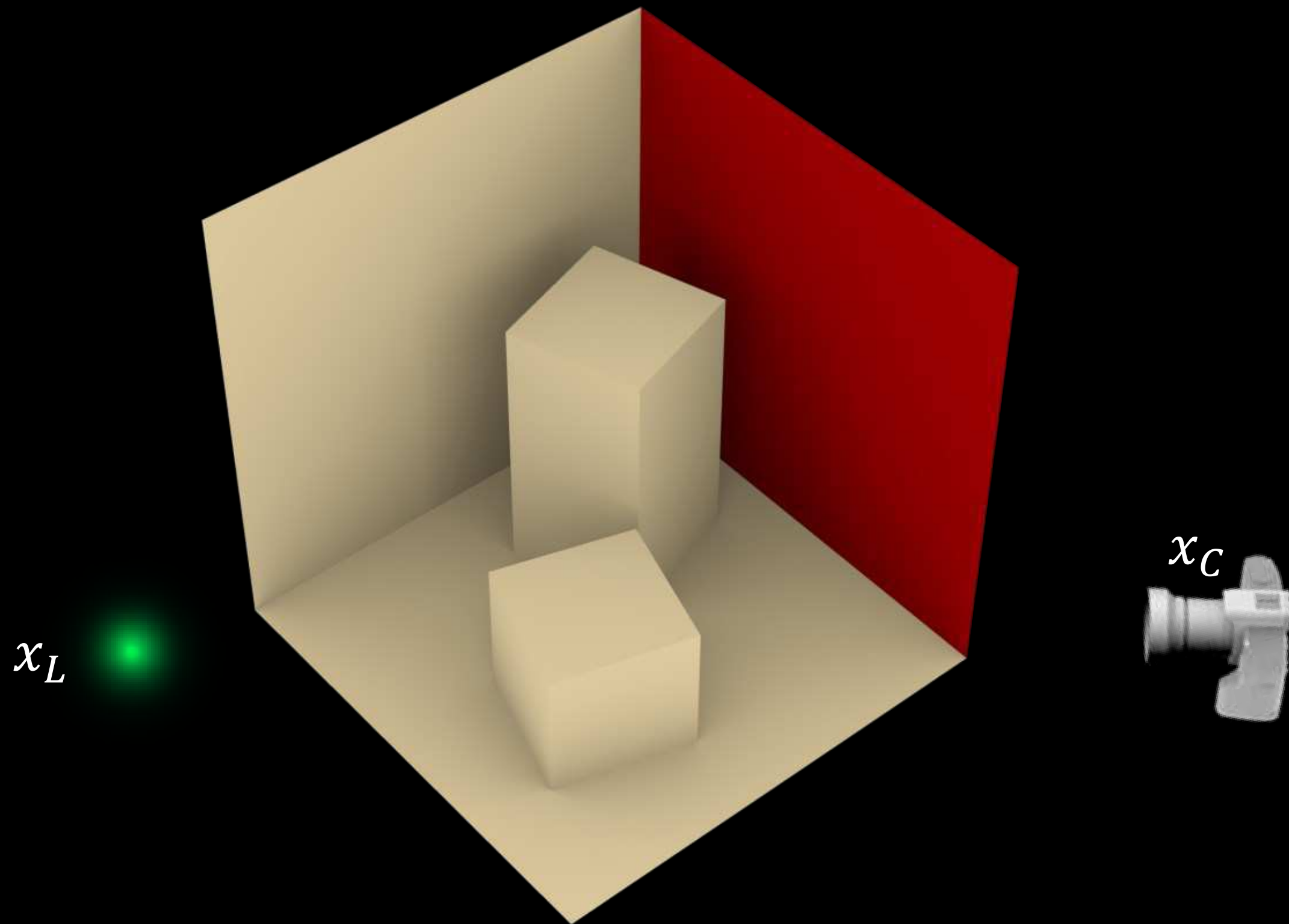


# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

# Path sampling for time-gated case

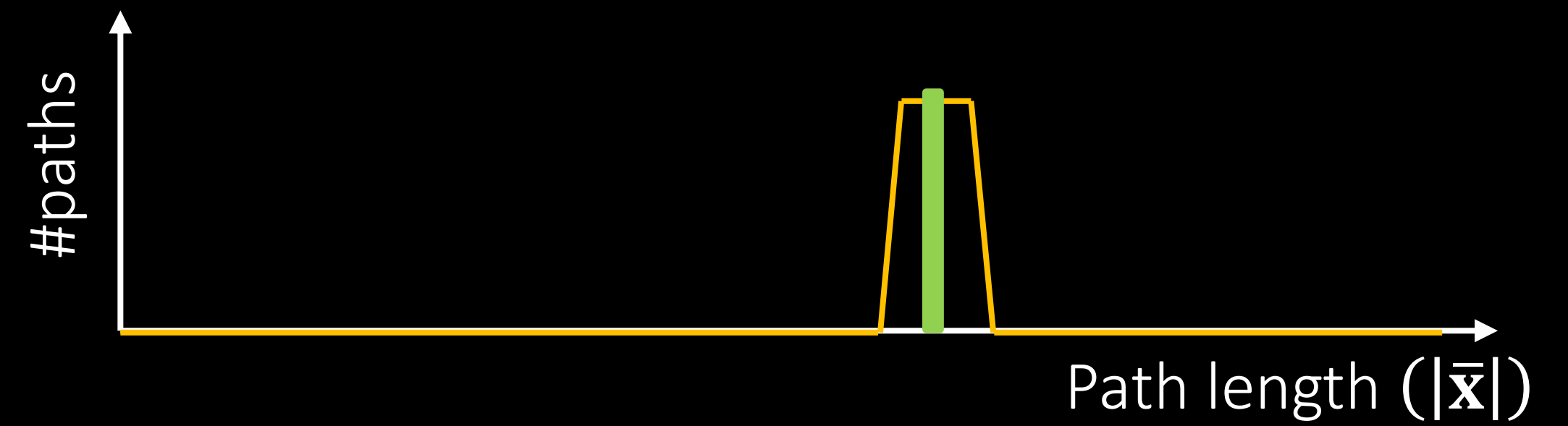
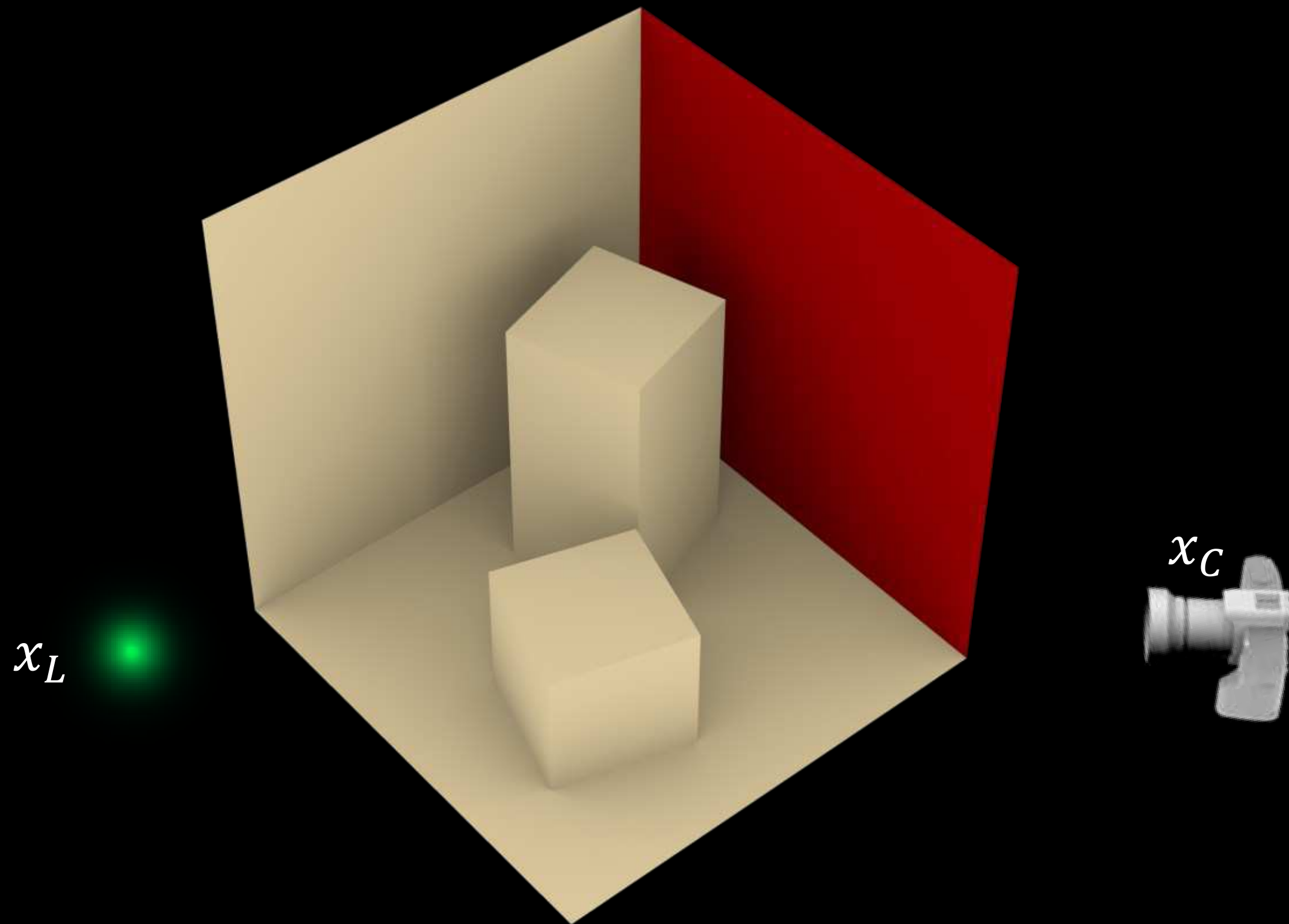


step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$



# Path sampling for time-gated case

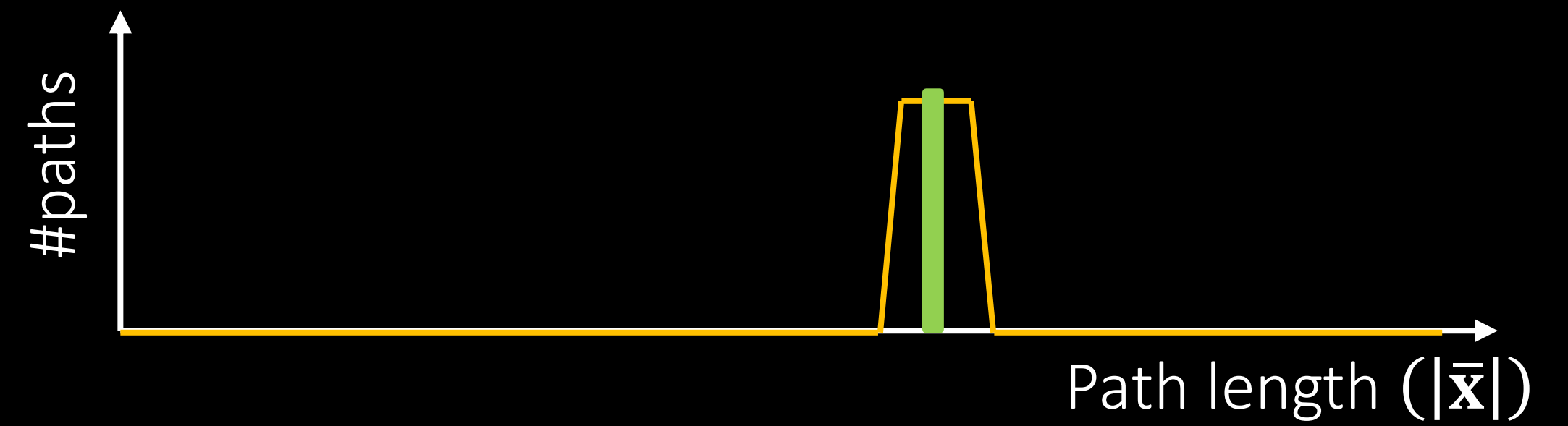
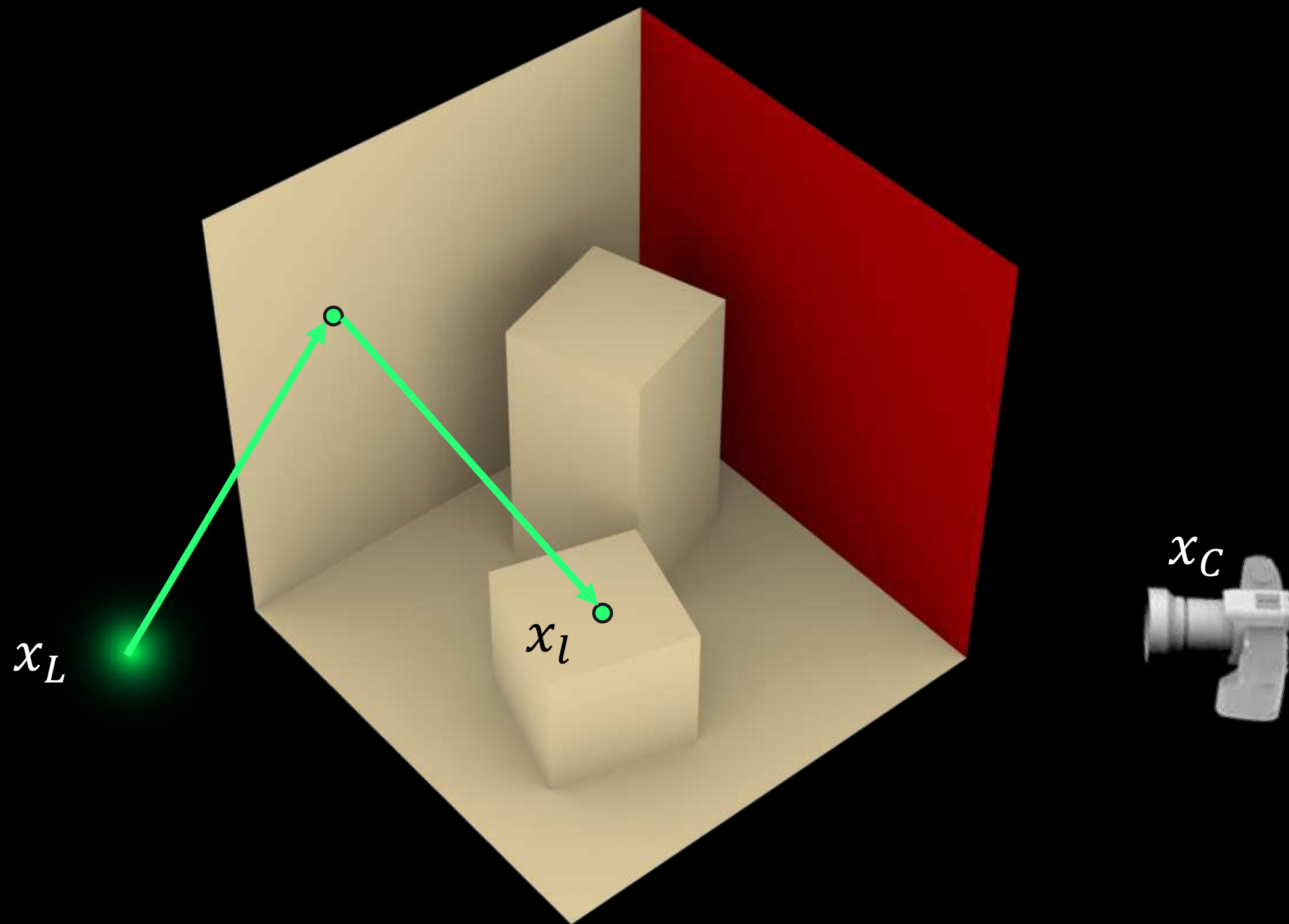


step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

bidirectional path tracing (BDPT)

# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

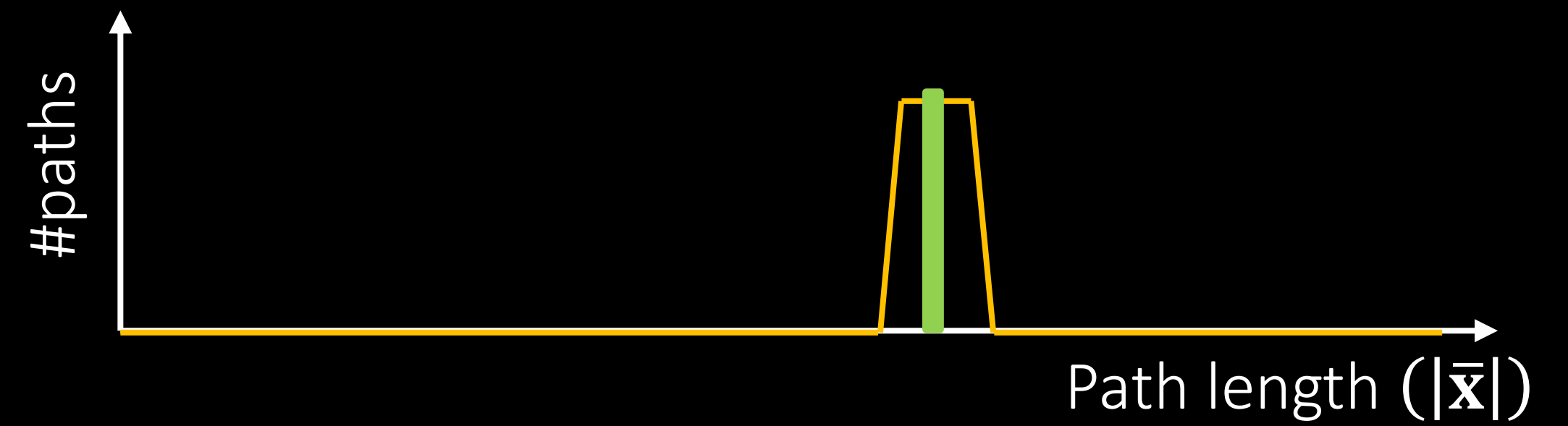
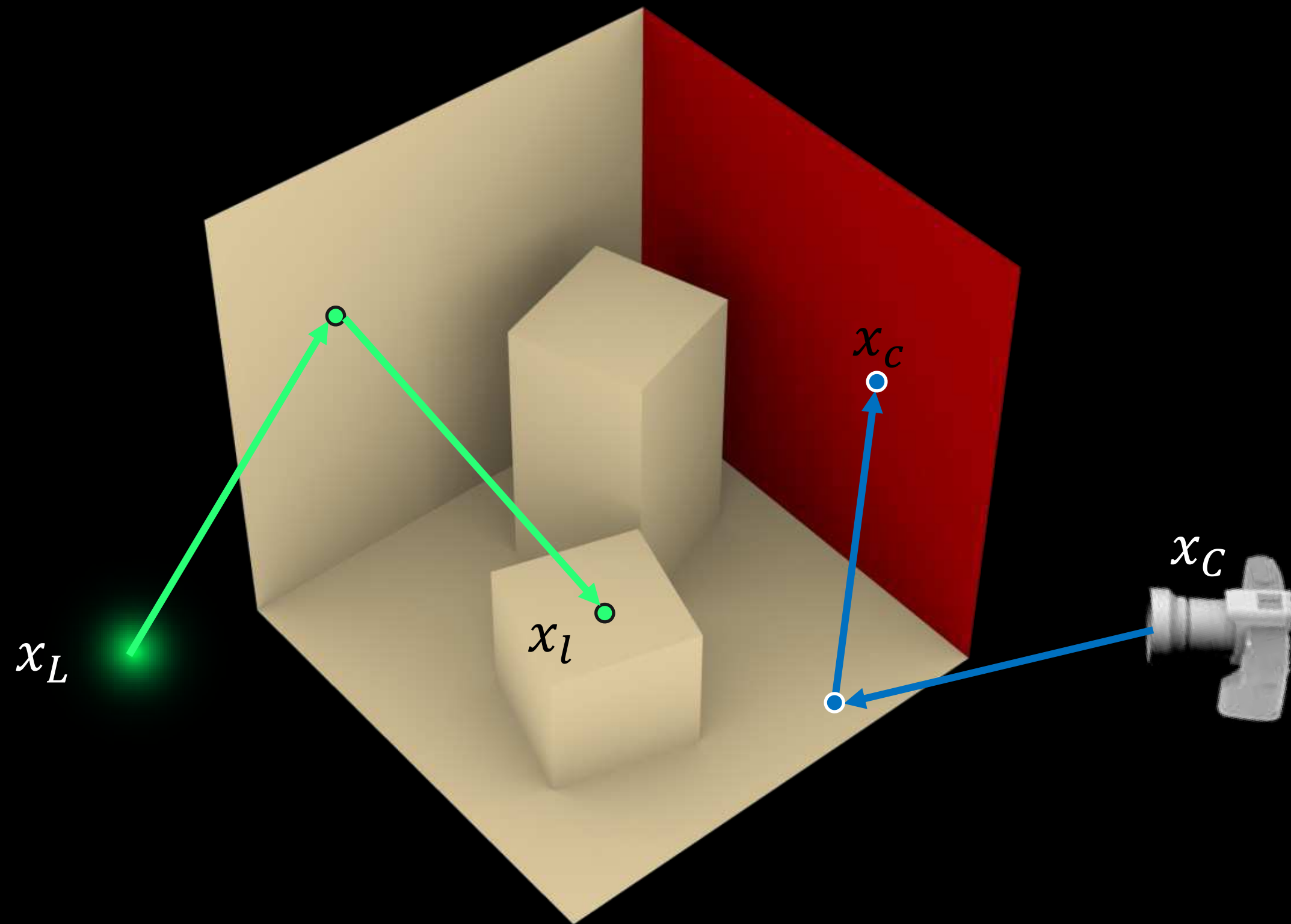
step 2: generate path with target length  $|\bar{x}|$

bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path



# Path sampling for time-gated case



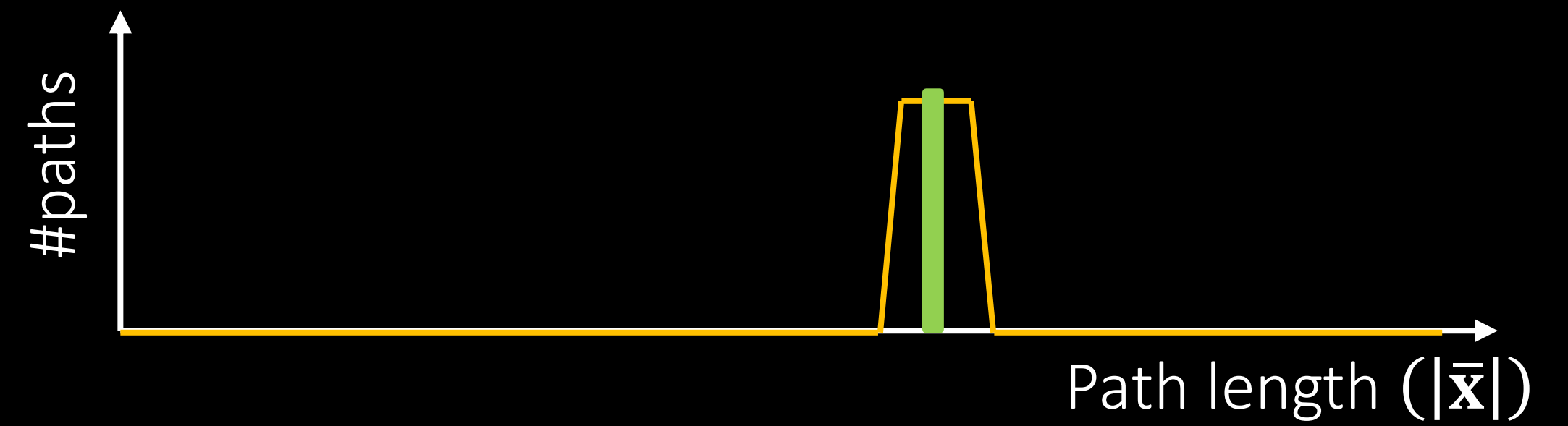
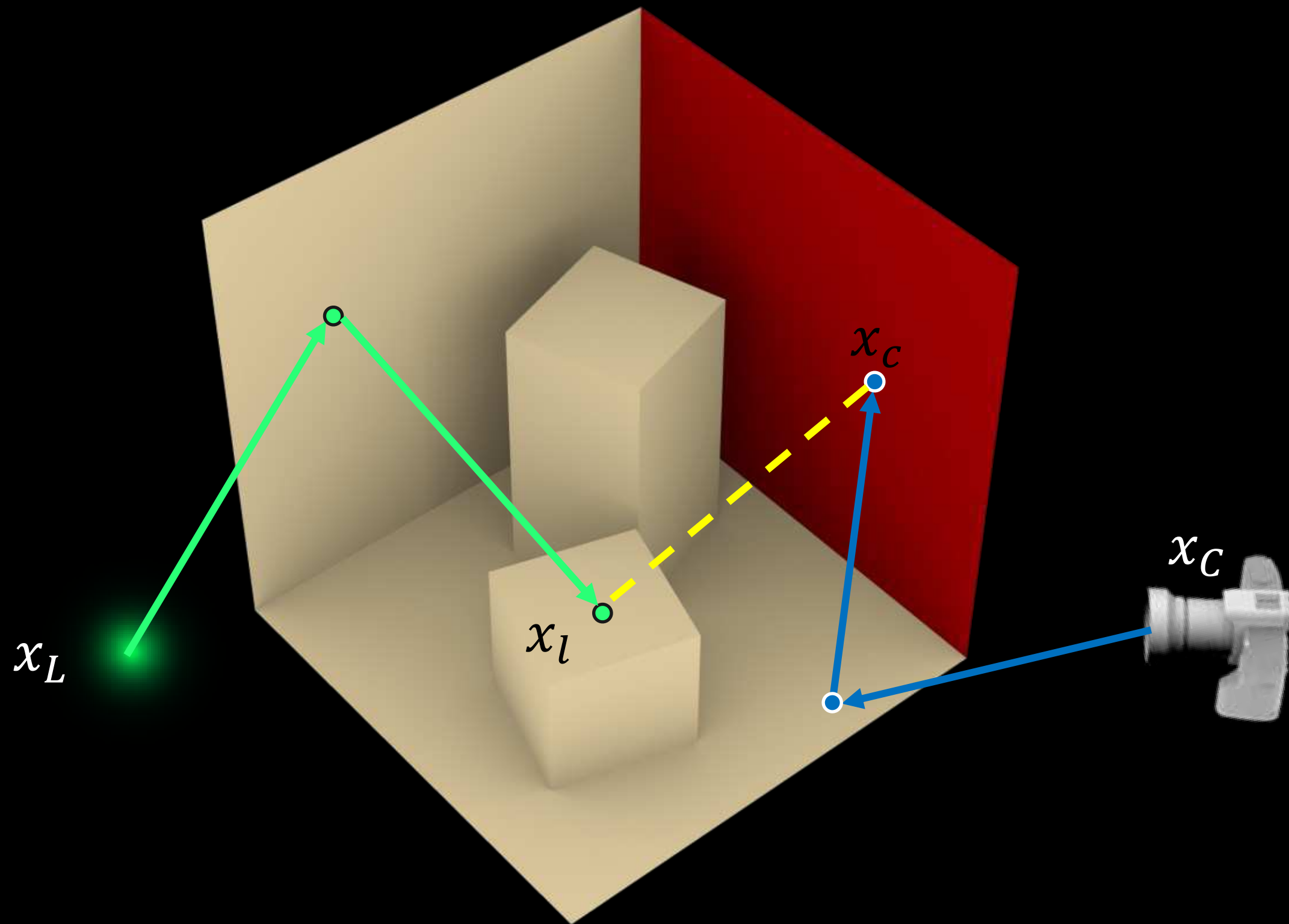
step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path

# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

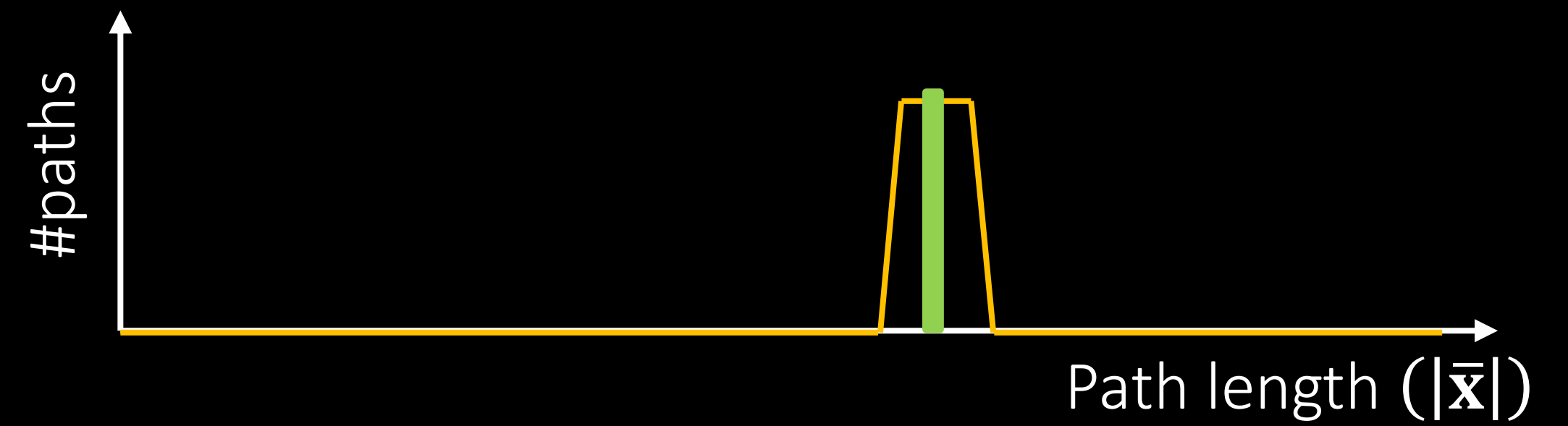
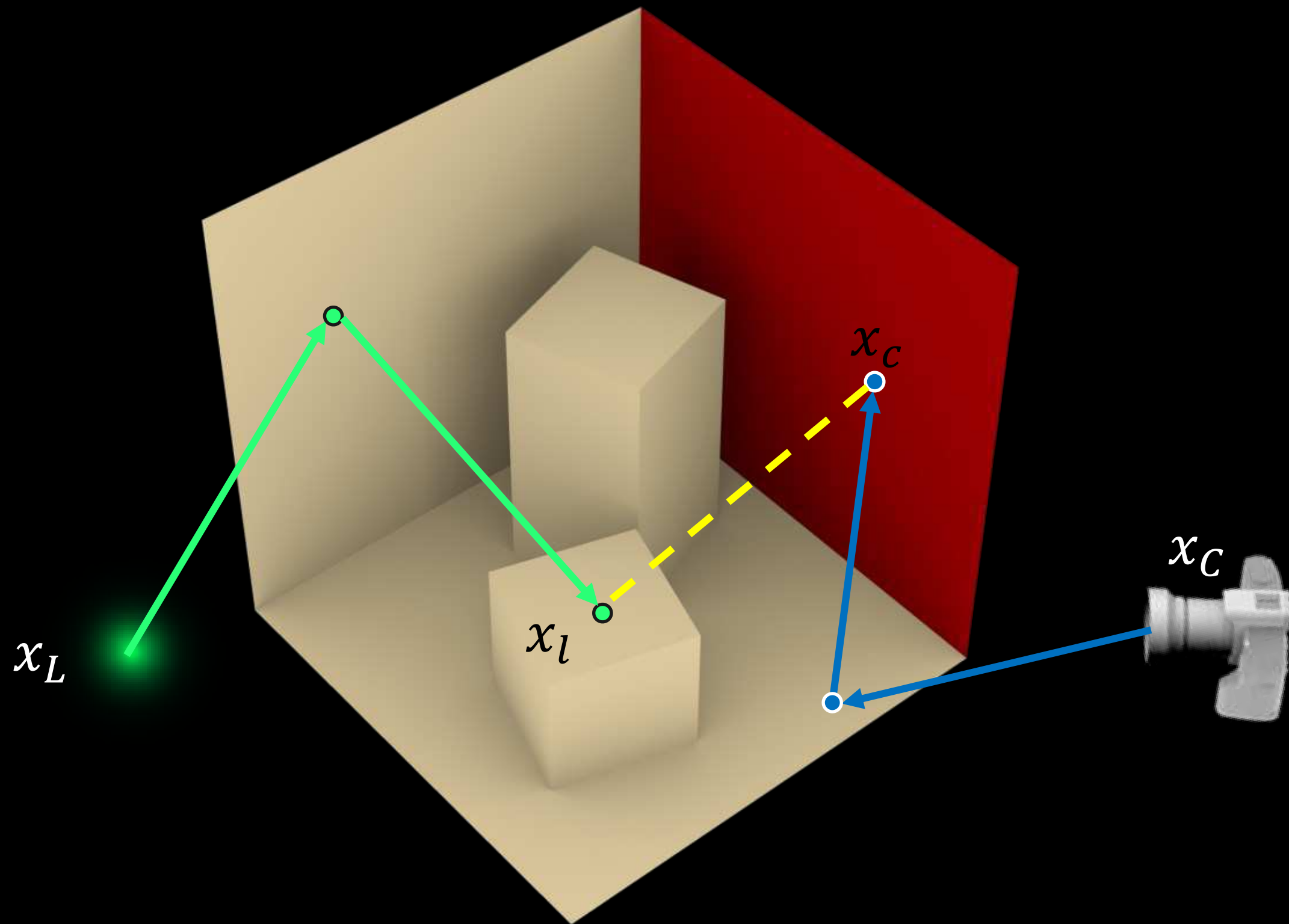
bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path

join source sub-path end  $x_s$  and camera sub-path end  $x_c$



# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

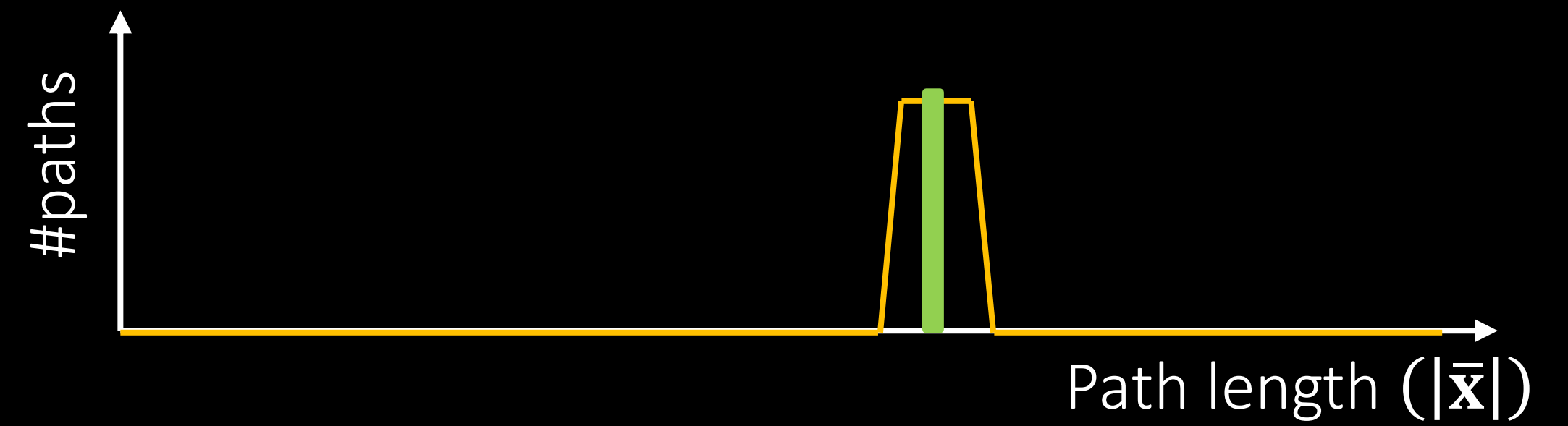
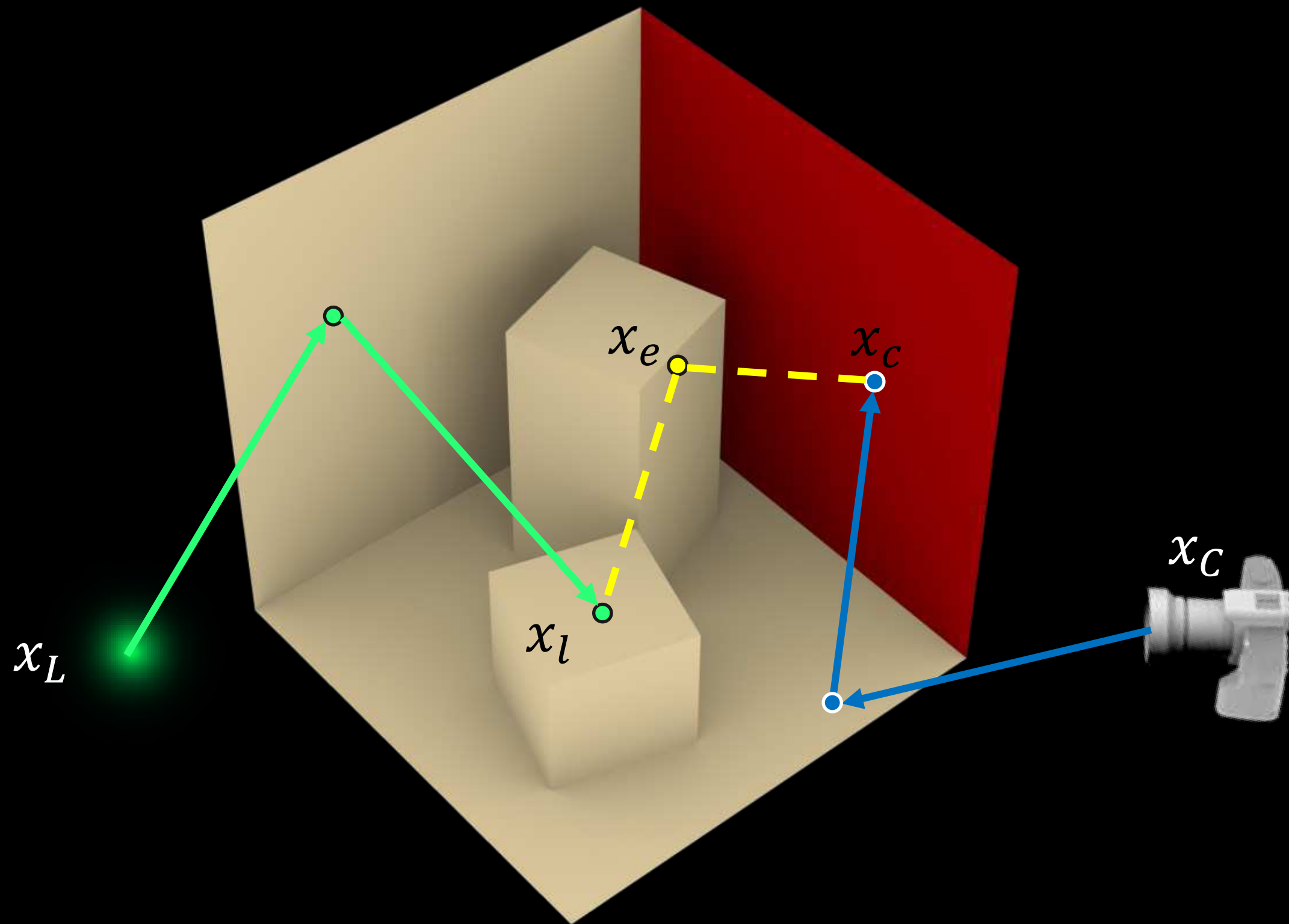
step 2: generate path with target length  $|\bar{x}|$

bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path

~~join source sub path end  $x_s$  and camera sub path end  $x_c$~~

# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

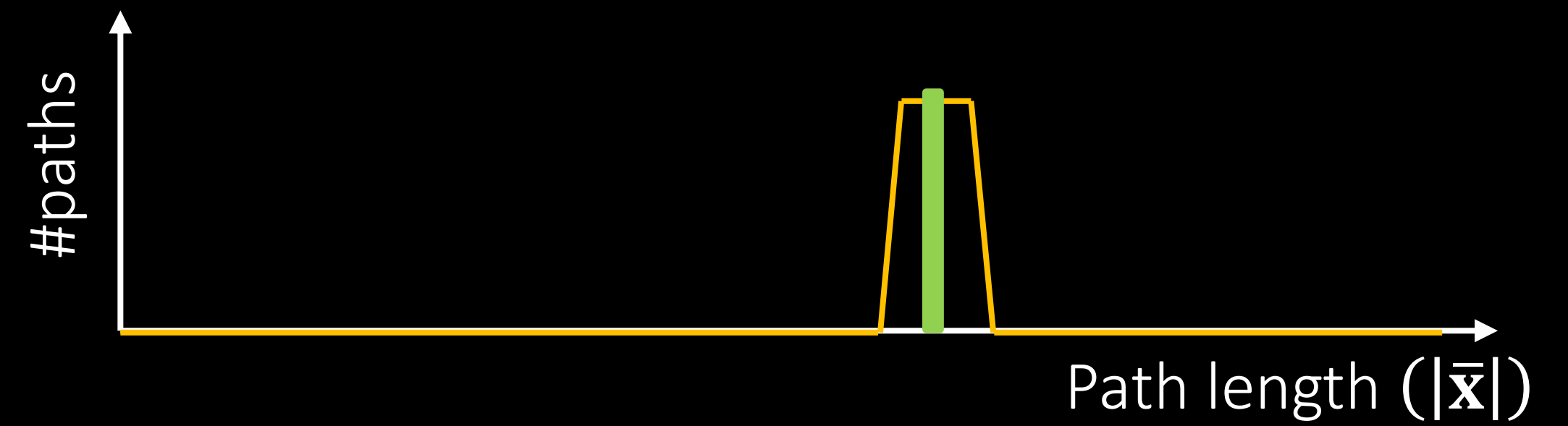
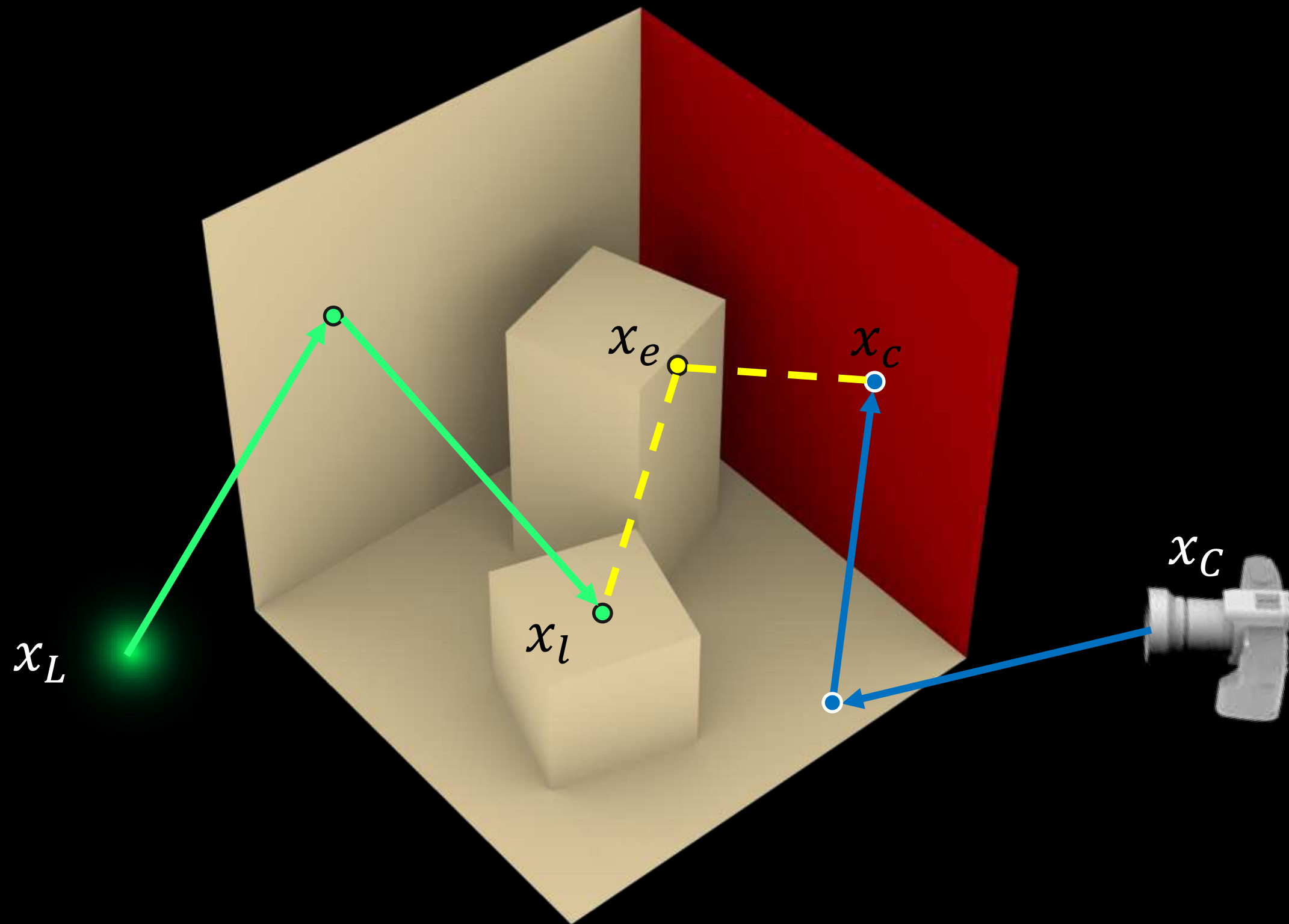
bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path

join  $x_l$  and  $x_c$  via connecting vertex ( $x_e$ )



# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

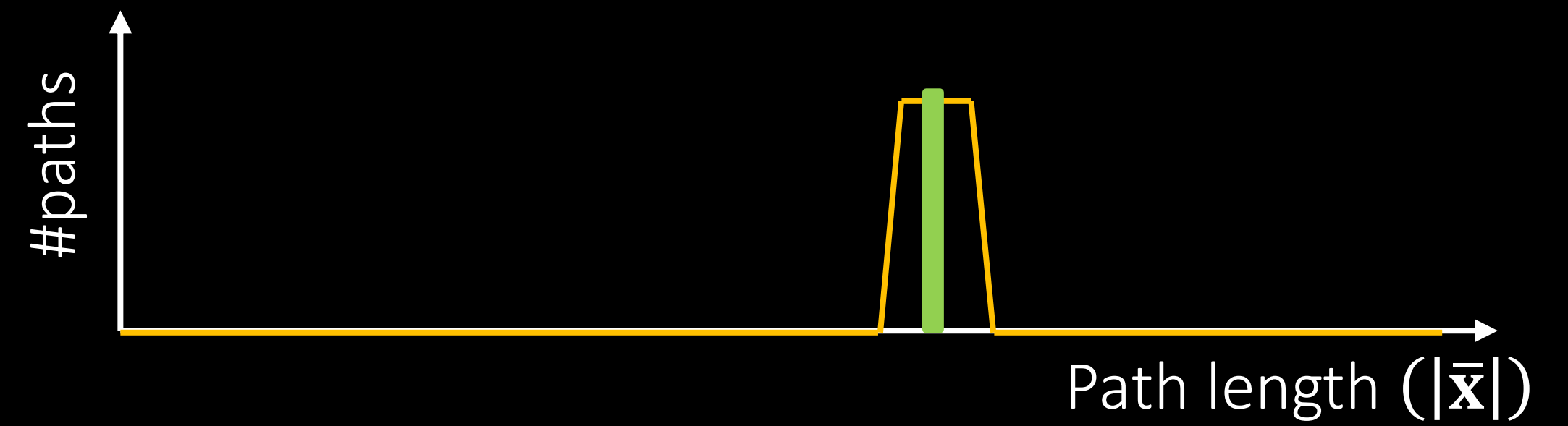
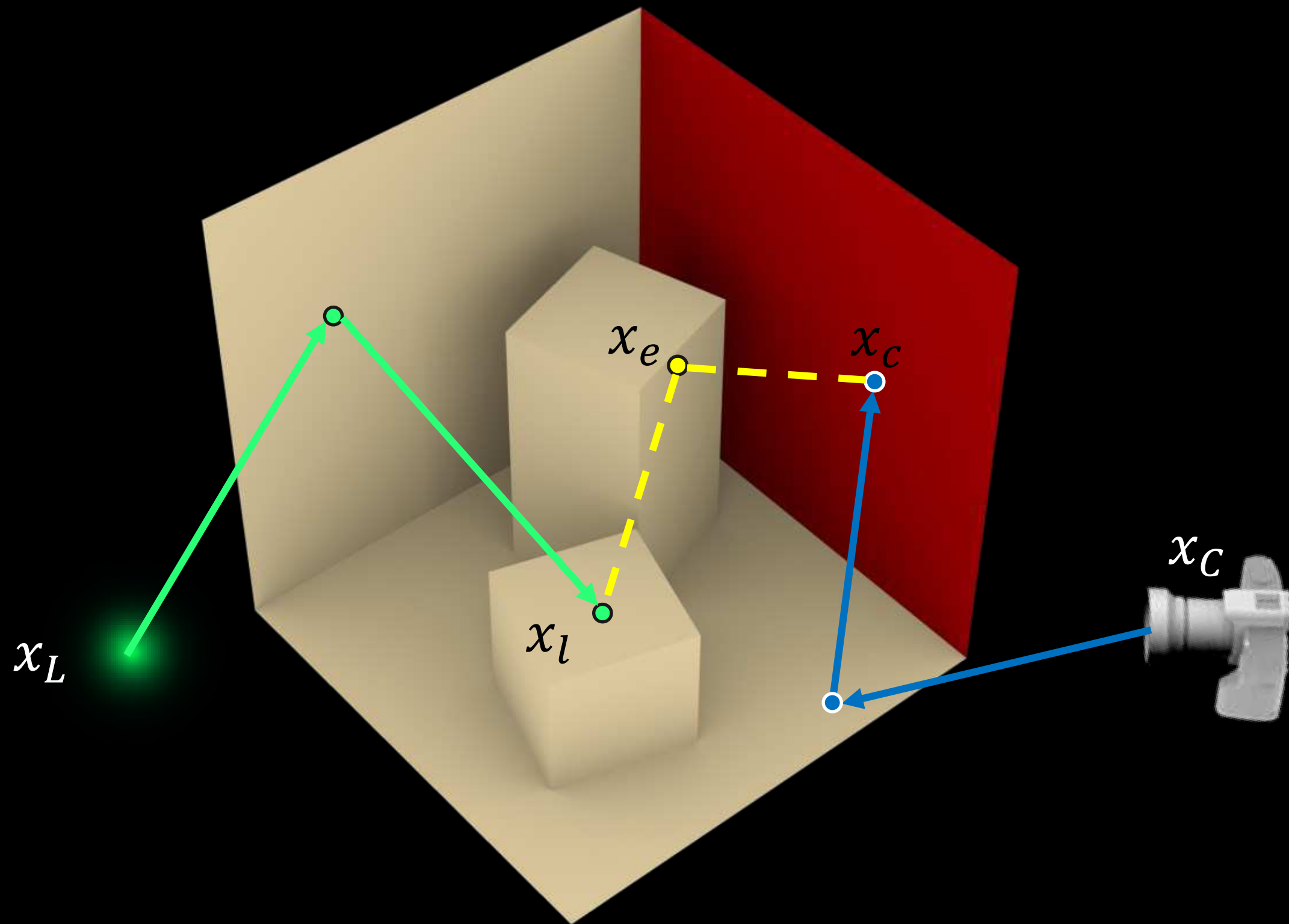
bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path

join  $x_l$  and  $x_c$  via connecting vertex ( $x_e$ )

$$\left. \begin{array}{l} \text{source sub-path length} + \\ \text{sensor sub-path length} + \\ |x_e \rightarrow x_l| + |x_e \rightarrow x_c| \end{array} \right\} = |\bar{x}|$$

# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

bidirectional path tracing (BDPT)

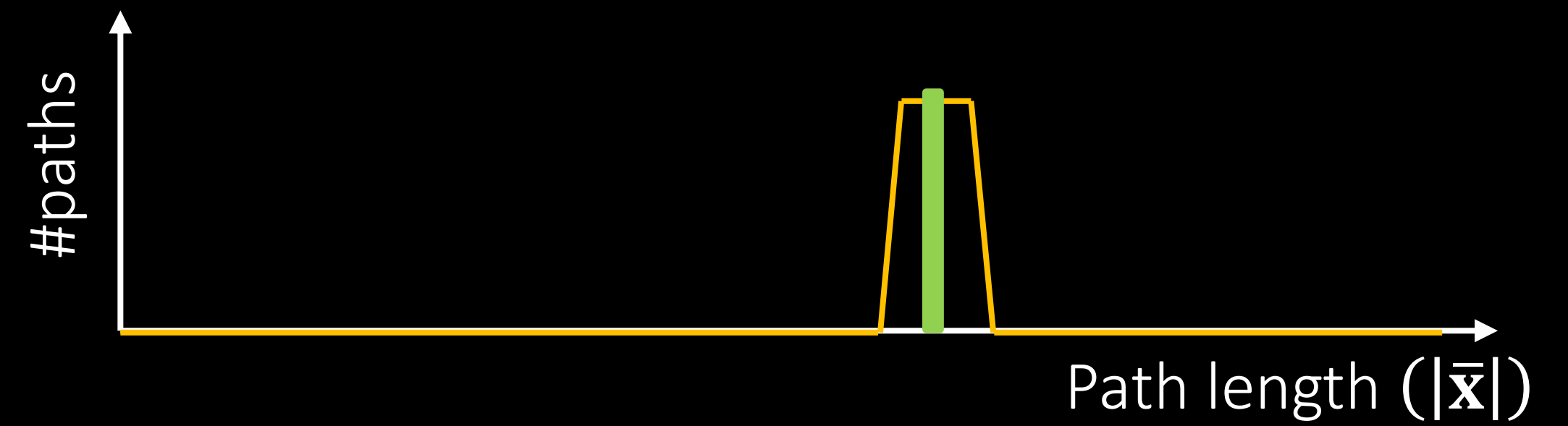
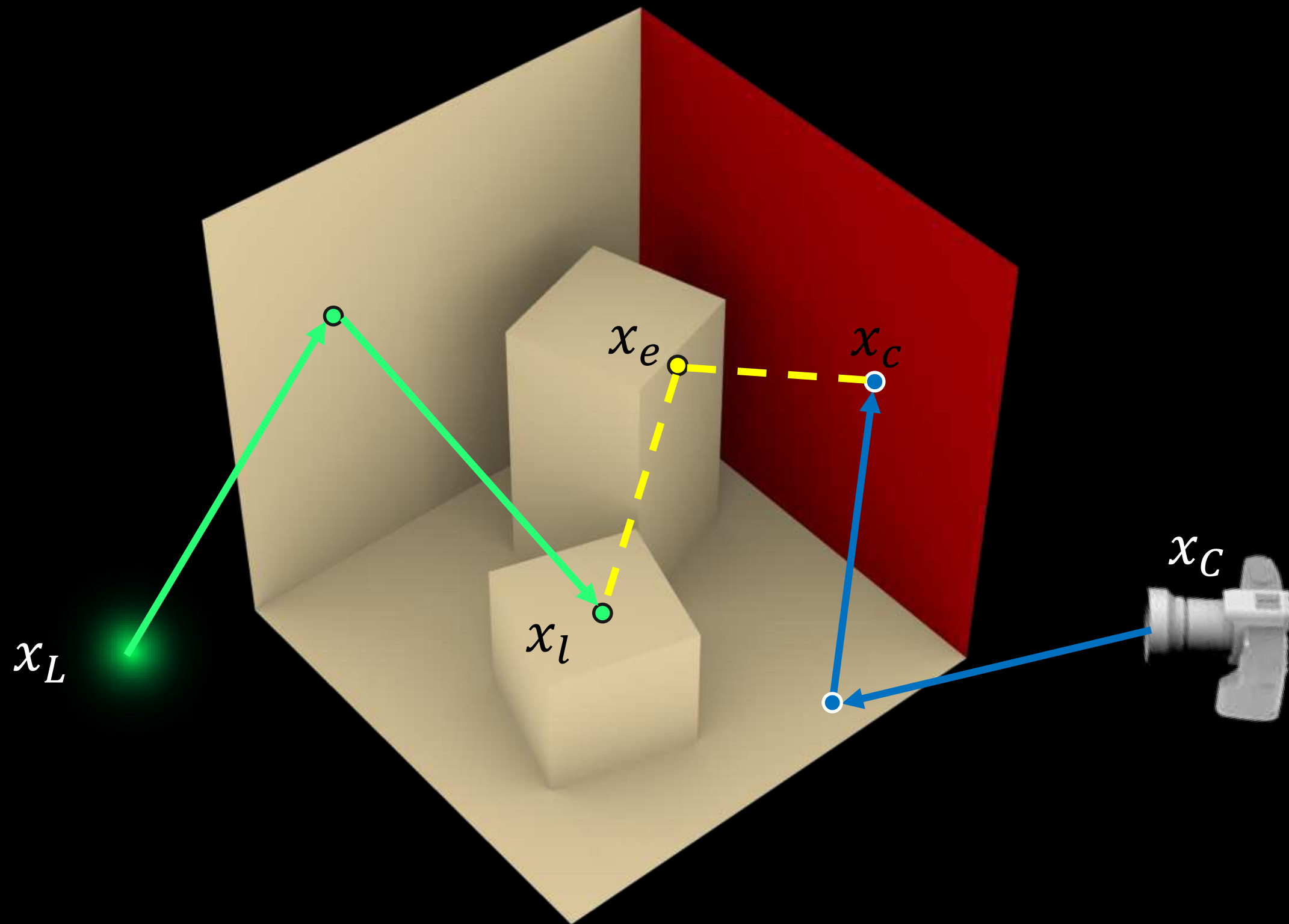
generate light sub-path, camera sub-path

join  $x_l$  and  $x_c$  via connecting vertex ( $x_e$ )

$$|x_e \rightarrow x_l| + |x_e \rightarrow x_c| = |\bar{x}| - \text{source + sensor sub-path lengths}$$



# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

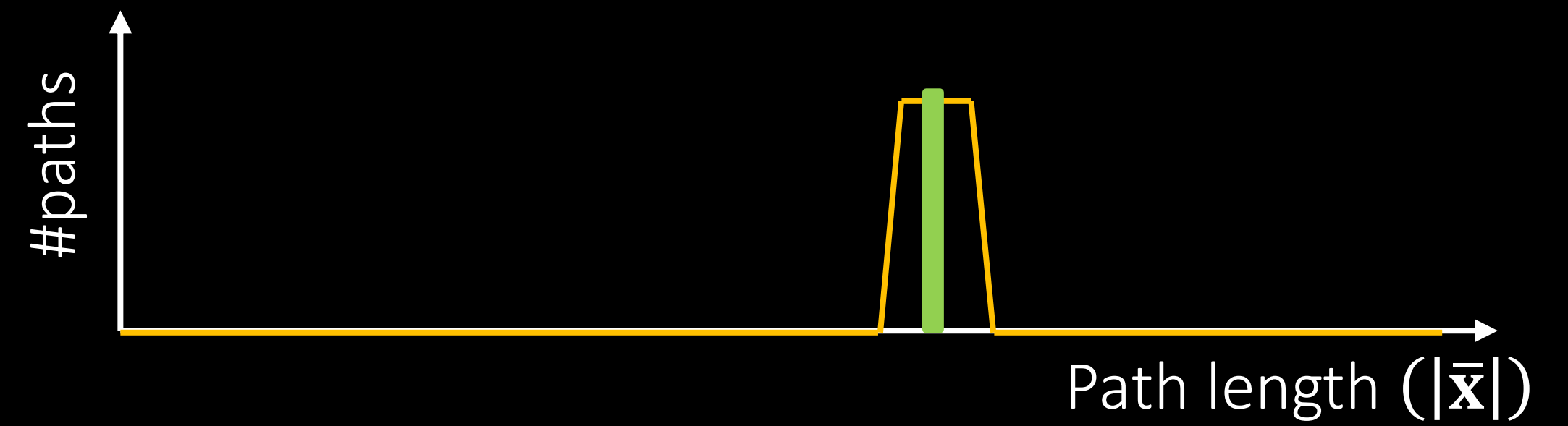
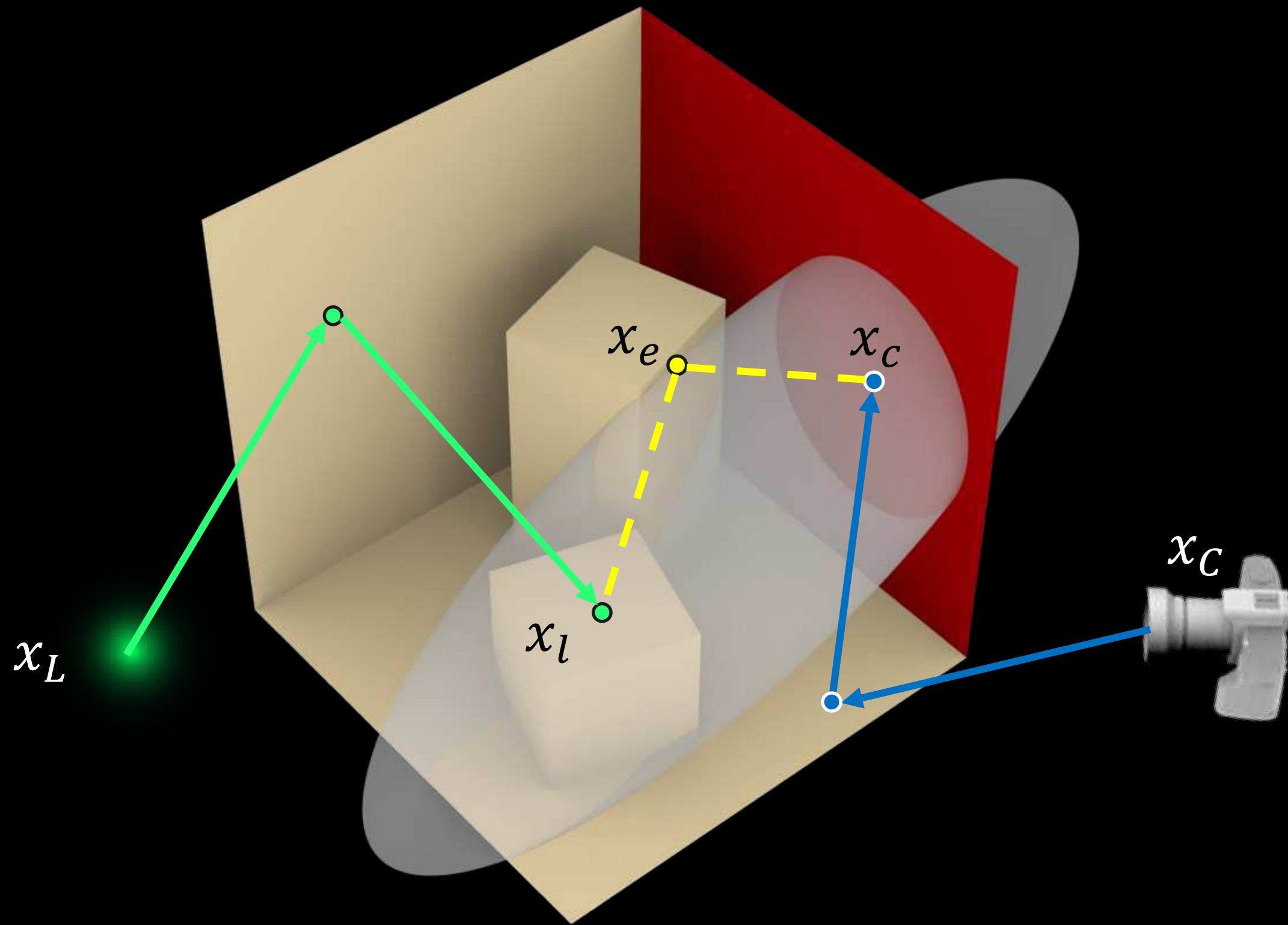
bidirectional path tracing (BDPT)

generate light sub-path, camera sub-path

join  $x_l$  and  $x_c$  via connecting vertex ( $x_e$ )

Definition of an ellipsoid:  $|x_e \rightarrow x_l| + |x_e \rightarrow x_c| = |\bar{x}| - \text{source} + \text{sensor}$   
sub-path lengths

# Path sampling for time-gated case



step 1: sample path length  $|\bar{x}|$

step 2: generate path with target length  $|\bar{x}|$

bidirectional path tracing (BDPT)

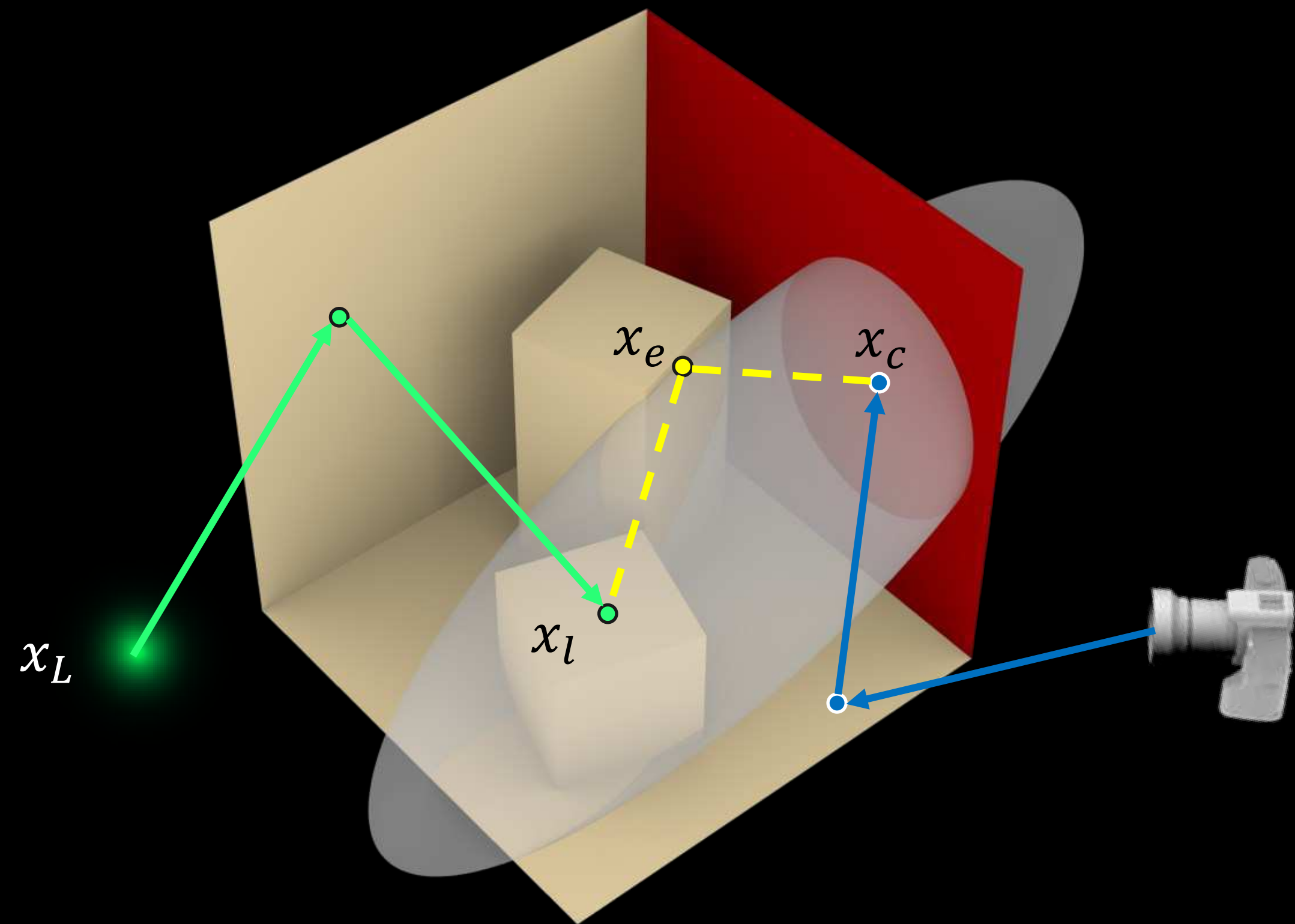
generate light sub-path, camera sub-path

join  $x_l$  and  $x_c$  via connecting vertex ( $x_e$ )

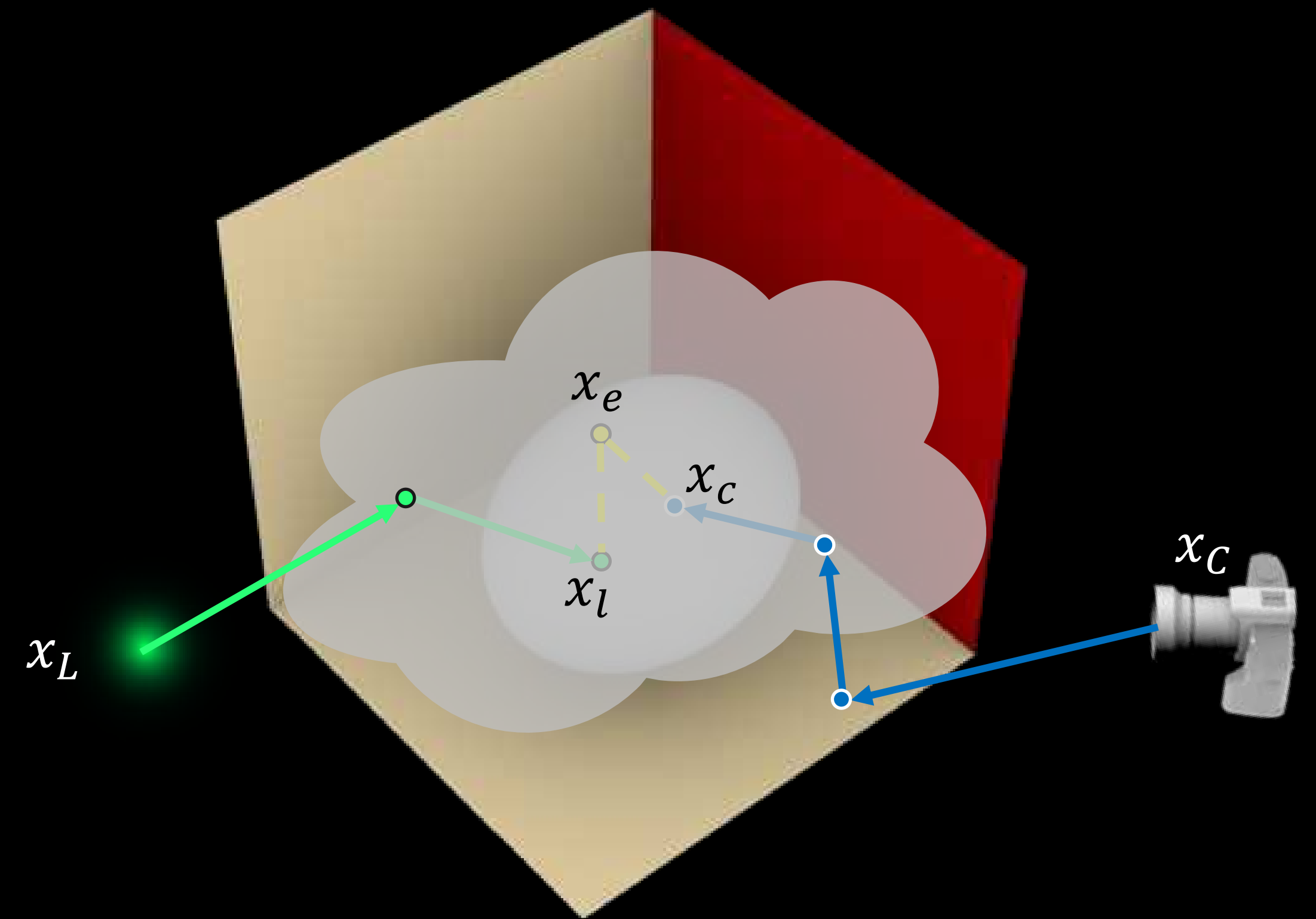
Definition of an ellipsoid:  $|x_e \rightarrow x_l| + |x_e \rightarrow x_c| = |\bar{x}| - \text{source + sensor sub-path lengths}$



# Path sampling for time-gated case

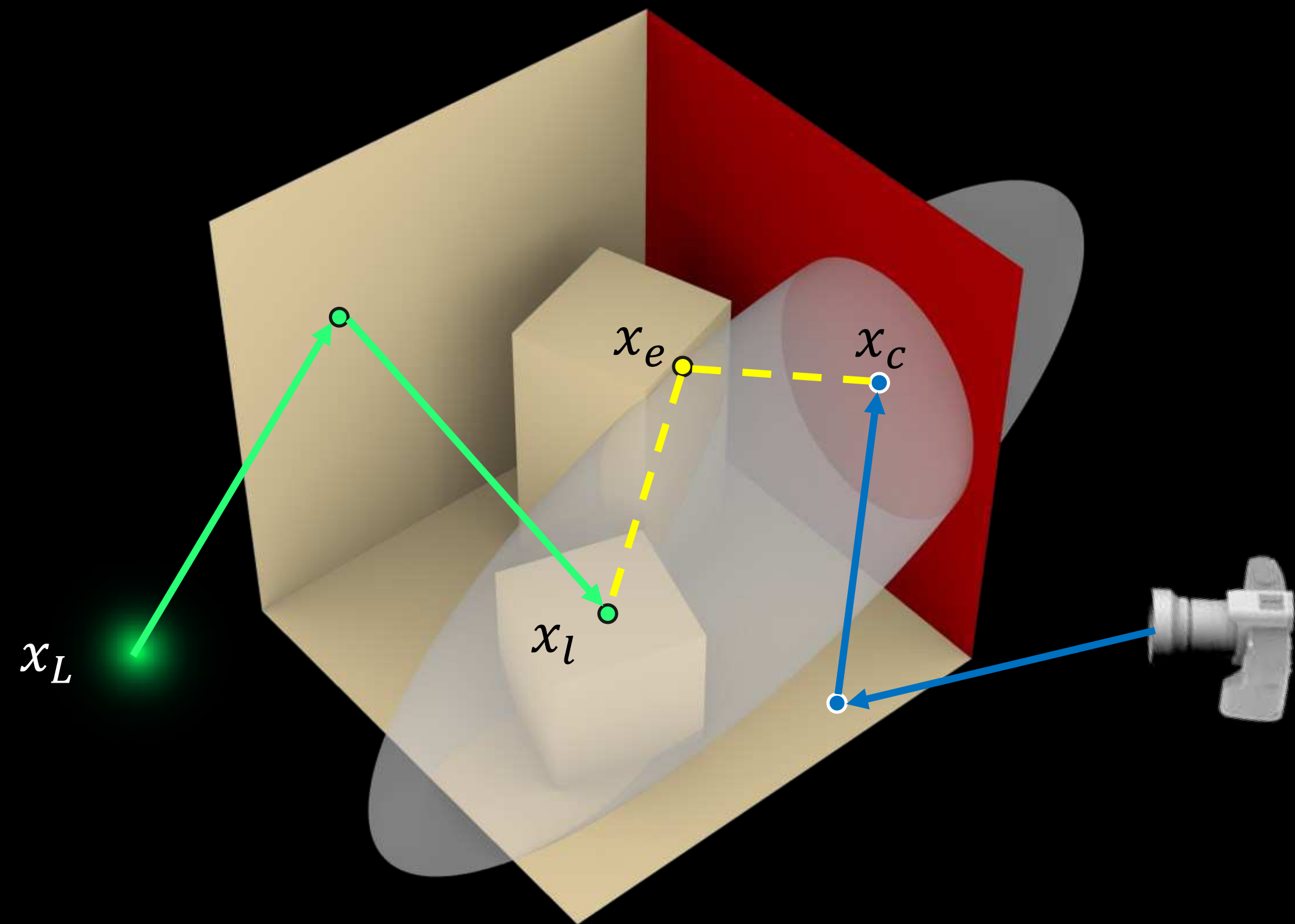


surface case

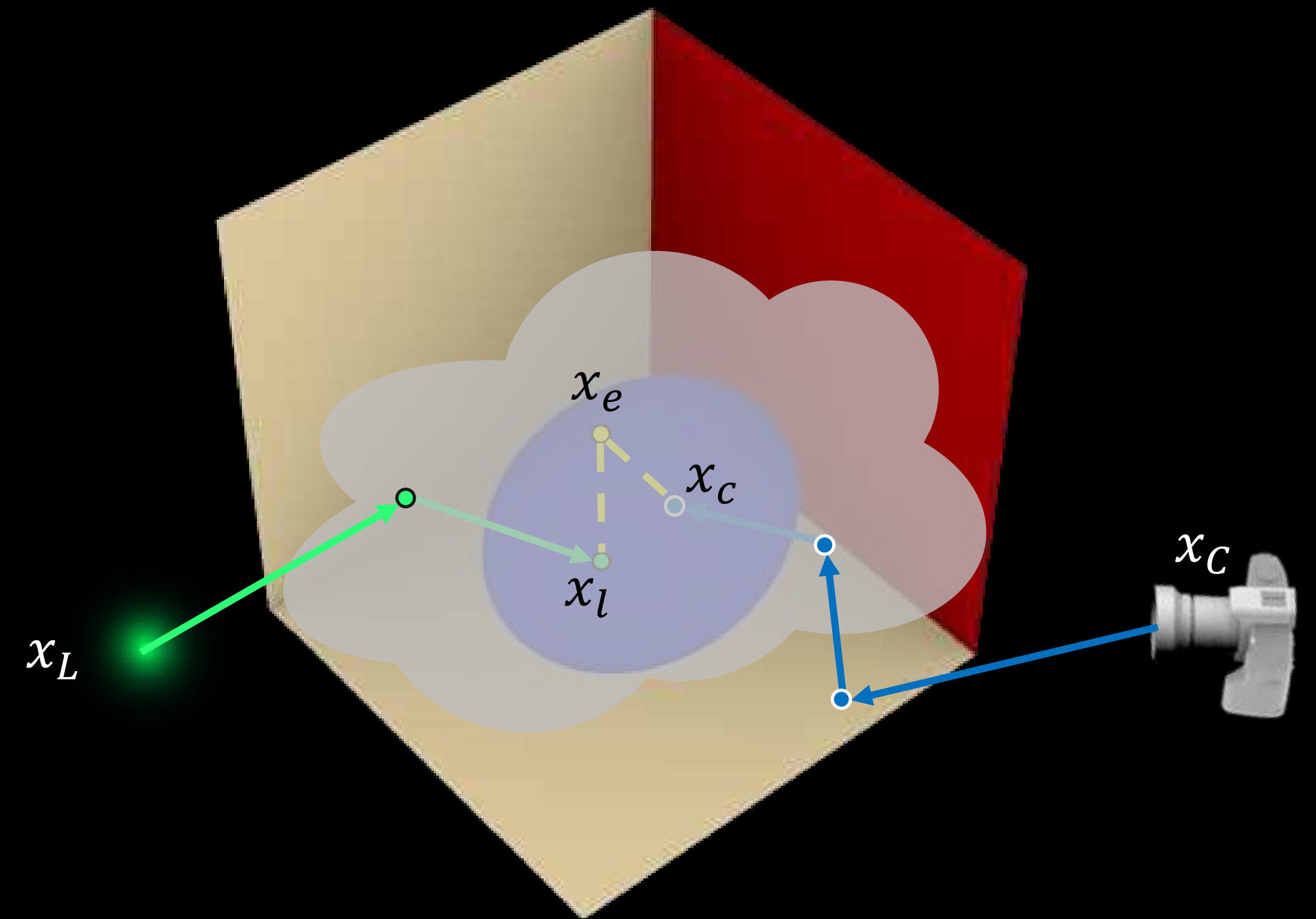


volumetric case

# Path sampling for time-gated case



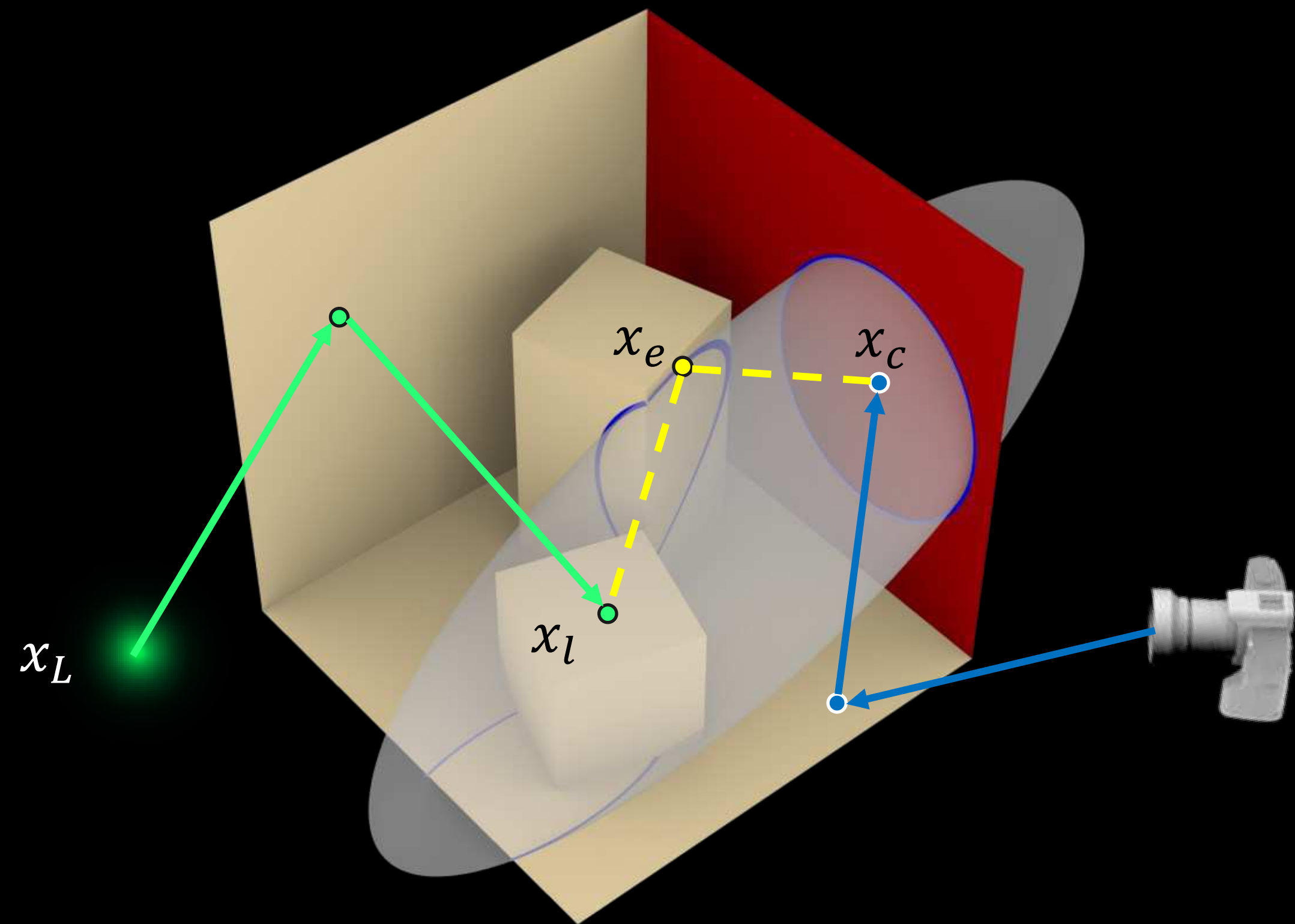
surface case



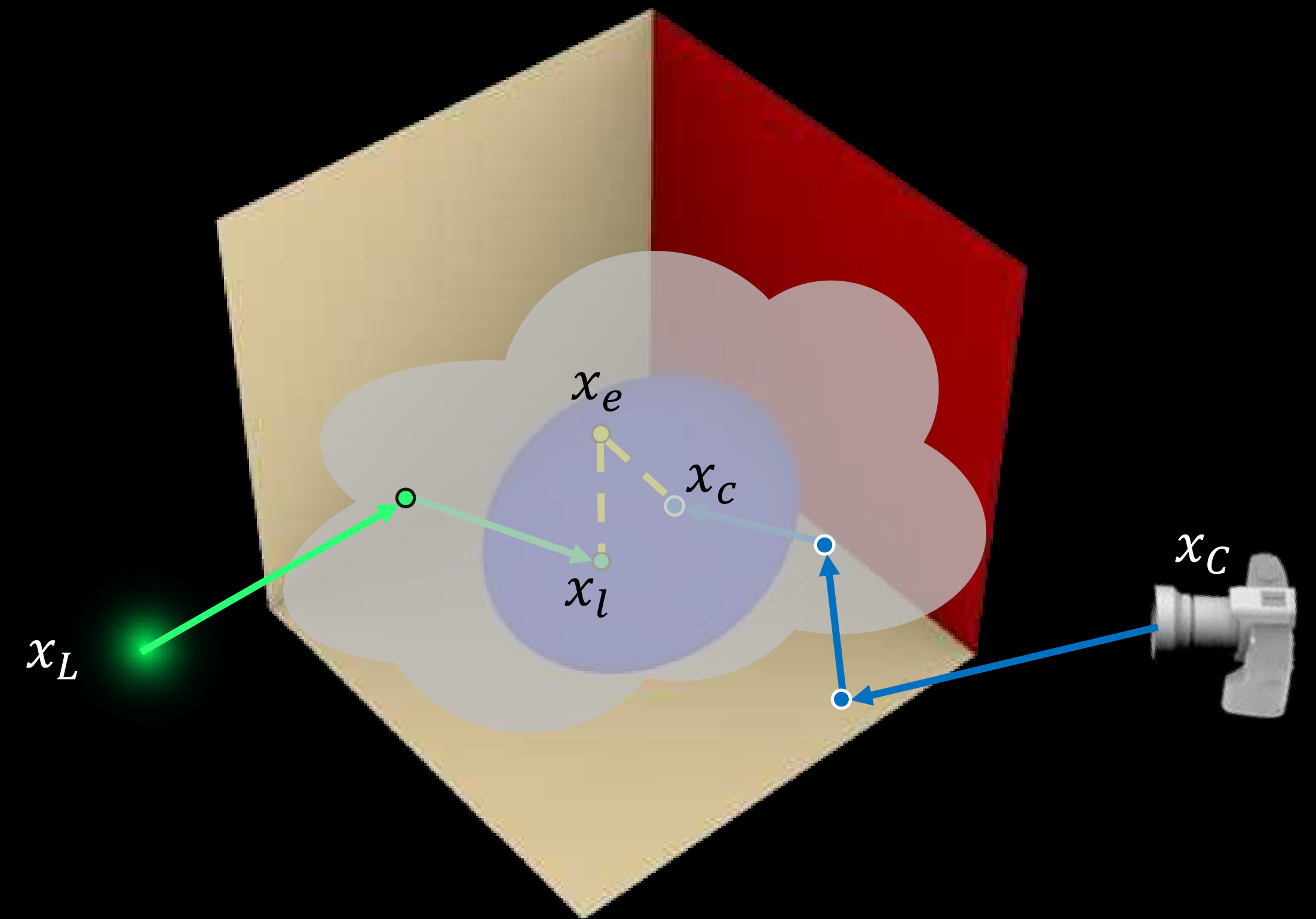
volumetric case



# Path sampling for time-gated case



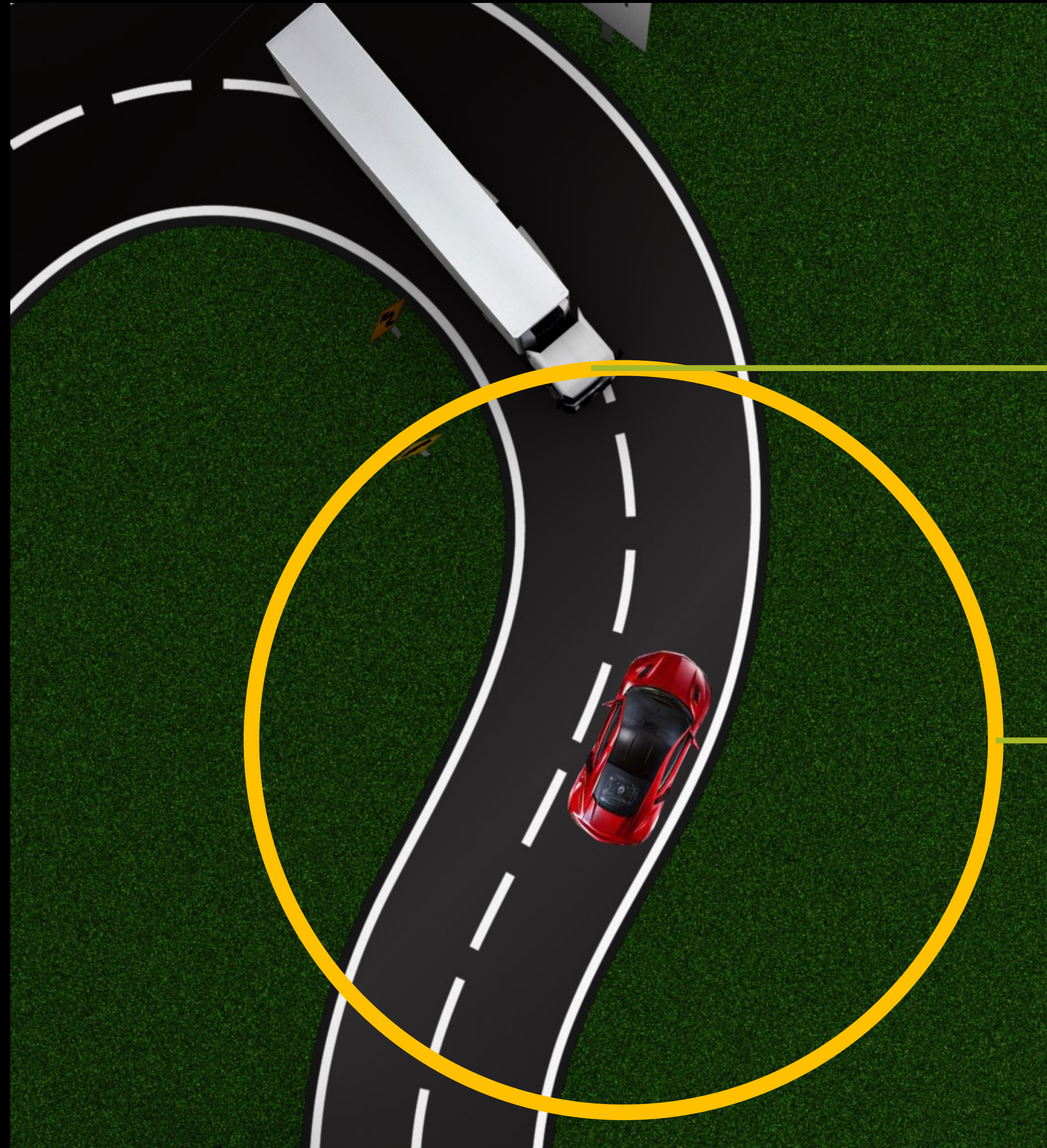
surface case



# volumetric case



# Application: proximity detector for cars



proximity detected

virtual light curtain



# Application: proximity detector for cars

road scene

standard BDPT

BDPT w/ ellipsoidal  
connections

Gate width: 200 ps (1.14% scene)  
Rendering time: 10s per frame

# Application: proximity detector for cars

road scene

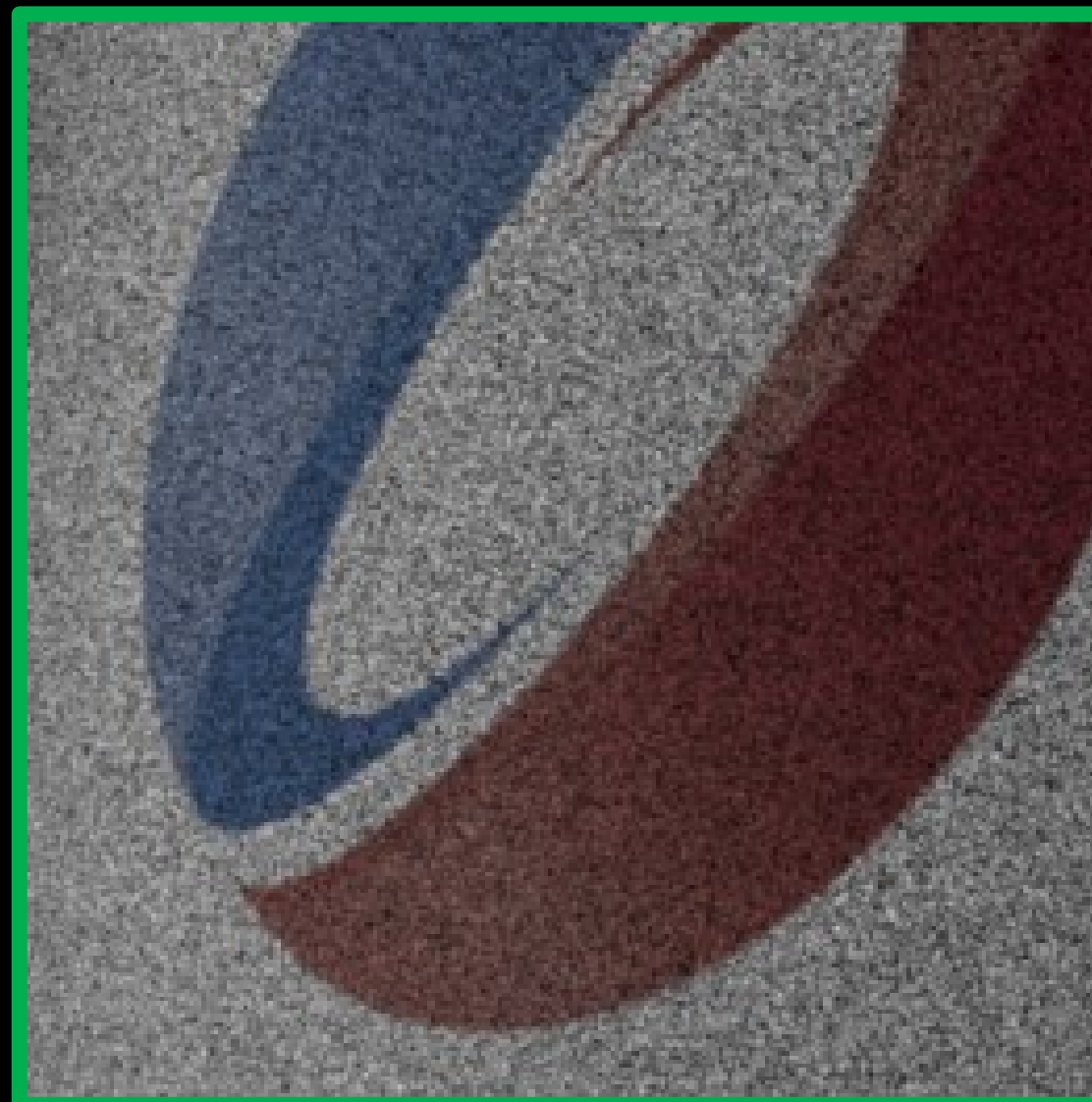
standard BDPT

BDPT w/ ellipsoidal connections

time = 1.34s



time = 1.74s





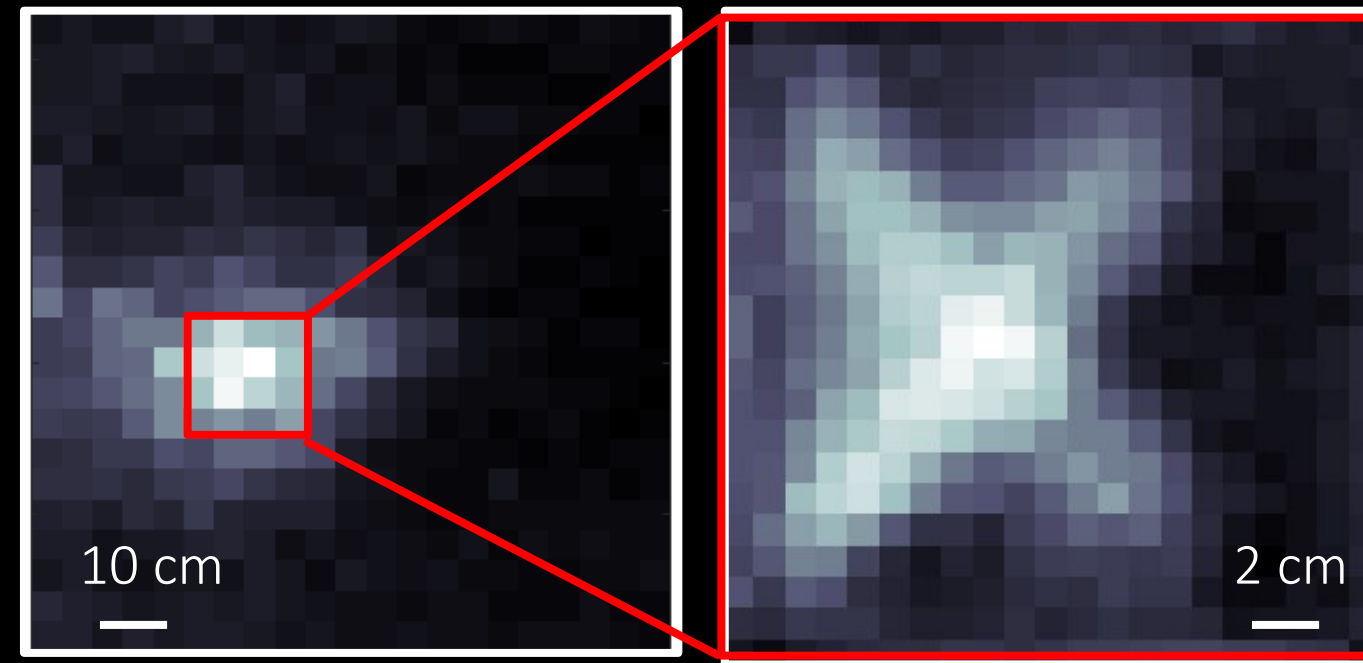
# Imaging projects using this renderer

Mitsuba based  
open source implementation

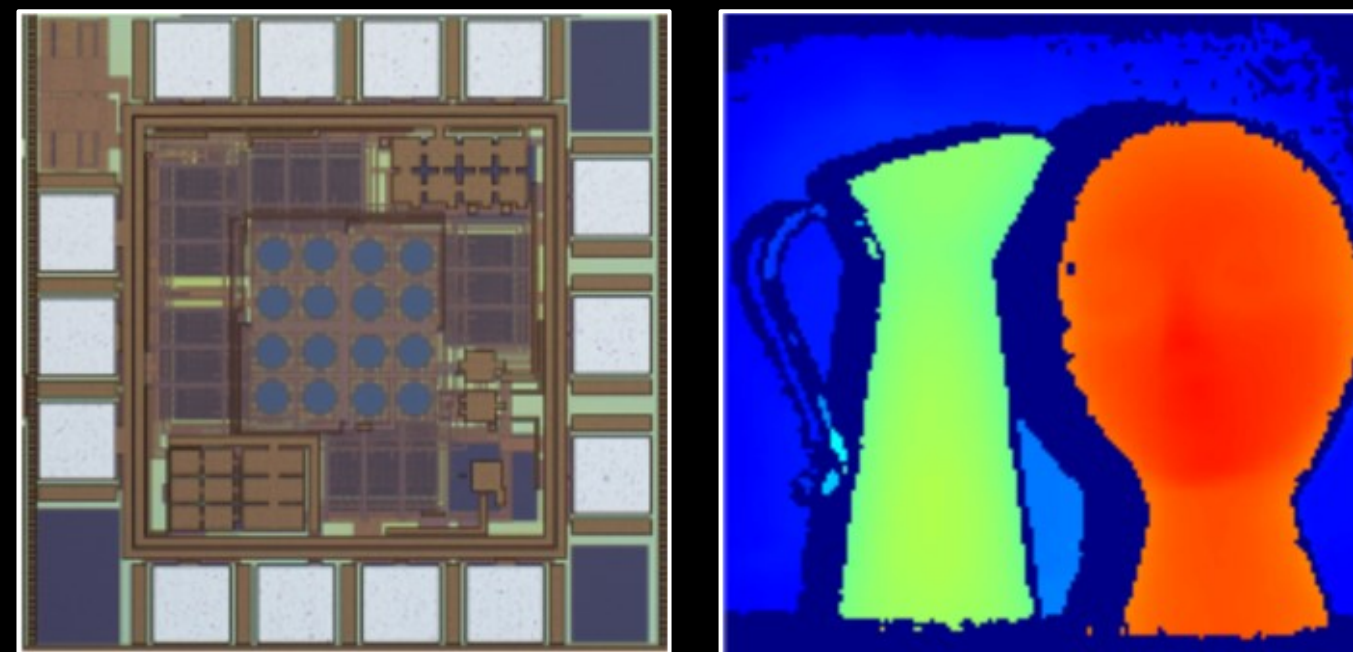


<http://bit.ly/32Uzm1m>

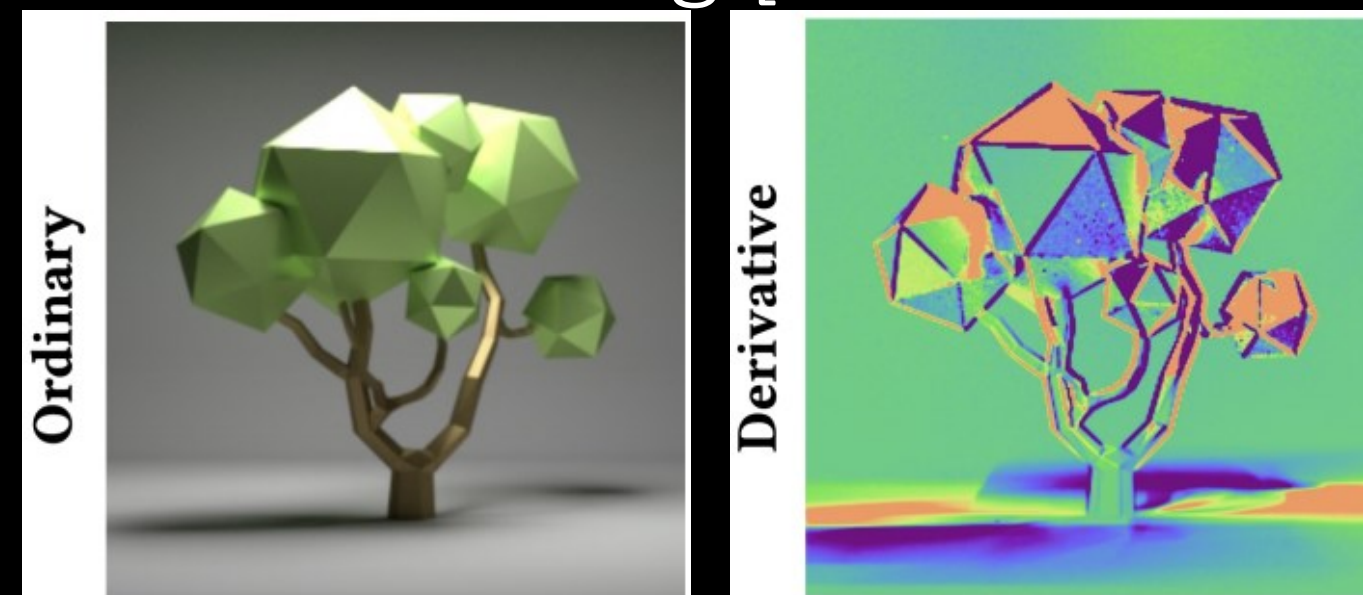
non-line-of-sight



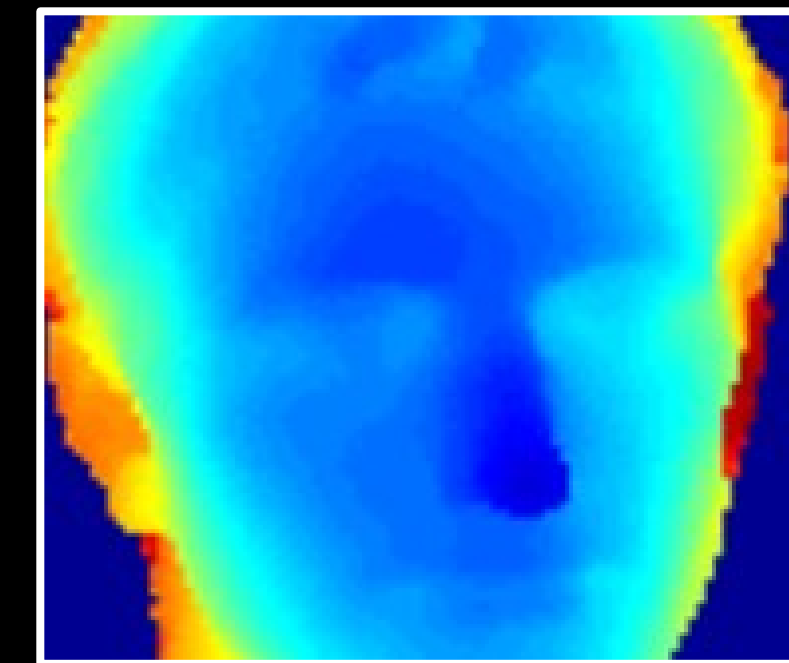
sensor design [White et al. 2022]



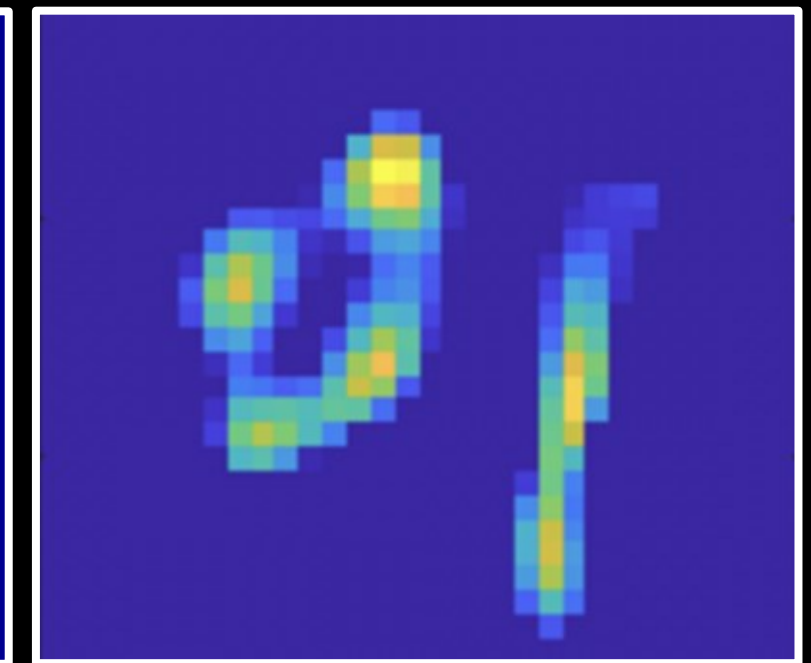
inverse rendering [Wu et al. 2021]



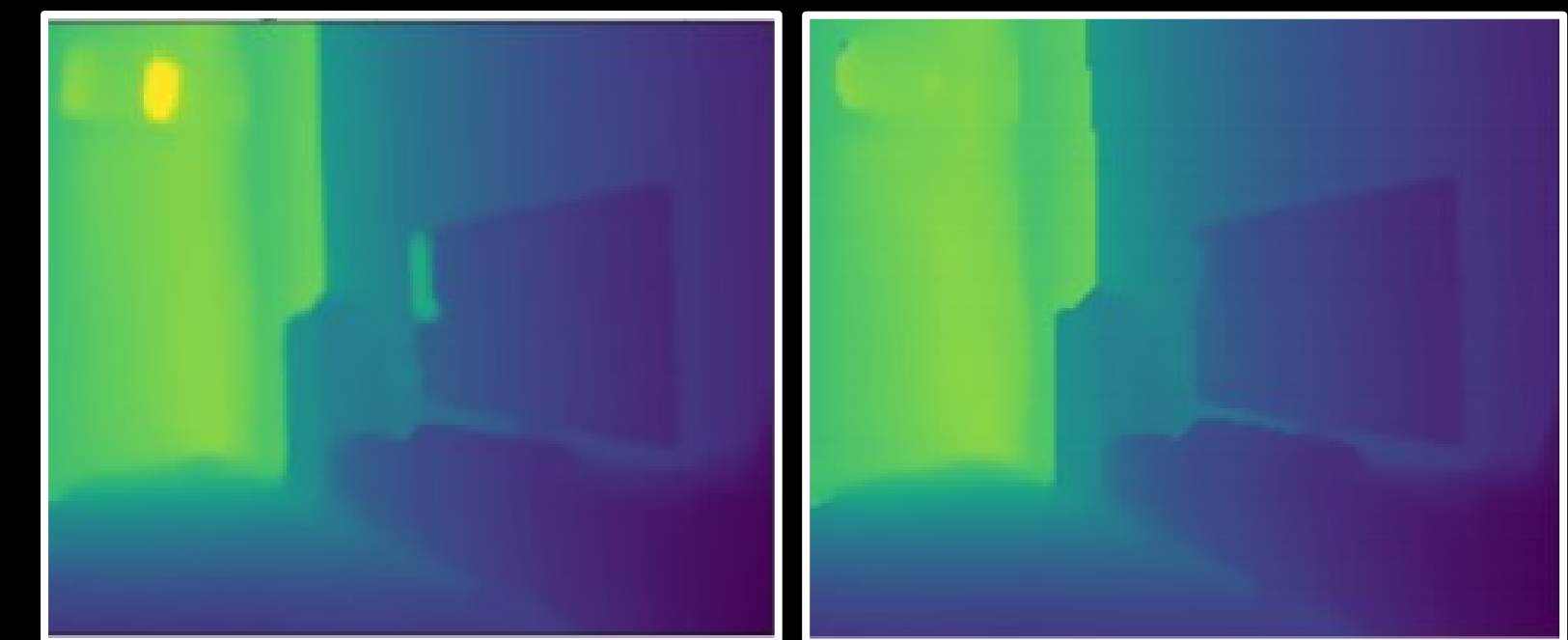
depth sensing



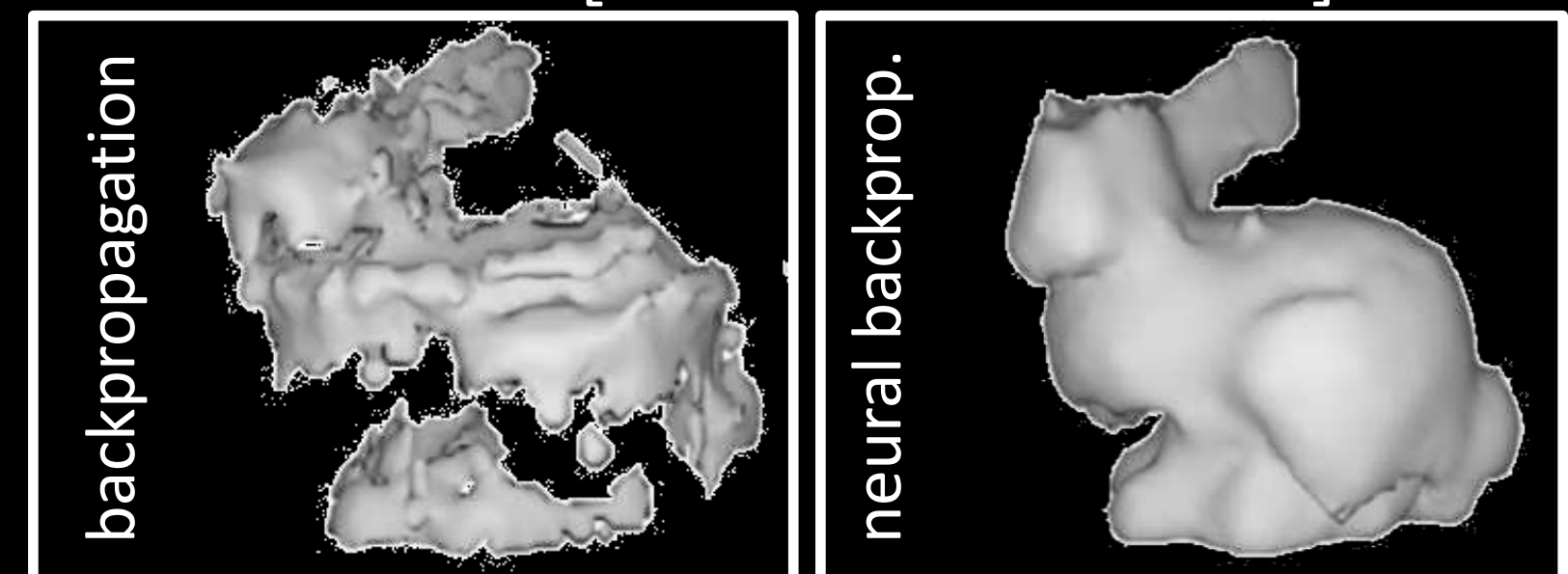
tissue imaging



deep learning [Barragan et al. 2021]



SONAR [Reed et al. 2023]



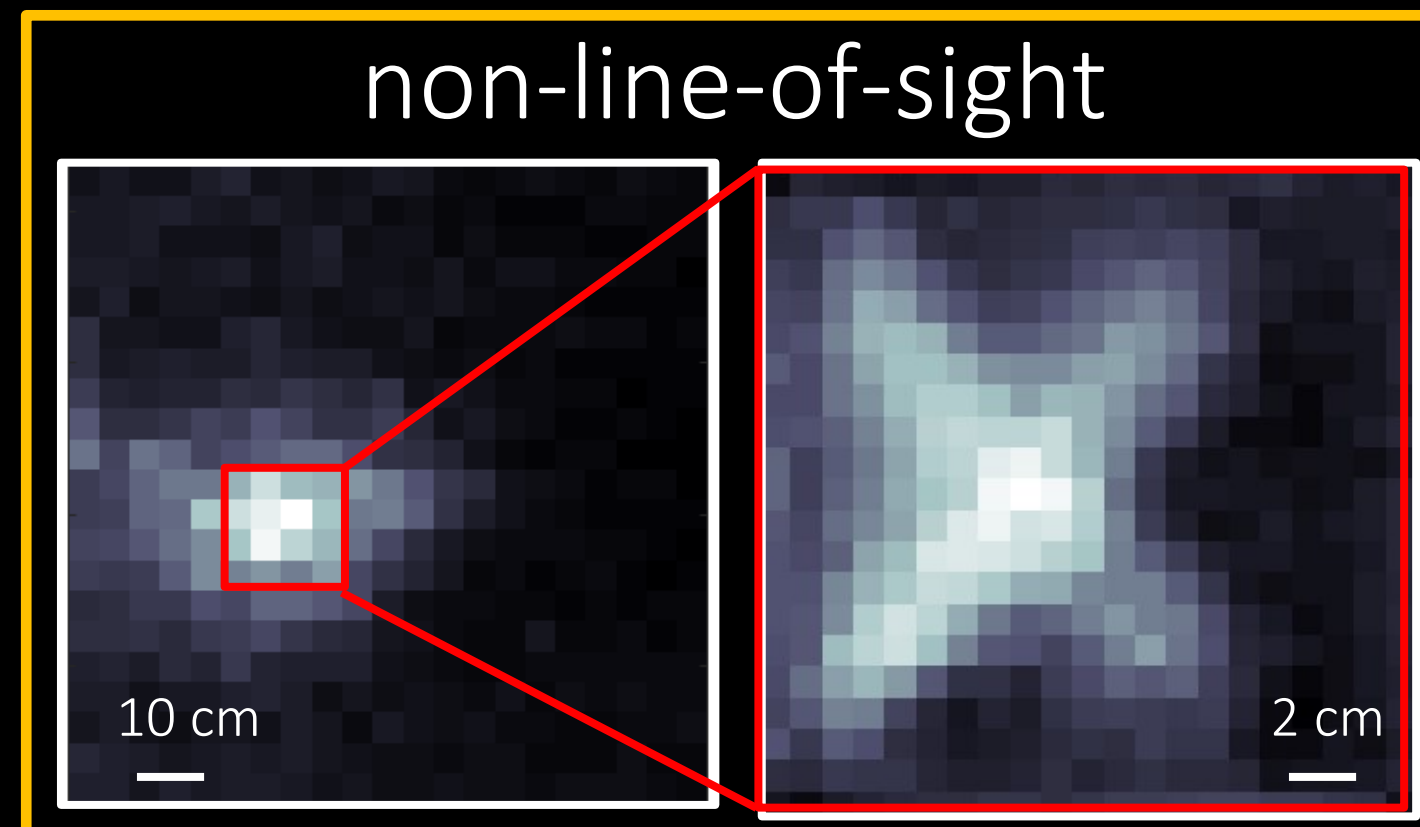


# Imaging projects using this renderer

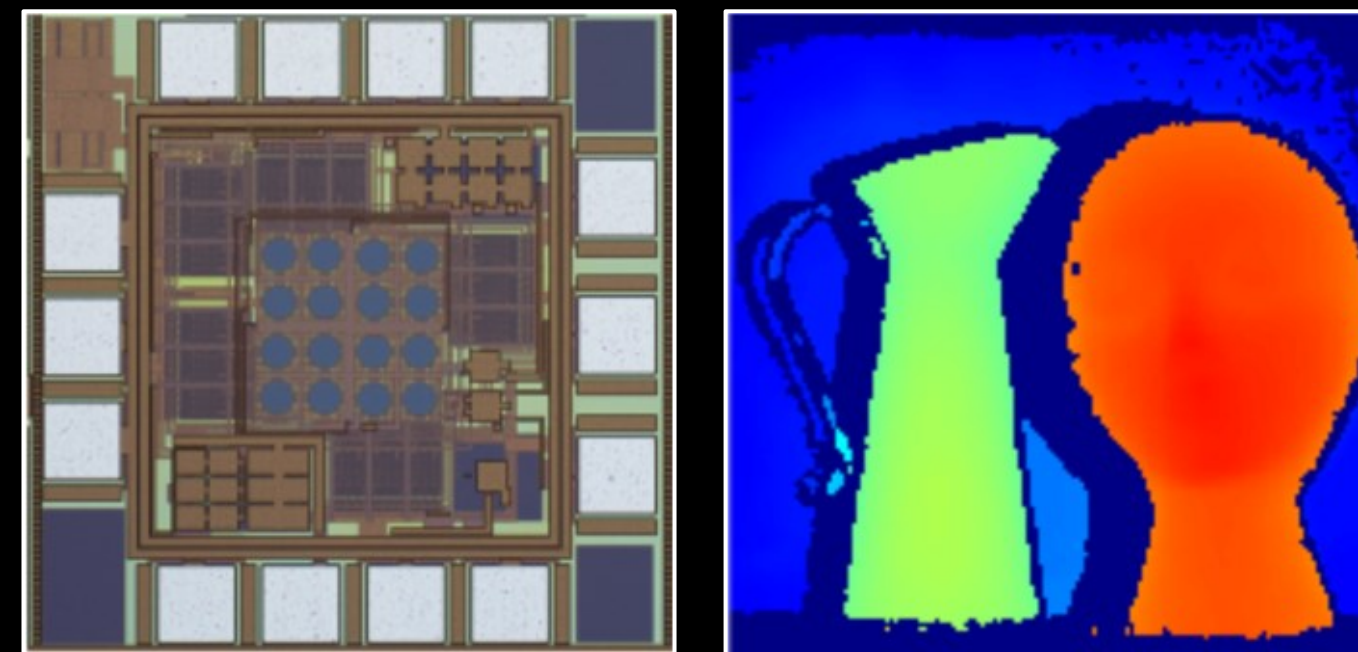
Mitsuba based  
open source implementation



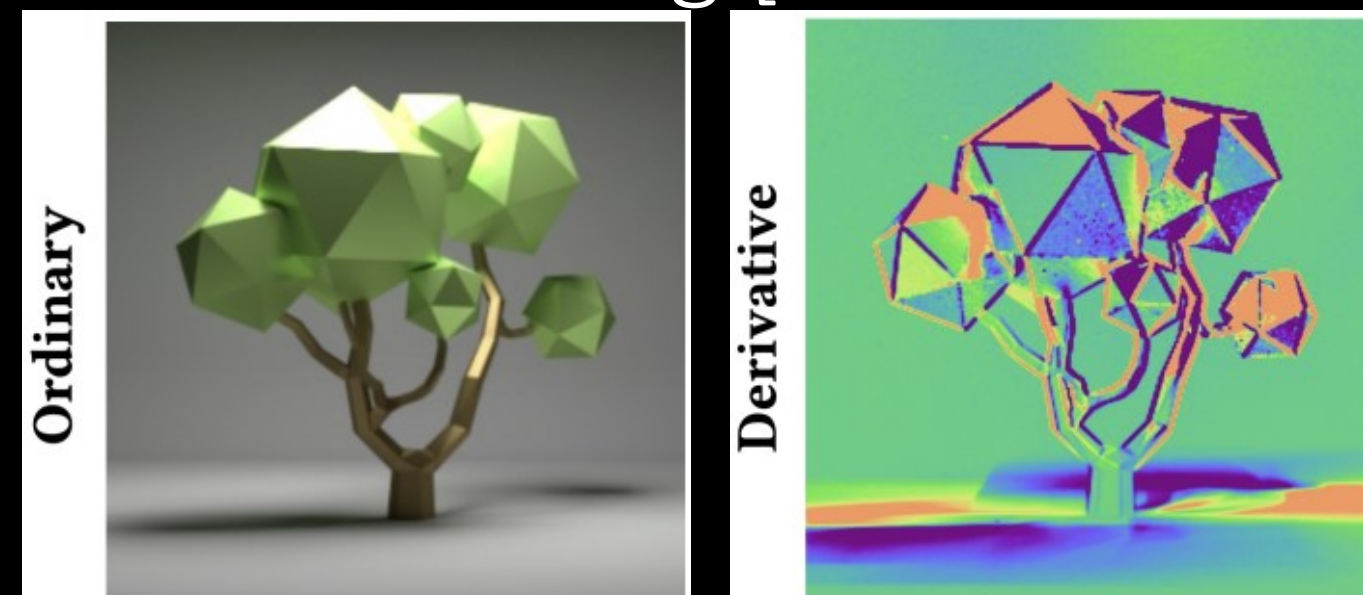
<http://bit.ly/32Uzm1m>



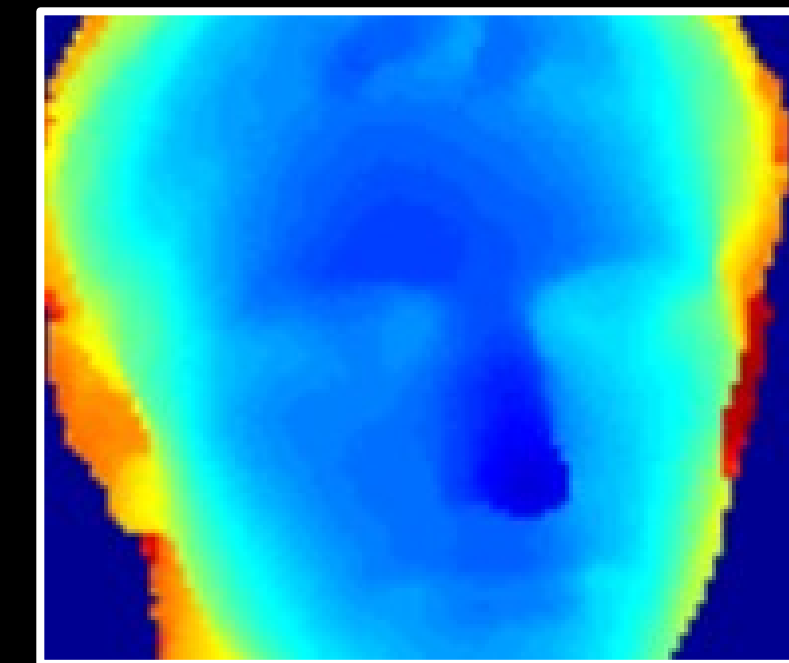
sensor design [White et al. 2022]



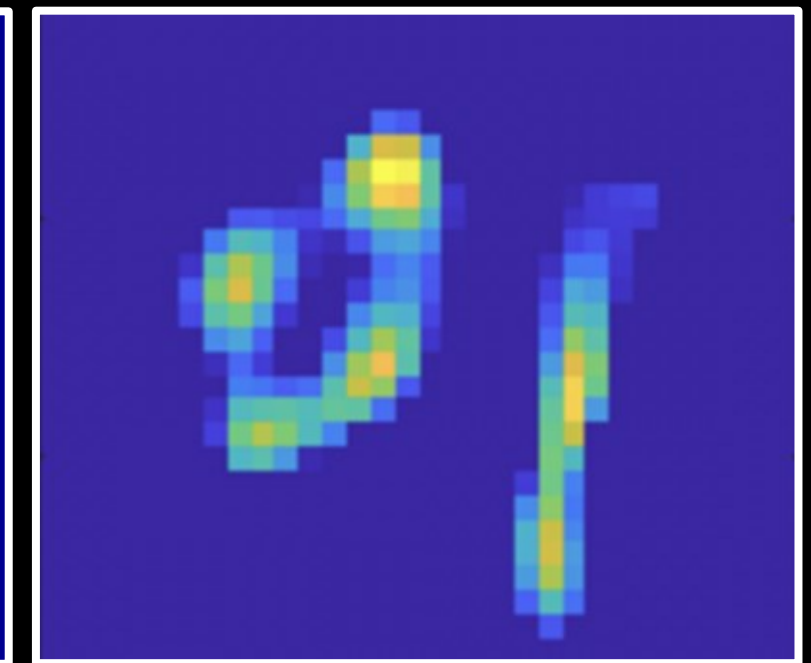
inverse rendering [Wu et al. 2021]



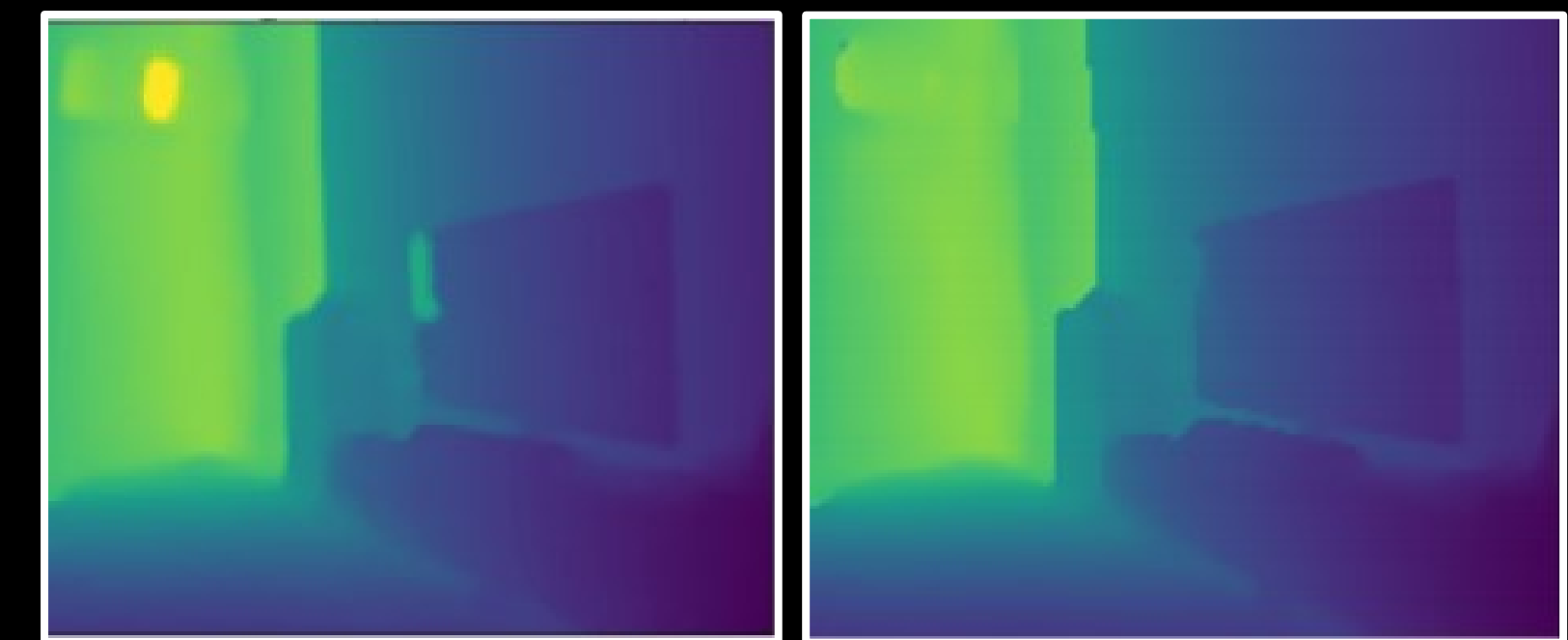
depth sensing



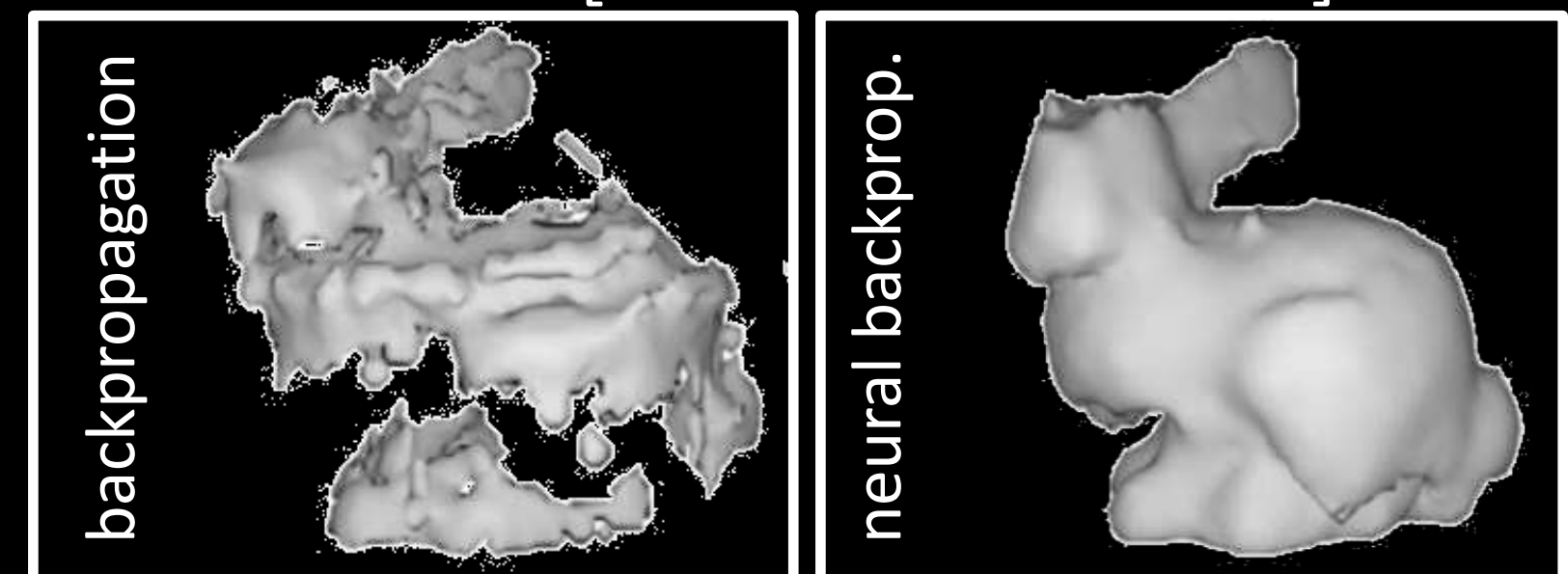
tissue imaging



deep learning [Barragan et al. 2021]



SONAR [Reed et al. 2023]





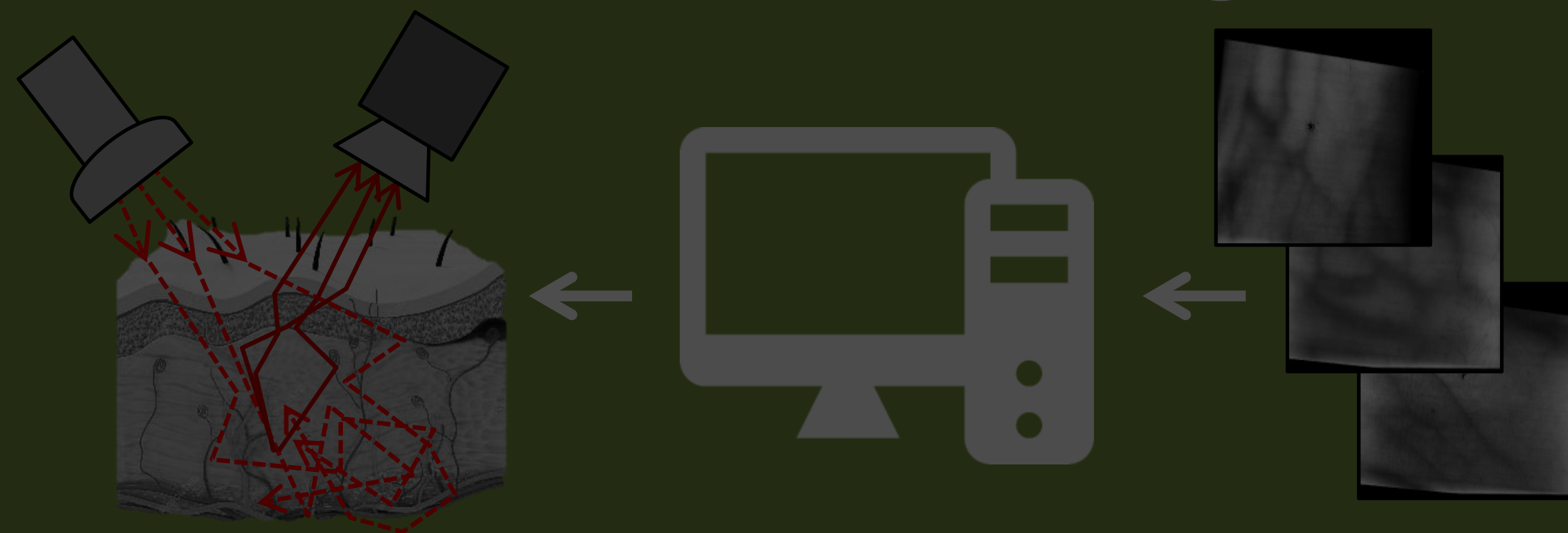
# Physics-based rendering and its applications to computational imaging

## forward rendering

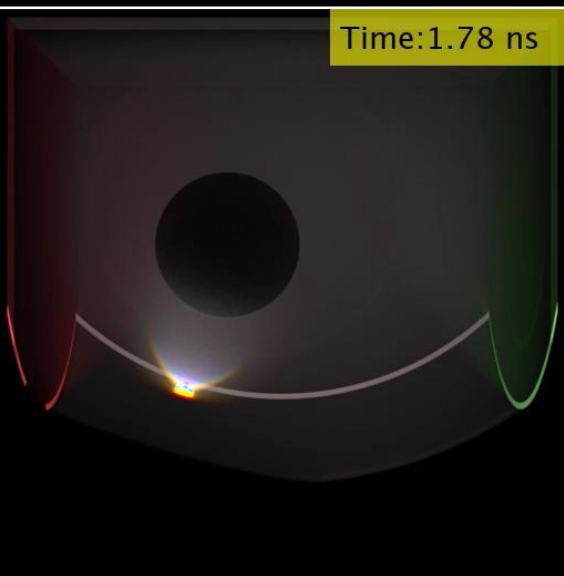


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

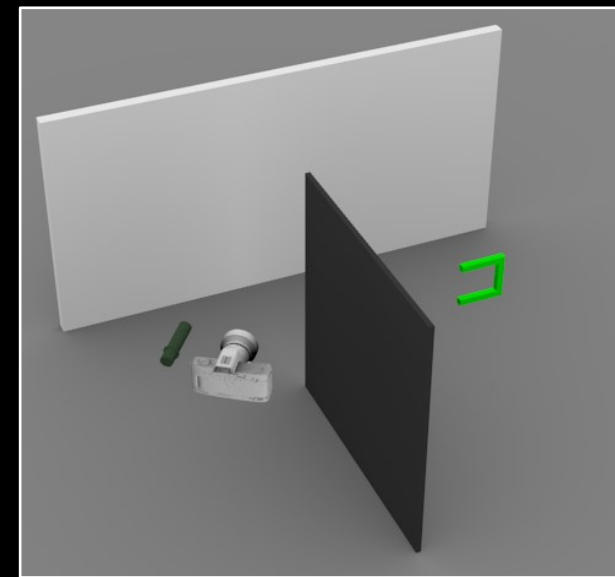
## inverse rendering



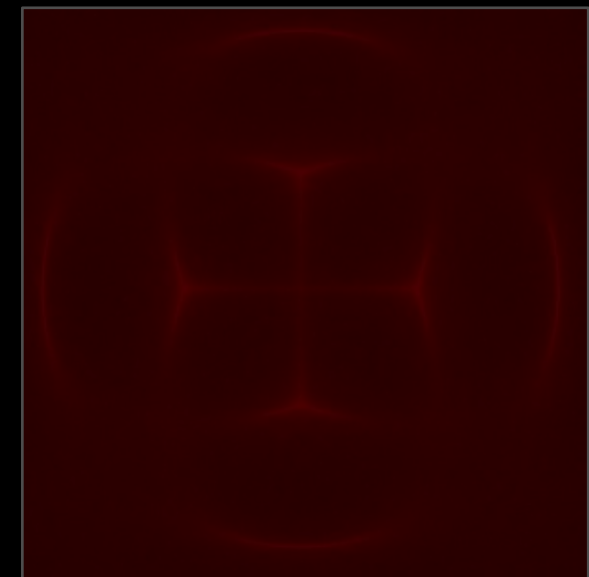
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



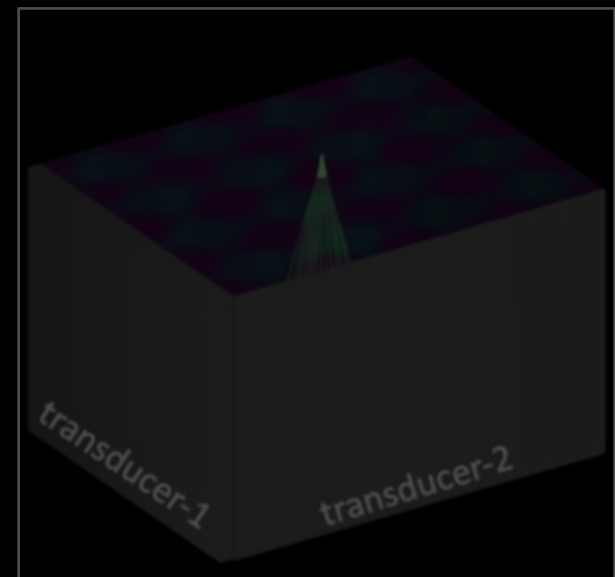
time-of-flight  
imaging



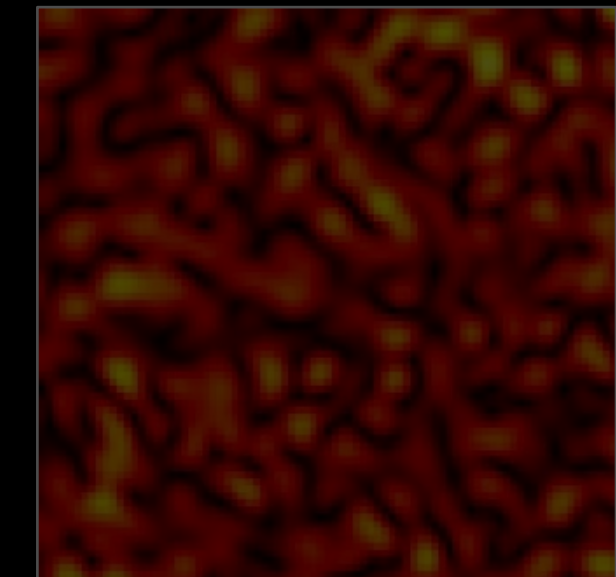
non-line-of-sight  
imaging



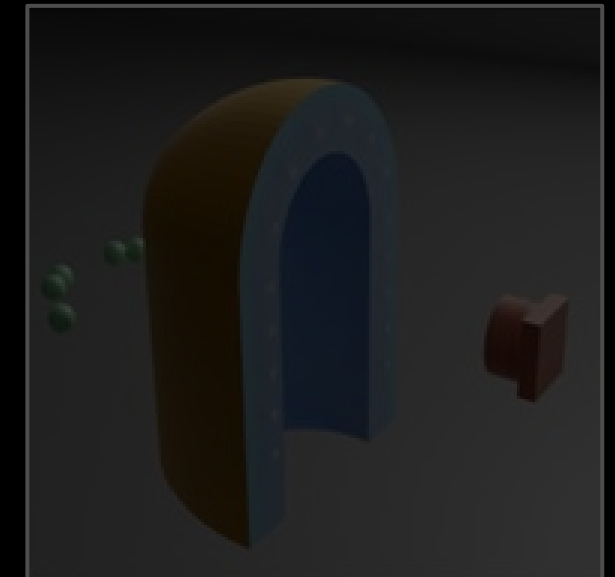
acousto-optic  
lensing



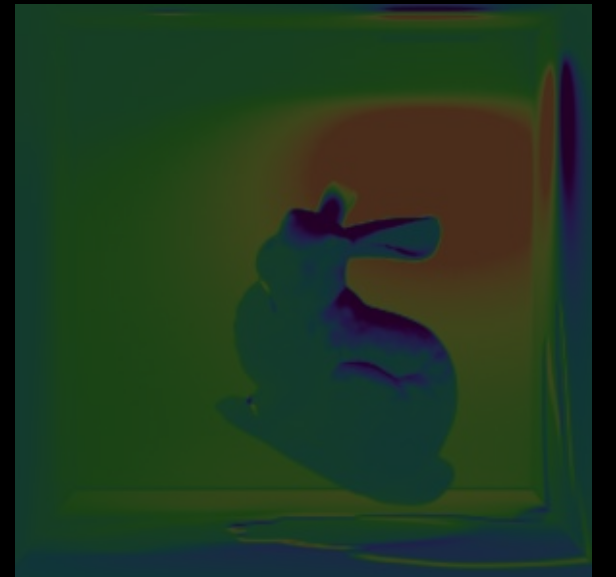
ultrafast light  
scanning



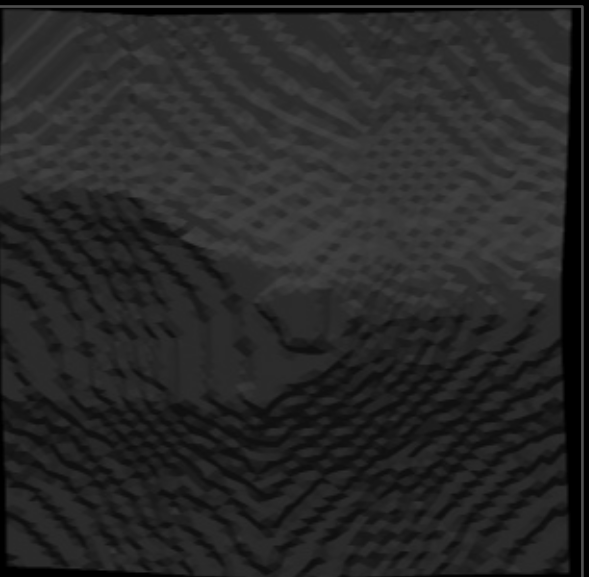
speckle  
imaging



tactile sensor  
design

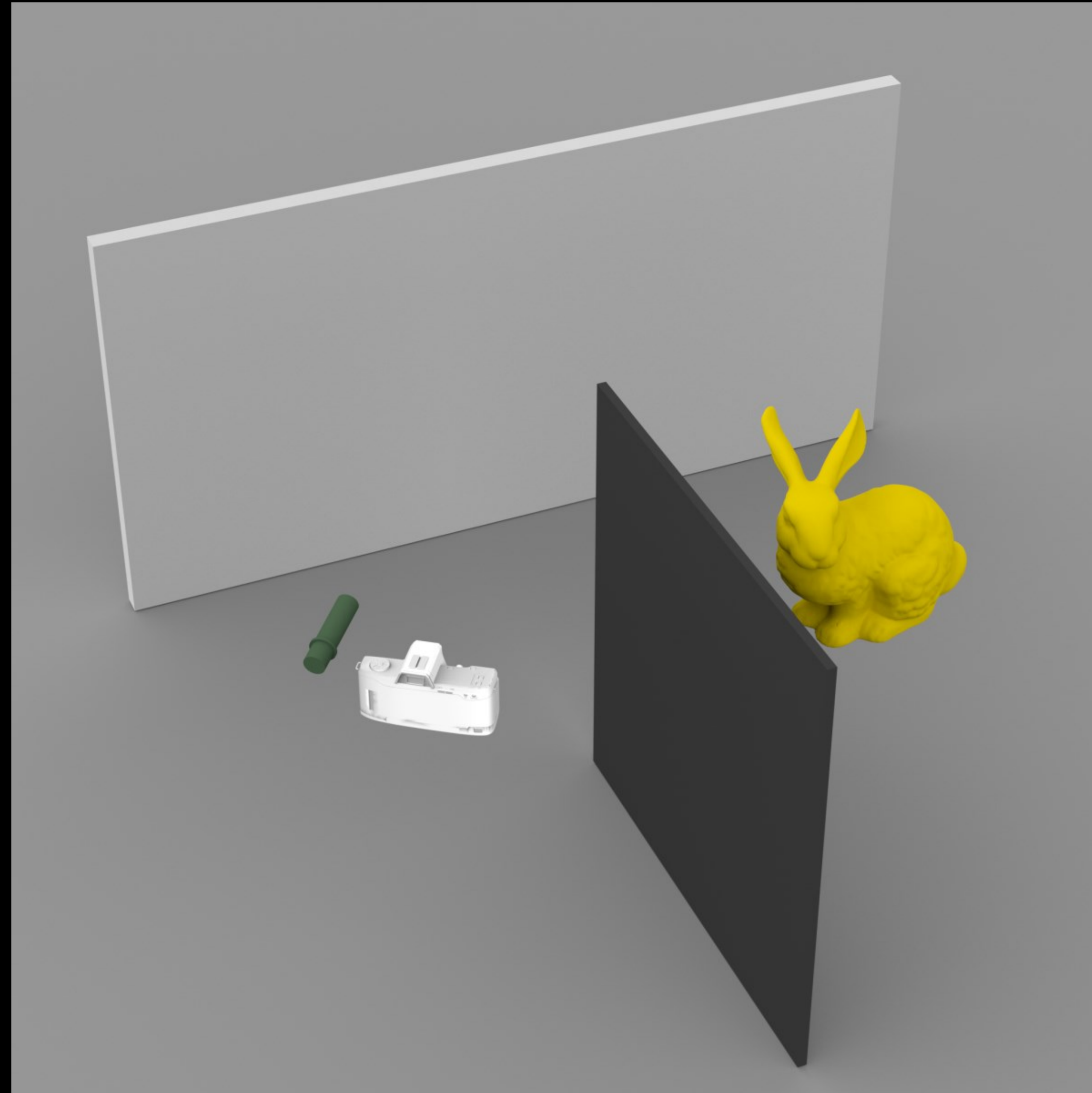


differentiable  
renderer



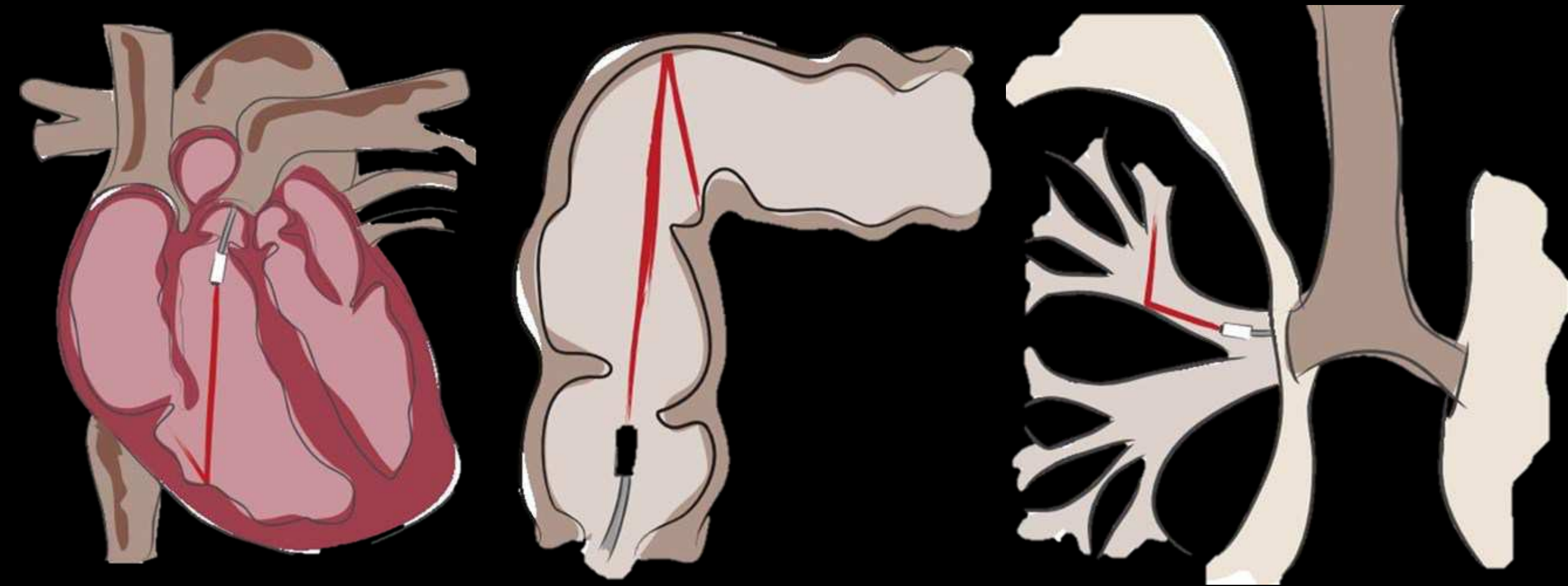
inverse  
problems

# Non-line-of-sight imaging

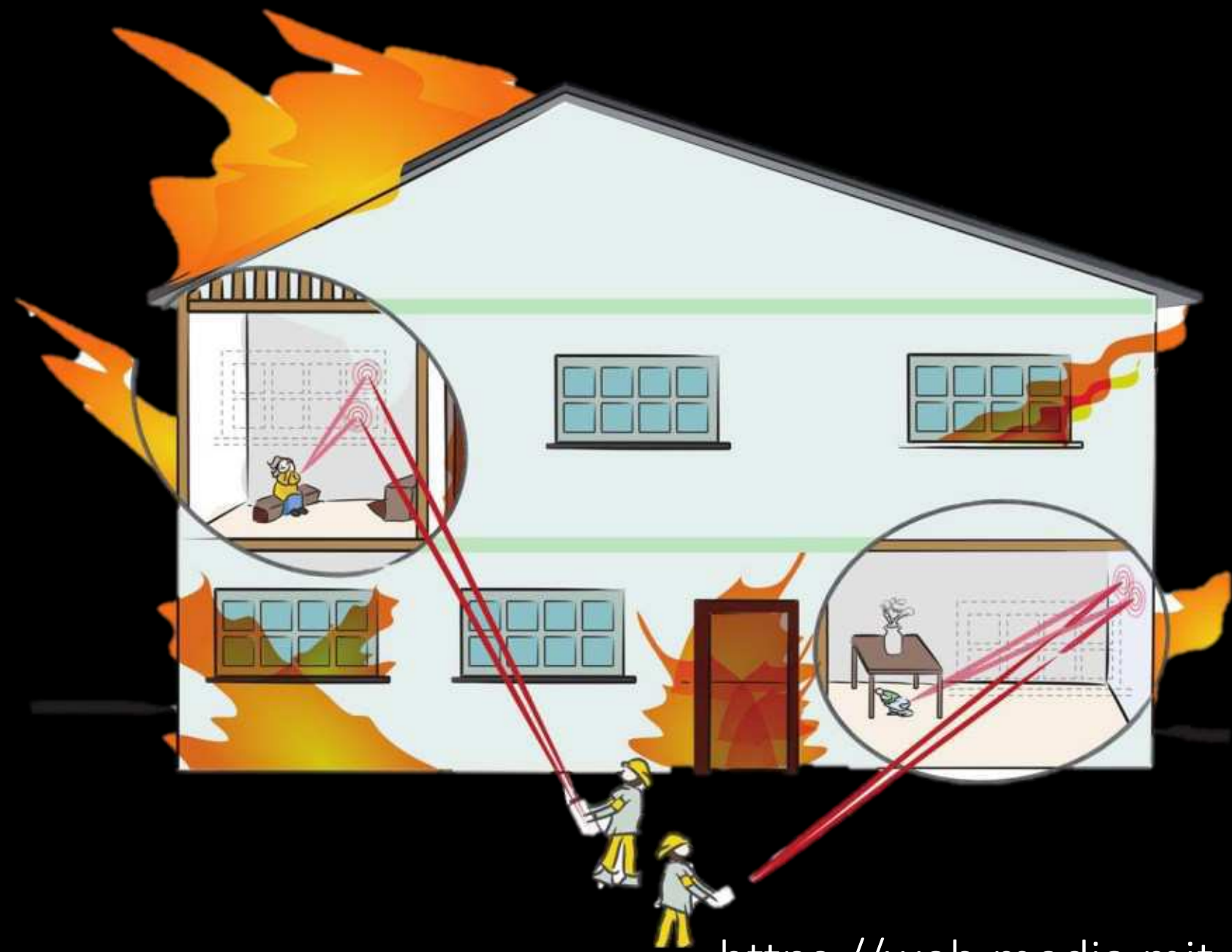




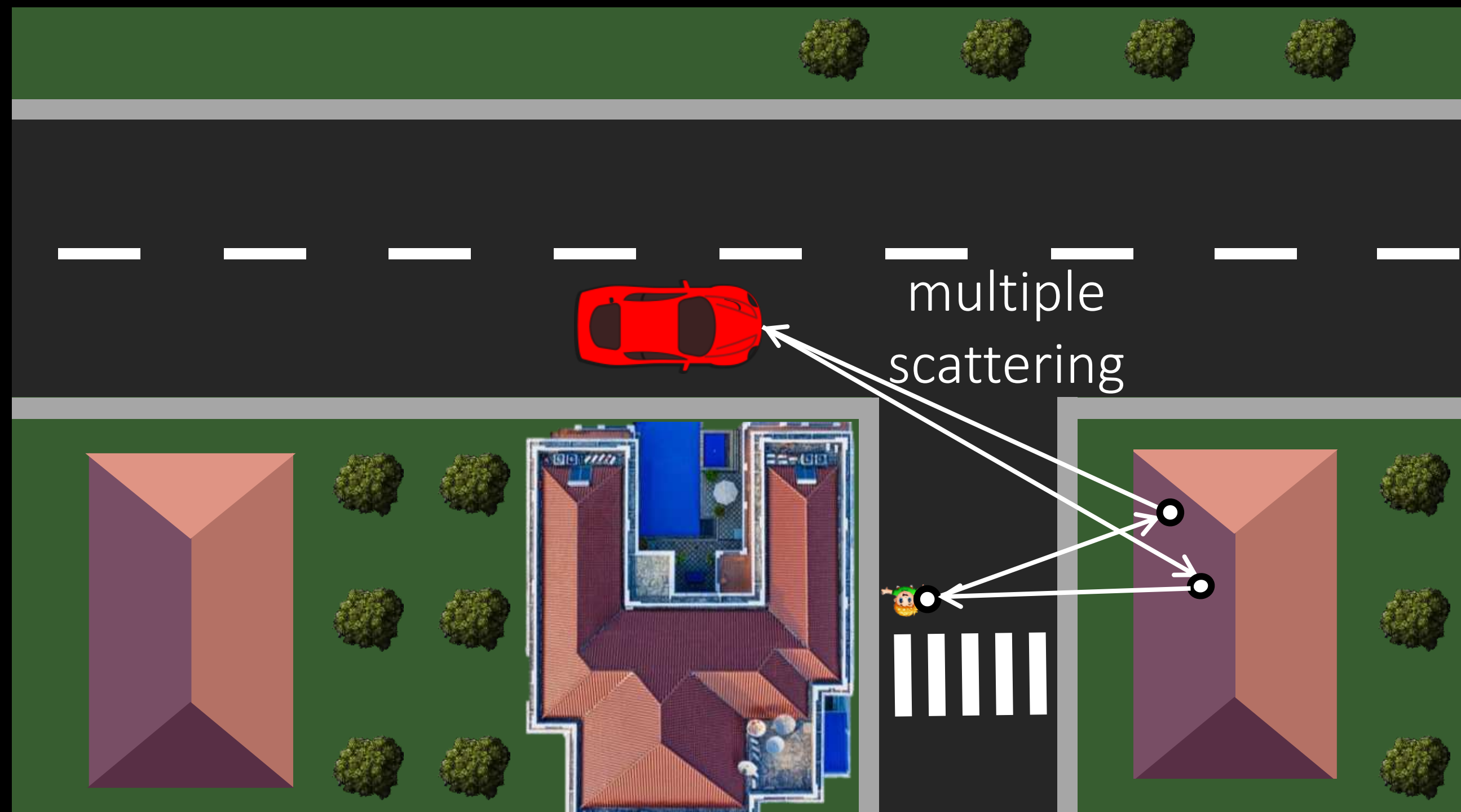
# Applications



<https://web.media.mit.edu/~raskar/cornar/>



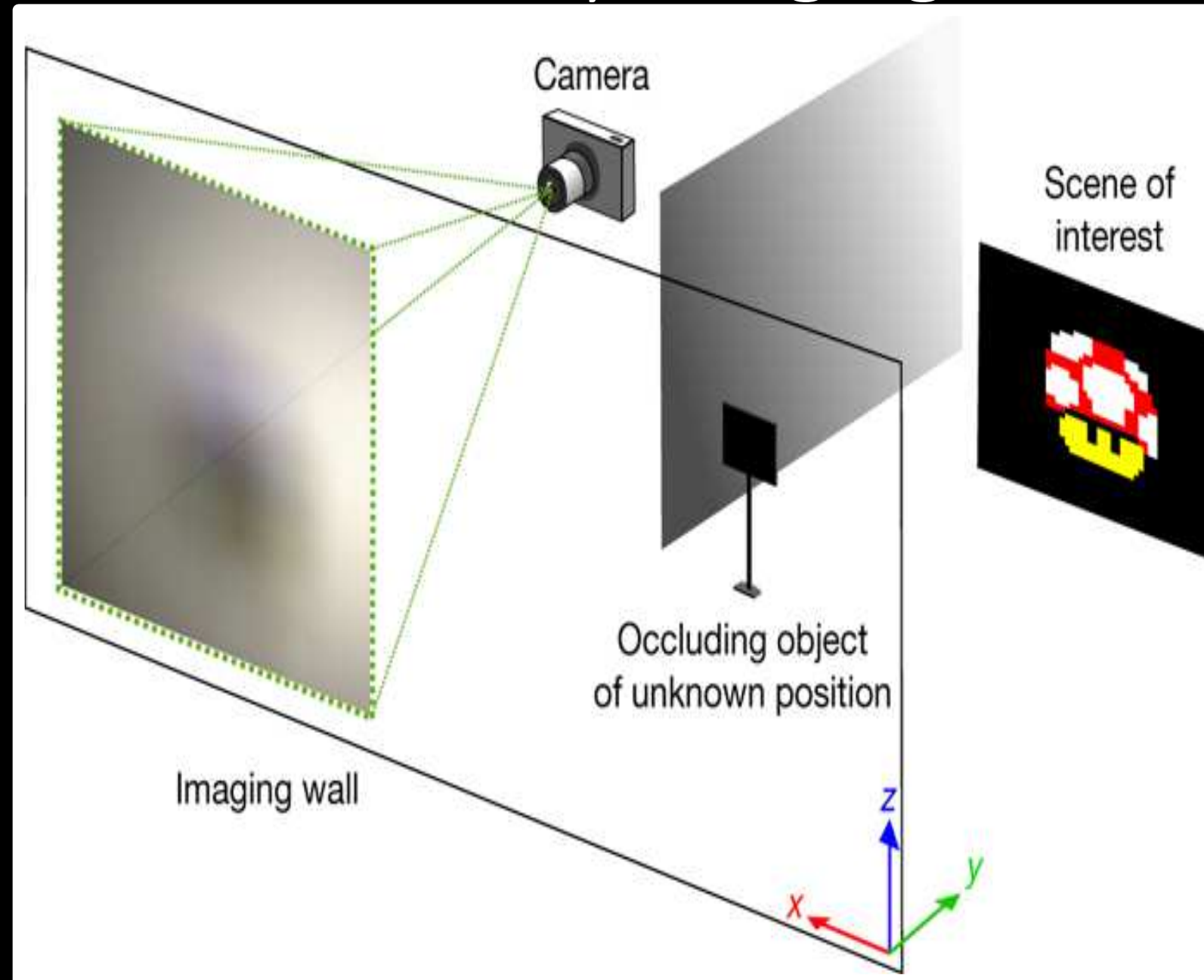
<https://web.media.mit.edu/~raskar/cornar/>





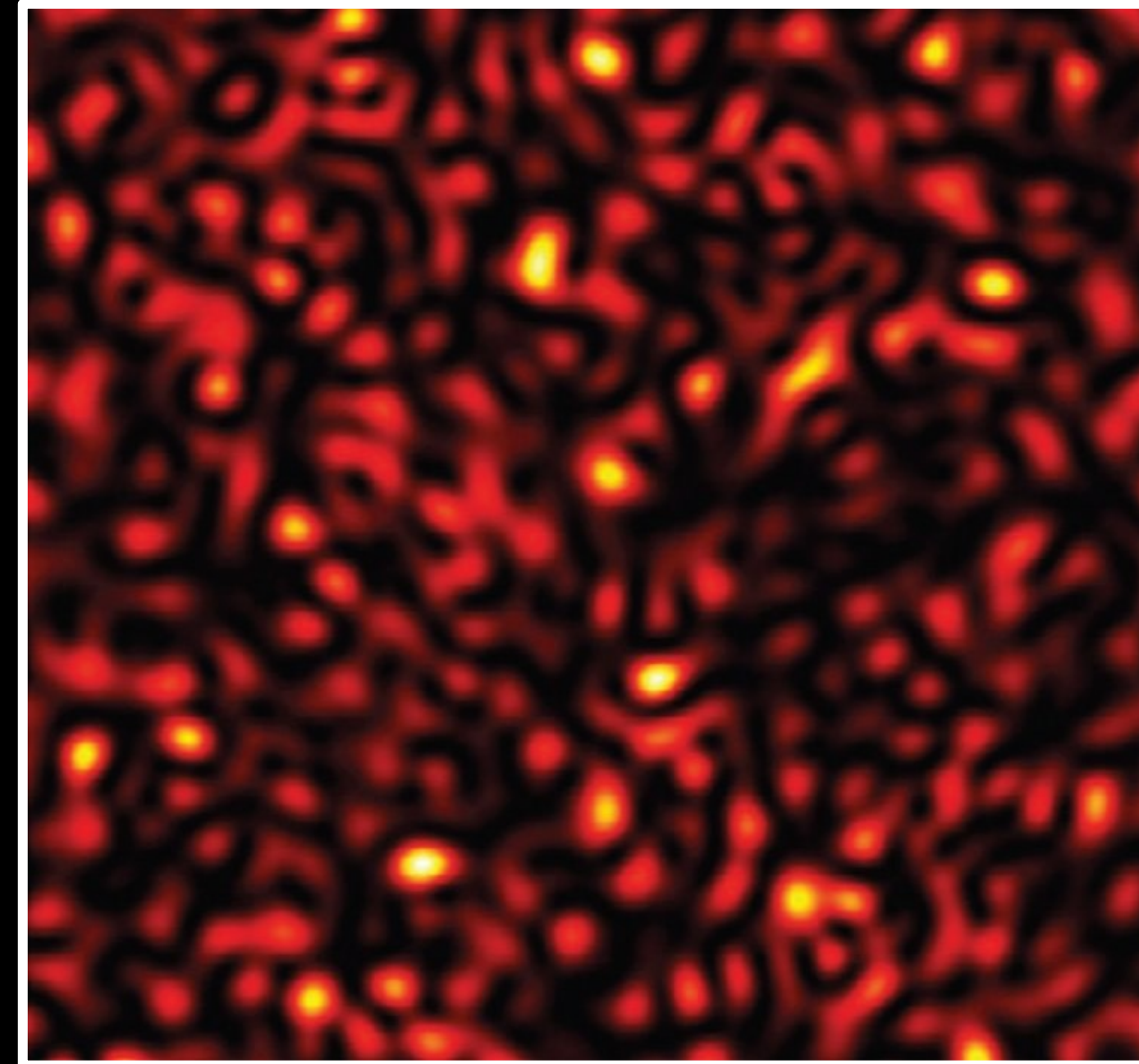
# A lot of research on non-line-of-sight imaging

intensity imaging



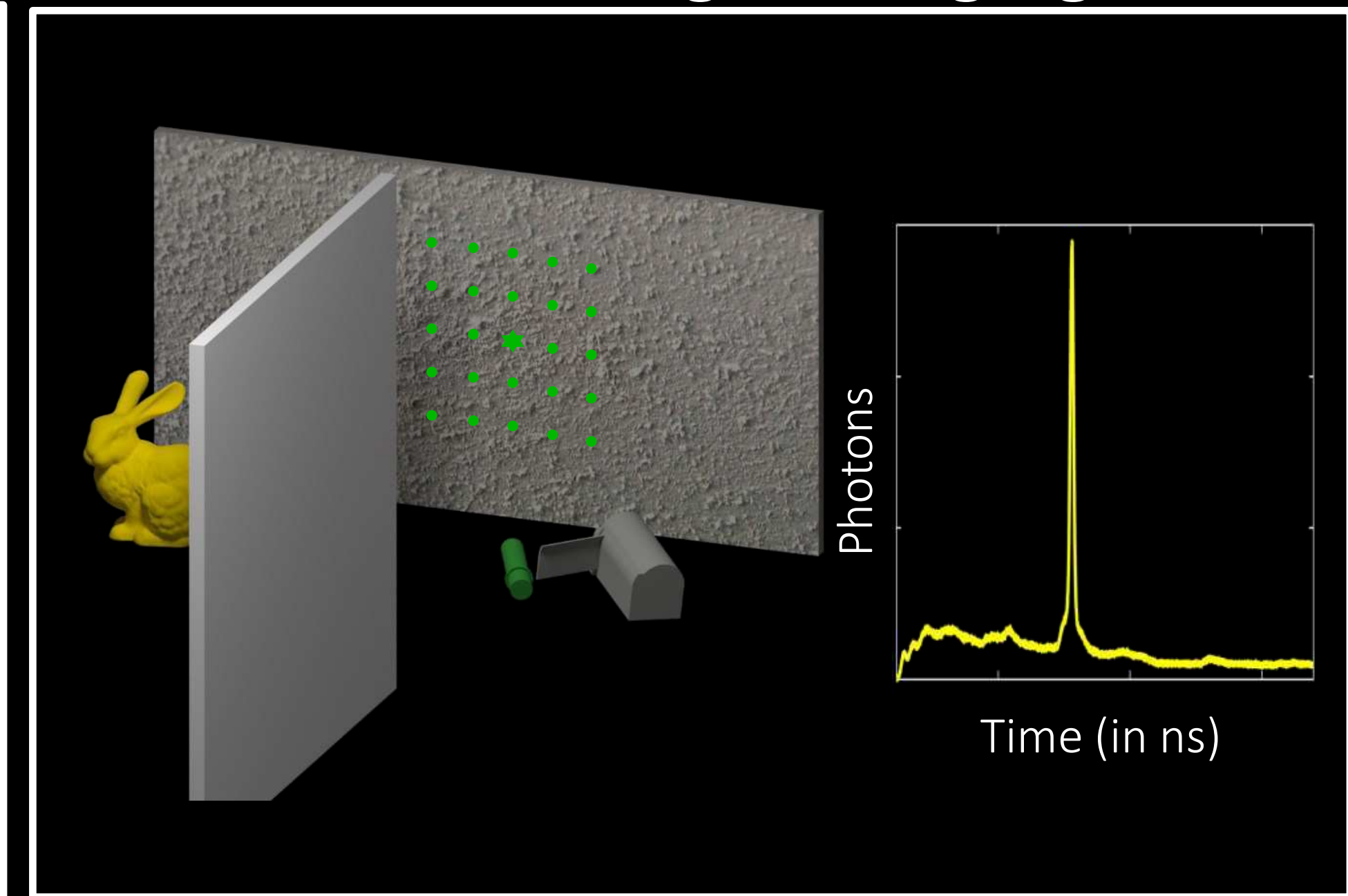
Bouman et al., ICCV 2017  
Saunders et al., Nature 2019  
Saunders et al., COSI 2019  
Maeda et al., ICCP 2019  
Lin et al., COSI 2020  
Sharma, ICCV 2021

coherence imaging



Katz et al., Nat. Photonics 2014  
Lei et al., CVPR, 2019  
Boger-Lombard, Nat. Com. 2019  
Metzler et al., Optica 2020  
Willomitzer et al., Nat. Com. 2021  
Chen et al. SPIE 2022

time-of-flight imaging



Velten et al., Nat. comm. 2012  
Toole et al., Nature 2018  
Liu et al., Nature 2019, Nat. comm. 2020  
Rapp et al. Nat. comm. 2020  
Xin et al., CVPR 2019  
Nam et al., Nat. comm. 2021



# How do we image in line-of-sight?

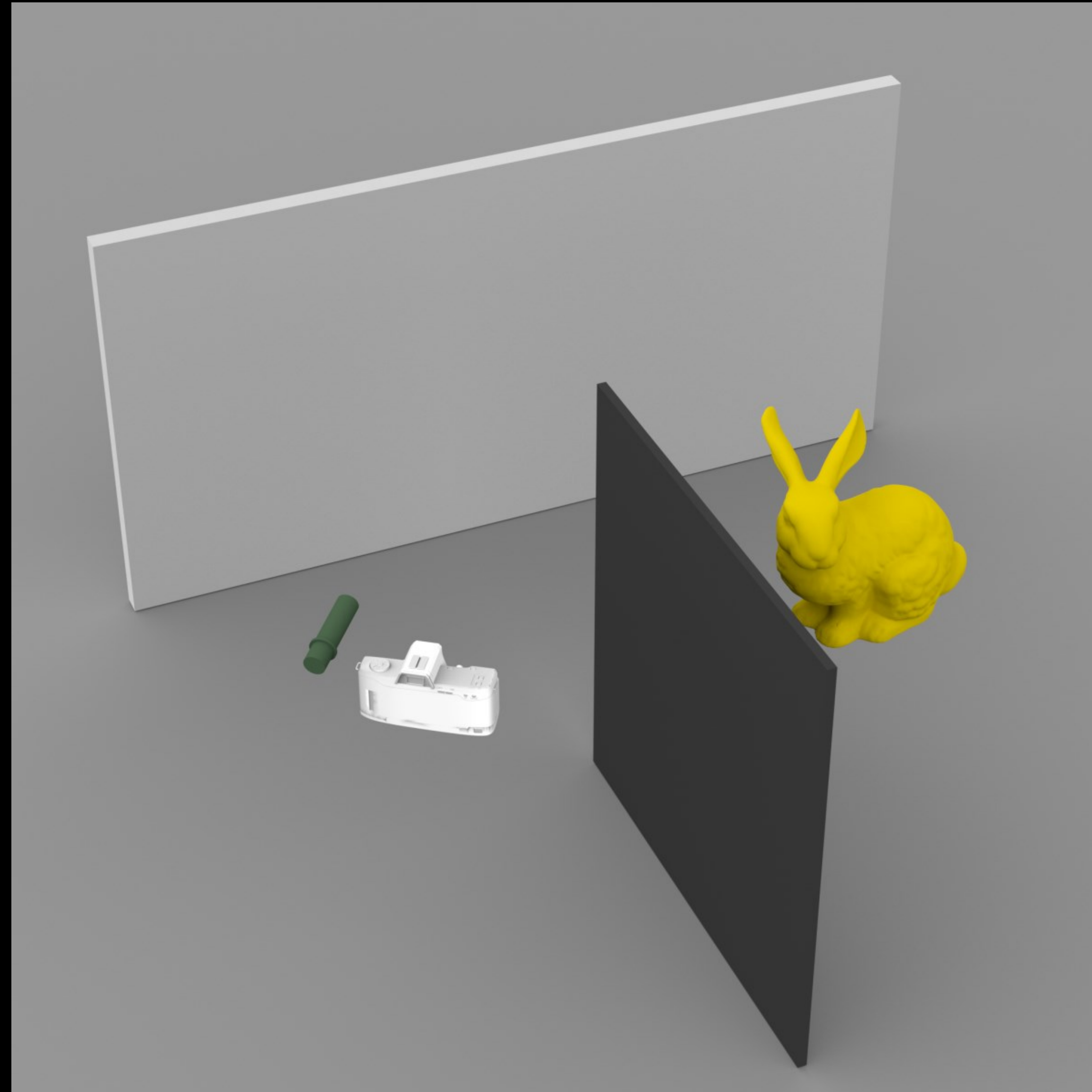


# How do we image in line-of-sight?

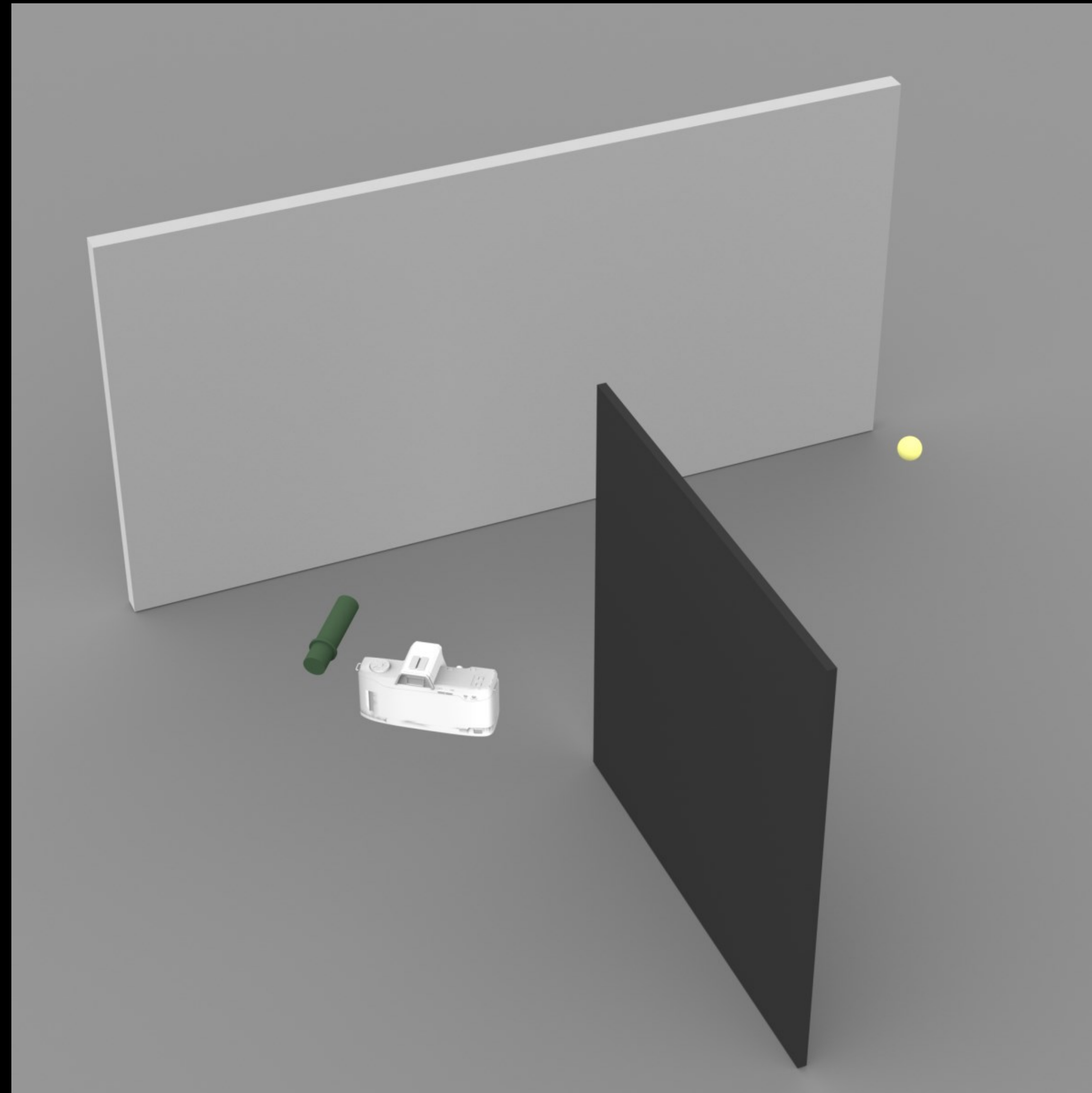




# How do we image in non-line-of-sight?

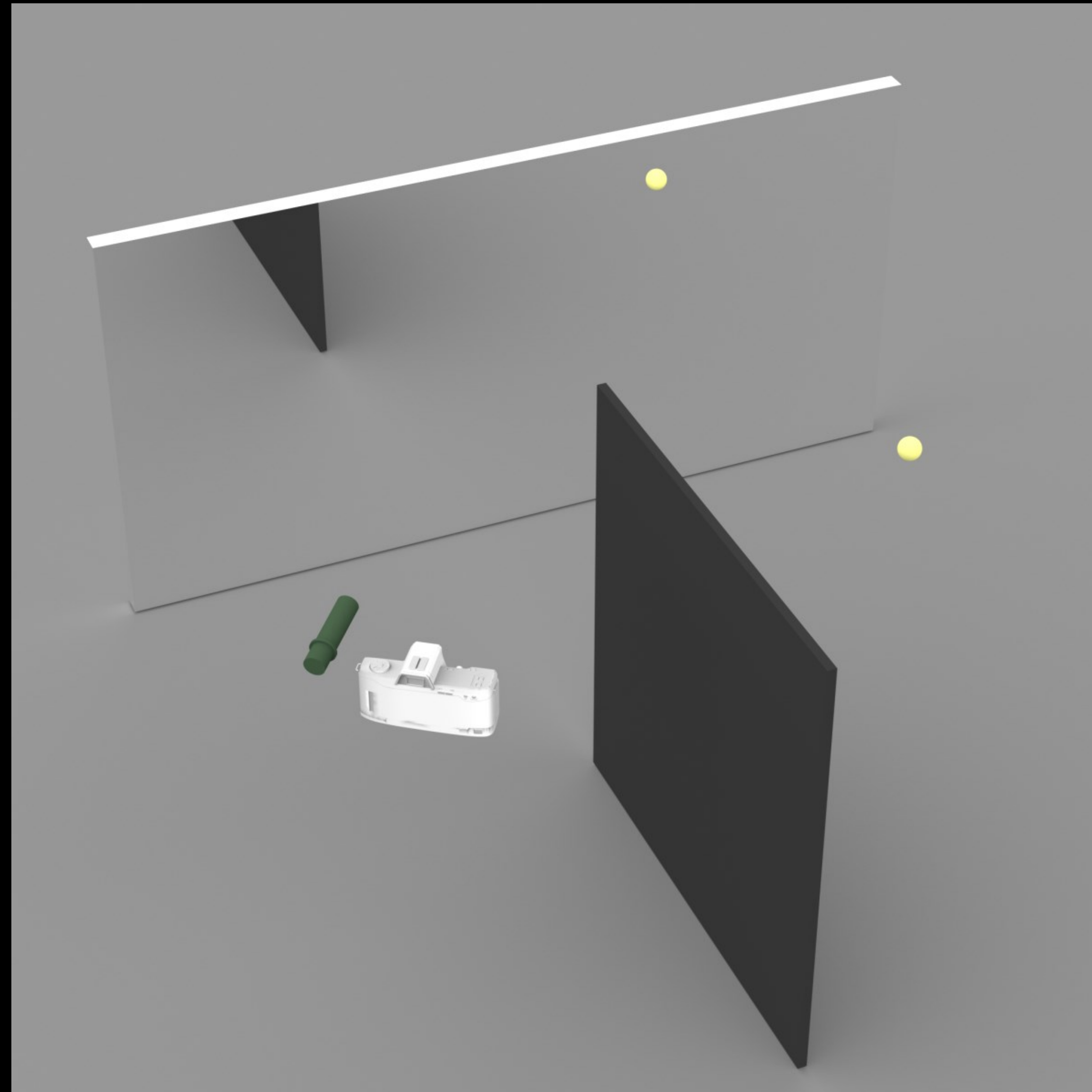


# How do we focus on a voxel?

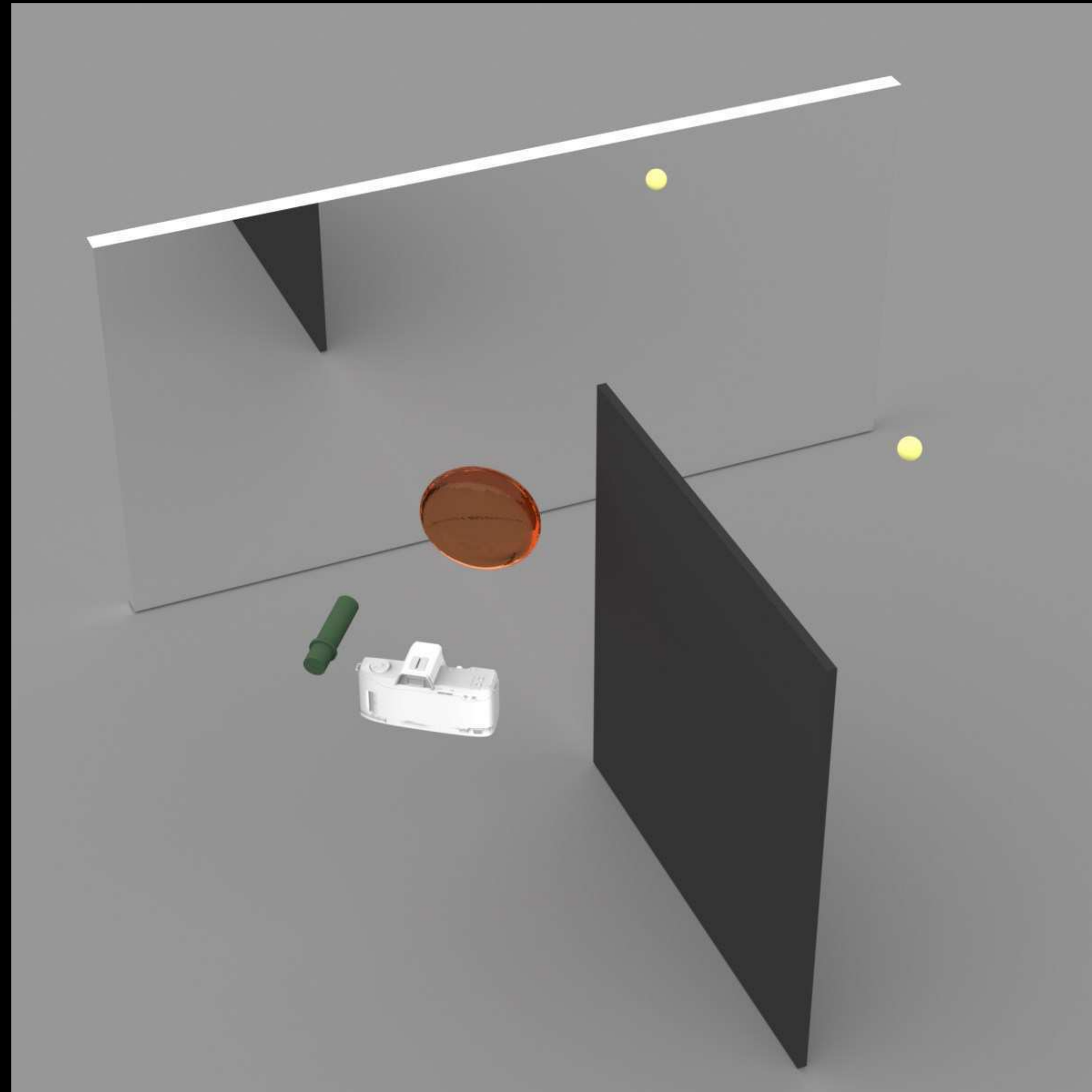




# How do we focus on a voxel?

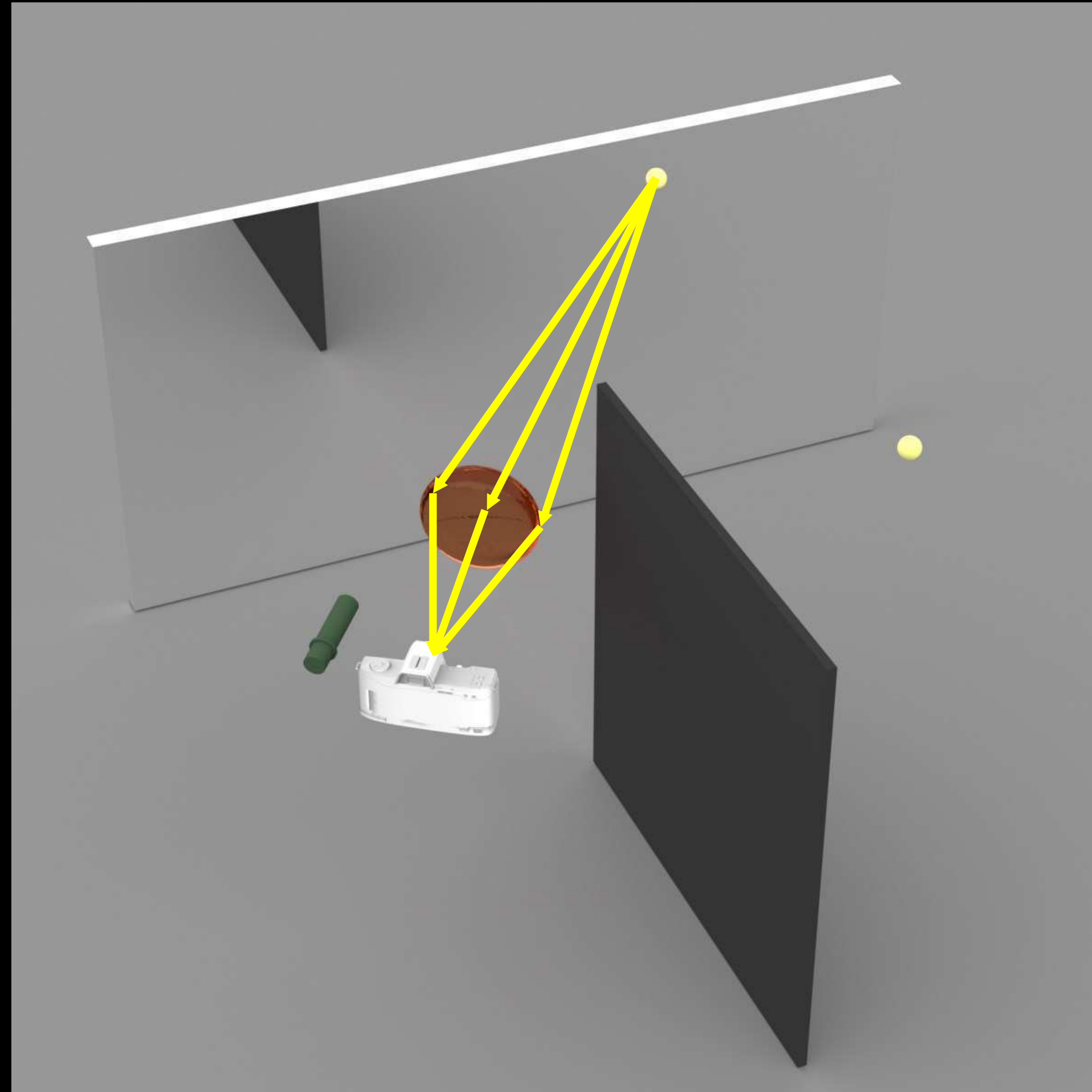


# How do we focus on a voxel?

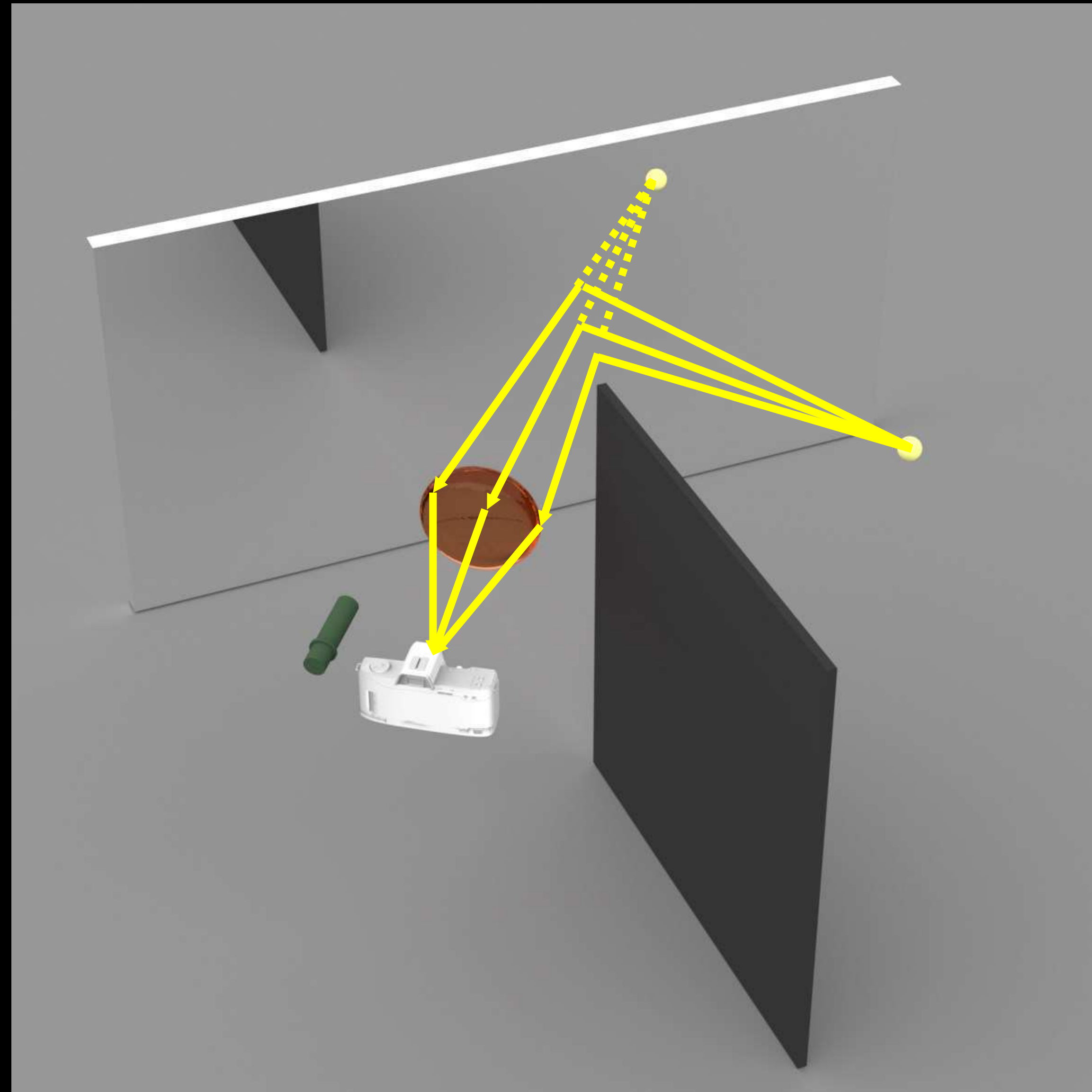




# How do we focus on a voxel?

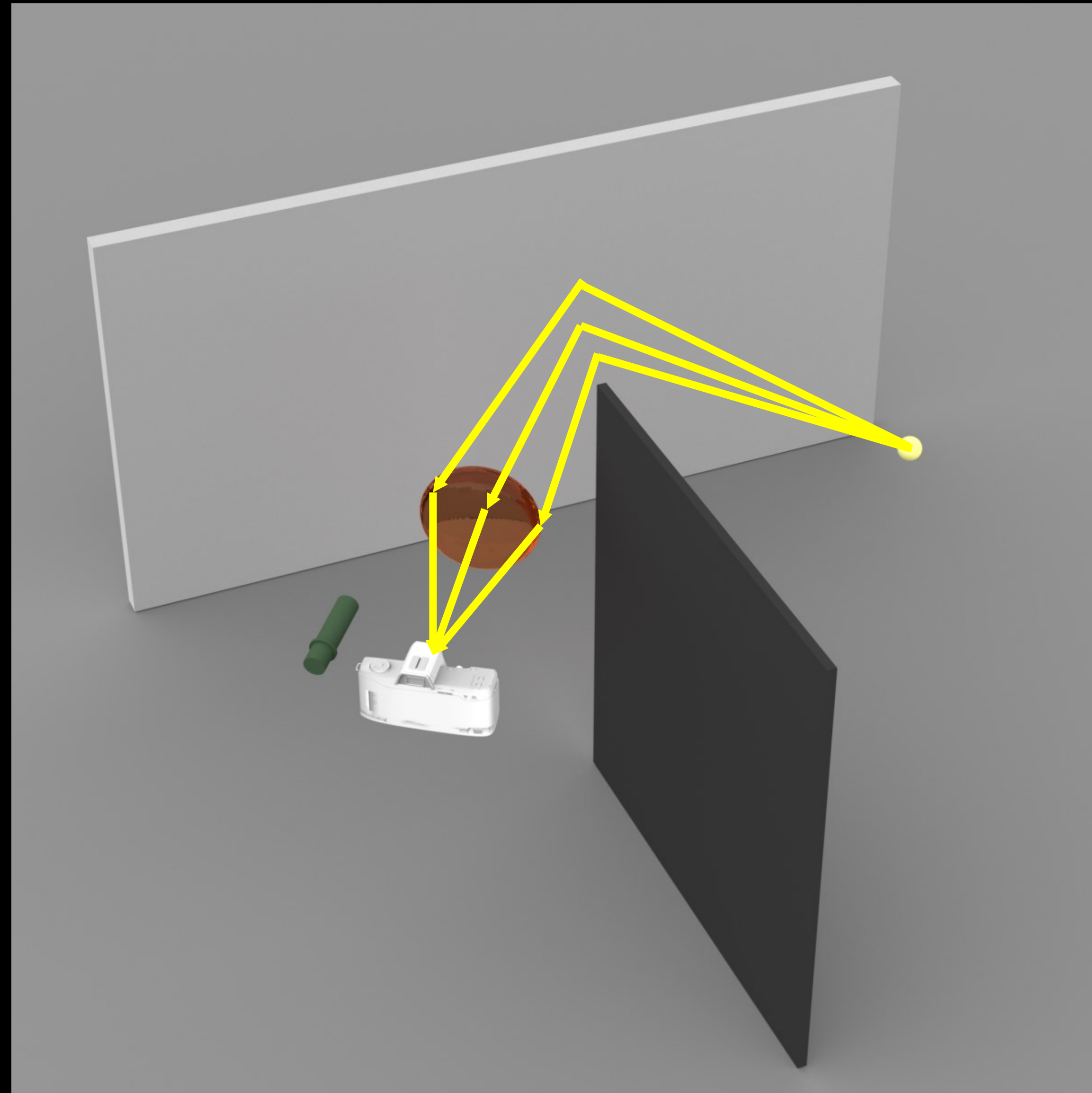


# How do we focus on a voxel?



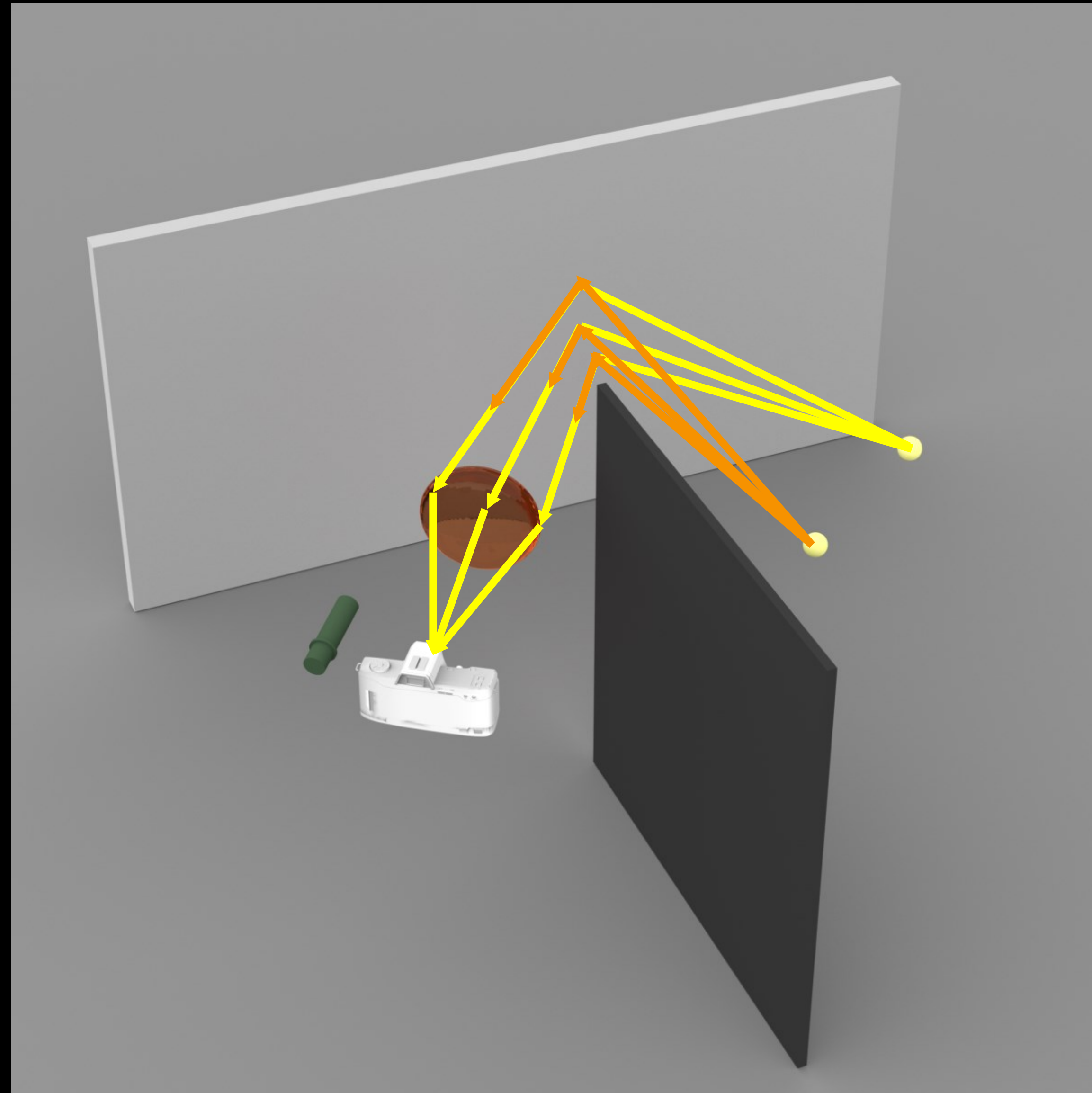


# How do we focus on a voxel?



# How do we focus on a voxel?

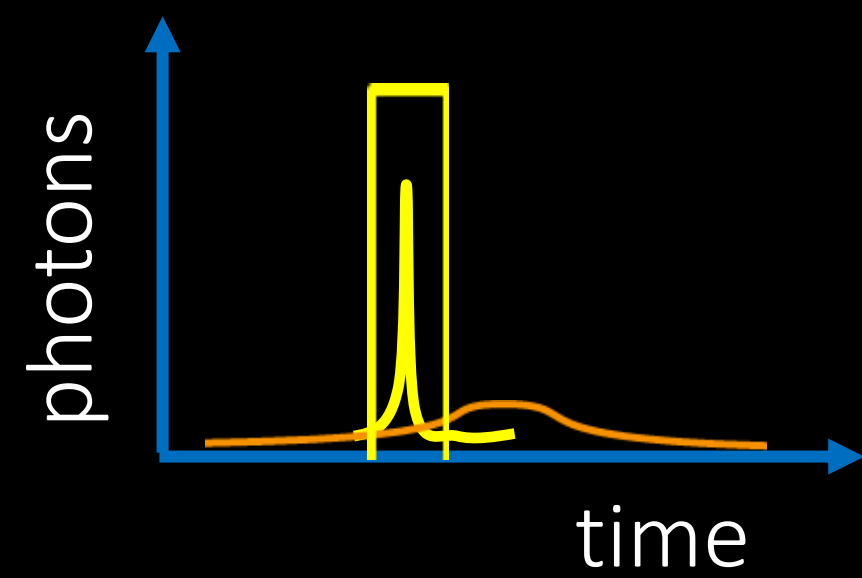
challenge:  
out of focus voxels



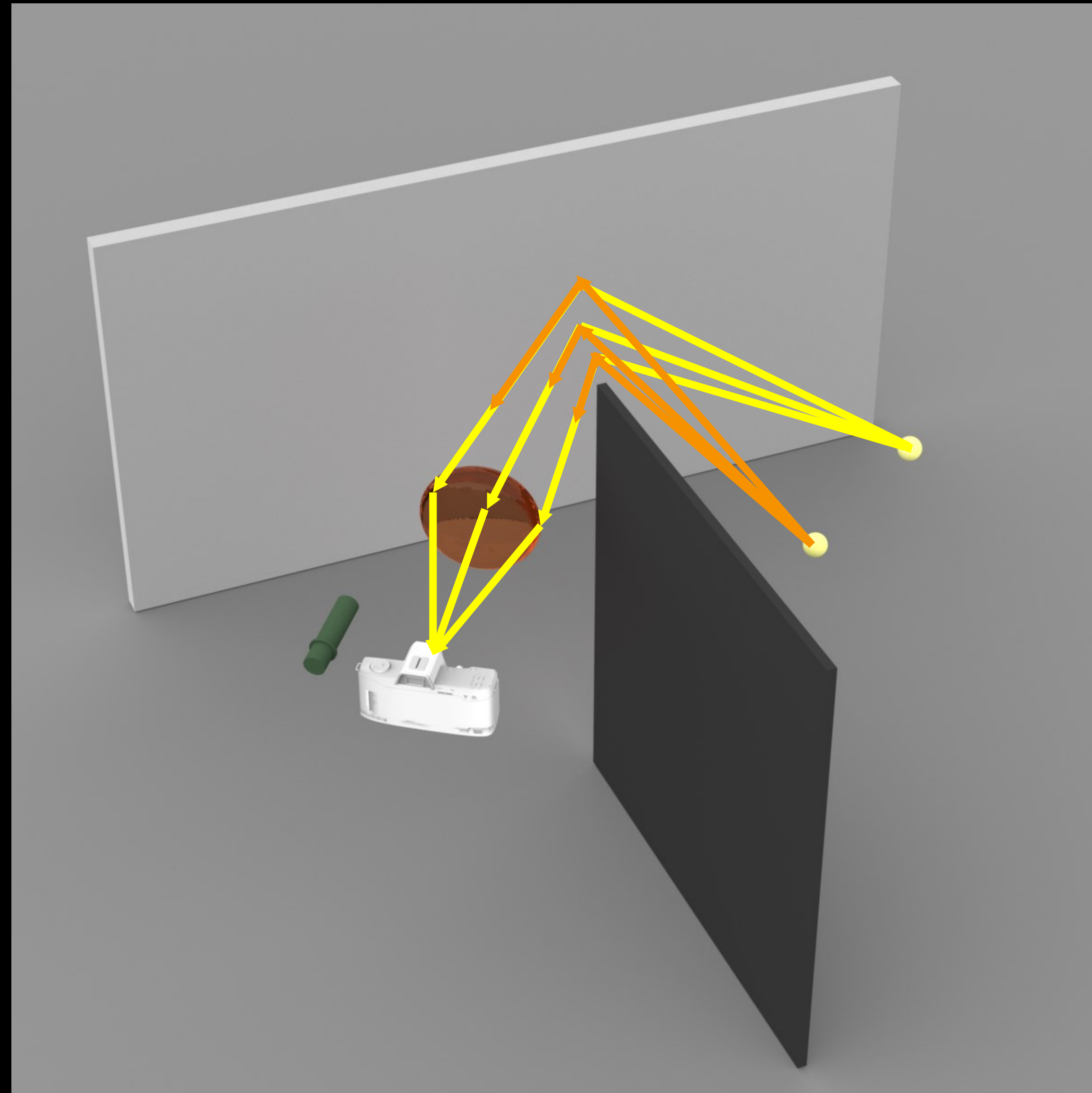


# How do we focus on a voxel?

challenge:  
out of focus voxels

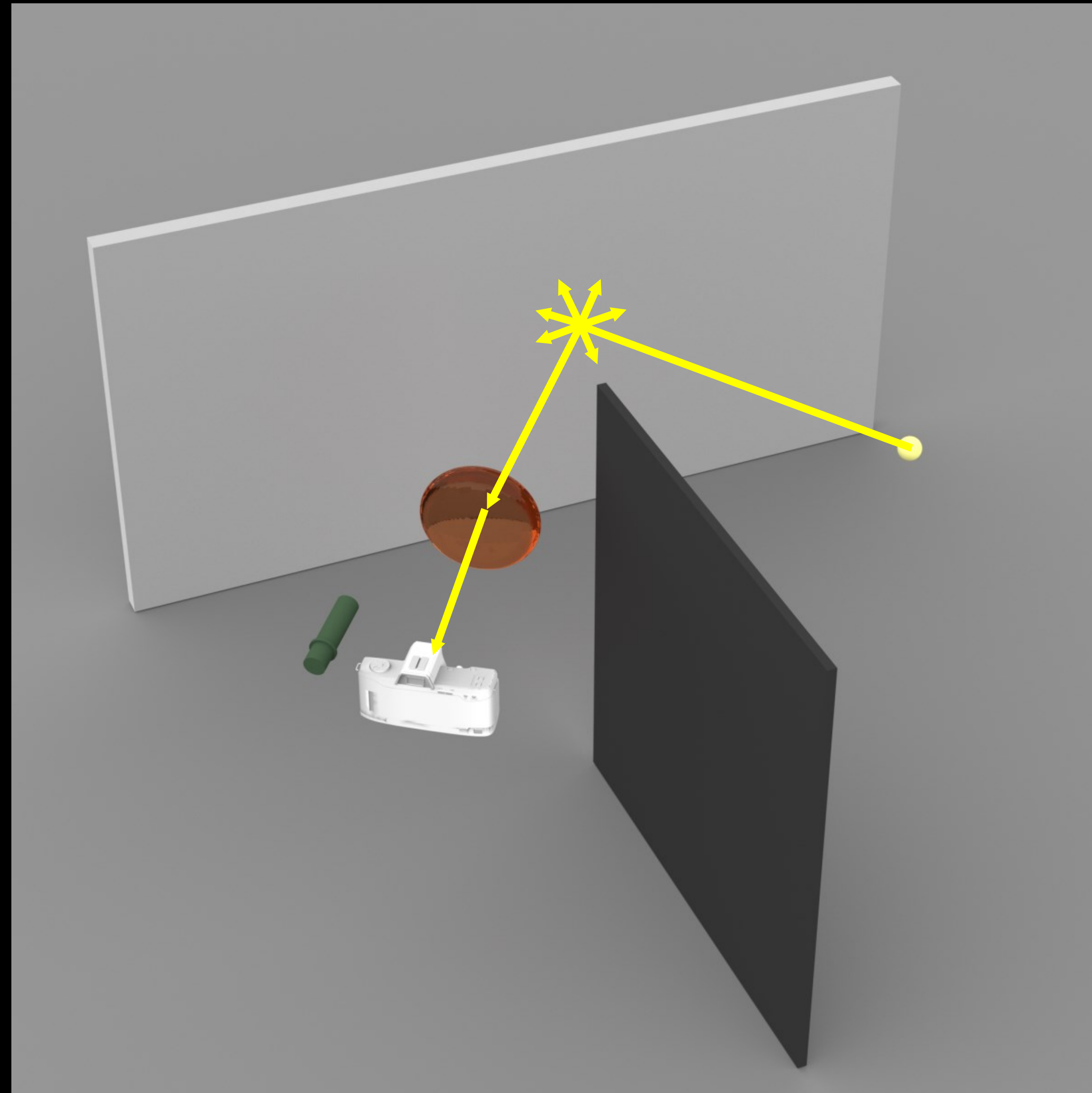


solution:  
time-gate photons



# How do we focus on a voxel?

challenge:  
non-specular photons

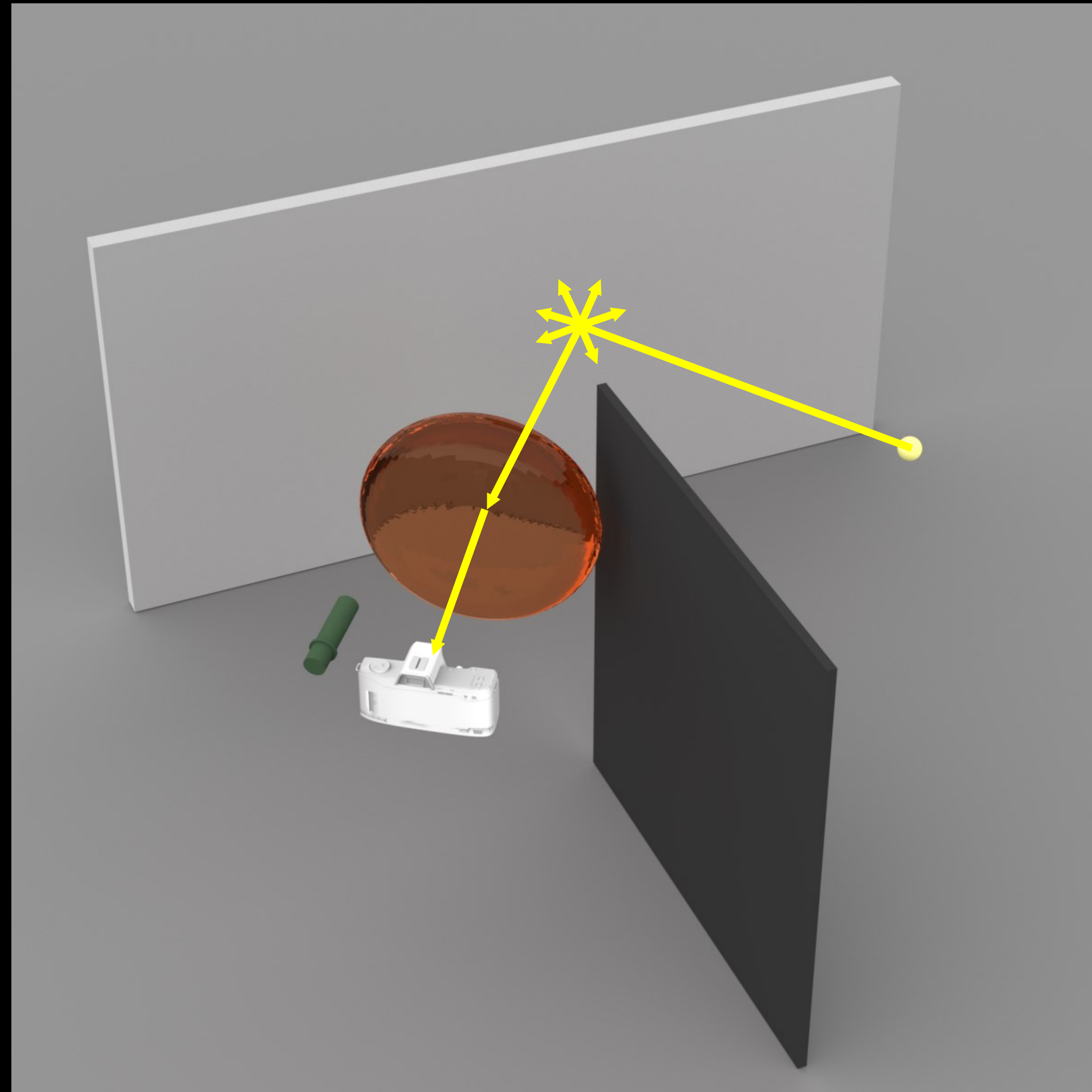




# How do we focus on a voxel?

challenge:  
non-specular photons

solution:  
use a large lens?  
expensive !!



# How do we focus on a voxel?

challenge:

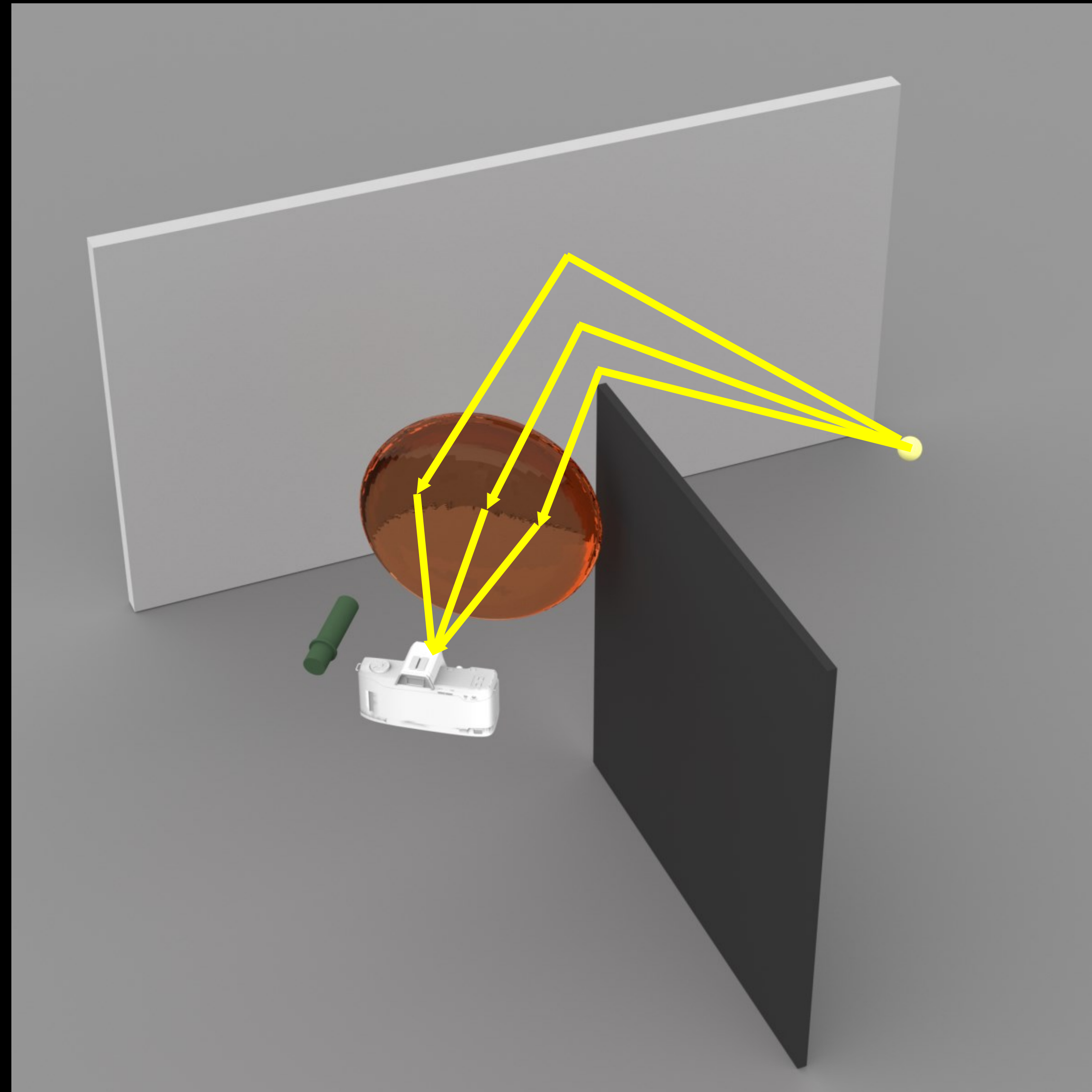
non-specular photons

solution:

use a large lens?

expensive !!

what does a lens do?  
delays rays such that  
they reach detector at  
same time instant

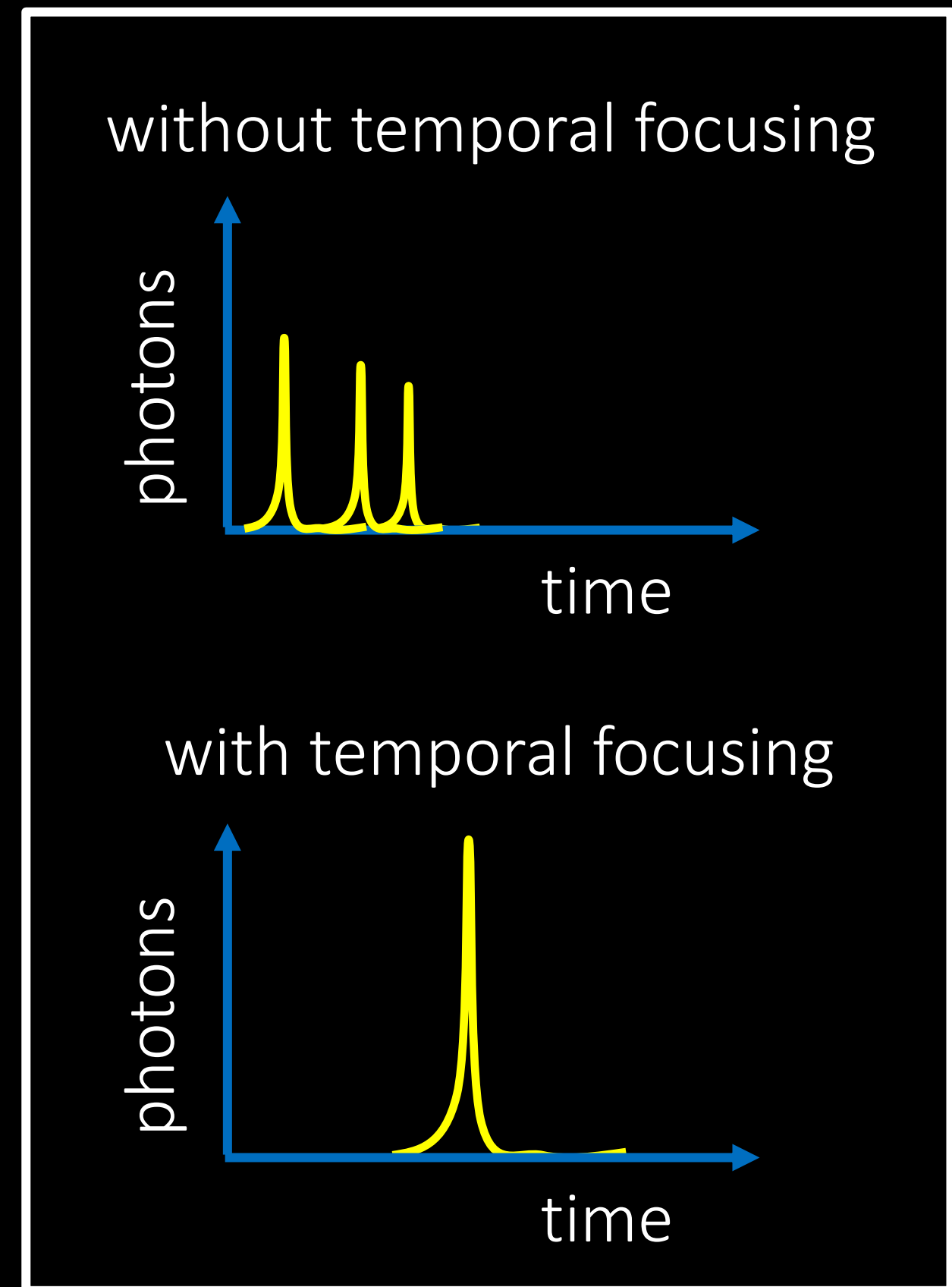
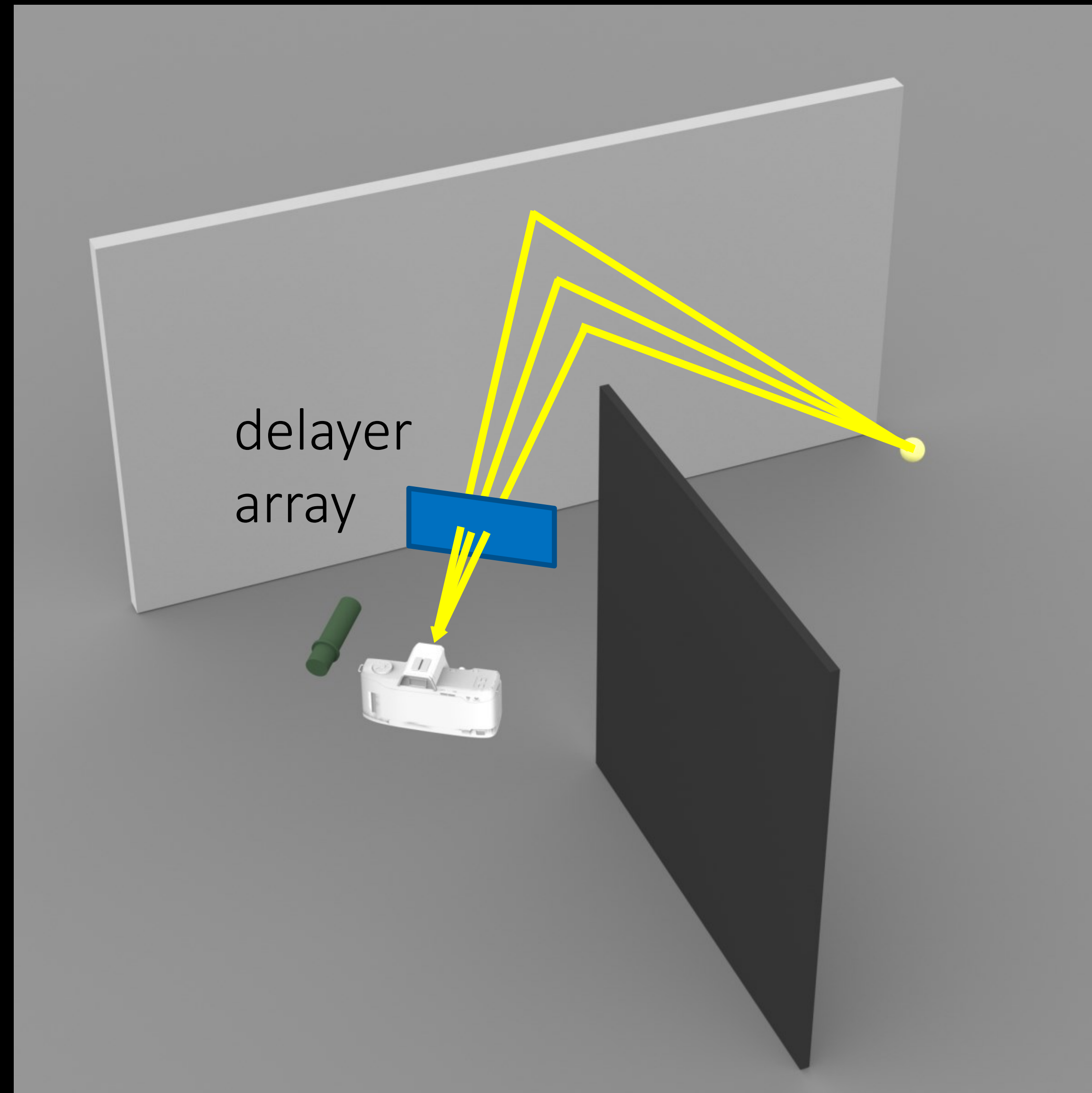




# Temporal focusing: imitate the lens

challenge:  
non-specular photons

solution:  
use a large lens?  
expensive !!  
temporal focusing:  
imitate large lens



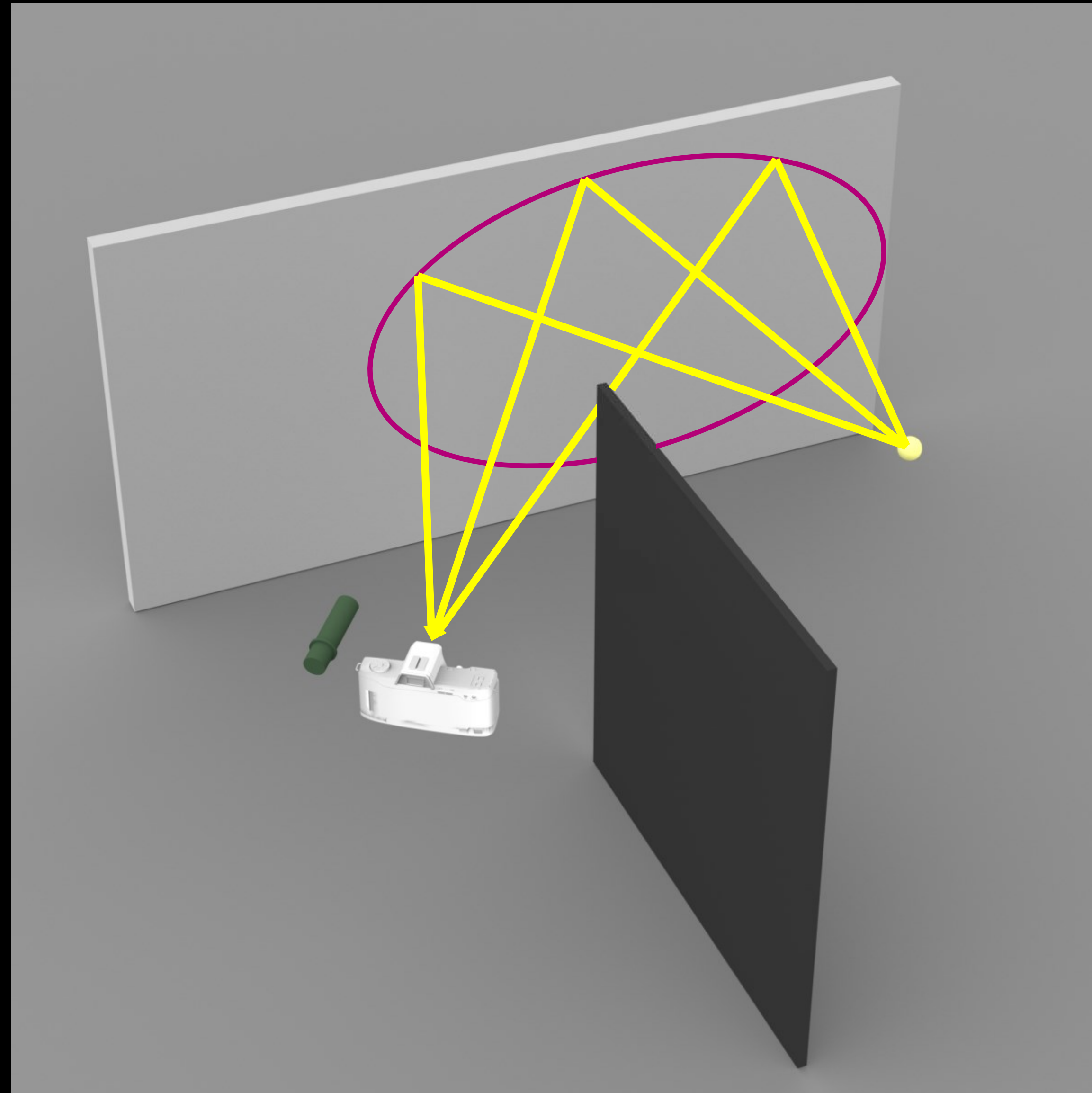
# Temporal focusing: imitate the lens

challenge:  
non-specular photons

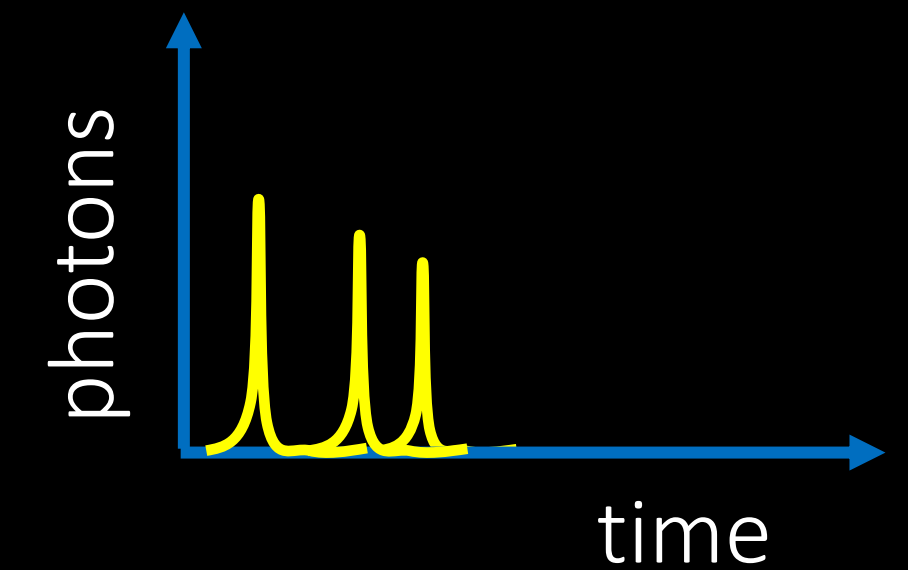
solution:

use a large lens?  
expensive !!

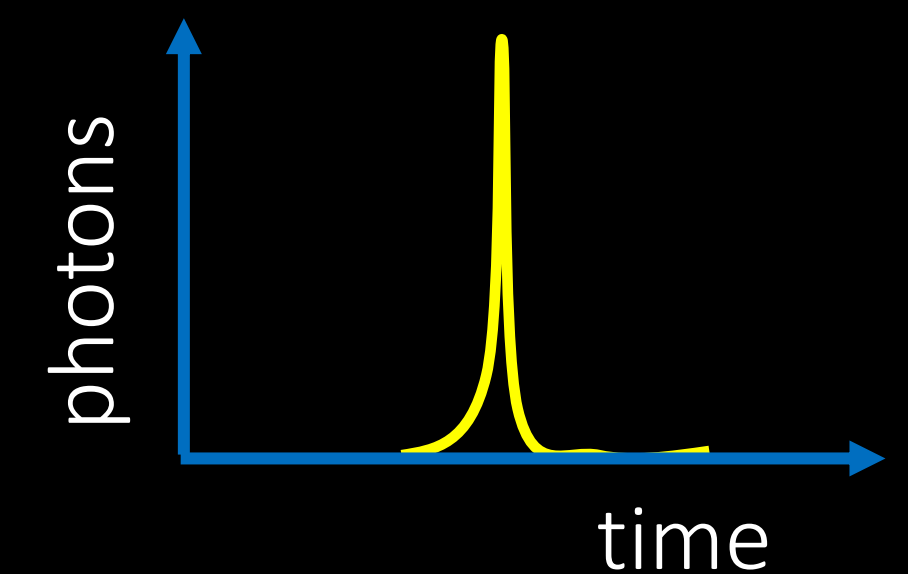
temporal focusing:  
imitate large lens



without temporal focusing



with temporal focusing





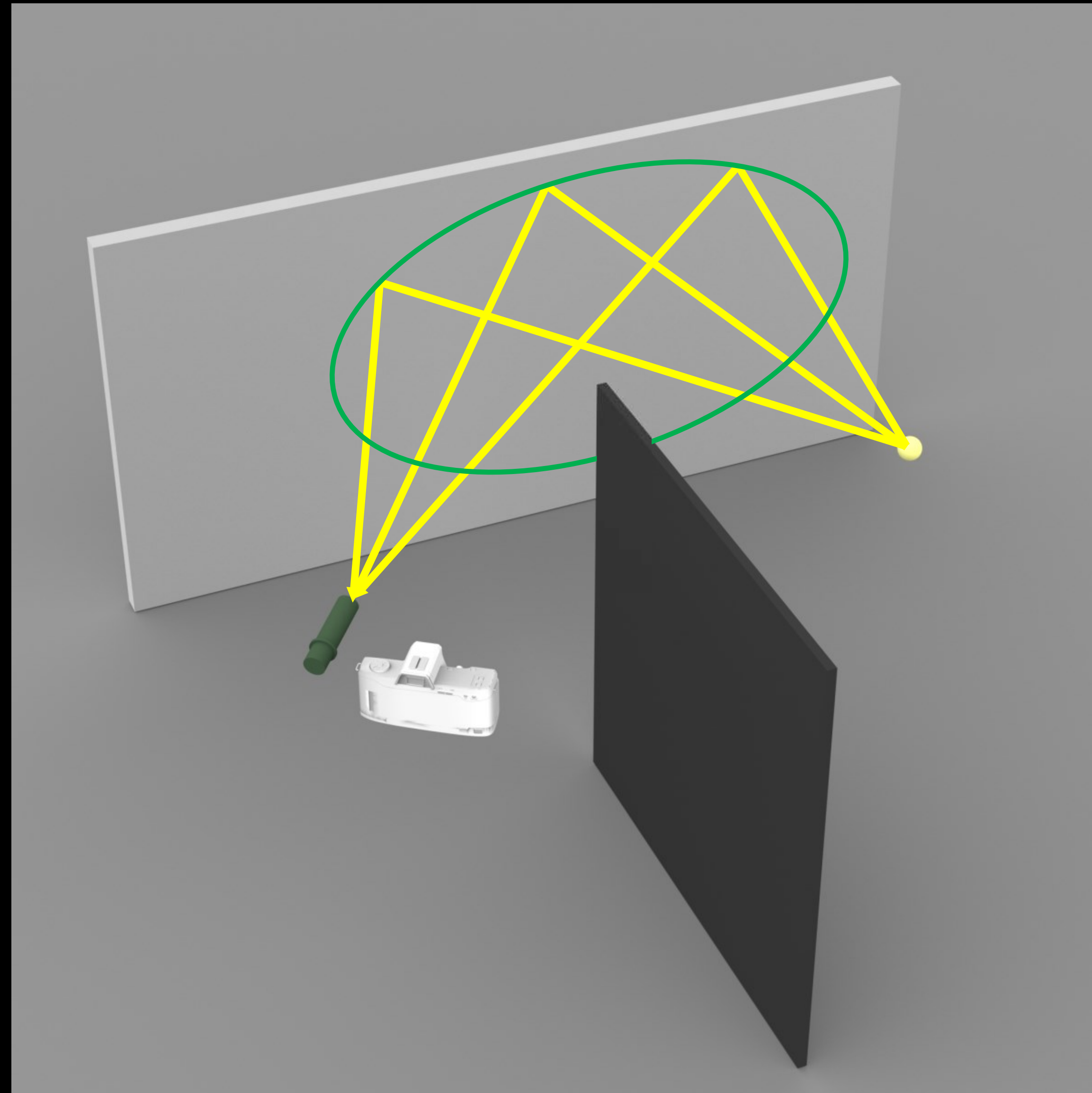
# Temporal focusing: Illumination should also be an ellipse

challenge:  
non-specular photons

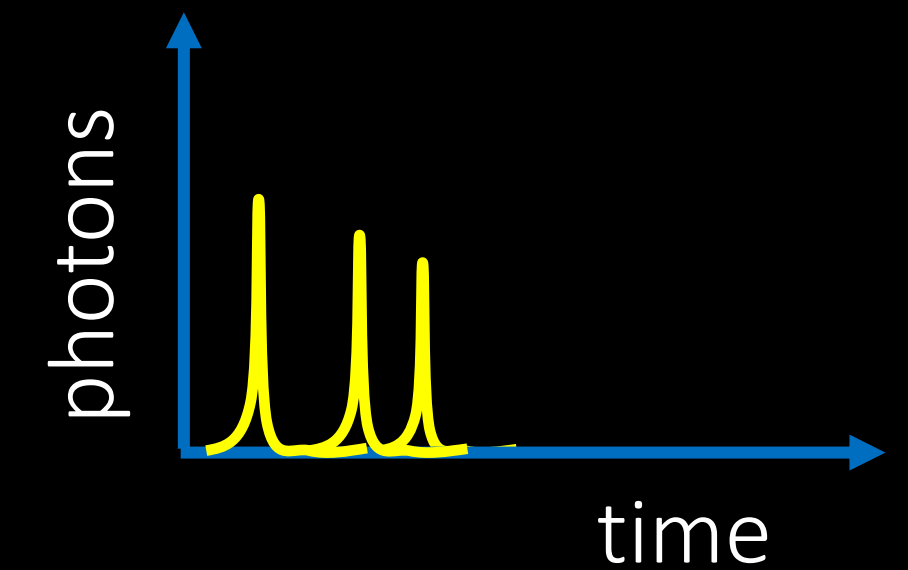
solution:

use a large lens?  
expensive !!

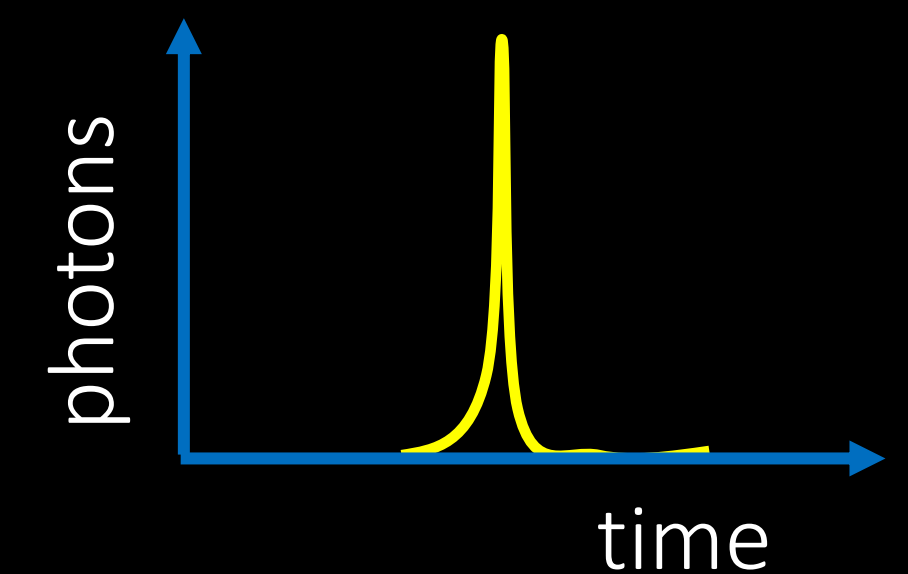
temporal focusing:  
imitate large lens



without temporal focusing

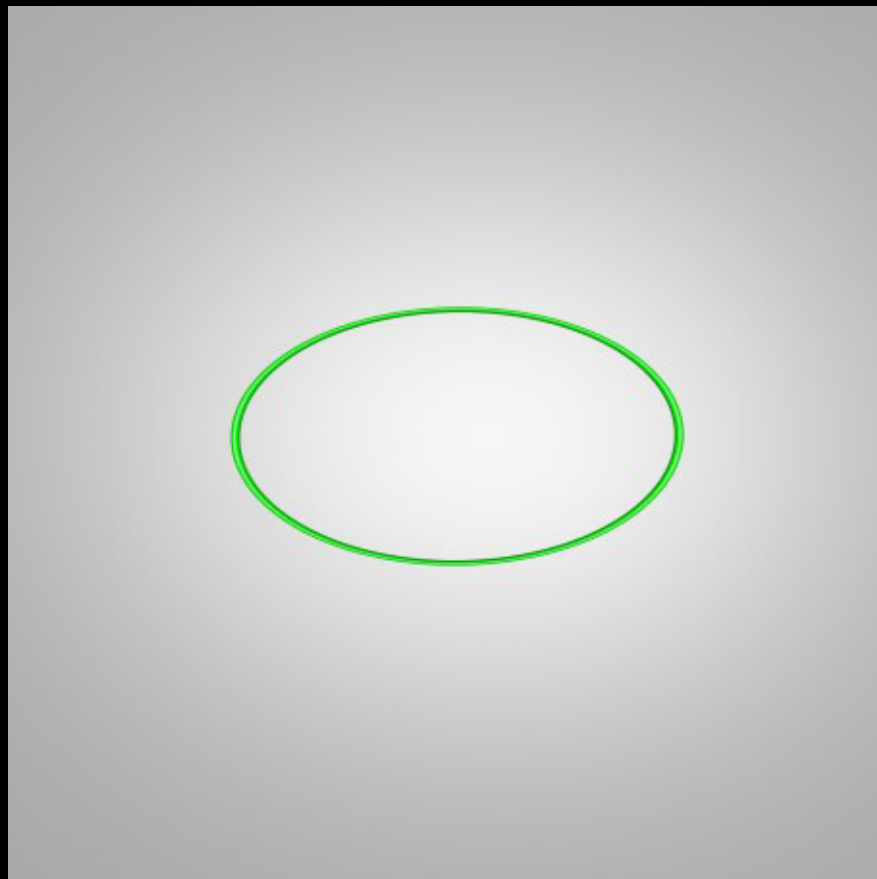
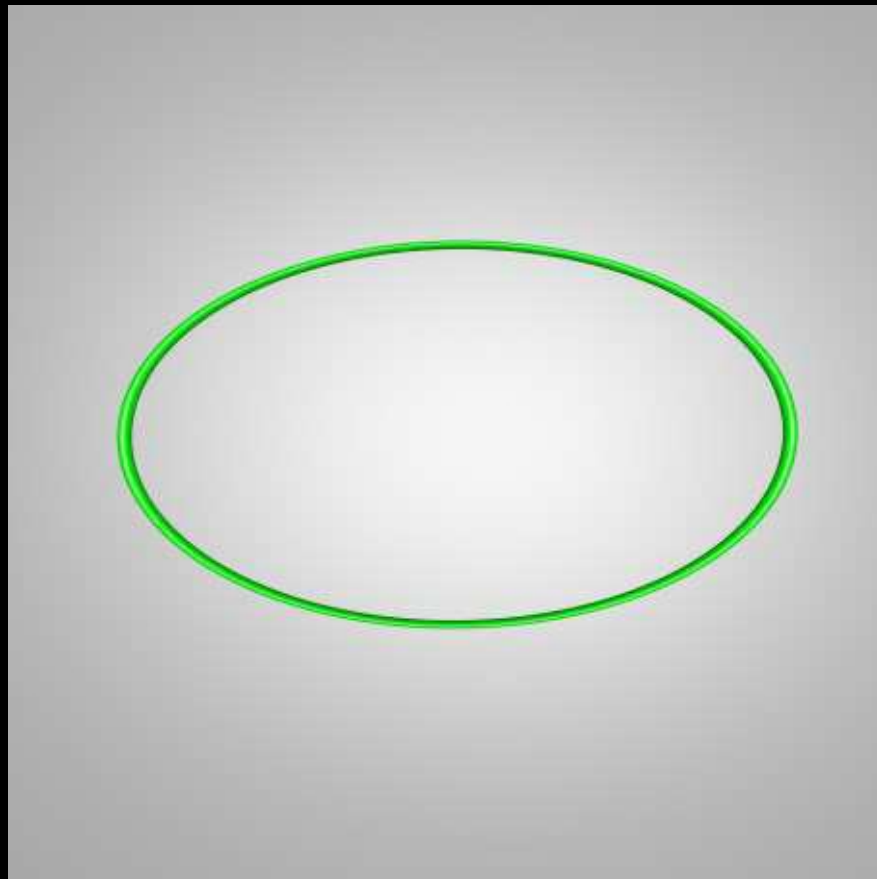


with temporal focusing

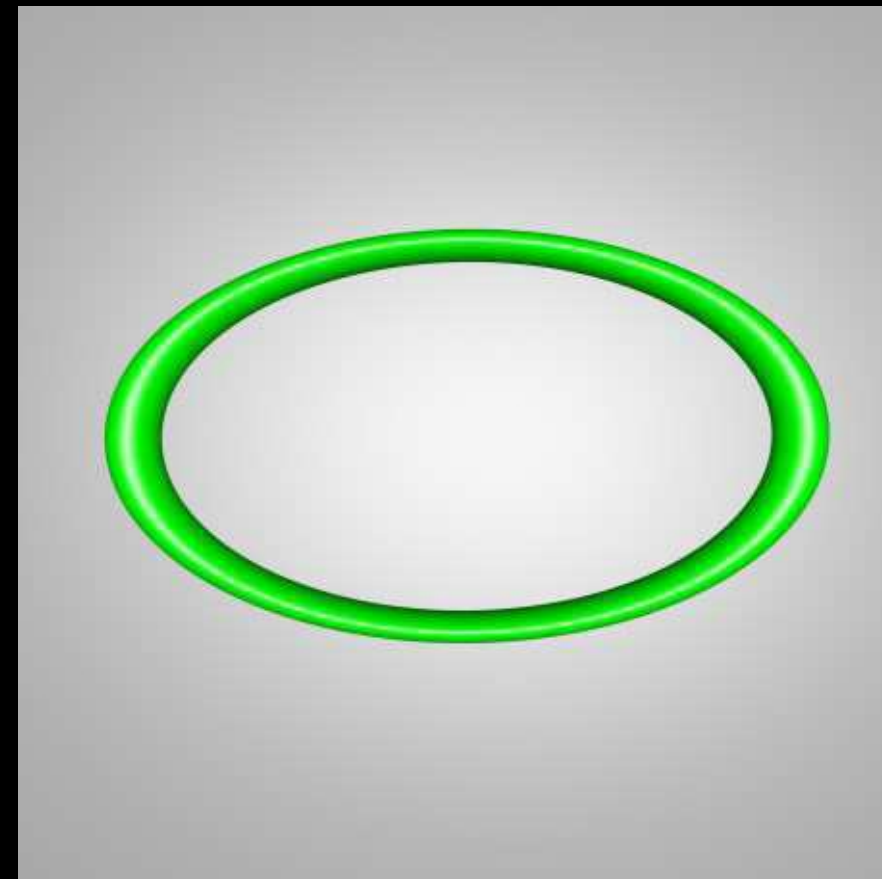
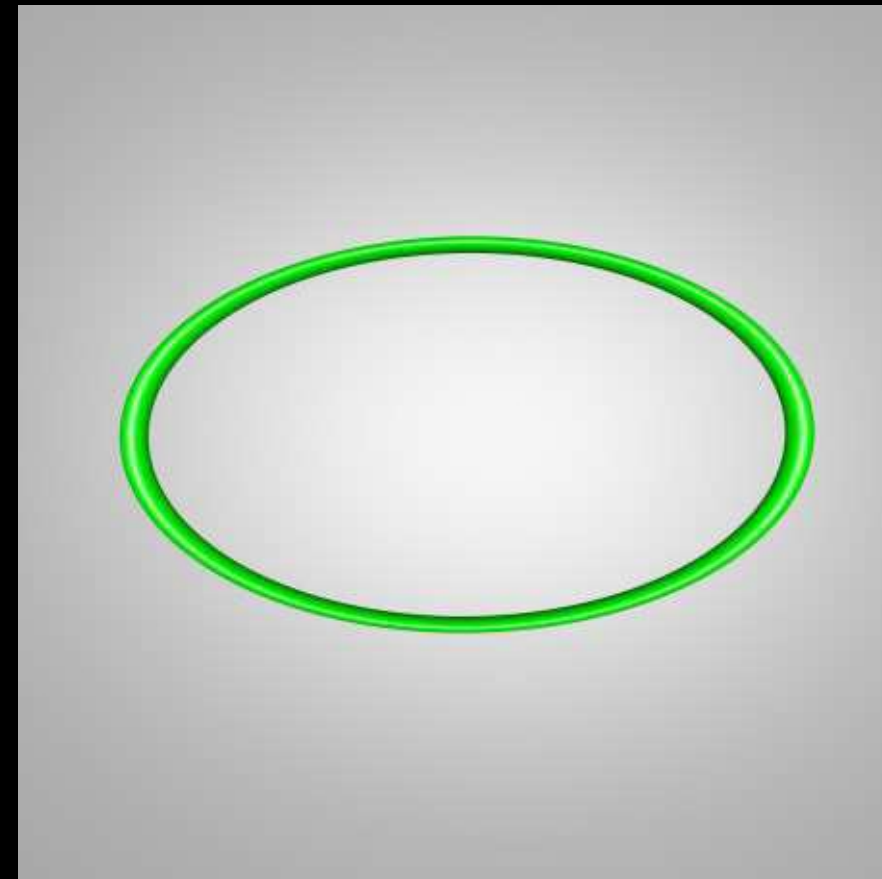


# Design choices for temporal focusing

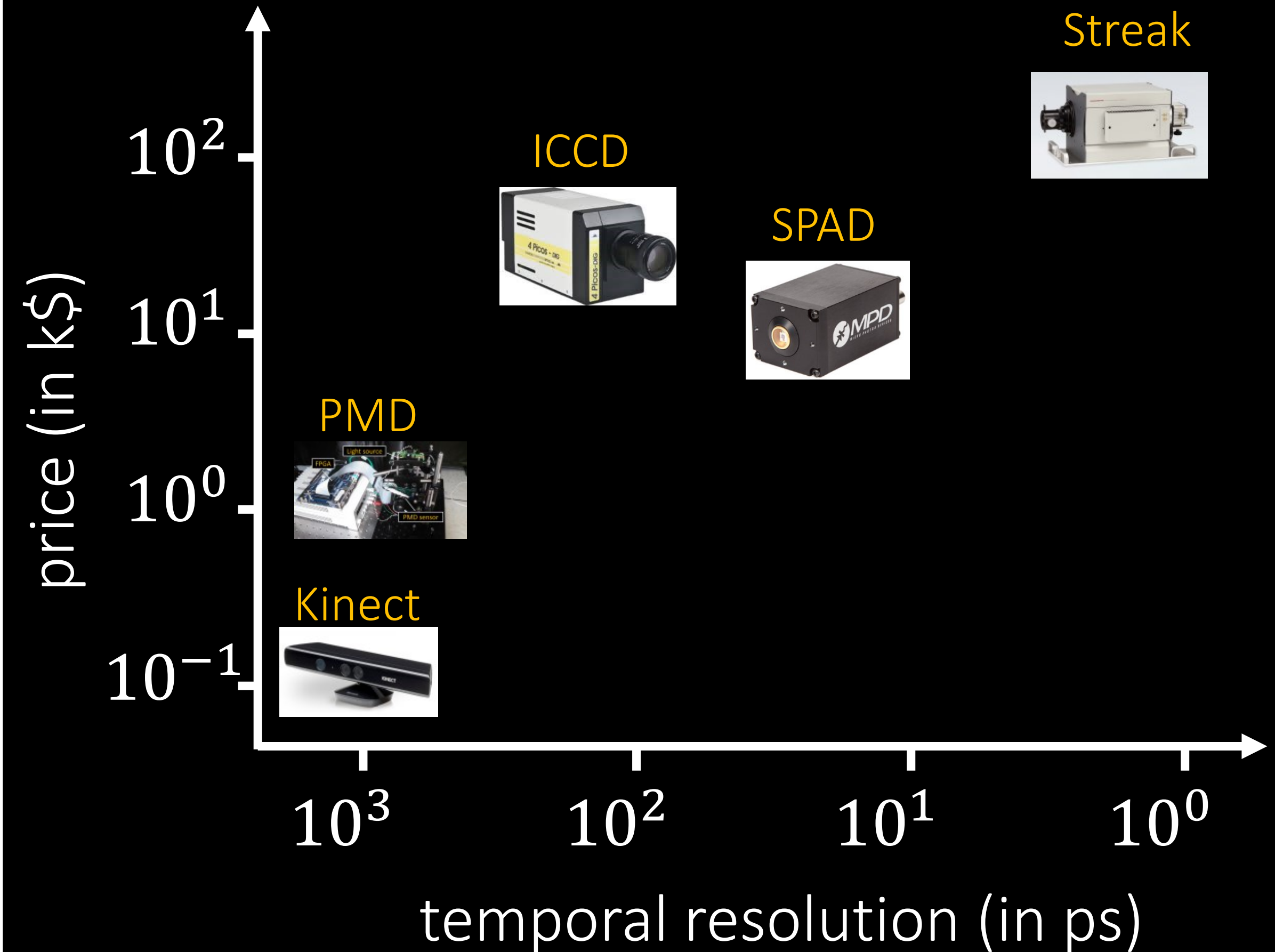
ellipse size



ellipse thickness

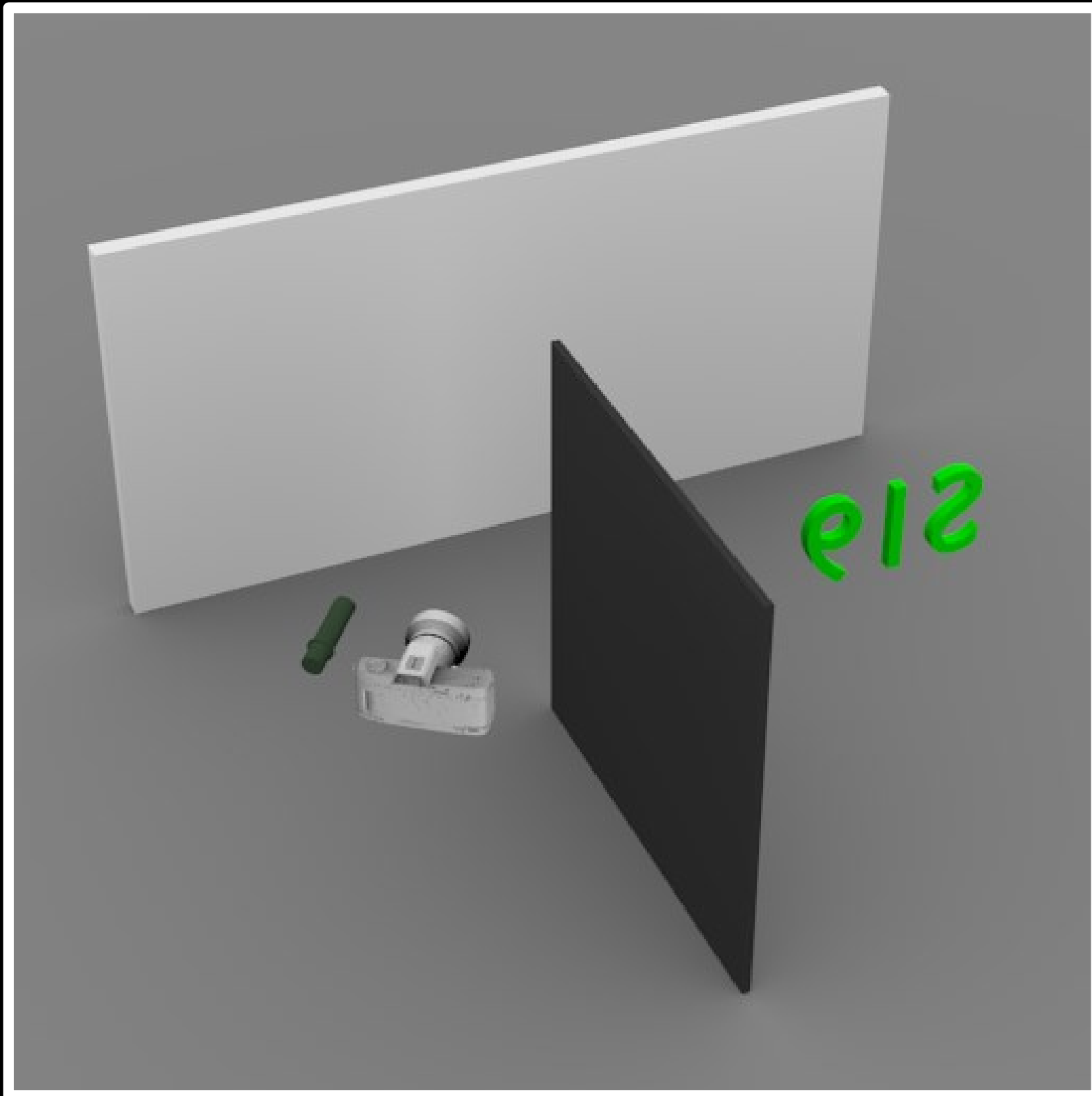


time-of-flight cameras



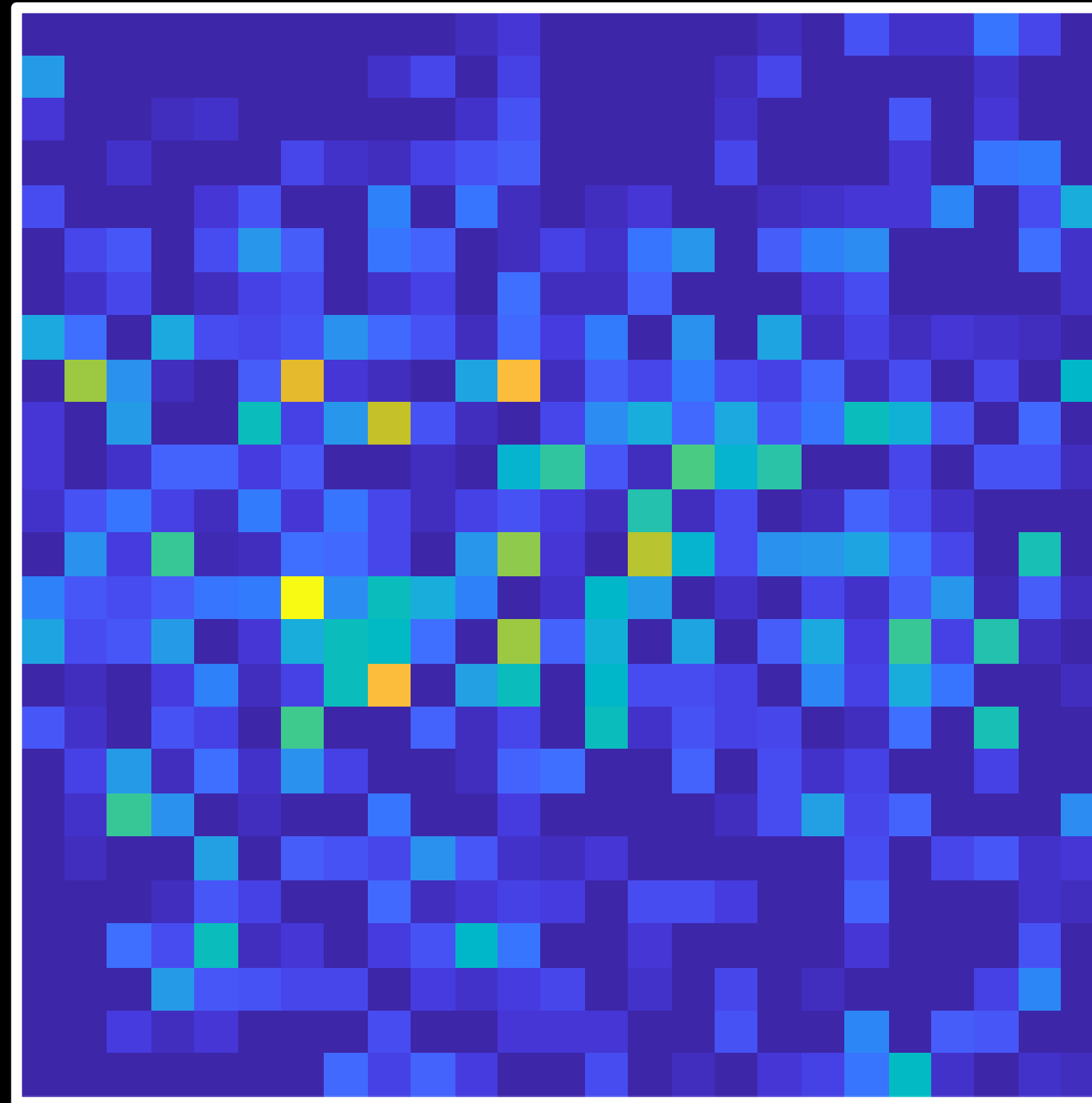


# Rendering non-line-of-sight imaging by temporal focusing

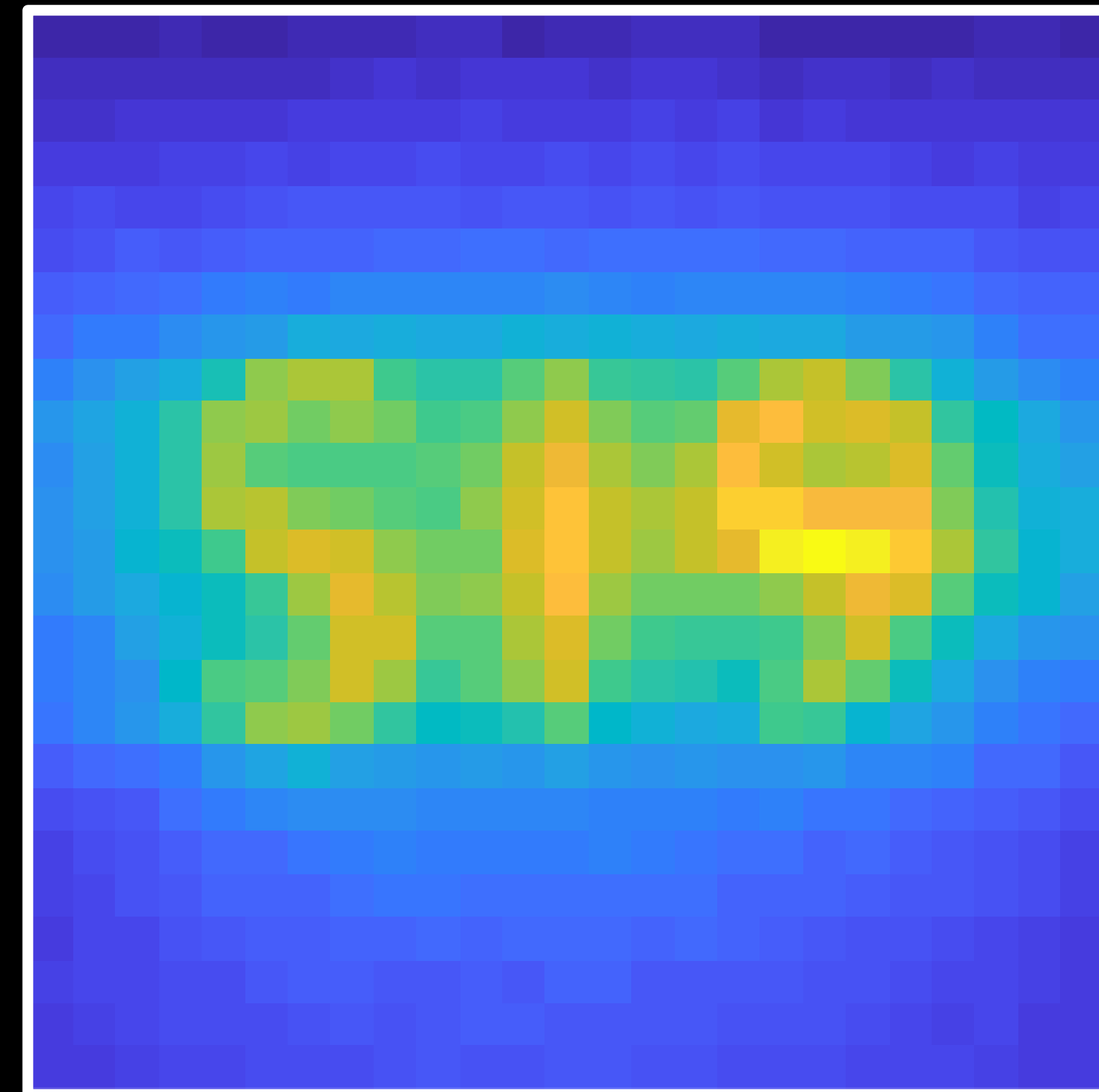


scene

Gate width: 4 ps (0.4% scene)  
Rendering time: 3 hr



standard BDPT



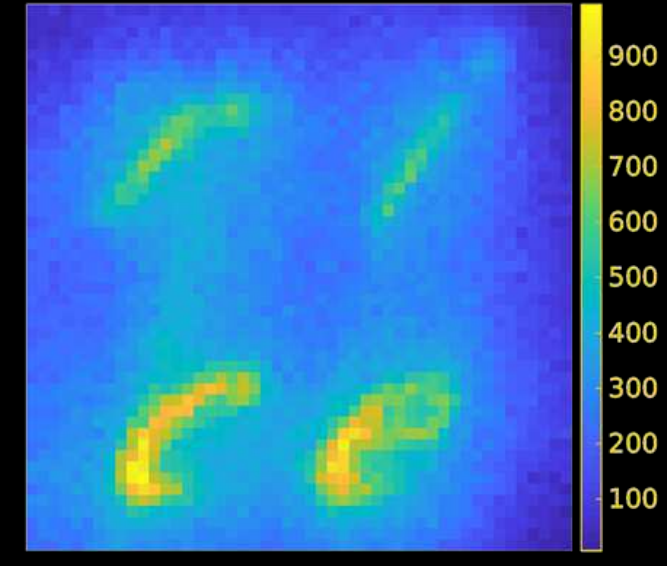
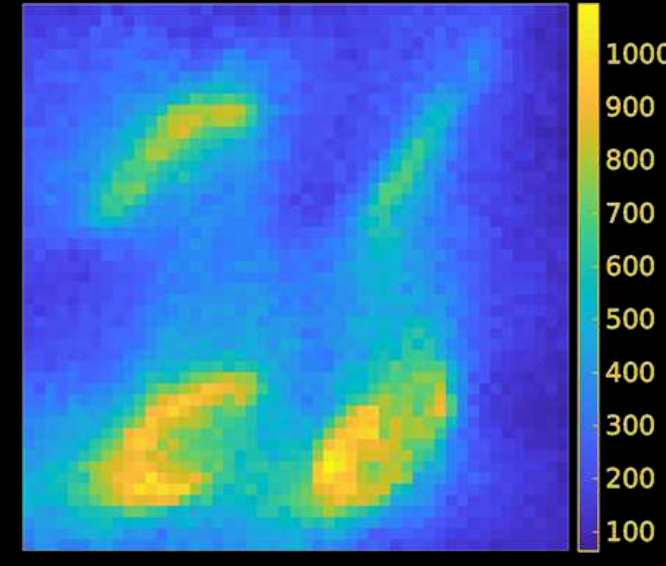
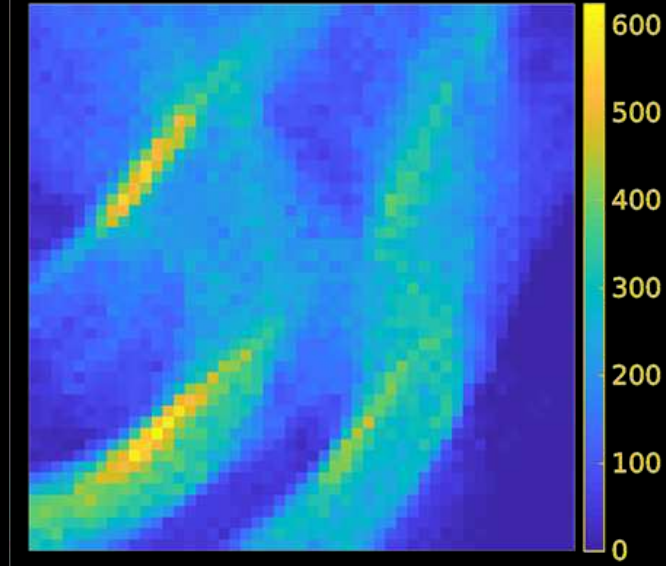
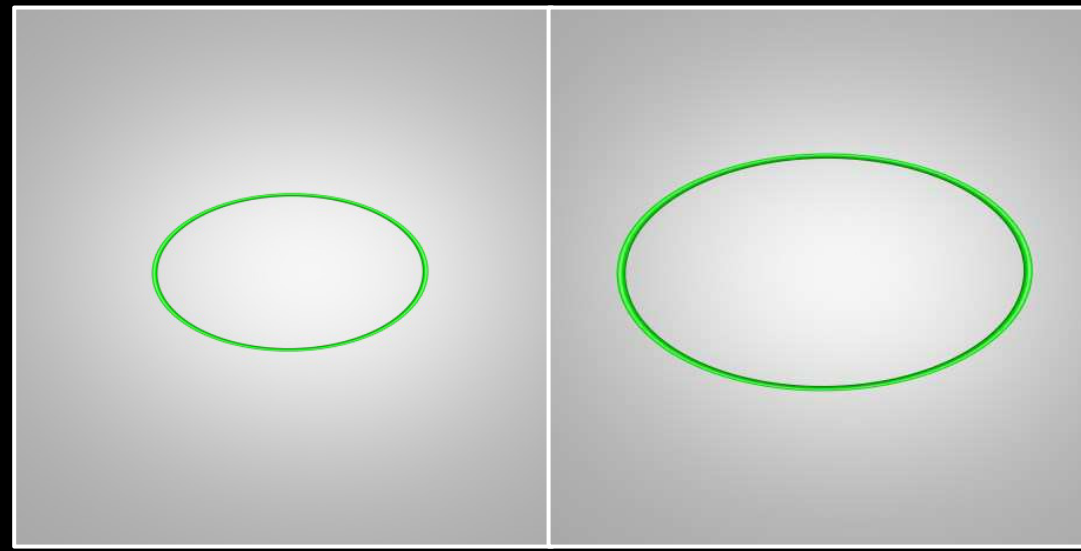
BDPT w/ ellipsoidal connections

\* simulation results

# Design choices

ellipse

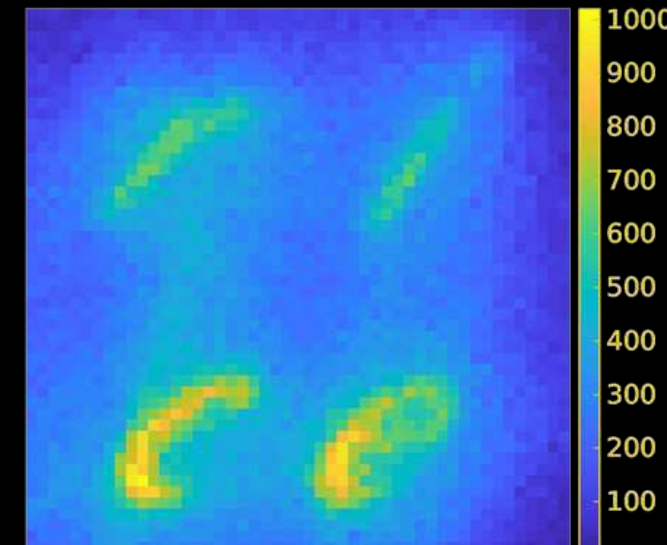
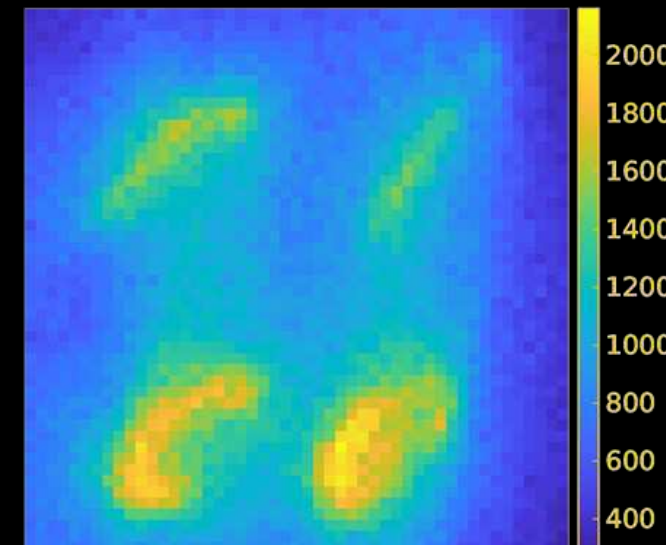
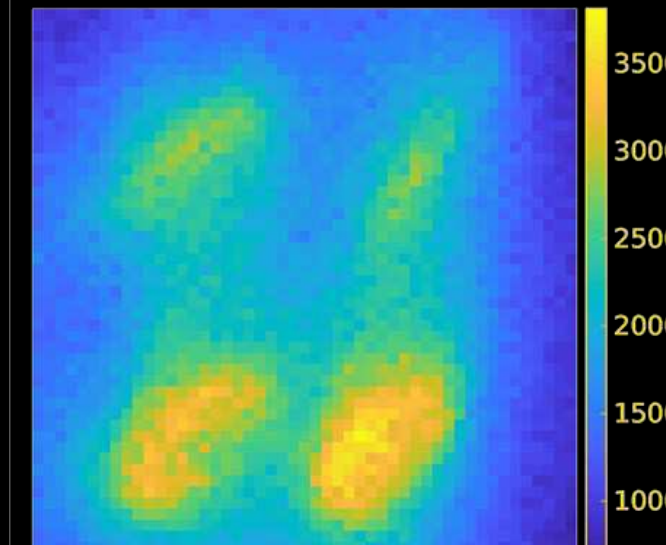
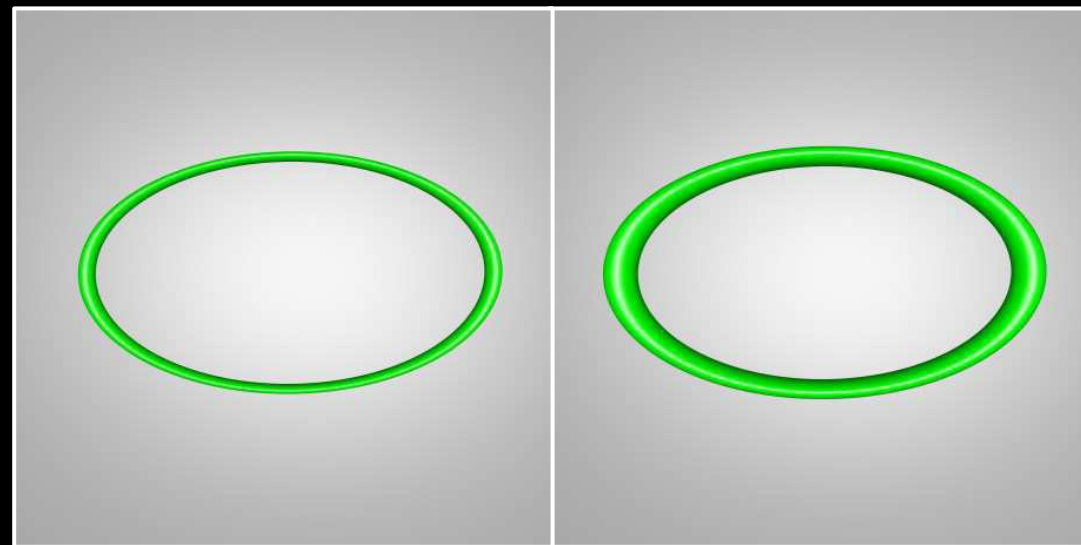
size



large ellipses result in better resolution

ellipse

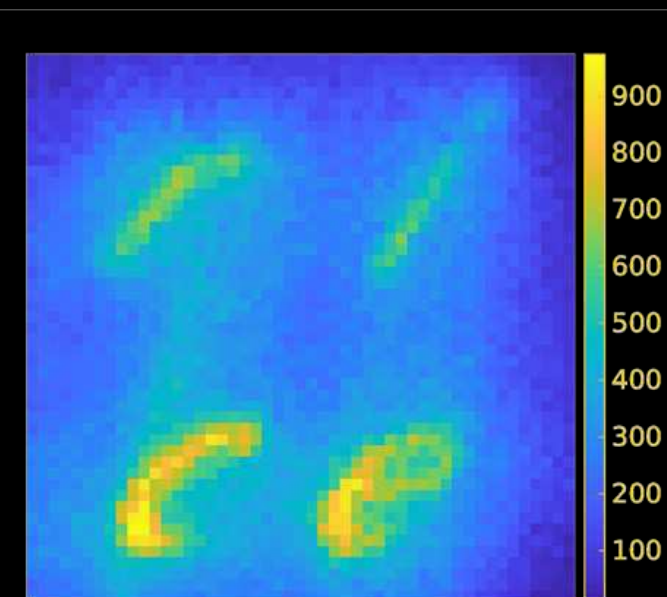
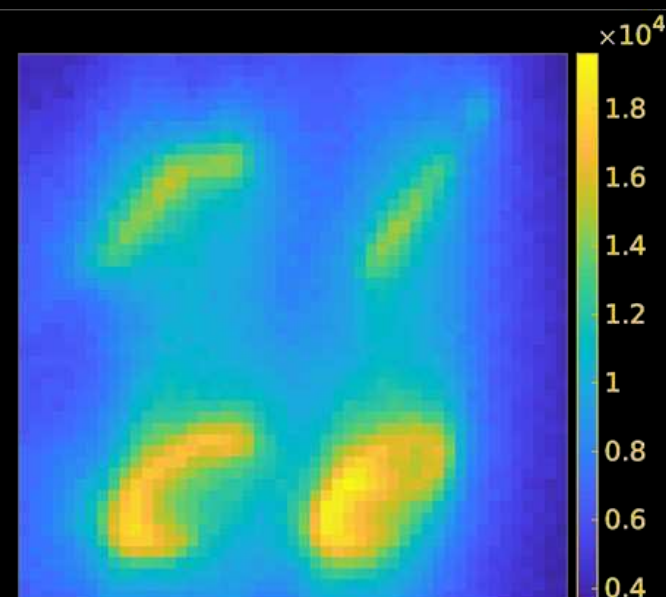
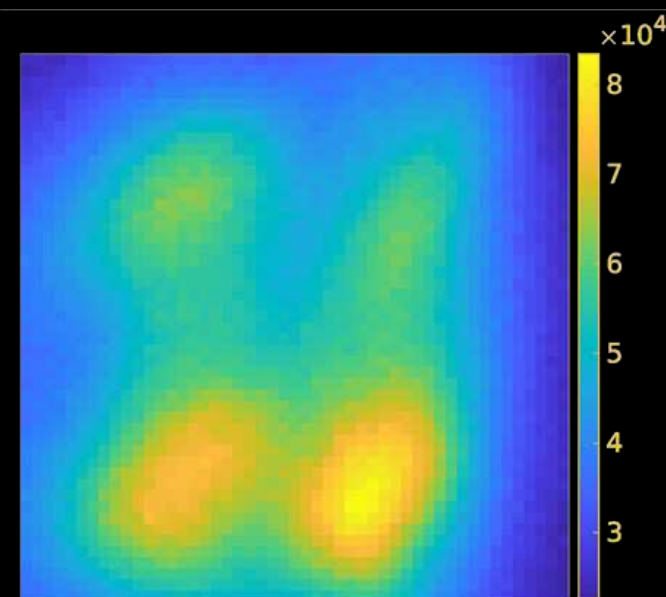
thickness



thinner ellipses result in better resolution, but loses light

temporal

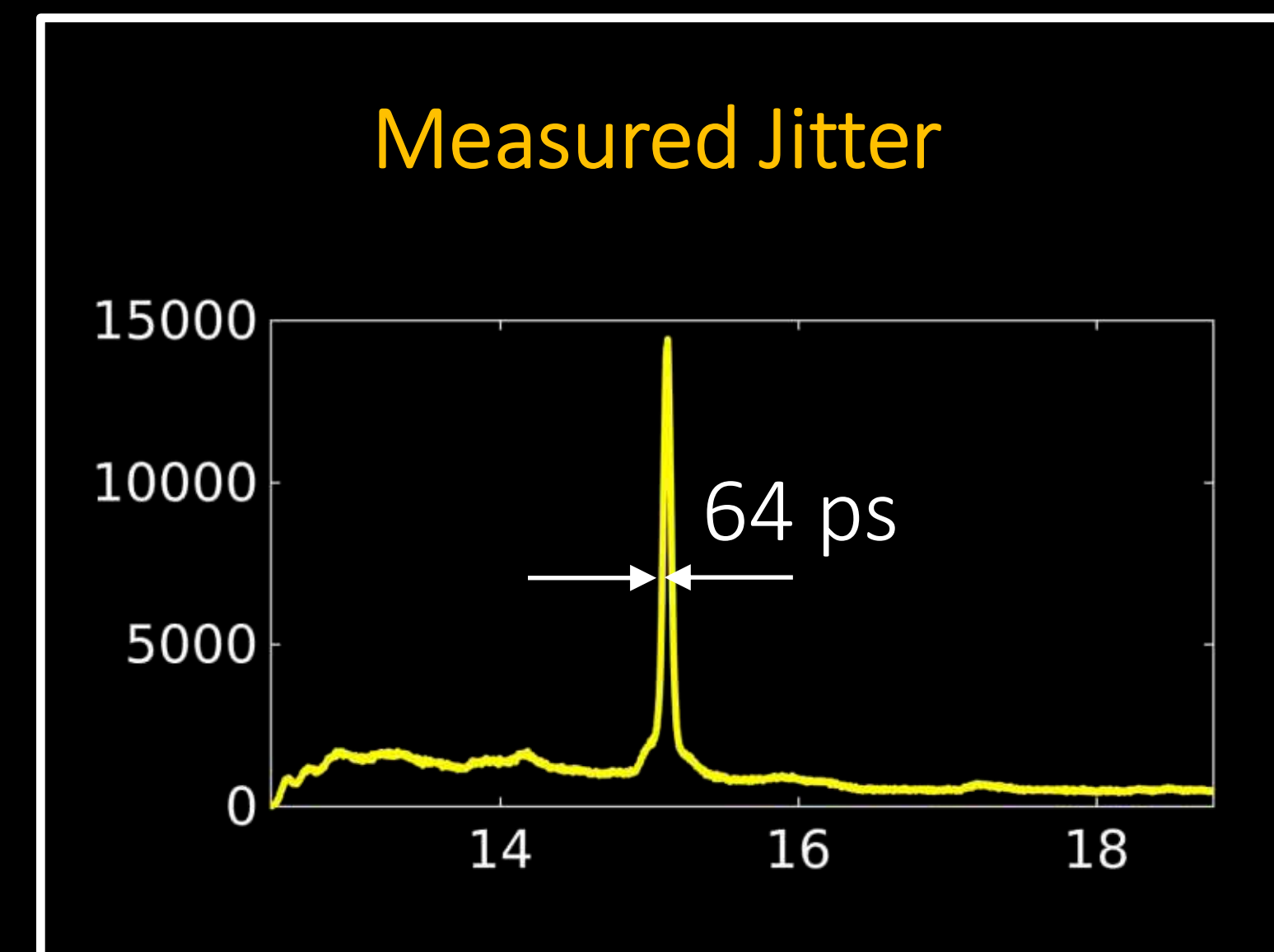
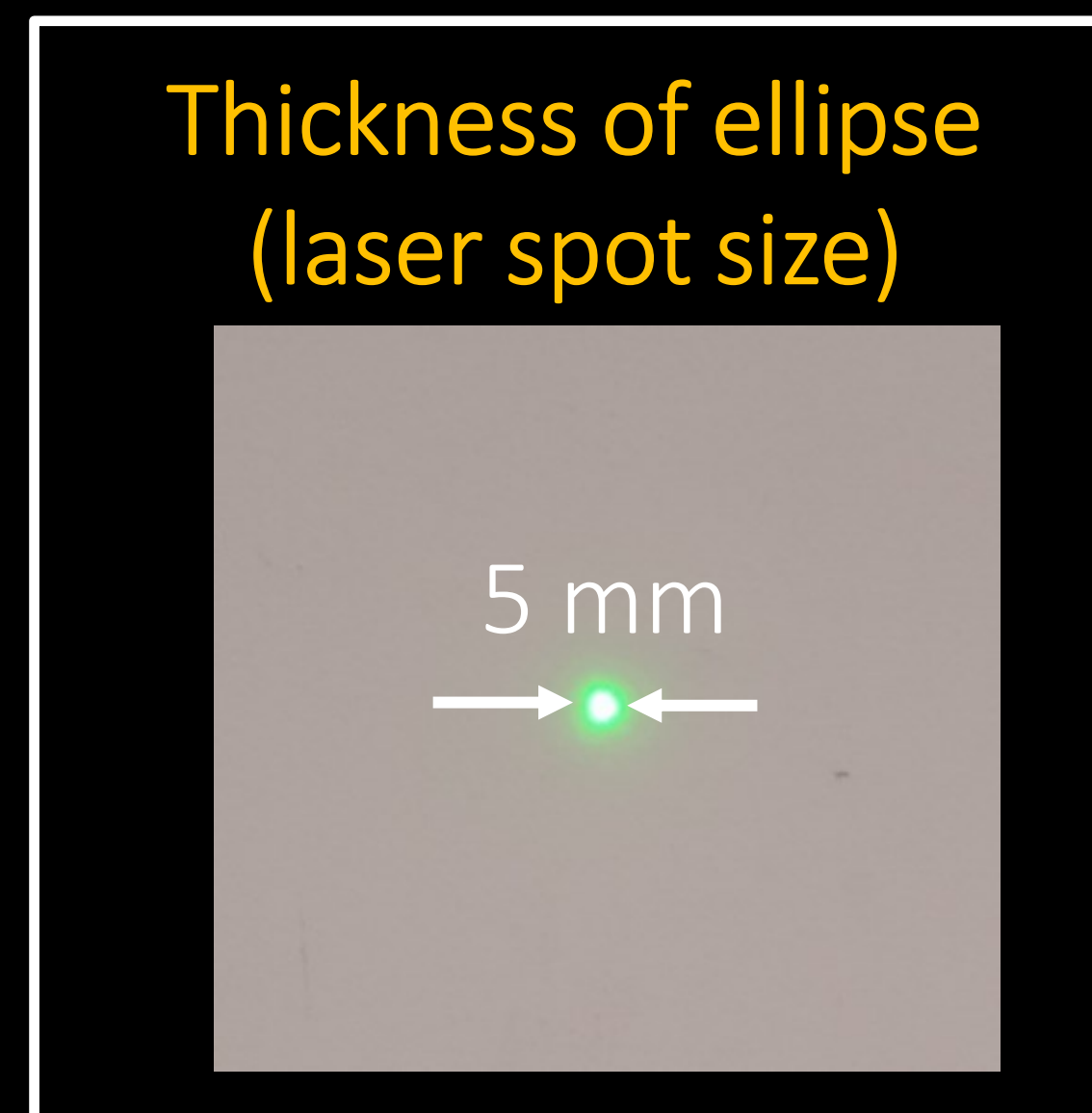
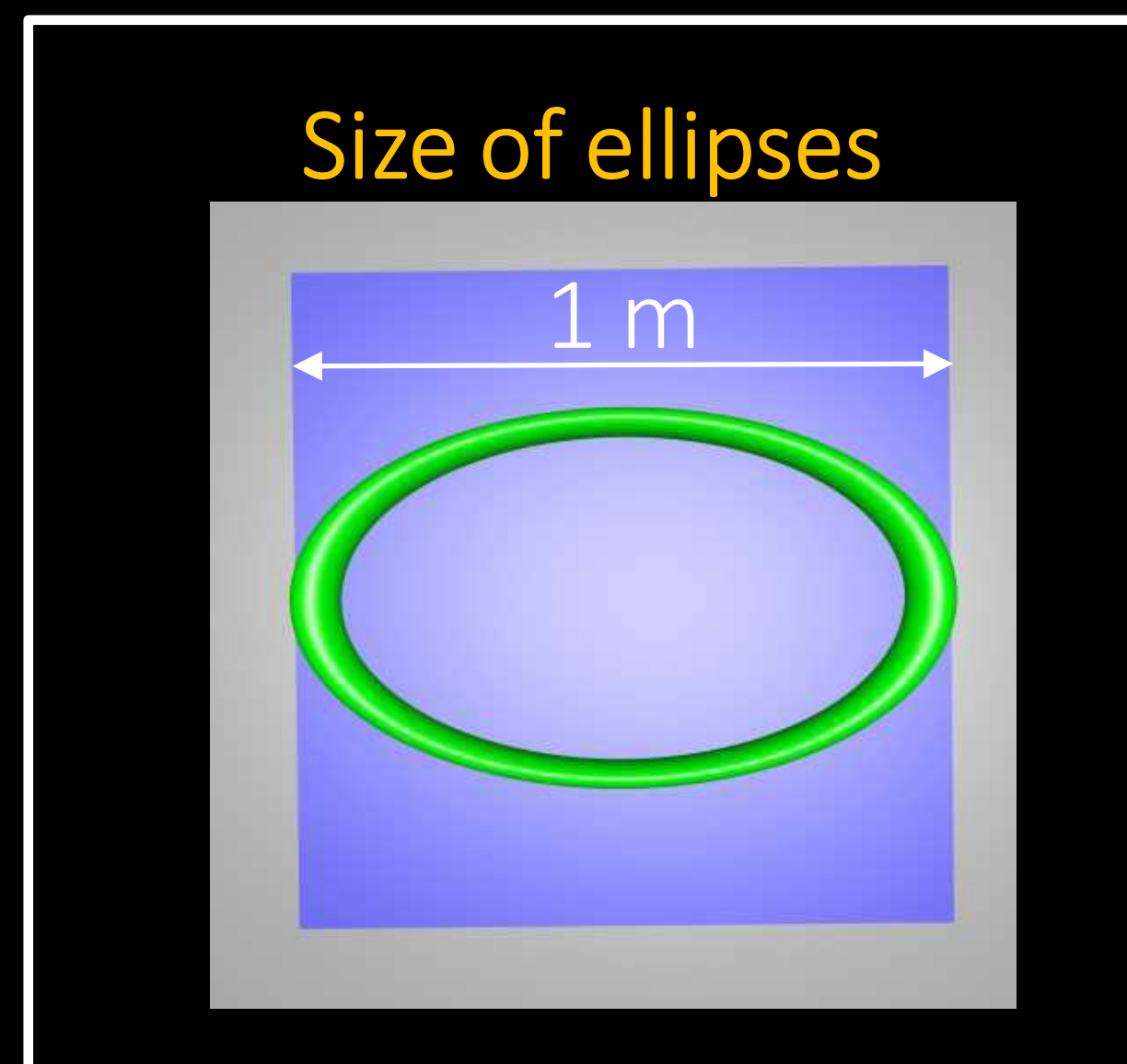
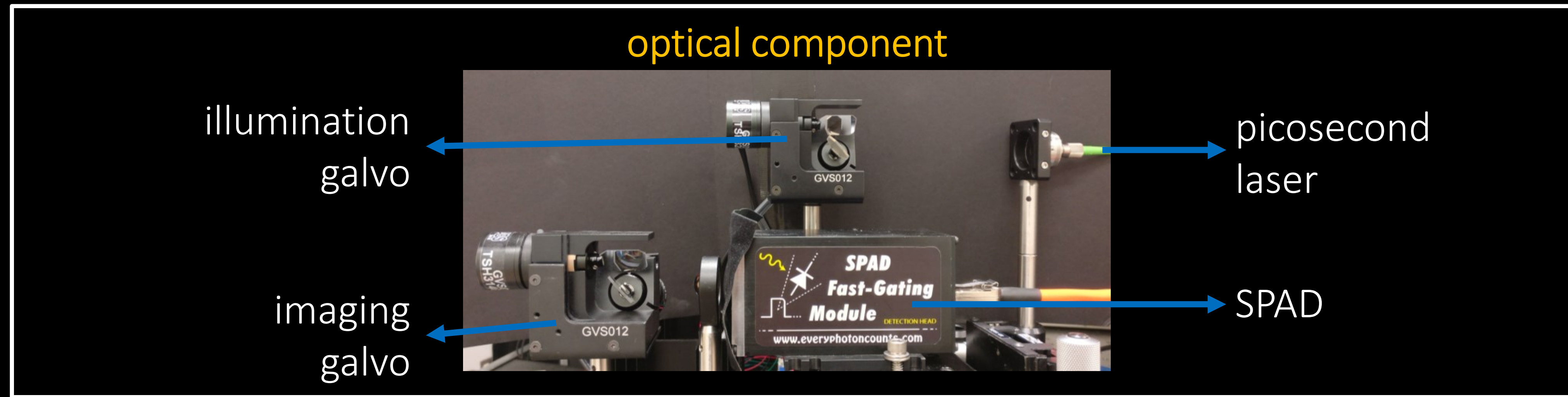
resolution



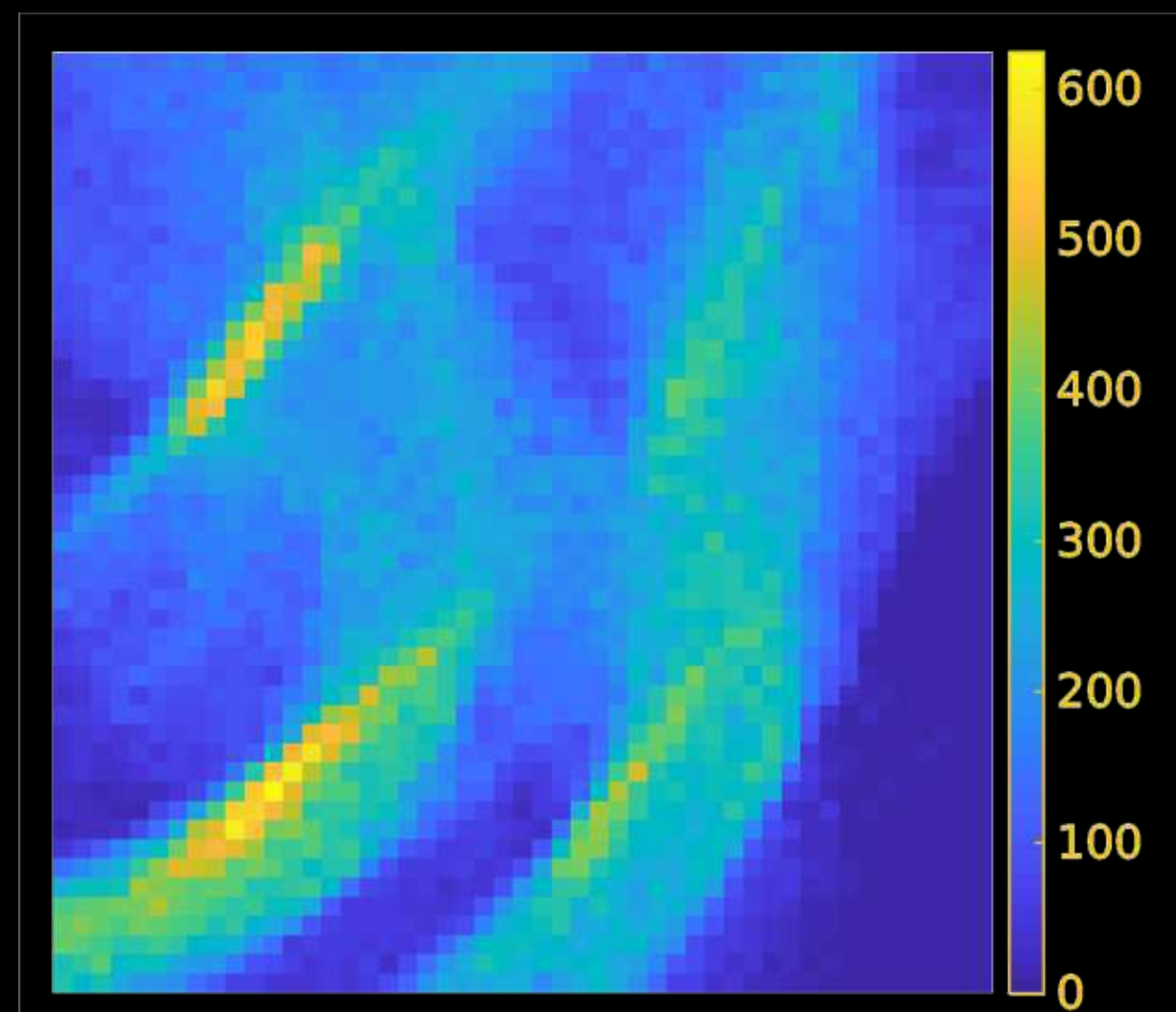
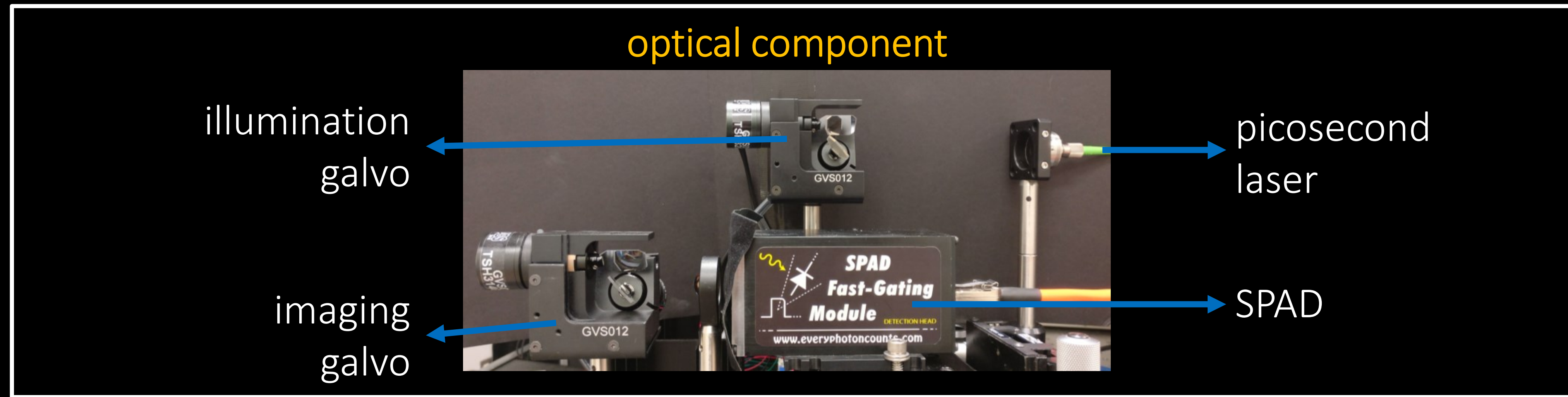
high resolution time-gate results in better resolution, but loses light



# Hardware prototype

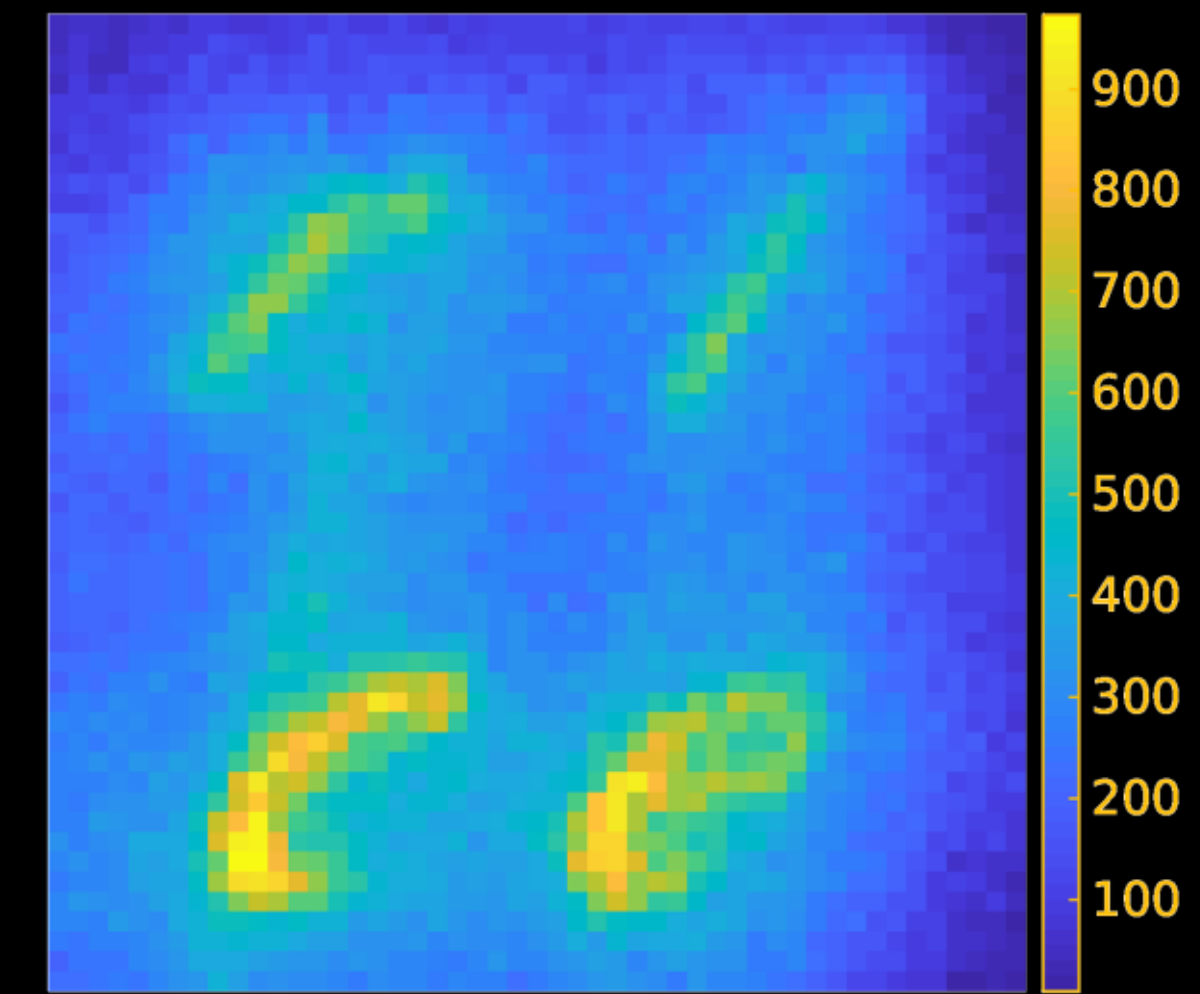


# Hardware prototype



unoptimized

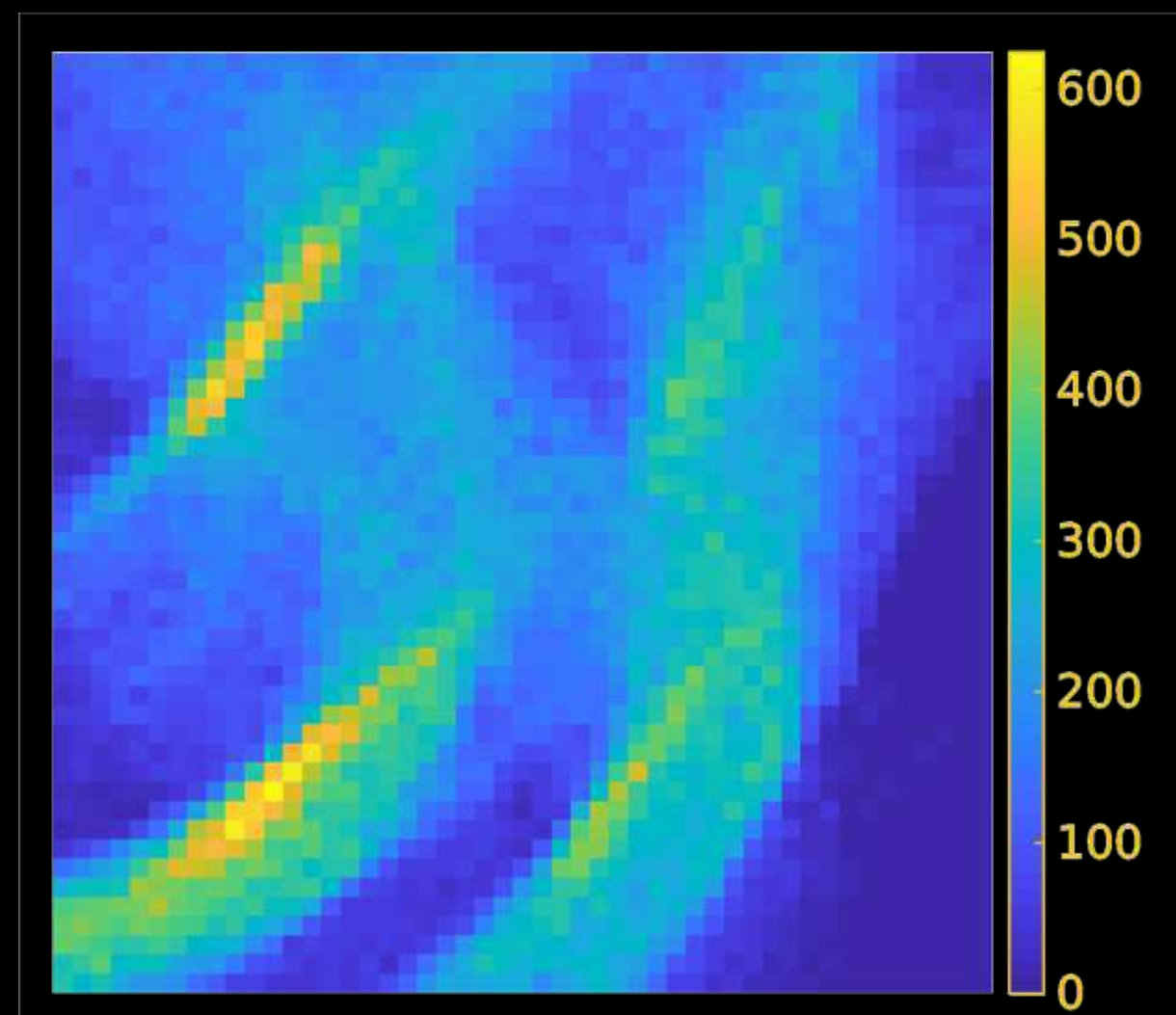
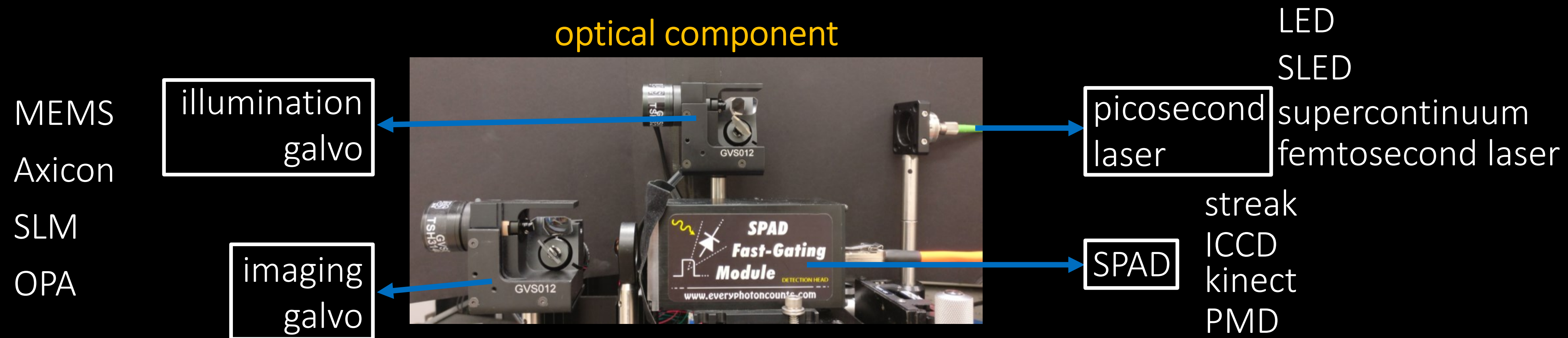
renderer driven  
optimization



optimized

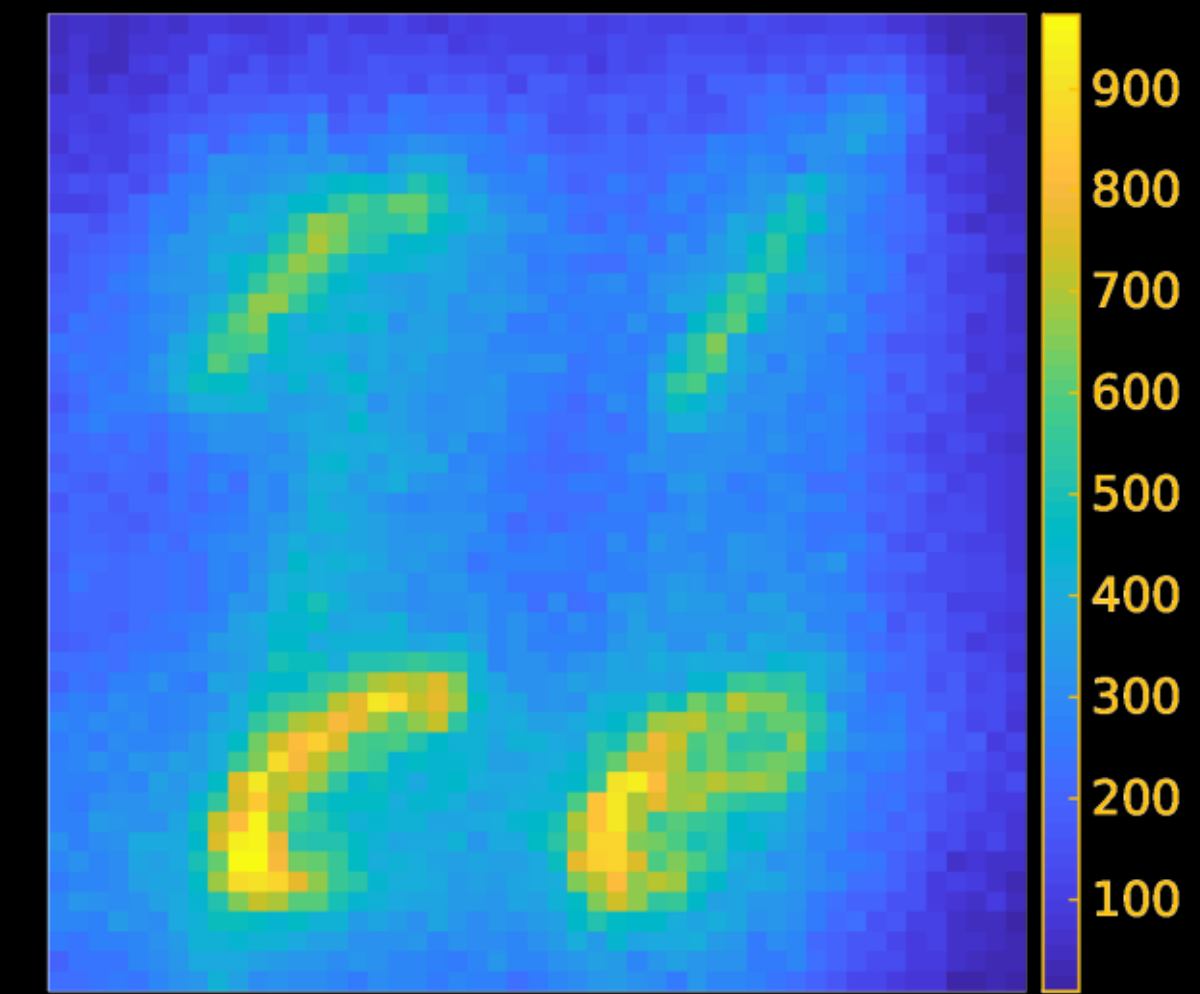


# Hardware prototype



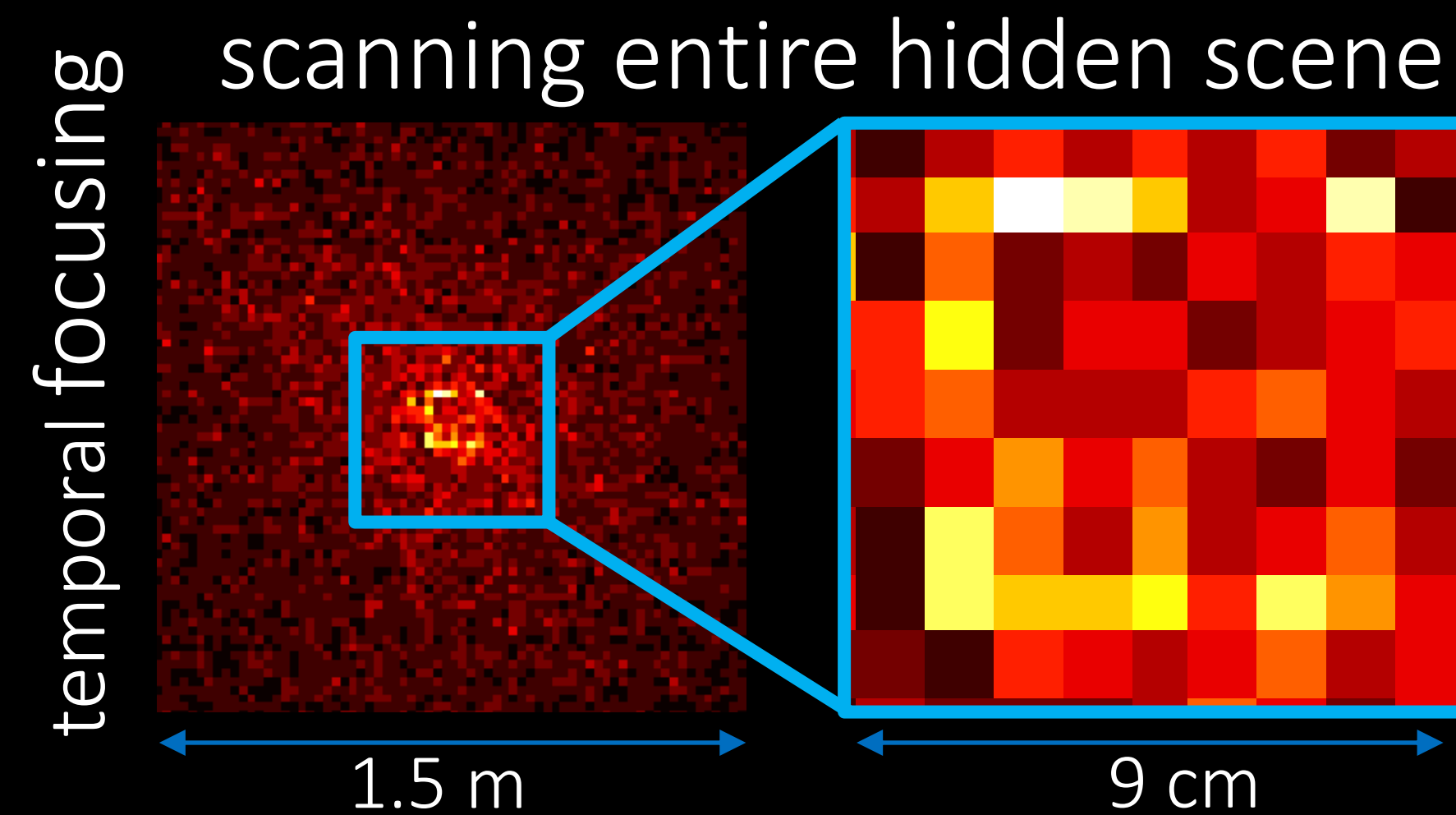
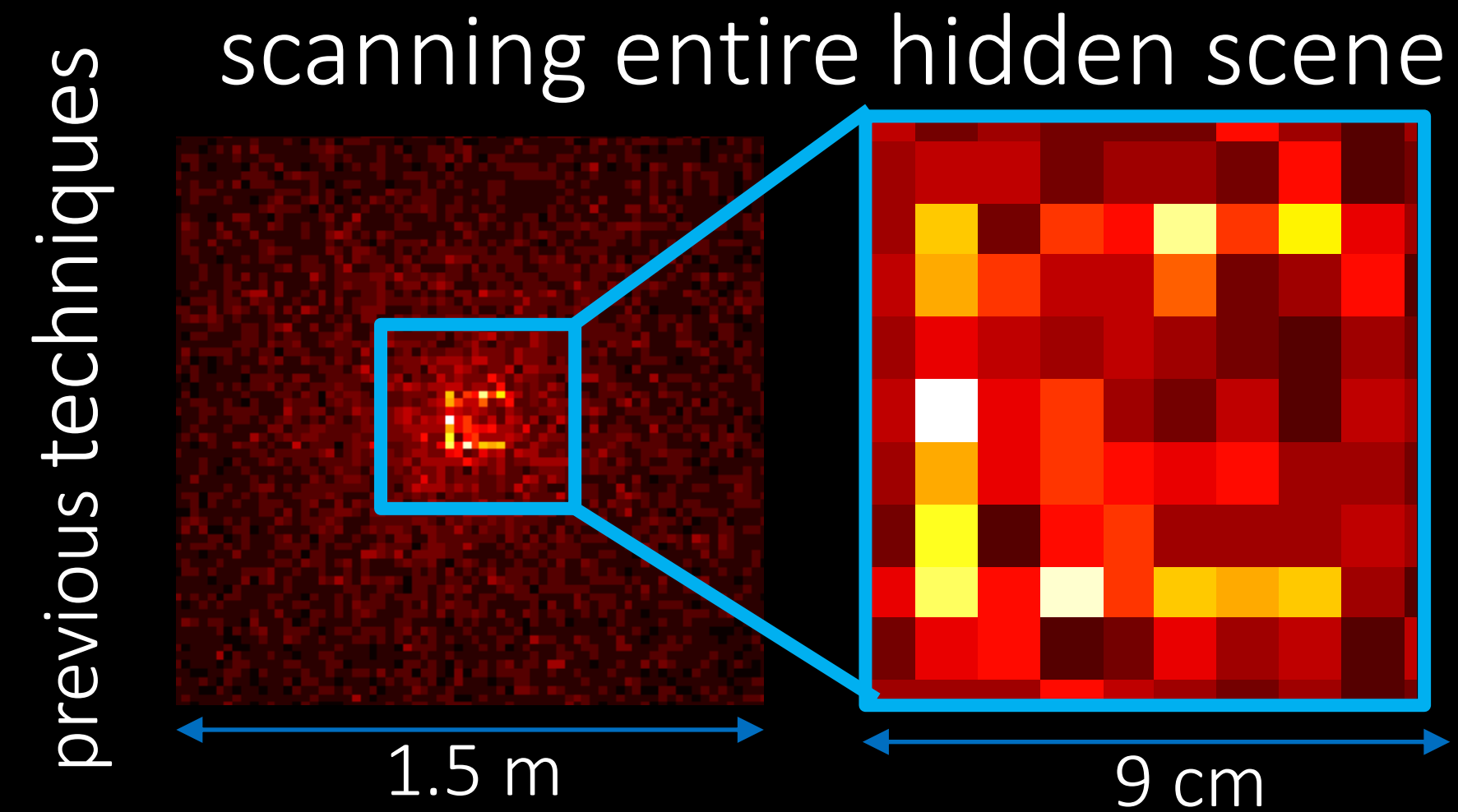
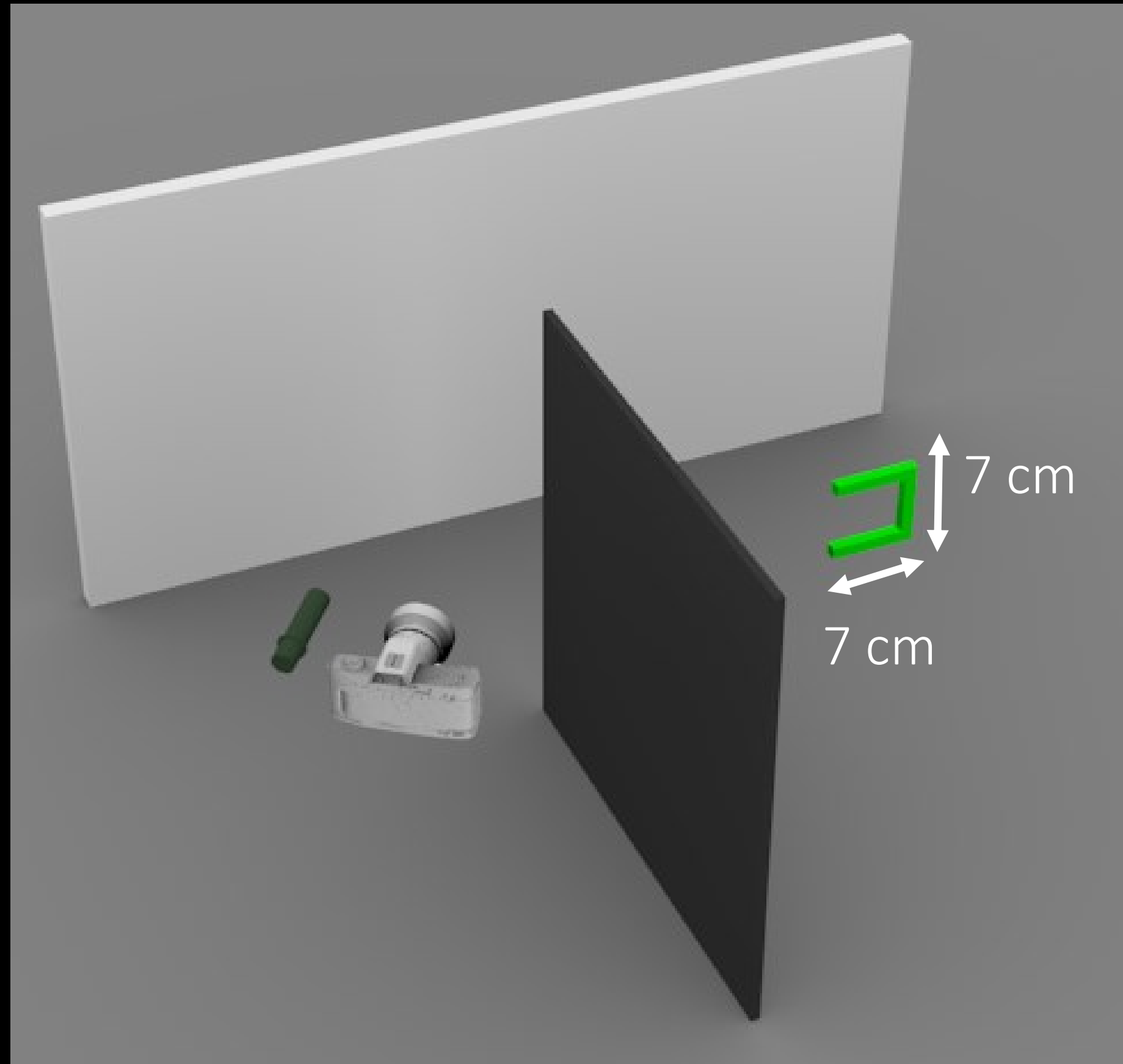
unoptimized

renderer driven  
optimization



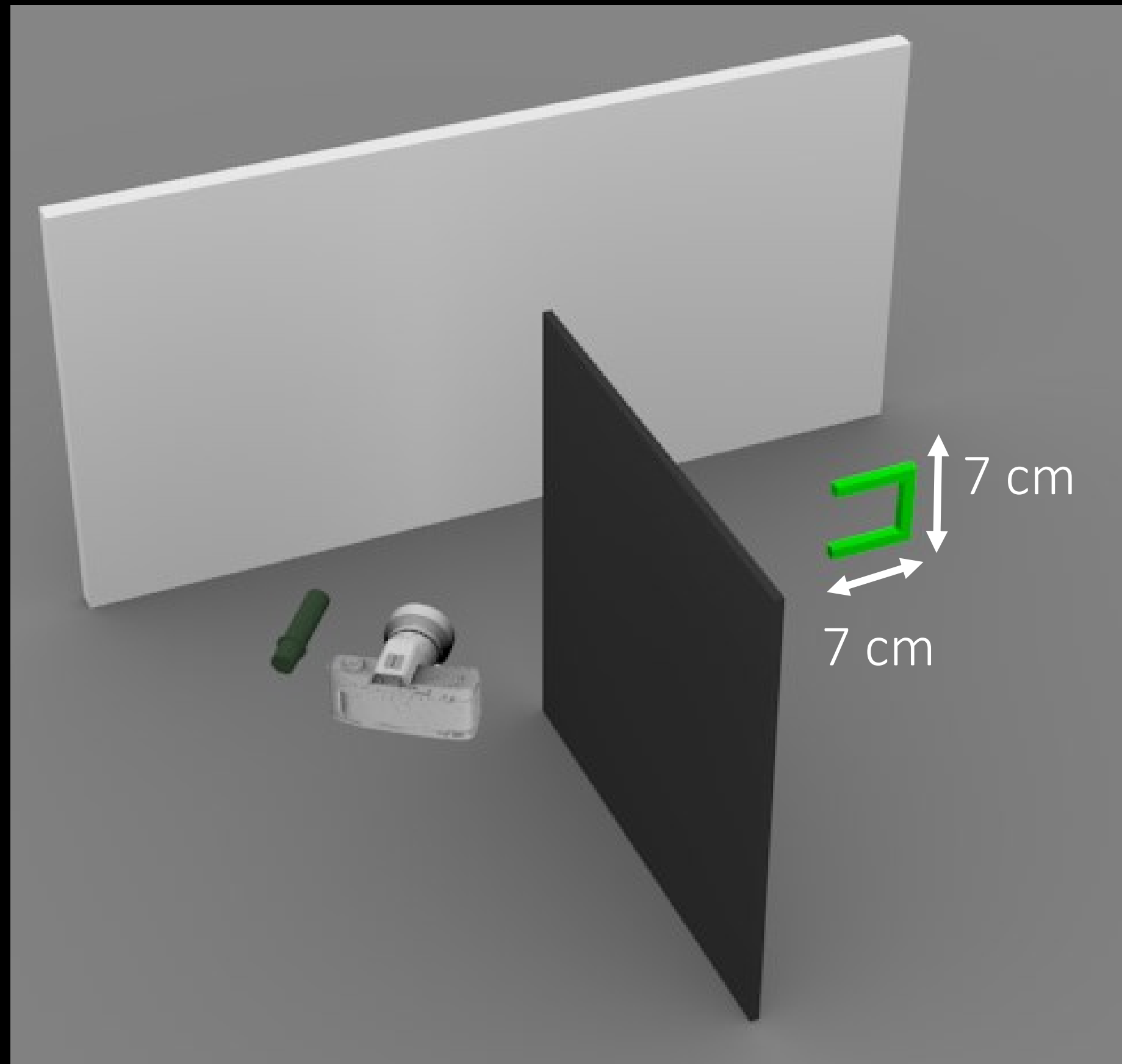
optimized

# Results: scanning limited ROI

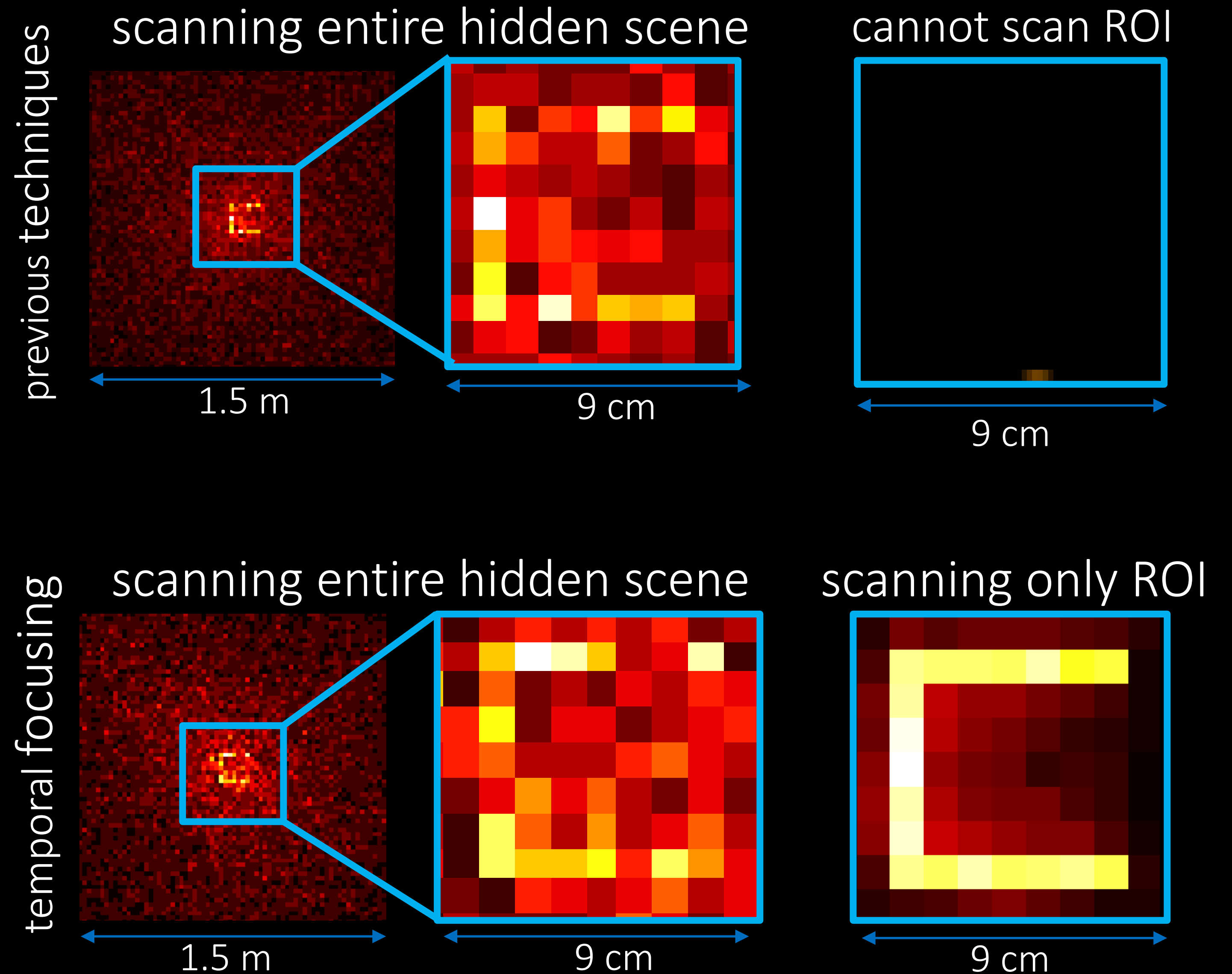




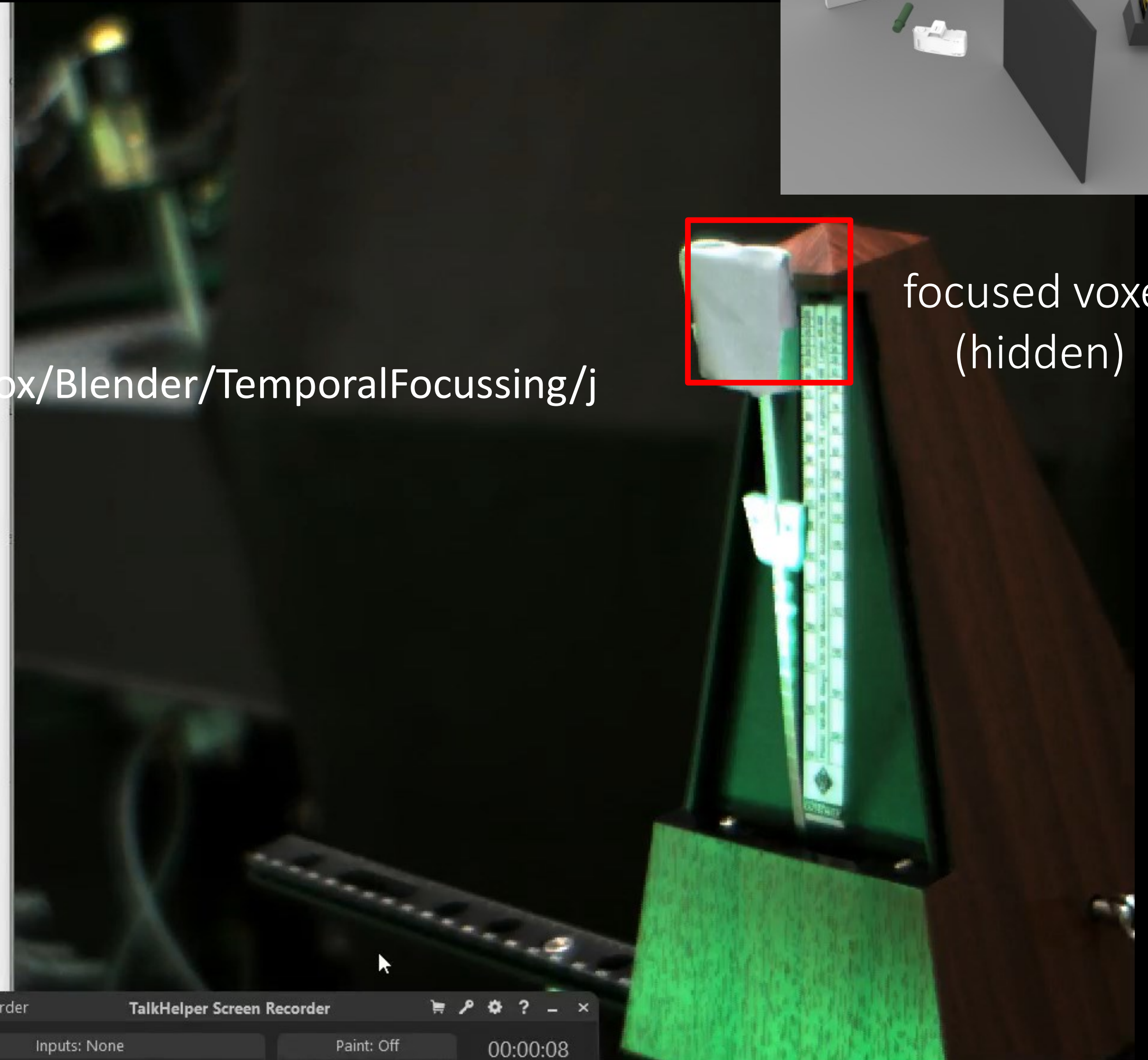
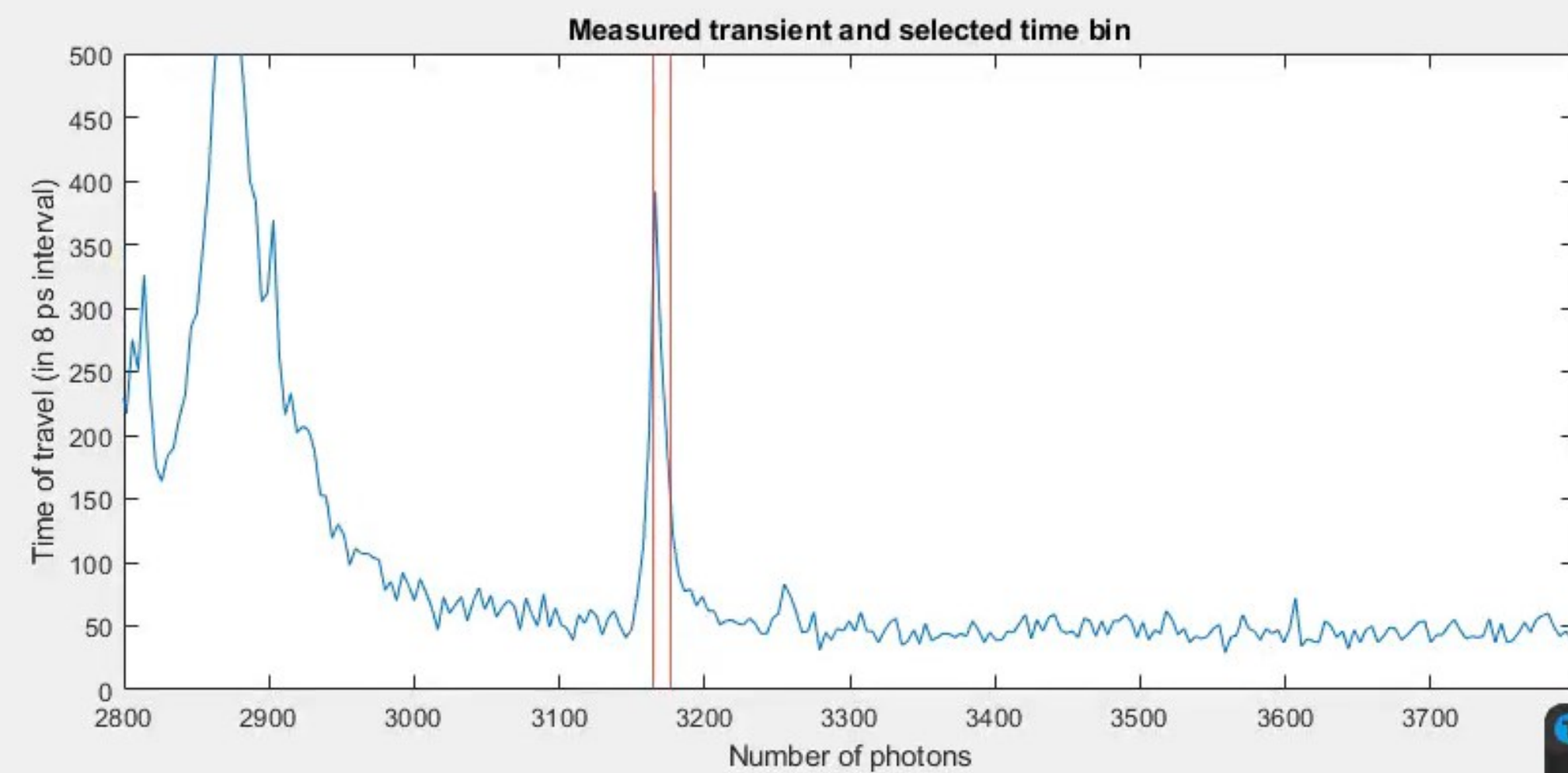
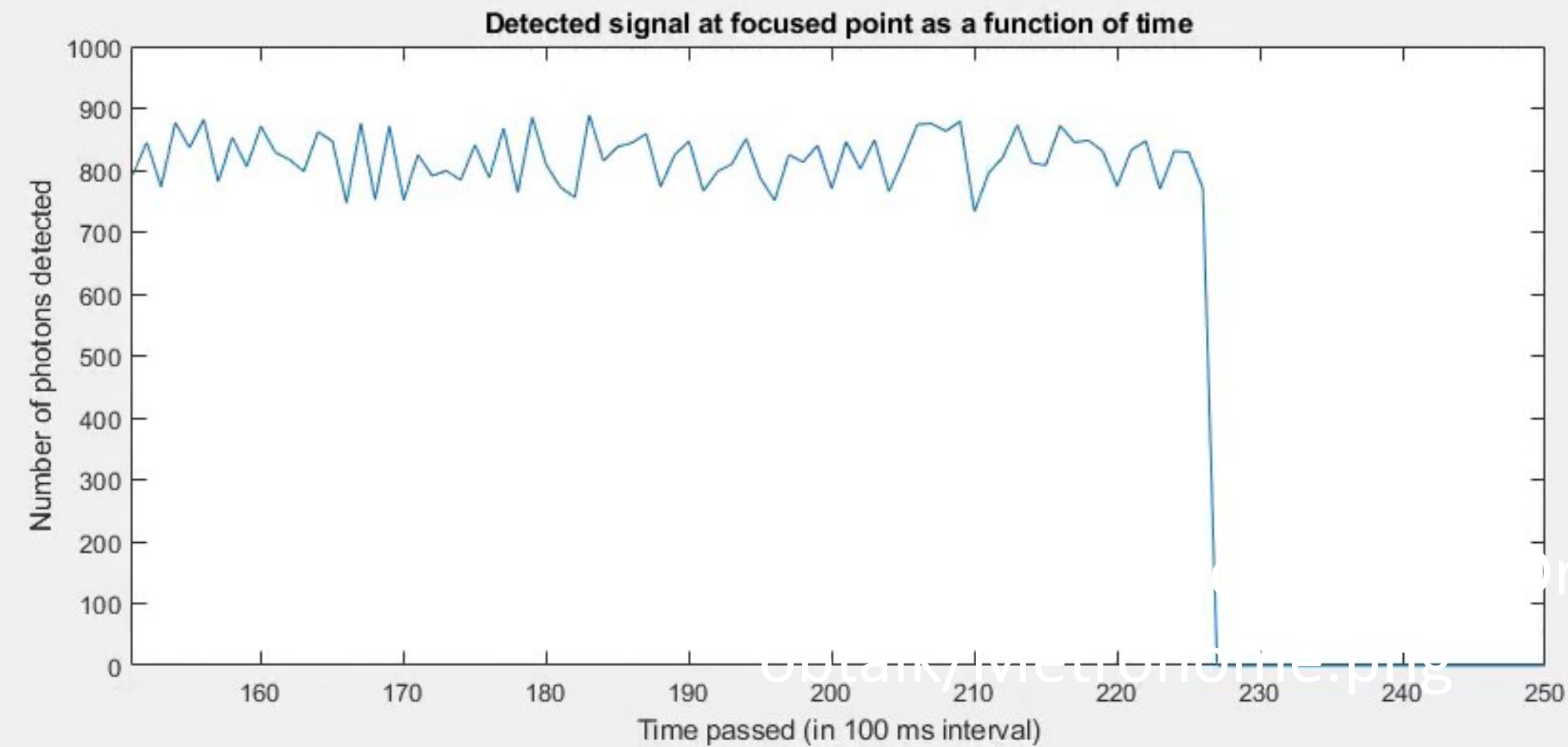
# Results: scanning limited ROI



SNR of temporal focusing is  
> 10× higher for small ROI




# Results: real-time occupancy detection





# Physics-based rendering and its applications to computational imaging


## forward rendering



The diagram illustrates the forward rendering process. On the left, a 3D scene with a textured surface and a light source is shown. Red dashed lines represent light rays originating from the light source, hitting the surface, and reflecting towards a camera. An arrow points from this scene to a computer icon, which then points to a stack of three grayscale images, representing the rendered output.

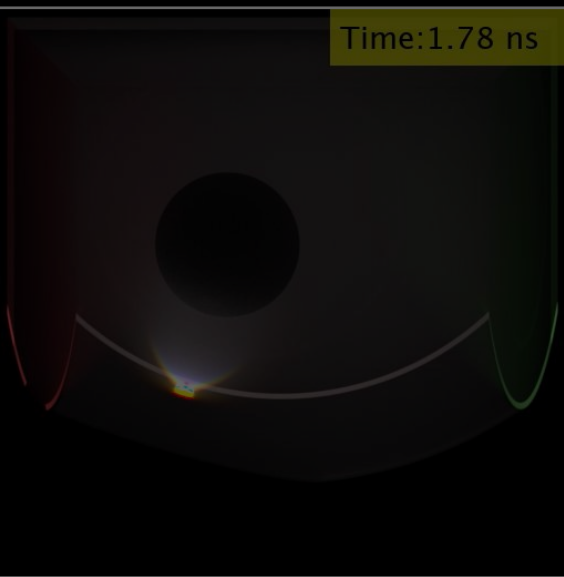
- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

## inverse rendering

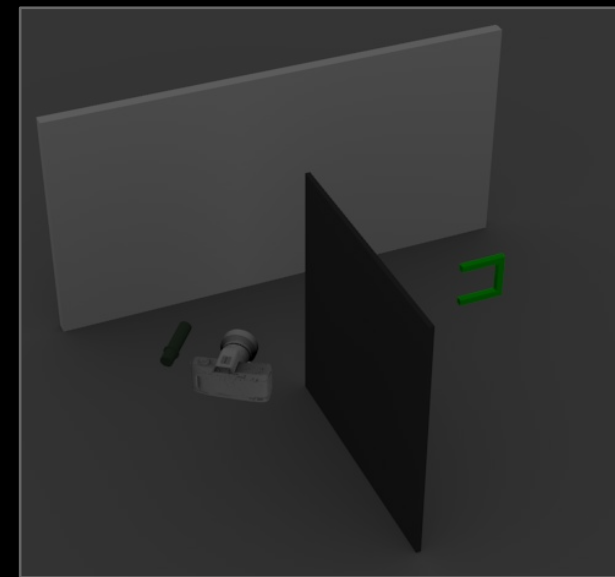


The diagram illustrates the inverse rendering process. On the left, a 3D scene is shown with red dashed lines representing light rays. An arrow points from this scene to a computer icon, which then points to a stack of three grayscale images. This represents the process of inferring the scene parameters from the observed images.

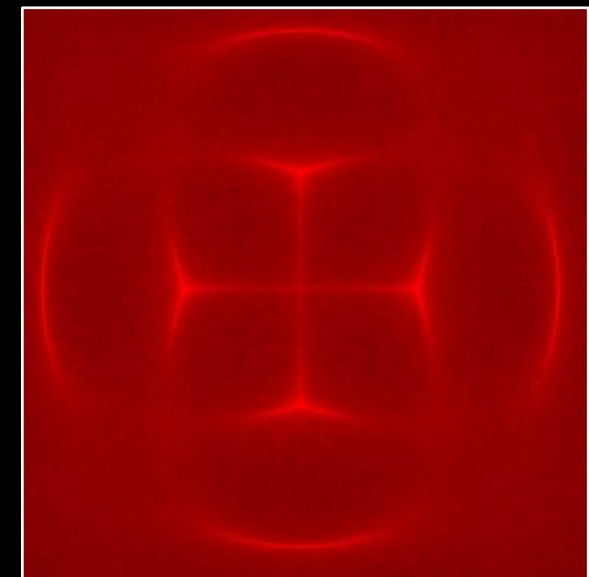
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



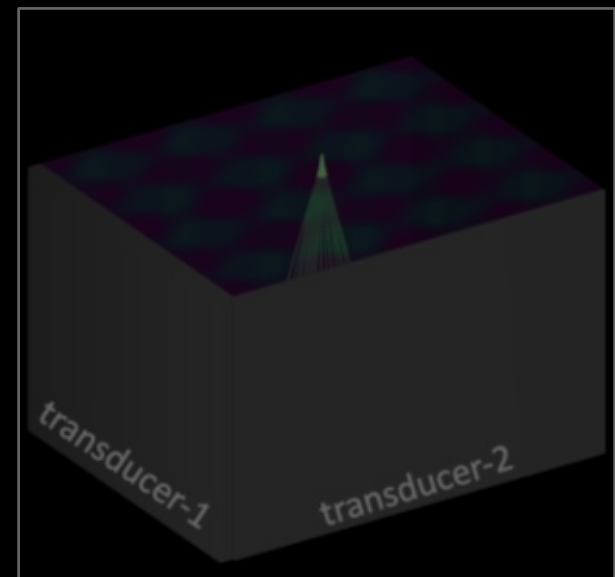
time-of-flight  
imaging



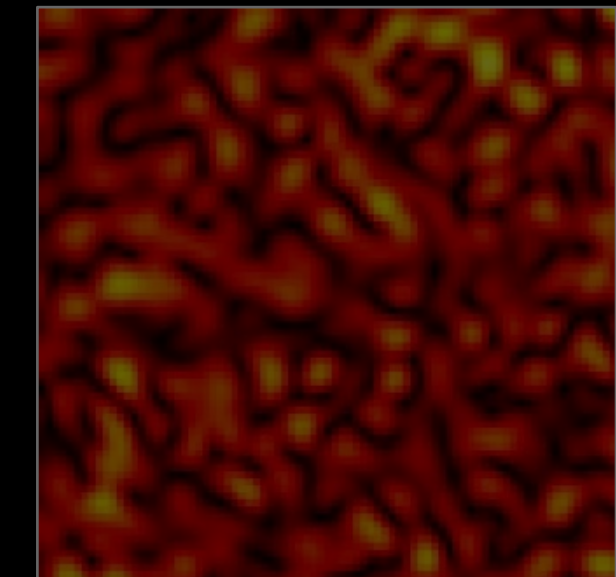
non-line-of-sight  
imaging



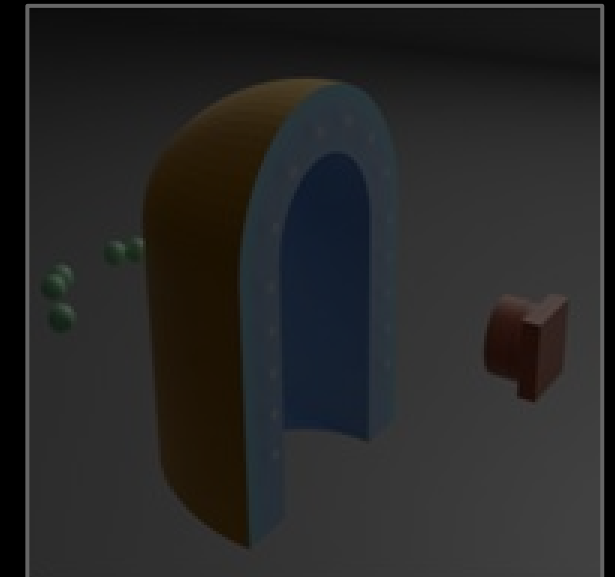
acousto-optic  
lensing



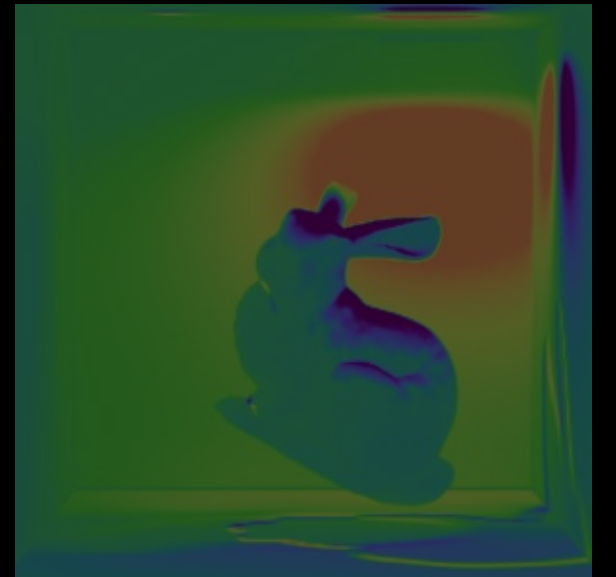
ultrafast light  
scanning



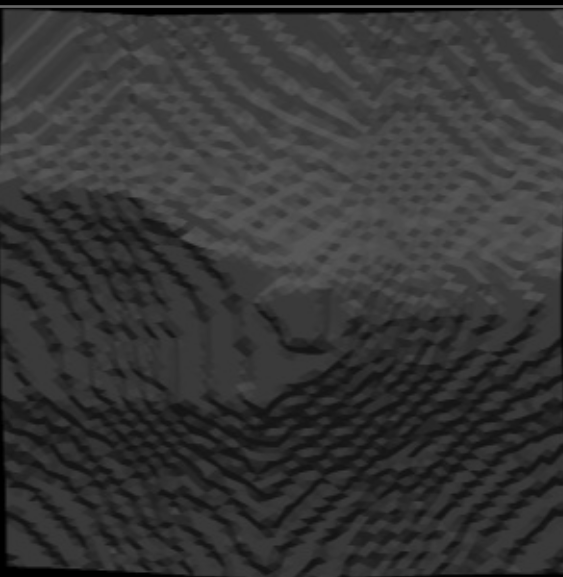
speckle  
imaging



tactile sensor  
design

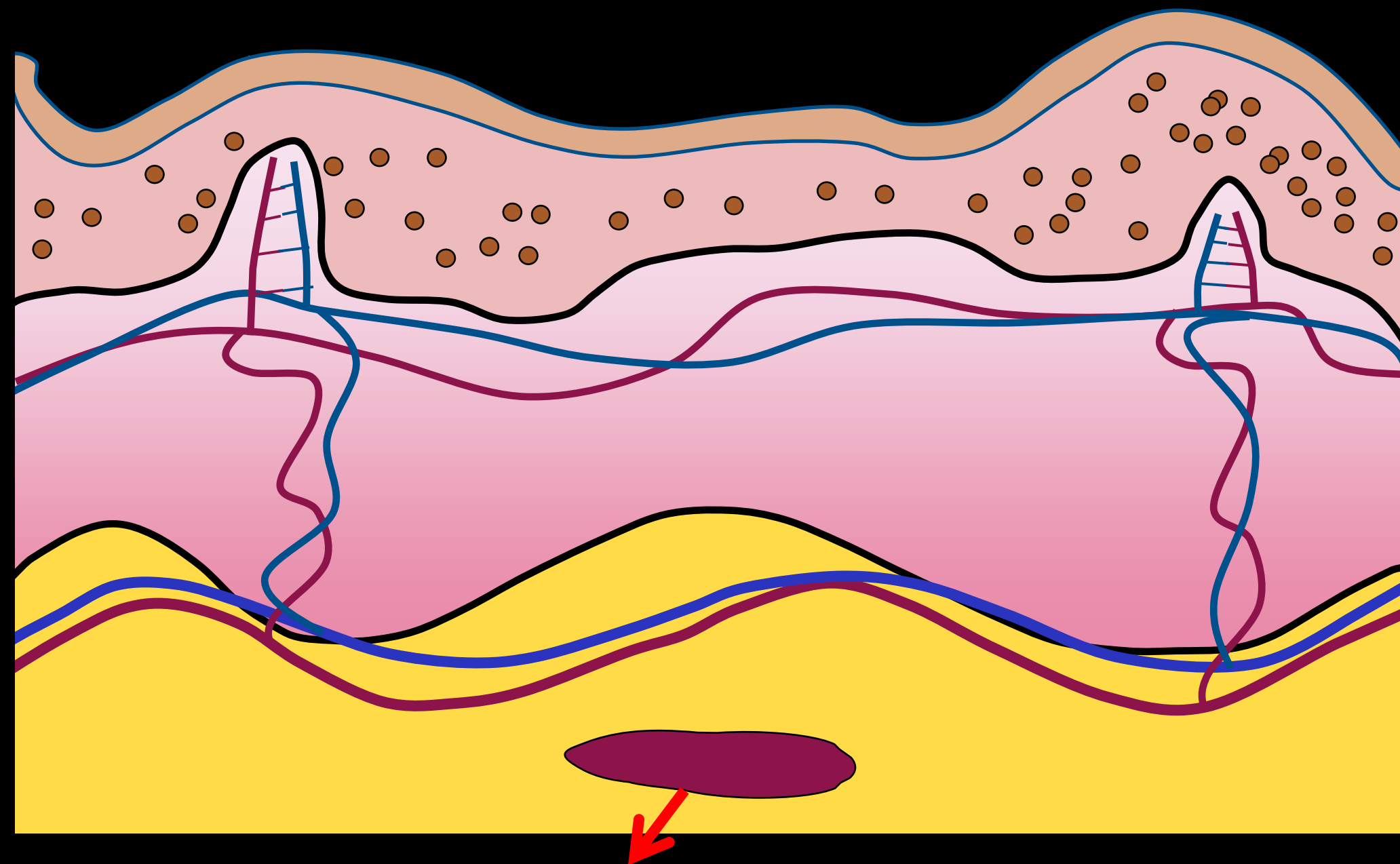


differentiable  
renderer



inverse  
problems

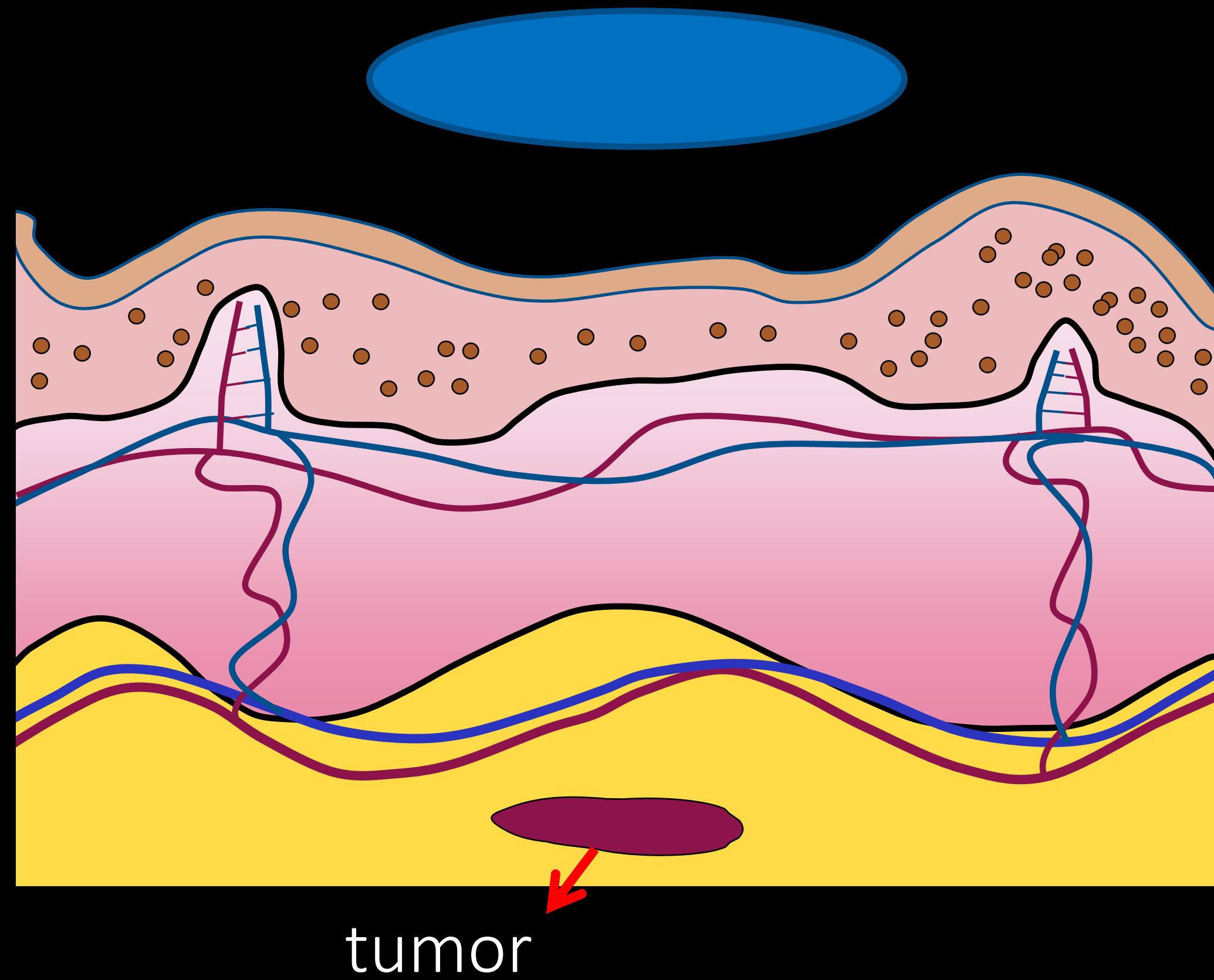
# Focusing light inside tissue



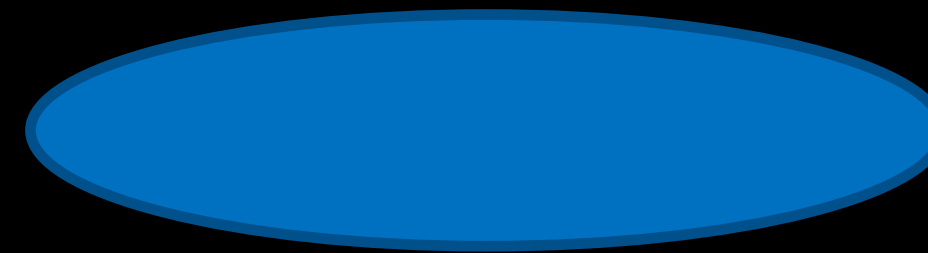
tumor



# Focusing light inside tissue



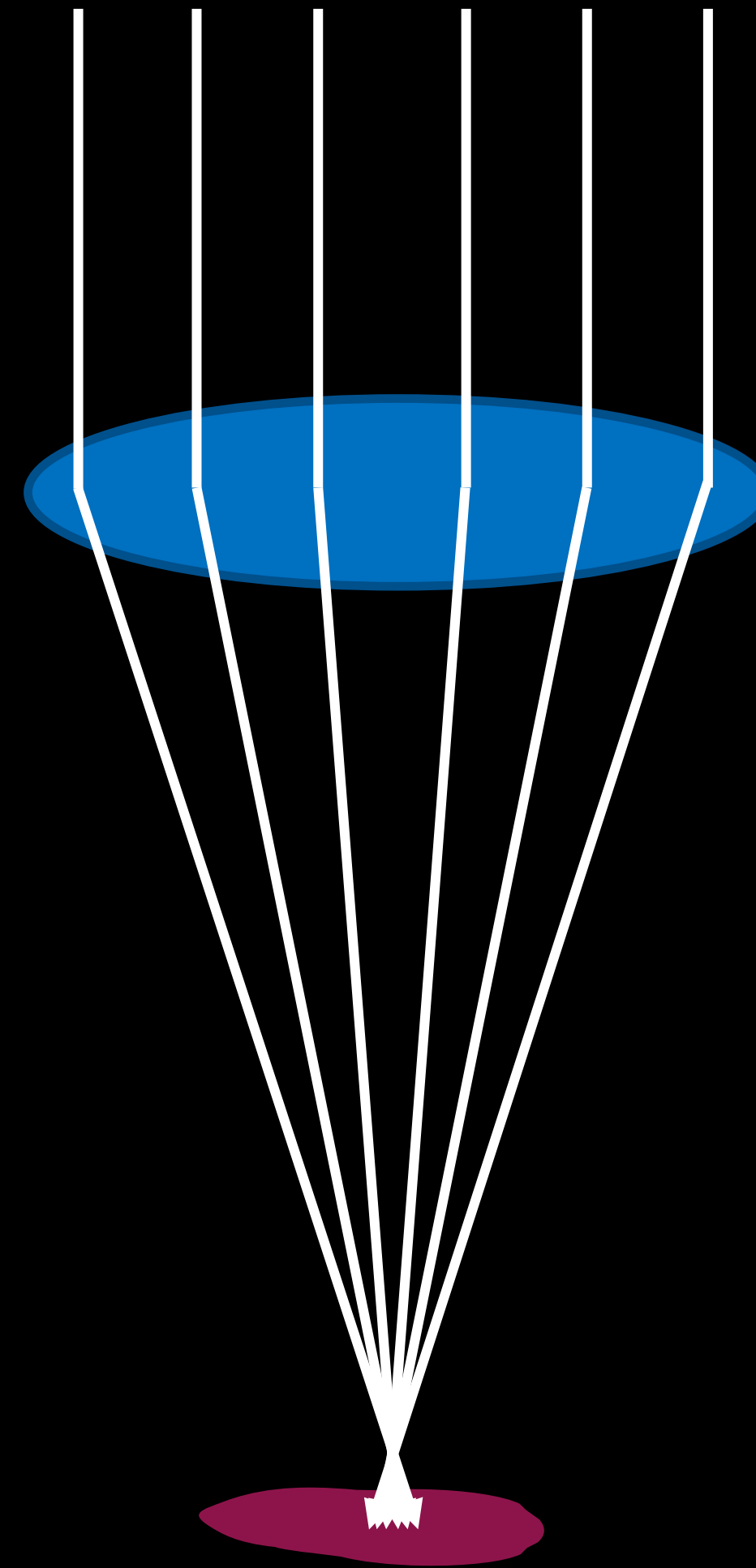
# Focusing light inside tissue



tumor

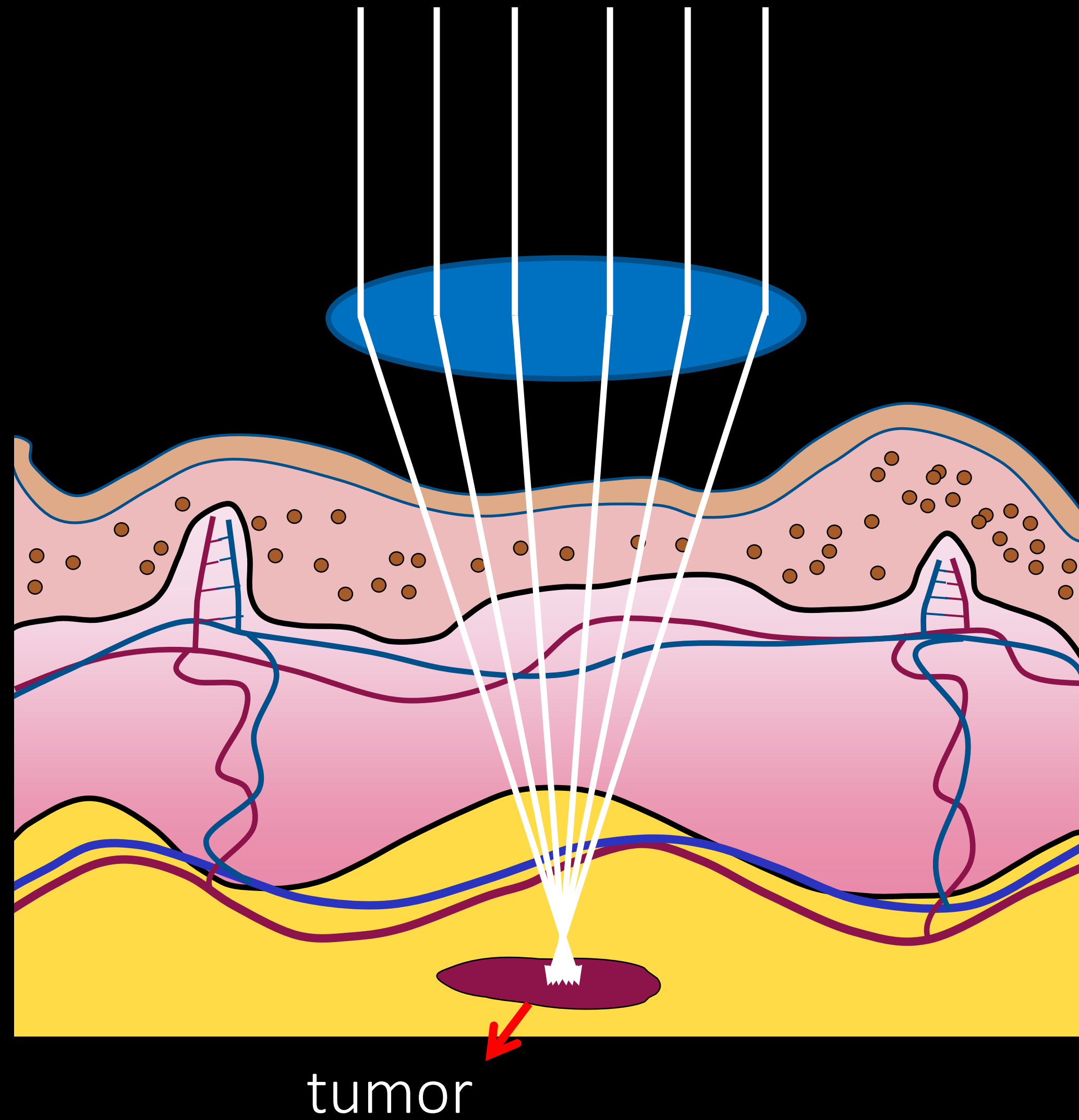


# Focusing light inside tissue



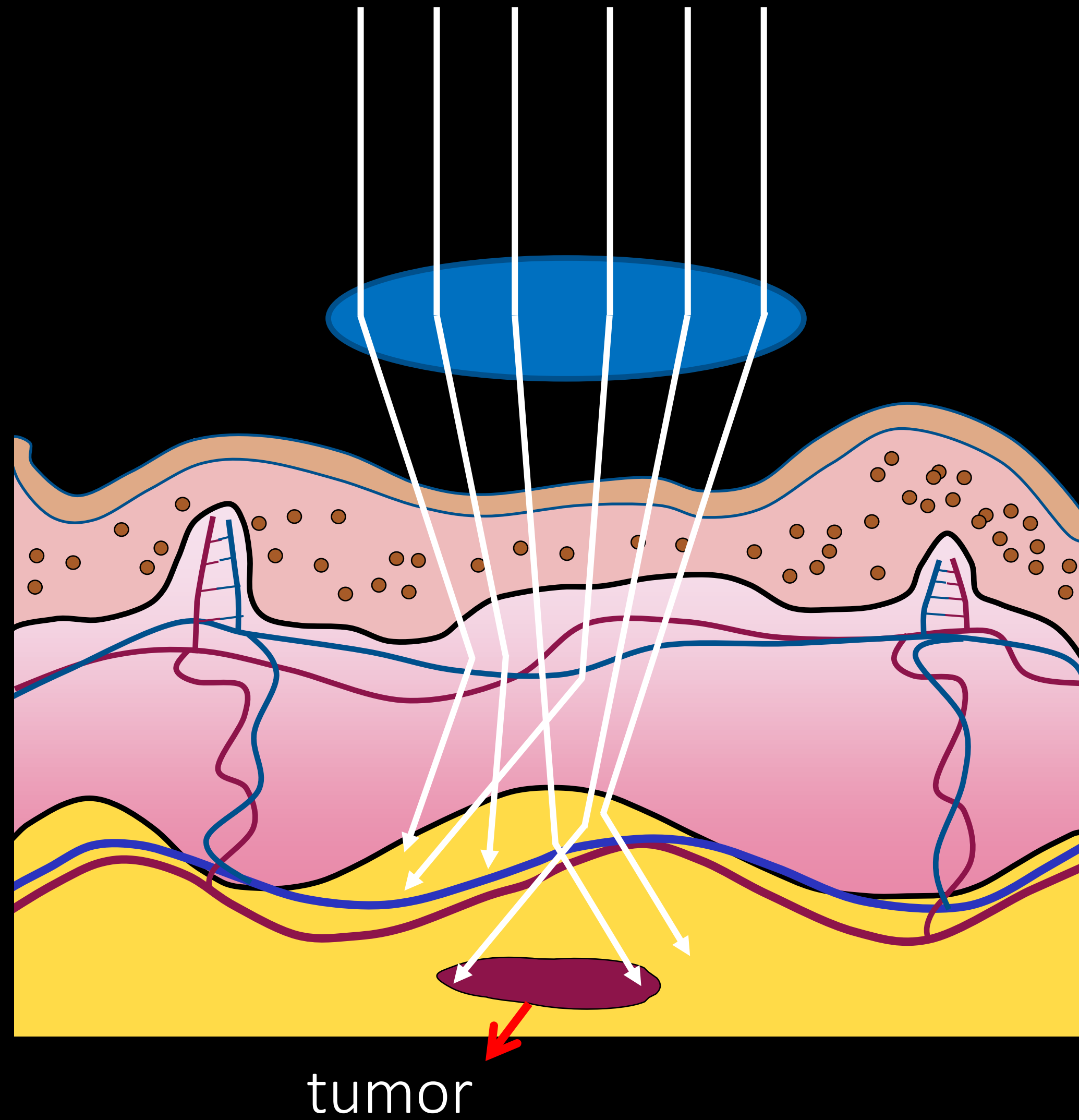
tumor

# Focusing light inside tissue

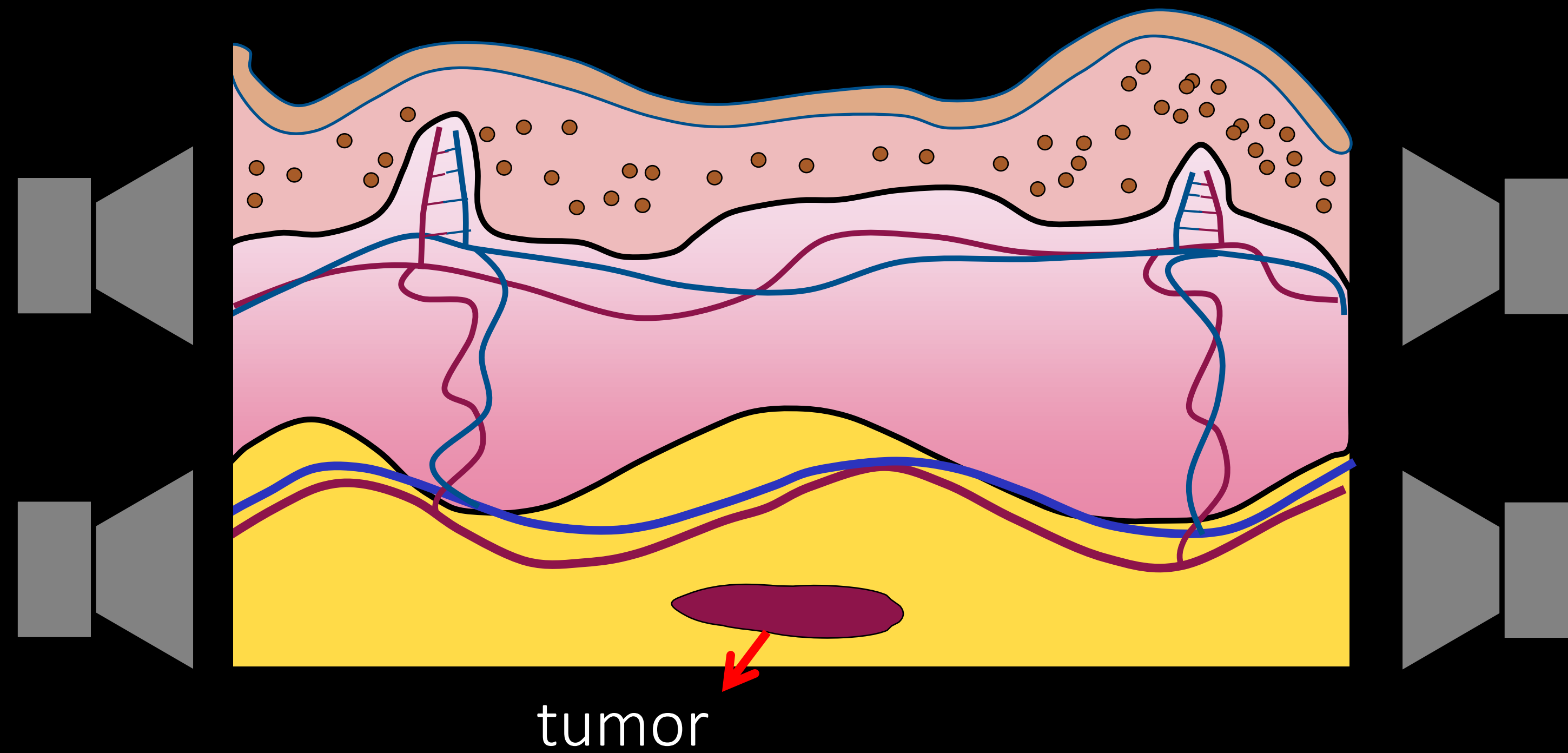




# Focusing light inside tissue

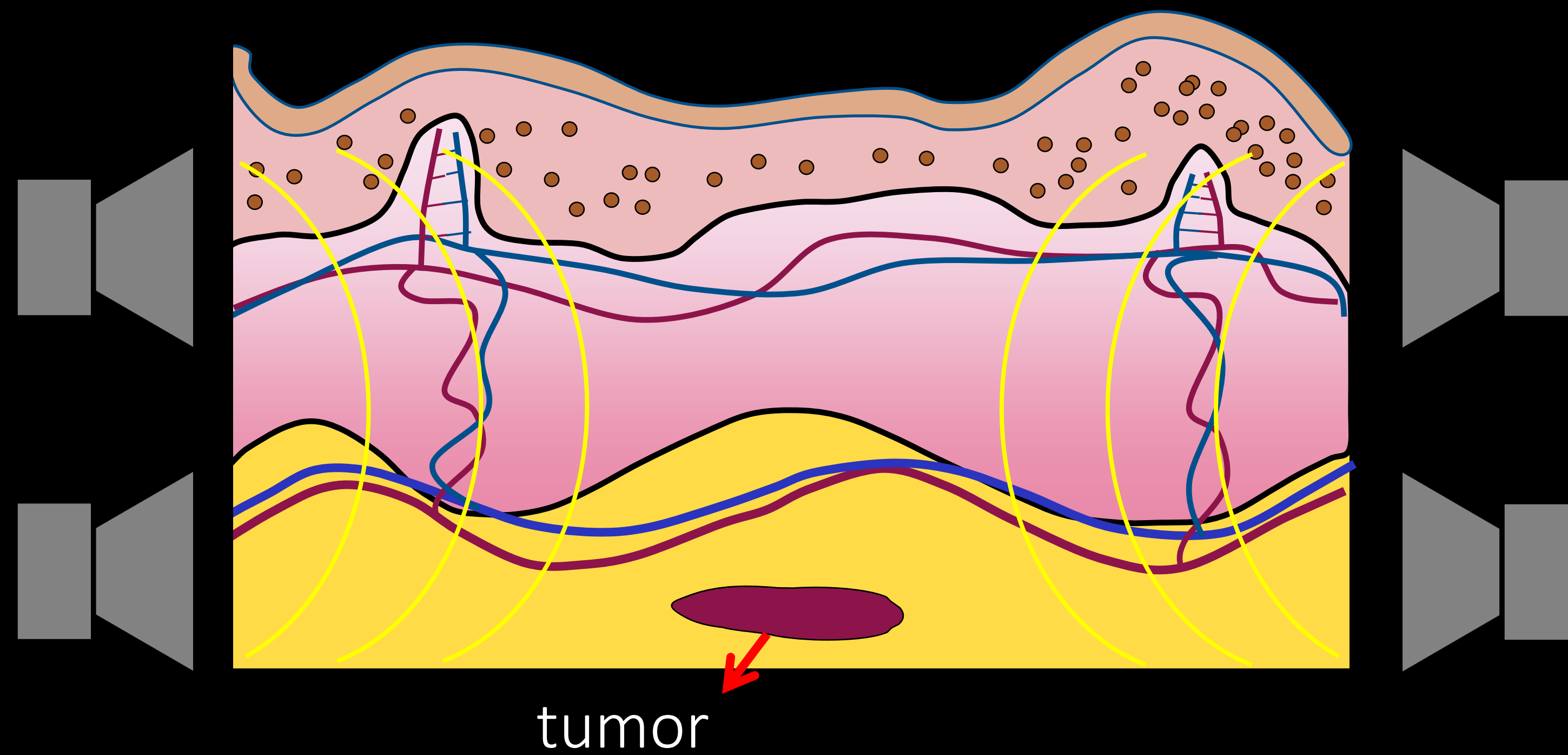


# Focusing light inside tissue

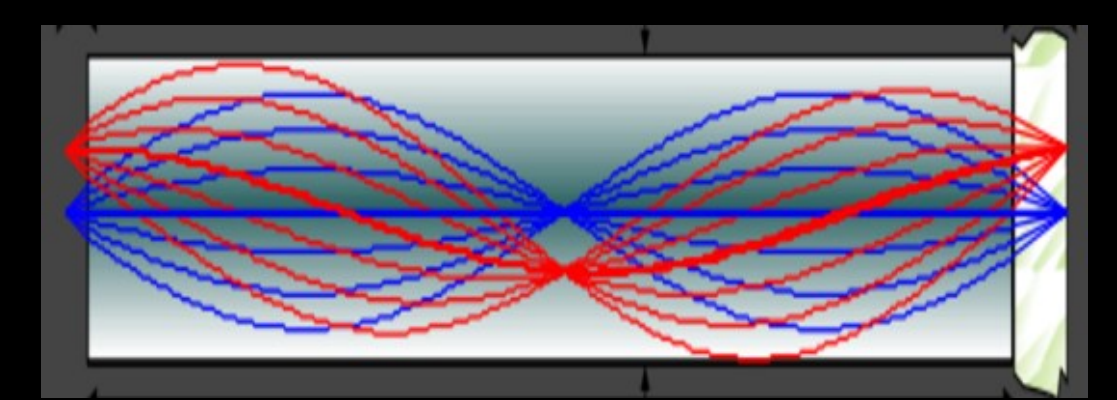




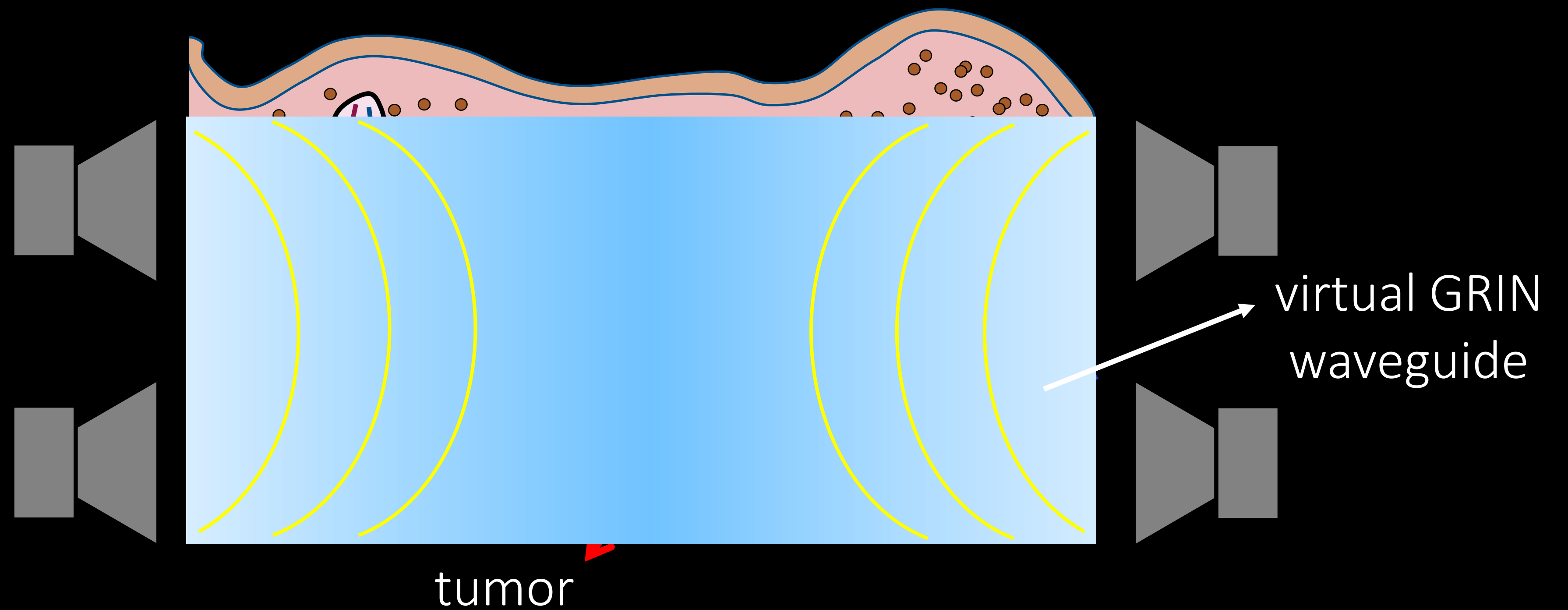
# Focusing light inside tissue



# Focusing light inside tissue

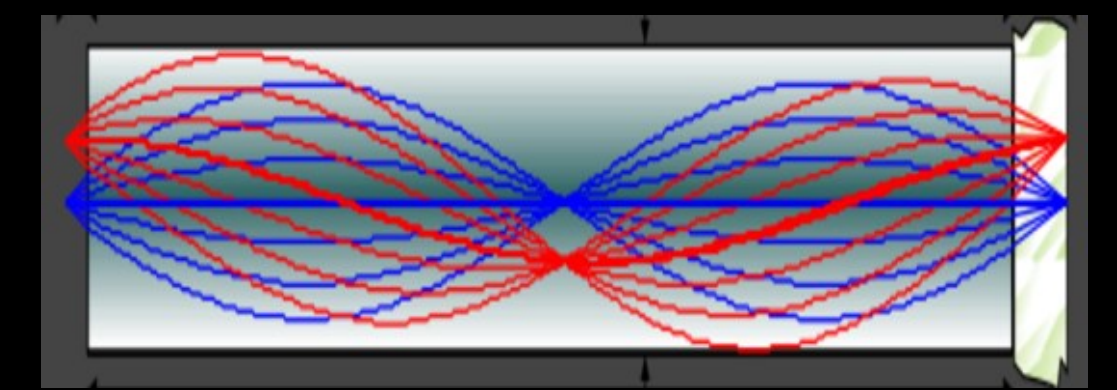


Gradient Refractive  
Index (GRIN) waveguide

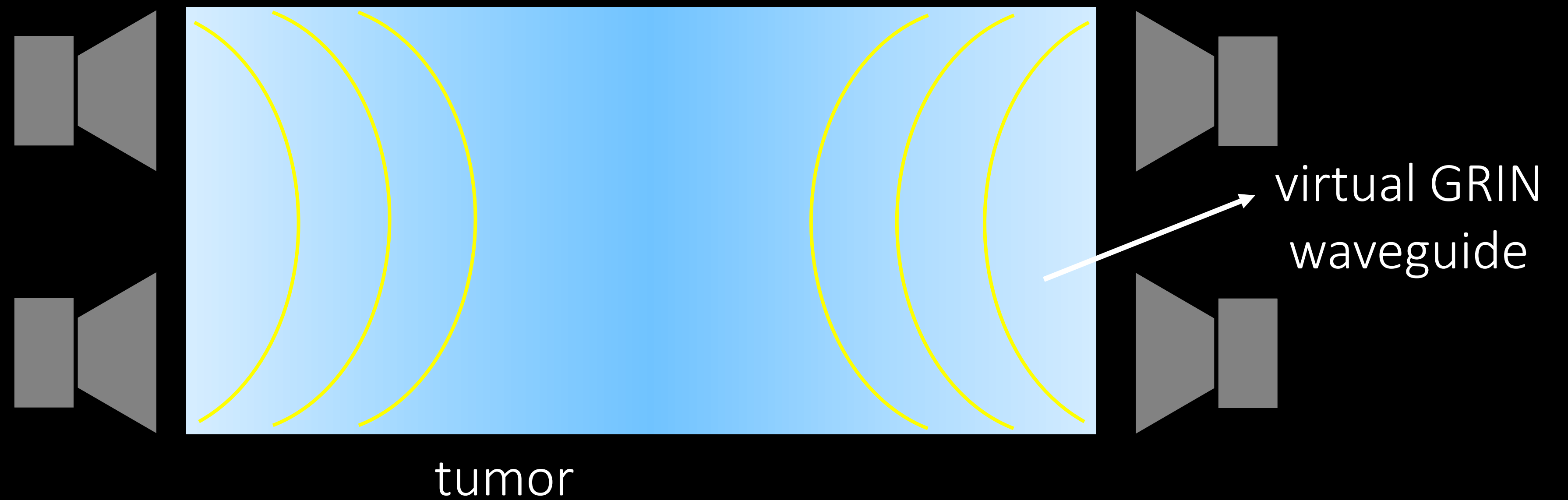




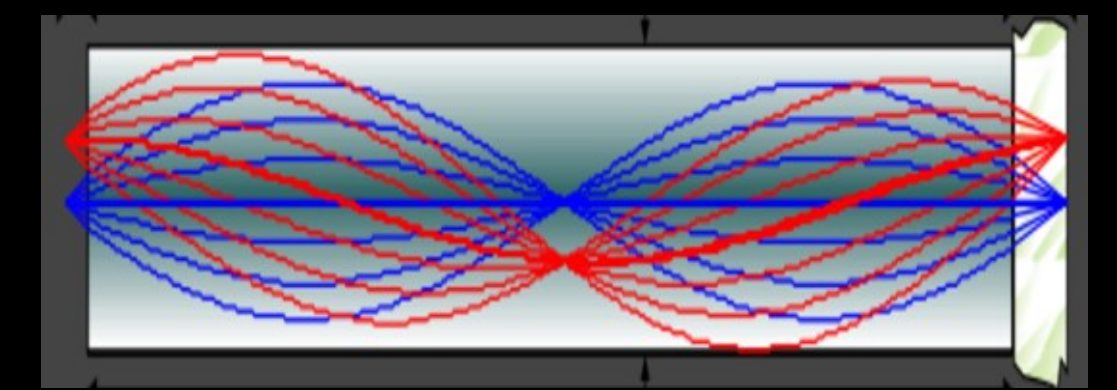
# Focusing light inside tissue



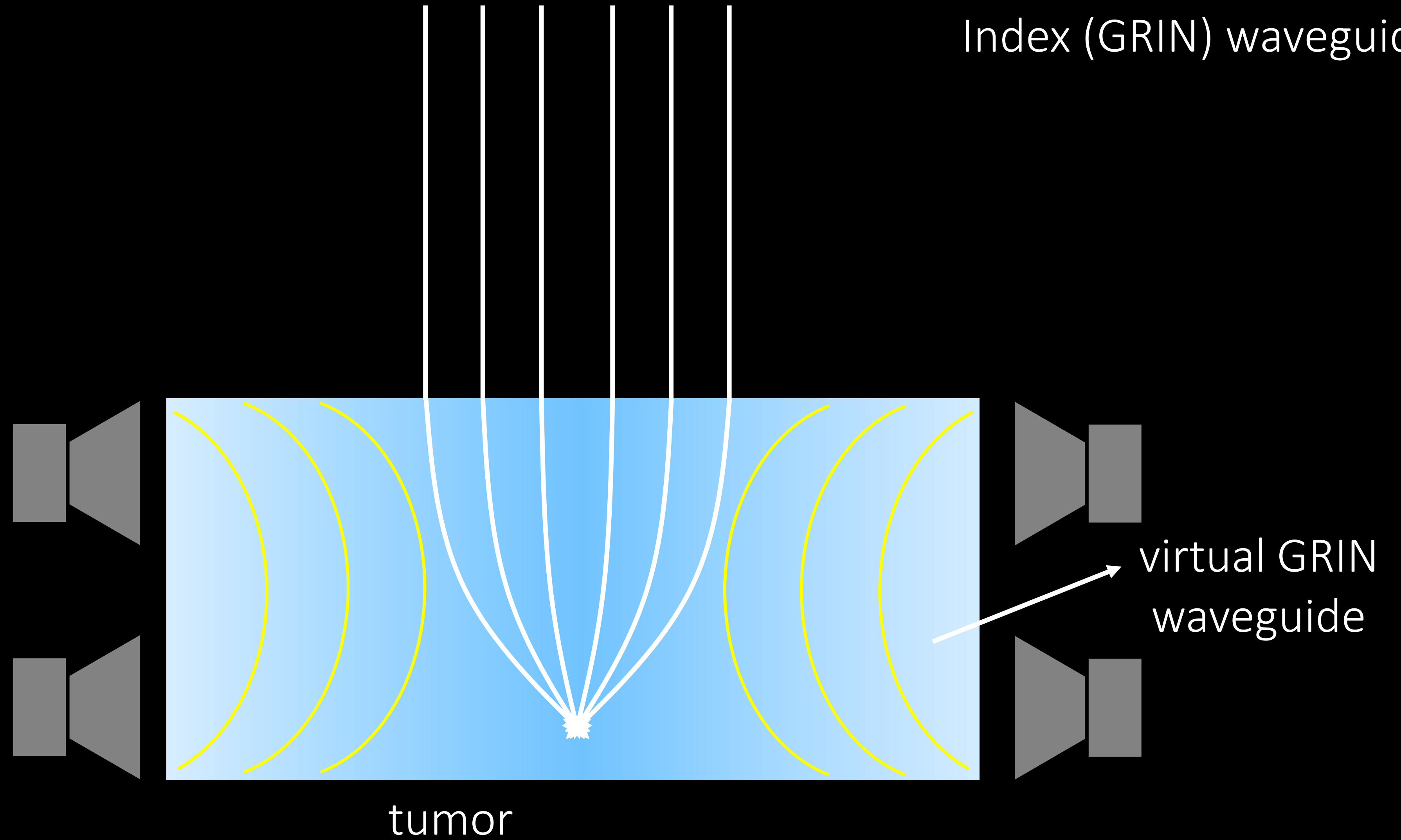
Gradient Refractive  
Index (GRIN) waveguide



# Focusing light inside tissue

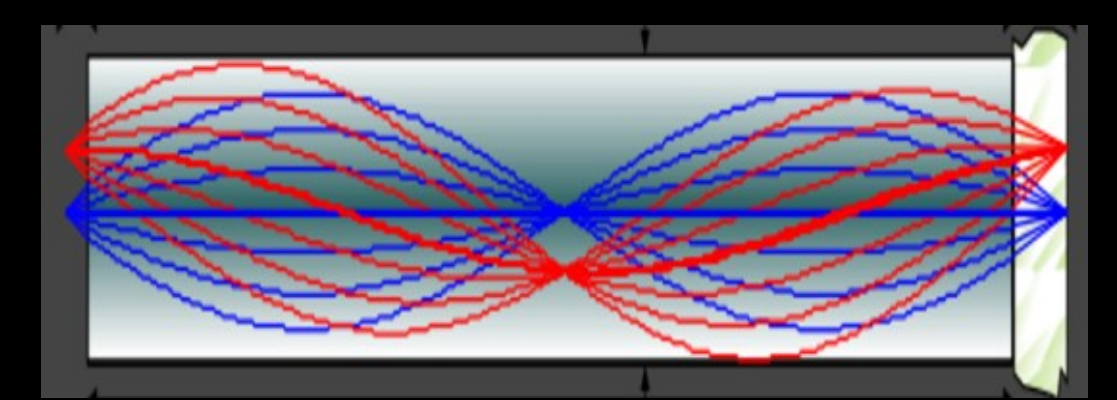


Gradient Refractive  
Index (GRIN) waveguide

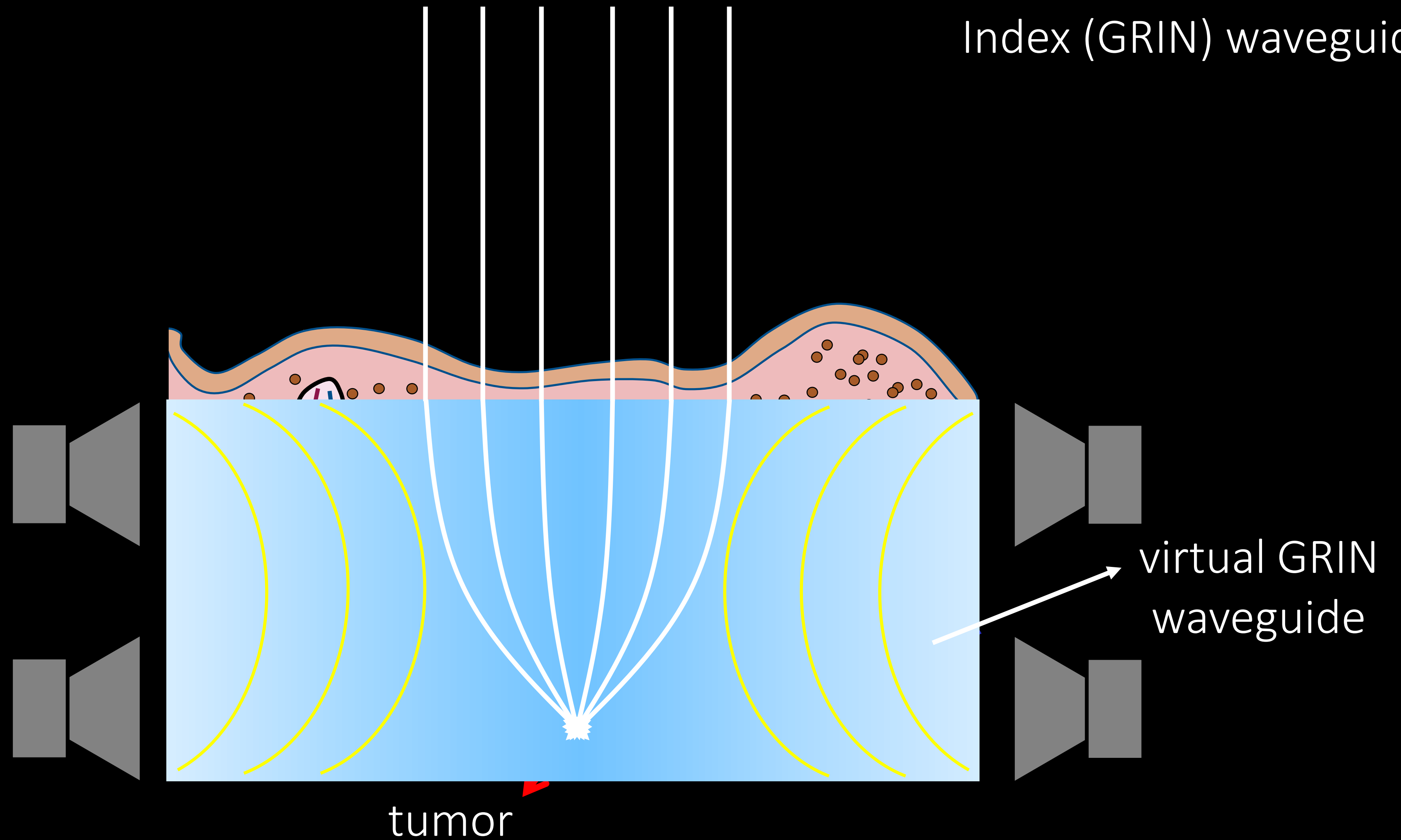




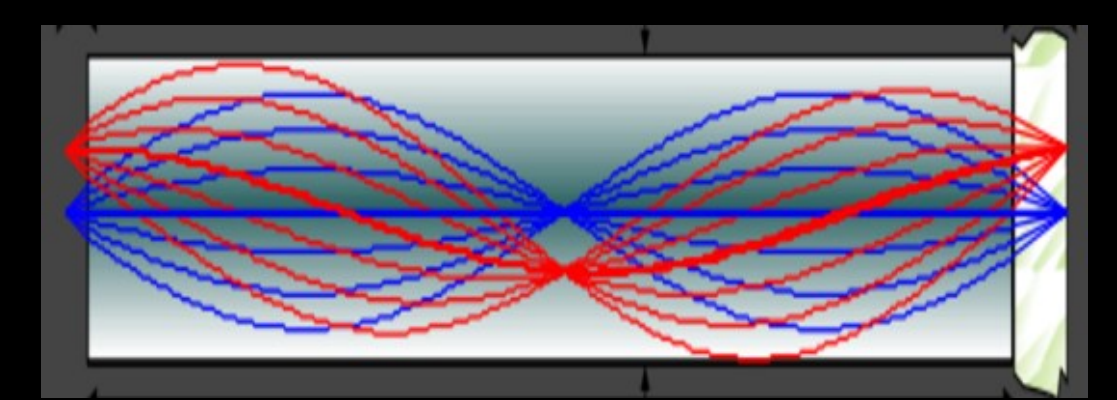
# Focusing light inside tissue



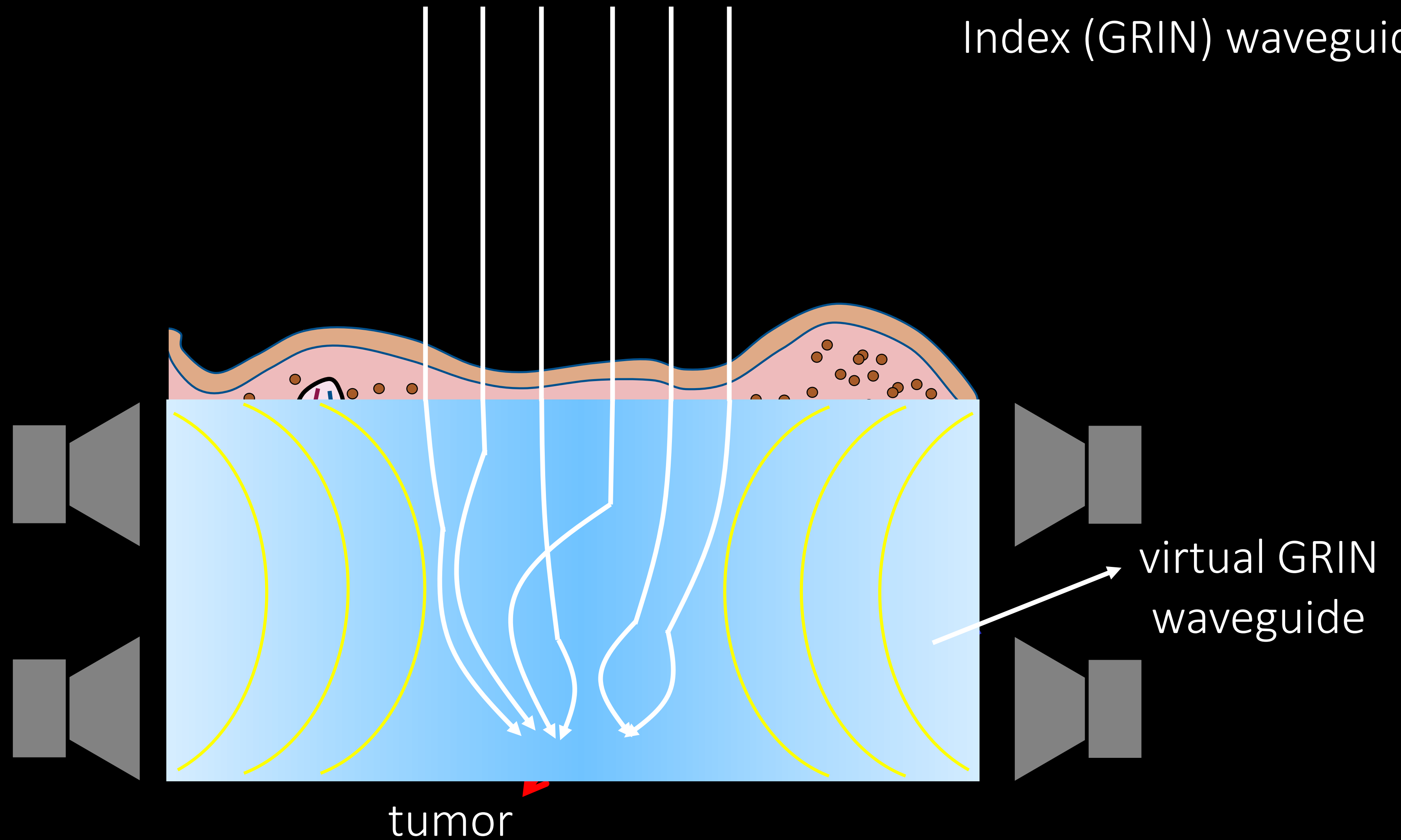
Gradient Refractive  
Index (GRIN) waveguide



# Focusing light inside tissue

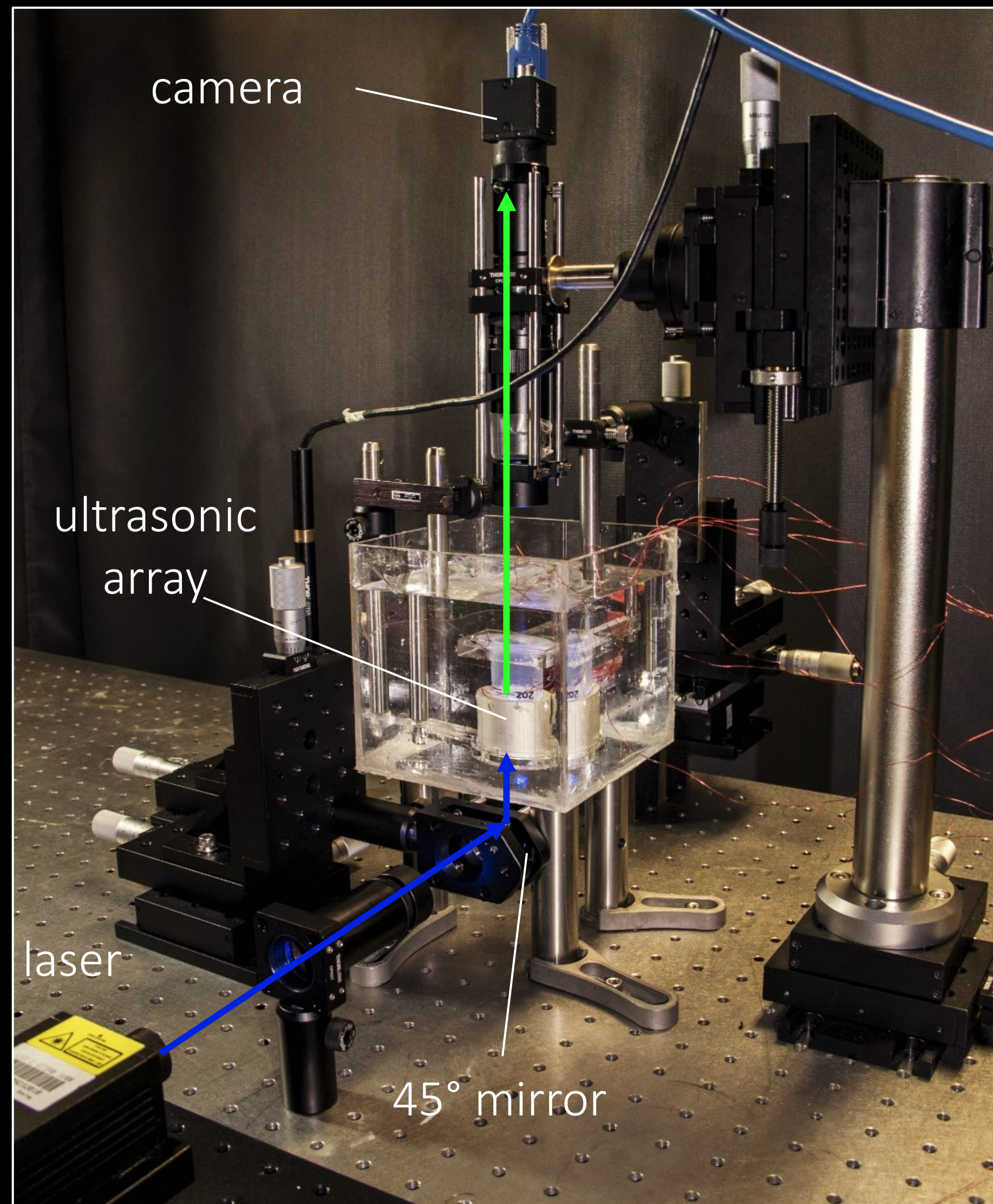


Gradient Refractive  
Index (GRIN) waveguide





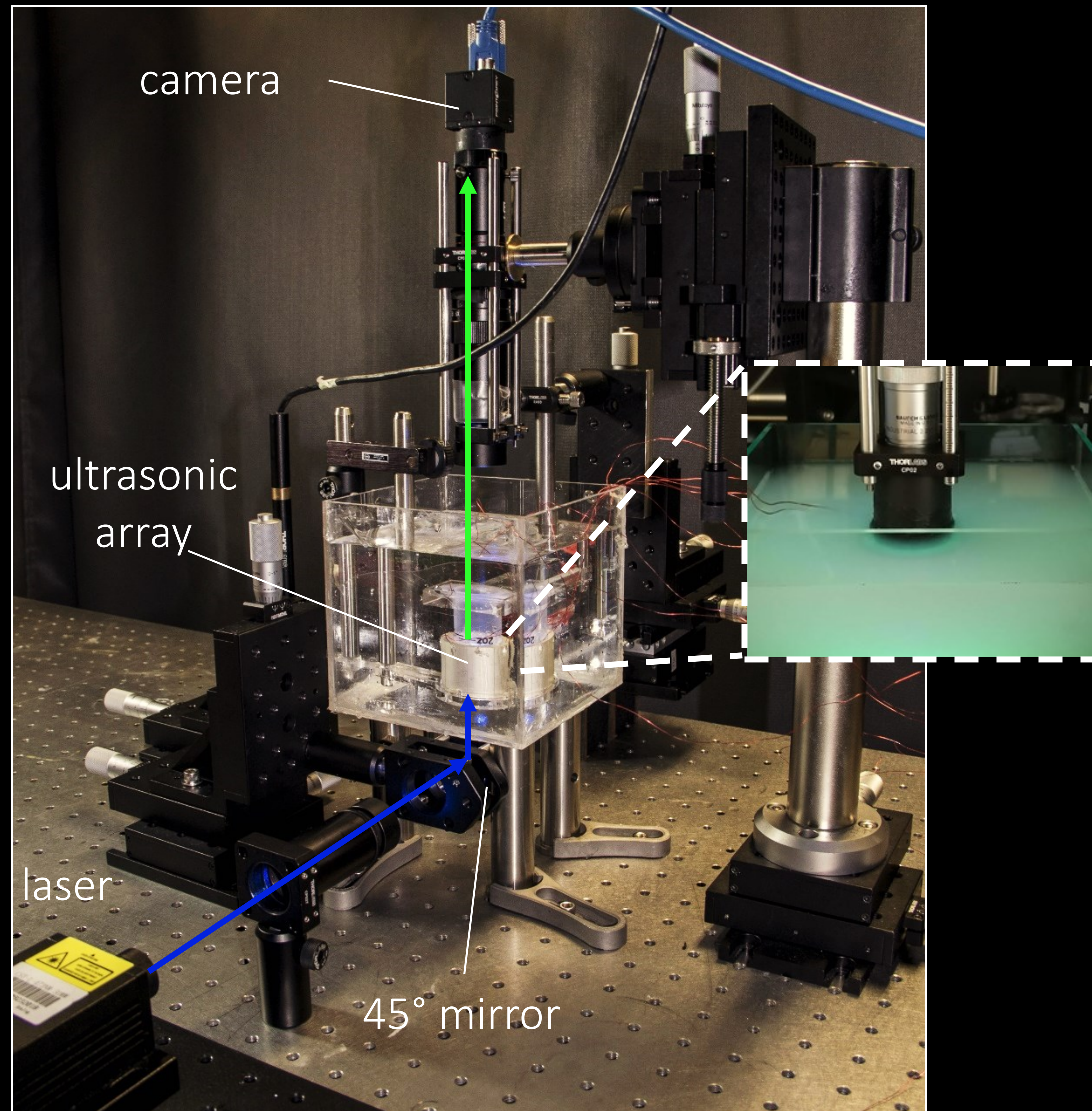
# Ultrasonic light guiding inside tissue



[Chamanzar et al., Nat. Comm. 2019]  
[Karimi et al., Optics Express, 2019]  
[Scopelliti et al., LSA, 2019]



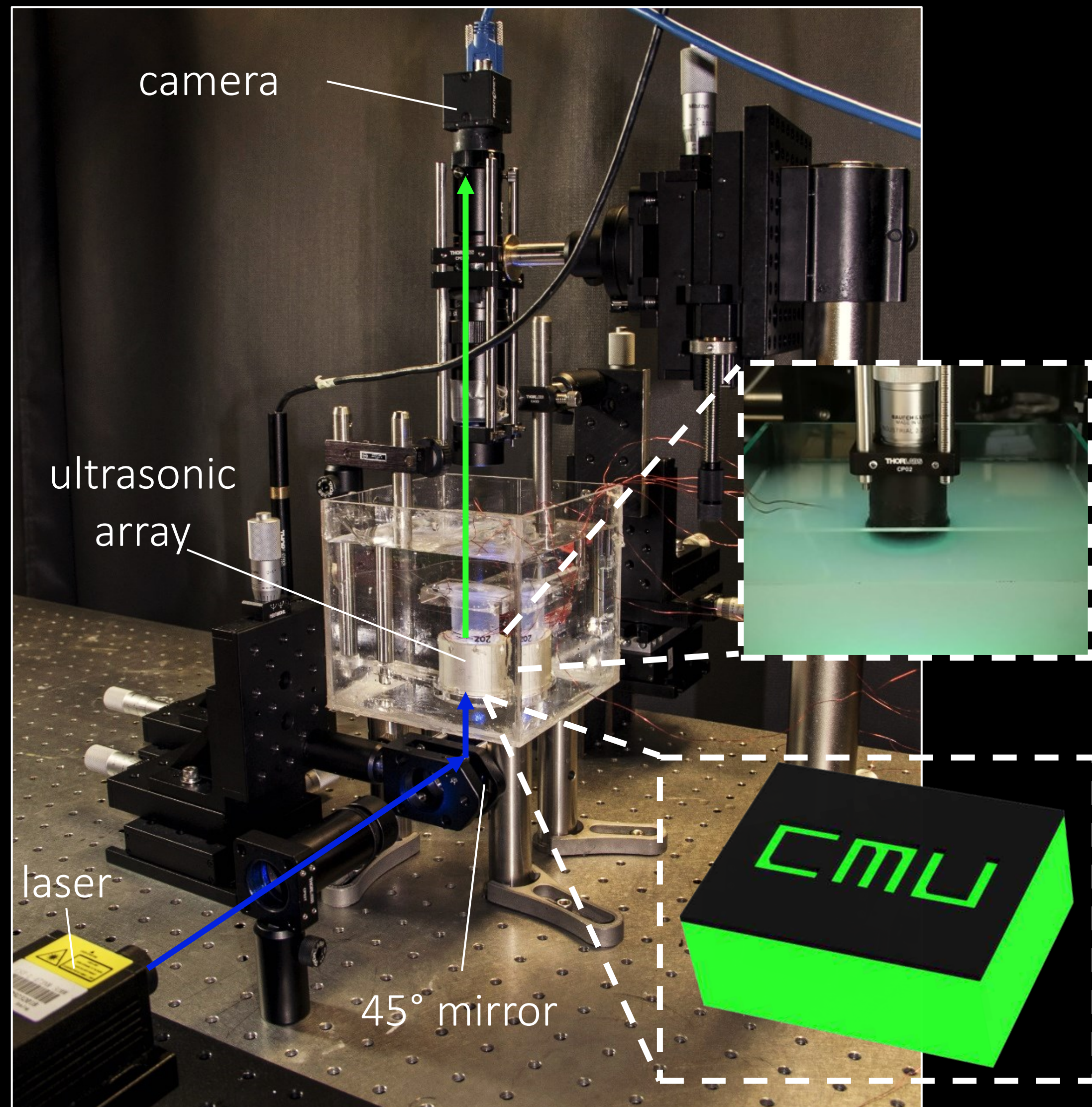
# Ultrasonic light guiding inside tissue



[Chamanzar et al., Nat. Comm. 2019]  
[Karimi et al., Optics Express, 2019]  
[Scopelliti et al., LSA, 2019]



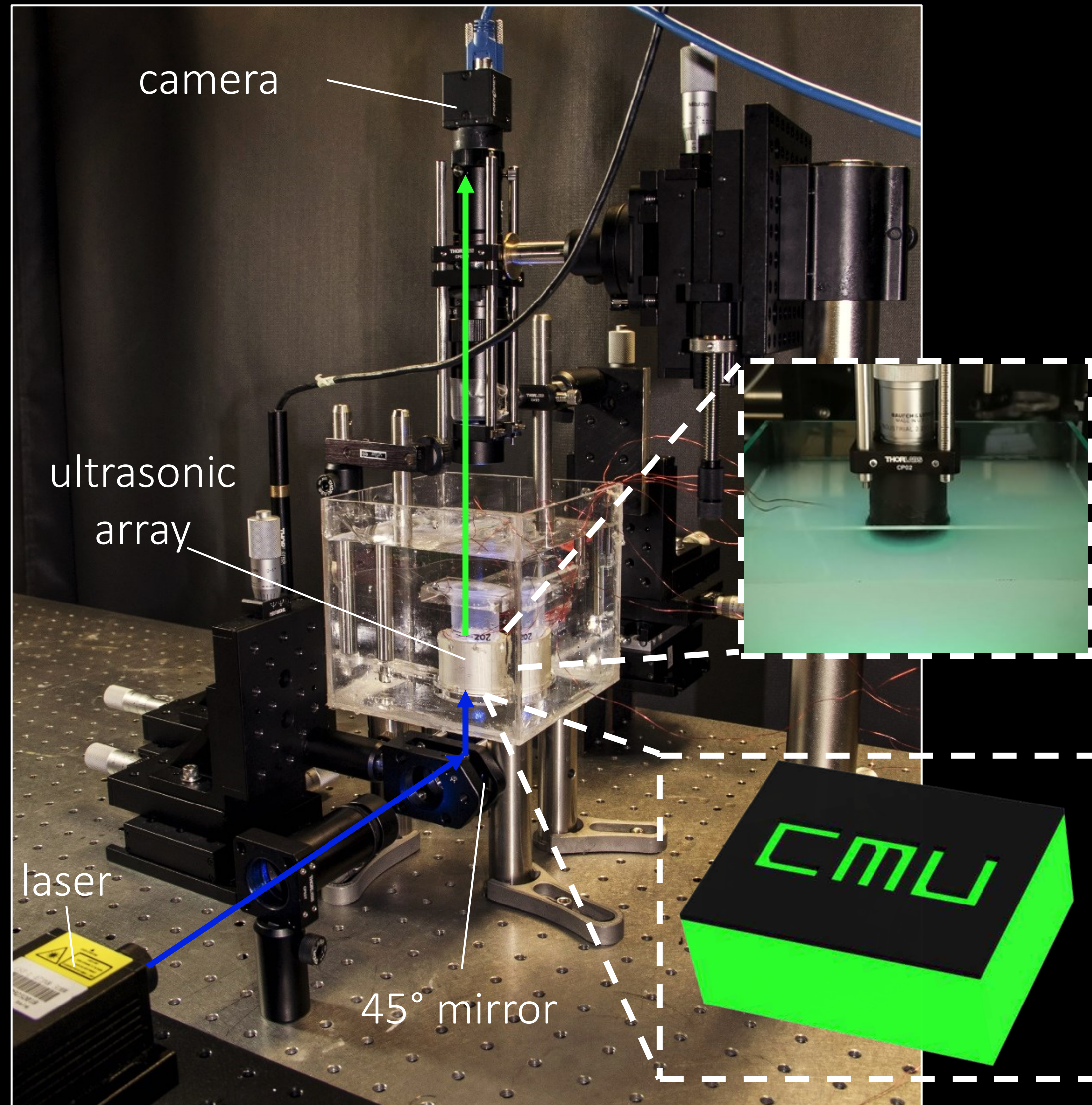
# Ultrasonic light guiding inside tissue



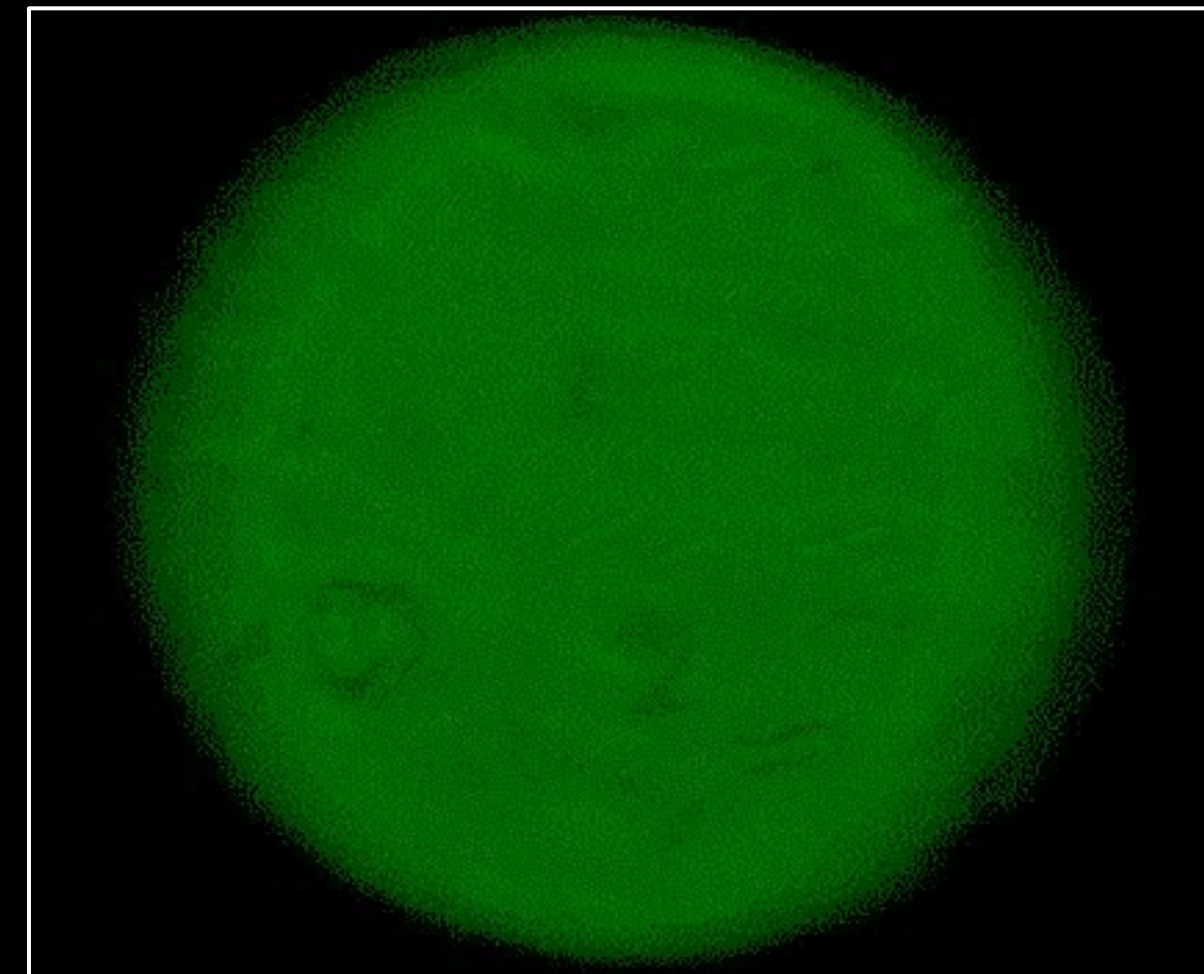
[Chamanzar et al., Nat. Comm. 2019]  
[Karimi et al., Optics Express, 2019]  
[Scopelliti et al., LSA, 2019]



# Ultrasonic light guiding inside tissue



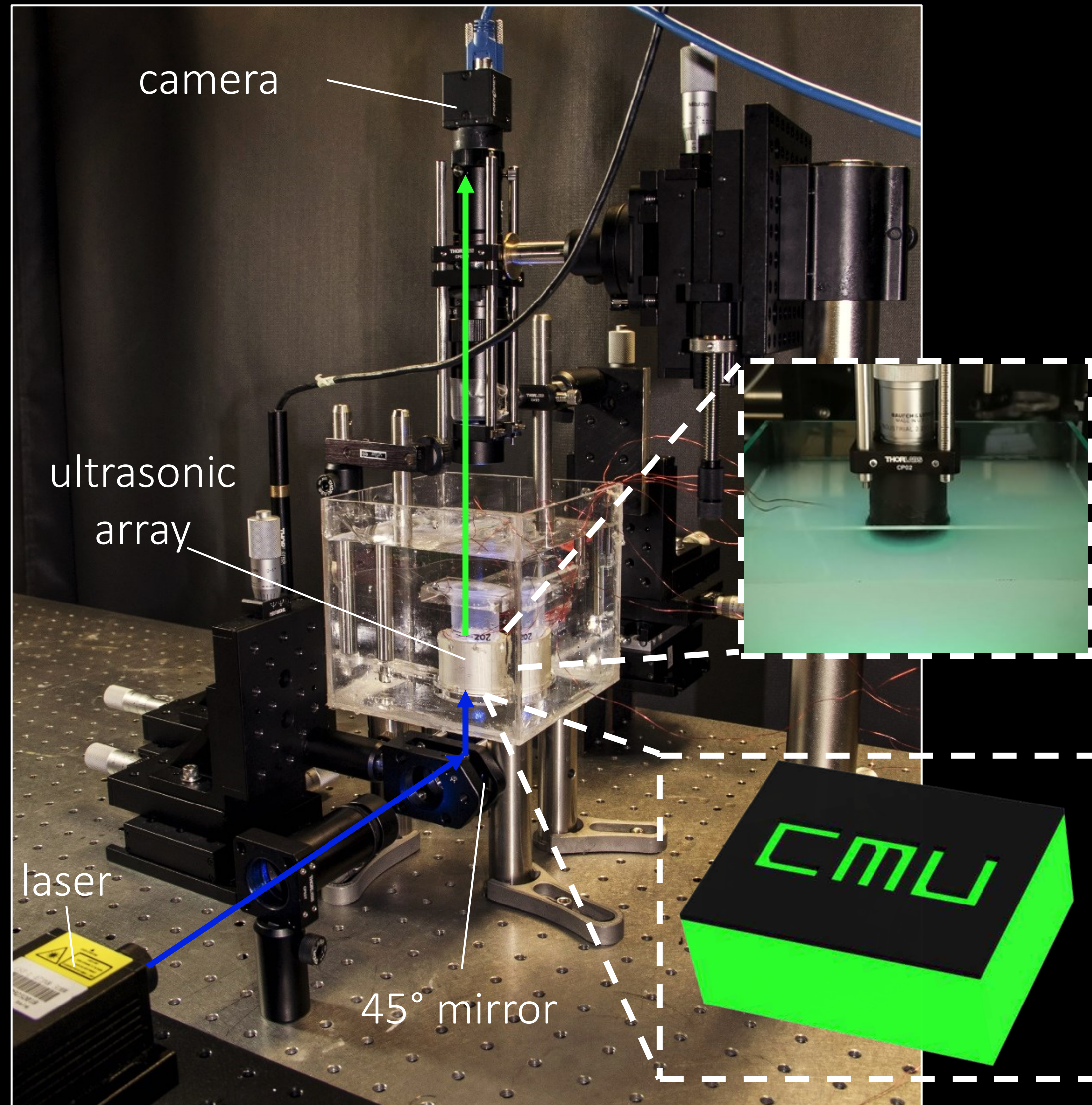
ultrasound off



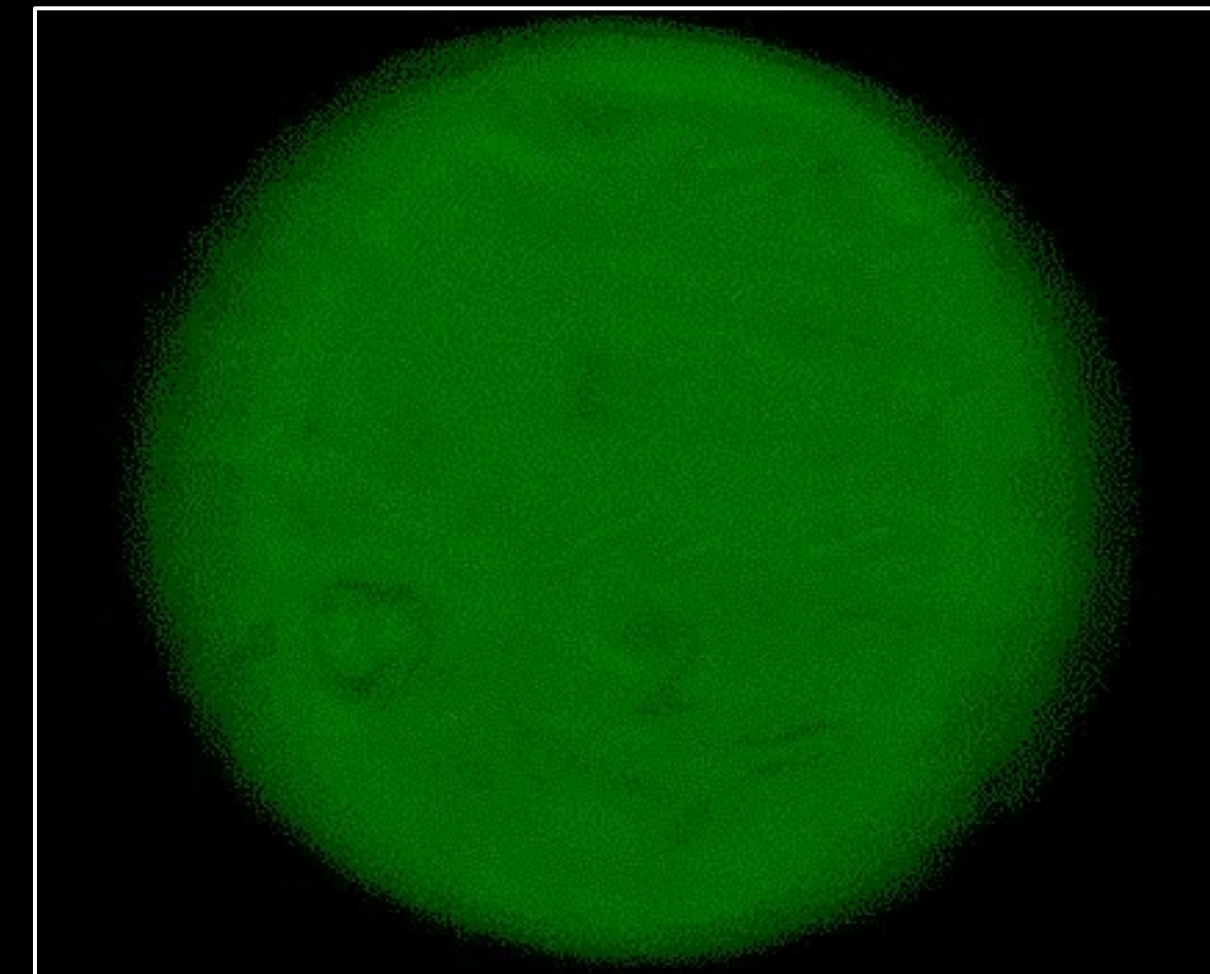
[Chamanzar et al., Nat. Comm. 2019]  
[Karimi et al., Optics Express, 2019]  
[Scopelliti et al., LSA, 2019]



# Ultrasonic light guiding inside tissue



ultrasound off



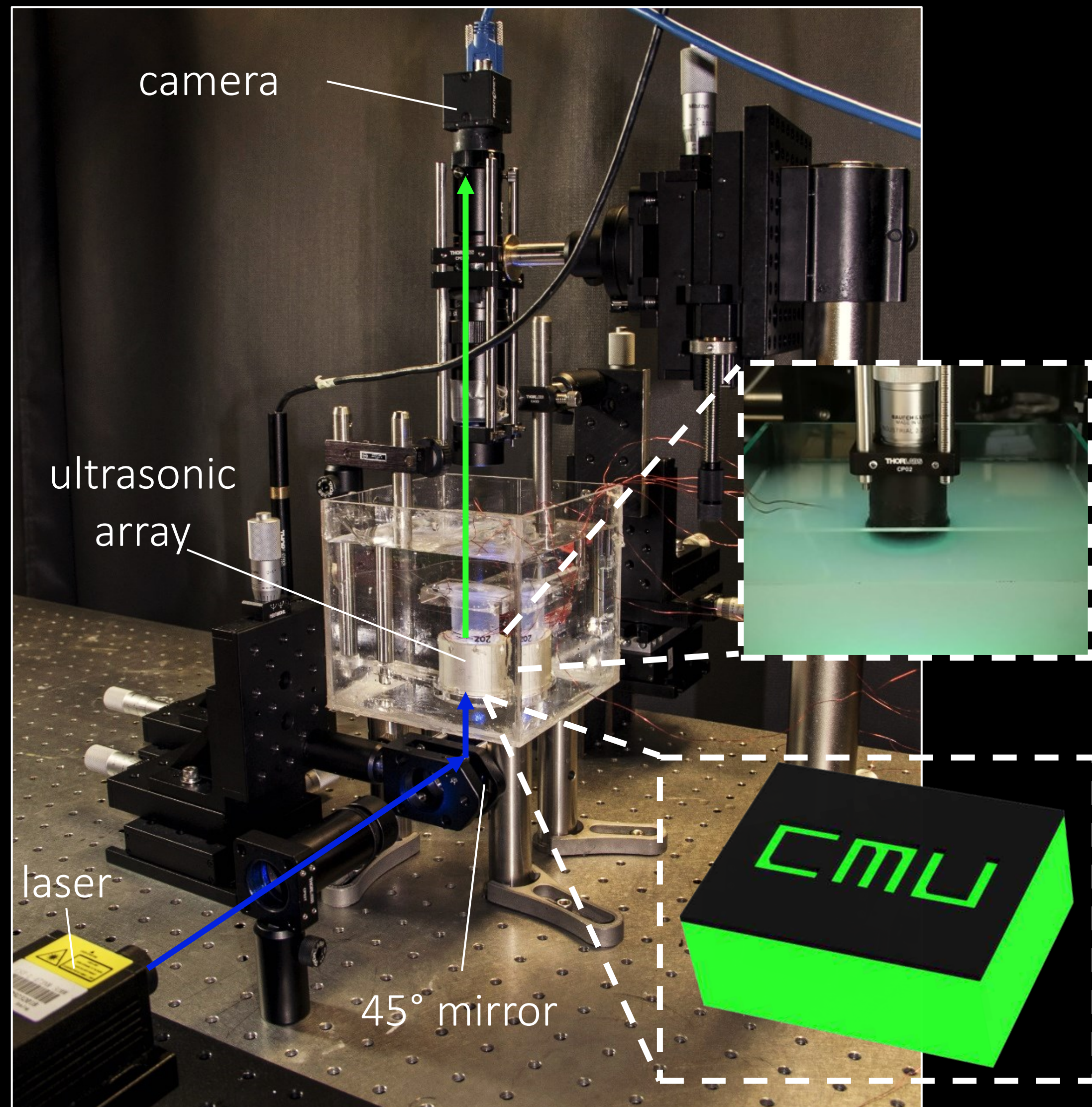
ultrasound on



[Chamanzar et al., Nat. Comm. 2019]  
[Karimi et al., Optics Express, 2019]  
[Scopelliti et al., LSA, 2019]



# Ultrasonic light guiding inside tissue

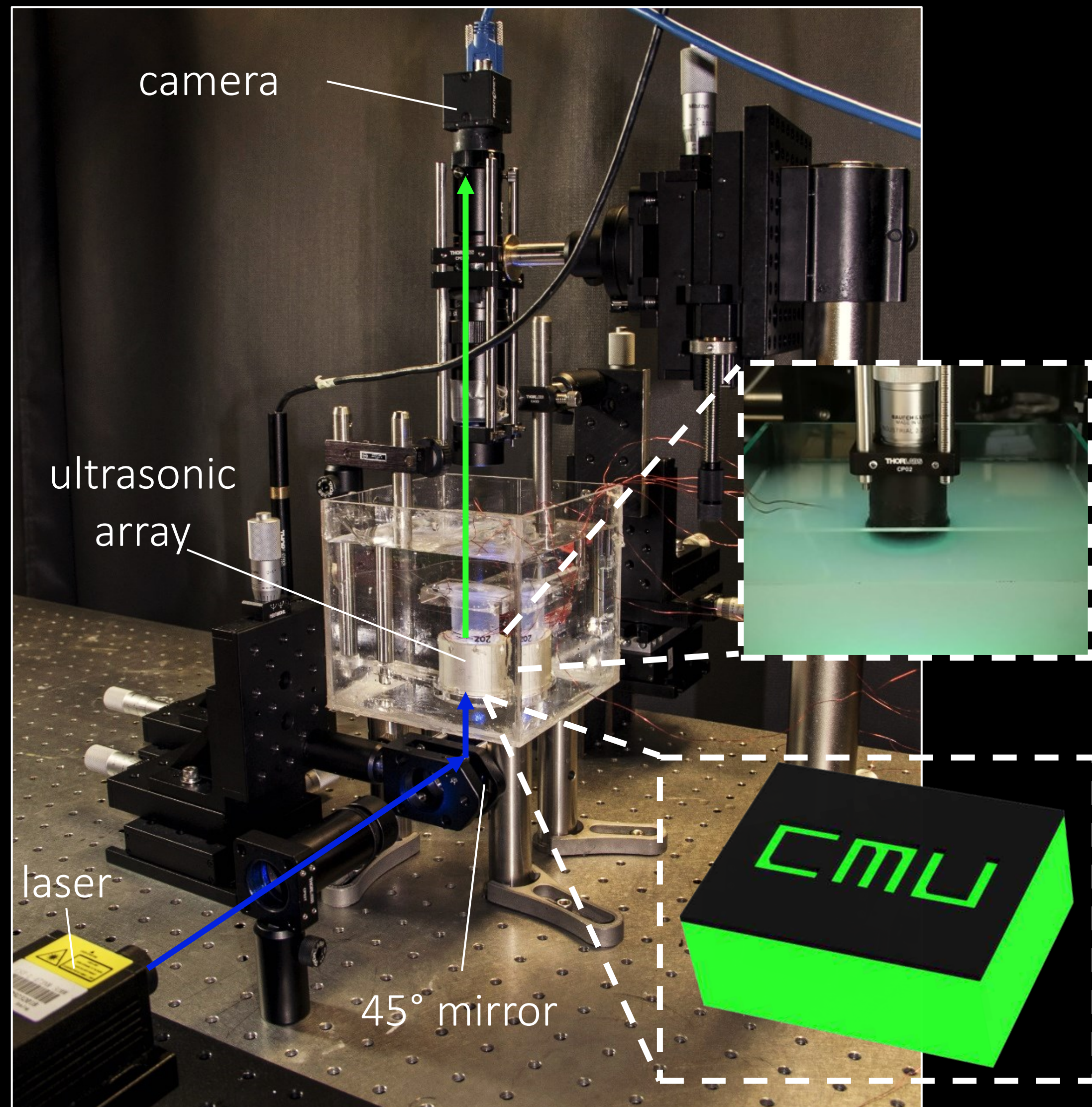


High-dimensional, highly-non-linear design problem:

- ultrasound frequency
- ultrasound voltage
- placement of transducers
- shape of waveguides
- waveform shape
- and more...



# Ultrasonic light guiding inside tissue



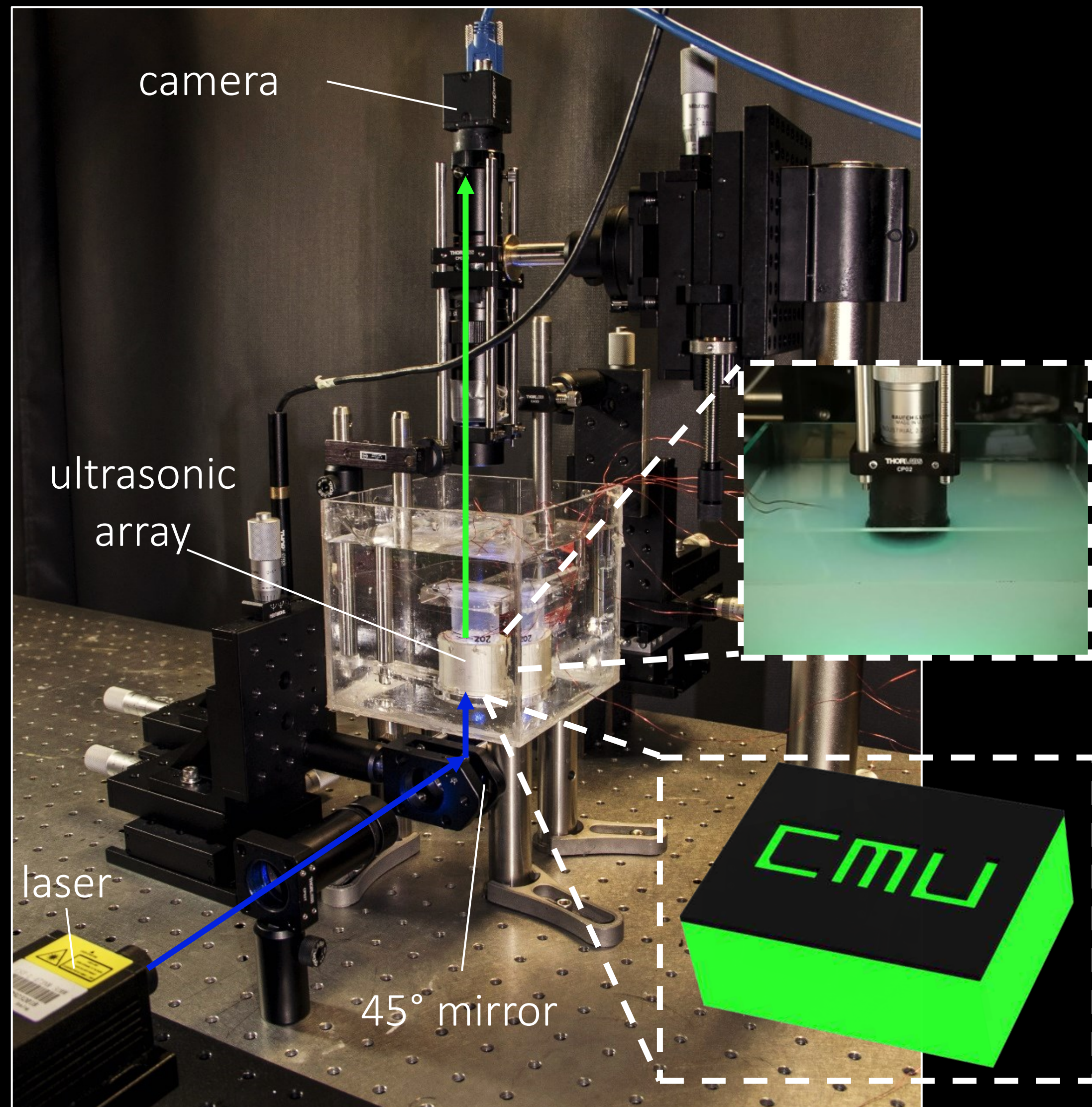
High-dimensional, highly-non-linear design problem:

- ultrasound frequency
- ultrasound voltage
- placement of transducers
- shape of waveguides
- waveform shape
- and more...

Efficiently explore using rendering



# Ultrasonic light guiding inside tissue



High-dimensional, highly-non-linear design problem:

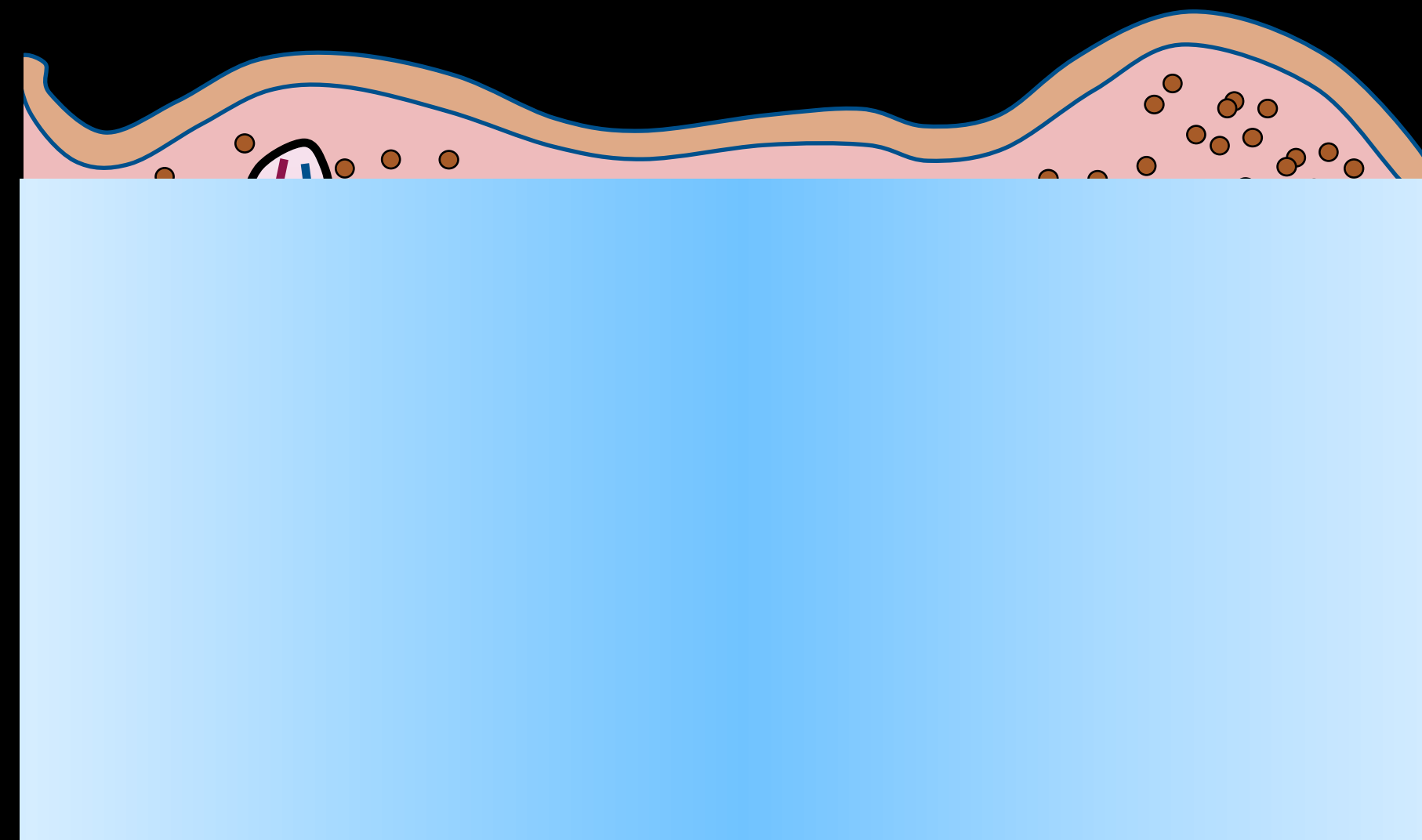
- ultrasound frequency
- ultrasound voltage
- placement of transducers
- shape of waveguides
- waveform shape
- and more...

Efficiently explore using rendering

Build first rendering algorithm



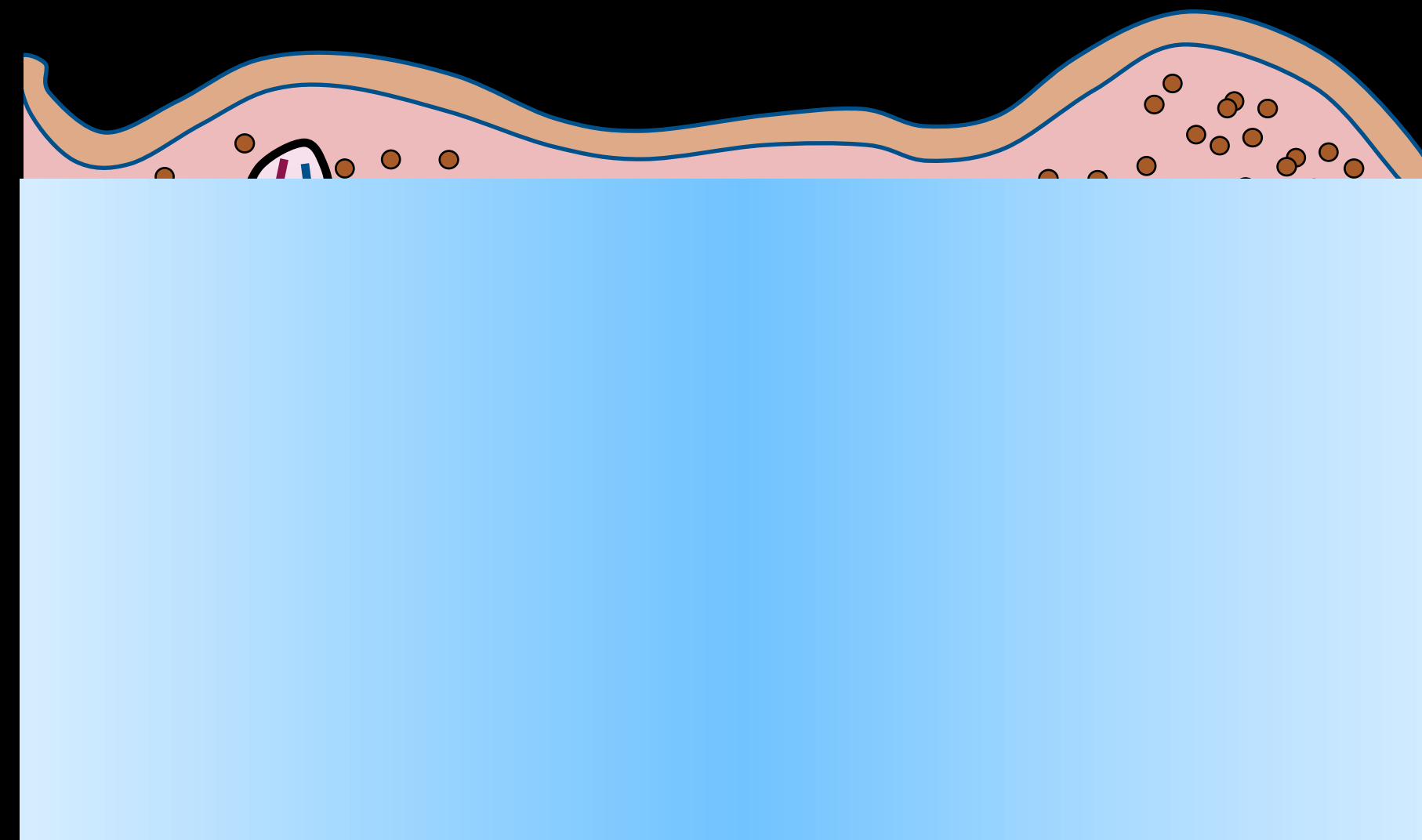
# Rendering continuous refraction and scattering



# Rendering continuous refraction and scattering

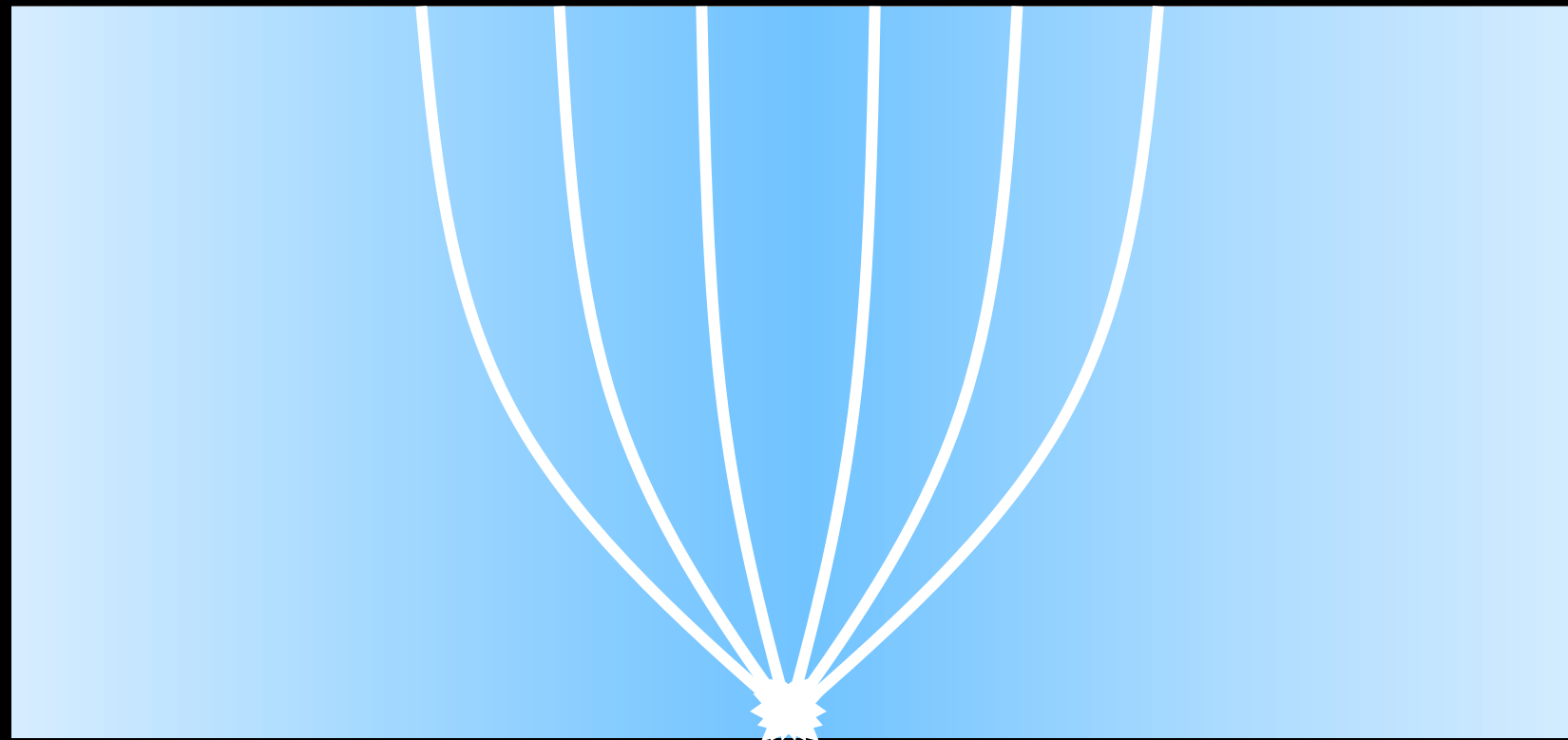


continuous refraction

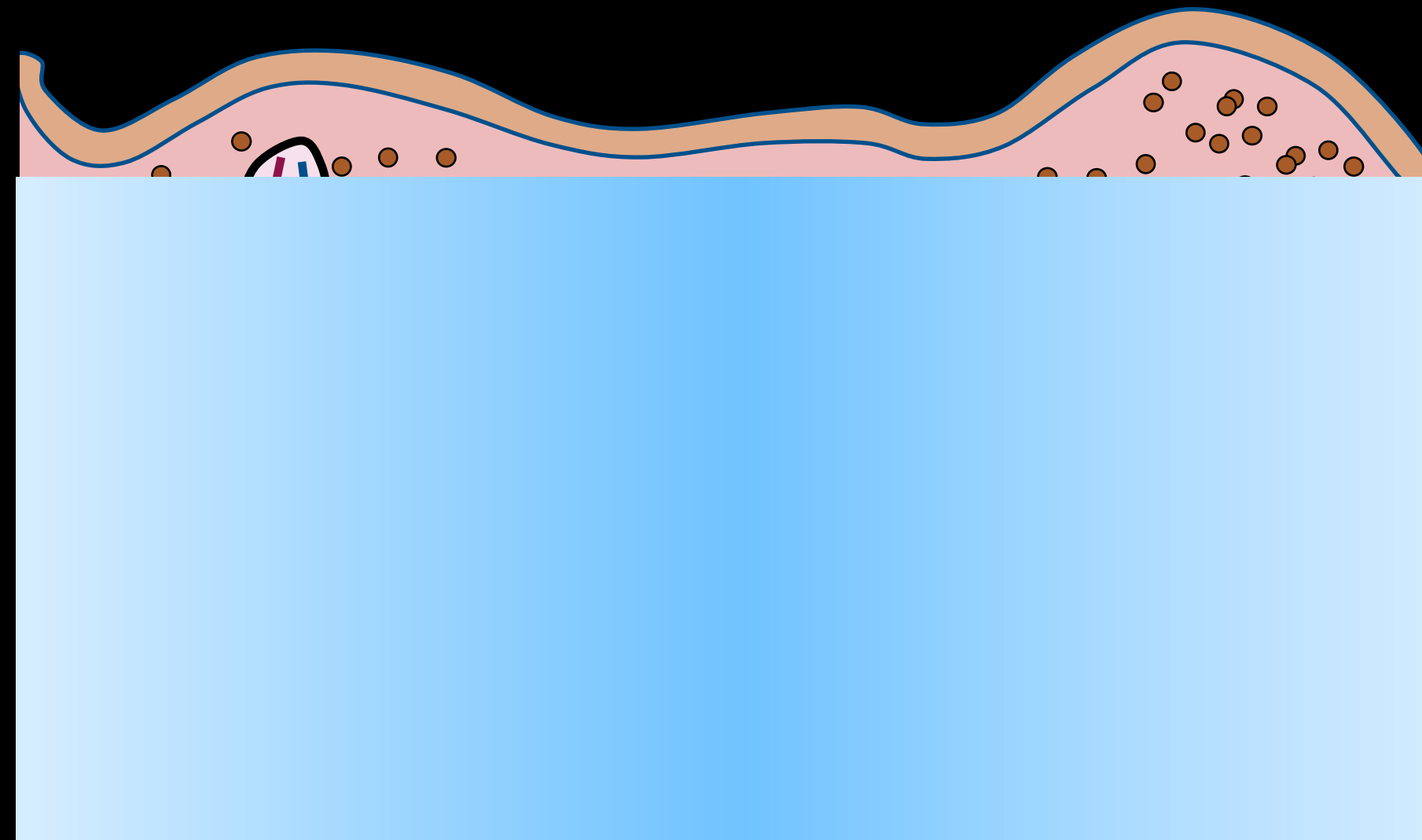




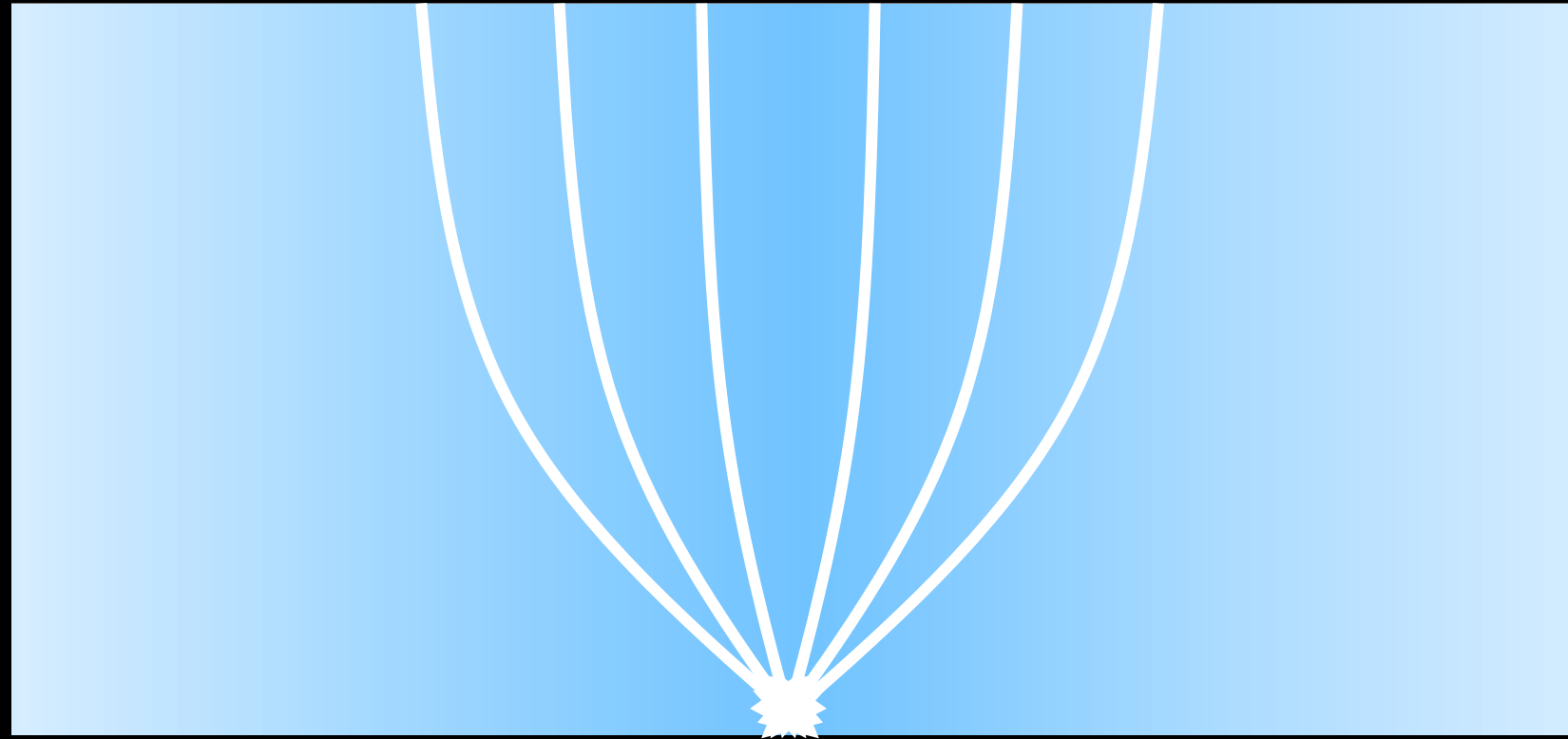
# Rendering continuous refraction and scattering



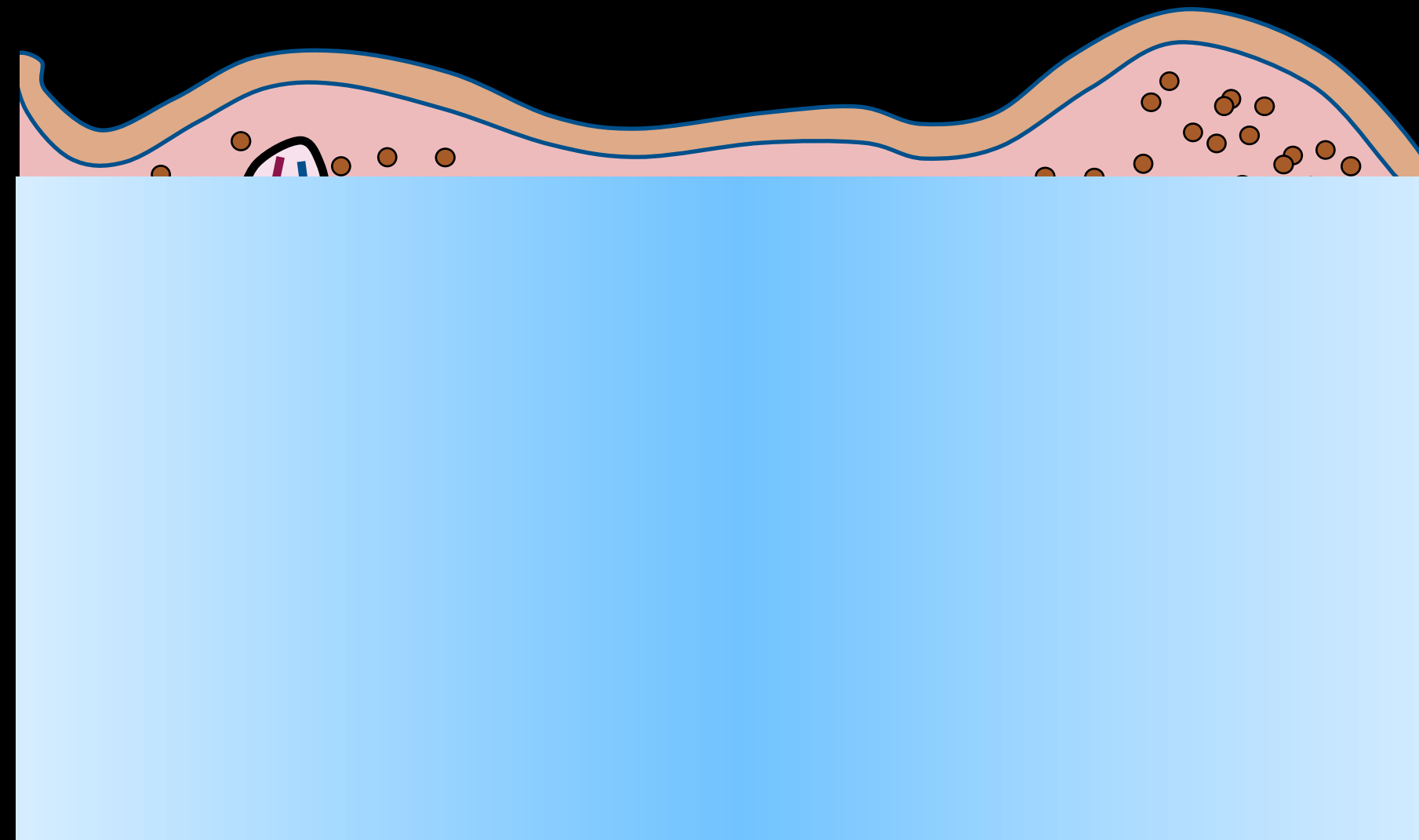
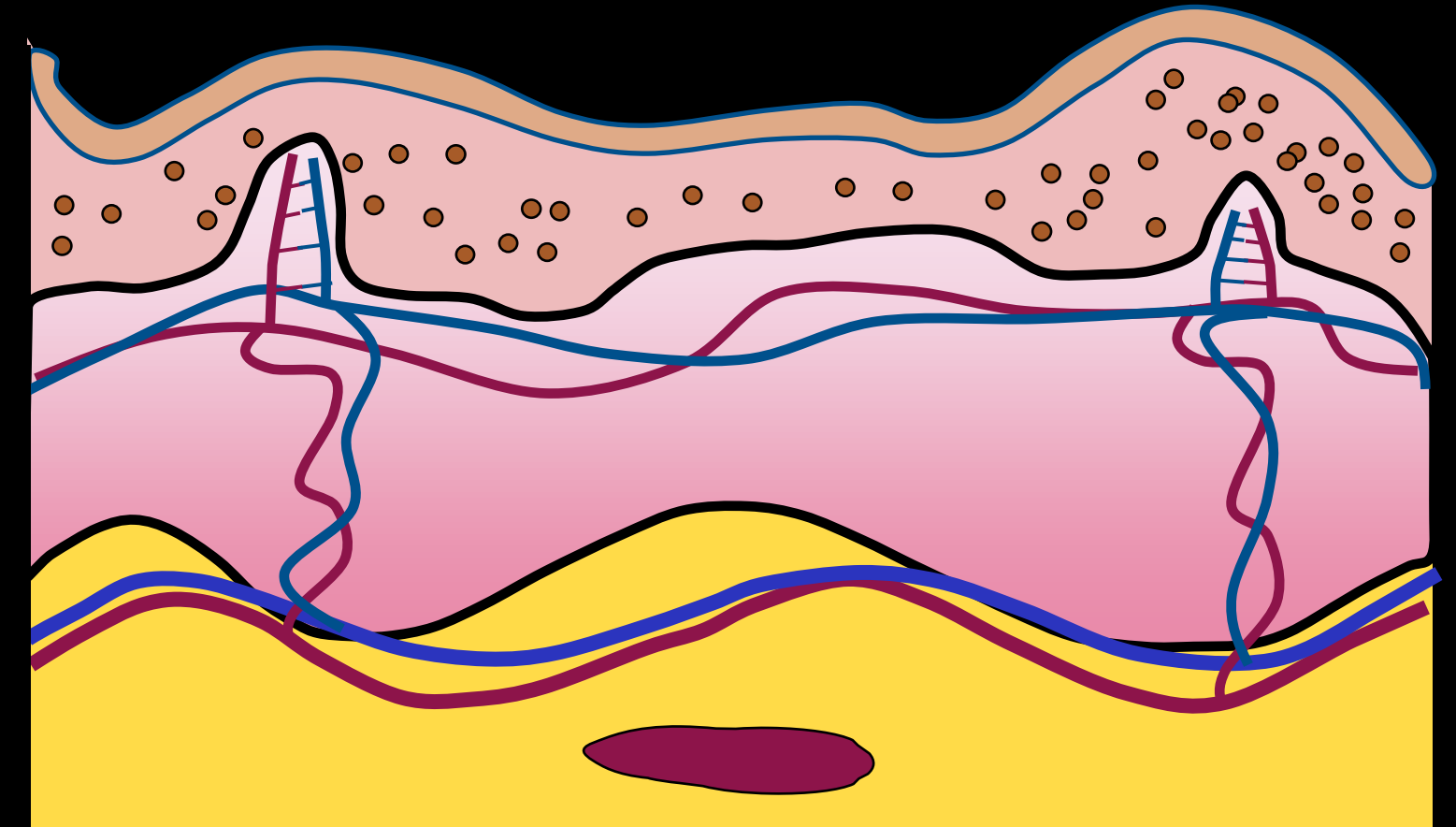
continuous refraction



# Rendering continuous refraction and scattering

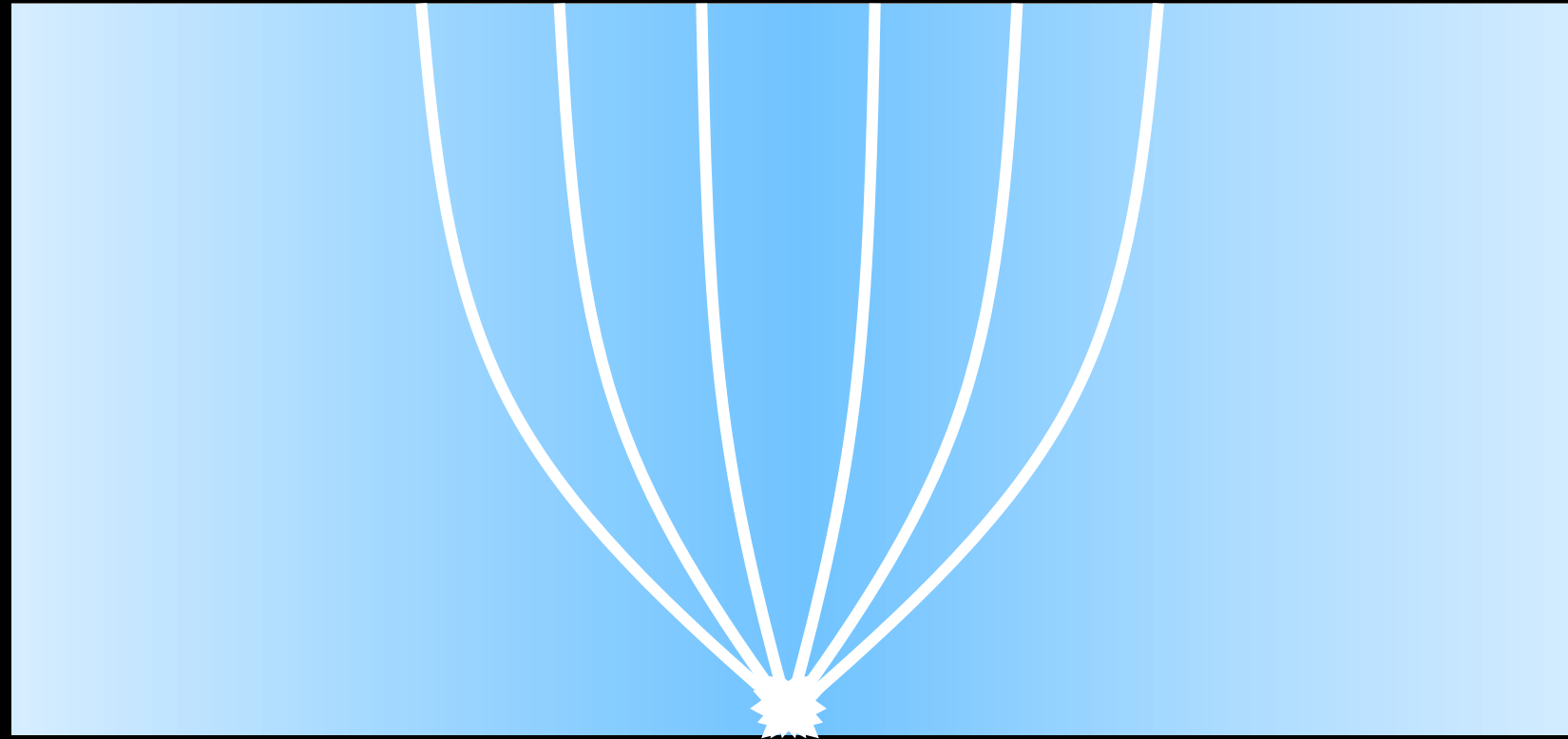


continuous refraction

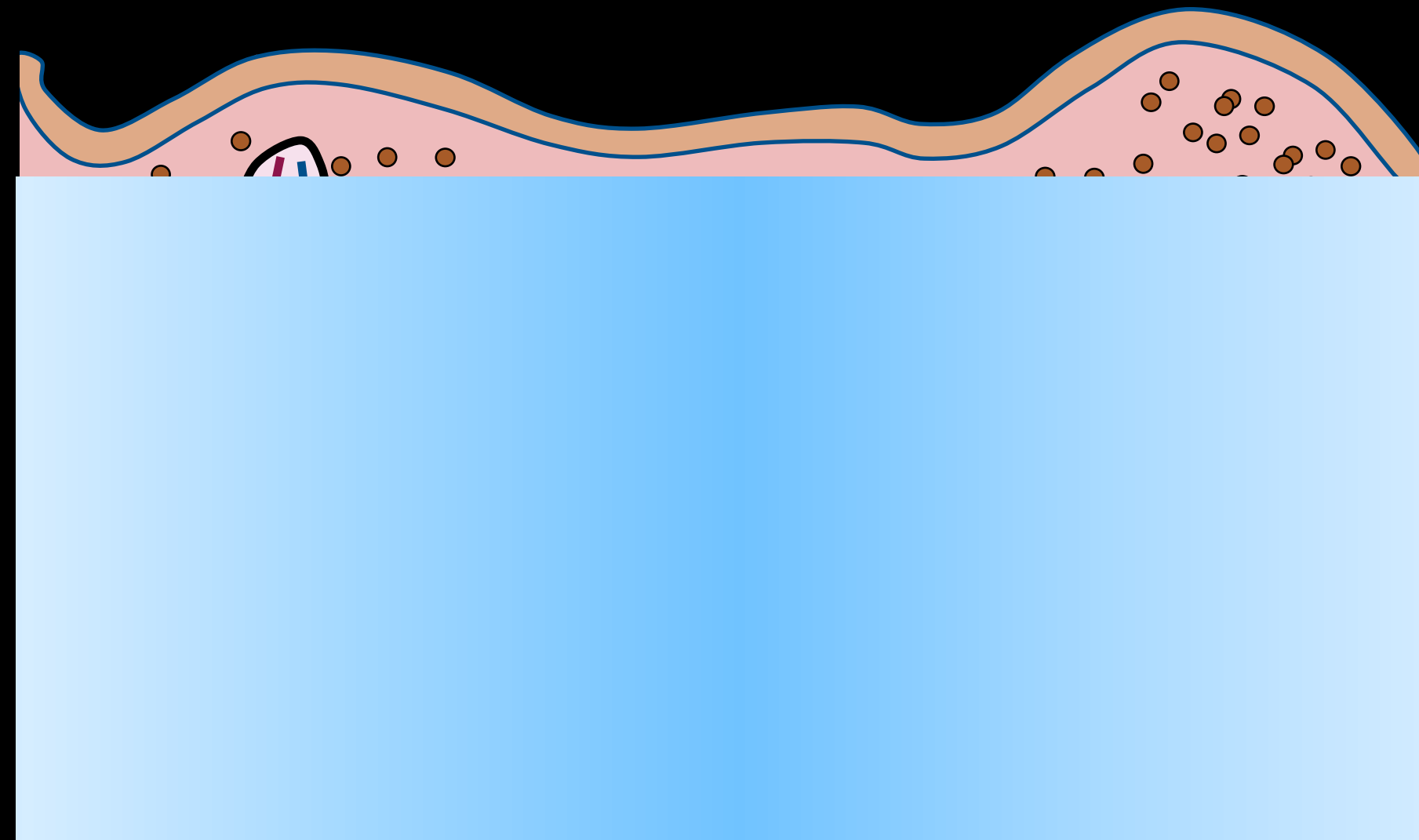
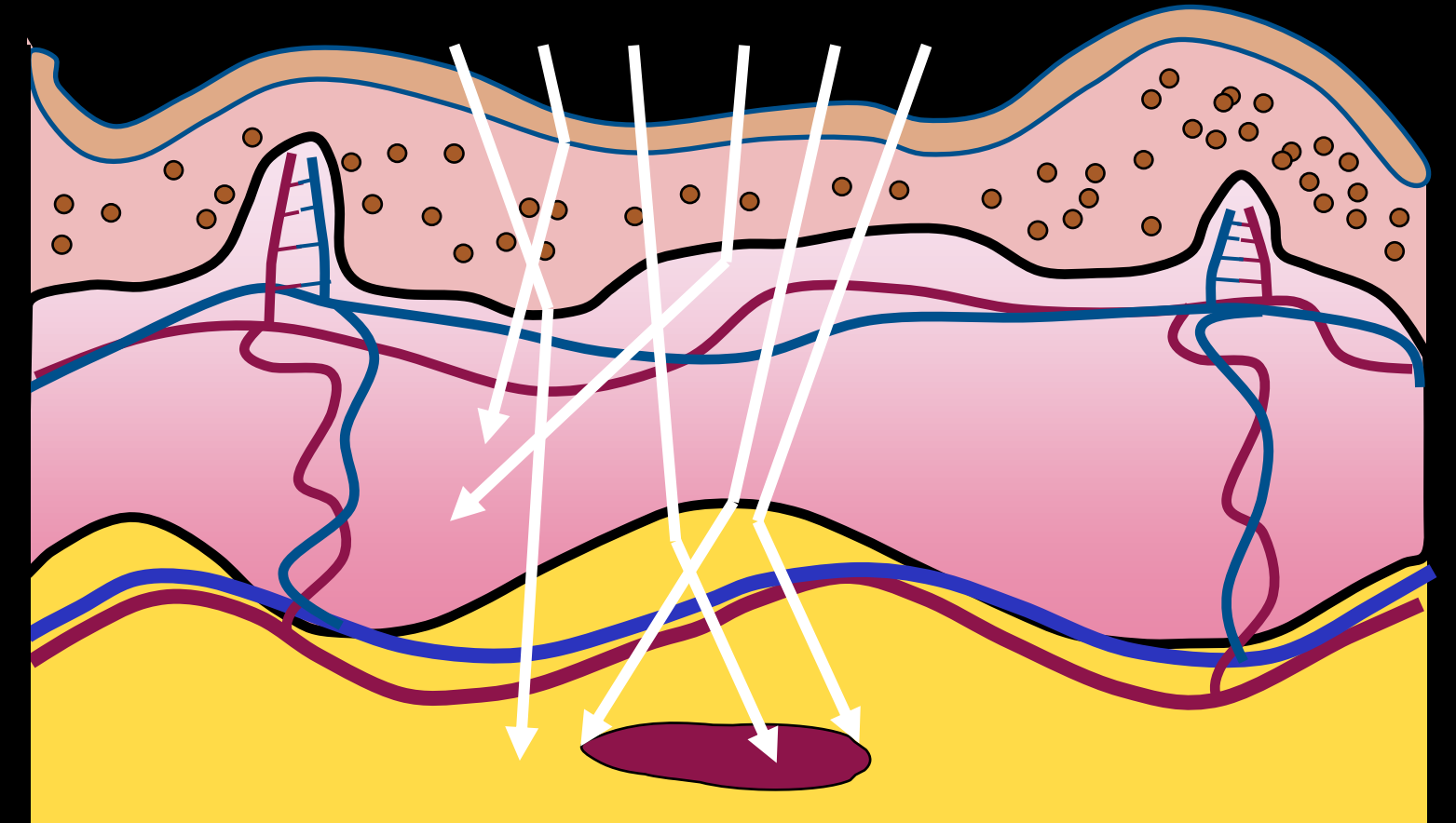




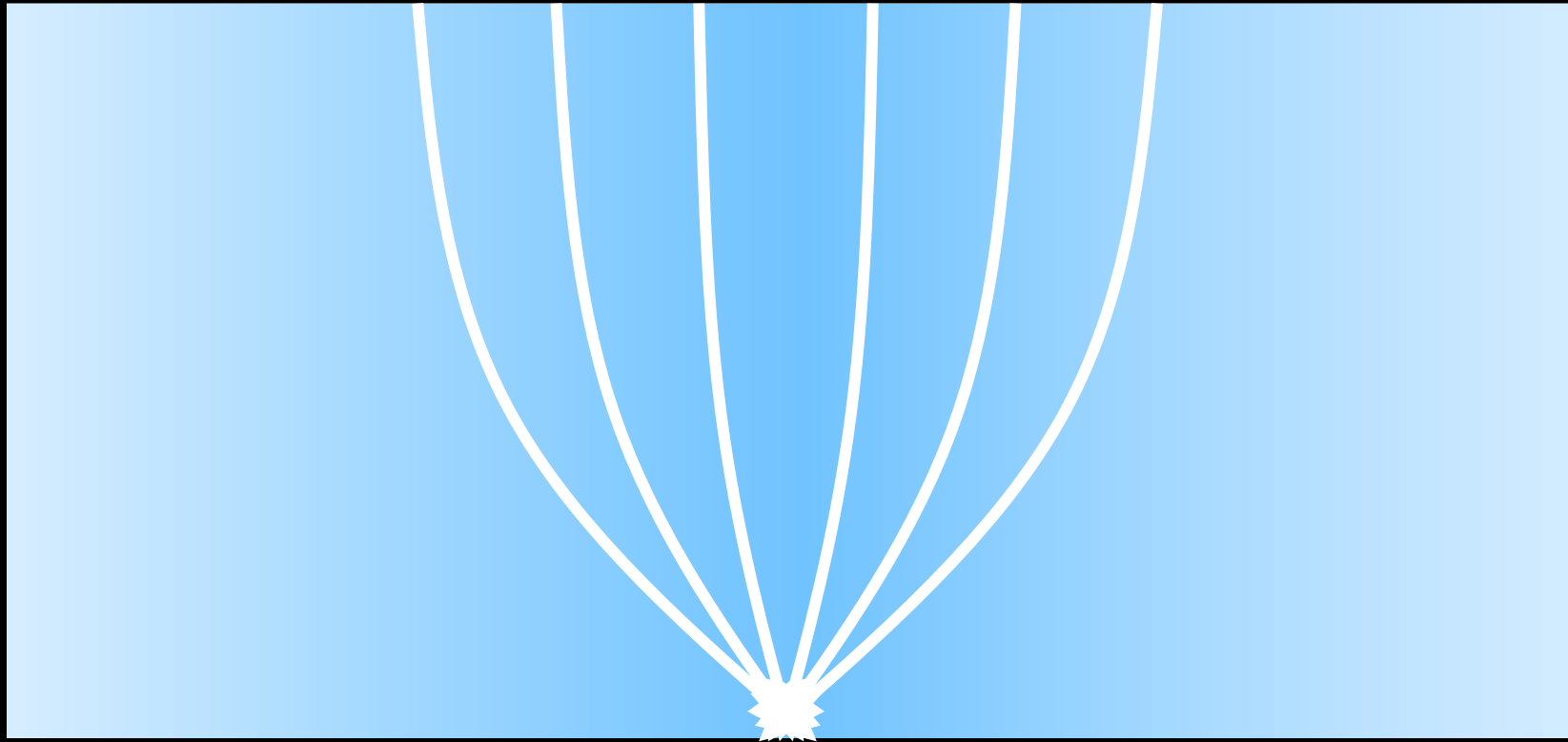
# Rendering continuous refraction and scattering



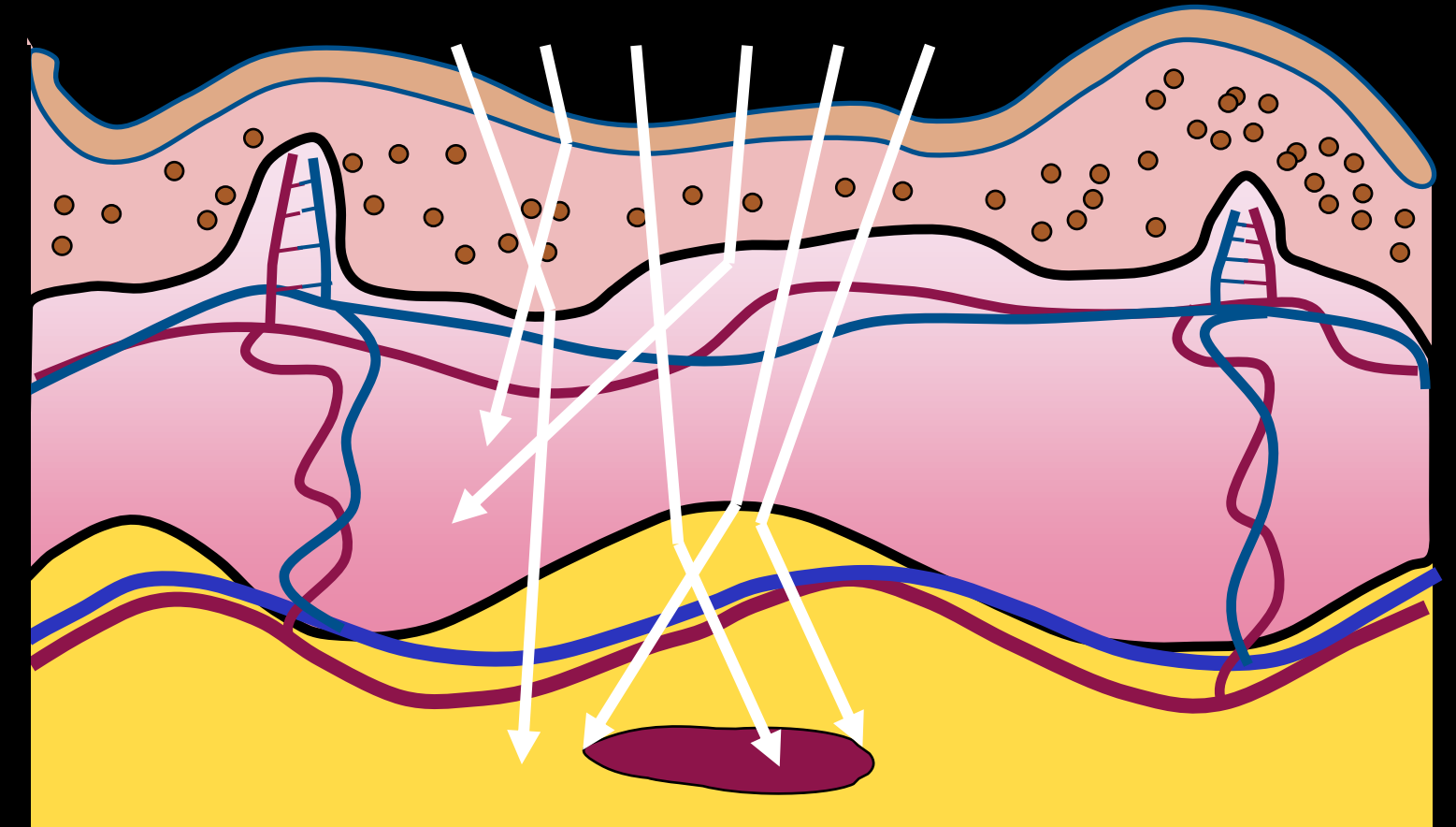
continuous refraction  
+  
scattering



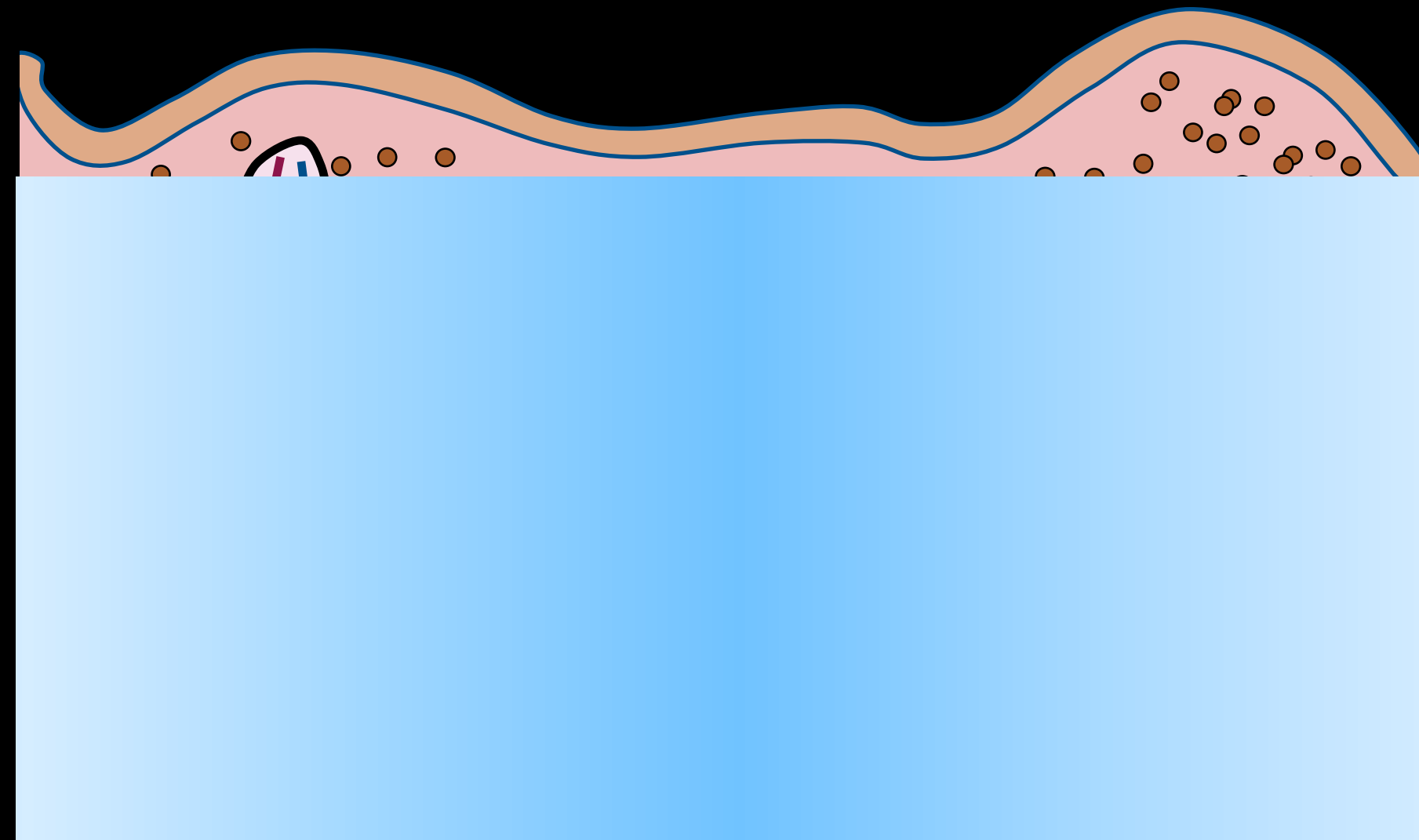
# Rendering continuous refraction and scattering



continuous refraction  
+  
scattering



[Kravtsov and Orlov, Book, 1990]  
[Gröller, Visual Comp., 1995]  
[Stam and Languénou, Rend. Techn., 1996]  
[Weiskopf et al., Com. Graph. forum, 2004]  
[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]



[Chandrasekhar, book, 1960]  
[Lenoble, book, 1985]  
[Lafortune and Willems, 1996]  
[Cammarrano and Jensen, 2002]  
[Gutierrez et al., Com. and Graph. 2006]  
[Jarosz et al., Comp. Graph. forum, 2008]  
[Jakob et al., ToG, 2010]  
[Jarosz et al., ToG, 2011]  
[Pediredla et al., JBO, 2016]  
[Novak et al., Comp. Graph. forum, 2018]  
[Bitterli et al., ToG, 2018]



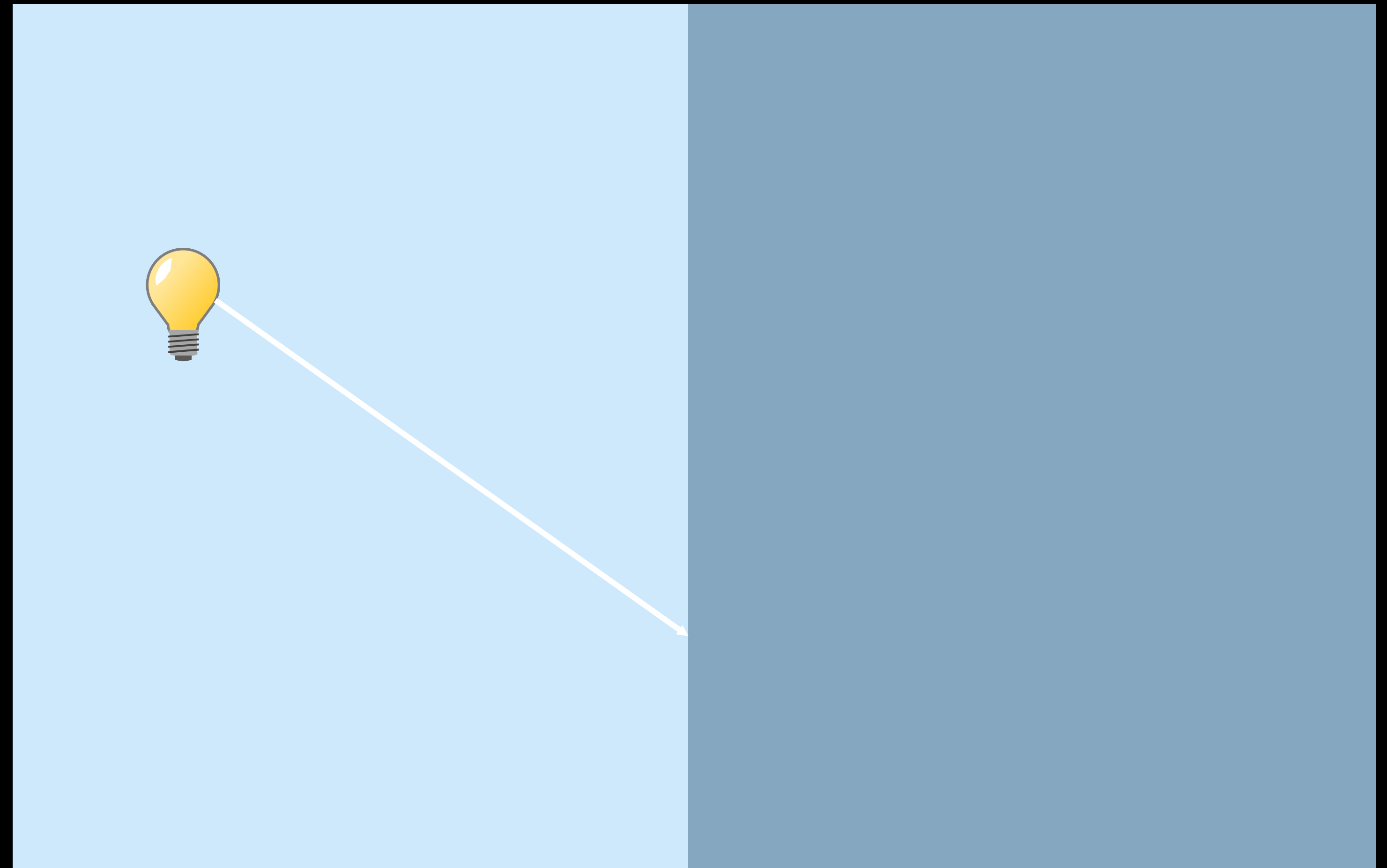
# Rendering continuous refraction ~~and scattering~~

[Kravtsov and Orlov, Book, 1990]  
[Gröllner, Visual Comp., 1995]  
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[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]



# Rendering continuous refraction ~~and scattering~~

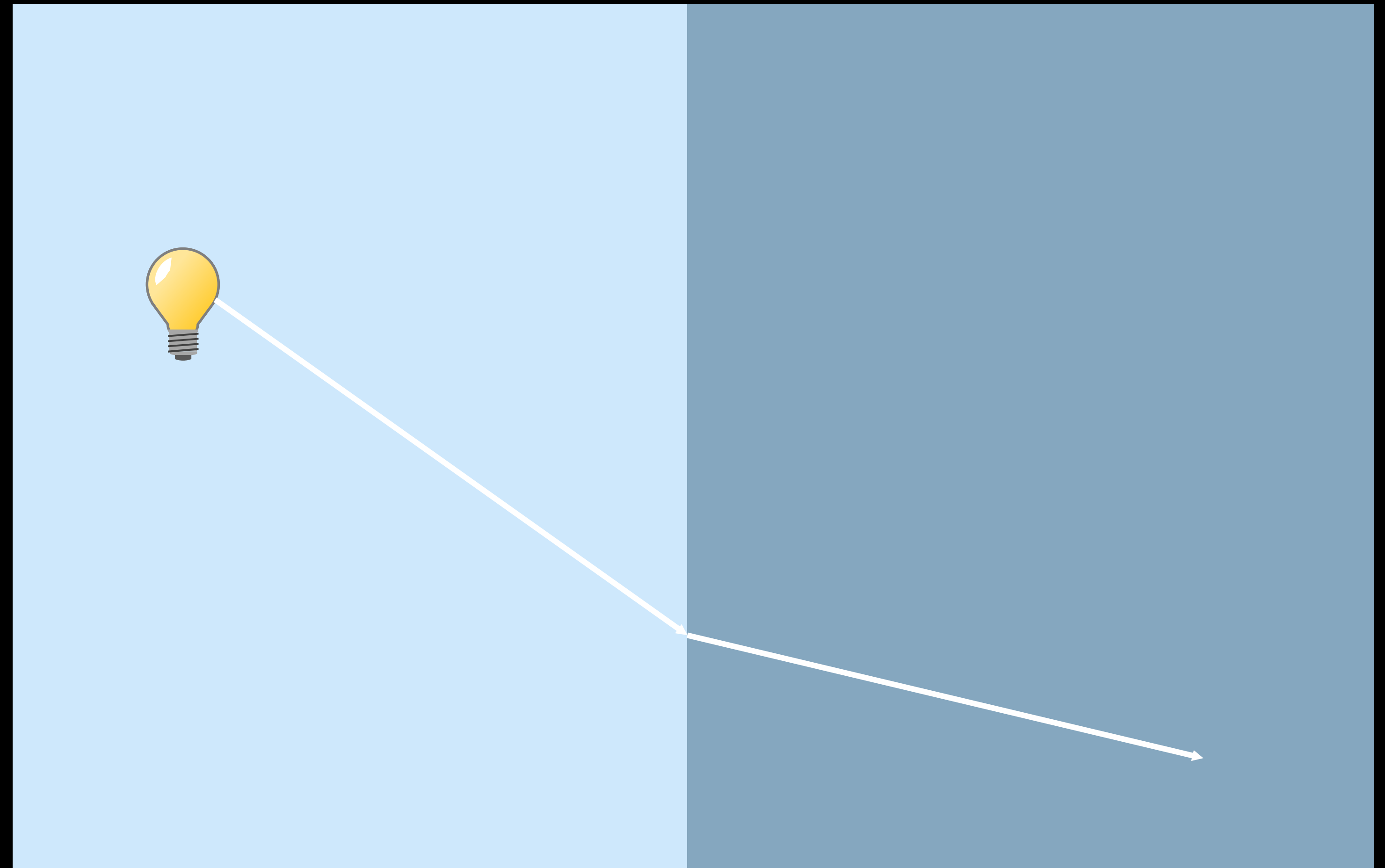
[Kravtsov and Orlov, Book, 1990]  
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[Stam and Languénou, Rend. Techn., 1996]  
[Weiskopf et al., Com. Graph. forum, 2004]  
[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]





# Rendering continuous refraction ~~and scattering~~

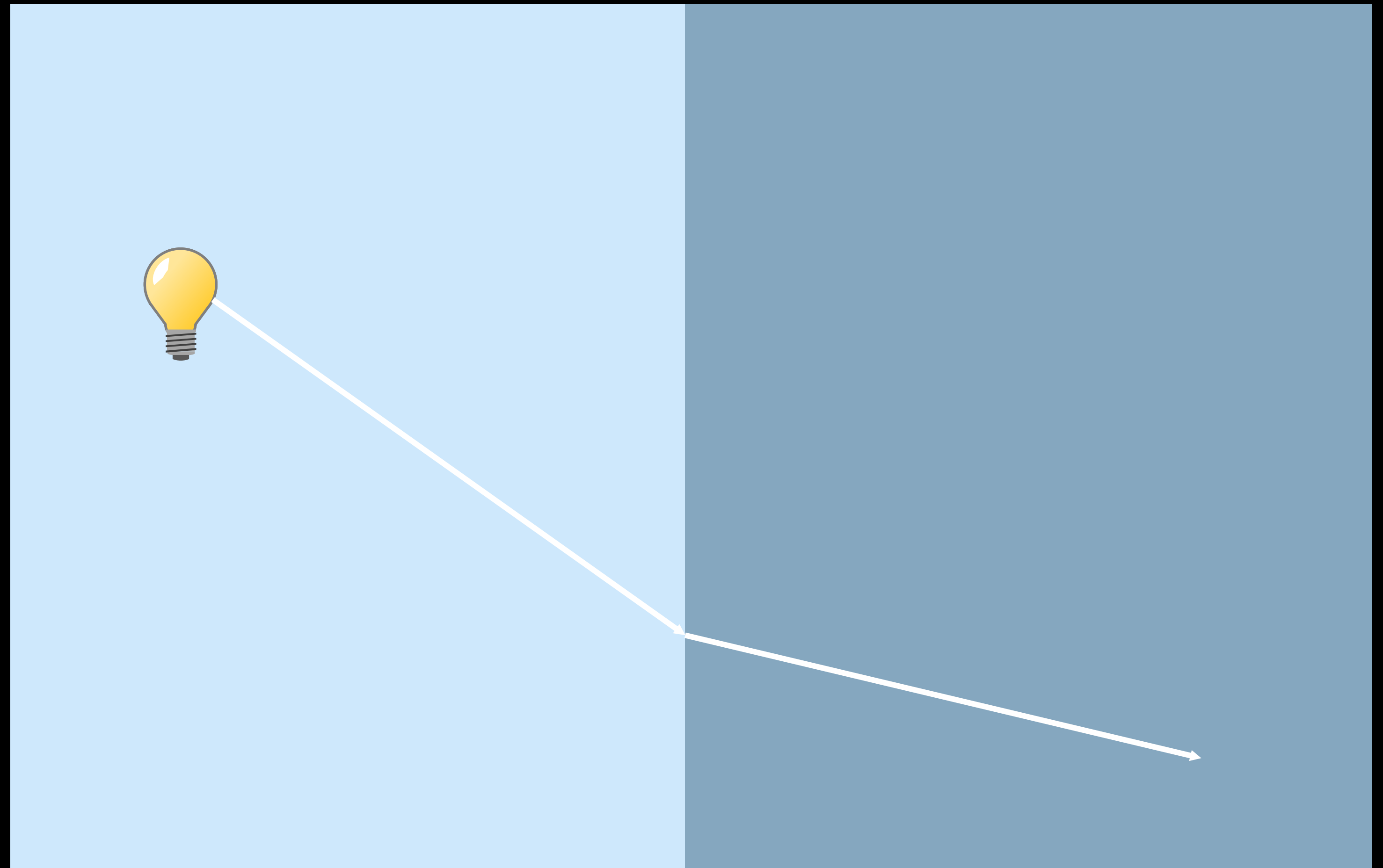
[Kravtsov and Orlov, Book, 1990]  
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[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]



# Rendering continuous refraction ~~and scattering~~

## Snell's law

[Kravtsov and Orlov, Book, 1990]  
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[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]

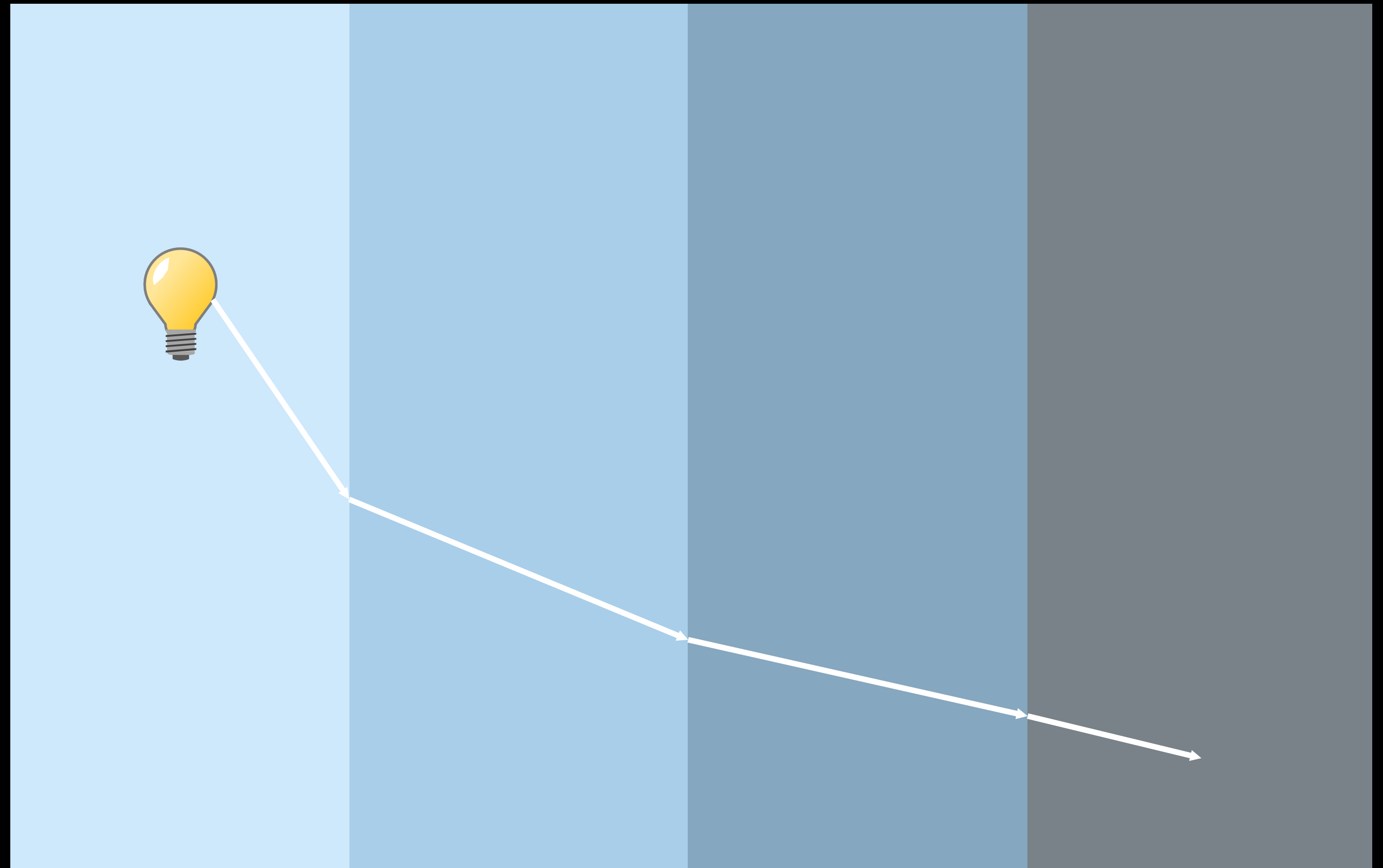




# Rendering continuous refraction ~~and scattering~~

## Snell's law

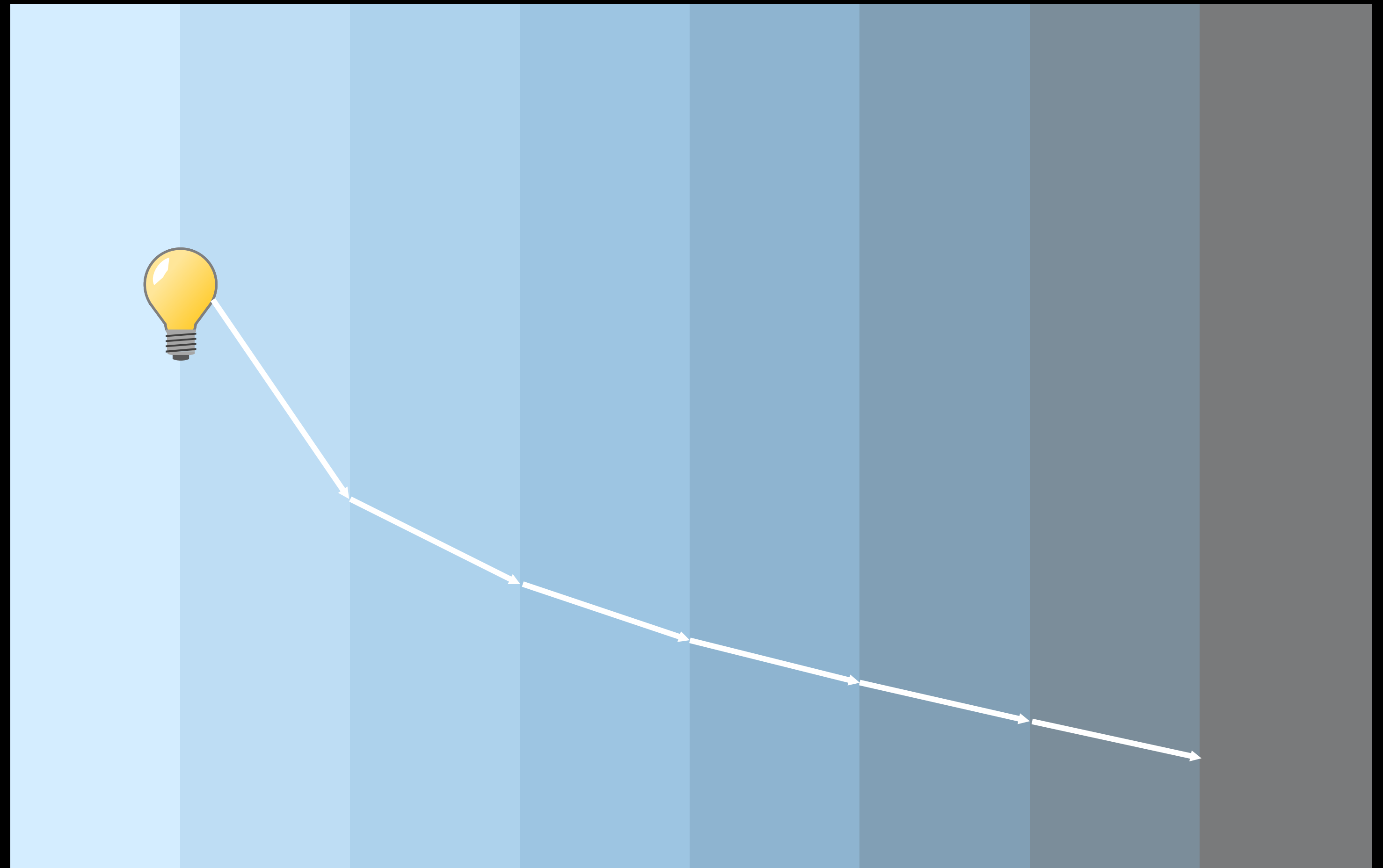
[Kravtsov and Orlov, Book, 1990]  
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# Rendering continuous refraction ~~and scattering~~

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[Pedrotti et al., Book, 2017]  
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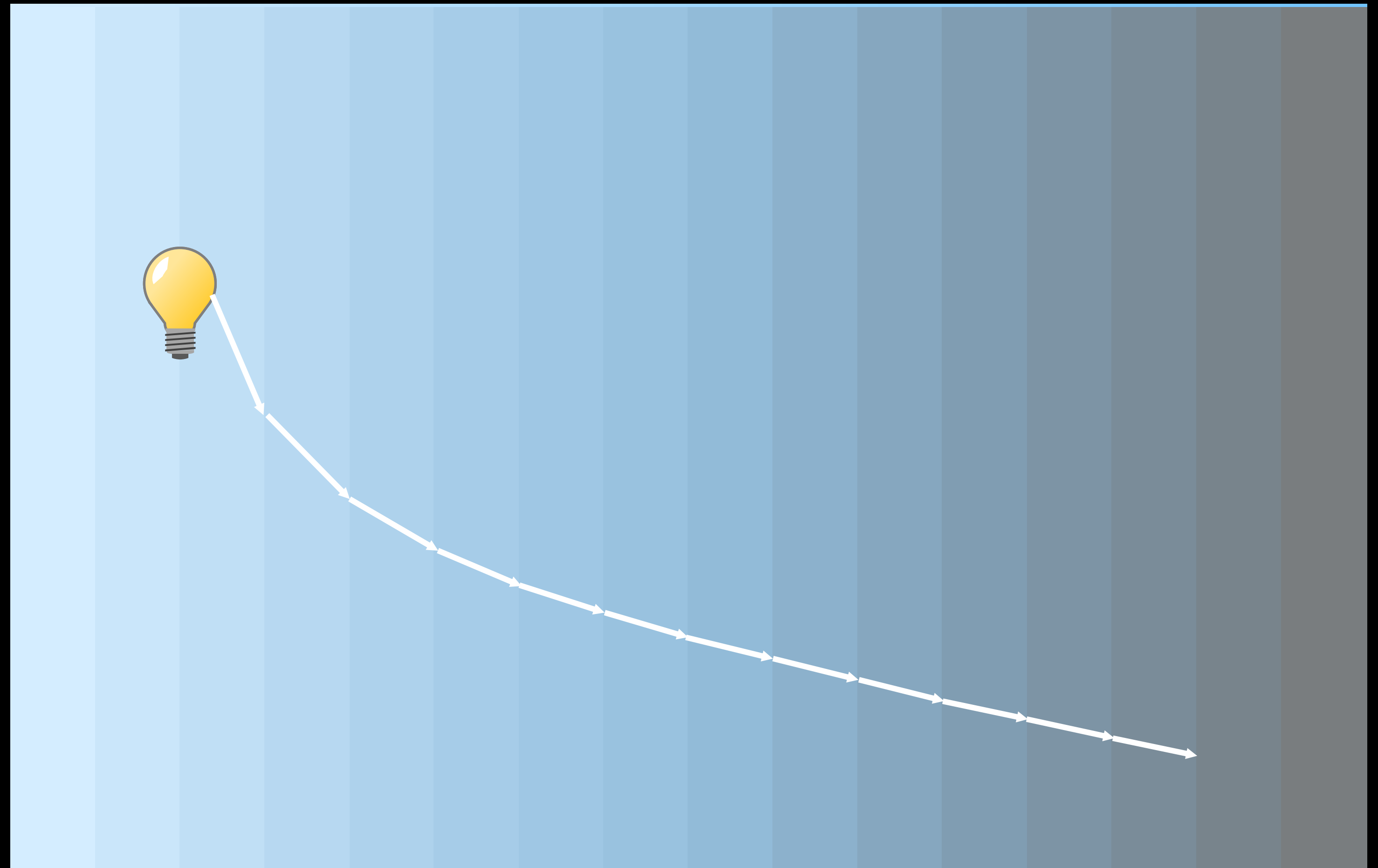




# Rendering continuous refraction ~~and scattering~~

## Snell's law

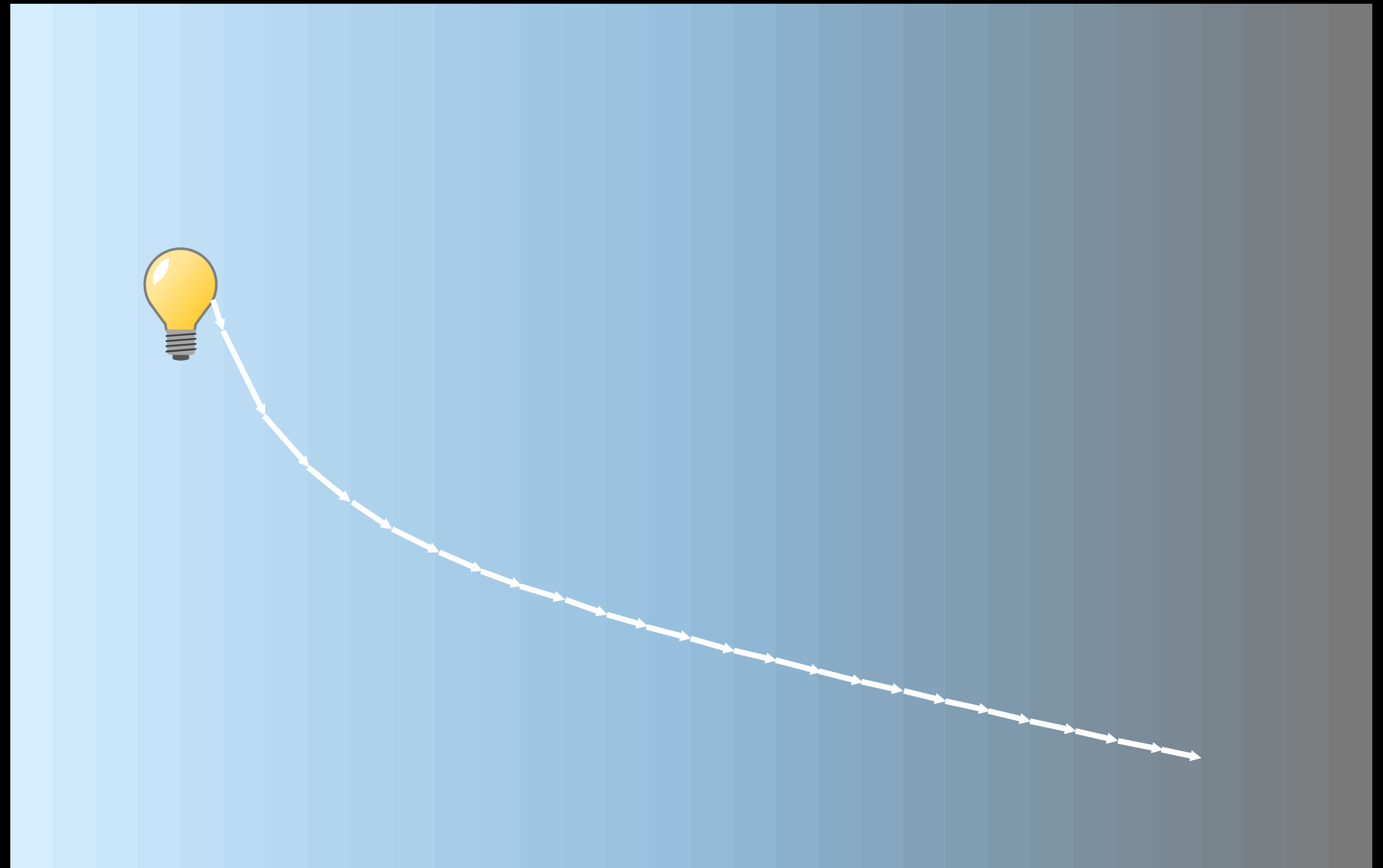
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# Rendering continuous refraction ~~and scattering~~

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[Gutierrez et al., In Rend. Techn., 2005]  
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[Atcheson et al., ToG, 2008]  
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[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]

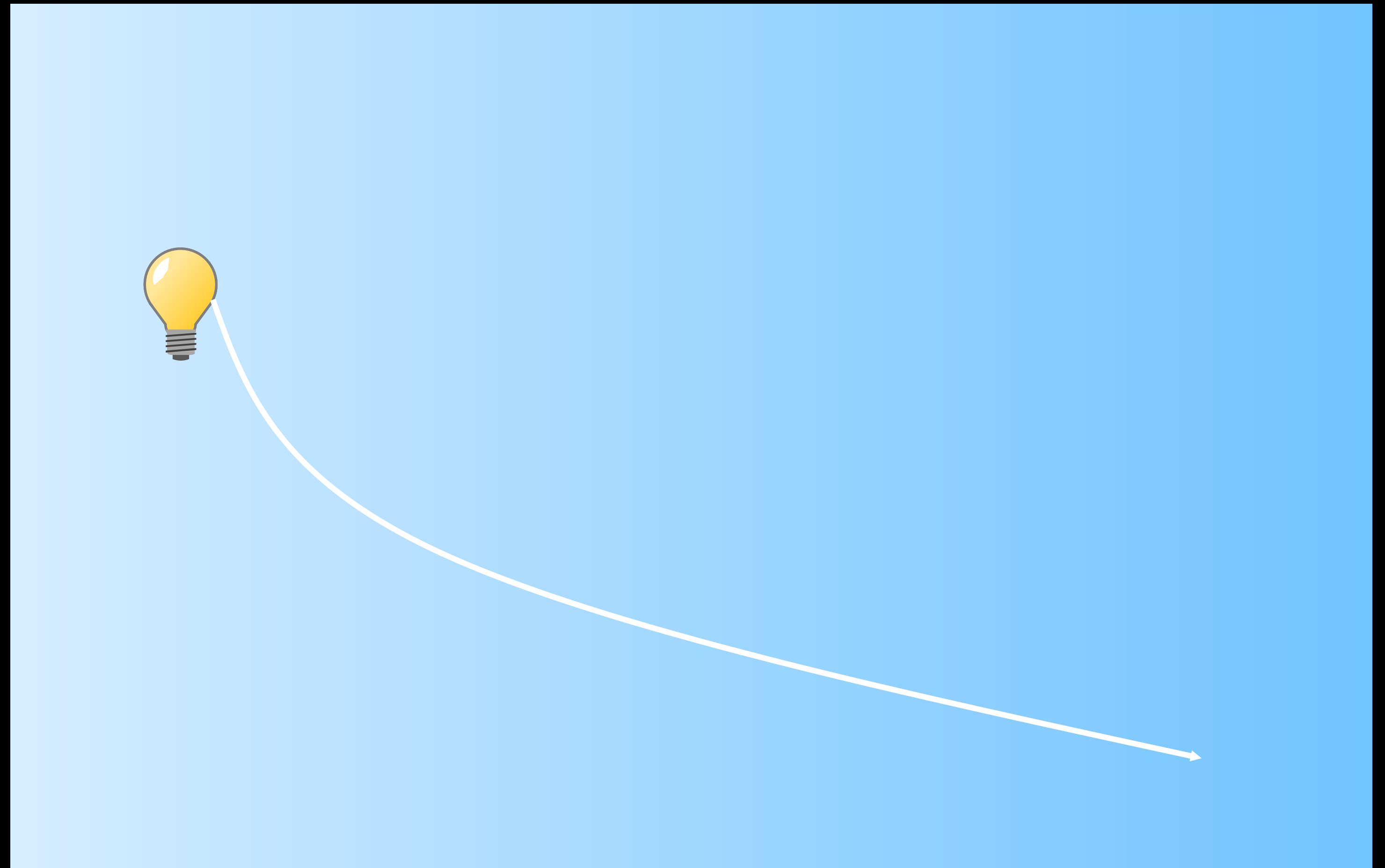




# Rendering continuous refraction ~~and scattering~~

## refractive ray tracing

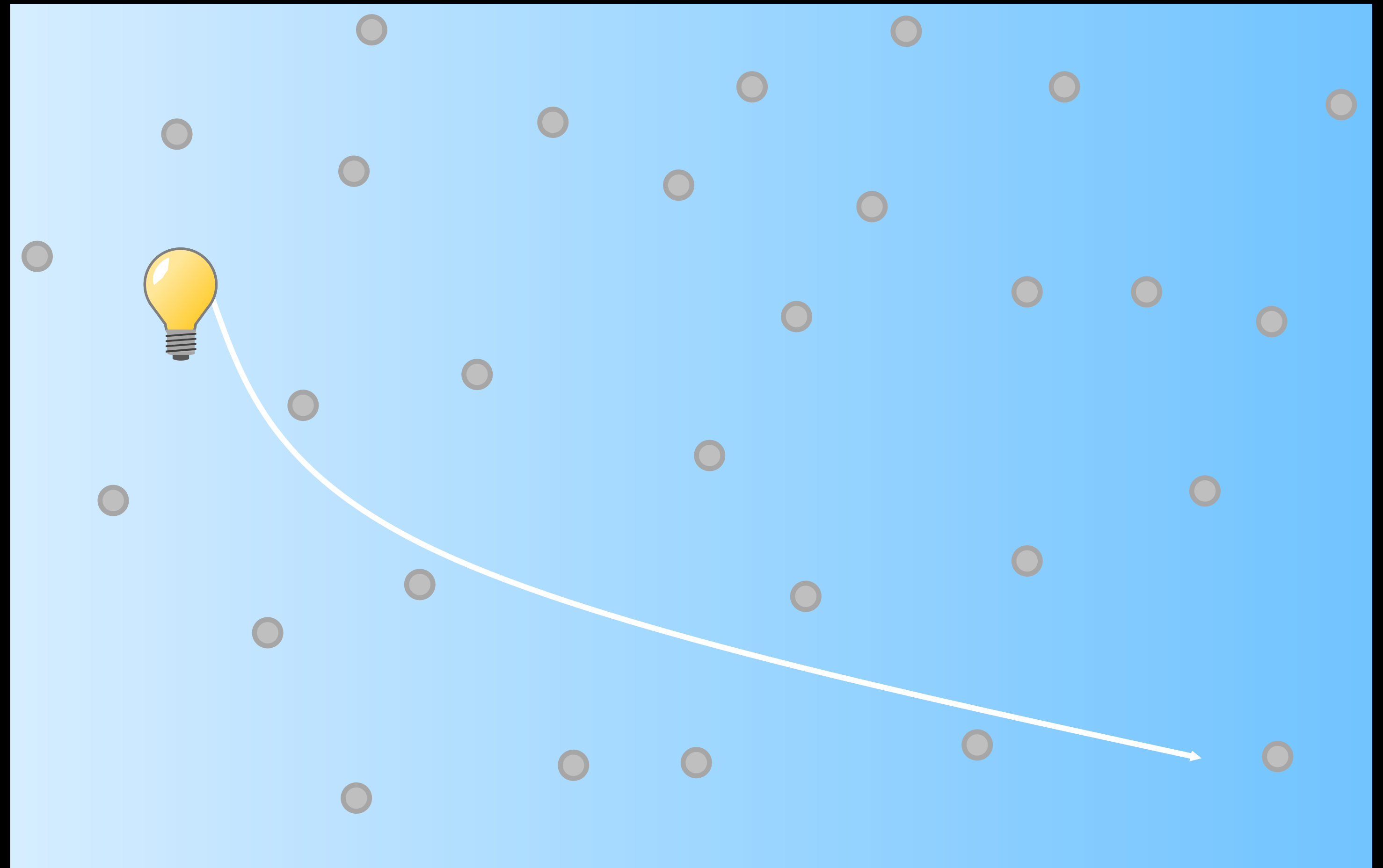
[Kravtsov and Orlov, Book, 1990]  
[Gröller, Visual Comp., 1995]  
[Stam and Languénou, Rend. Techn., 1996]  
[Weiskopf et al., Com. Graph. forum, 2004]  
[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]



# Rendering continuous refraction and scattering

## refractive ray tracing

[Kravtsov and Orlov, Book, 1990]  
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[Weiskopf et al., Com. Graph. forum, 2004]  
[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]

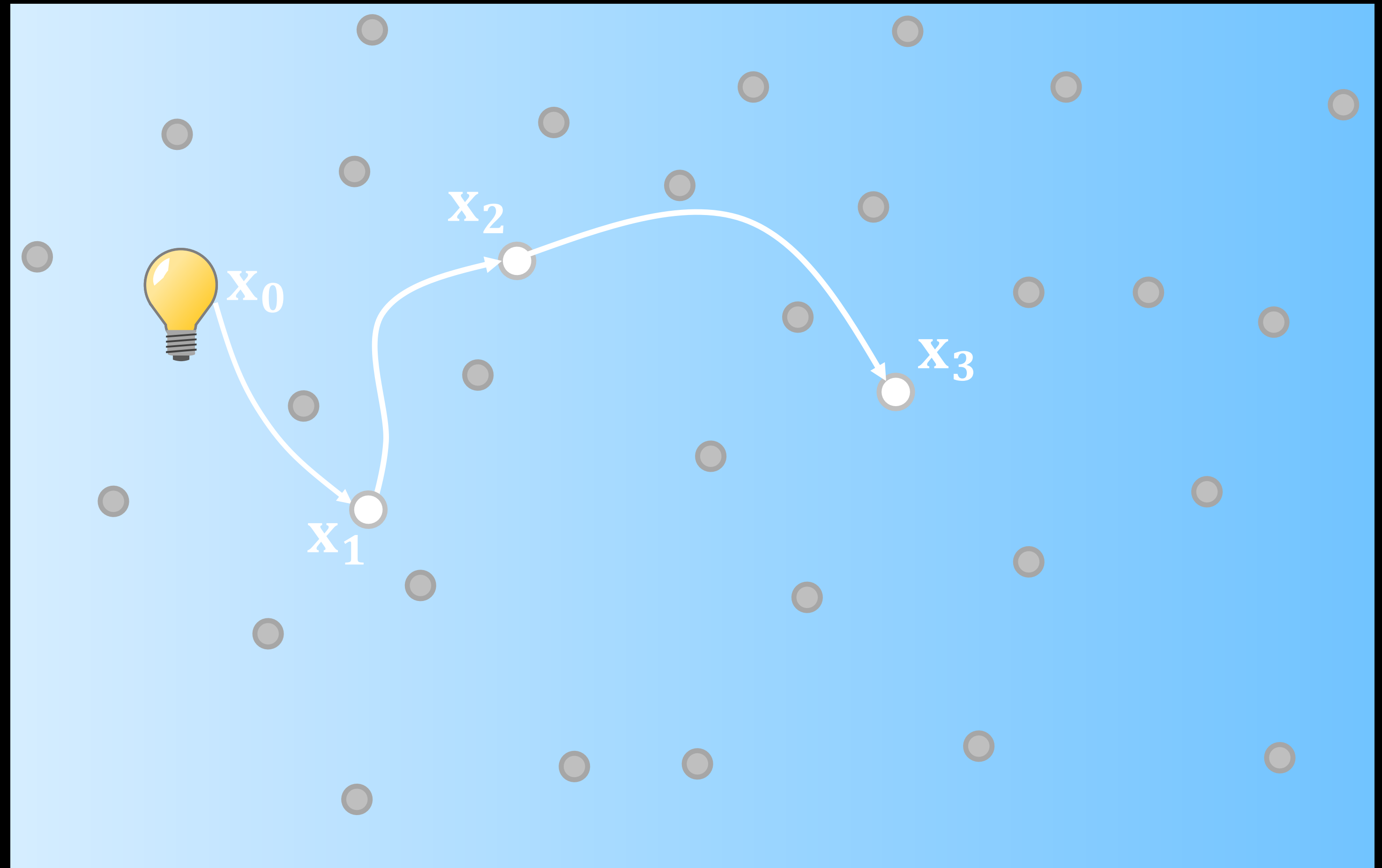




# Rendering continuous refraction and scattering

## refractive ray tracing

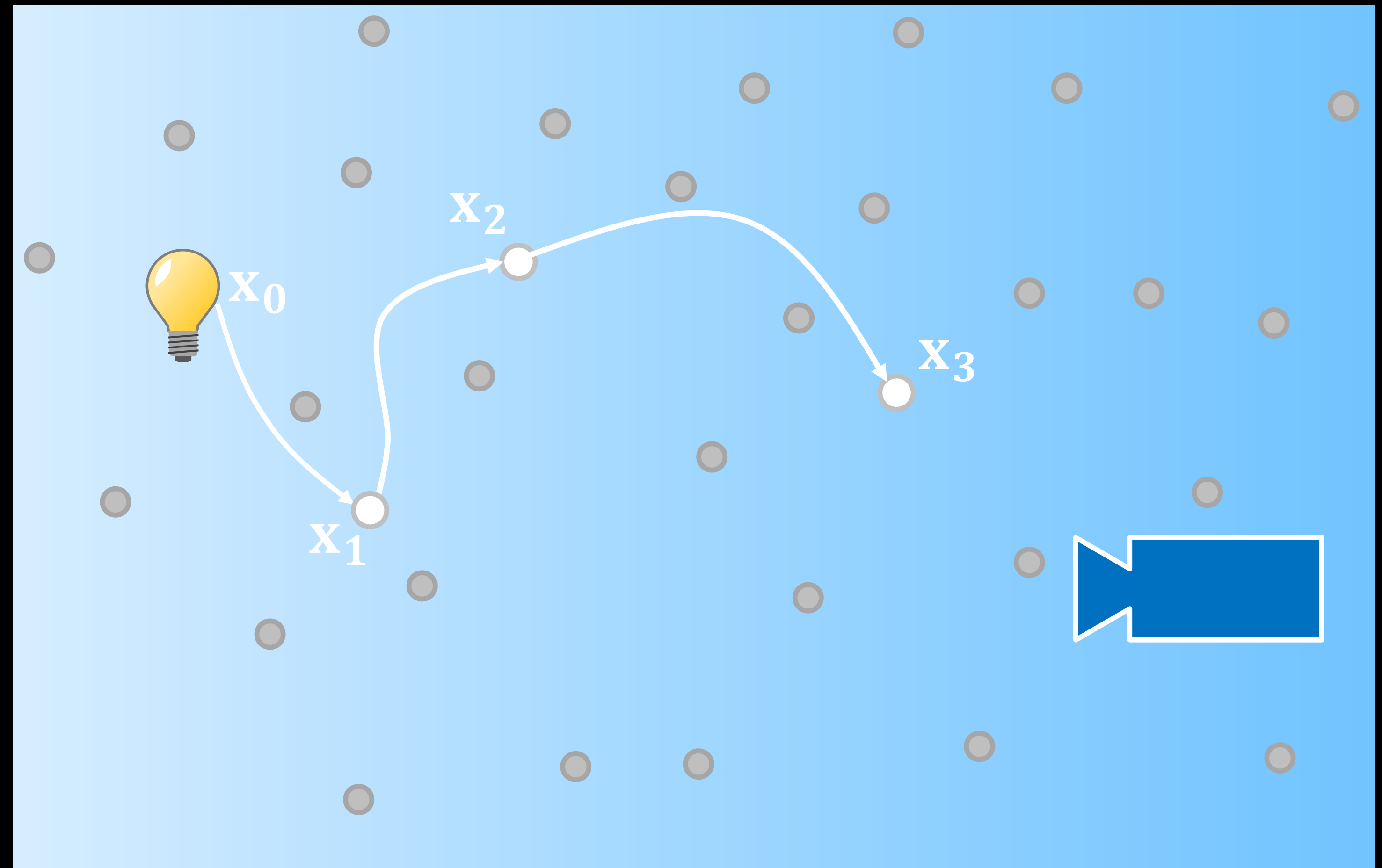
[Kravtsov and Orlov, Book, 1990]  
[Gröller, Visual Comp., 1995]  
[Stam and Languénou, Rend. Techn., 1996]  
[Weiskopf et al., Com. Graph. forum, 2004]  
[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]



# Rendering continuous refraction and scattering

## refractive ray tracing

[Kravtsov and Orlov, Book, 1990]  
[Gröllner, Visual Comp., 1995]  
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[Weiskopf et al., Com. Graph. forum, 2004]  
[Gutierrez et al., In Rend. Techn., 2005]  
[Ihrke et al., ToG, 2007]  
[Atcheson et al., ToG, 2008]  
[Ji et al., CVPR, 2013]  
[Pedrotti et al., Book, 2017]  
[Scopelliti et al., Nature LSA, 2019]

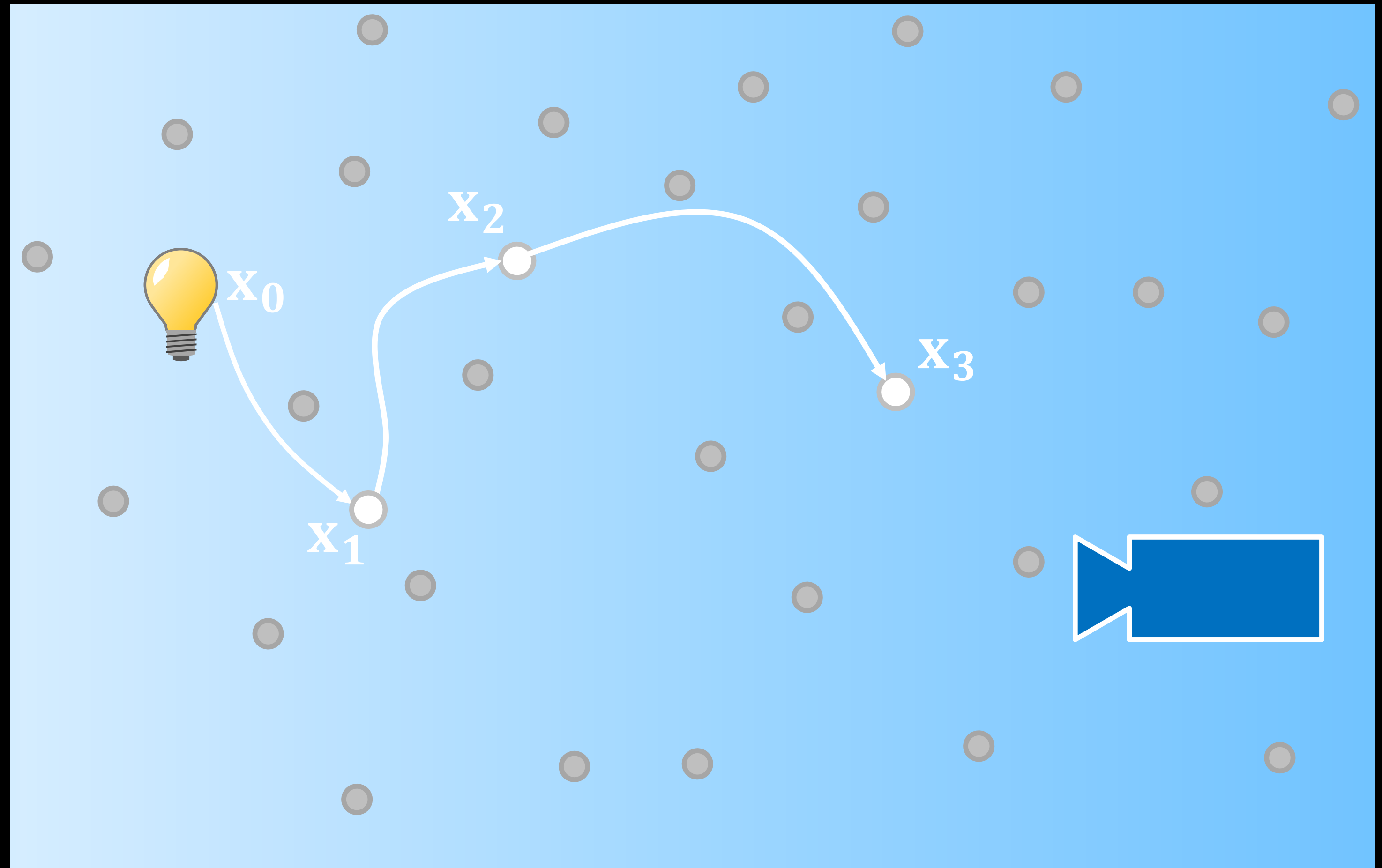




# Rendering continuous refraction and scattering

radiative transfer equation

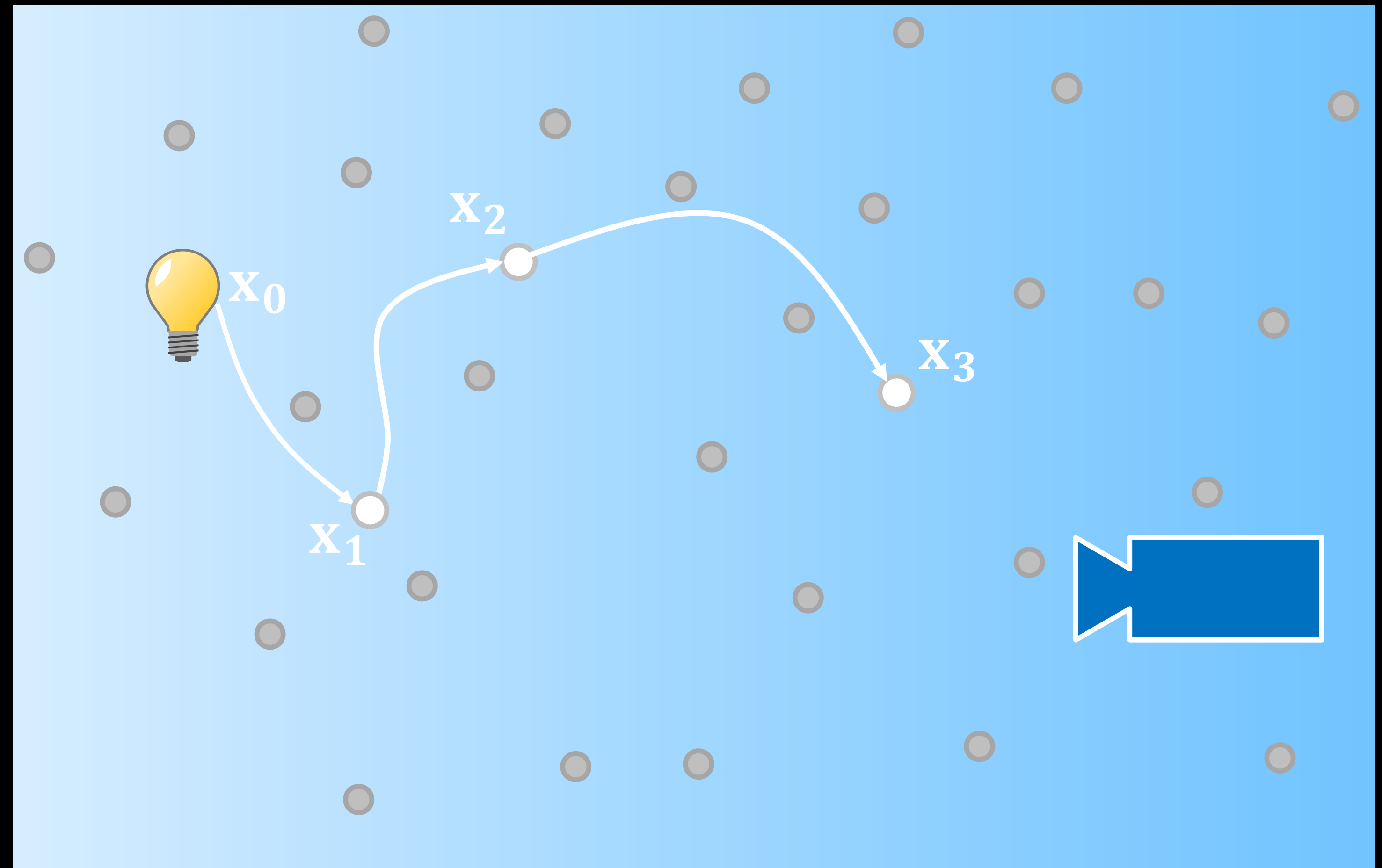
[Chandrasekhar, book, 1960]  
[Lenoble, book, 1985]  
[Lafortune and Willems, 1996]  
[Cammarano and Jensen, 2002]  
[Gutierrez et al., Com. and Graph. 2006]  
[Jarosz et al., Comp. Graph. forum, 2008]  
[Jakob et al., ToG, 2010]  
[Jarosz et al., ToG, 2011]  
[Pediredla et al., JBO, 2016]  
[Novak et al., Comp. Graph. forum, 2018]  
[Bitterli et al., ToG, 2018]



# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):



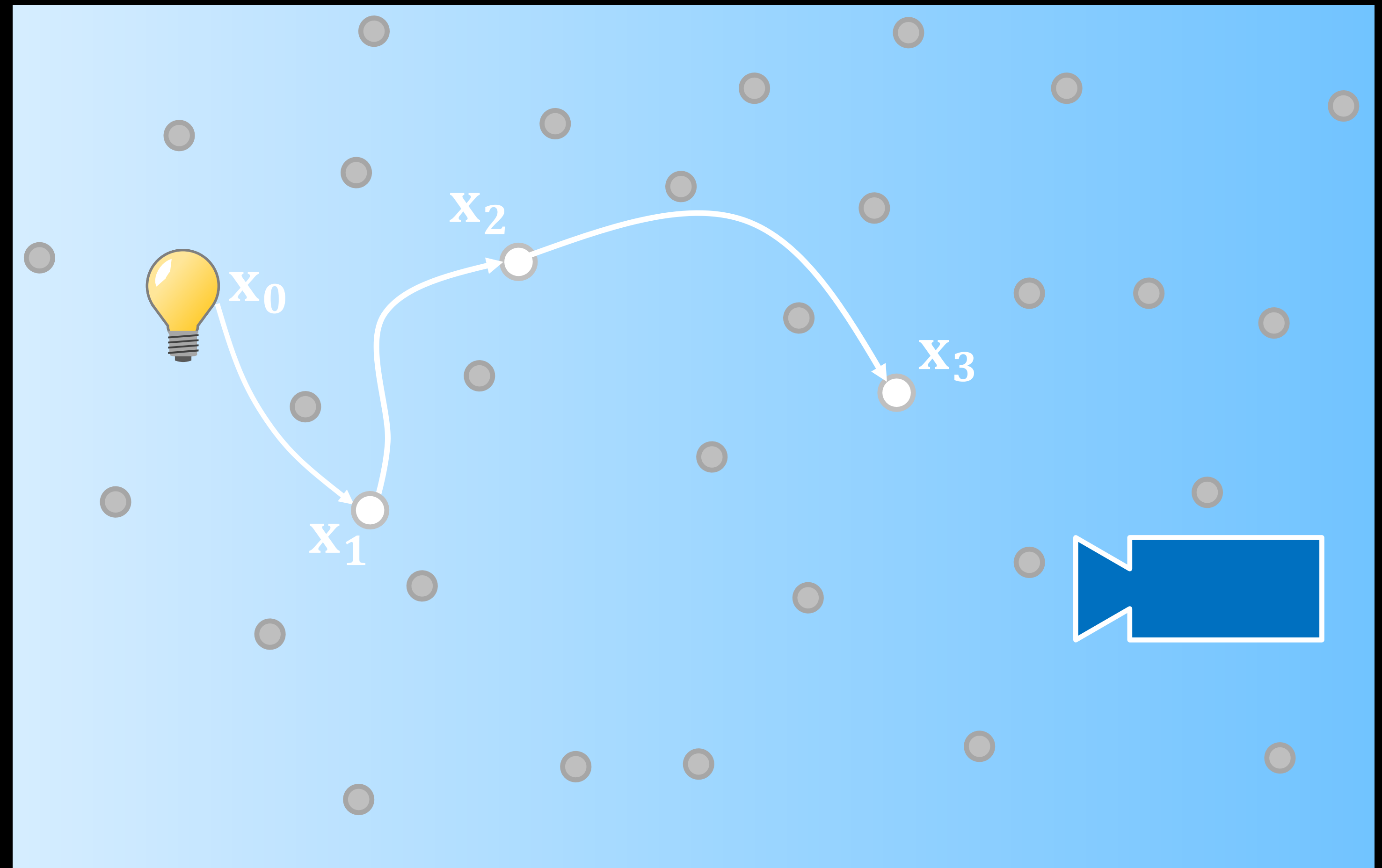


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath

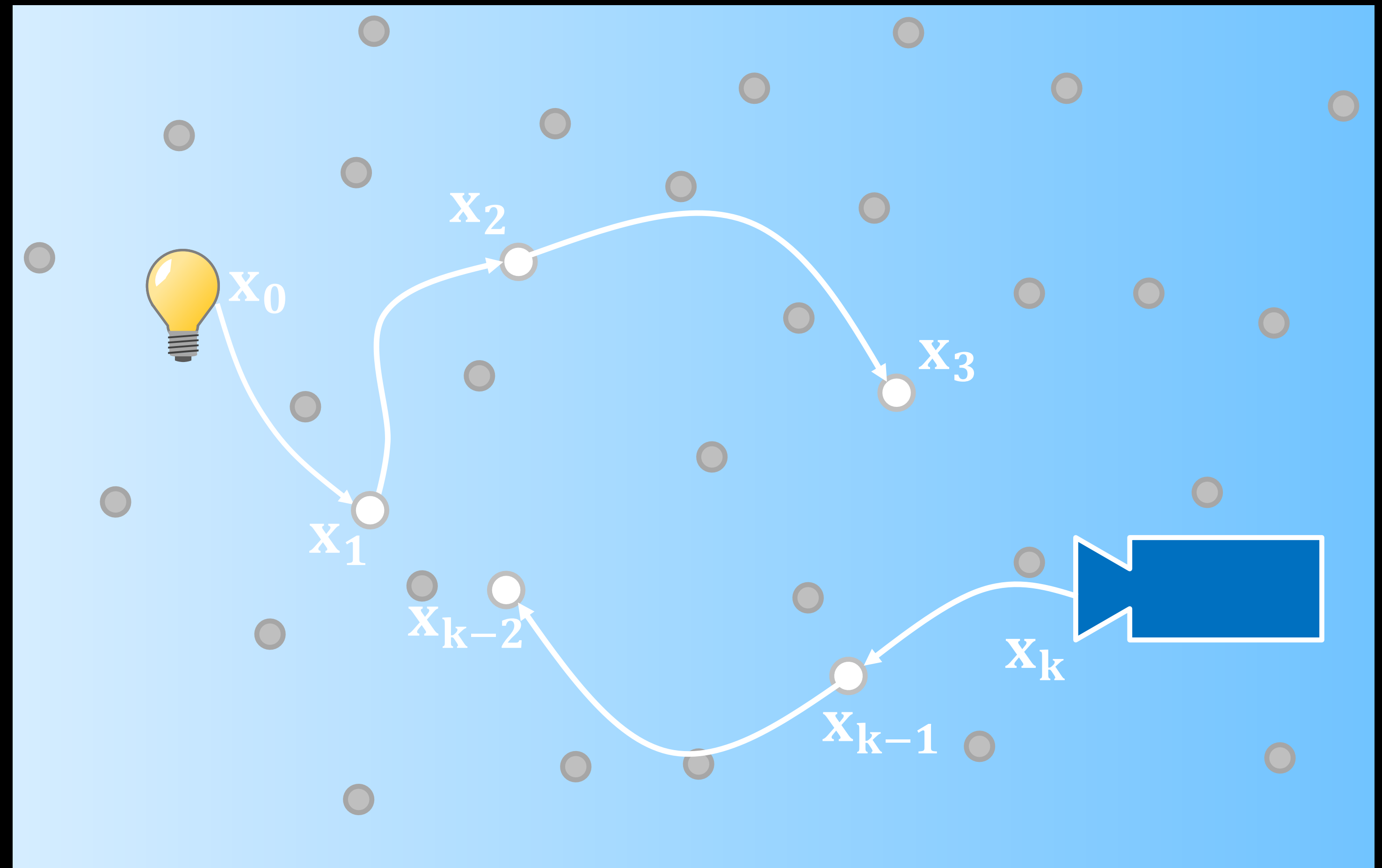


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath
2. trace a random sensor subpath



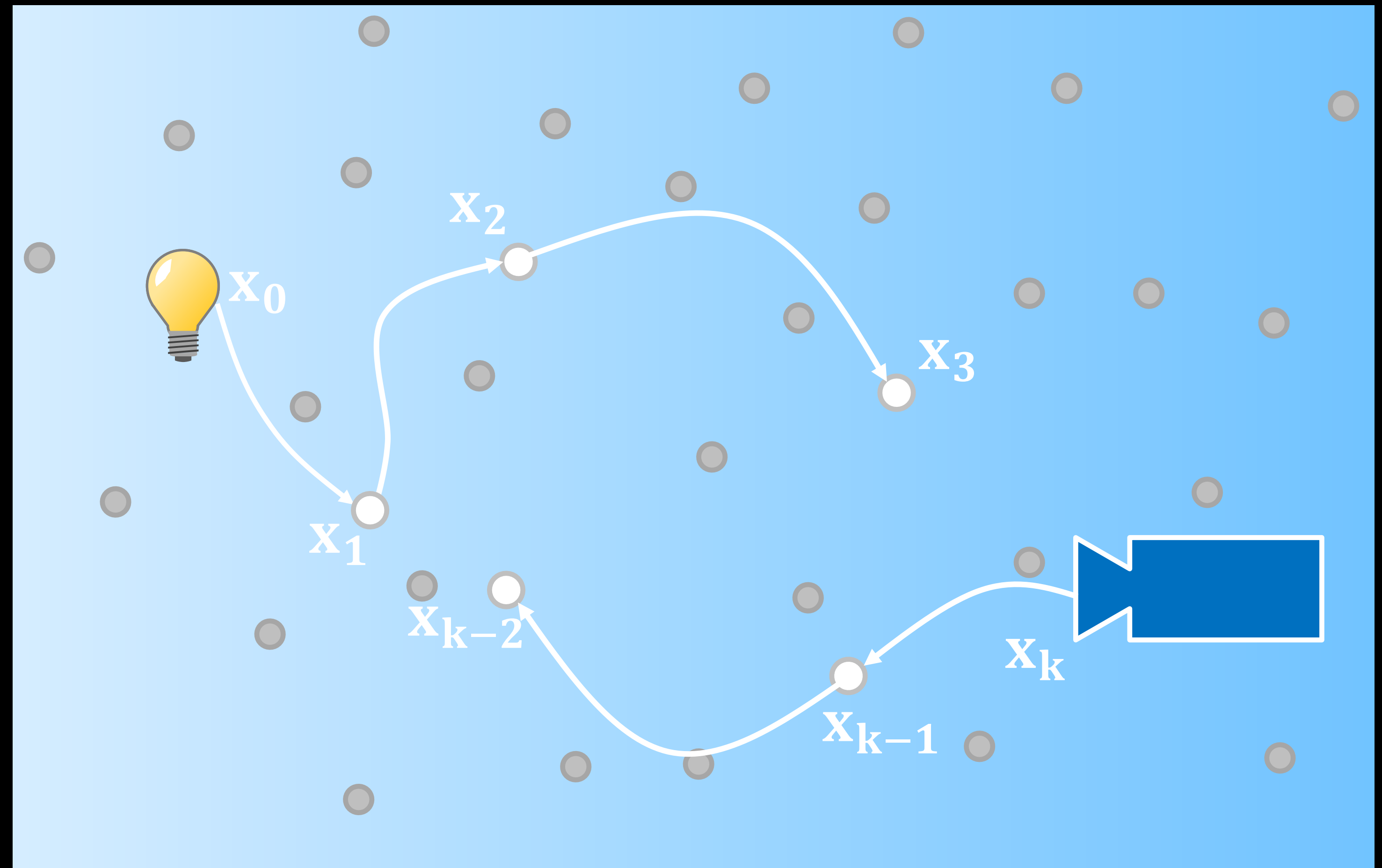


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath
2. trace a random sensor subpath
3. join vertices with a straight line

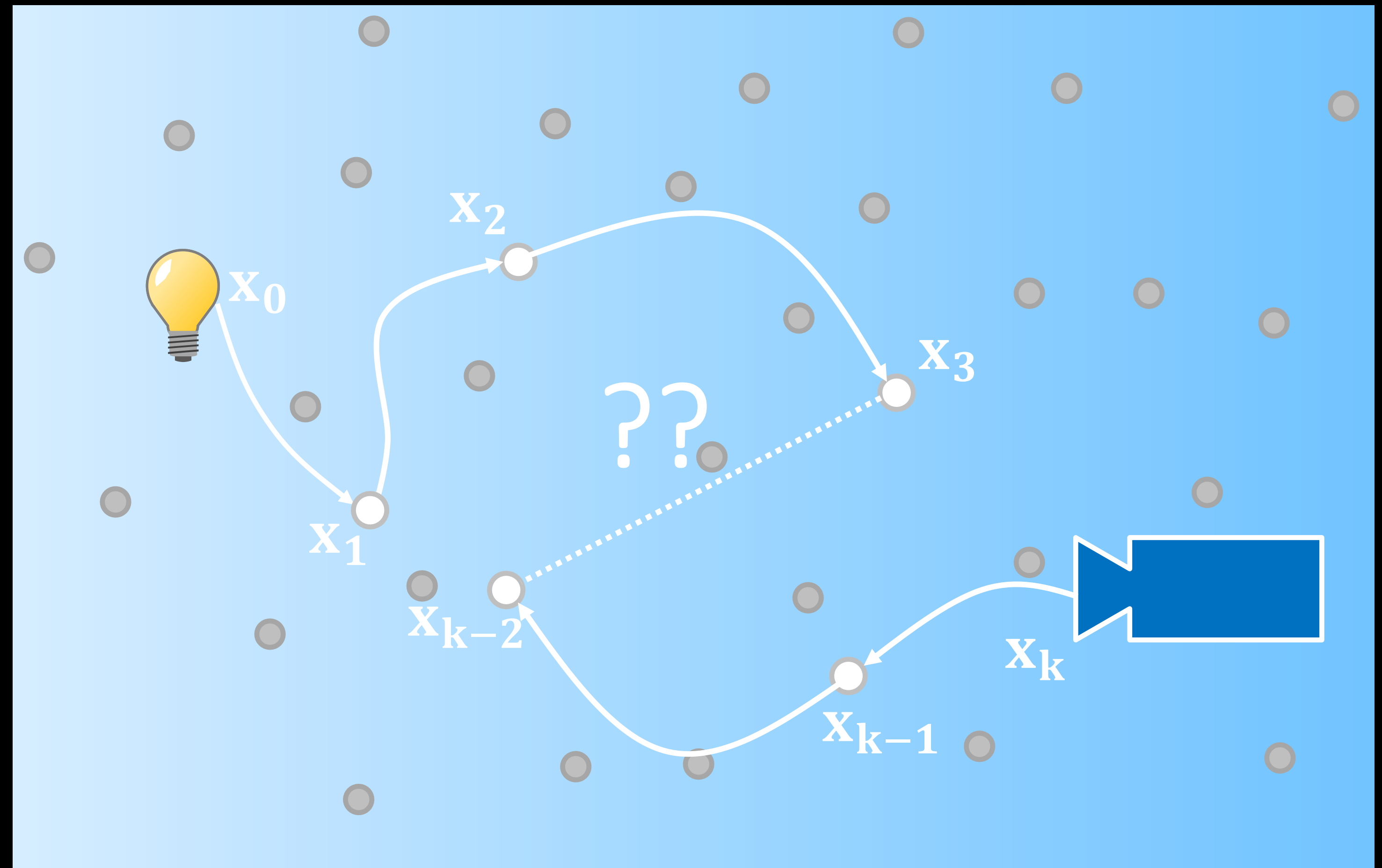


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath
2. trace a random sensor subpath
3. join vertices with a ~~straight line~~ curve



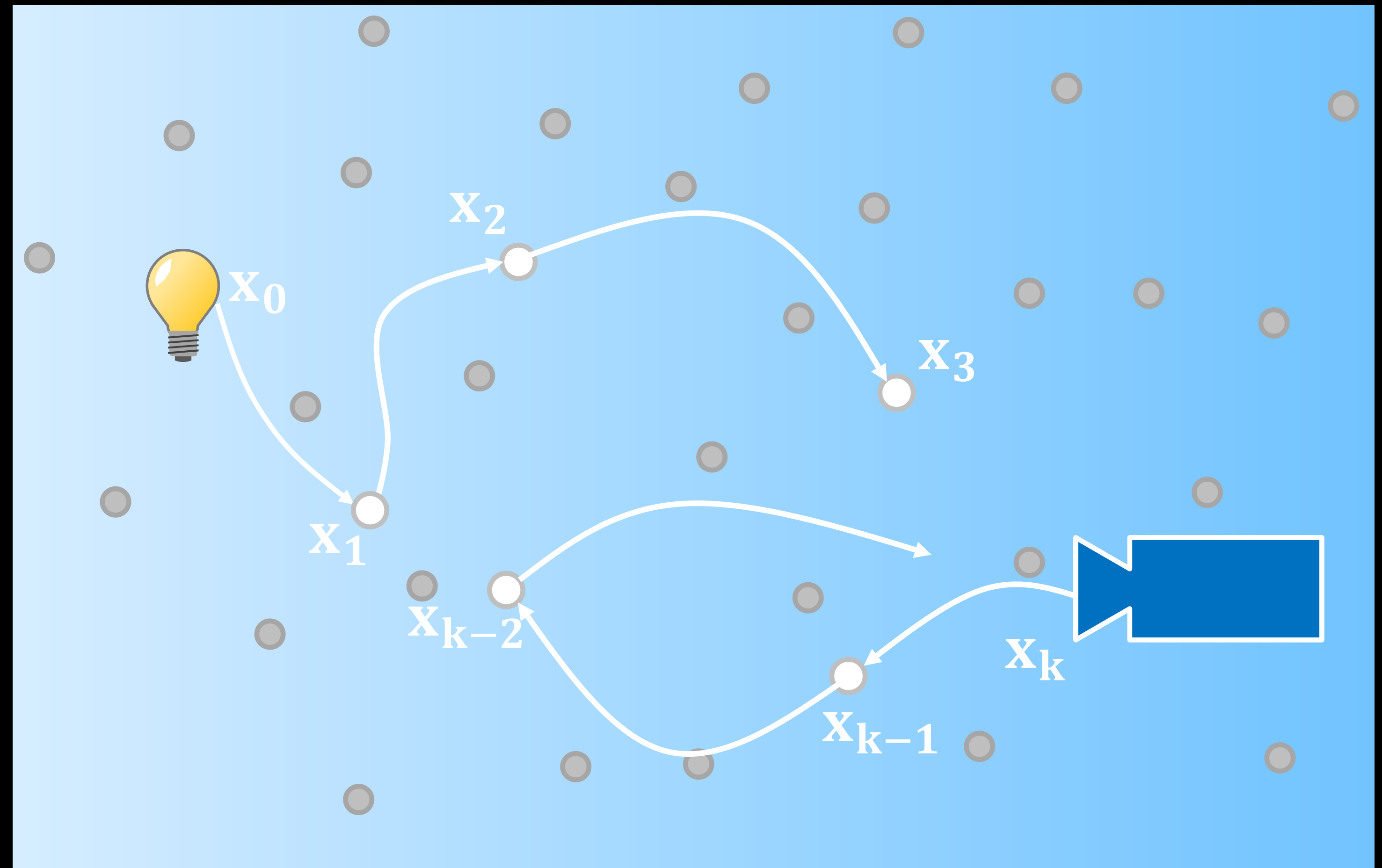


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath
2. trace a random sensor subpath
3. join vertices with a ~~straight line~~ curve

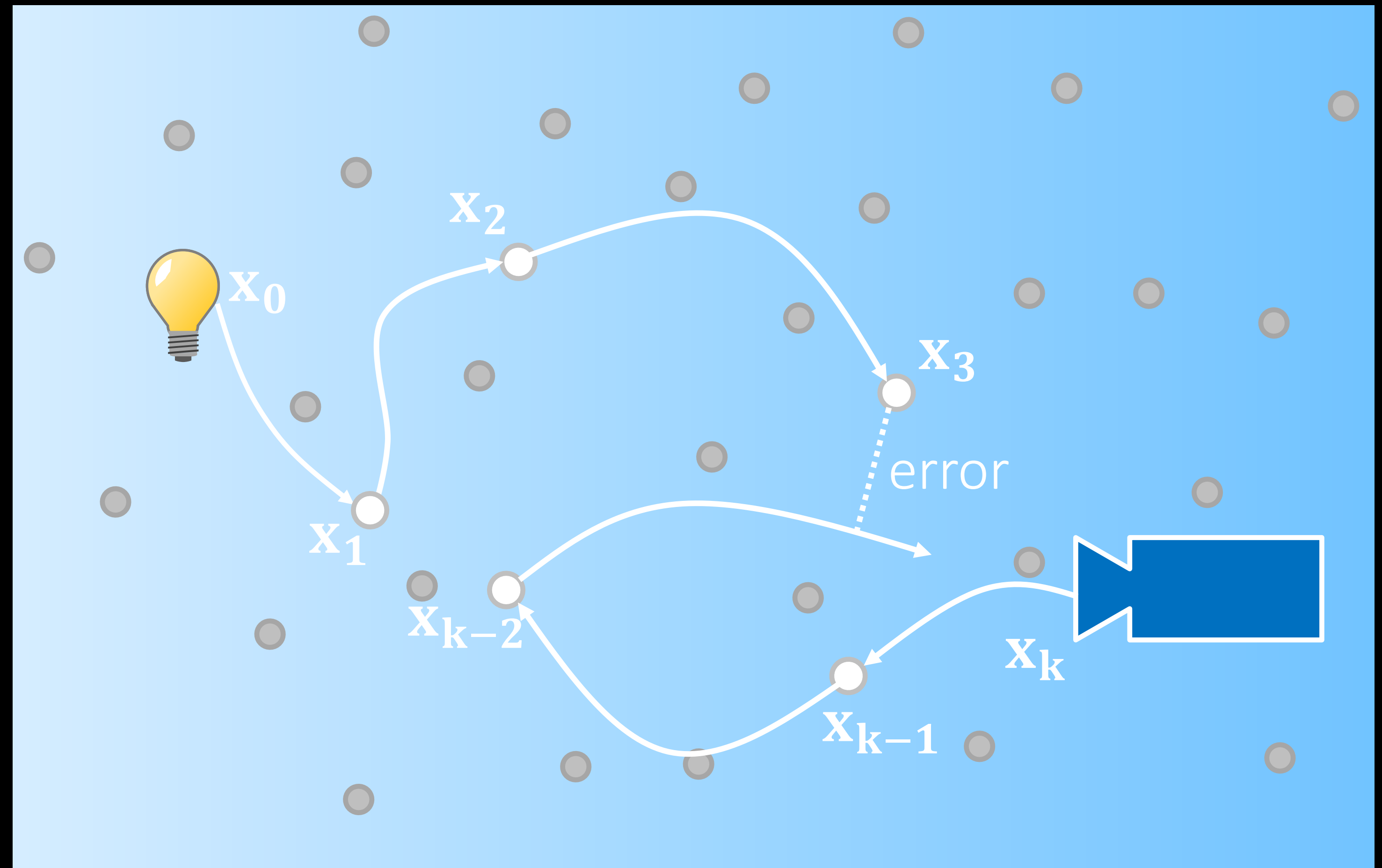


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath
2. trace a random sensor subpath
3. join vertices with a ~~straight line~~ curve



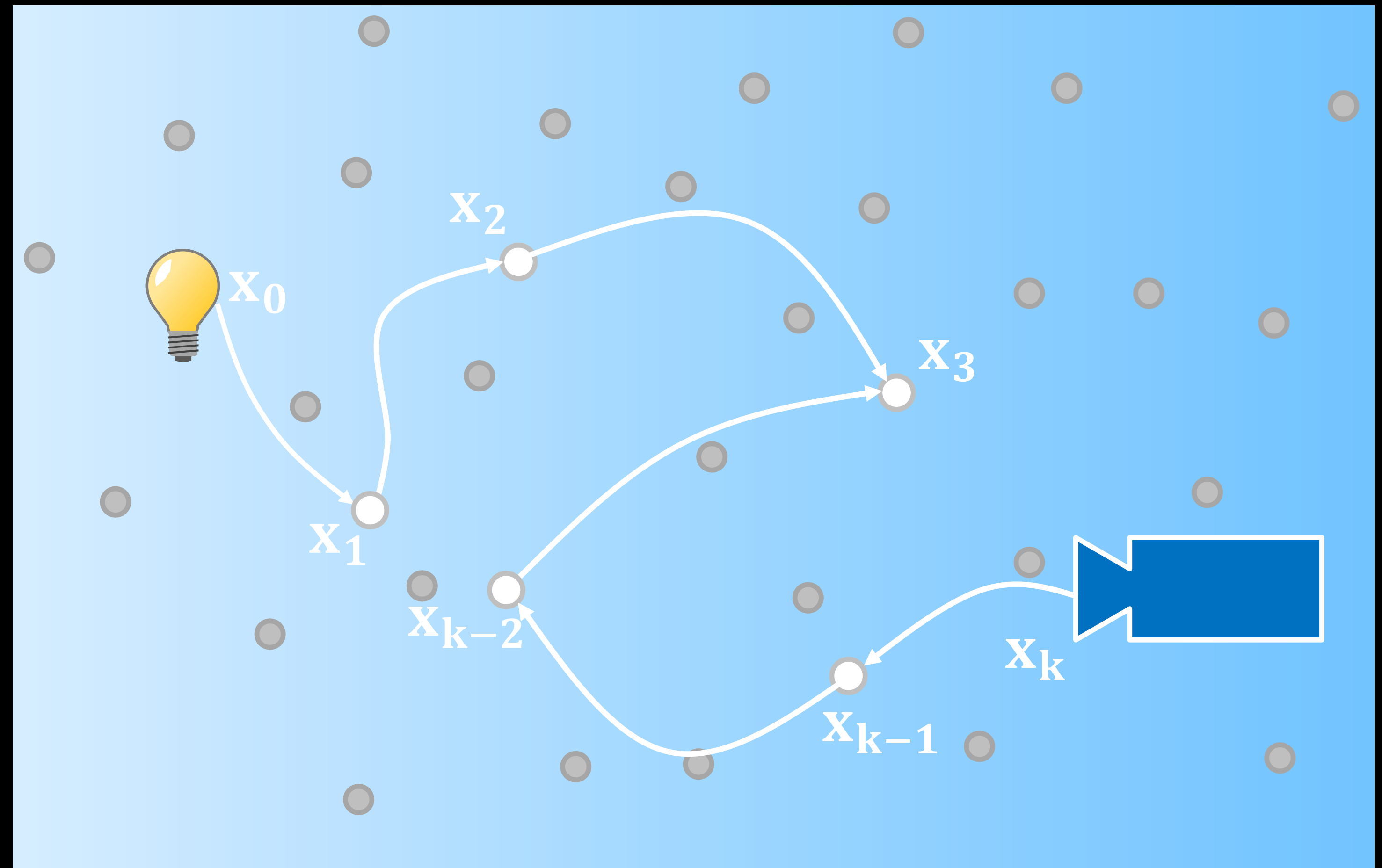


# Rendering continuous refraction and scattering

radiative transfer equation

bidirectional path tracing (BDPT):

1. trace a random emitter subpath
2. trace a random sensor subpath
3. join vertices with a ~~straight line~~ curve



# Rendering continuous refraction and scattering

$$\begin{aligned}\frac{d\mathcal{L}}{dv_0} &= (\mathbf{x}_{s^*} - \mathbf{y})^T \frac{d\mathbf{x}_{s^*}}{dv_0}, \\ \frac{d\mathbf{x}_{s^*}}{dv_0} &= \frac{\partial \mathbf{x}_{s^*}}{\partial v_0} + \frac{\partial \mathbf{x}_{s^*}}{\partial s^*} \frac{ds^*}{dv_0}, \\ g(\mathbf{x}_{s^*}) = 0 &\Rightarrow \frac{dg(\mathbf{x}_{s^*})}{d\mathbf{x}_{s^*}} \frac{d\mathbf{x}_{s^*}}{dv_0} = 0 \\ &\Rightarrow \frac{ds^*}{dv_0} = - \frac{\frac{dg(\mathbf{x}_{s^*})}{d\mathbf{x}_{s^*}} \frac{\partial \mathbf{x}_{s^*}}{\partial v_0}}{\frac{dg(\mathbf{x}_{s^*})}{d\mathbf{x}_{s^*}} \frac{\partial \mathbf{x}_{s^*}}{\partial s^*}}, \\ \frac{d\mathbf{x}_{s^*}}{dv_0} &= \left( I_{3 \times 3} - \frac{\frac{\partial \mathbf{x}_{s^*}}{\partial s^*} \frac{dg(\mathbf{x}_{s^*})}{d\mathbf{x}_{s^*}}}{\frac{dg(\mathbf{x}_{s^*})}{d\mathbf{x}_{s^*}} \frac{\partial \mathbf{x}_{s^*}}{\partial s^*}} \right) \frac{\partial \mathbf{x}_{s^*}}{\partial v_0}.\end{aligned}$$

## Algorithm 2: Symplectic integration for derivative tracing

**Input:**  $n(\mathbf{x})$ ,  $\nabla n(\mathbf{x})$ ,  $H_n(\mathbf{x})$ ,  $\text{ray}(\mathbf{x}, \mathbf{v})$ ,  $\frac{\partial \mathbf{x}_{s^*}}{\partial v_0} = O_3$ ,  $\frac{\partial \mathbf{v}_{s^*}}{\partial v_0} = I_3$ ,

$nSteps$ ,  $s$  = step size

**Output:**  $\frac{\partial \mathbf{x}_{s^*}}{\partial v_0}$ ,  $\frac{\partial \mathbf{v}_{s^*}}{\partial v_0}$

**for**  $i = 1 : nSteps$  **do**

$\text{ray.v} += 0.5s \nabla n(\text{ray.x})$ ;

$\frac{\partial \mathbf{v}_{s^*}}{\partial v_0} += 0.5s H_n(\text{ray.x}) \frac{\partial \mathbf{x}_{s^*}}{\partial v_0}$ ;

$\text{ray.x} += s \frac{\text{ray.v}}{n(\text{ray.x})}$ ;

$\frac{\partial \mathbf{x}_{s^*}}{\partial v_0} += s \left( - \frac{\mathbf{v}_s \nabla n(\text{ray.x})}{n(\text{ray.x})^2} \frac{\partial \mathbf{x}_s}{\partial v_0} + \frac{1}{n(\text{ray.x})} \frac{\partial \mathbf{v}_s}{\partial v_0} \right)$ ;

$\text{ray.v} += 0.5s \nabla n(\text{ray.x})$ ;

$\frac{\partial \mathbf{v}_{s^*}}{\partial v_0} += 0.5s H_n(\text{ray.x}) \frac{\partial \mathbf{x}_{s^*}}{\partial v_0}$ ;

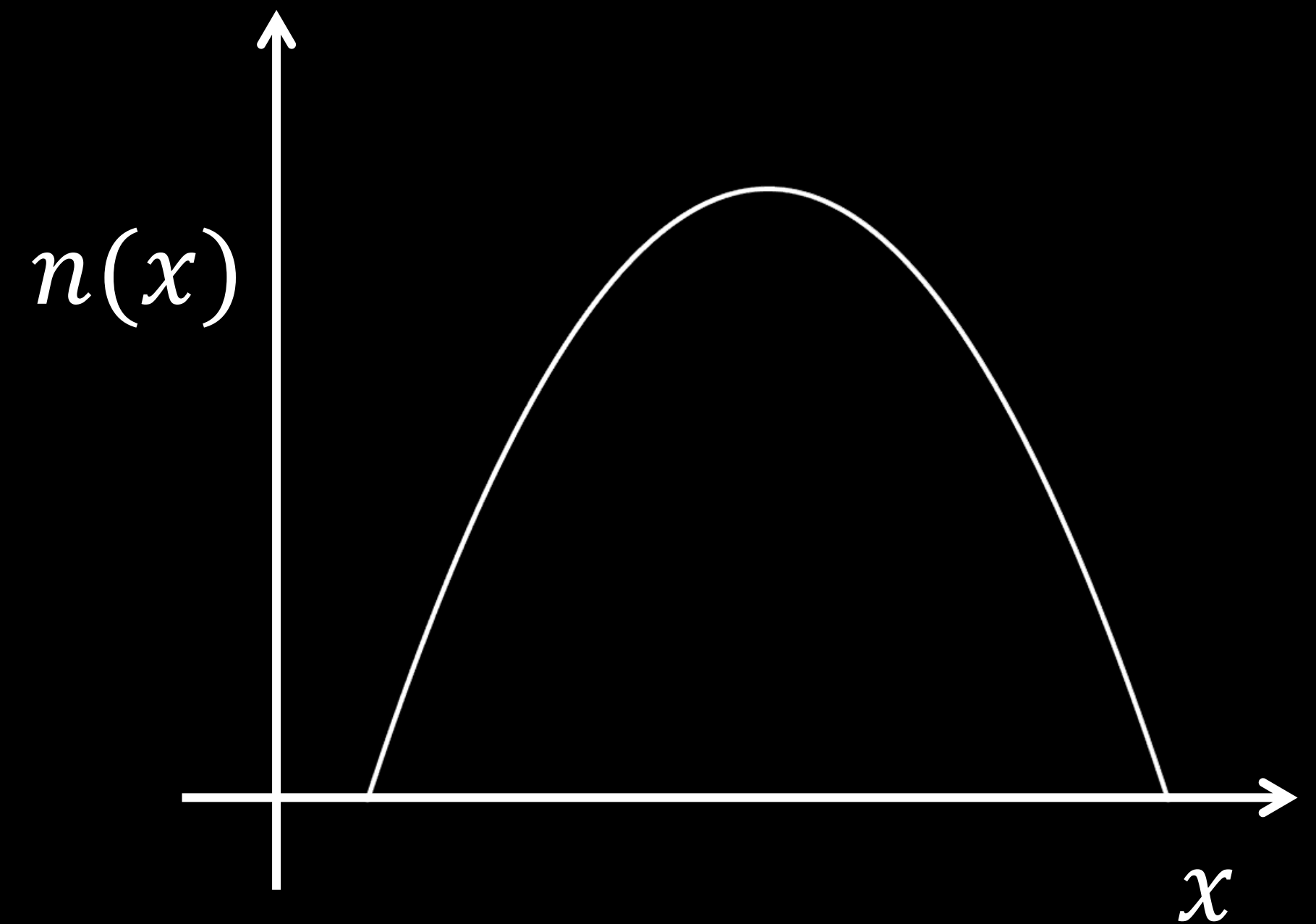
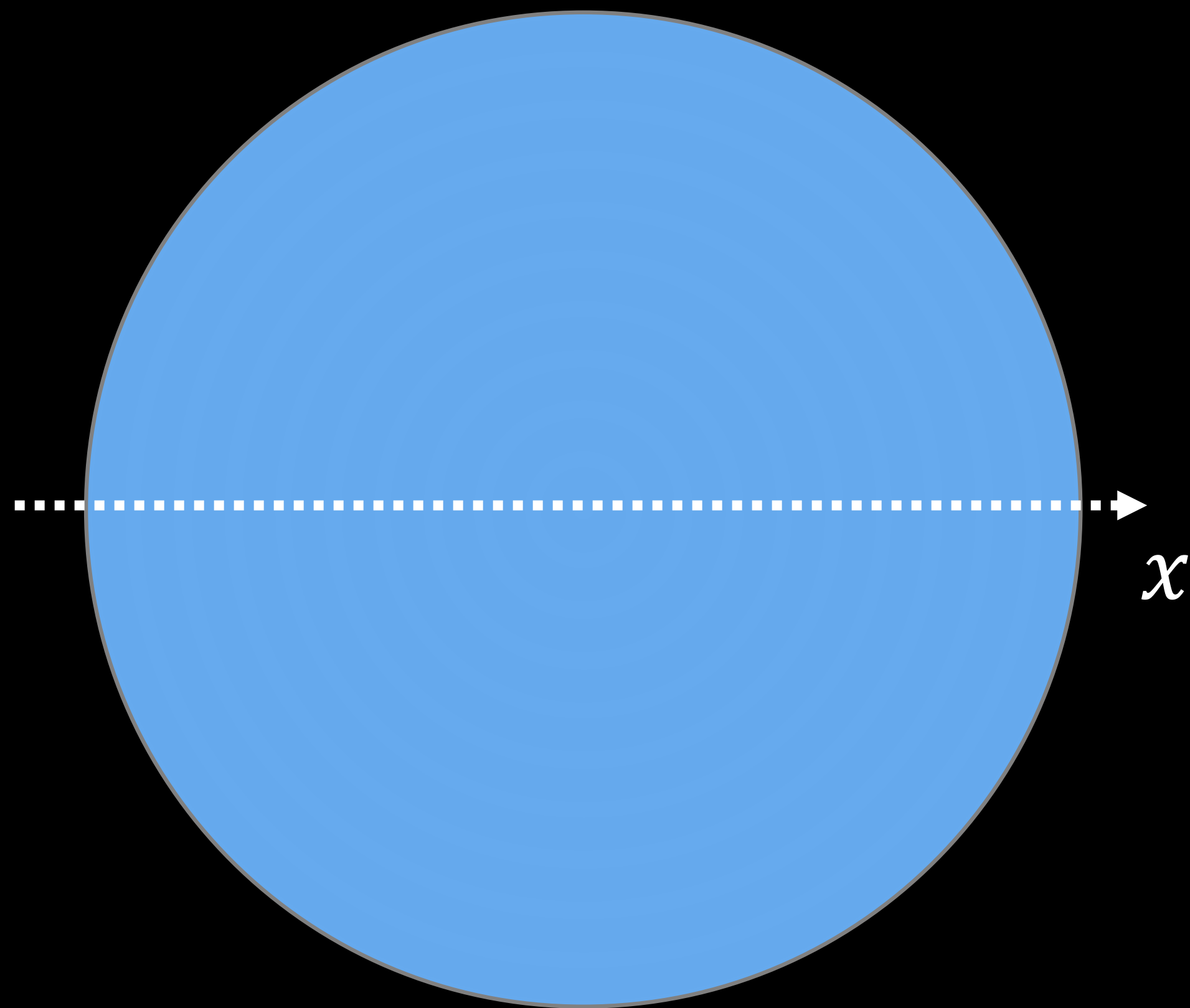
**end**



# Application: simulate Luneburg lenses

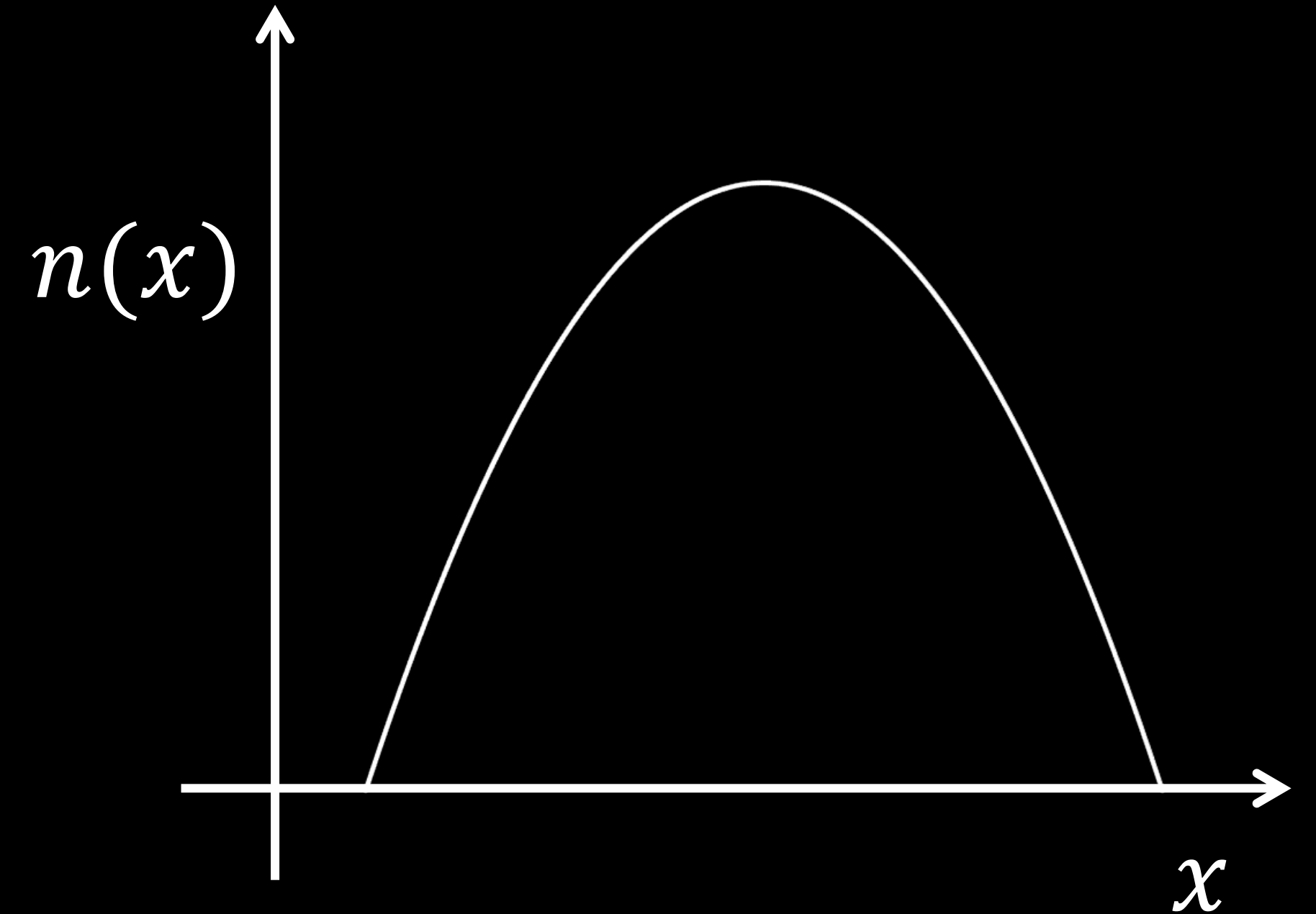
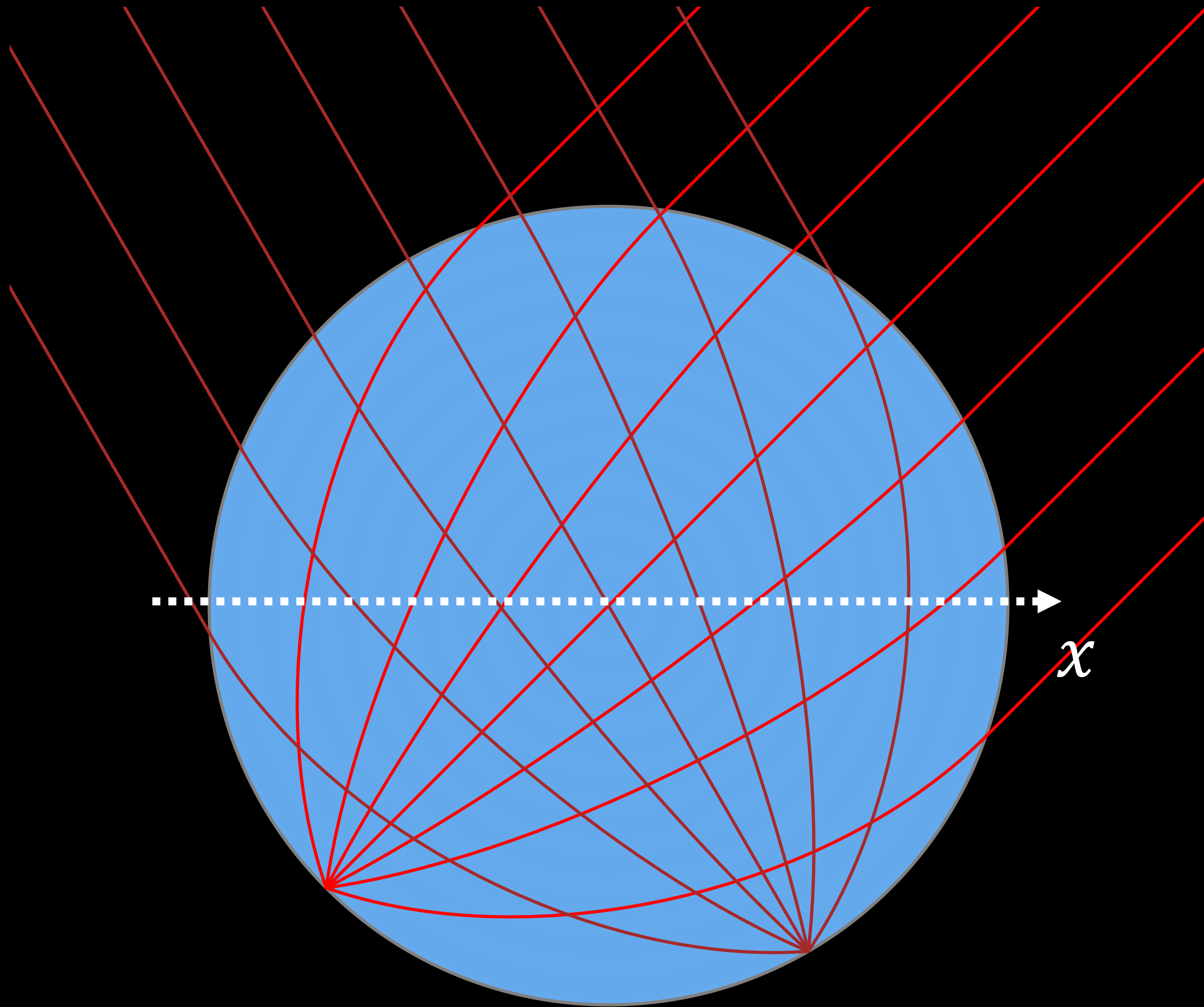


# Application: simulate Luneburg lenses

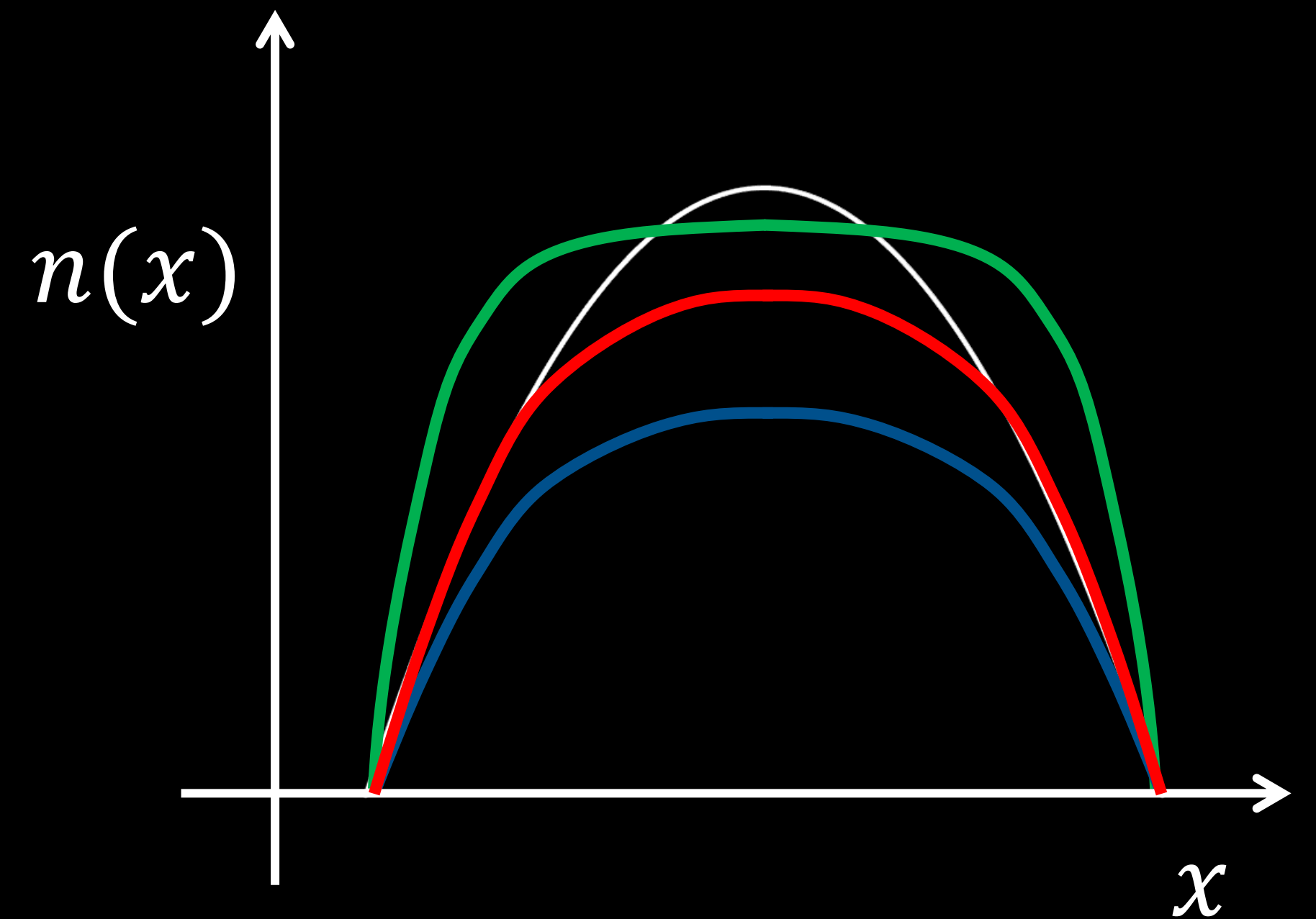
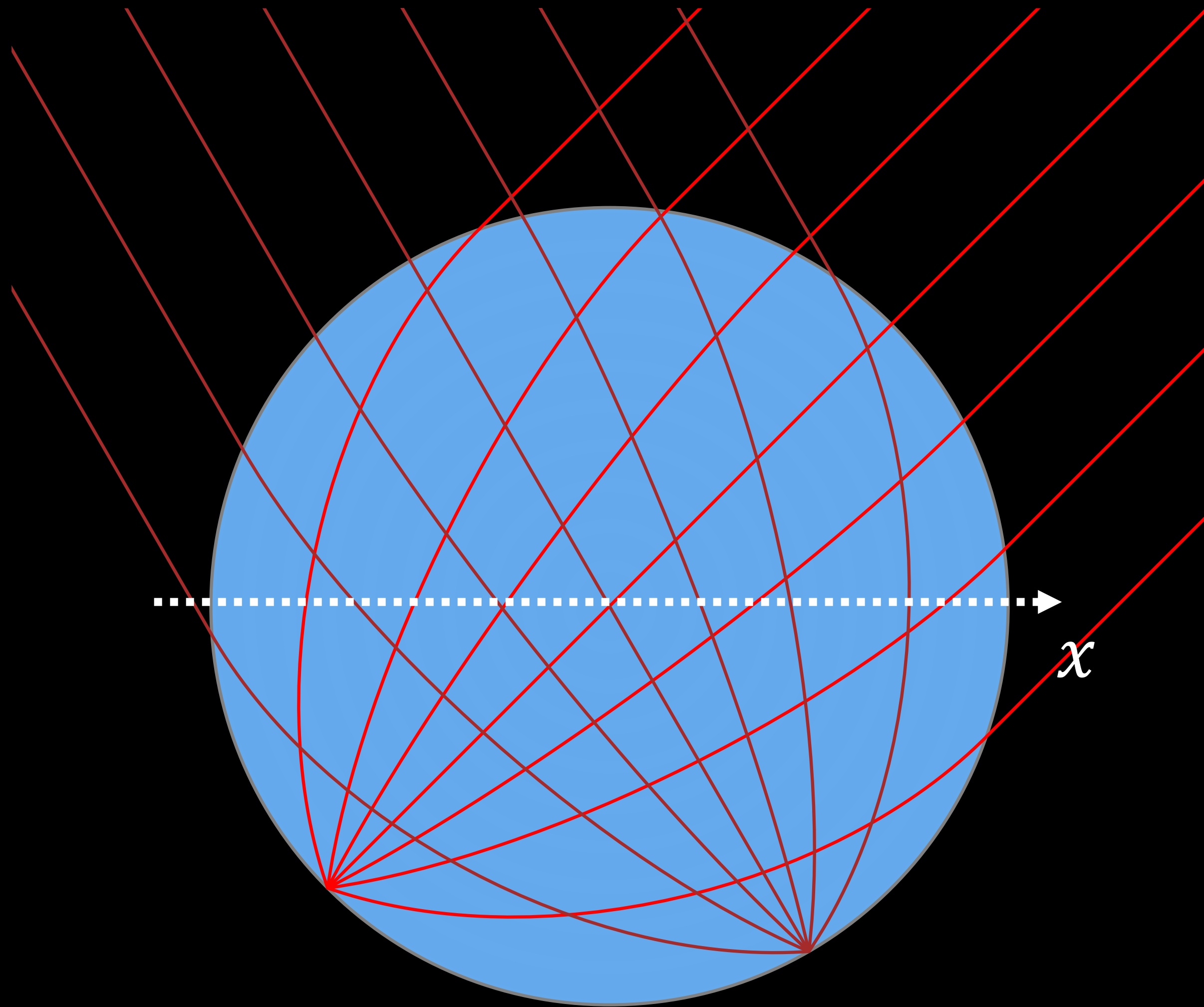




# Application: simulate Luneburg lenses

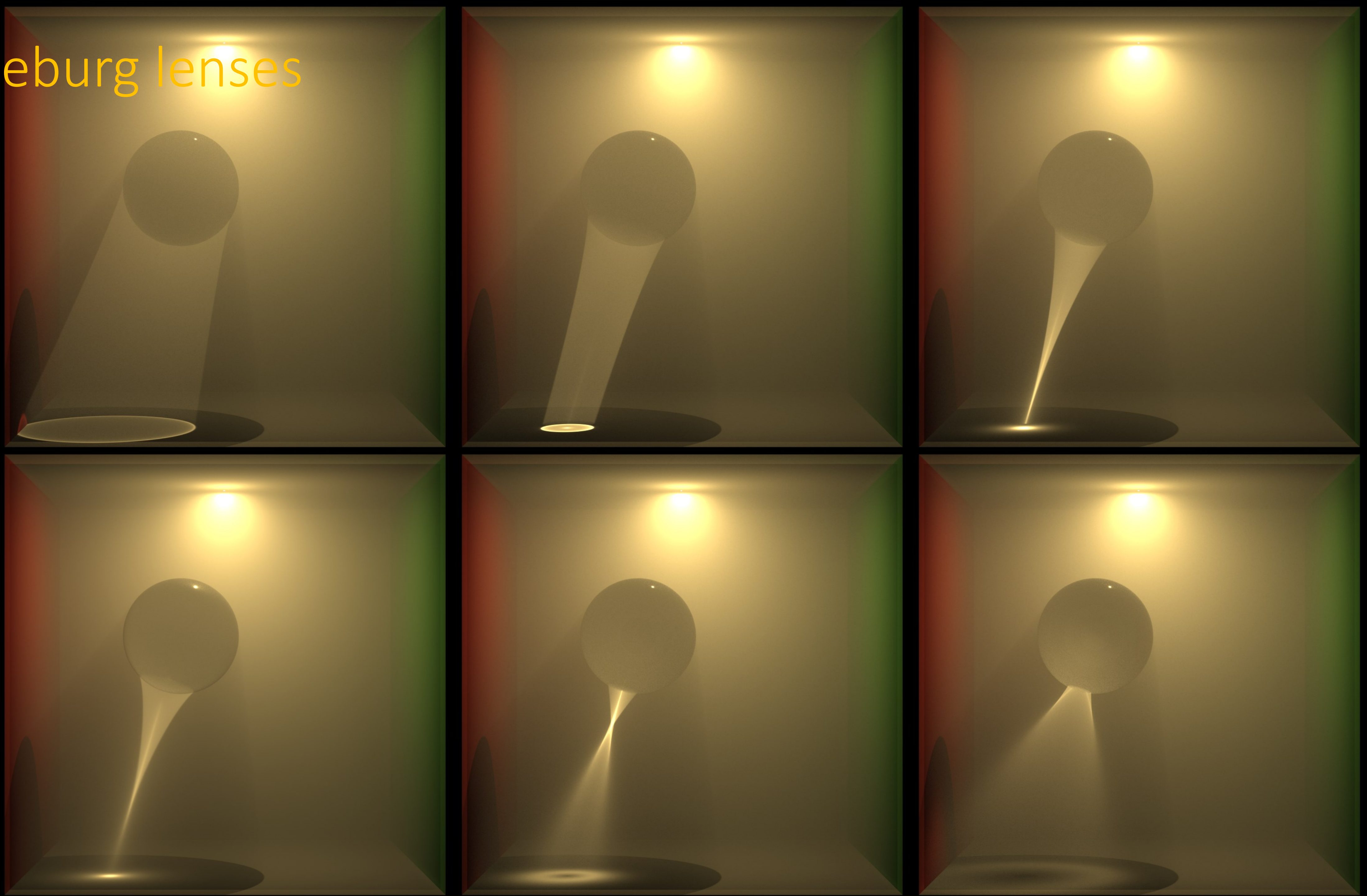


# Application: simulate Luneburg lenses





# Luneburg lenses



# Application: transient rendering

constant refractive index

continuous refractive index



# Application: transient rendering

constant refractive index

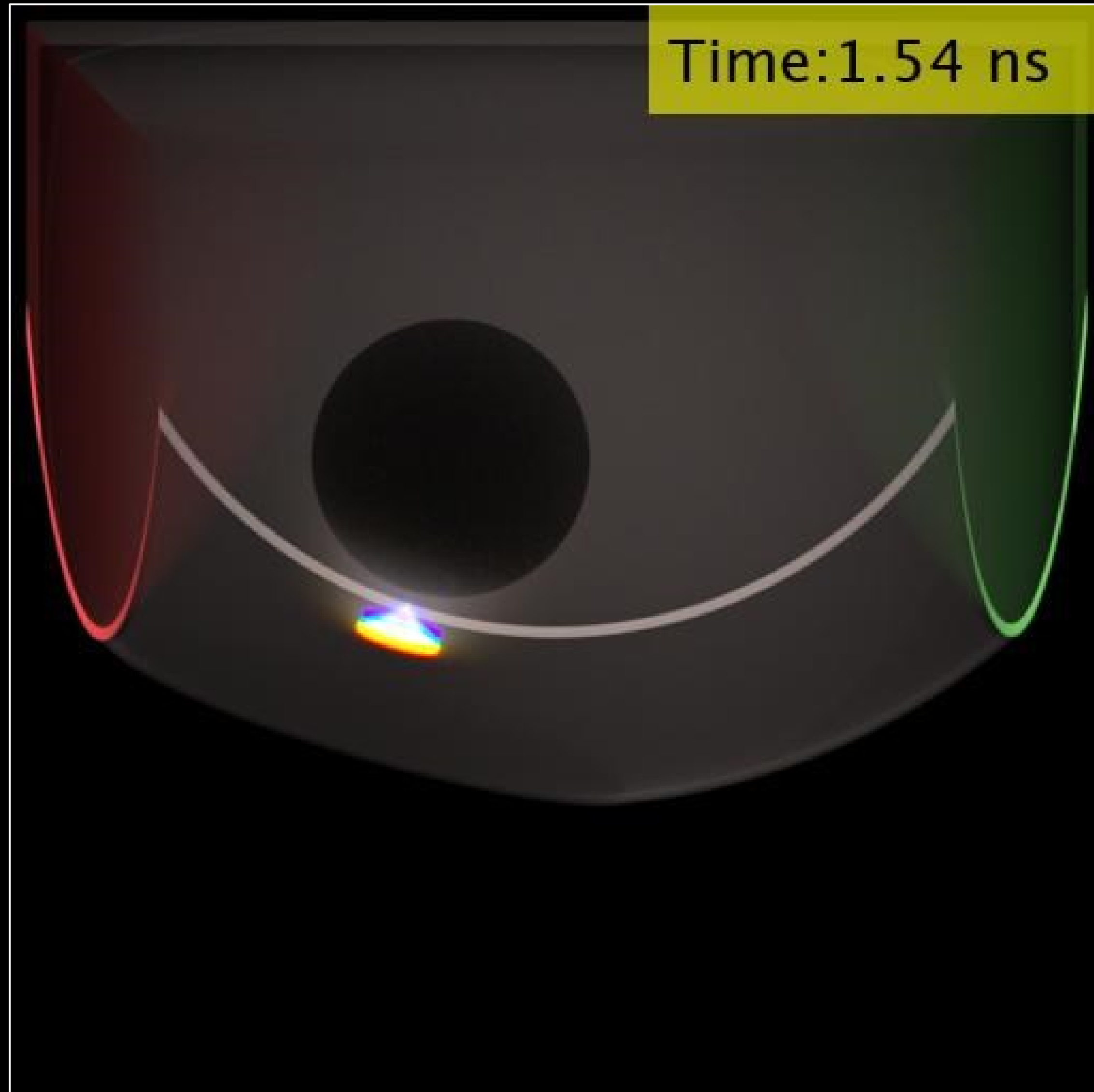
Time:0.02 ns

continuous refractive index

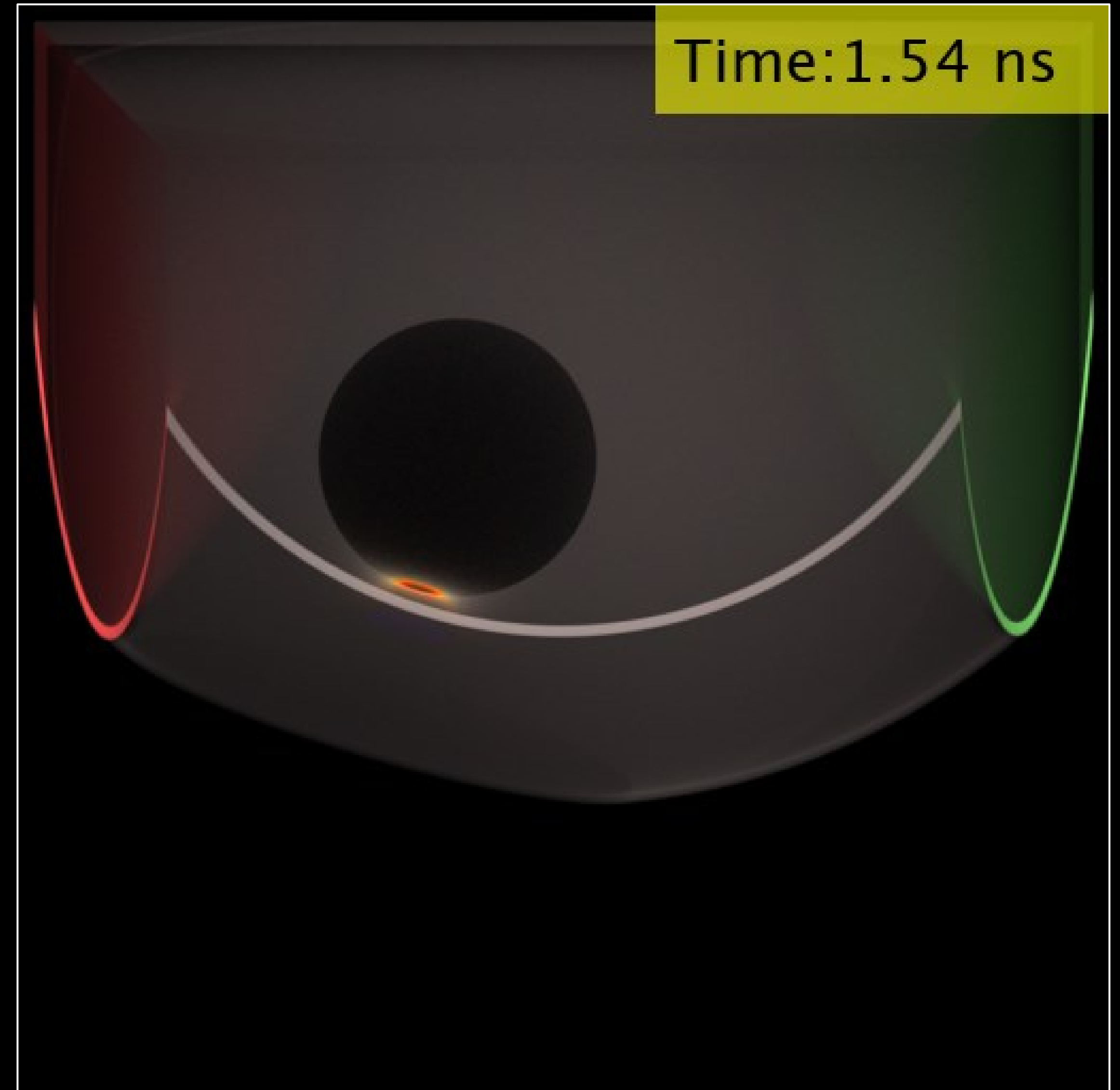
Time:0.02 ns

# Application: transient rendering

constant refractive index



continuous refractive index



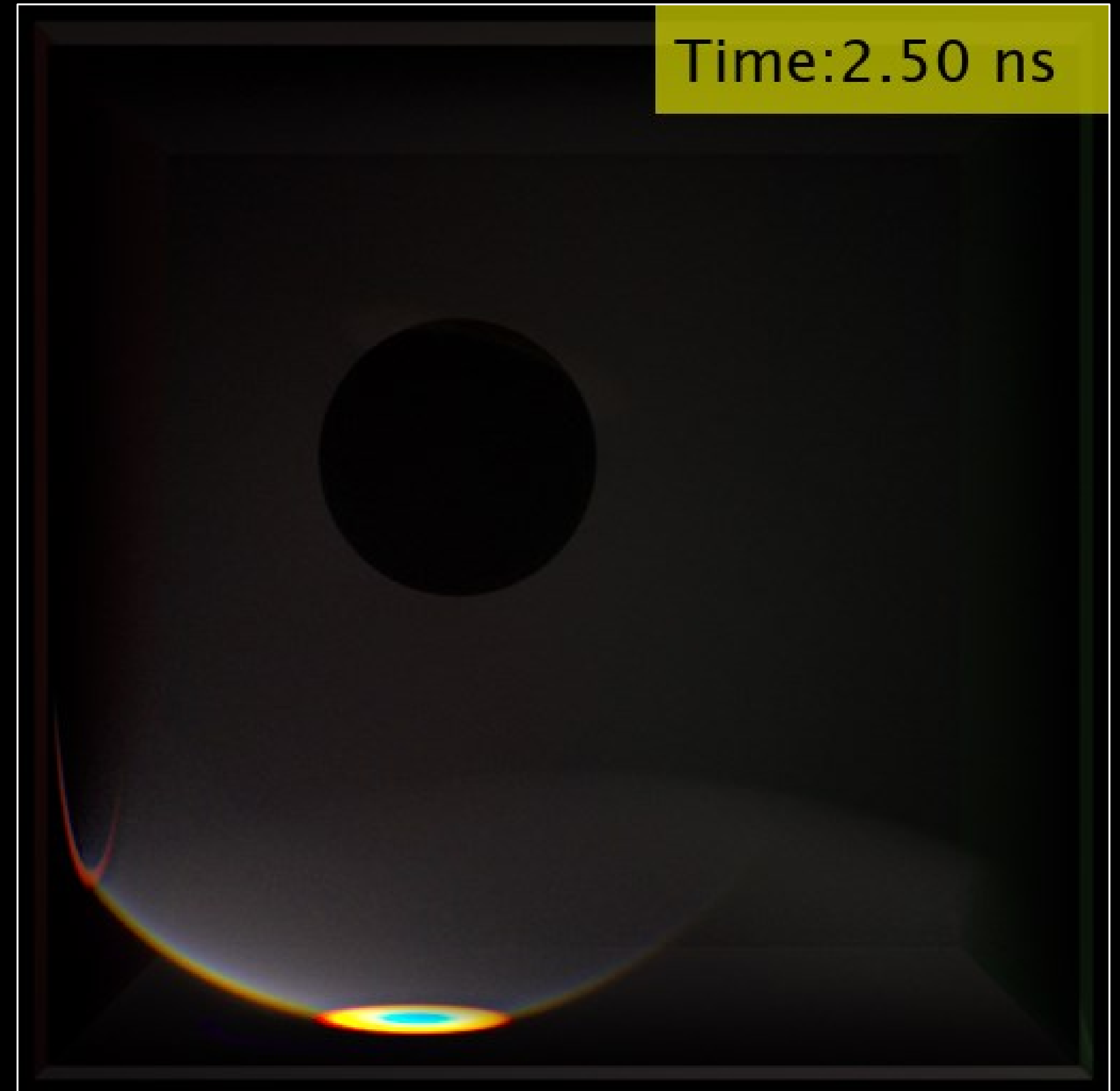


# Application: transient rendering

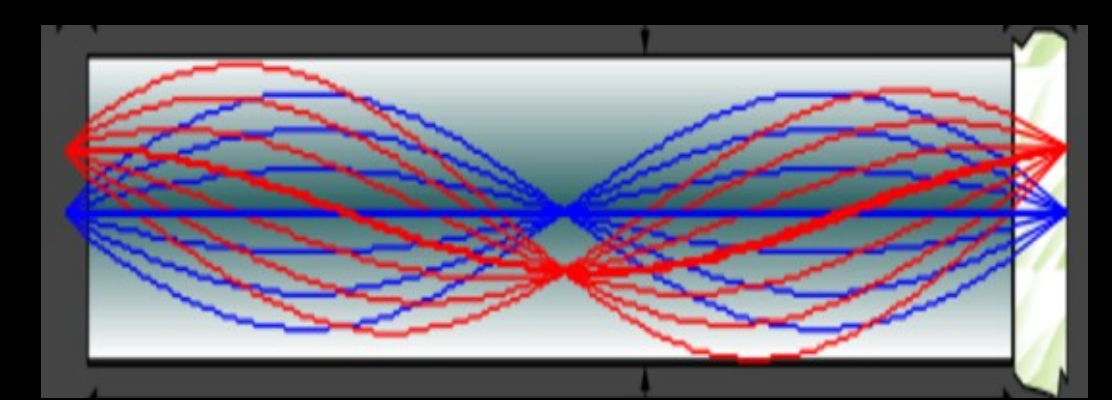
constant refractive index



continuous refractive index



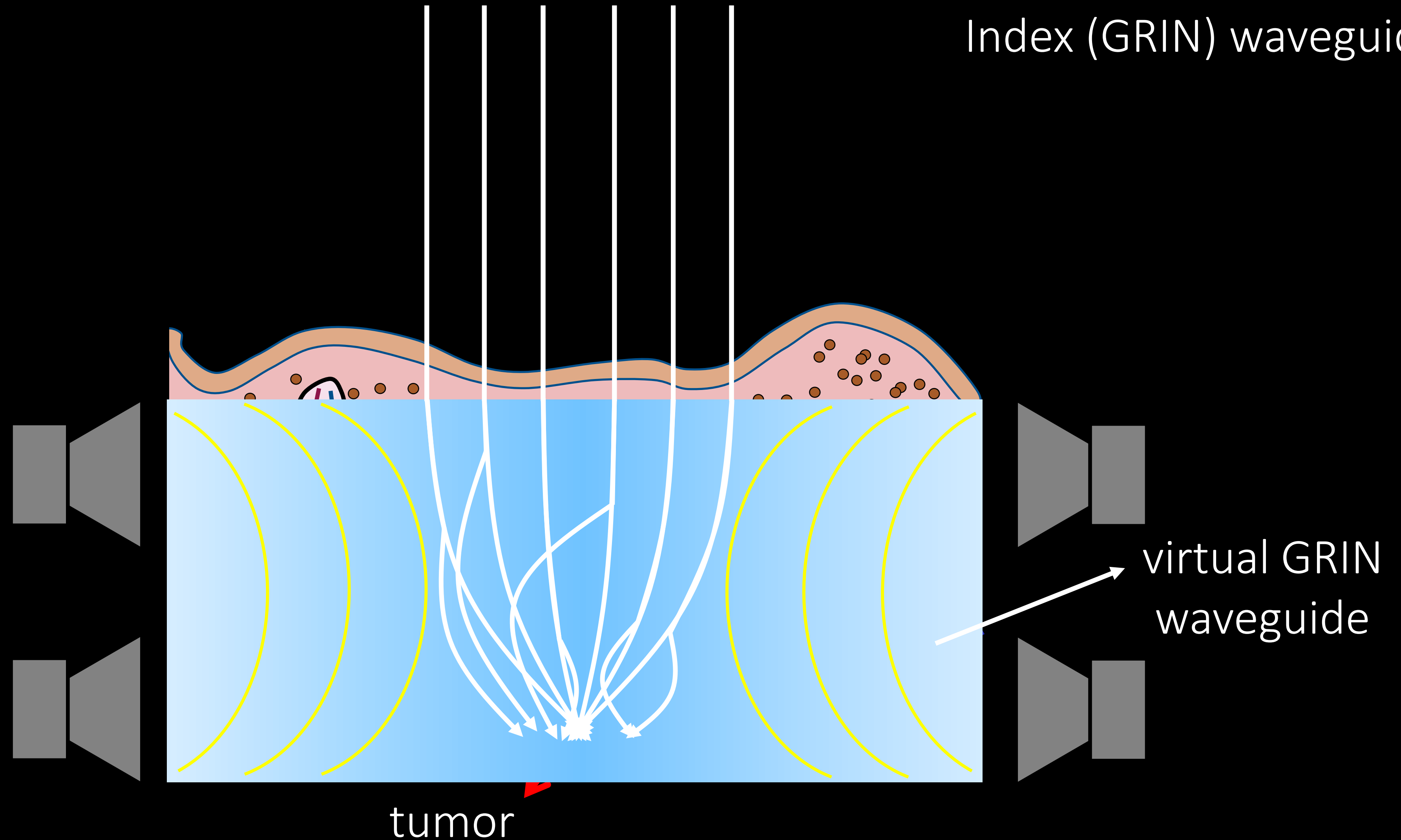
# Application: focusing light inside tissue



Gradient Refractive Index (GRIN) waveguide

High-dimensional, highly-non-linear design problem:

- ultrasound frequency
- ultrasound voltage
- placement of transducers
- waveform shape
- and more...

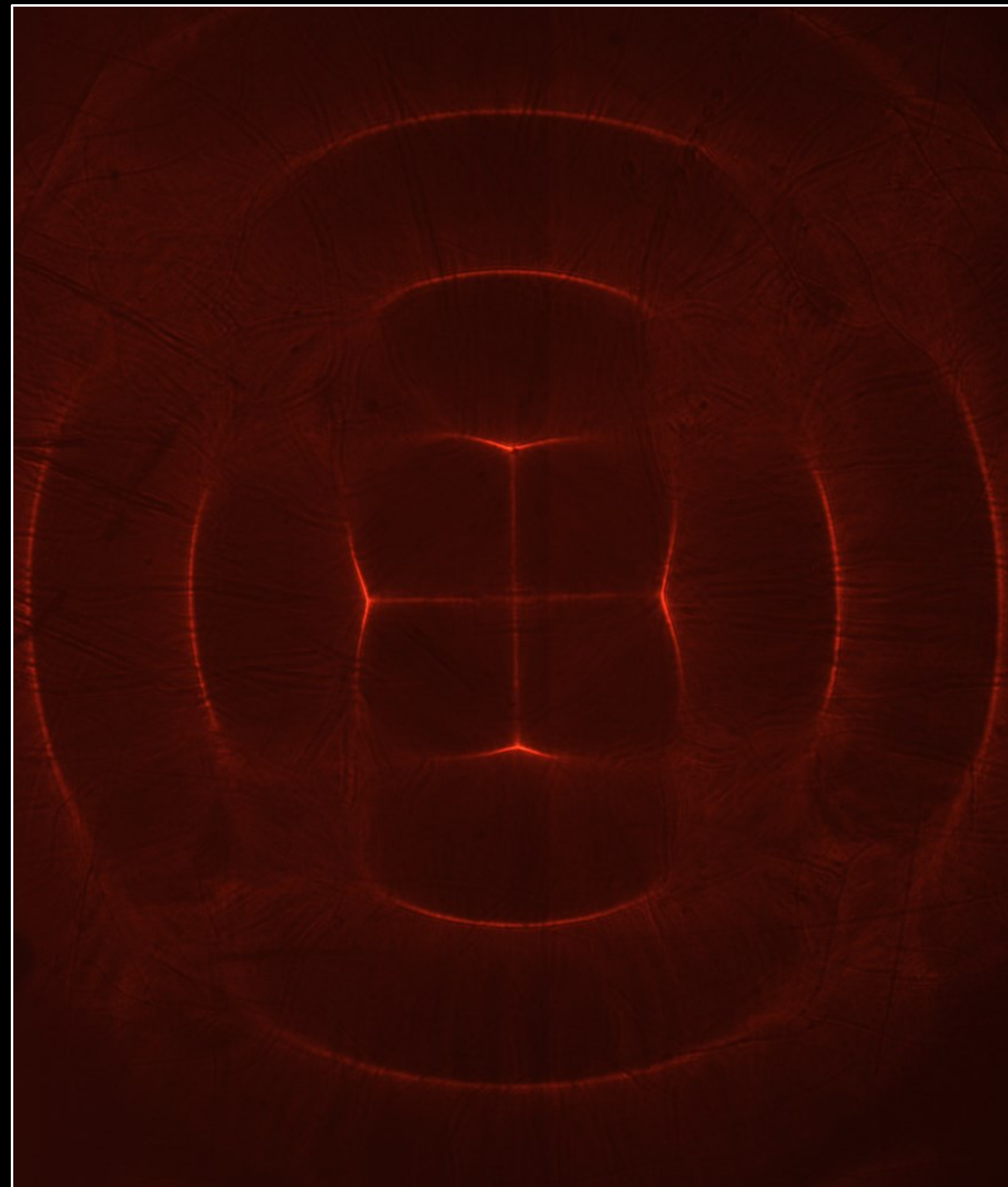


Efficiently explore using rendering

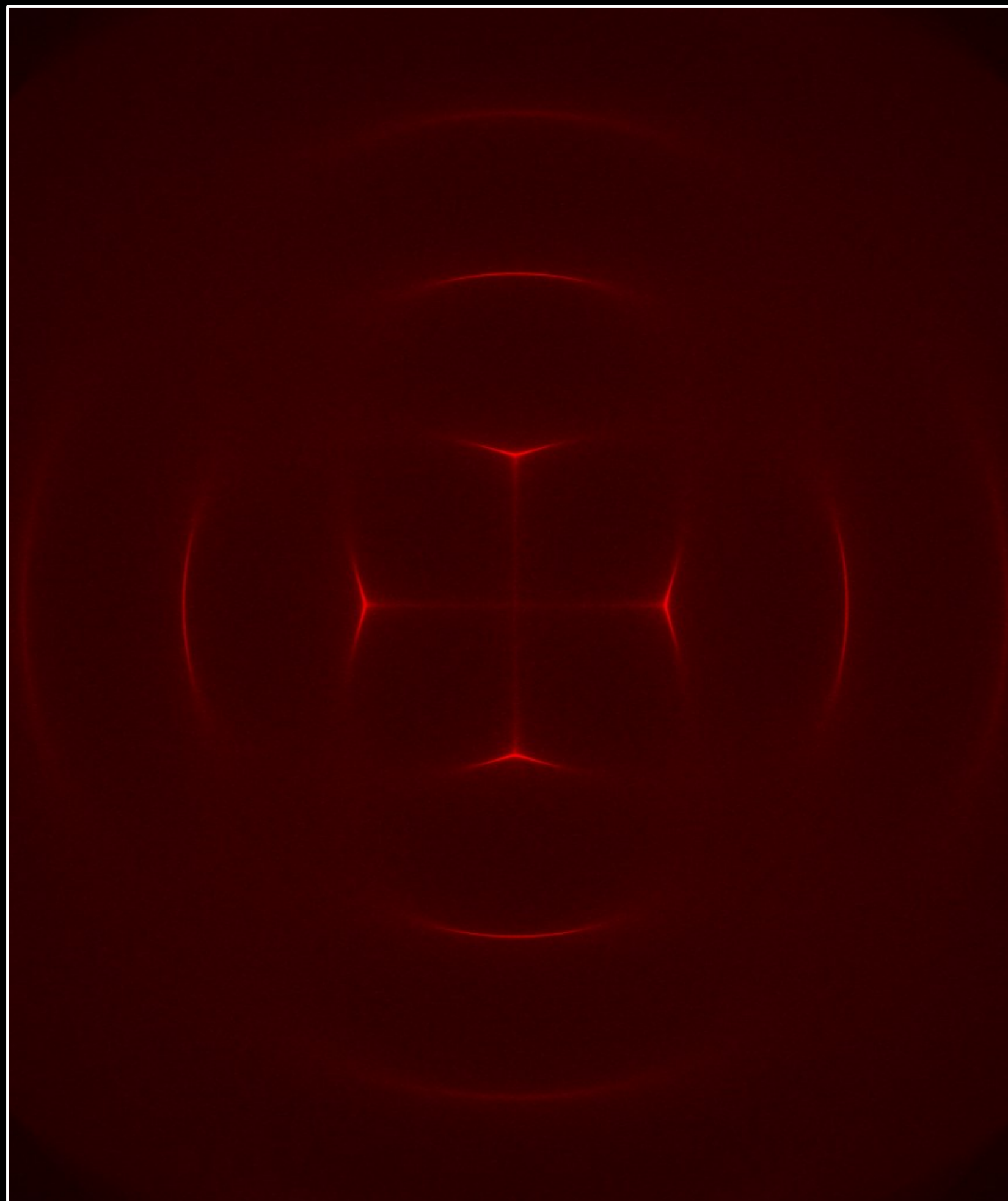


# Rendering virtual ultrasonic waveguides

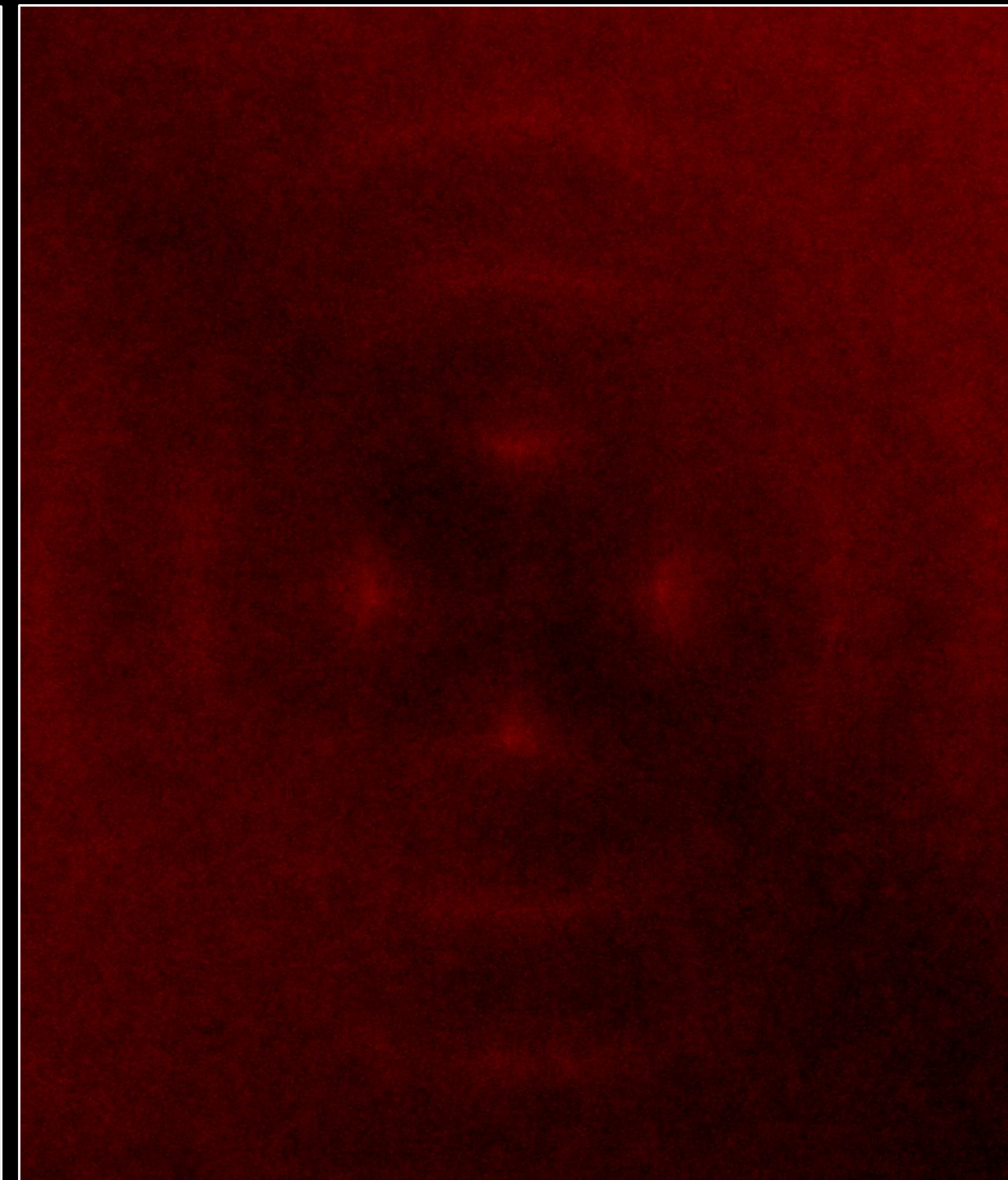
real measurement



BDPT  
(our technique)



photon mapping  
(previous technique)

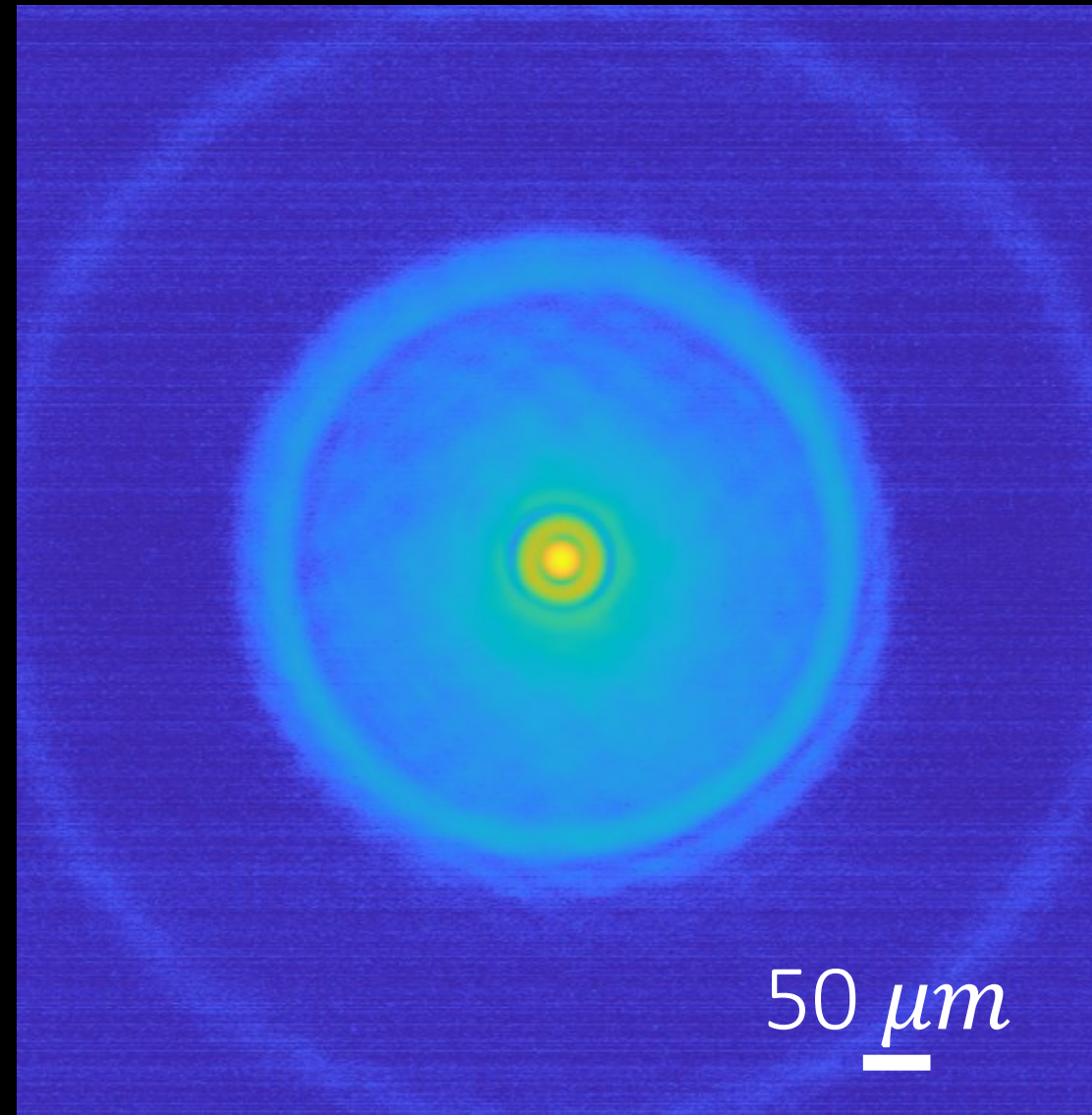




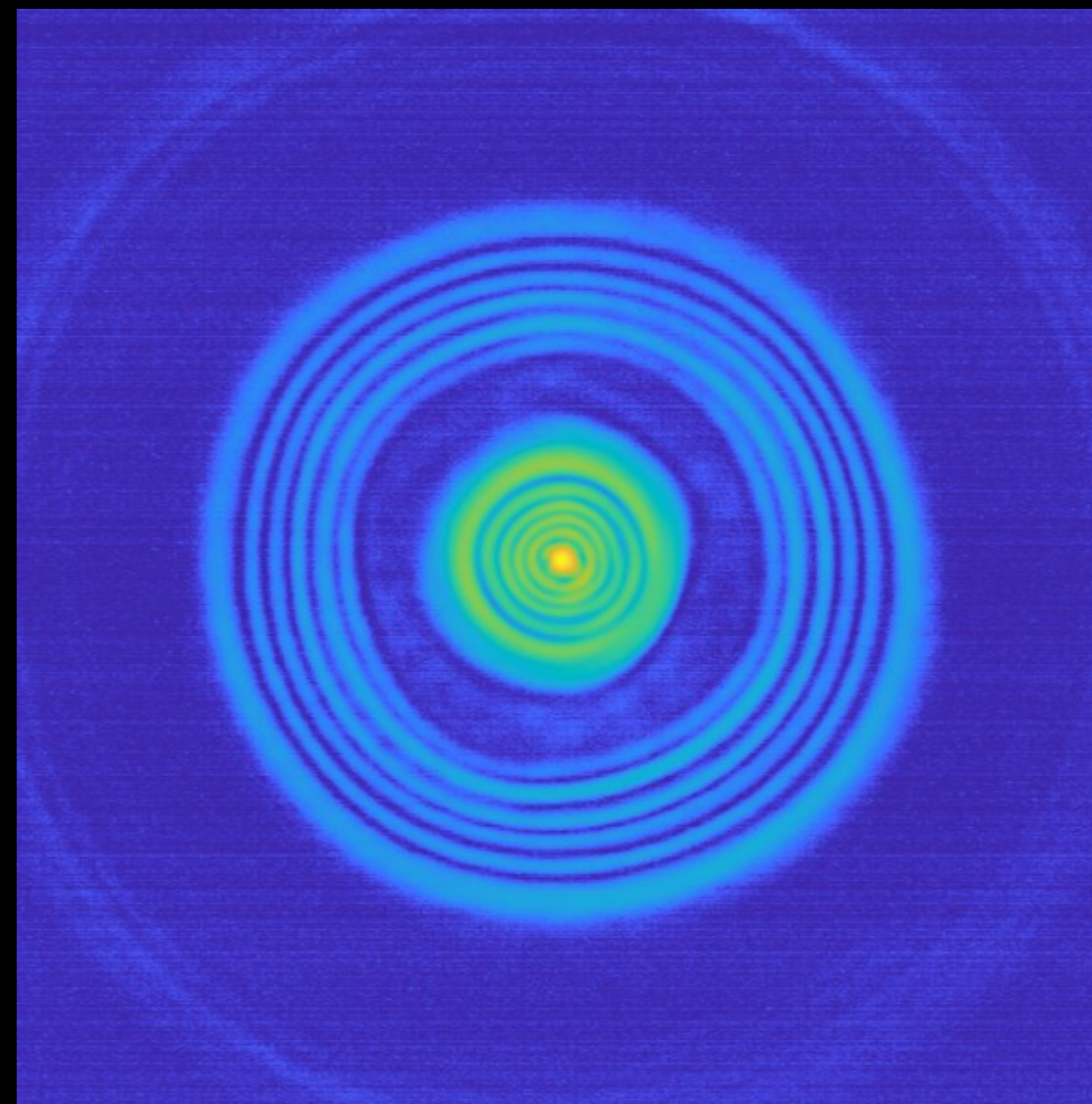
# Validation of simulated data

experimental data

first focus

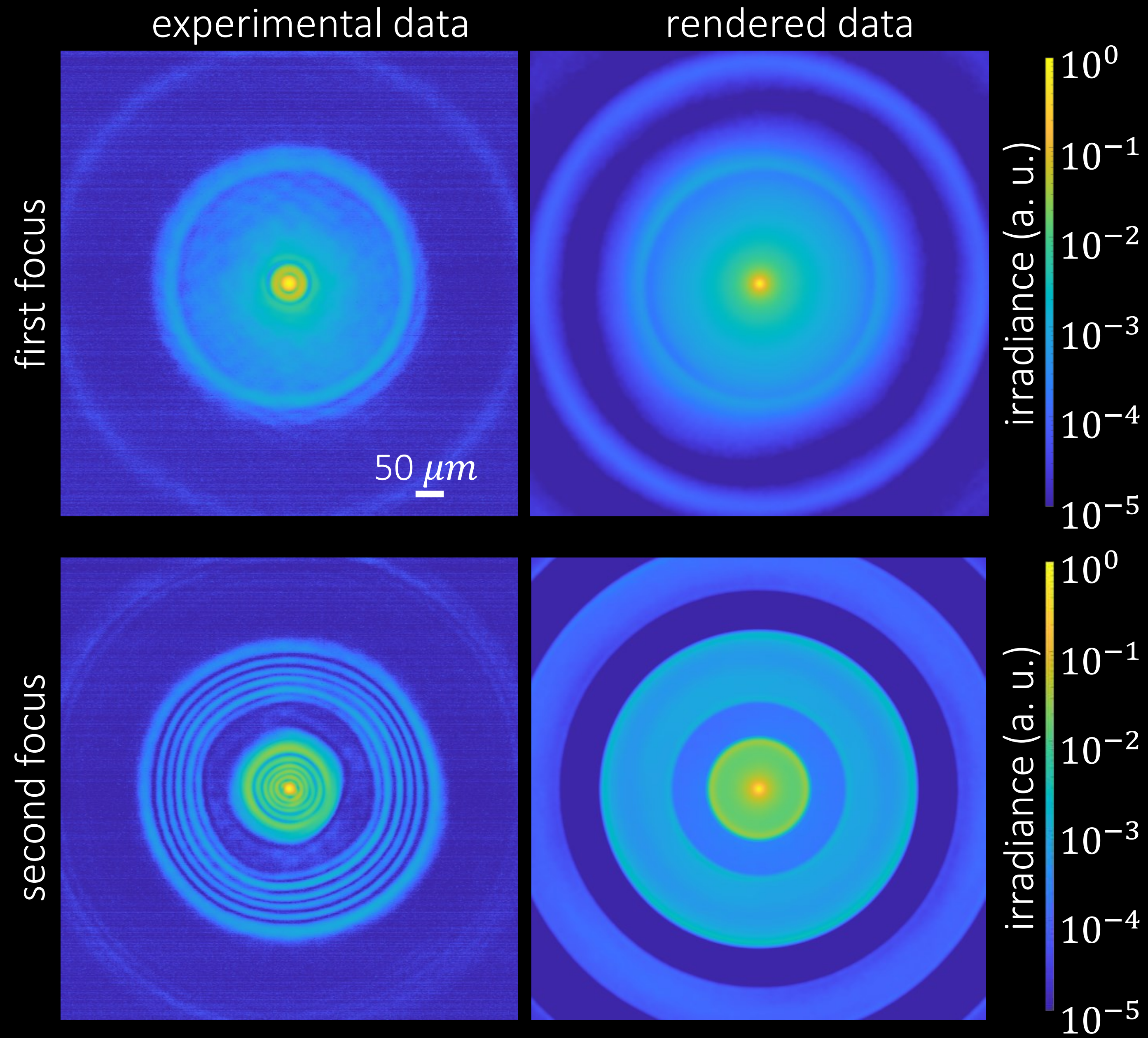


second focus



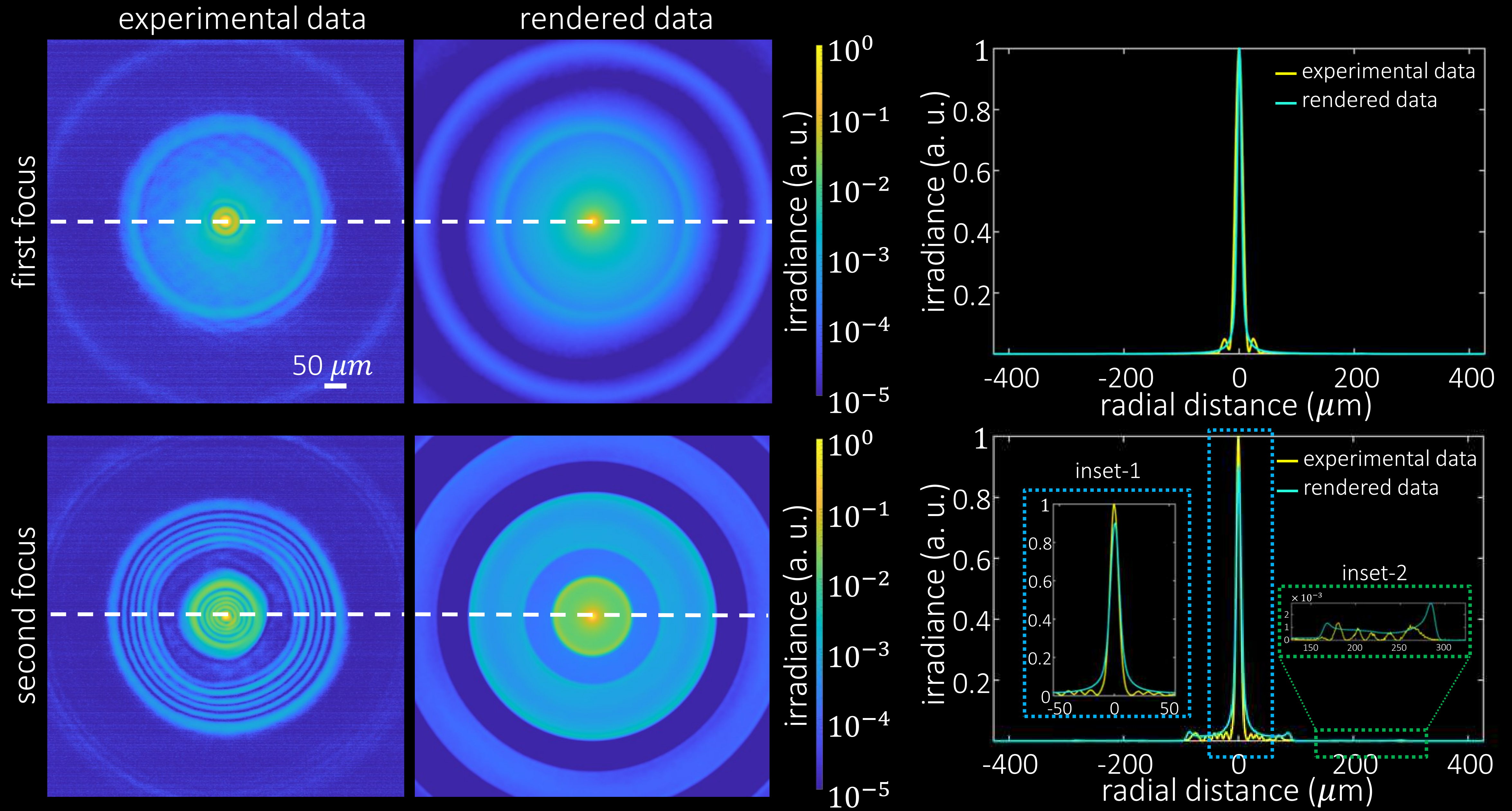


# Validation of simulated data



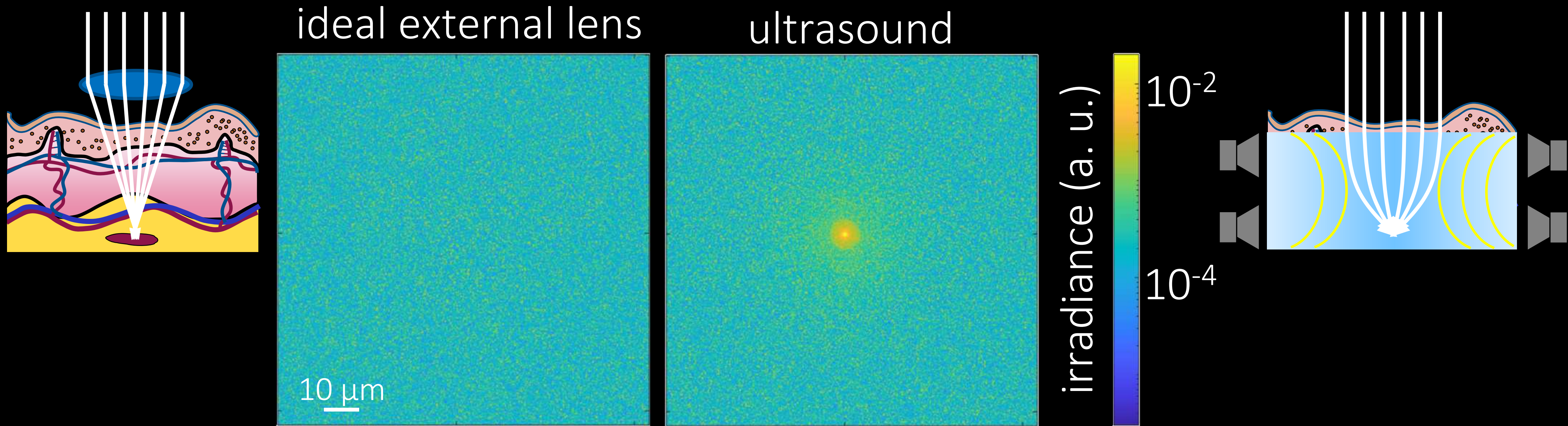


# Validation of simulated data





# Optimized configurations are better than ideal external lens



On human bladder (10 scattering lengths, 2.67 mm thick)

- 50% higher focusing performance than external lens.
- 300% higher focusing performance than previous designs.

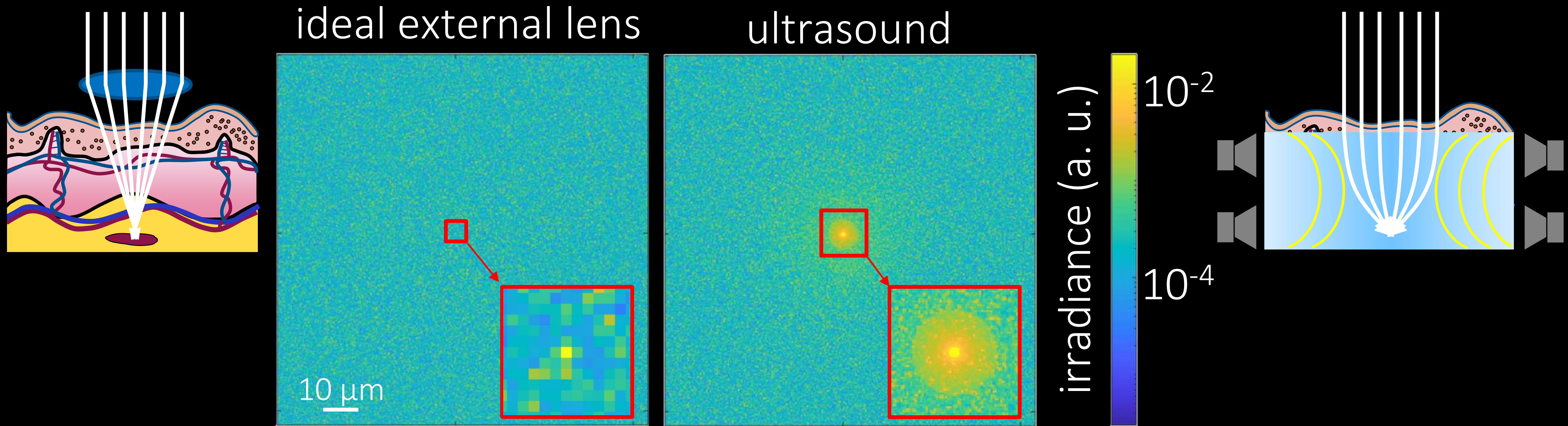
On brain tissue (50 scattering lengths, 7.5 mm thick)

- 15% higher focusing performance than external lens.
- Experimentally validated on tissue phantoms.

[Pediredla et al., to appear in Nature Communications]



# Optimized configurations are better than ideal external lens



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# Physics-based rendering and its applications to computational imaging


## forward rendering



The diagram illustrates the forward rendering process. On the left, a 3D scene with a textured surface and a light source is shown. Red dashed lines represent light rays originating from the light source, hitting the surface, and reflecting towards a camera. An arrow points from this scene to a computer icon, which then points to a stack of three grayscale images, representing the rendered output.

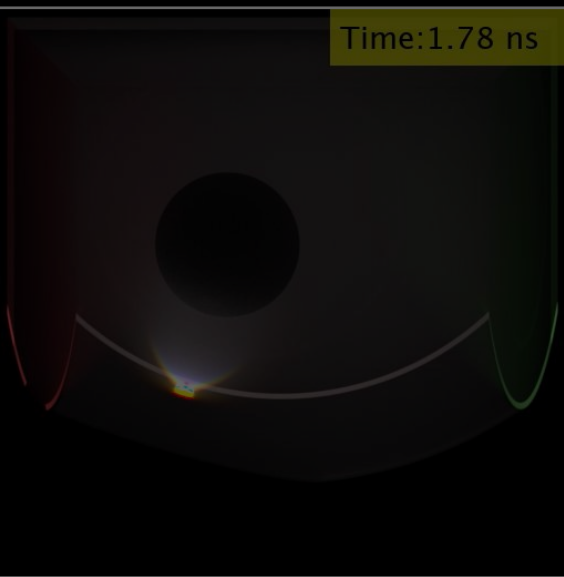
- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

## inverse rendering

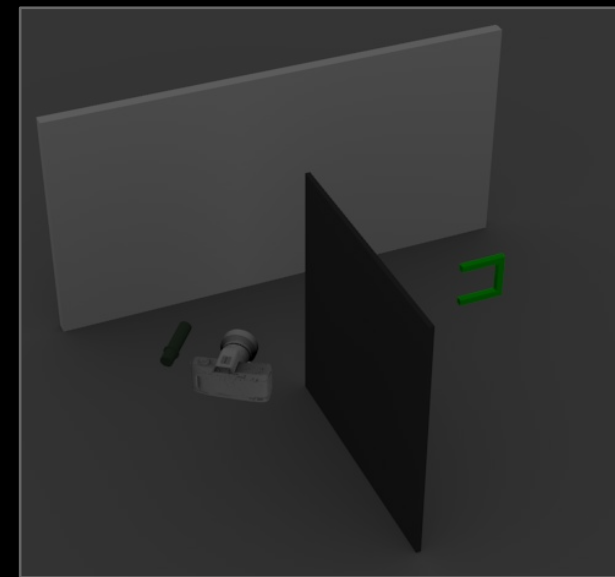


The diagram illustrates the inverse rendering process. On the left, a 3D scene is shown with red dashed lines representing light rays. An arrow points from this scene to a computer icon, which then points to a stack of three grayscale images. This represents the process of inferring the scene parameters from the observed images.

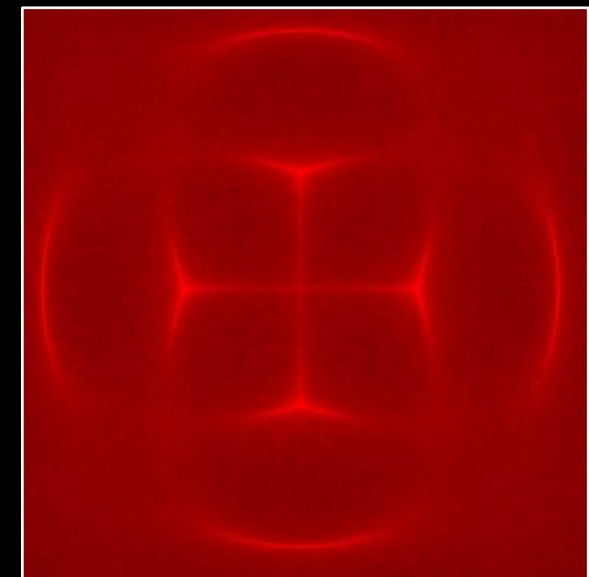
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



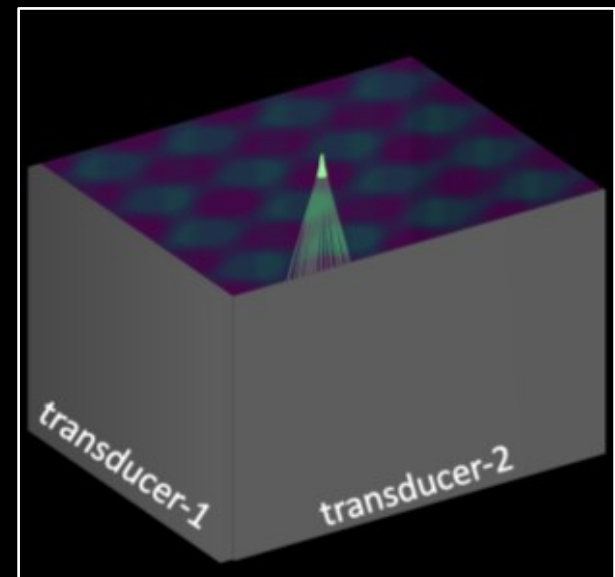
time-of-flight  
imaging



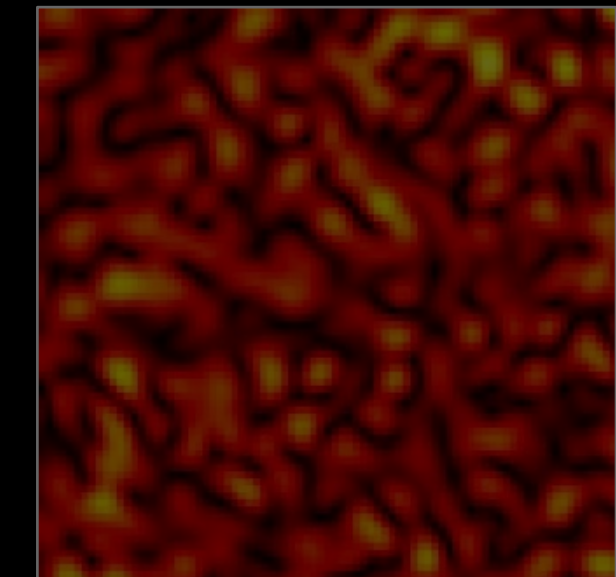
non-line-of-sight  
imaging



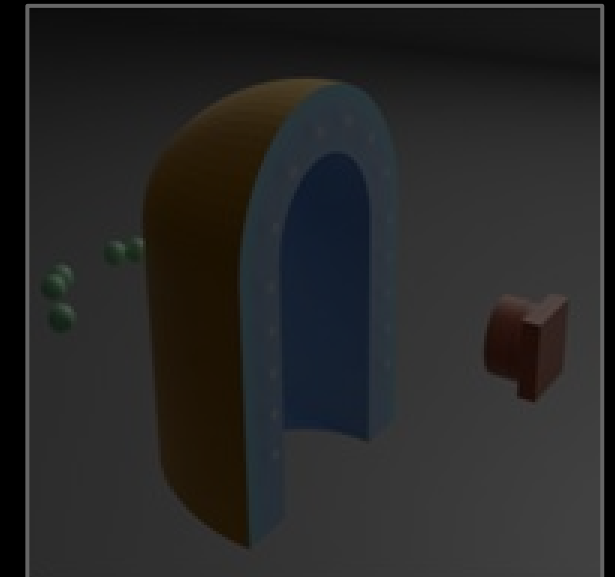
acousto-optic  
lensing



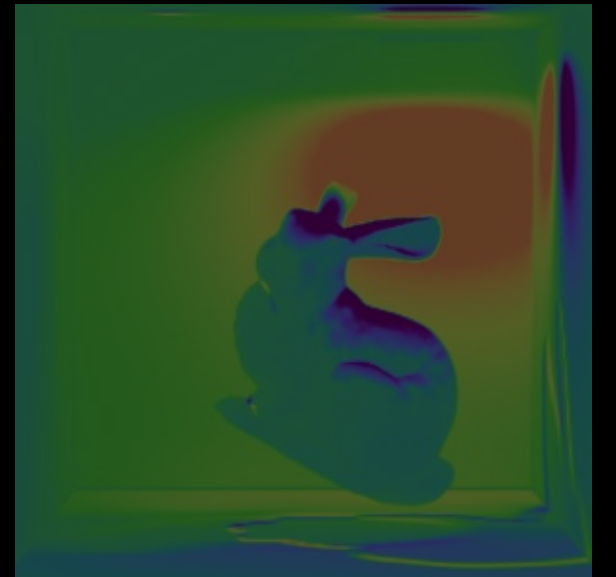
ultrafast light  
scanning



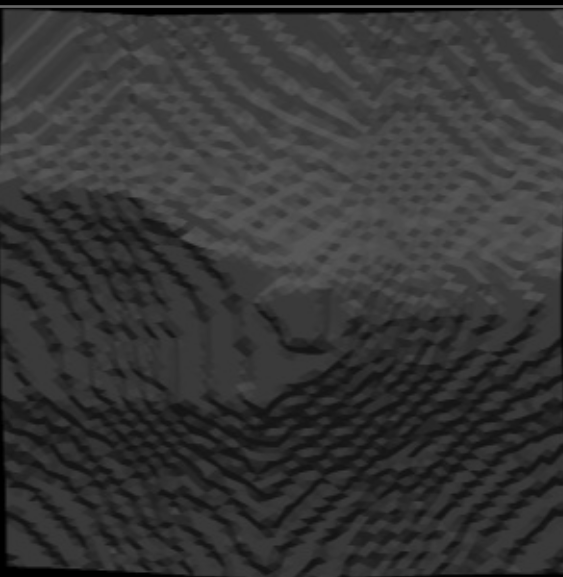
speckle  
imaging



tactile sensor  
design

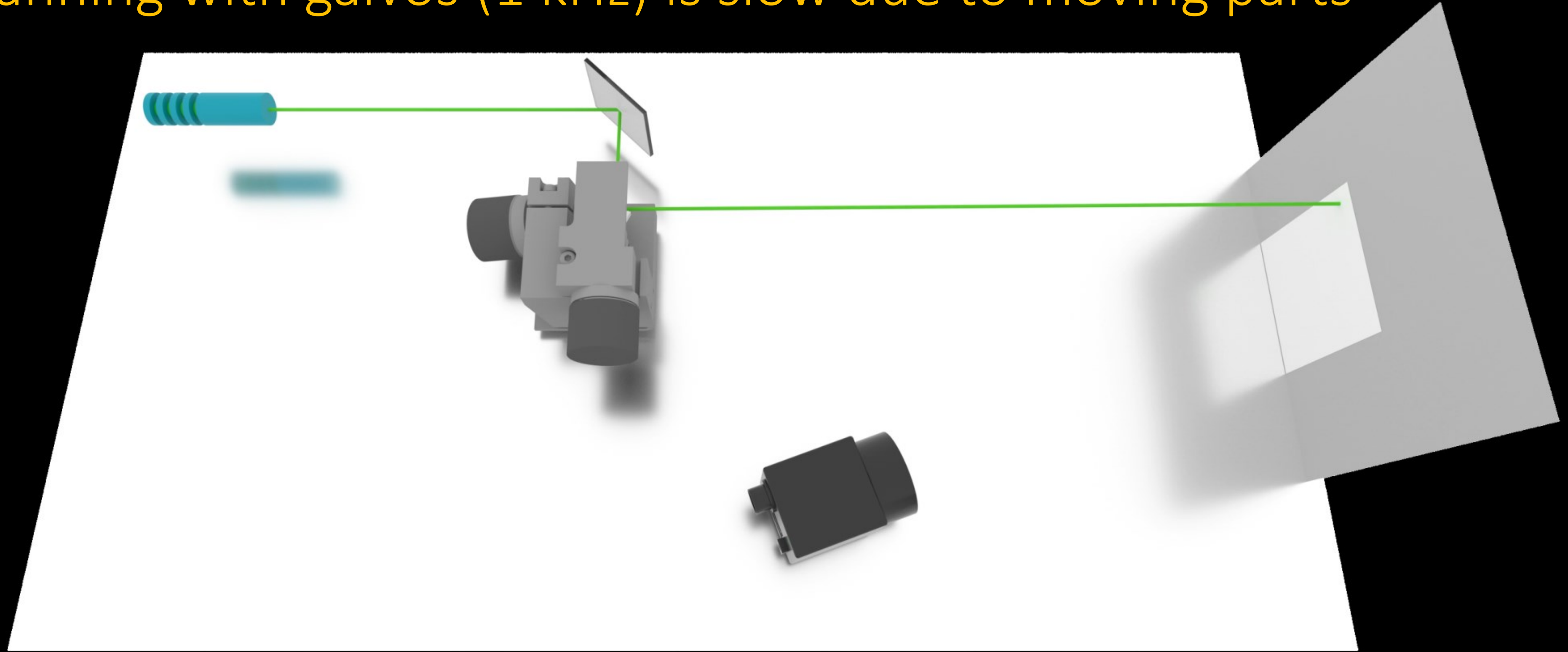


differentiable  
renderer



inverse  
problems

scanning with galvos (1 kHz) is slow due to moving parts



projector



microscopy

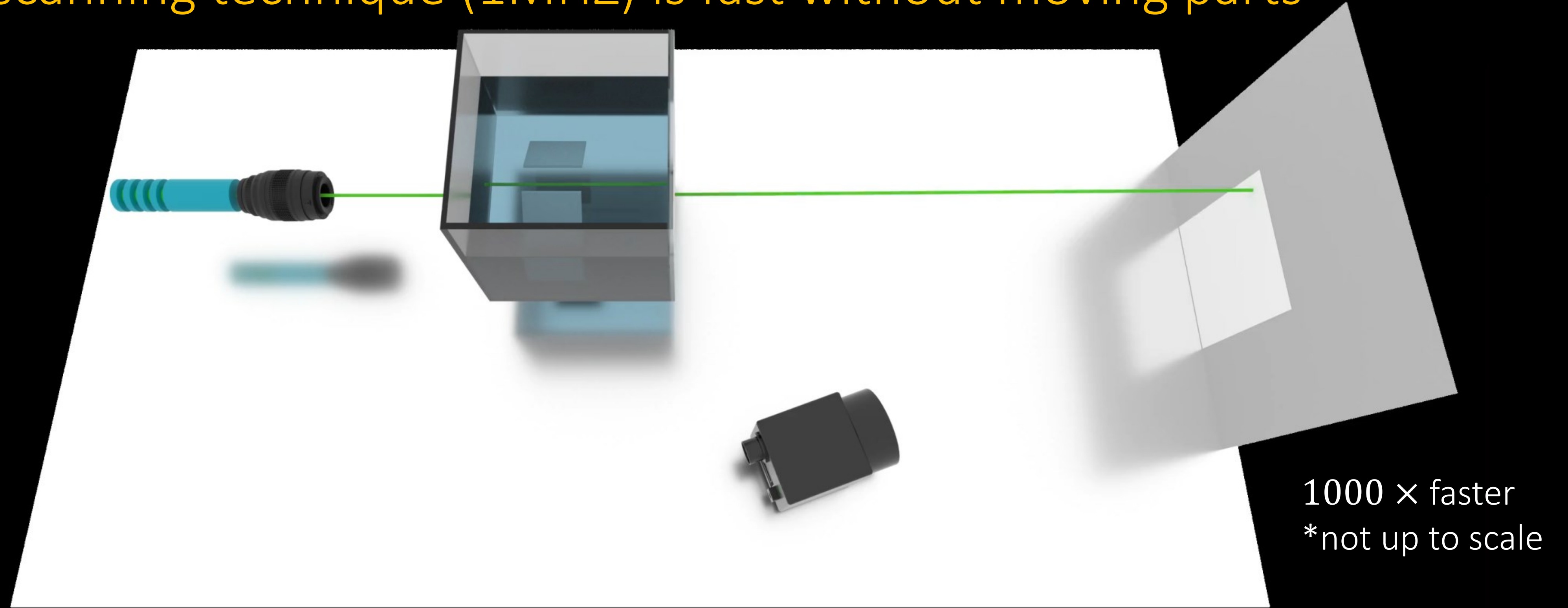


lidar





our scanning technique (1MHZ) is fast without moving parts



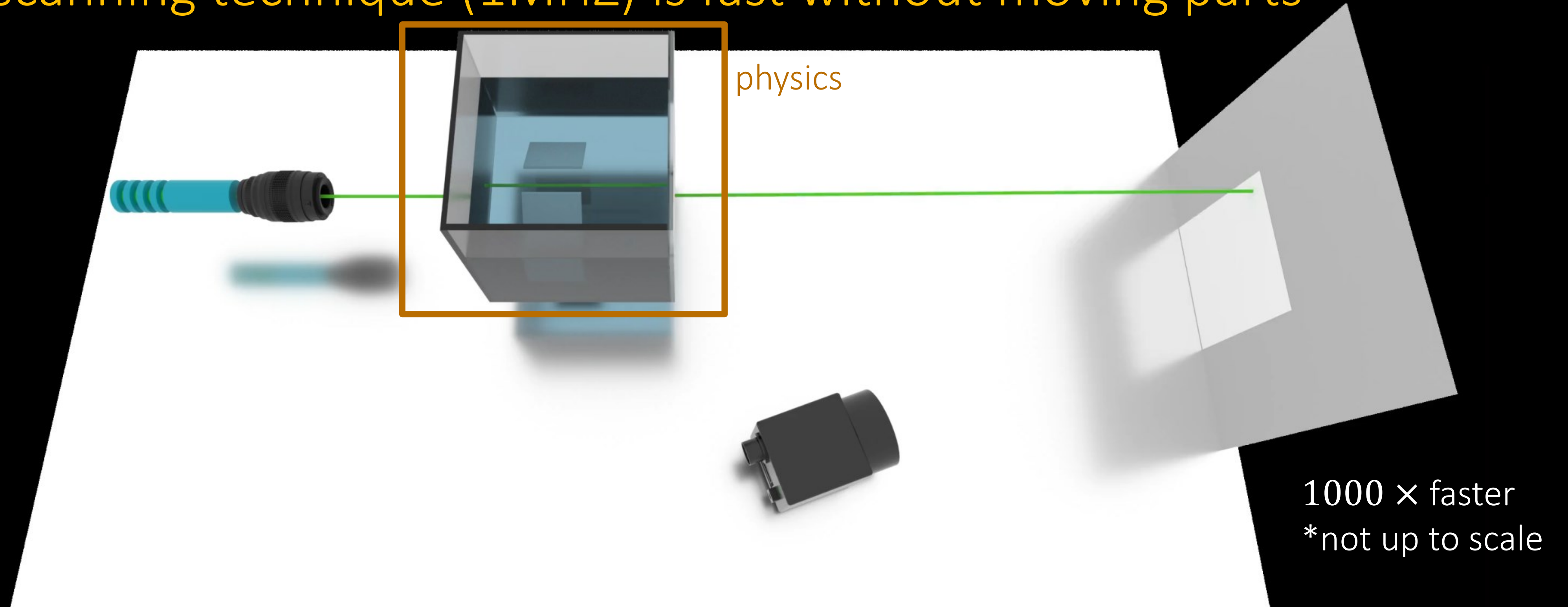
projector

microscopy

lidar



our scanning technique (1MHZ) is fast without moving parts



projector



microscopy

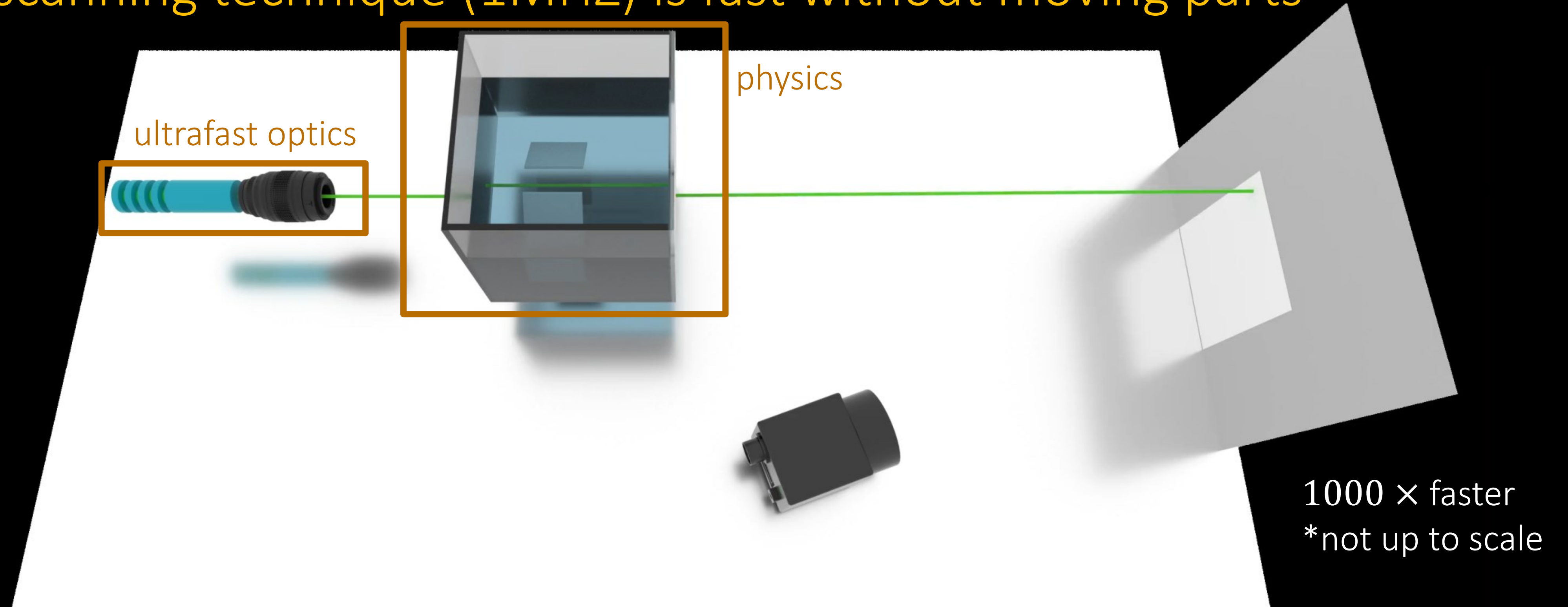


lidar





our scanning technique (1MHz) is fast without moving parts



projector



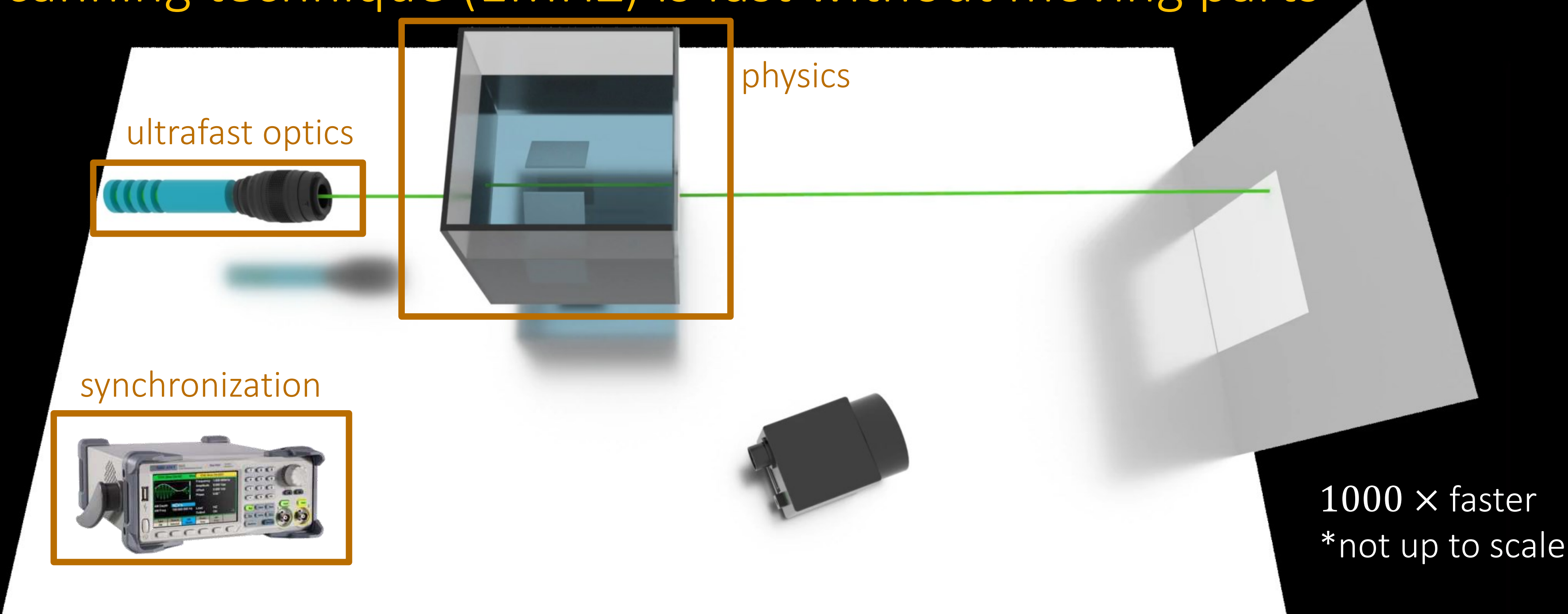
microscopy



lidar



our scanning technique (1MHZ) is fast without moving parts



projector



microscopy

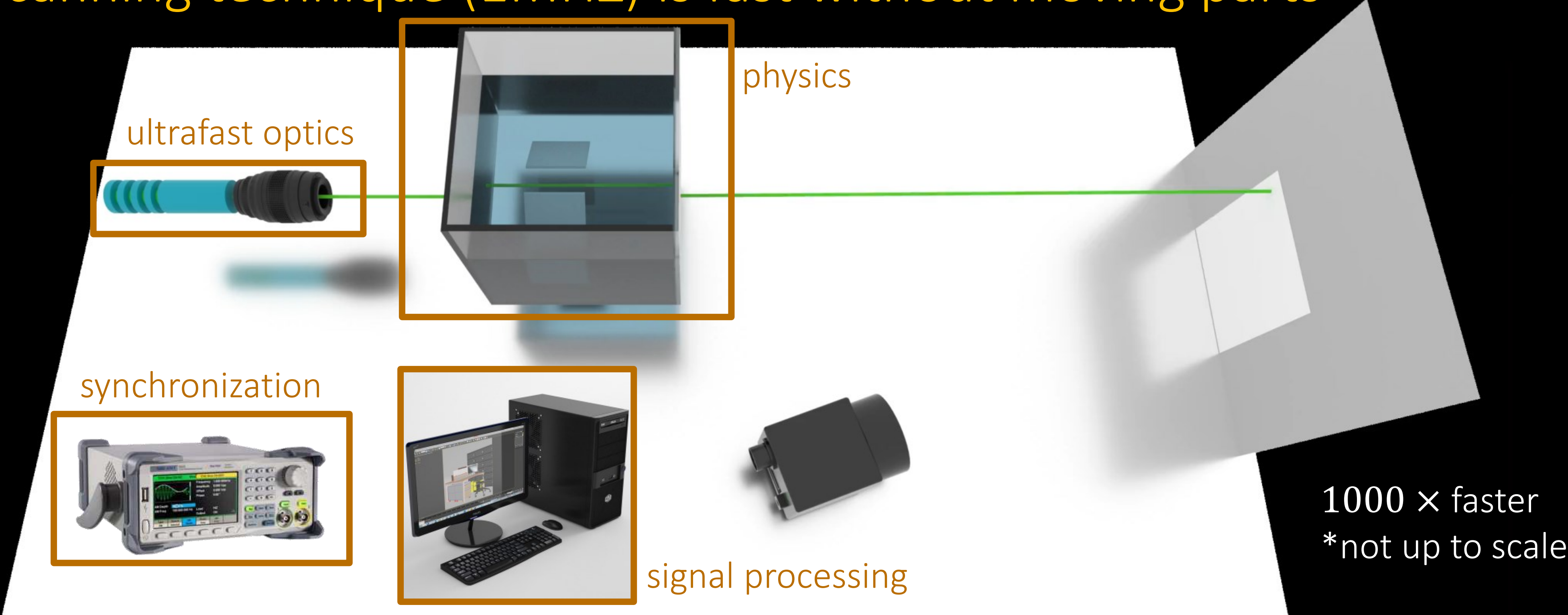


lidar





our scanning technique (1MHZ) is fast without moving parts



projector



microscopy



lidar



# Physics sound, light, and matter interaction



water

2D visualization



# Physics sound, light, and matter interaction



2D visualization

# Physics sound, light, and matter interaction



2D visualization



# Physics sound, light, and matter interaction

nm scale  
(exaggerated)



2D visualization

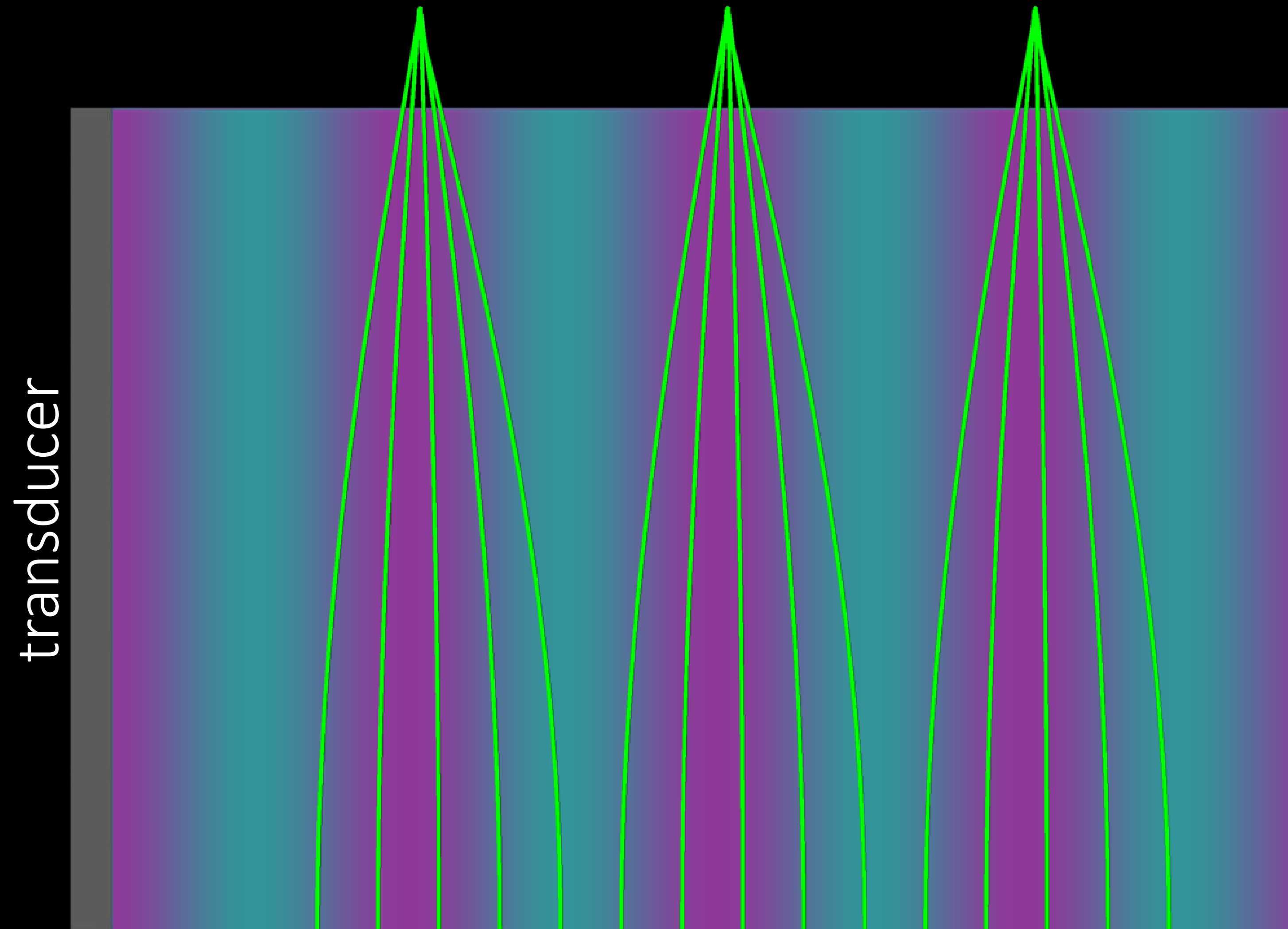
# Physics sound, light, and matter interaction



2D visualization

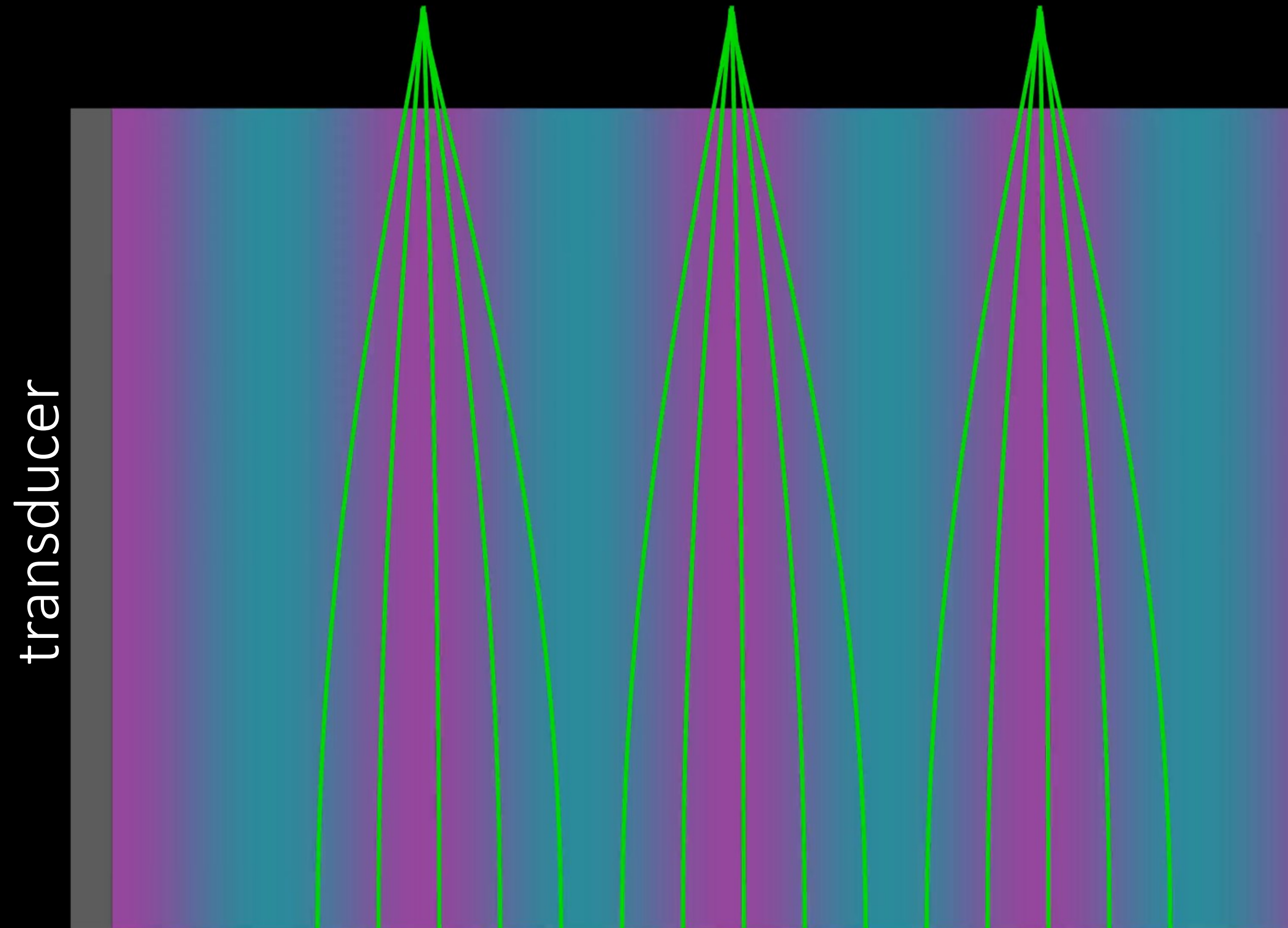


# Physics sound, light, and matter interaction



2D visualization

# Physics sound, light, and matter interaction

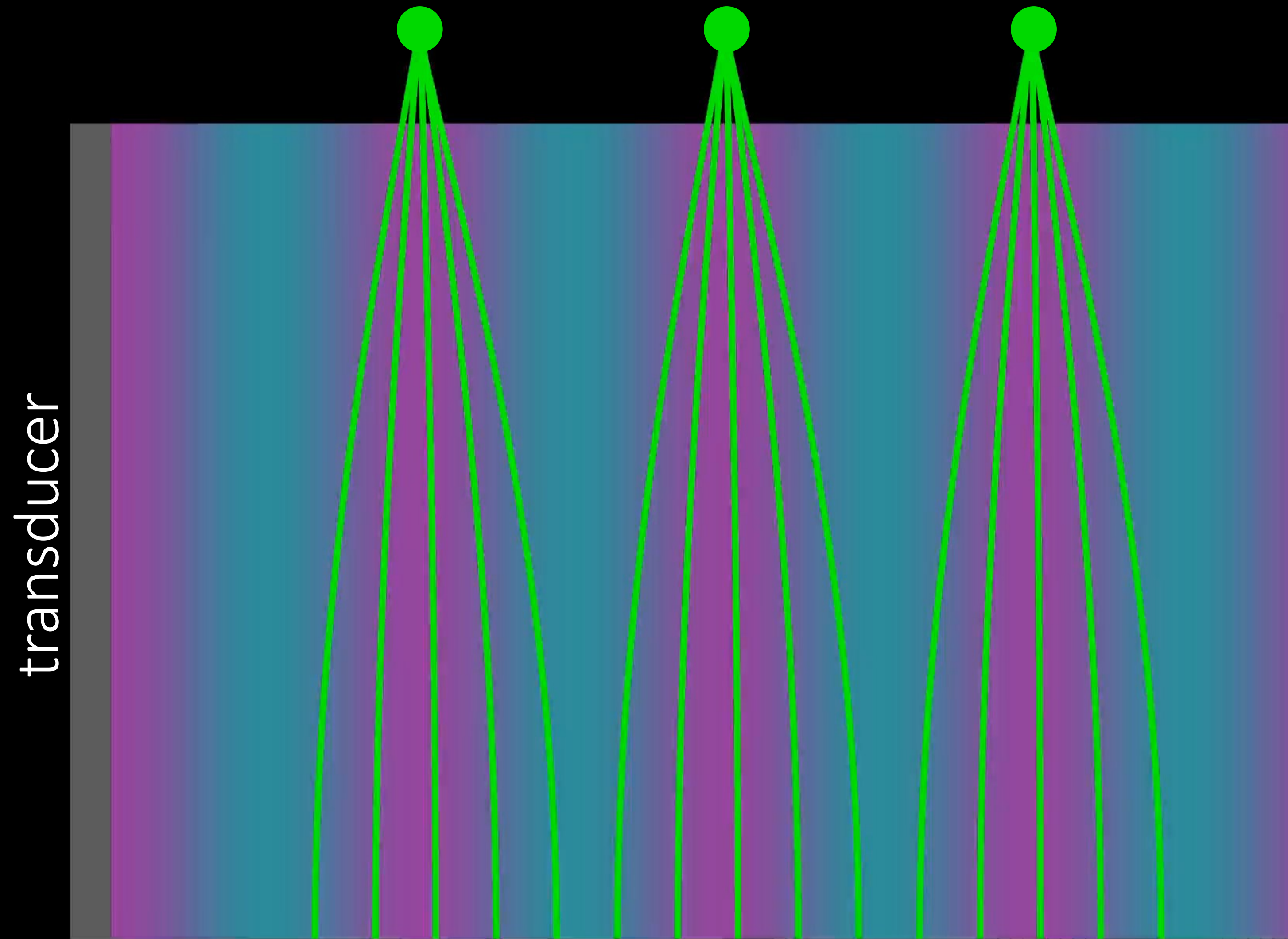


focus travels at the speed of 1.5 km/s

2D visualization



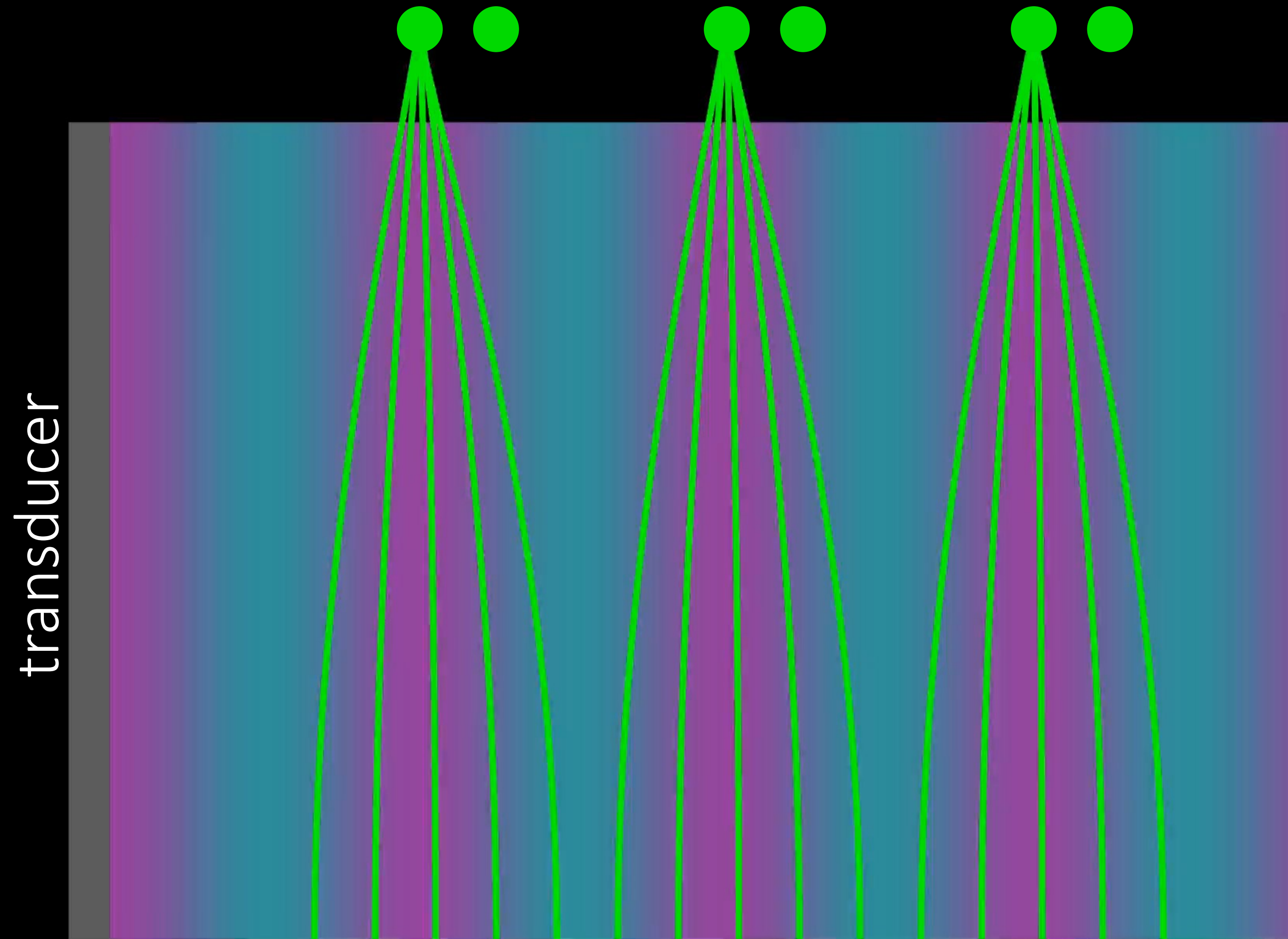
# Ultrafast optics and synchronization



ultrafast synchronized optics that illuminates same spot

2D visualization

# Ultrafast optics and synchronization

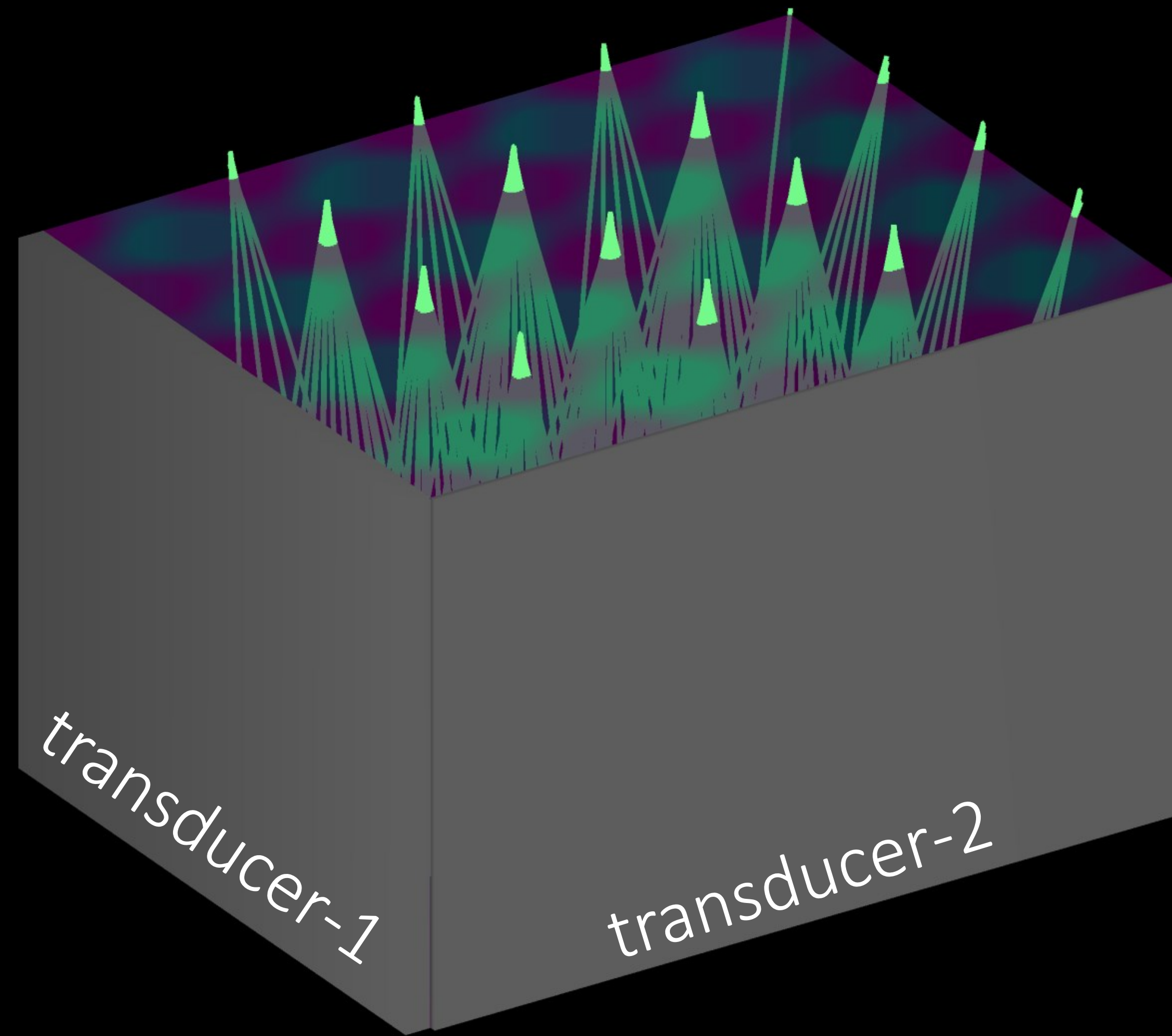


illuminating two spots per cycle

2D visualization

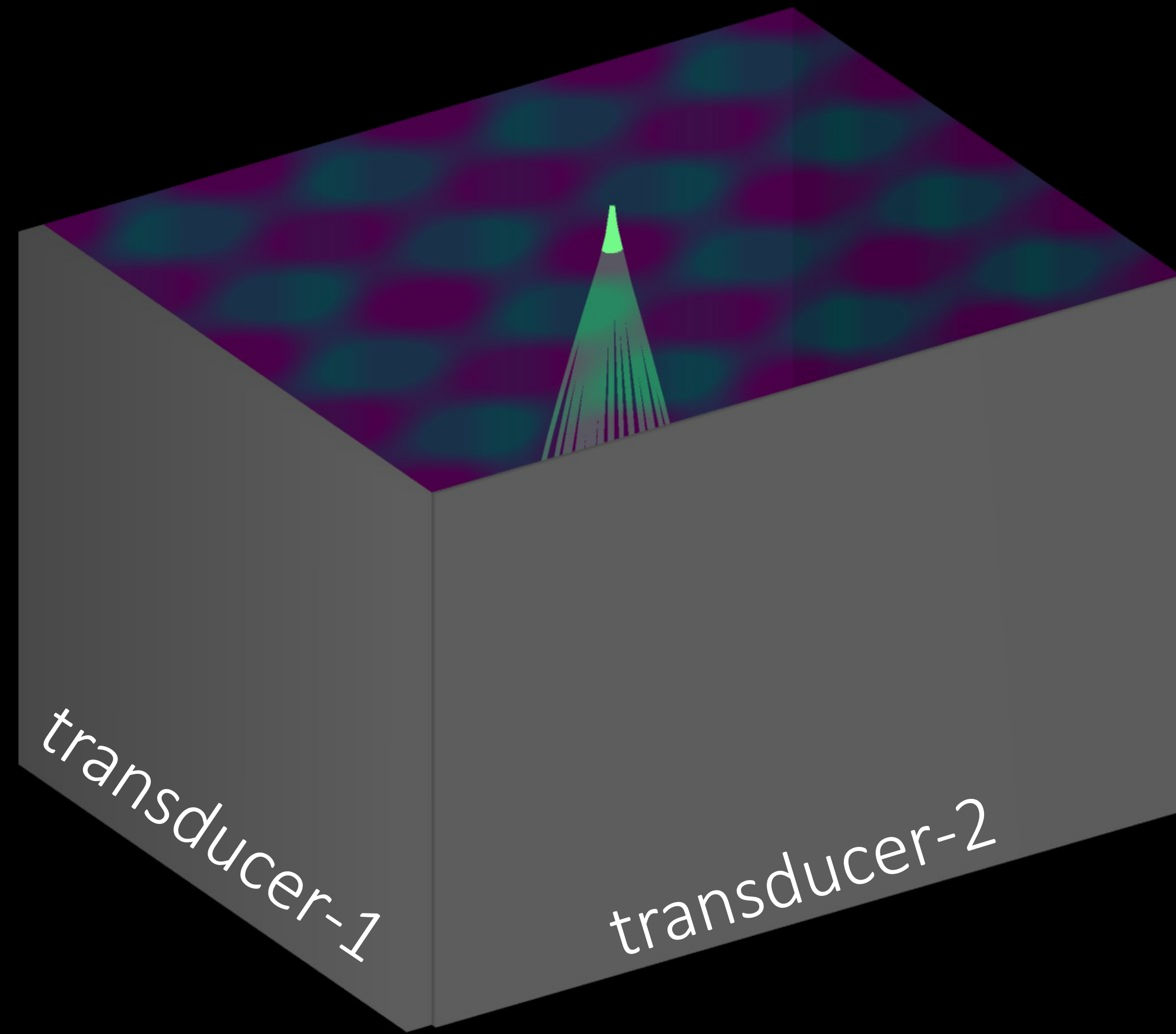


# Dot projector



2D visualization

# Dot projector

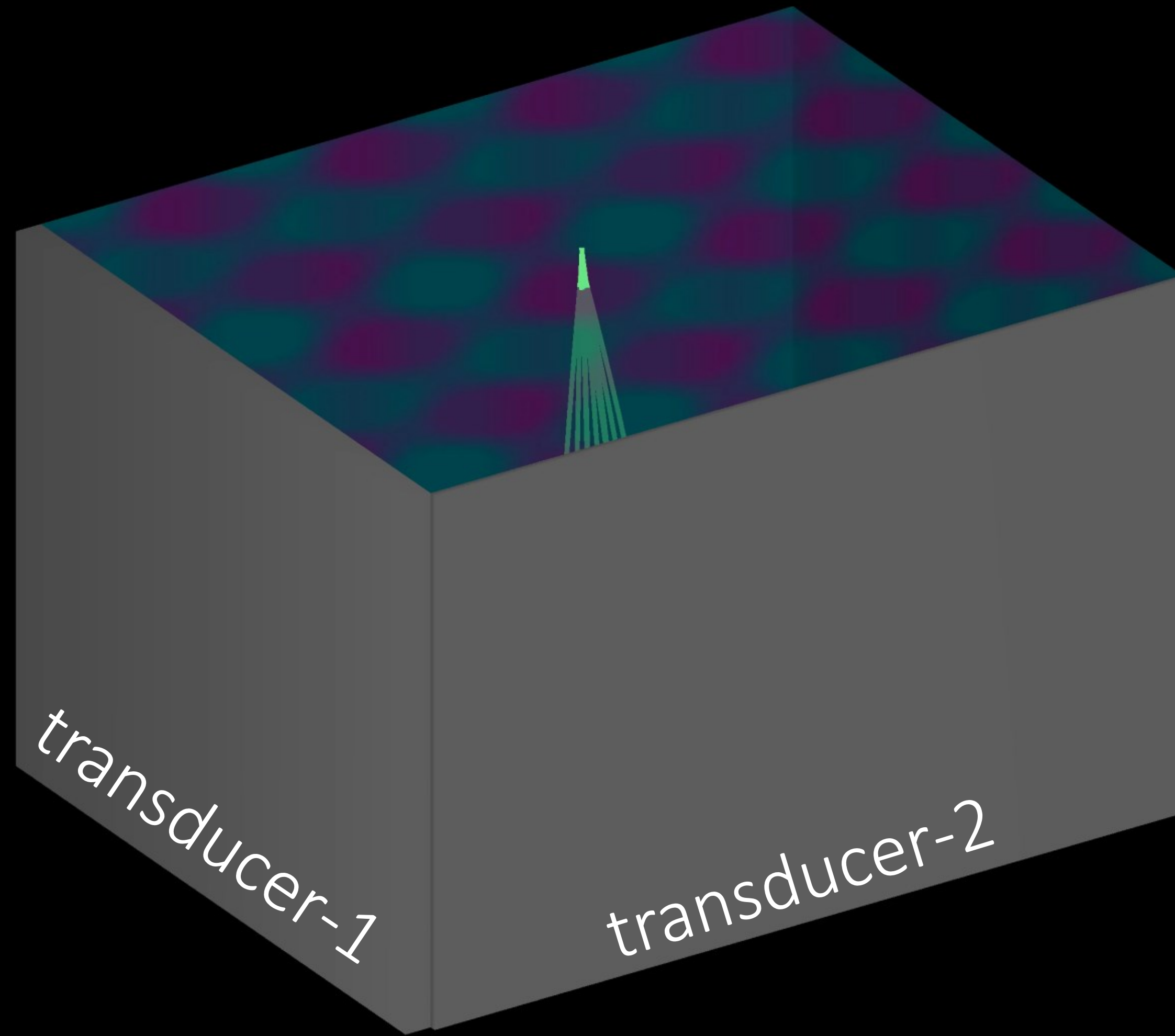
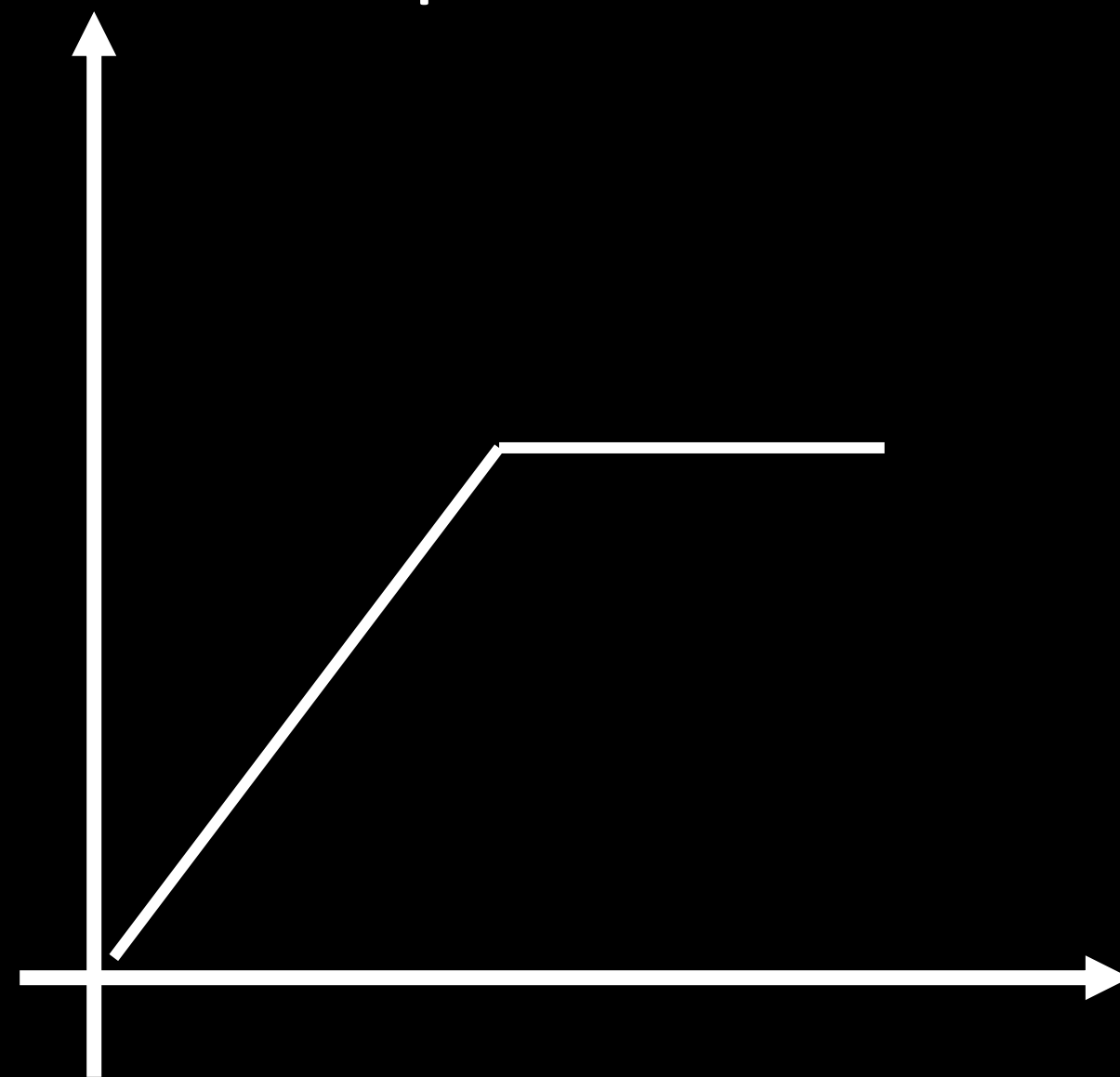


2D visualization

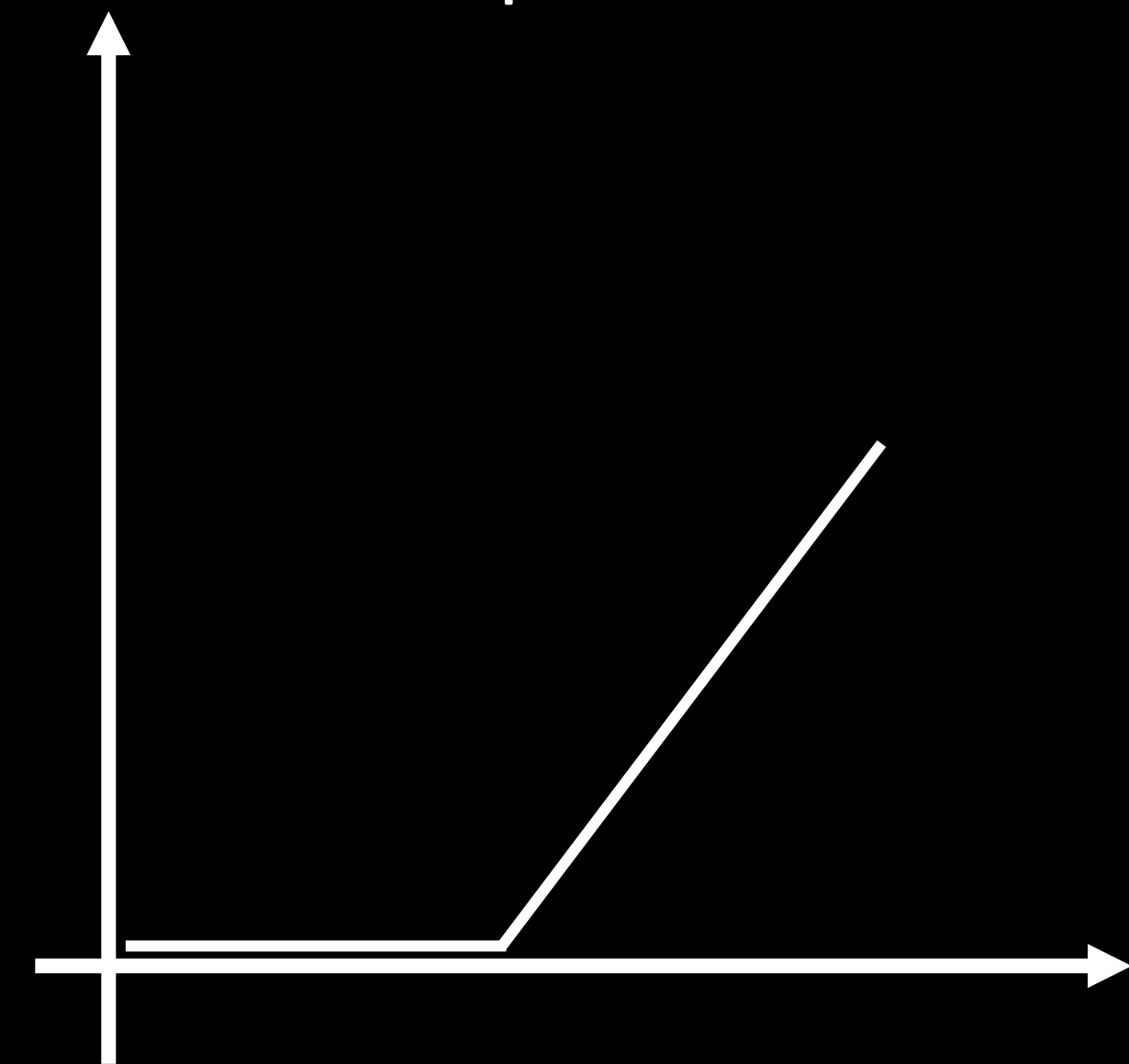


# Dot projector

transducer-1  
phase



transducer-2  
phase





# Hardware experiments

galvo

SPAD

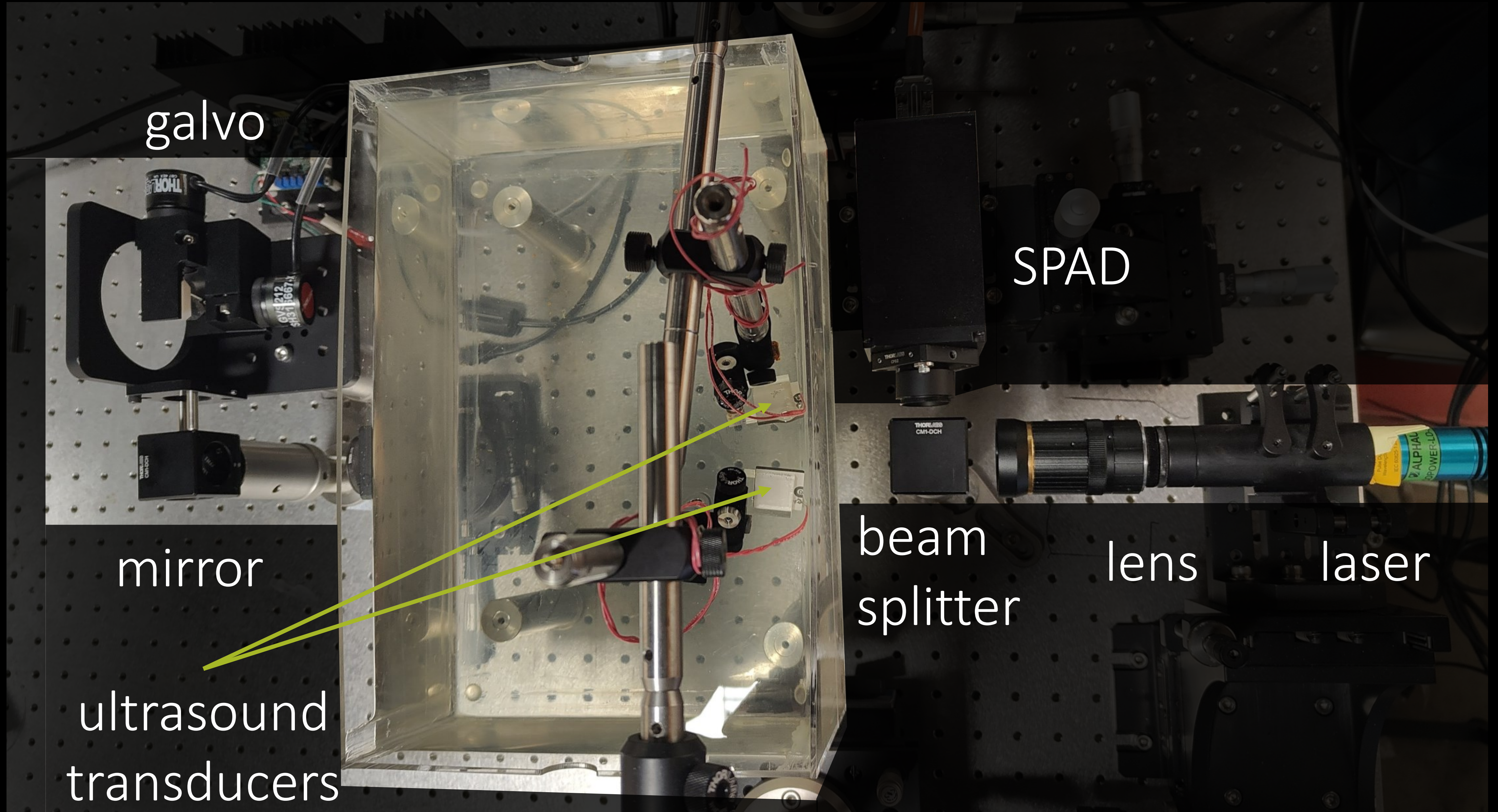
mirror

ultrasound  
transducers

beam  
splitter

lens

laser

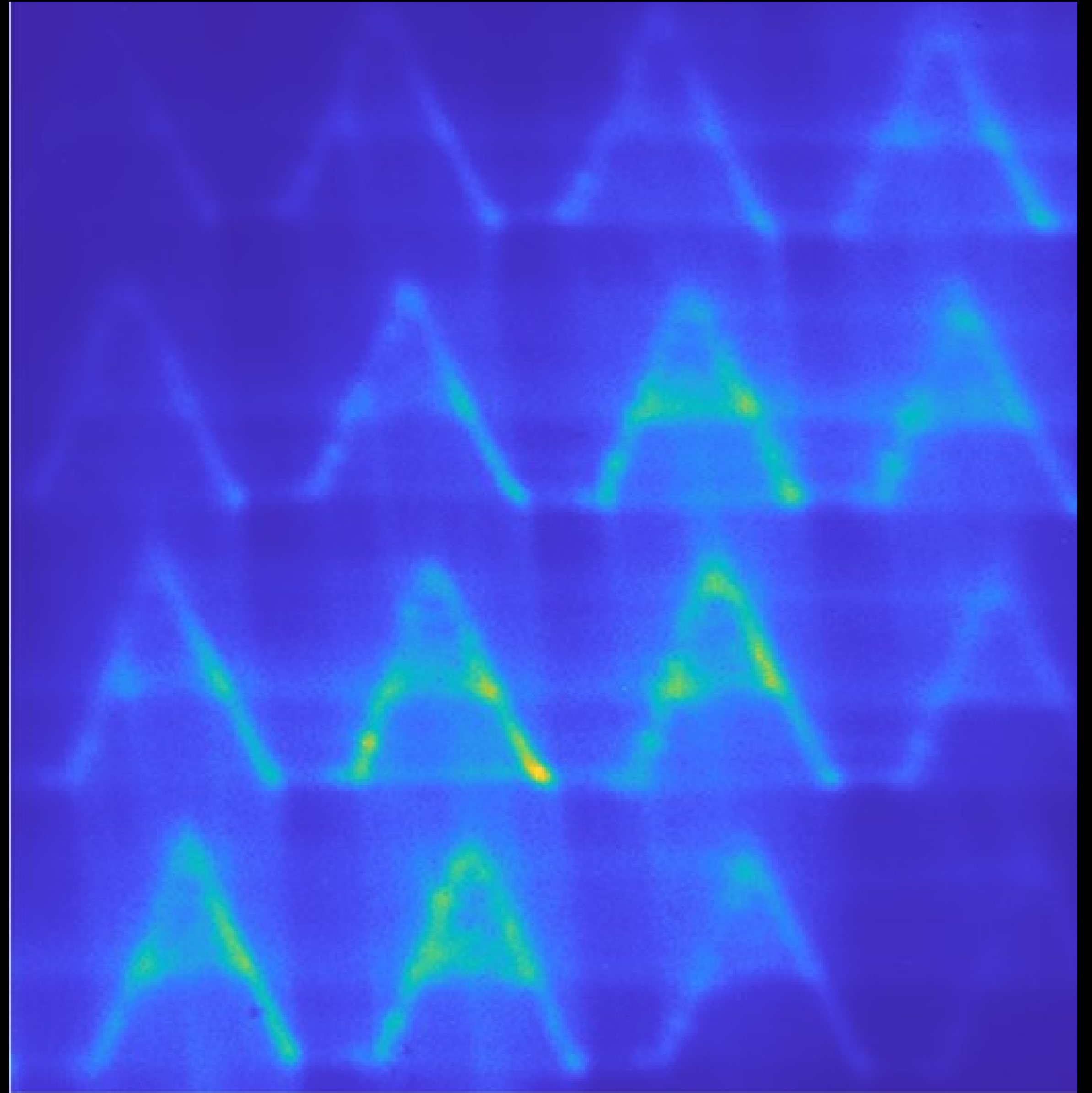
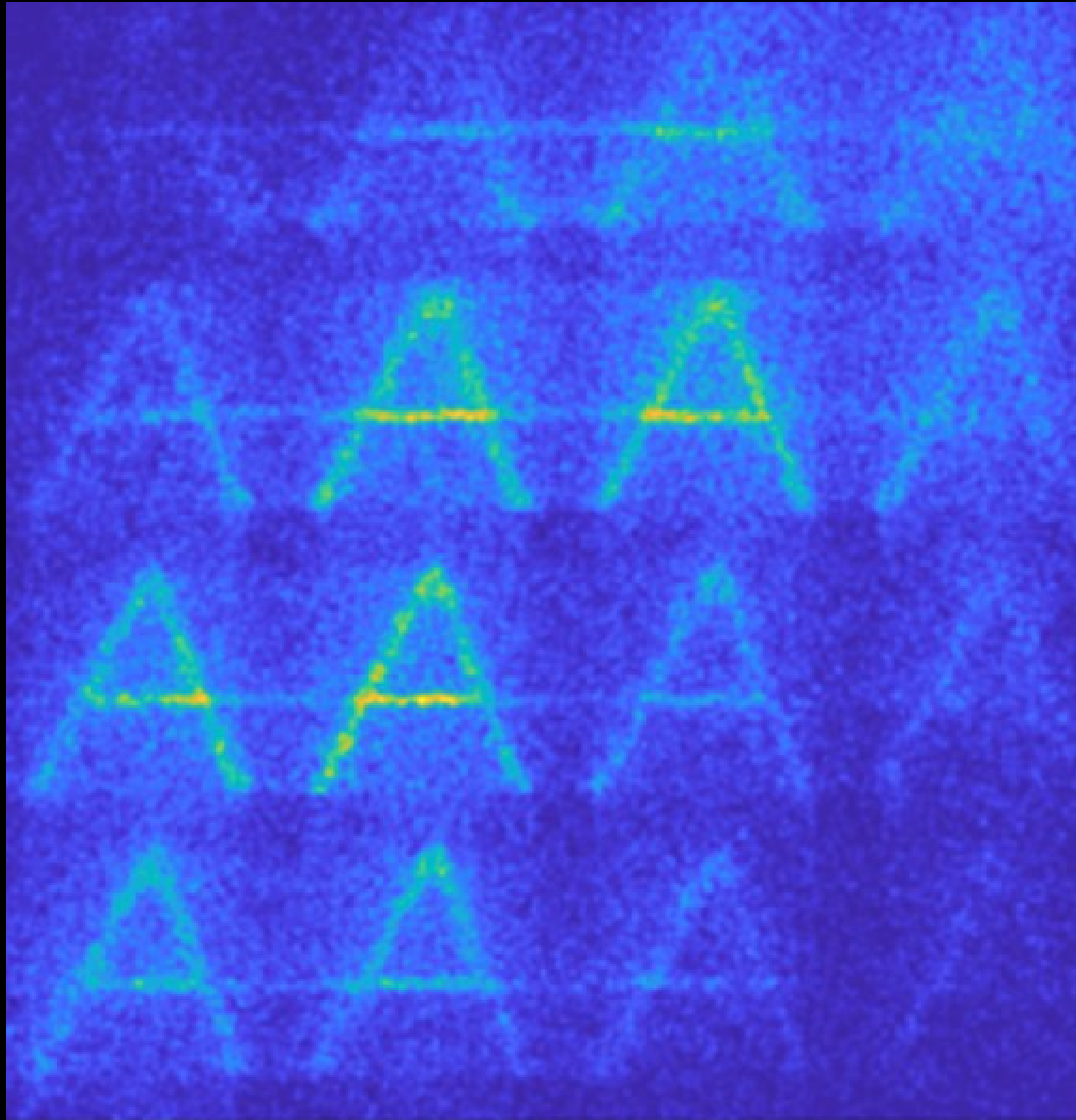




Renderer

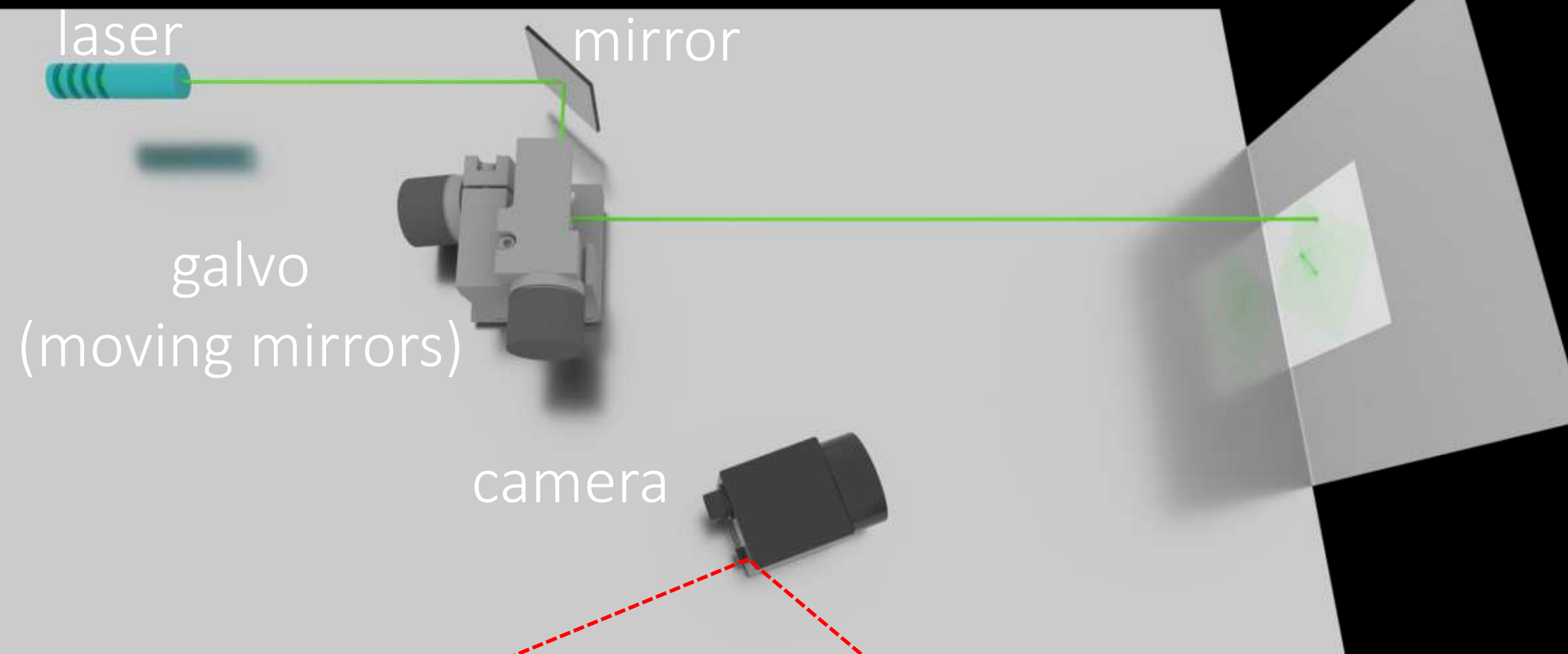
vs.

Hardware

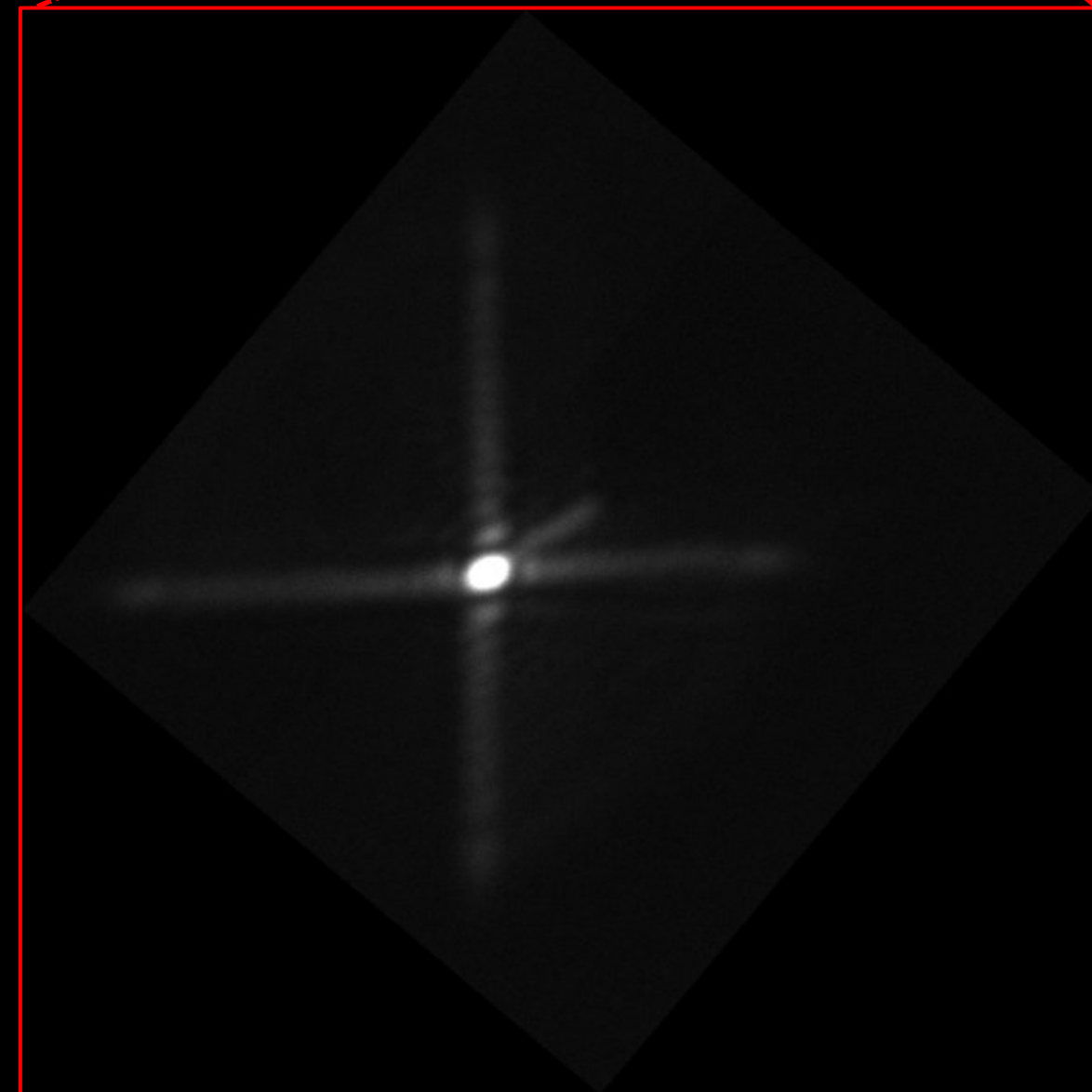


# Hardware results: dot projector

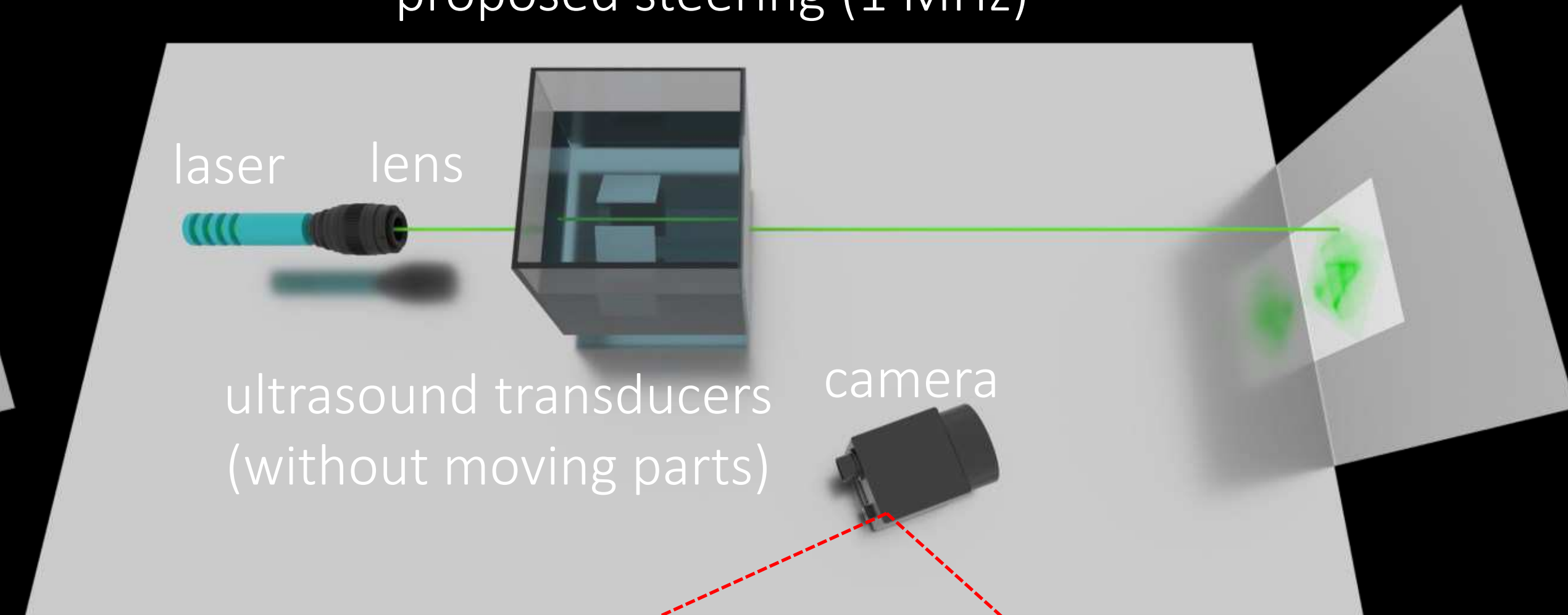
galvo steering (1 kHz)



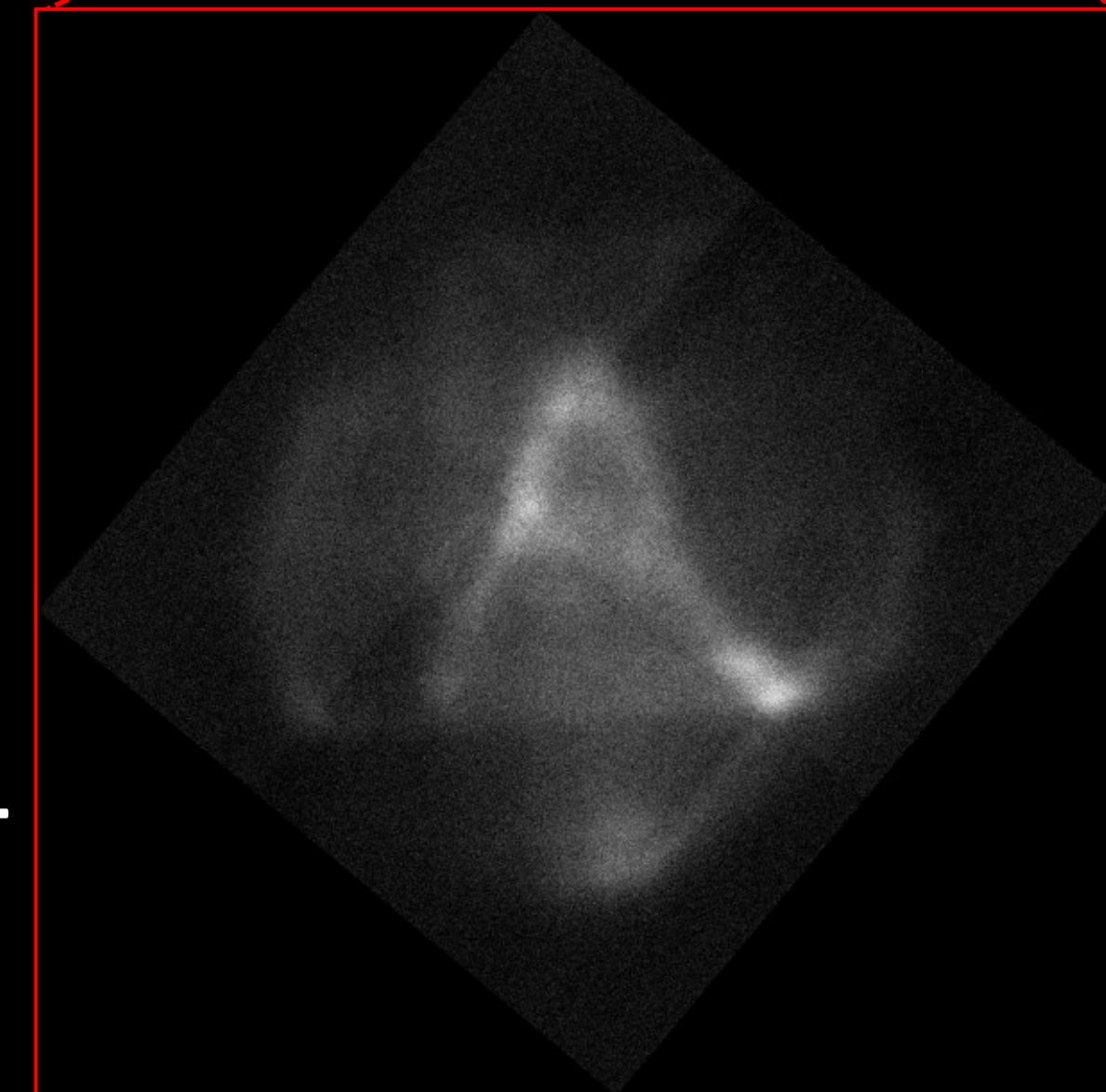
exposure= 1 ms



proposed steering (1 MHz)



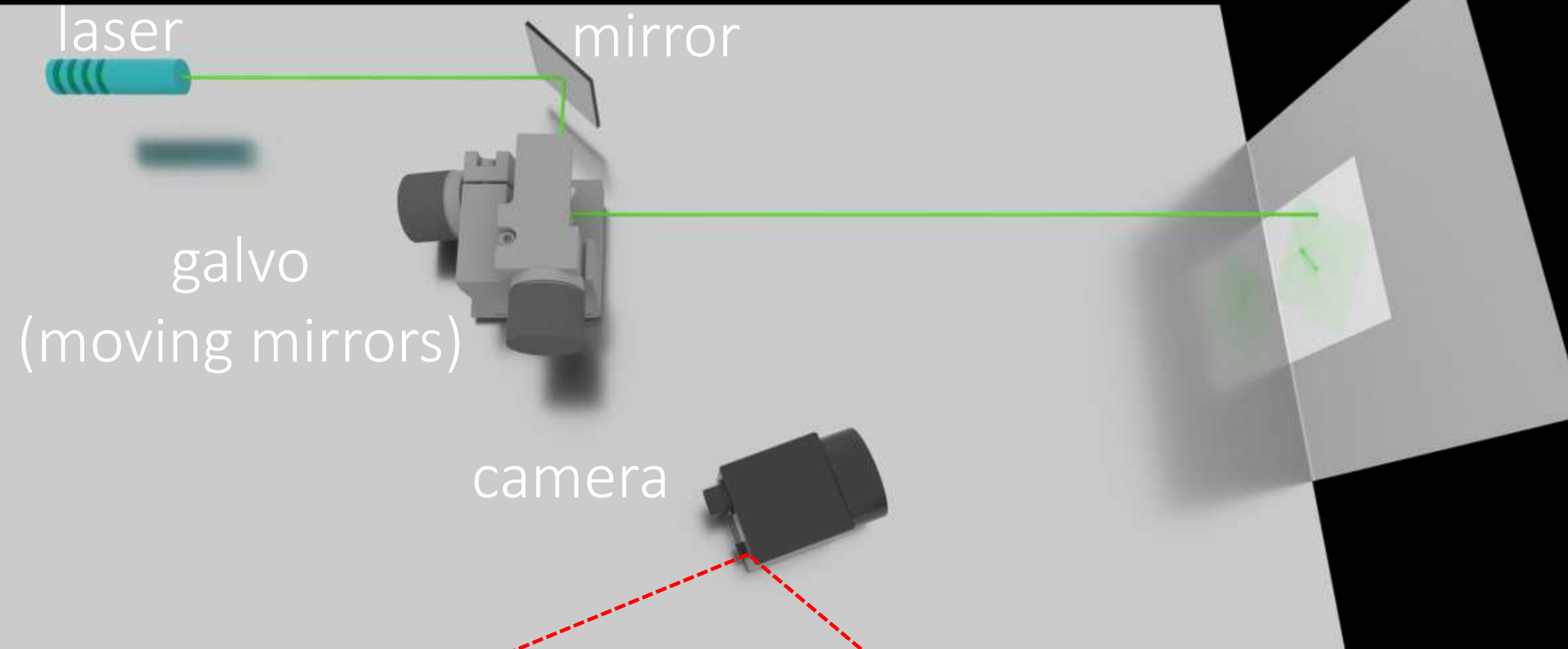
exposure= 1 ms



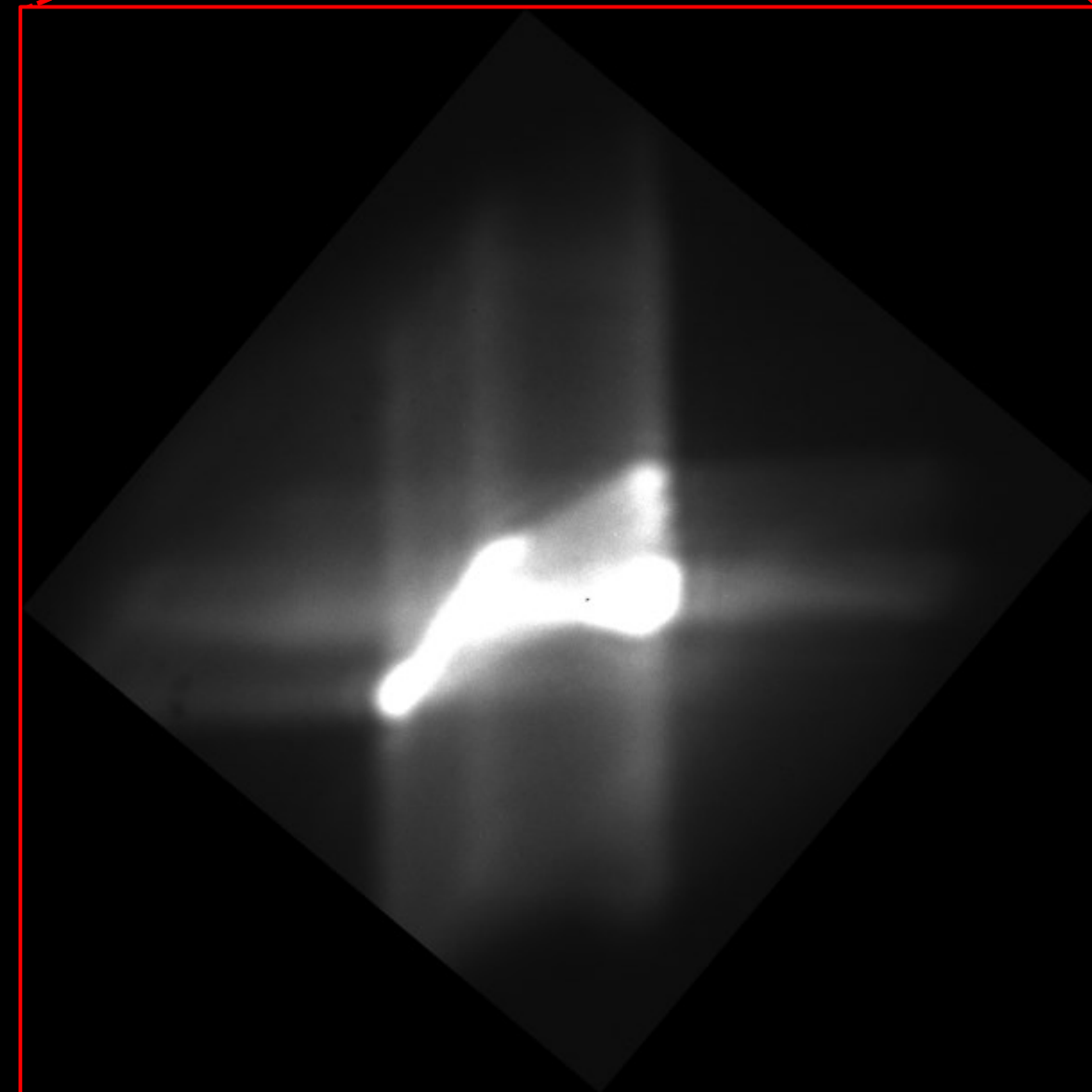


# Hardware results: dot projector

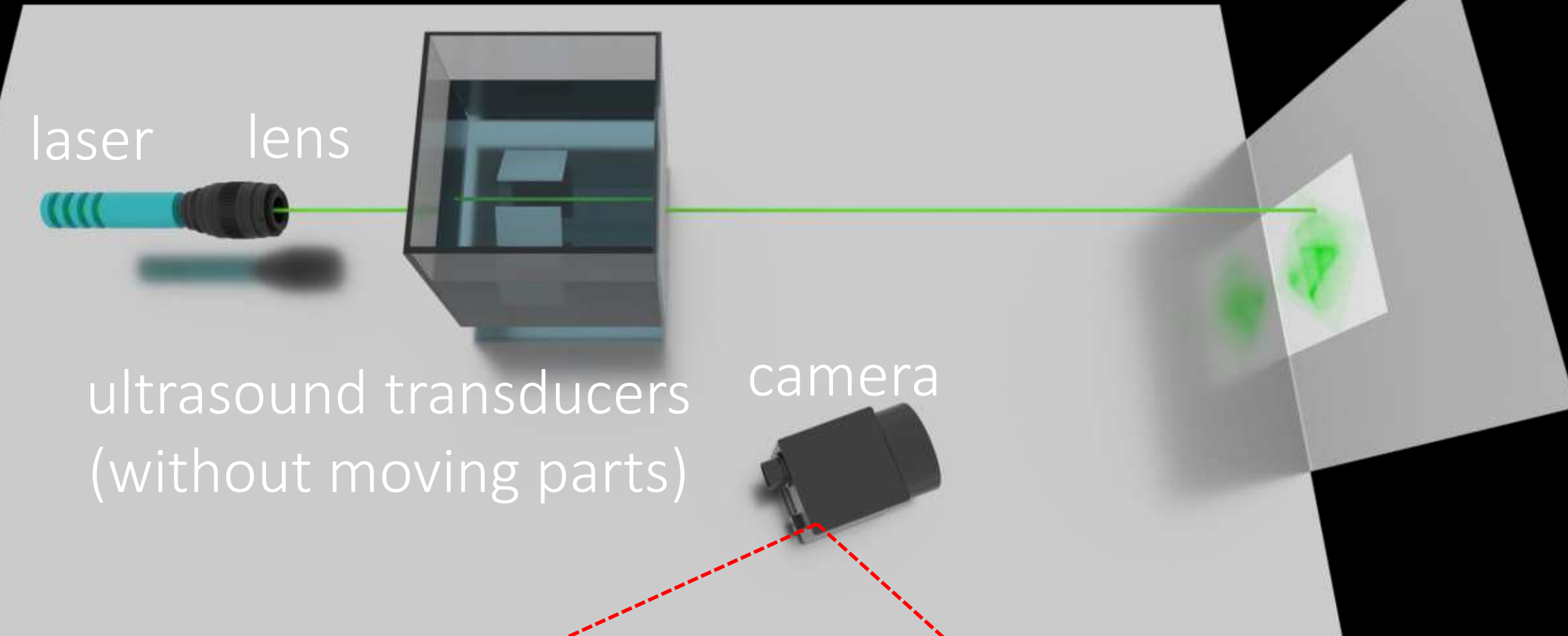
galvo steering (1 kHz)



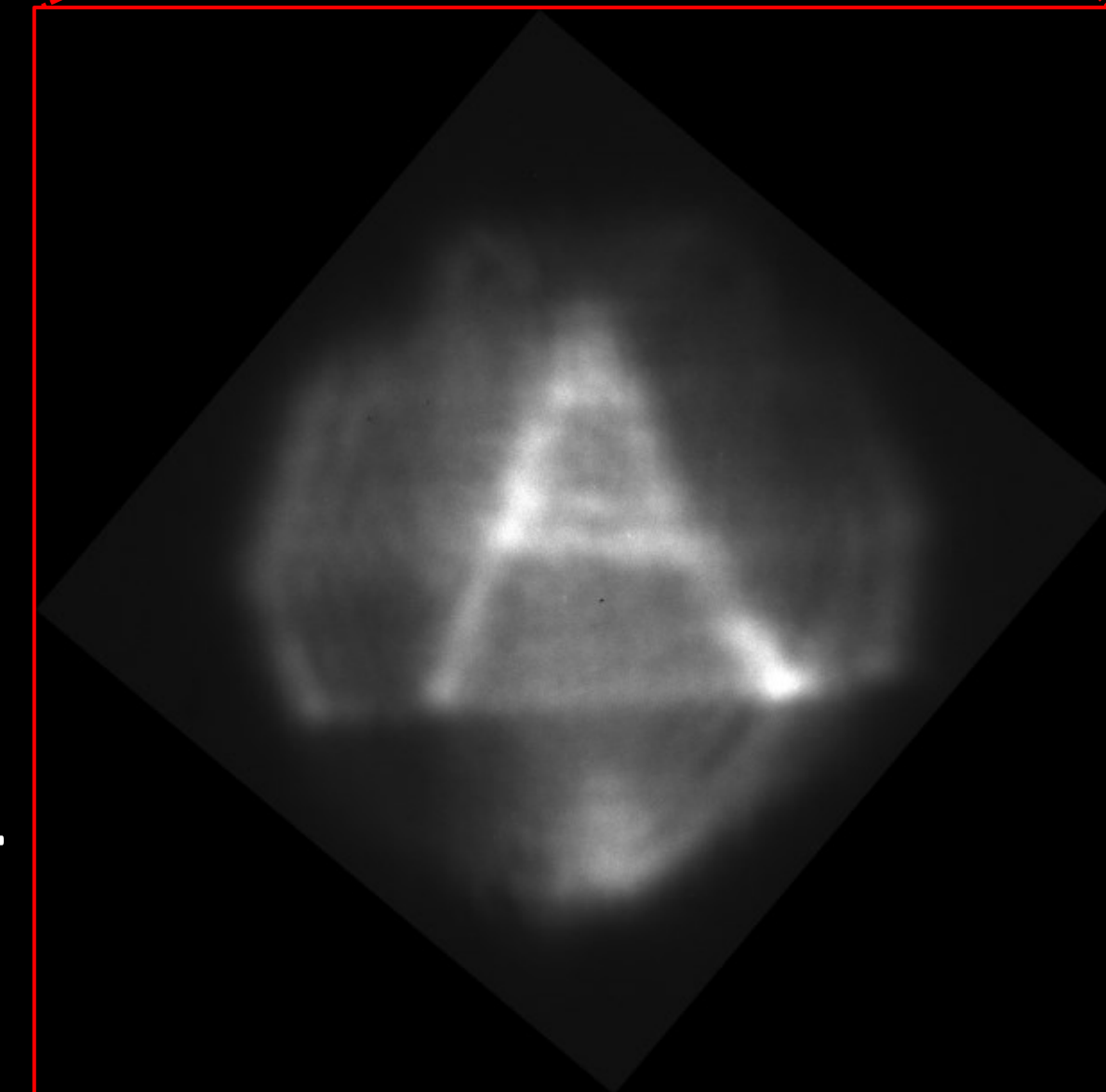
exposure= 50 ms



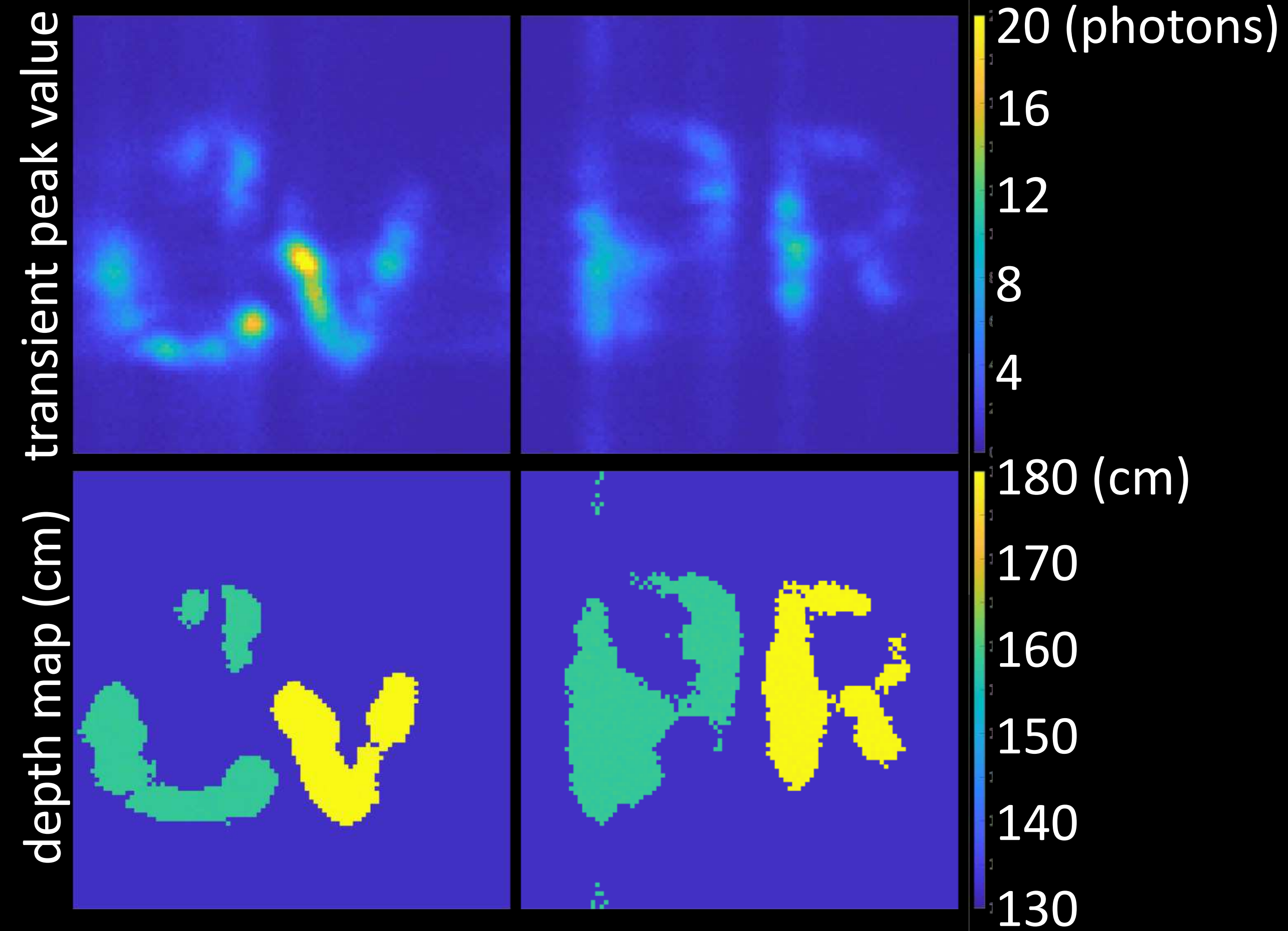
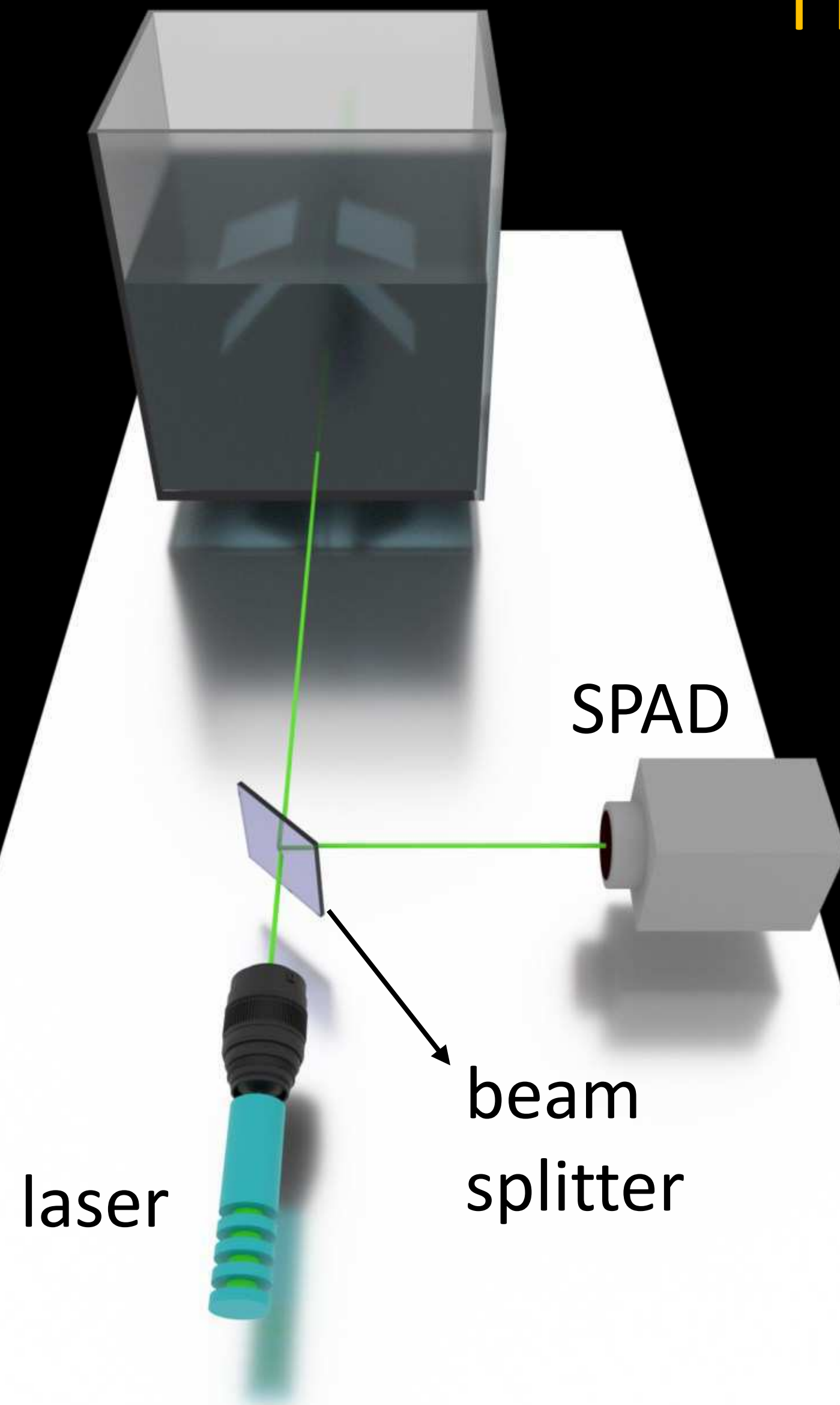
proposed steering (1 MHz)



exposure= 50 ms



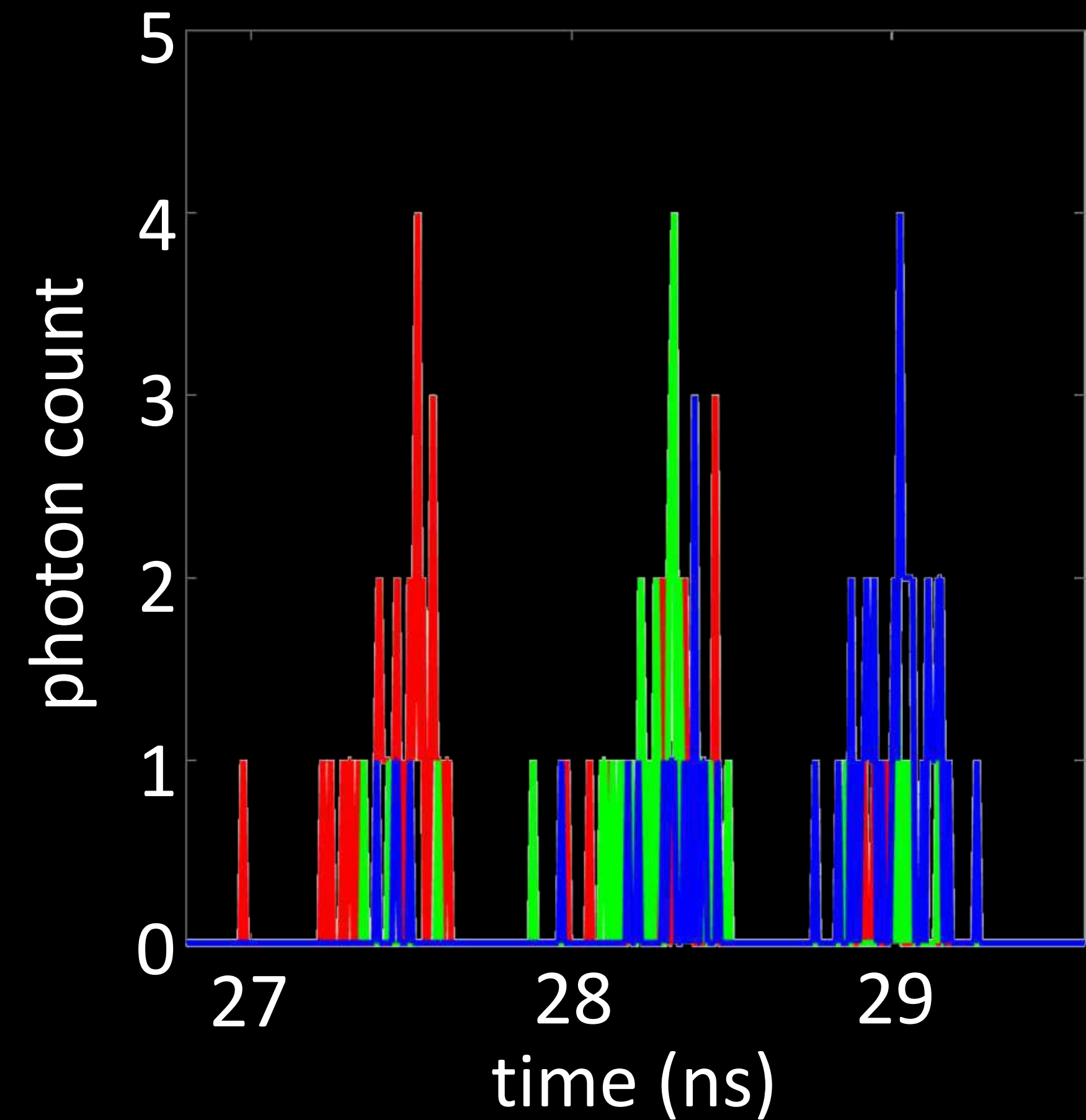
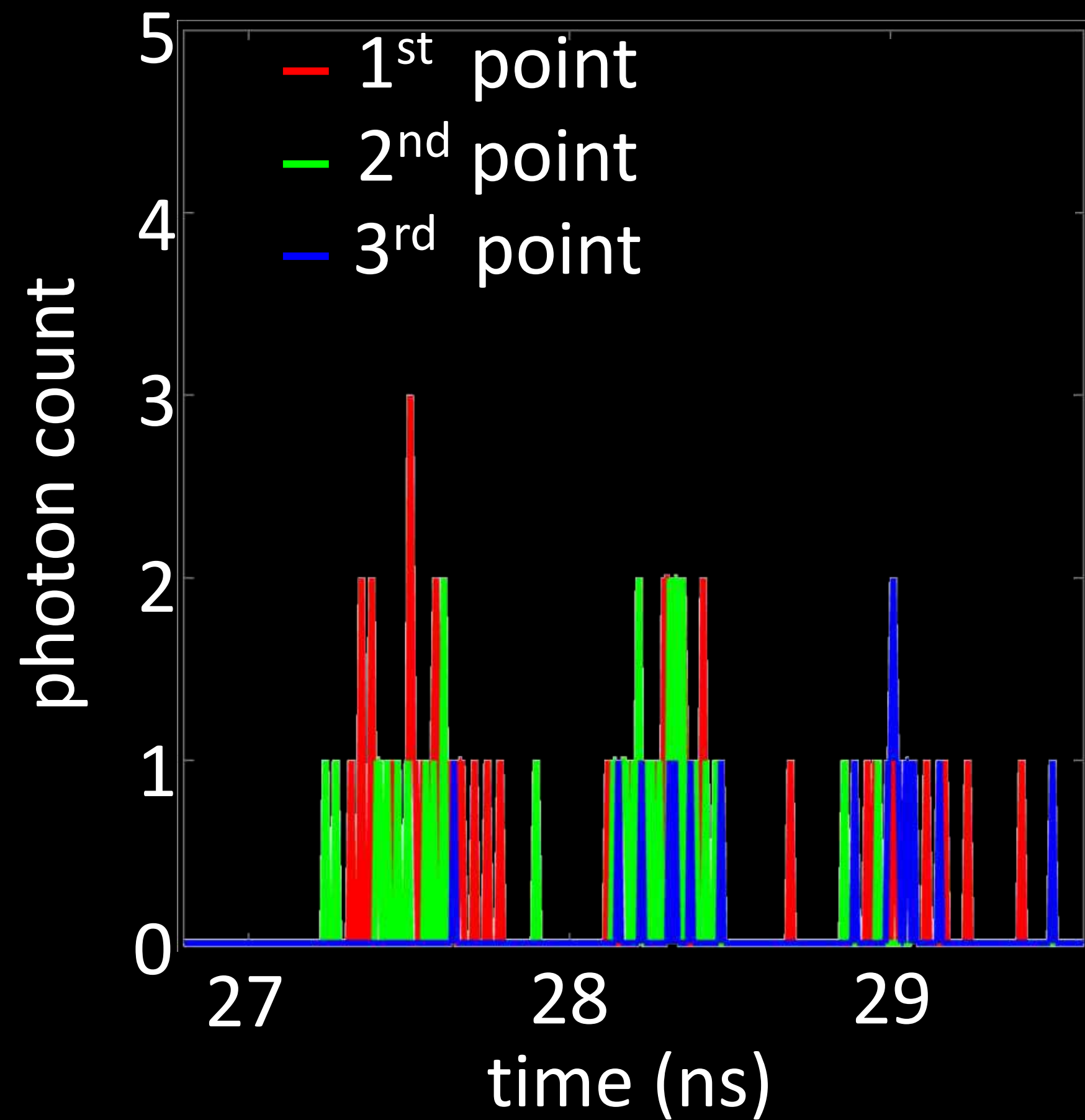
# Hardware results: Lidar



100 × faster than standard lidar



# Hardware results: adaptive depth measurement

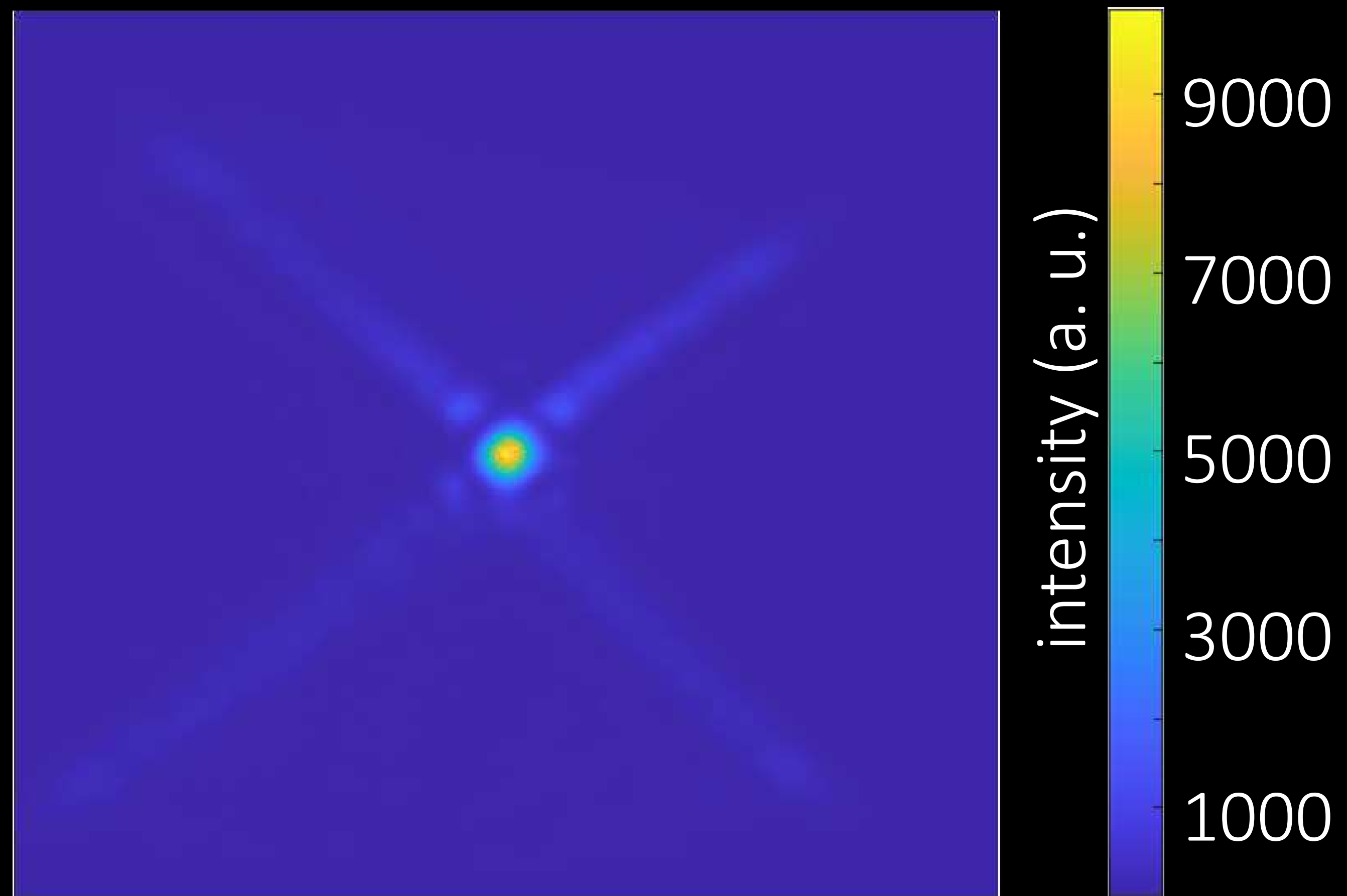
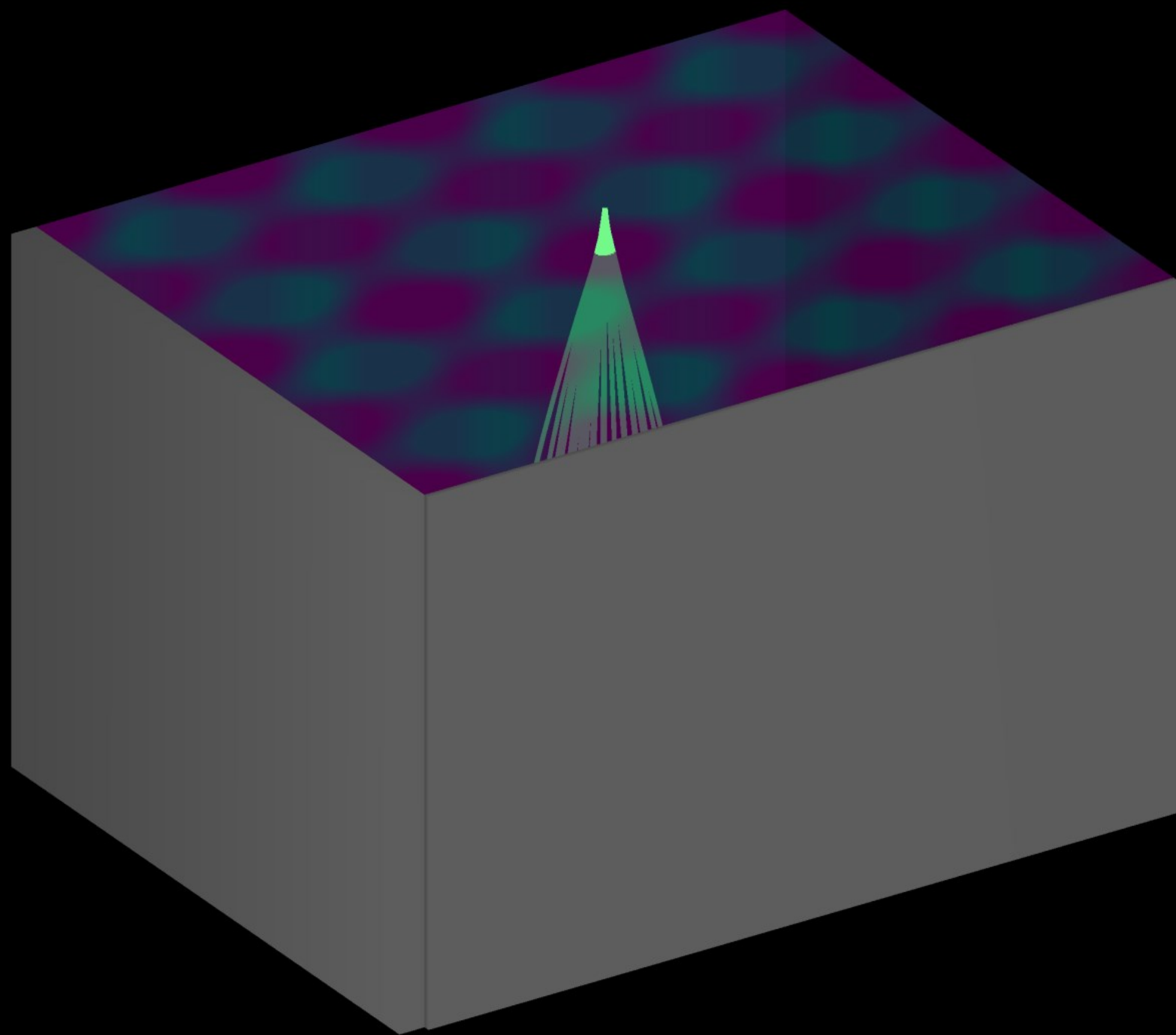


standard galvo (depth error = 51.3 cm)

our technique (depth error = 3.28 cm)

**15 × depth accuracy**

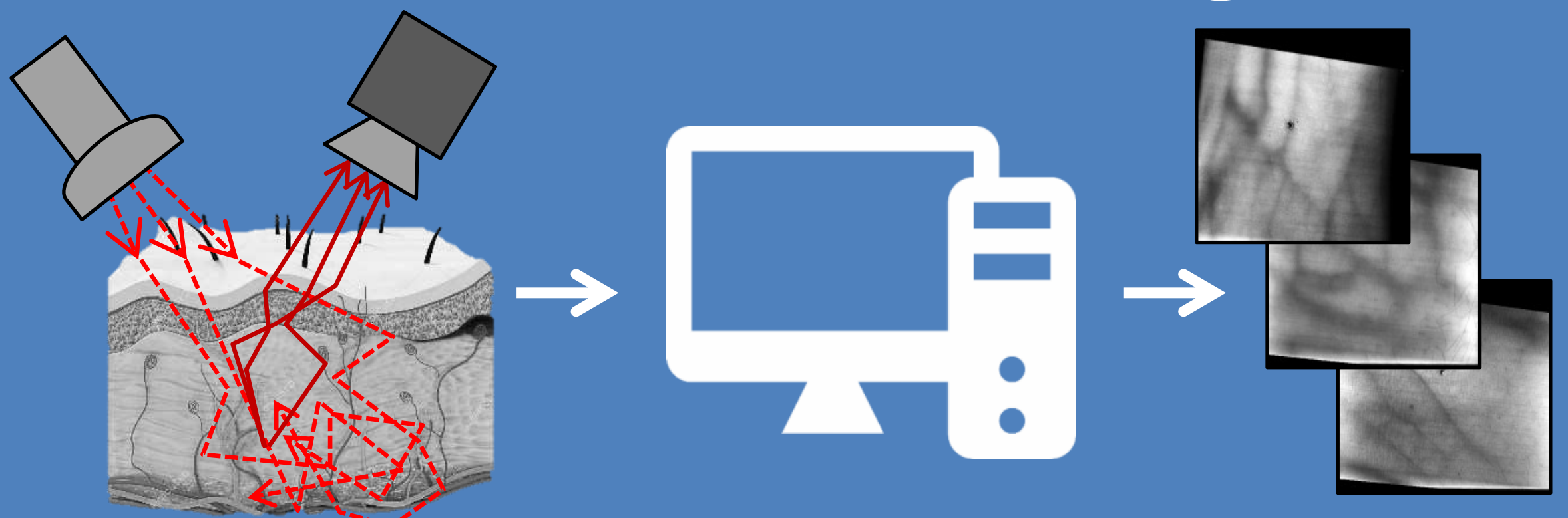
# Limitation





# Physics-based rendering and its applications to computational imaging


## forward rendering



The diagram illustrates the forward rendering process. On the left, a 3D model of a biological tissue sample is shown with red dashed lines representing light rays originating from a light source and reflecting off the surface and internal structures. An arrow points from this model to a central icon of a computer monitor and tower. Another arrow points from the computer icon to a stack of three grayscale images on the right, representing the rendered output.

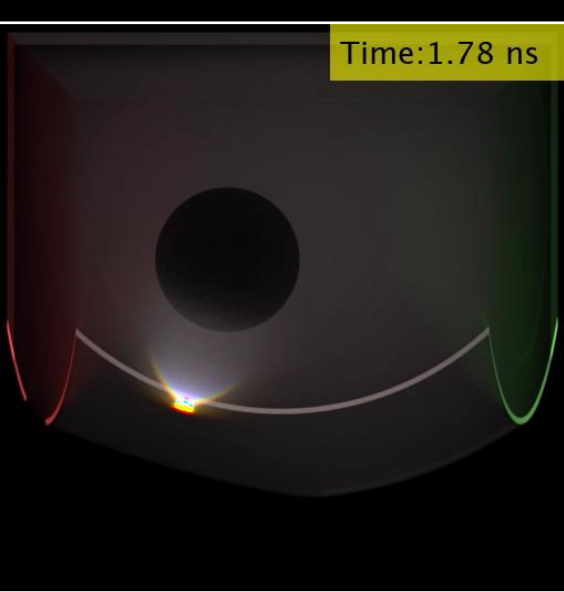
- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

## inverse rendering

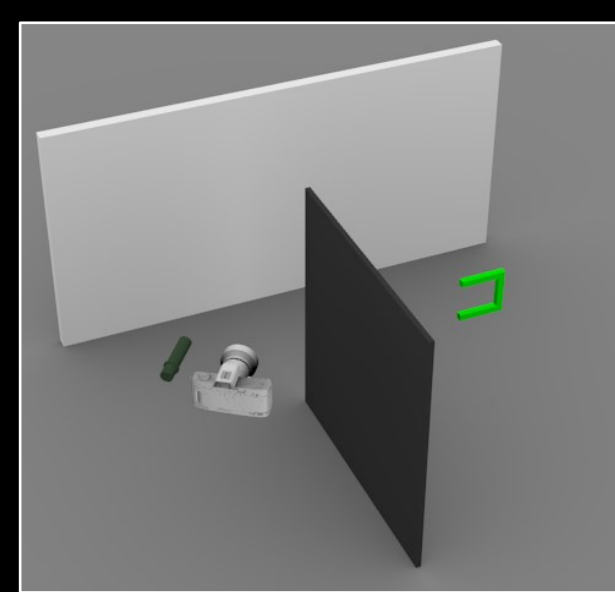


The diagram illustrates the inverse rendering process. On the left, a 3D model of a biological tissue sample is shown with red dashed lines representing light rays. An arrow points from this model to a central icon of a computer monitor and tower. Another arrow points from the computer icon to a stack of three grayscale images on the right, representing the rendered output.

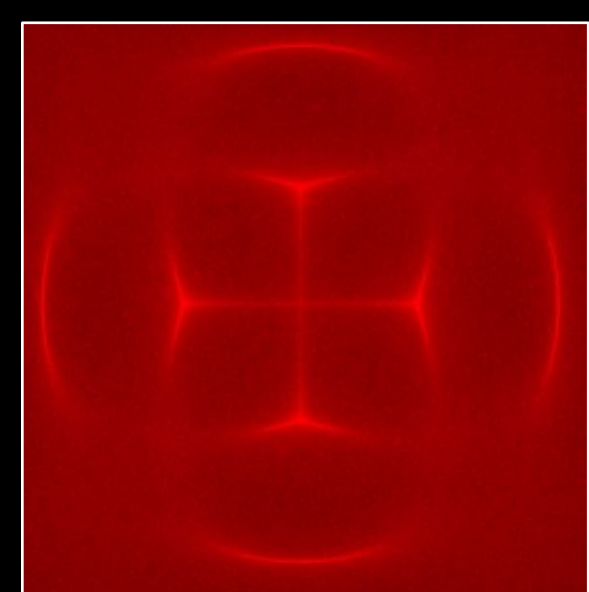
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



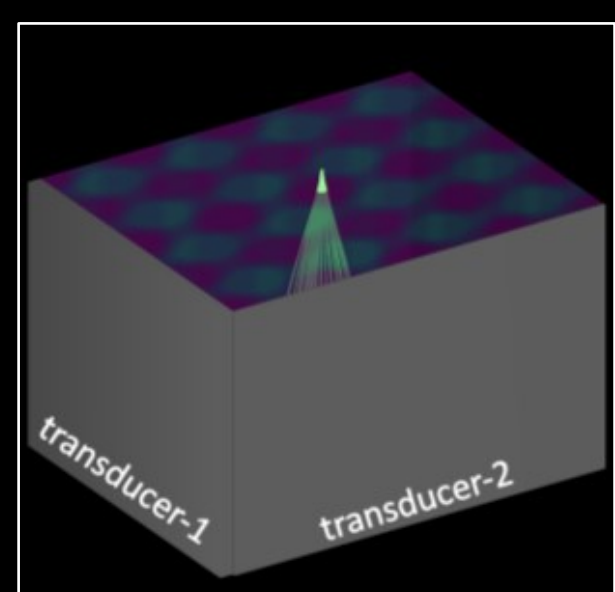
time-of-flight  
imaging



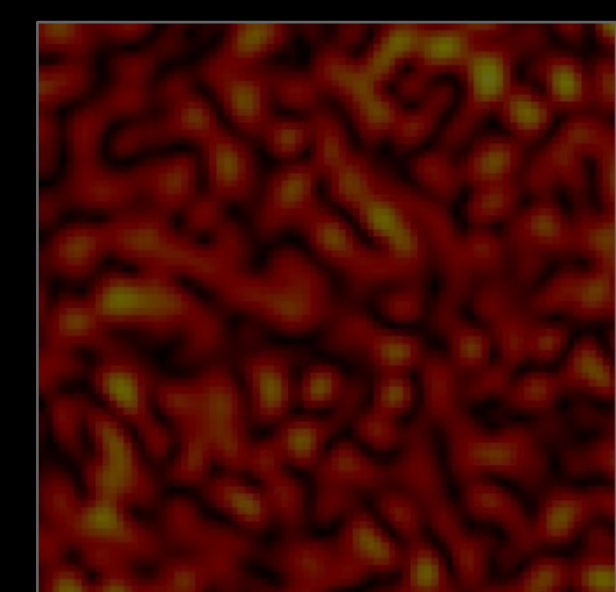
non-line-of-sight  
imaging



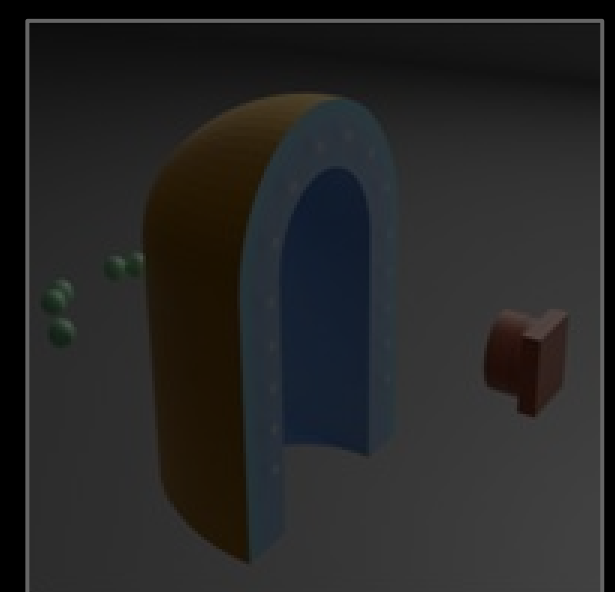
acousto-optic  
lensing



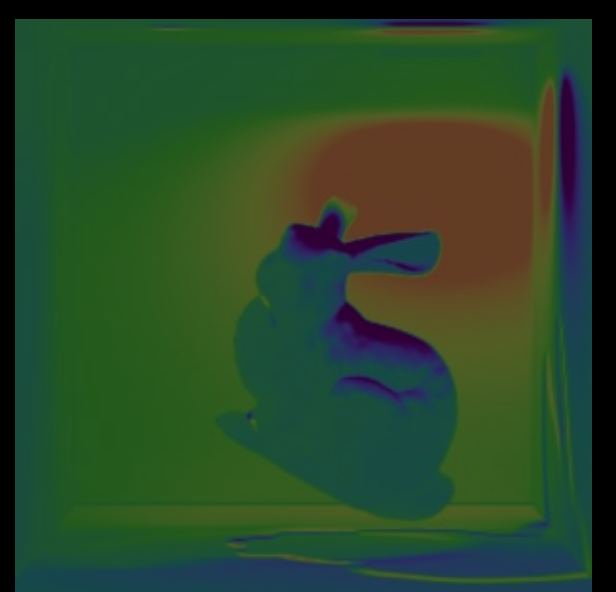
ultrafast light  
scanning



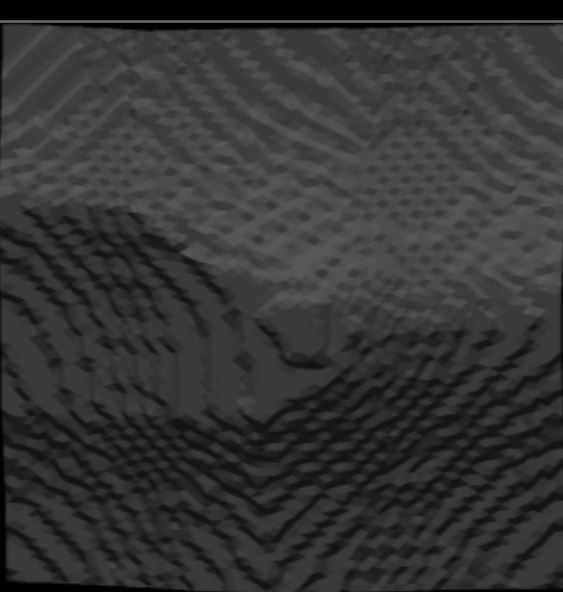
speckle  
imaging



tactile sensor  
design



differentiable  
renderer

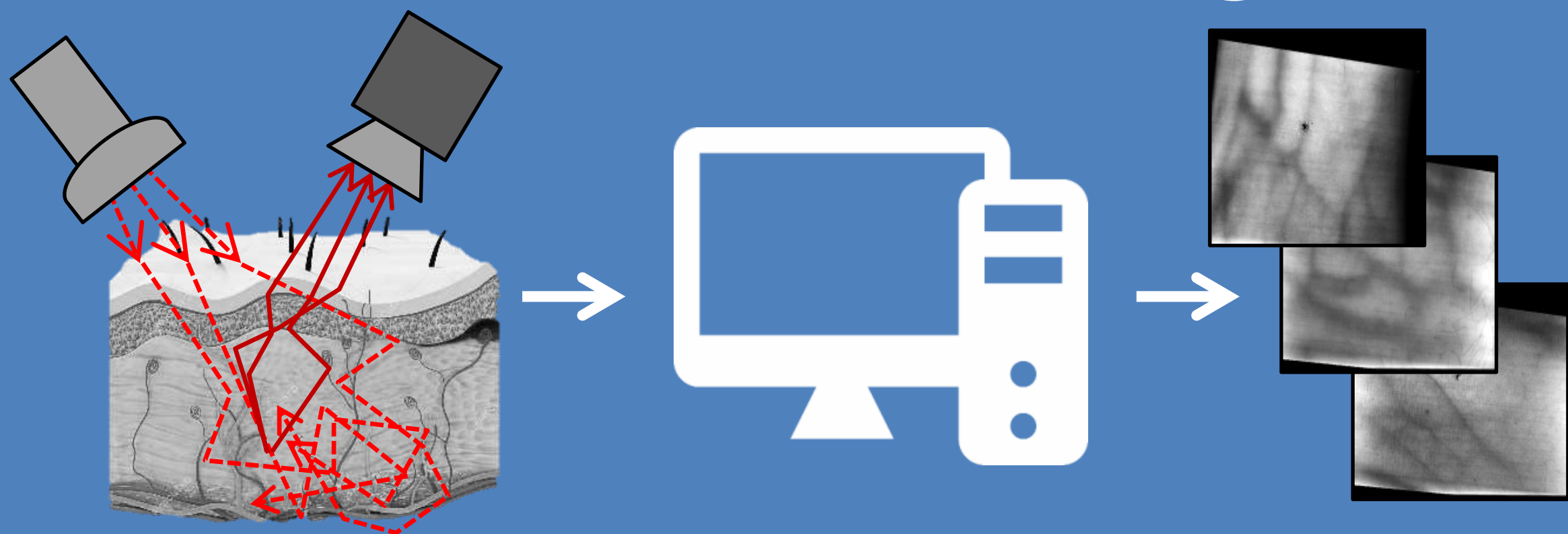


inverse  
problems



# Physics-based rendering and its applications to computational imaging

## forward rendering

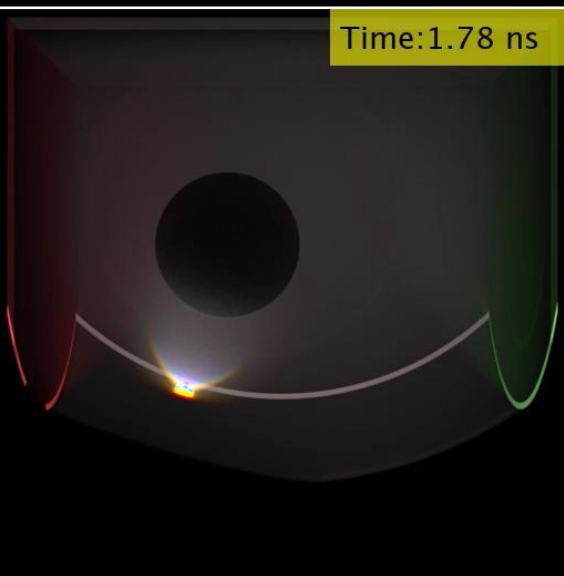


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

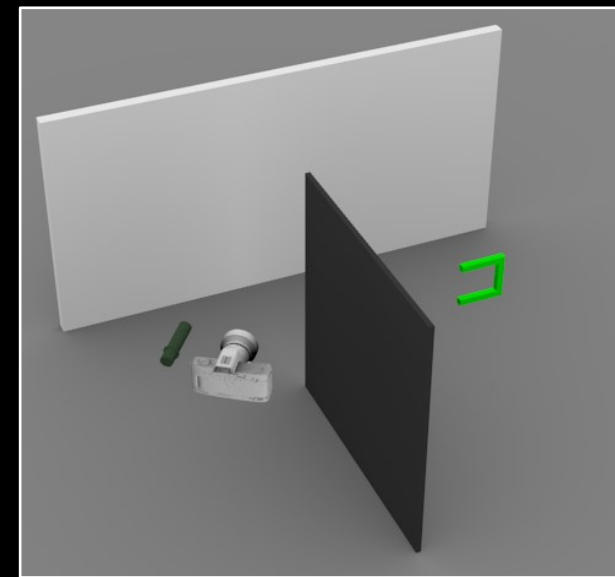
## inverse rendering



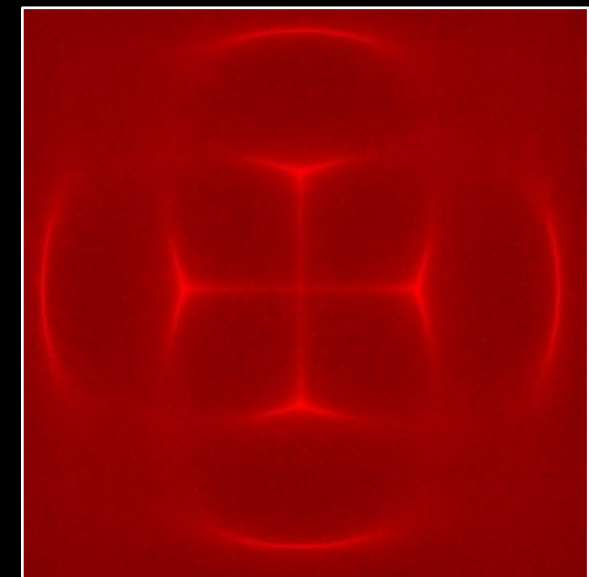
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



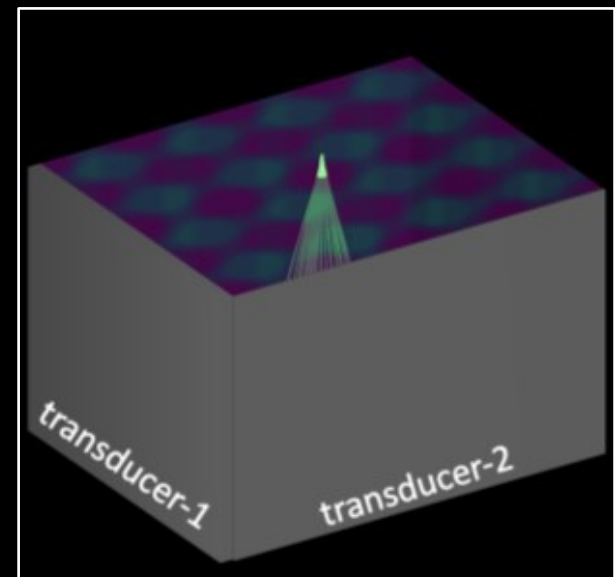
time-of-flight  
imaging



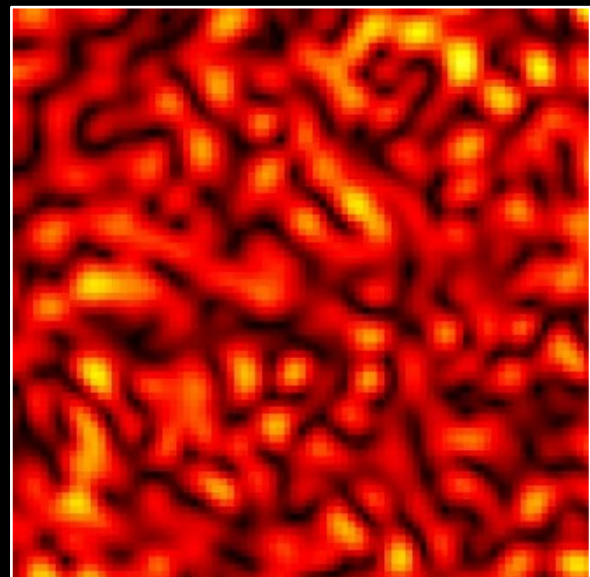
non-line-of-sight  
imaging



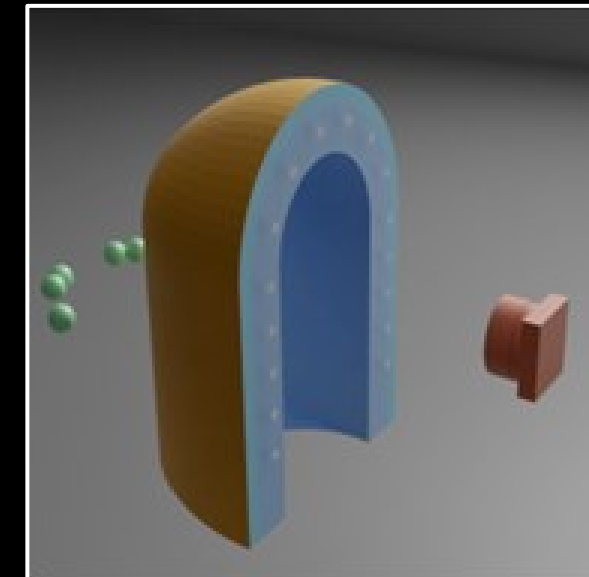
acousto-optic  
lensing



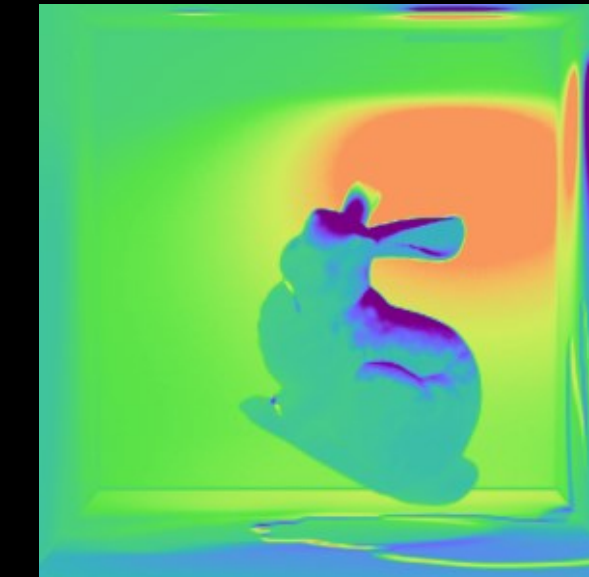
ultrafast light  
scanning



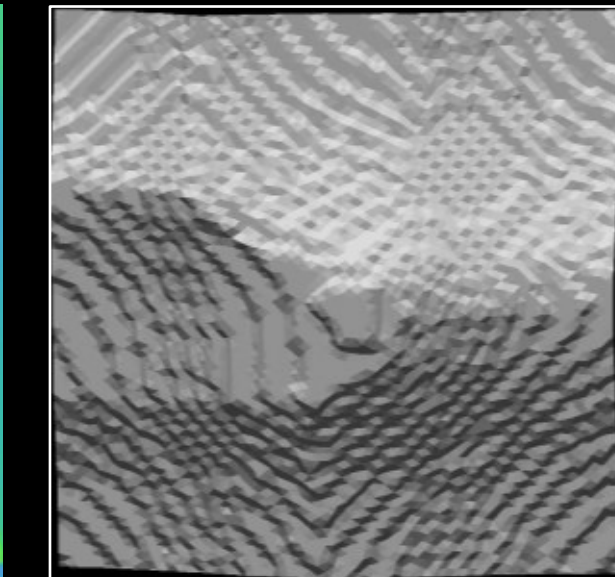
speckle  
imaging



tactile sensor  
design



differentiable  
renderer



inverse  
problems

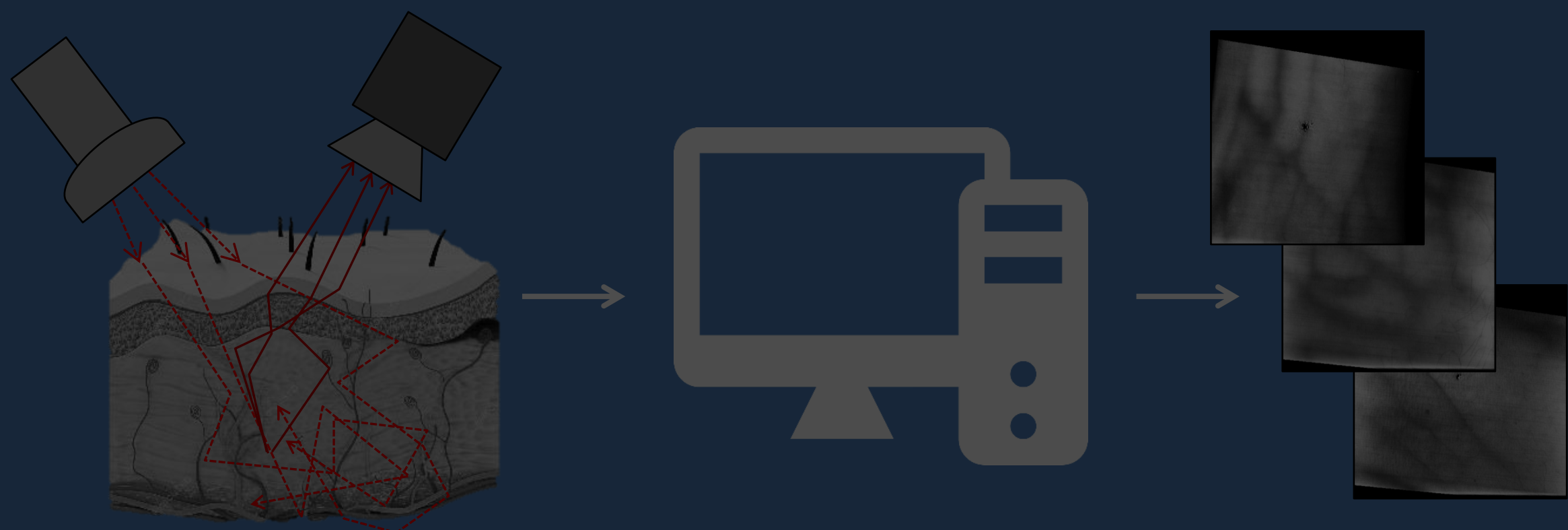


Coffee break  
See you at 3.30 PM



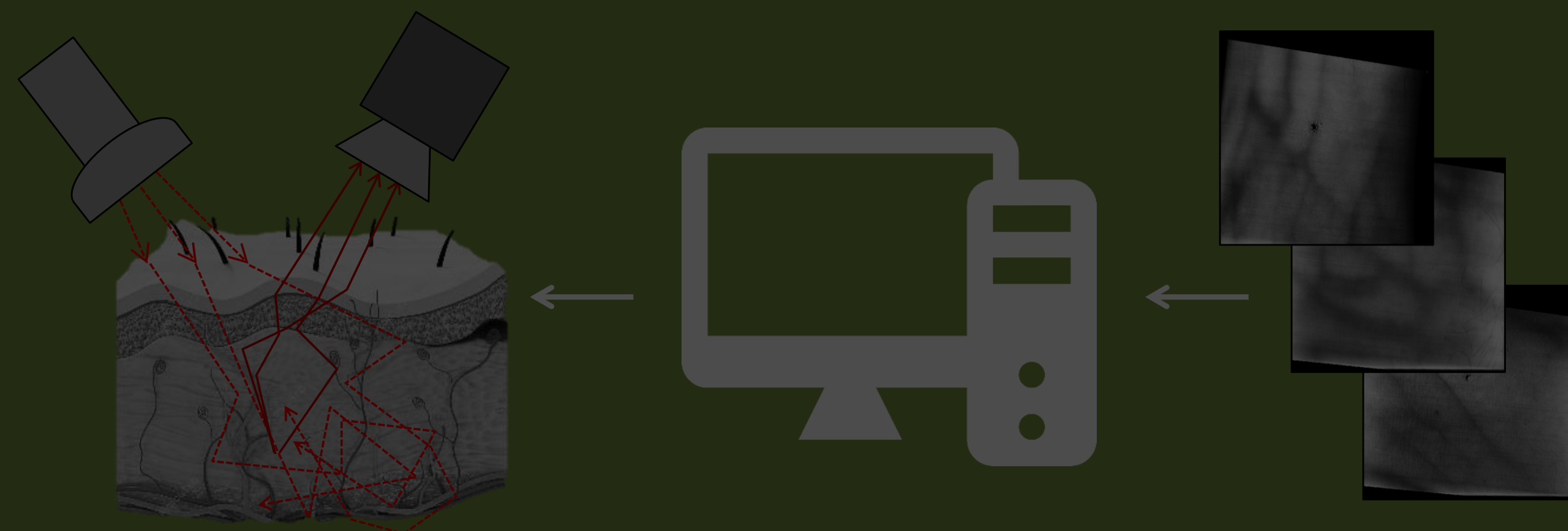
# Physics-based rendering and its applications to computational imaging

## forward rendering

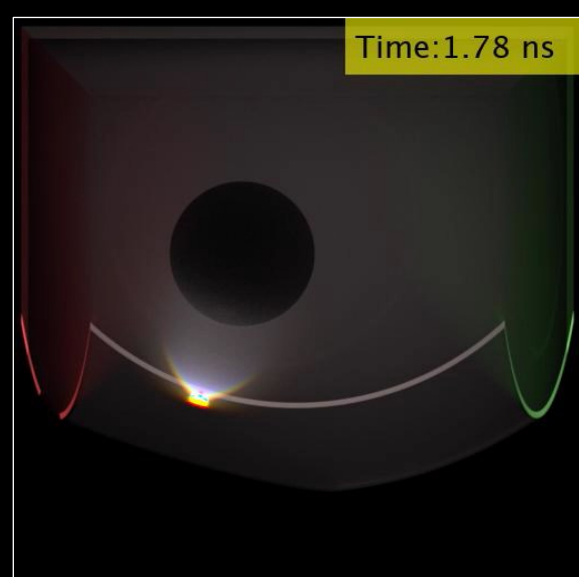


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

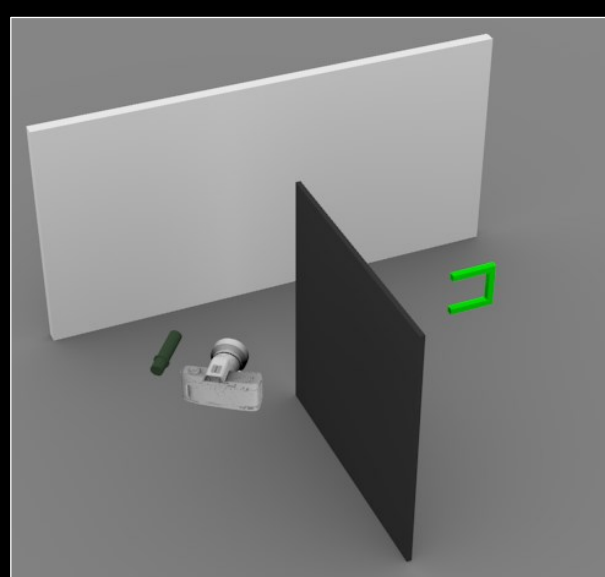
## inverse rendering



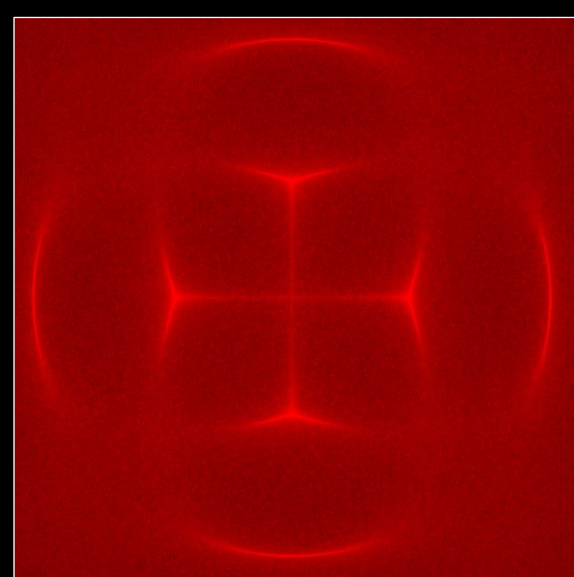
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



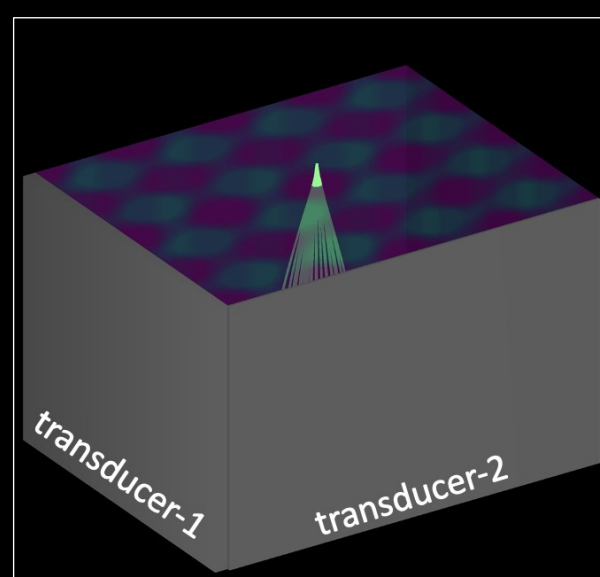
time-of-flight  
imaging



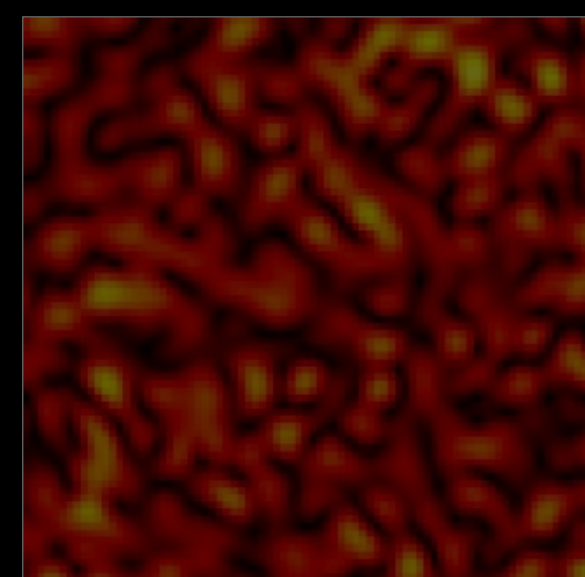
non-line-of-sight  
imaging



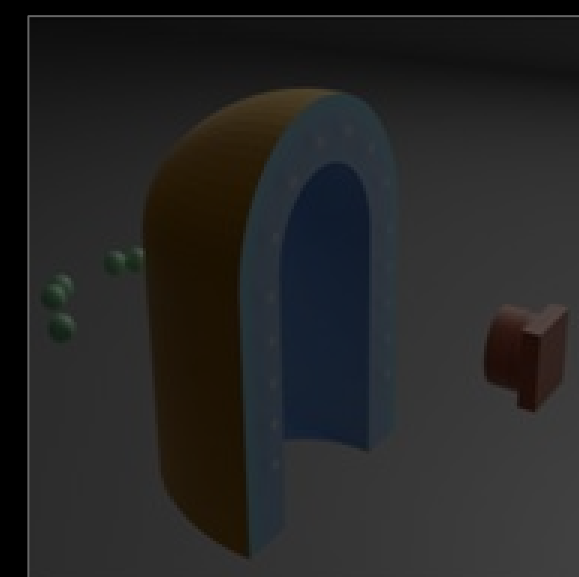
acousto-optic  
lensing



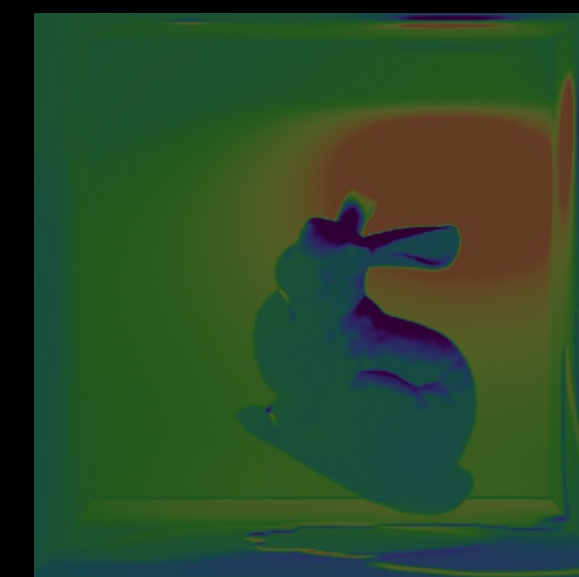
ultrafast light  
scanning



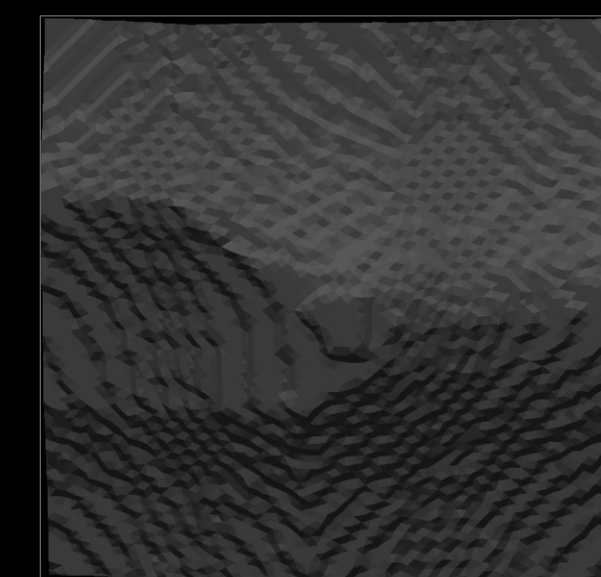
speckle  
imaging



tactile sensor  
design



differentiable  
renderer

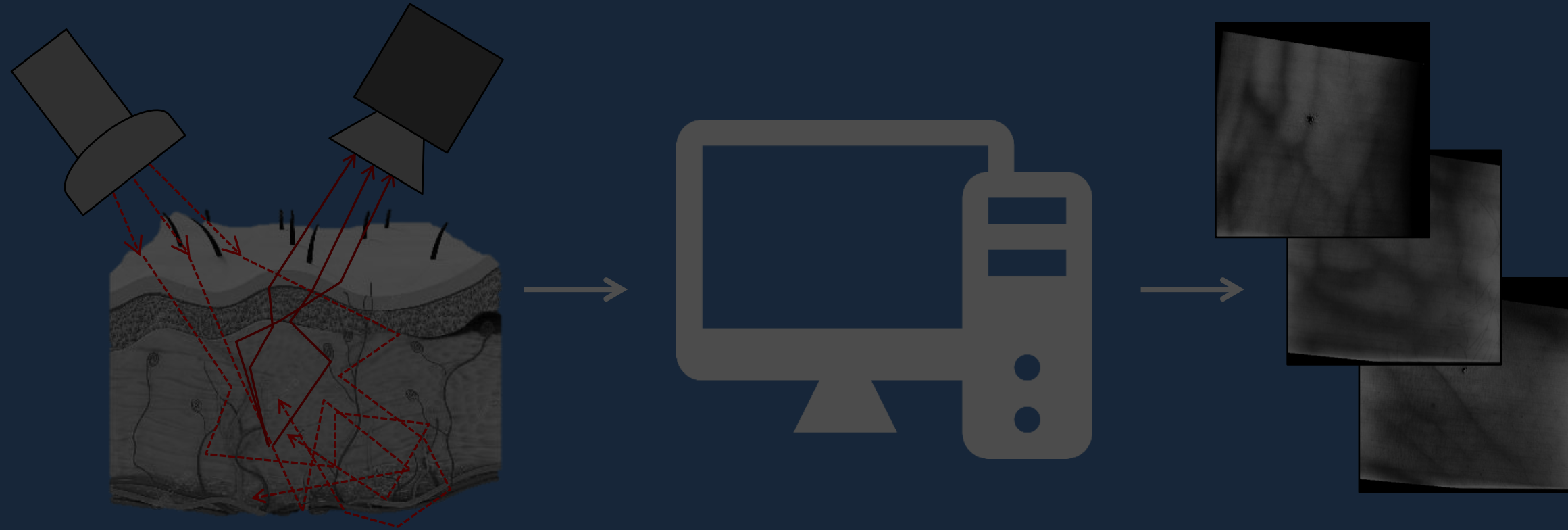


inverse  
problems



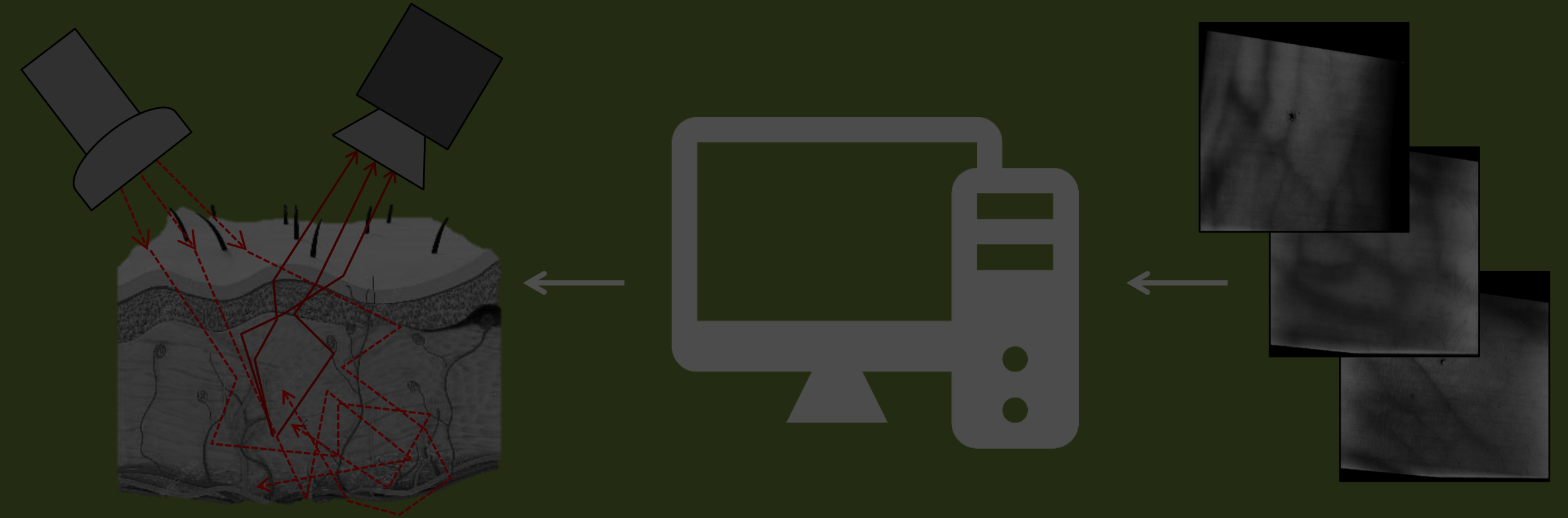
# Physics-based rendering and its applications to computational imaging

## forward rendering

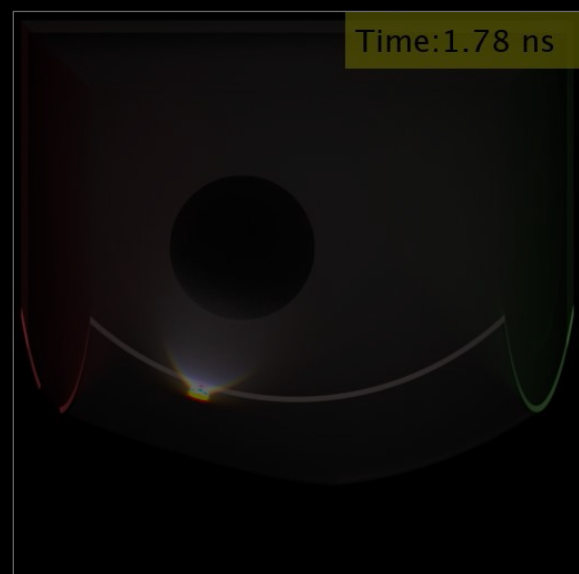


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

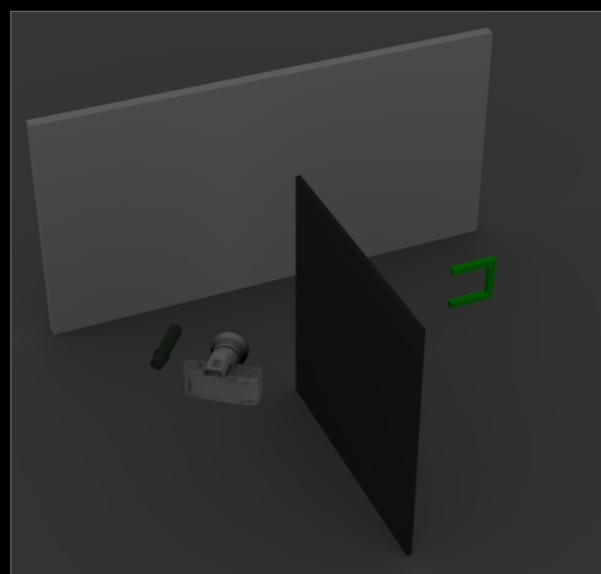
## inverse rendering



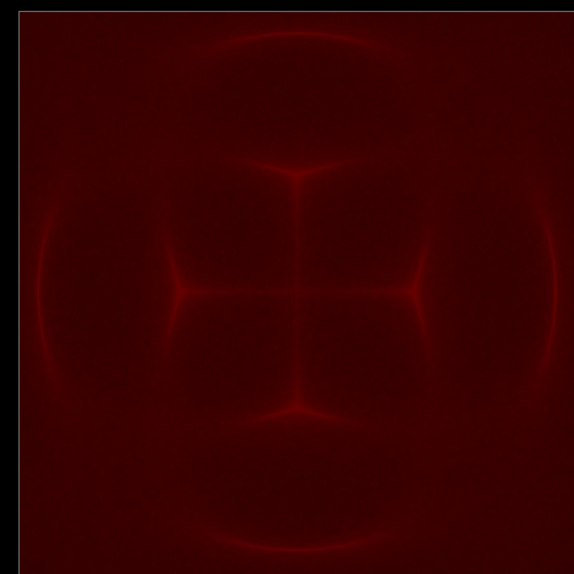
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



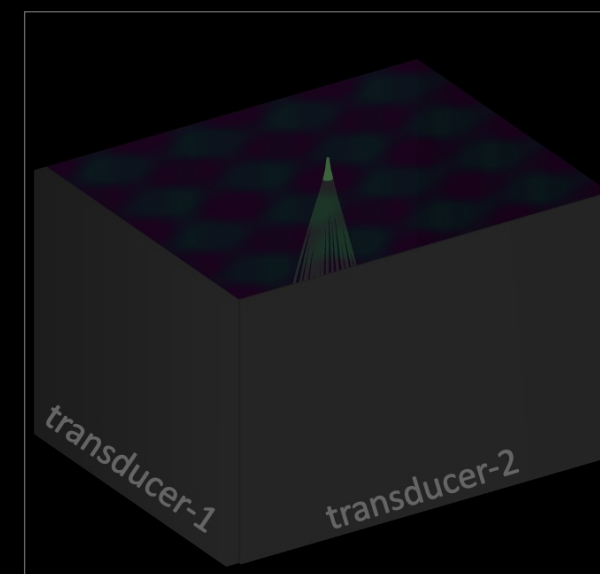
time-of-flight  
imaging



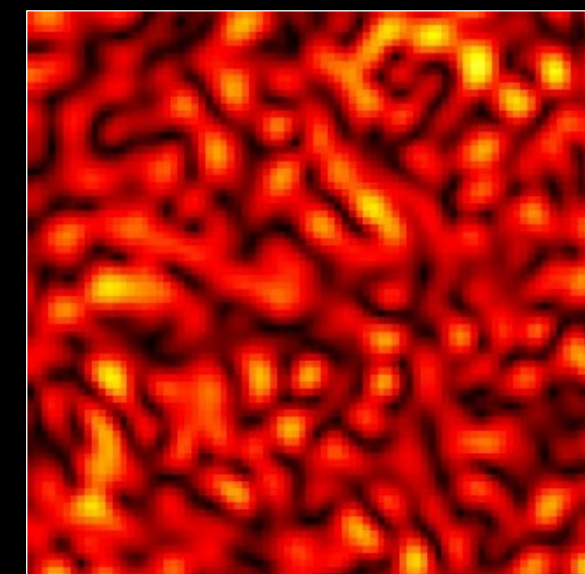
non-line-of-sight  
imaging



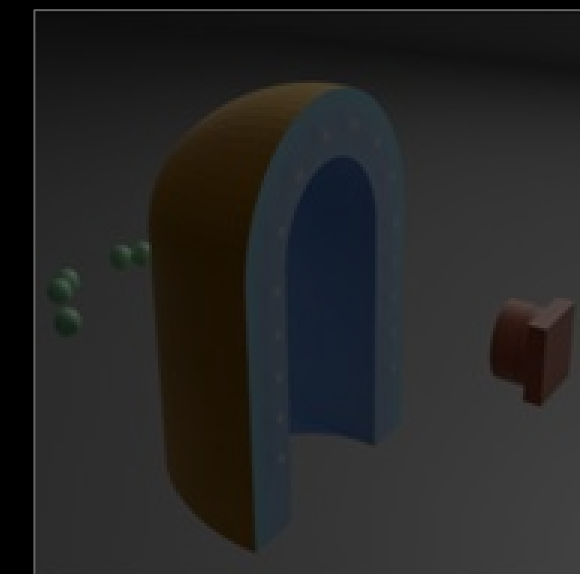
acousto-optic  
lensing



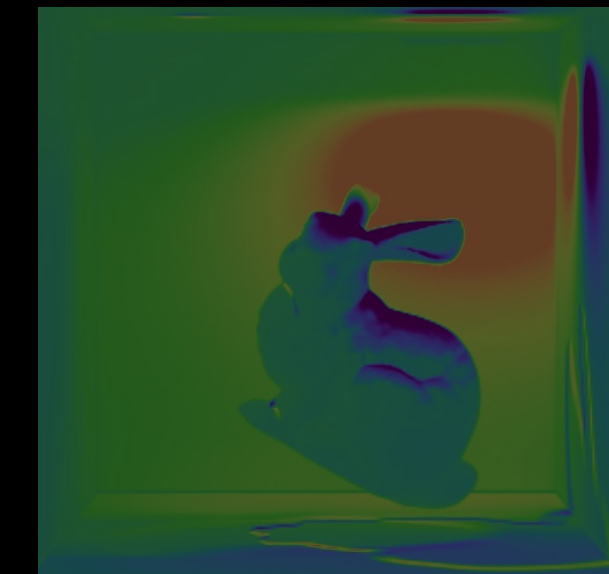
ultrafast light  
scanning



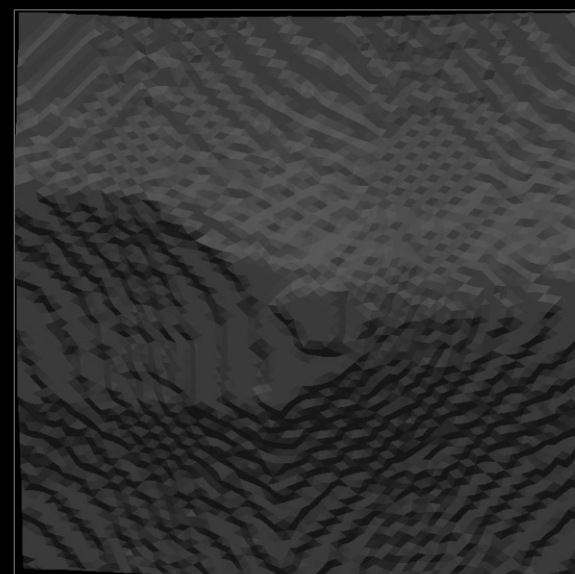
speckle  
imaging



tactile sensor  
design

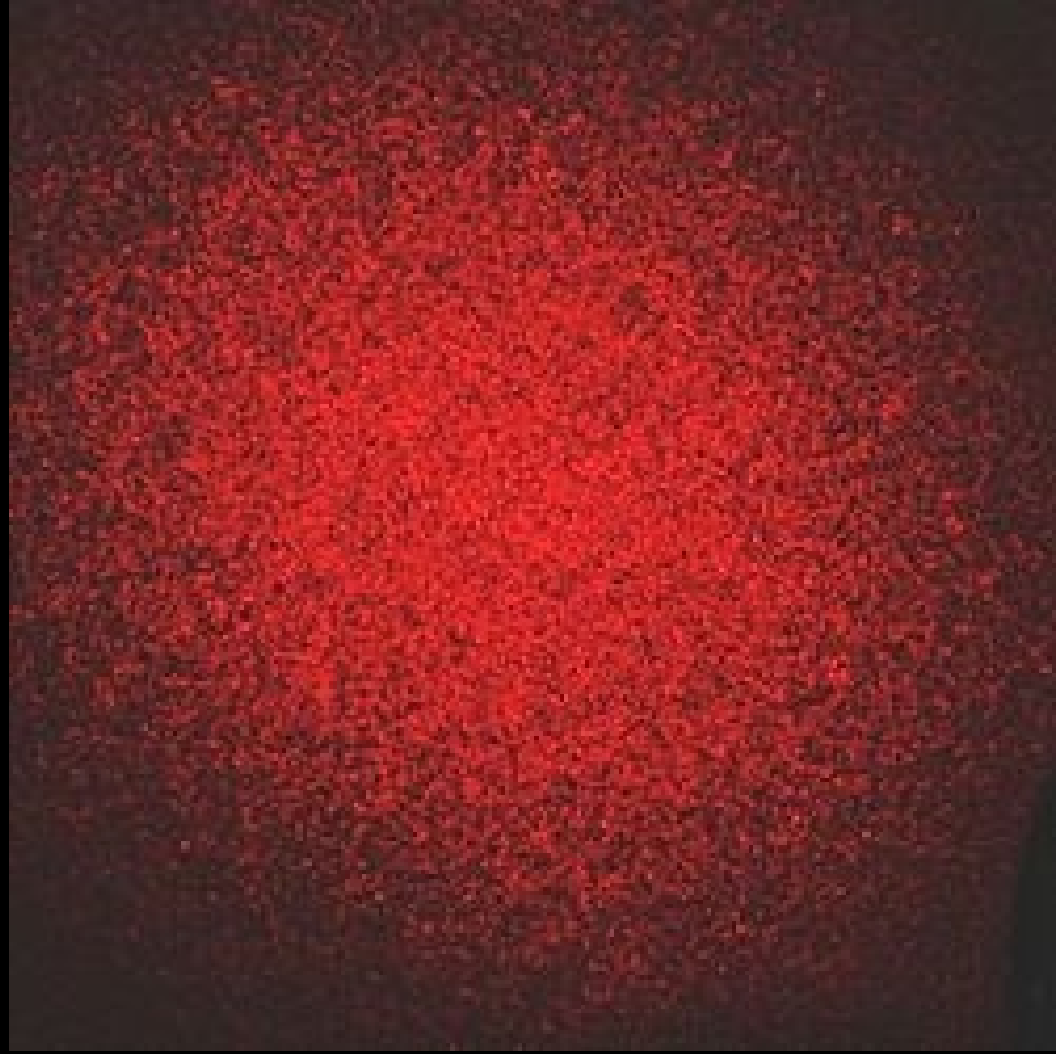


differentiable  
renderer



inverse  
problems

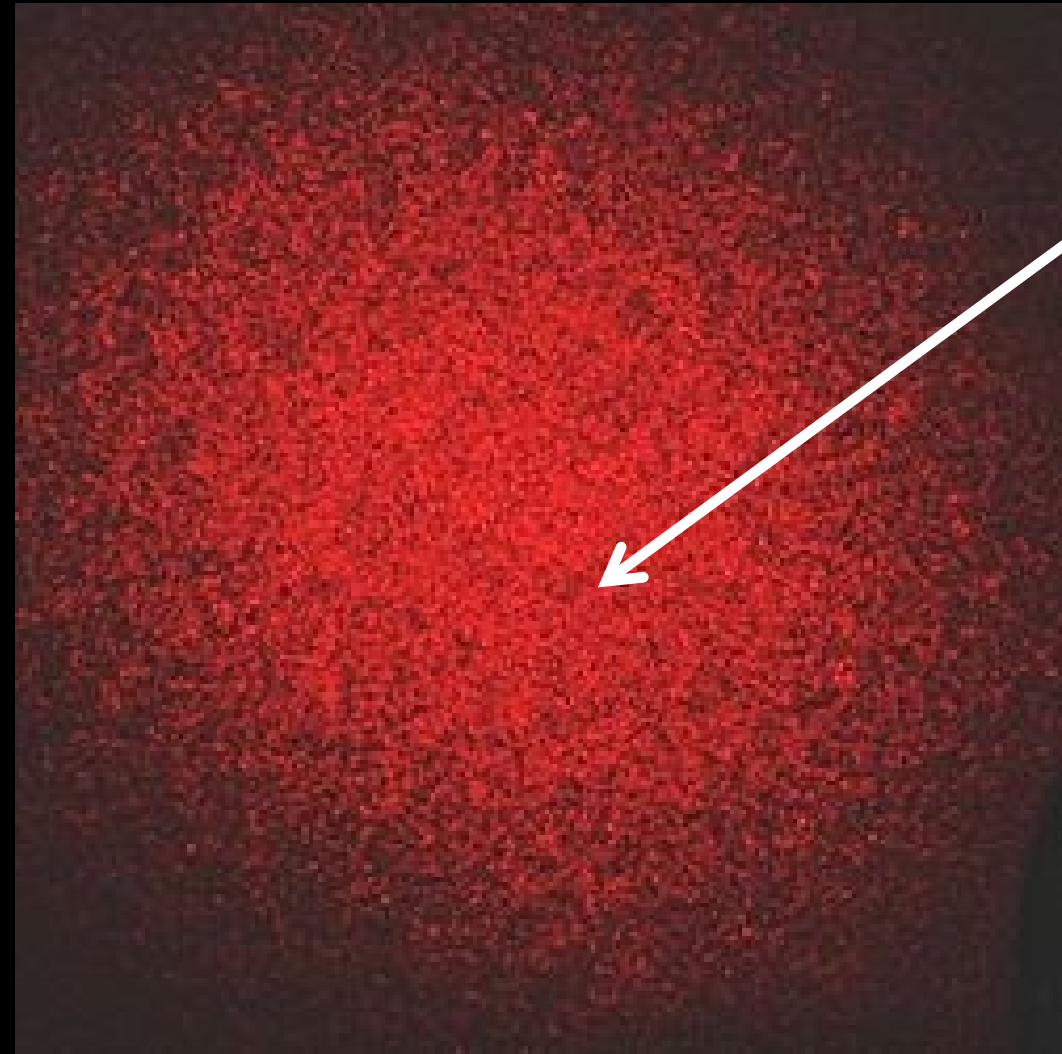
# Rendering wave-optics effects



what real laser  
images look like



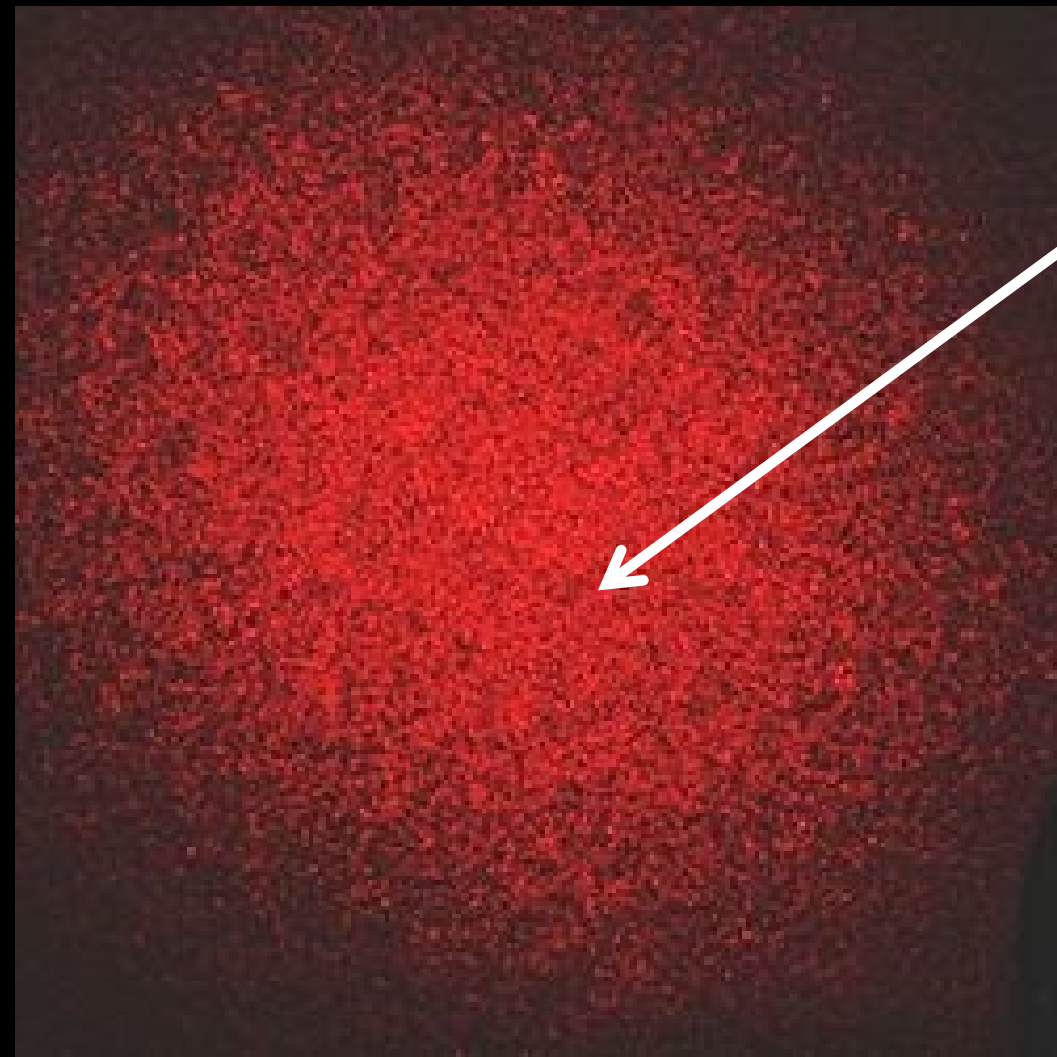
# Rendering wave-optics effects



speckle: noise-  
like pattern

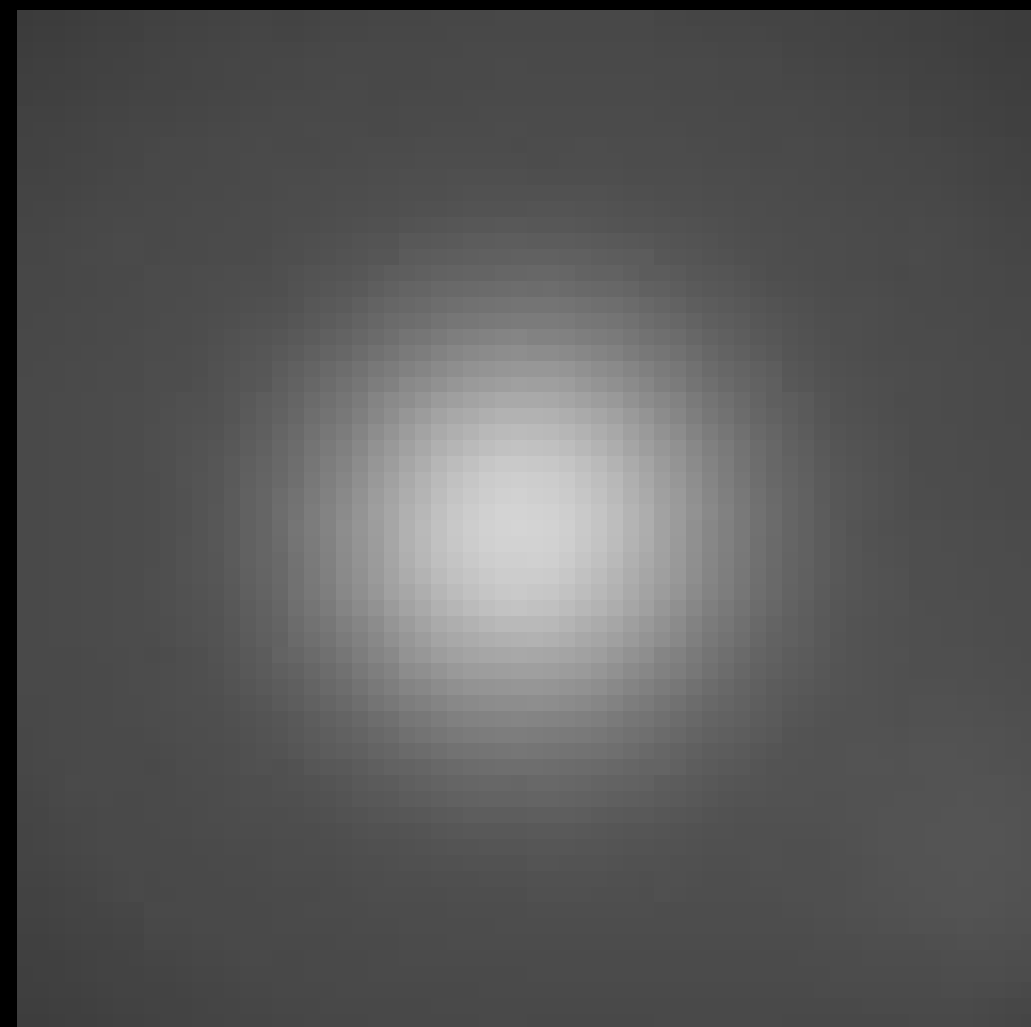
what real laser  
images look like

# Rendering wave-optics effects



speckle: noise-  
like pattern

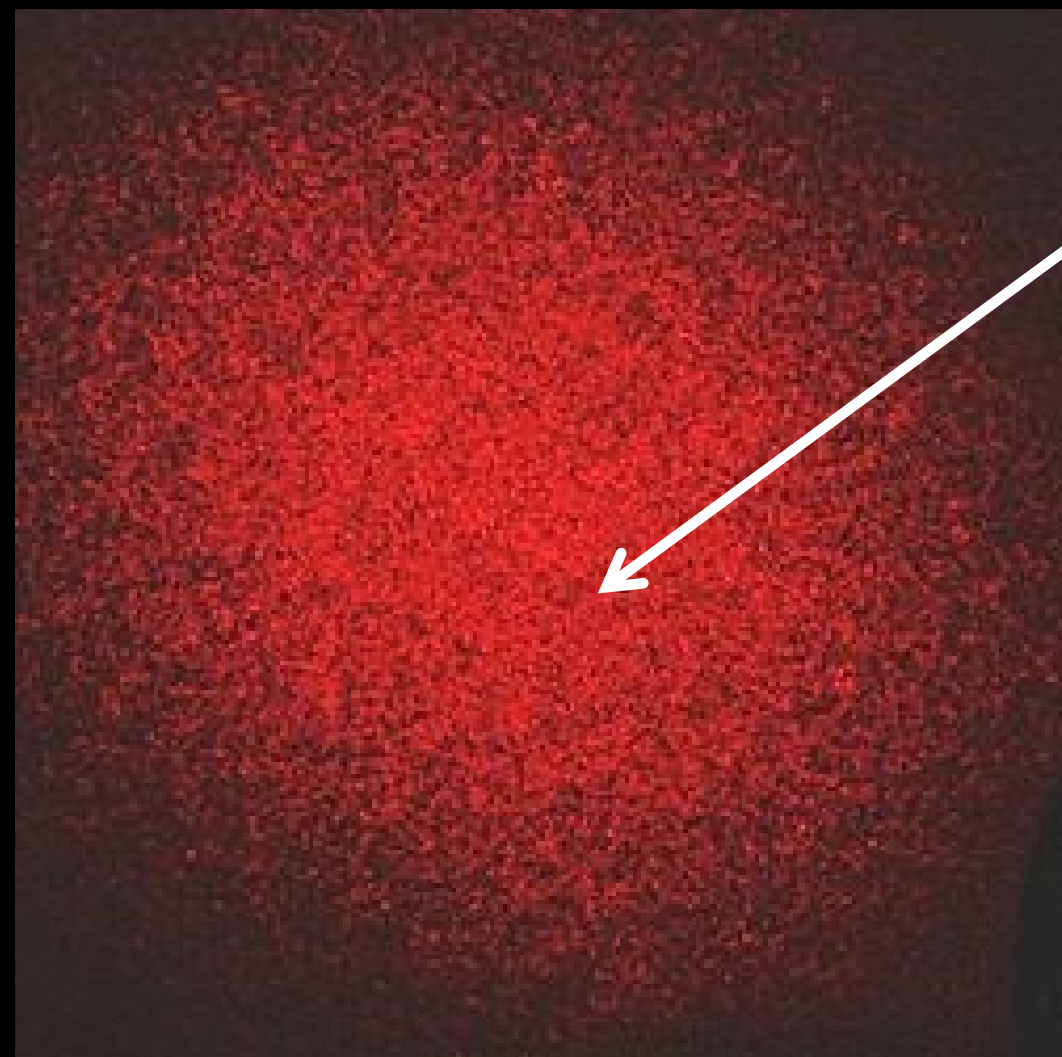
what real laser  
images look like



what standard  
Monte Carlo  
renderings look like

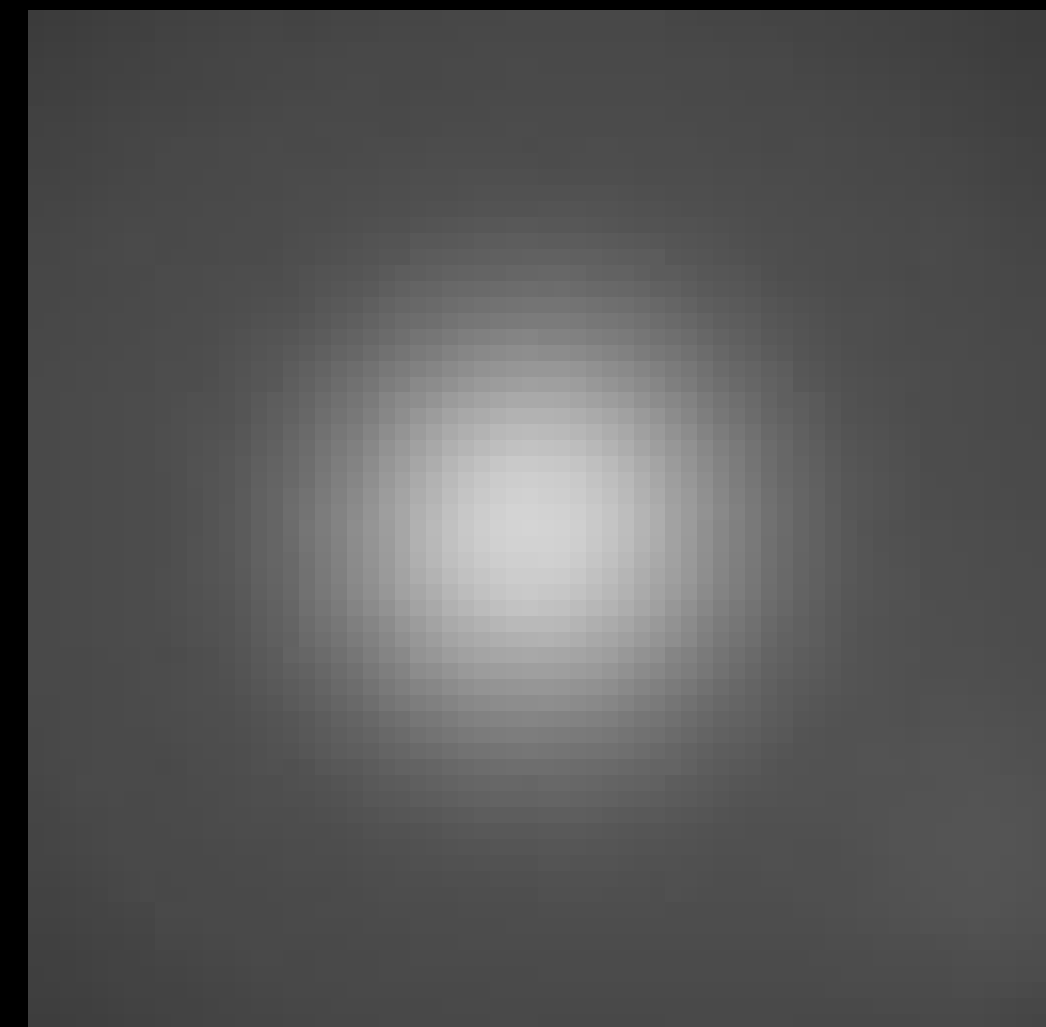


# Rendering wave-optics effects

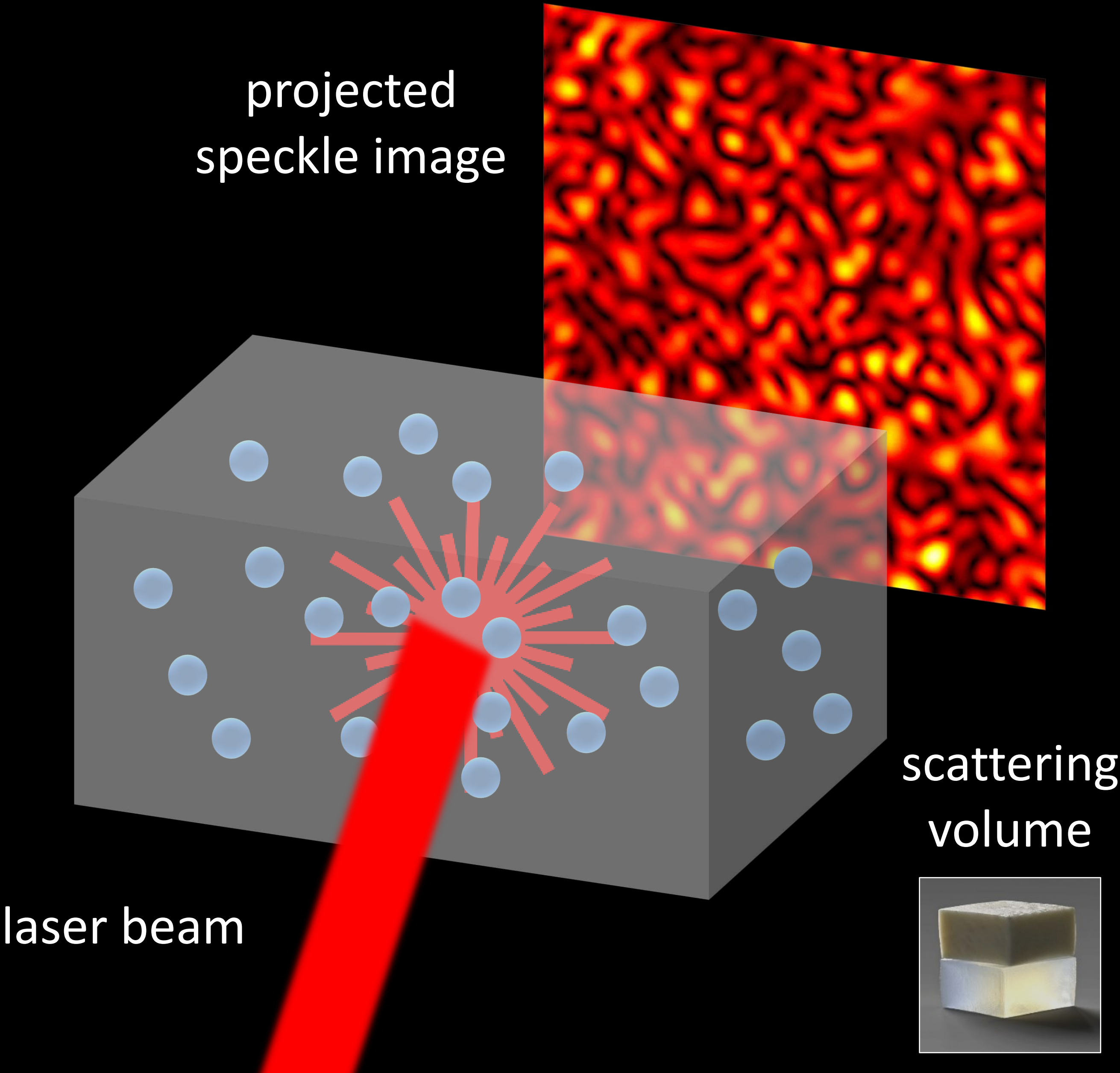


speckle: noise-like pattern

what real laser images look like

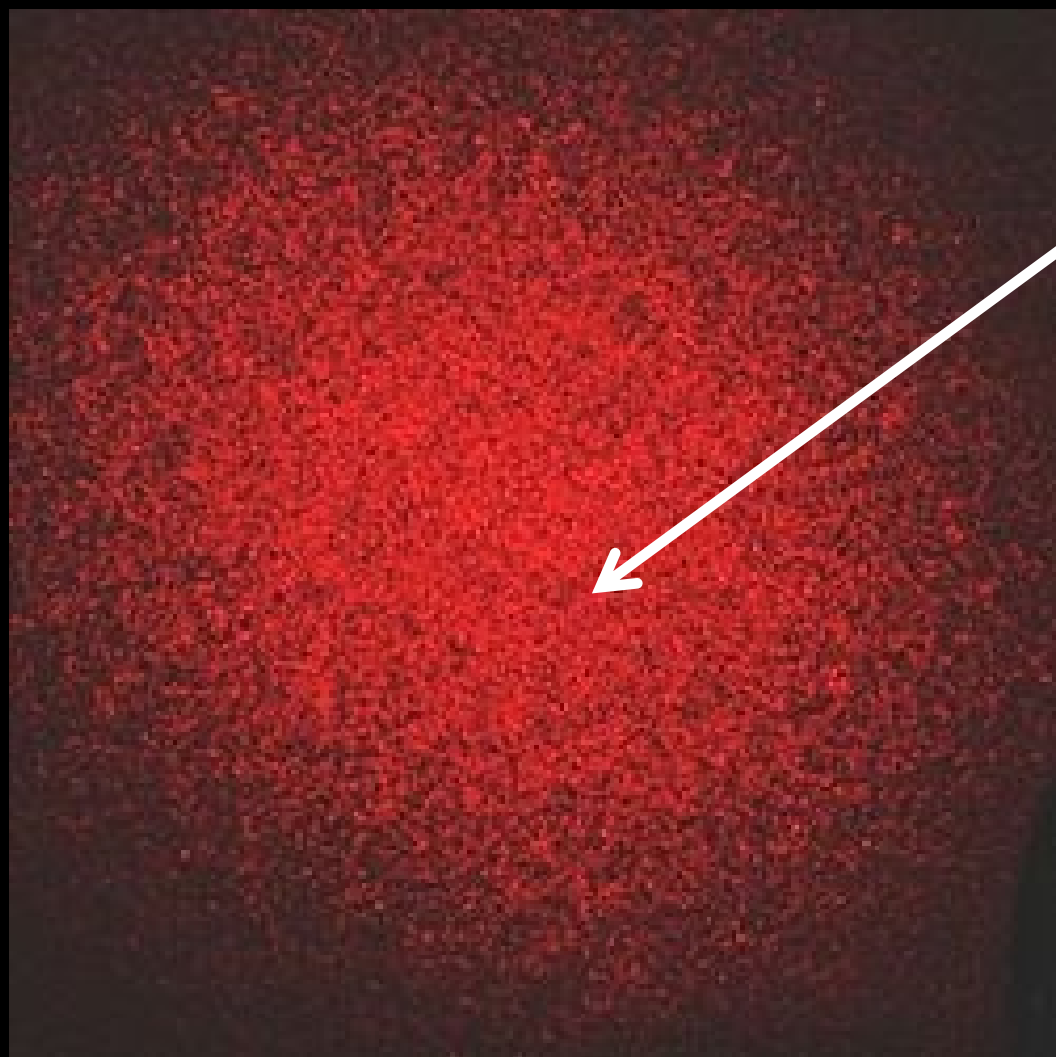


what standard Monte Carlo renderings look like



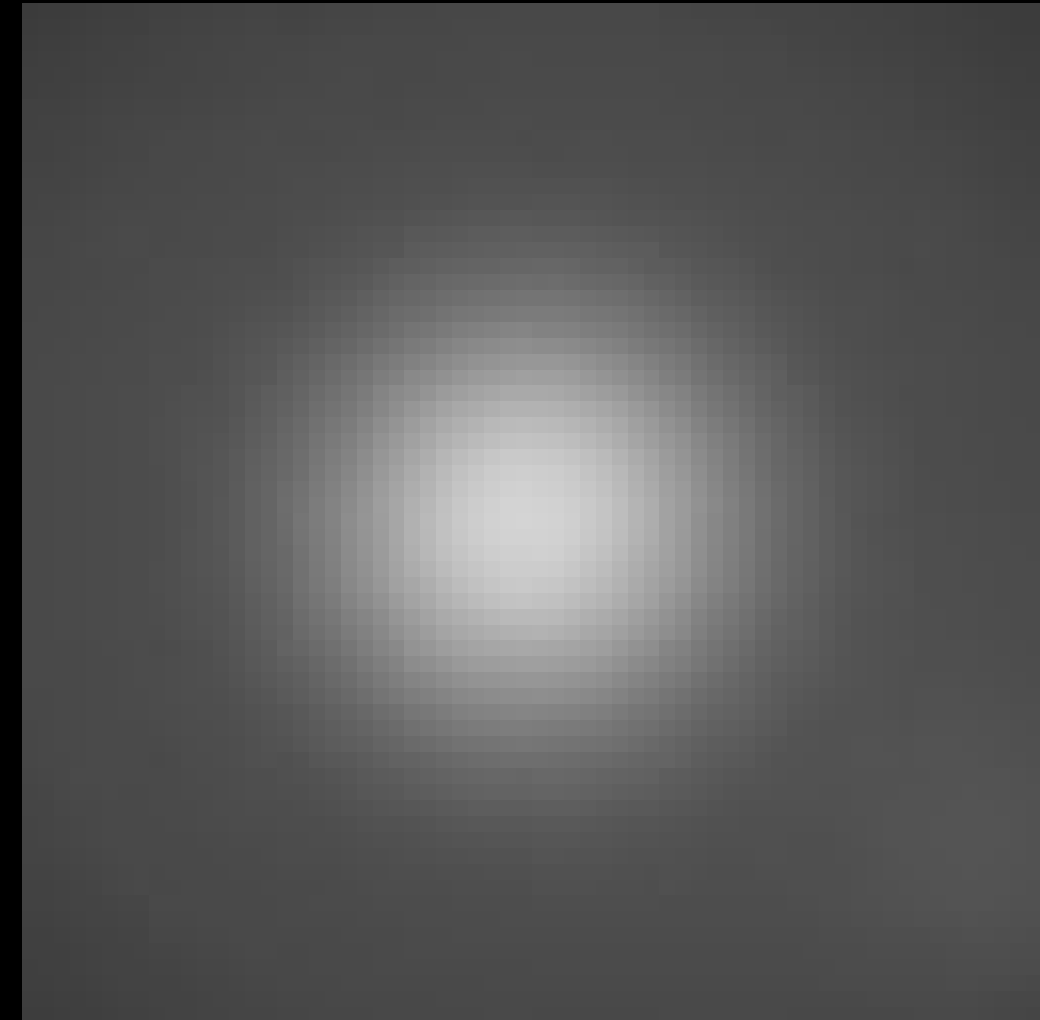


# Rendering wave-optics effects



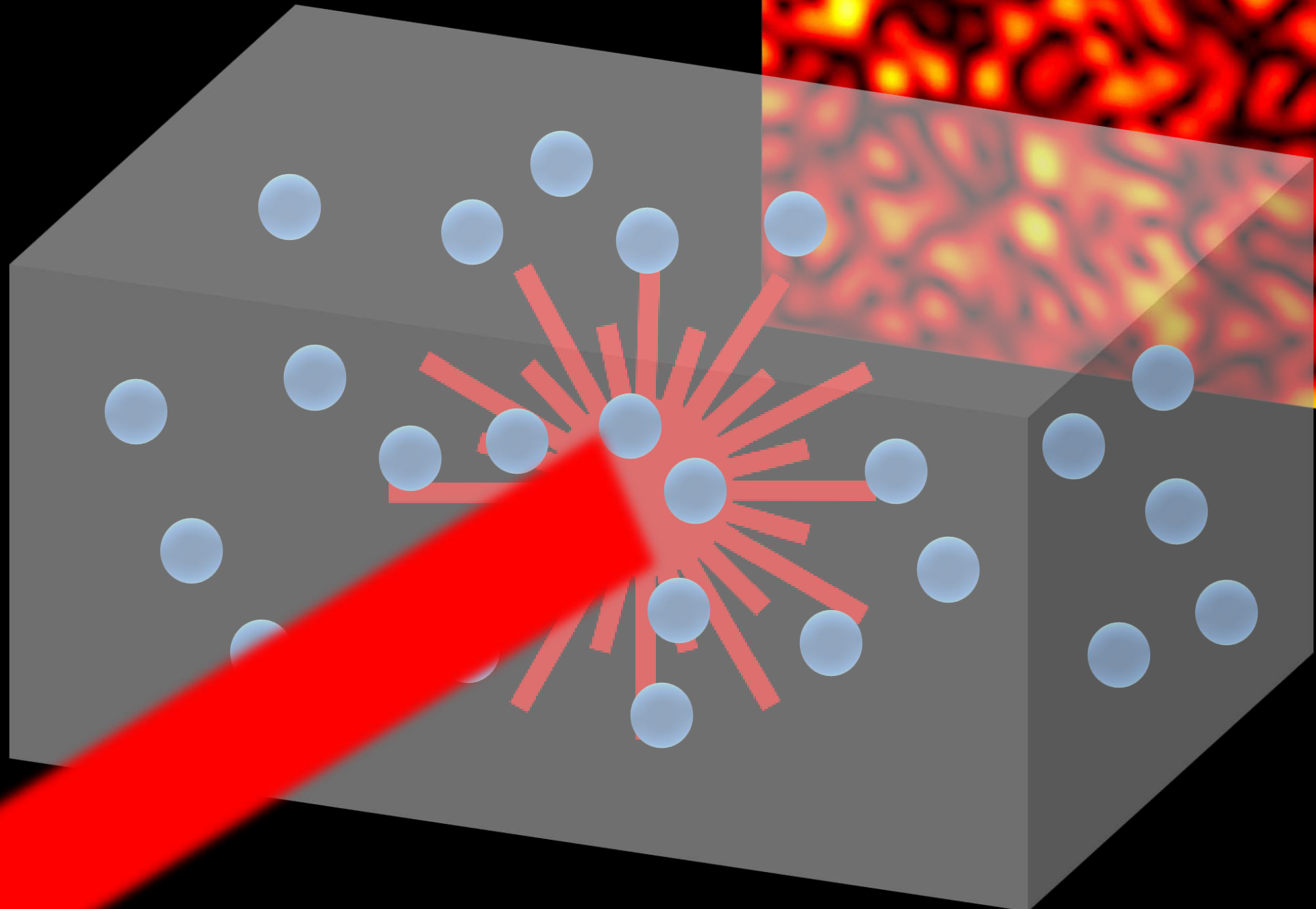
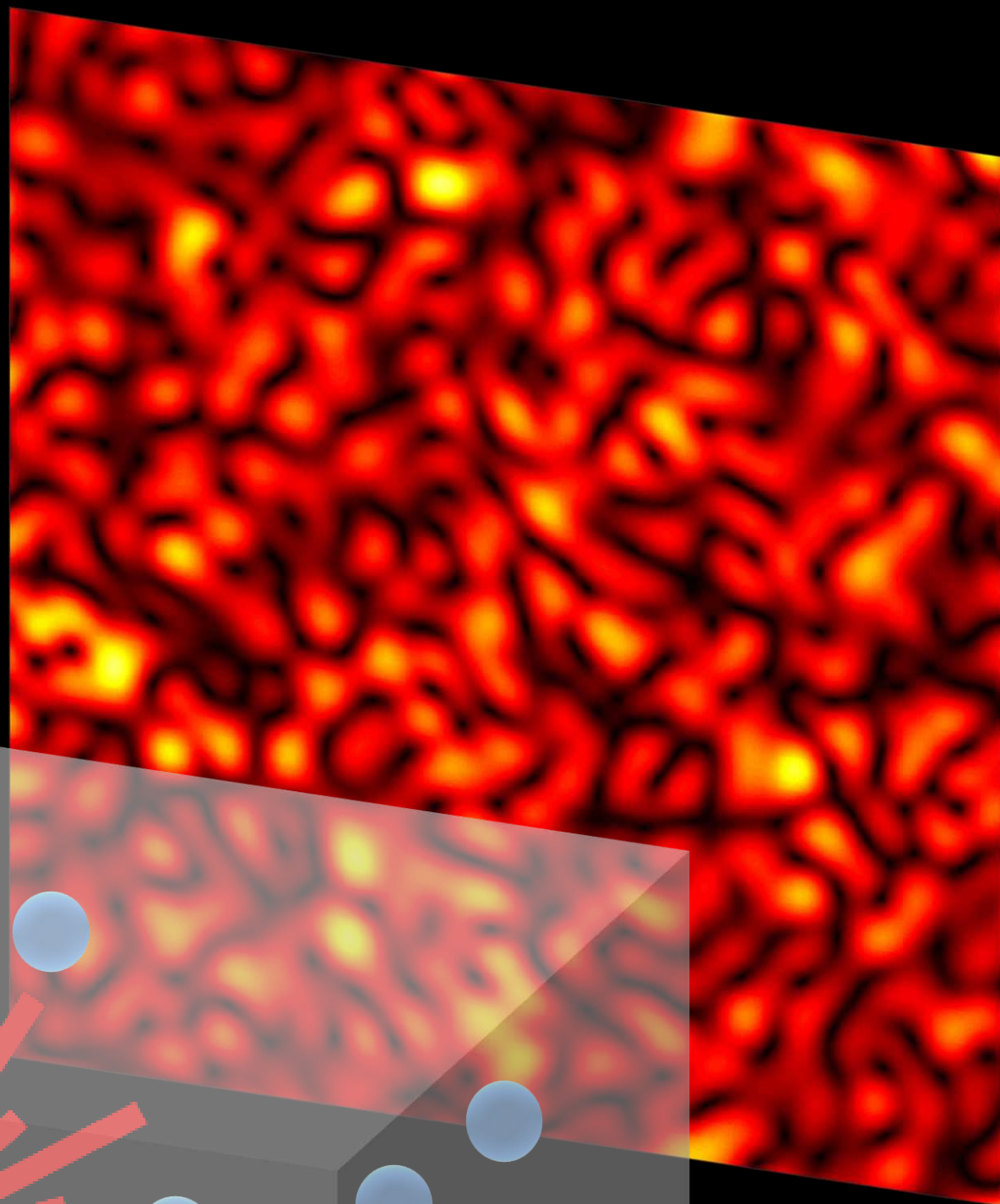
speckle: noise-like pattern

what real laser images look like



what standard Monte Carlo renderings look like

projected speckle image



laser beam

scattering volume

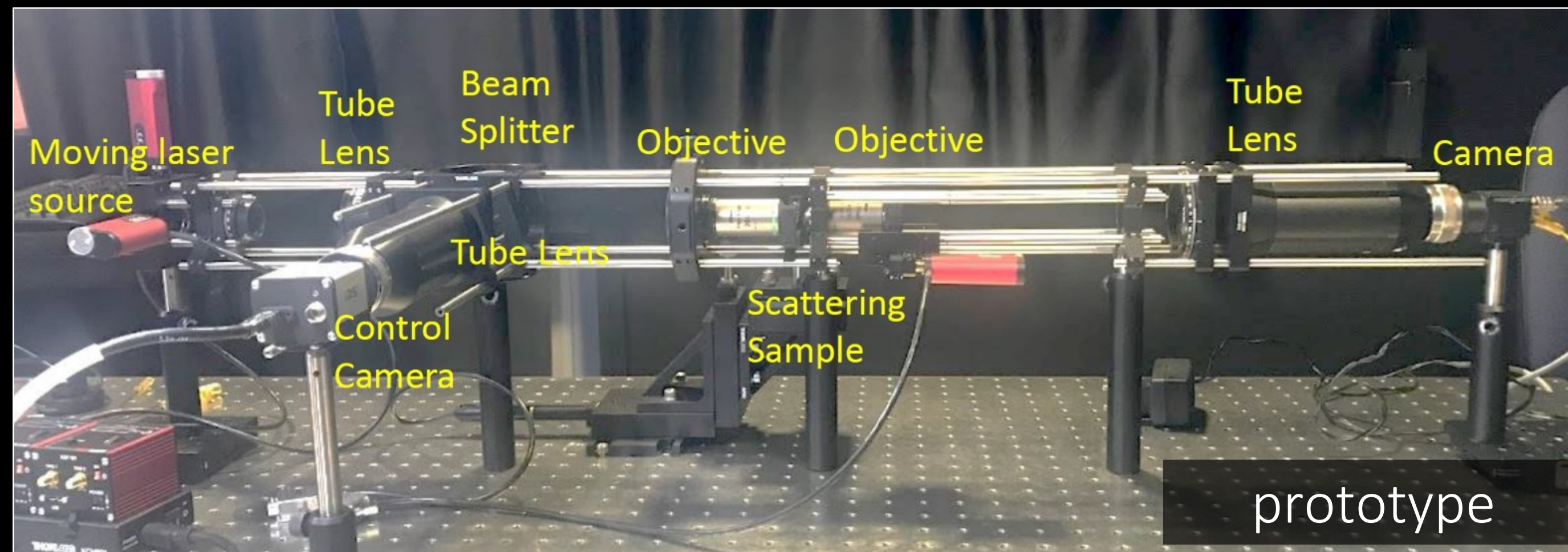
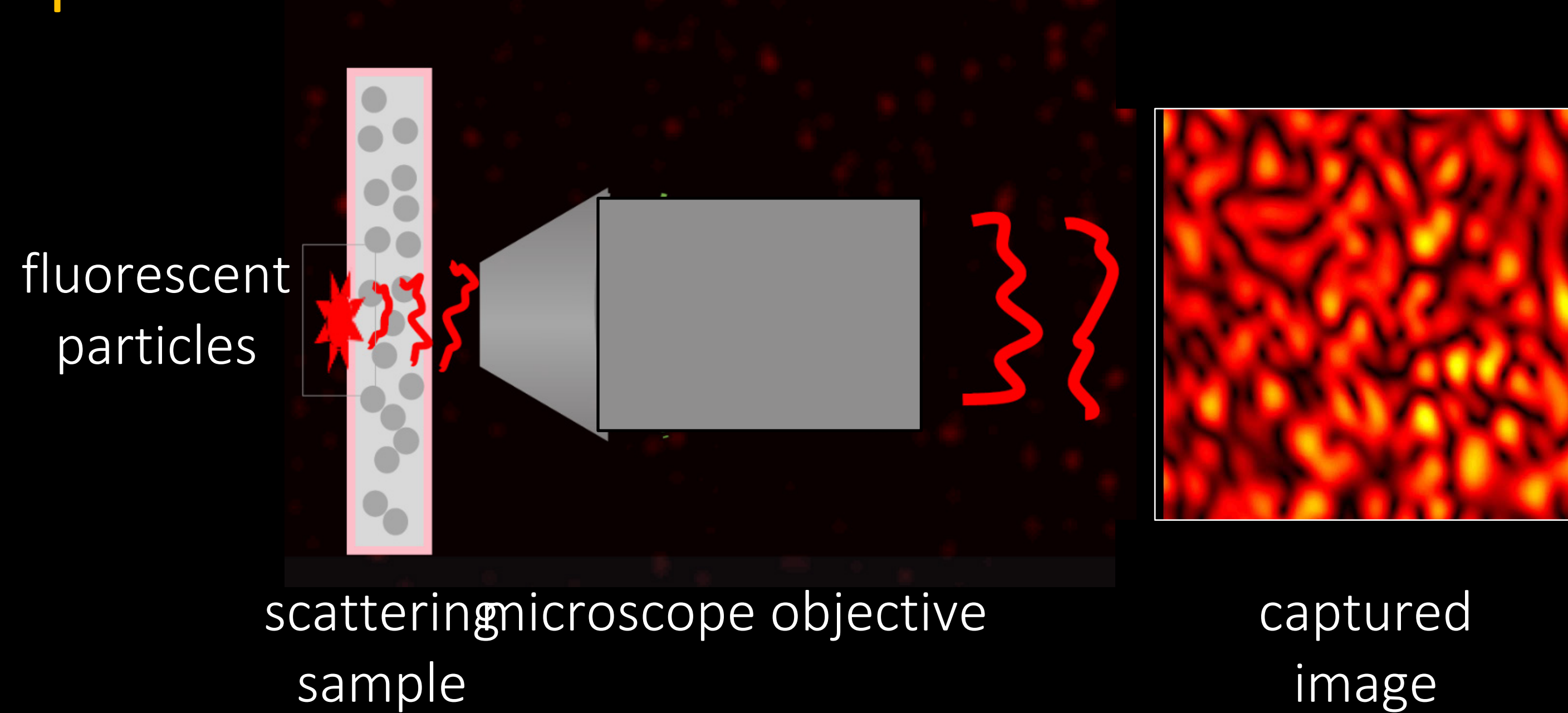






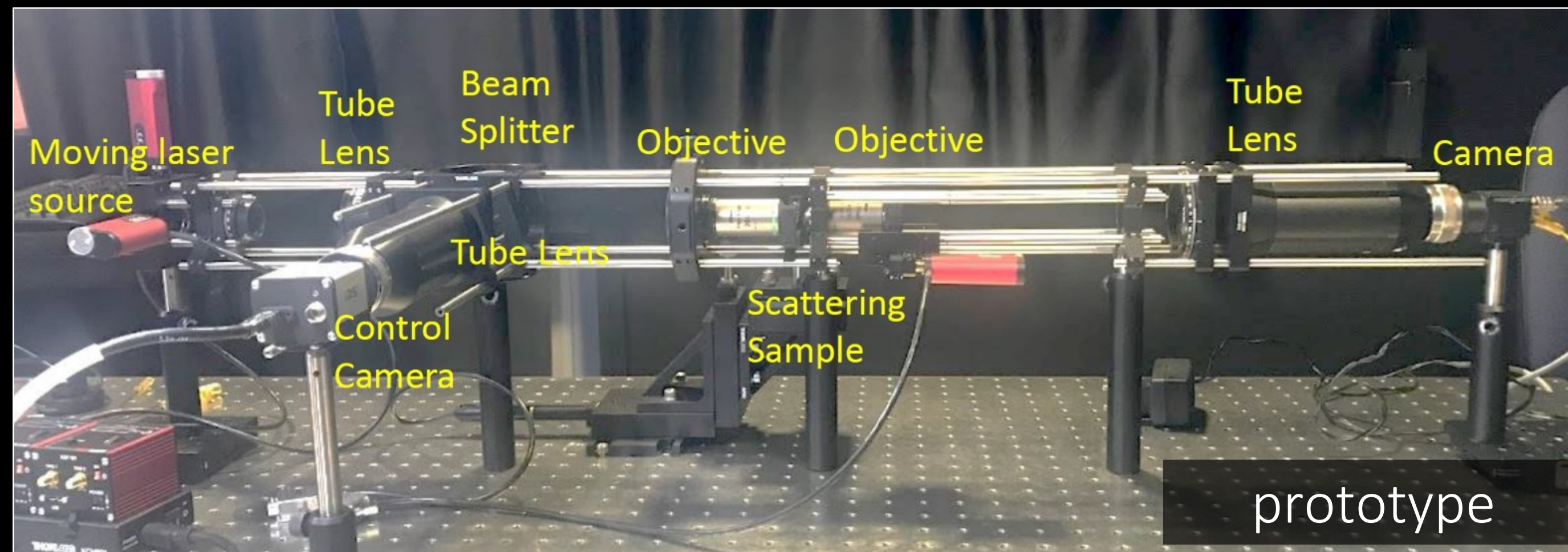
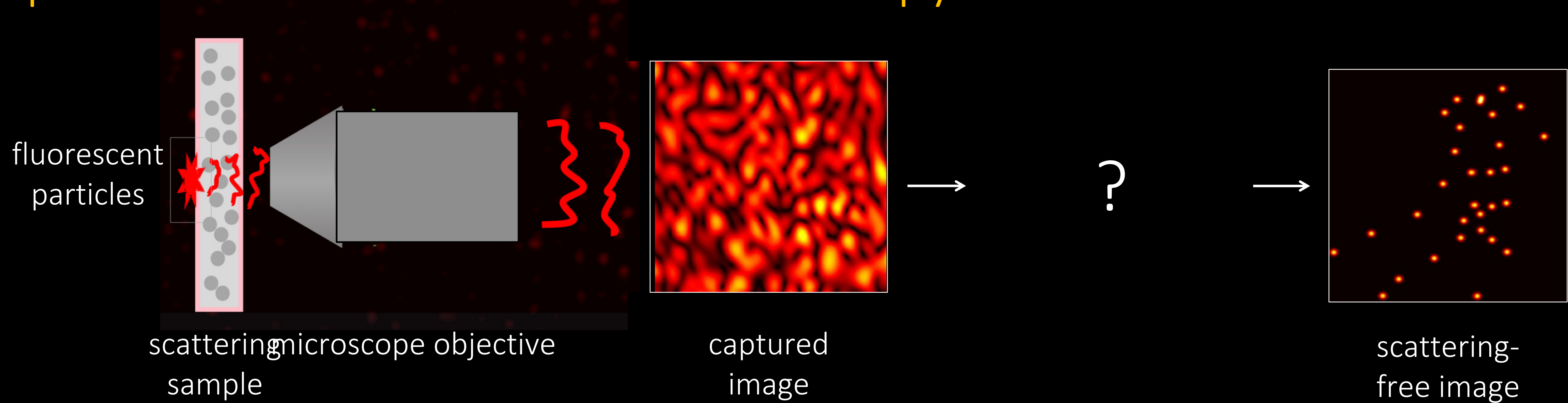


# Speckle-based fluorescence microscopy

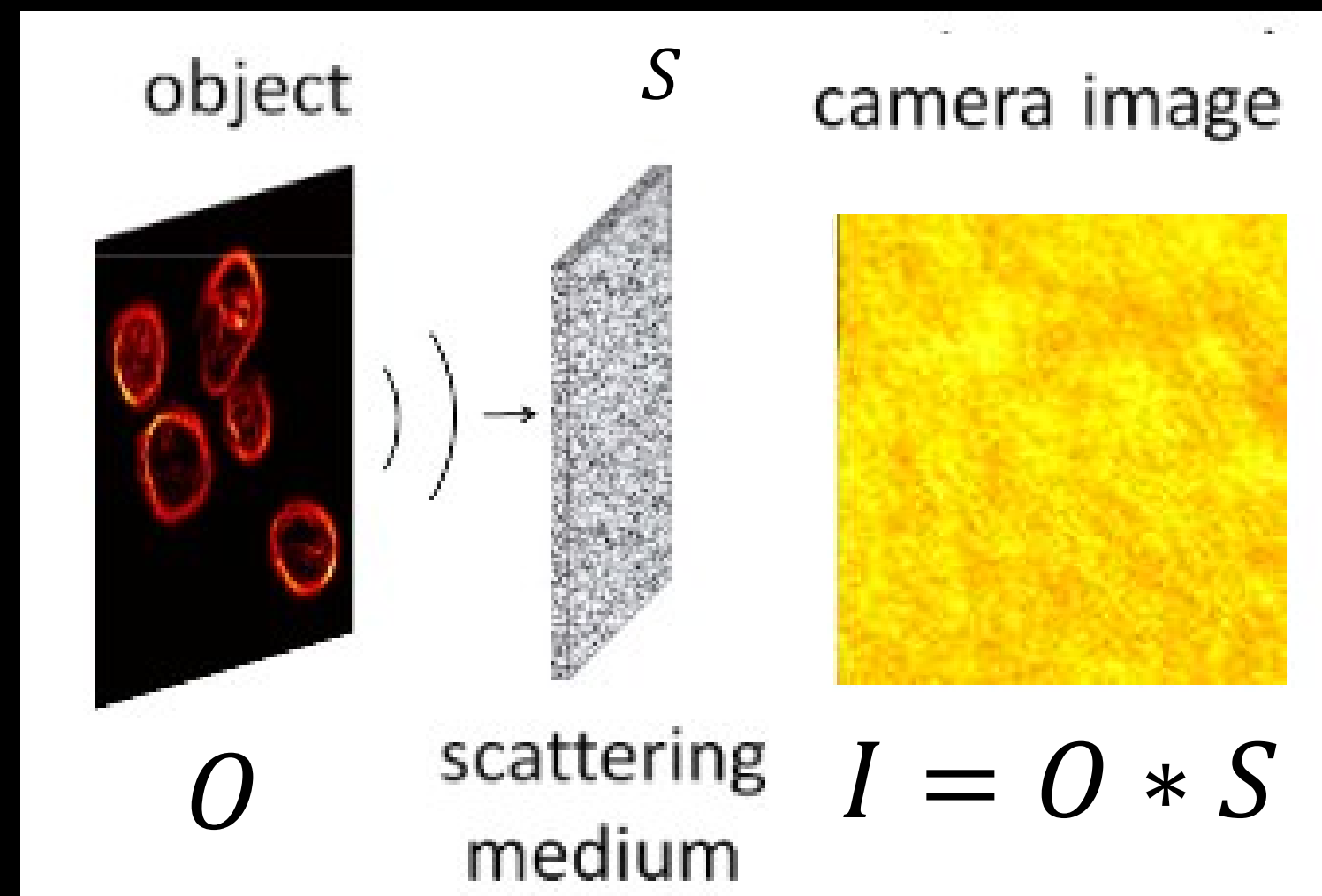
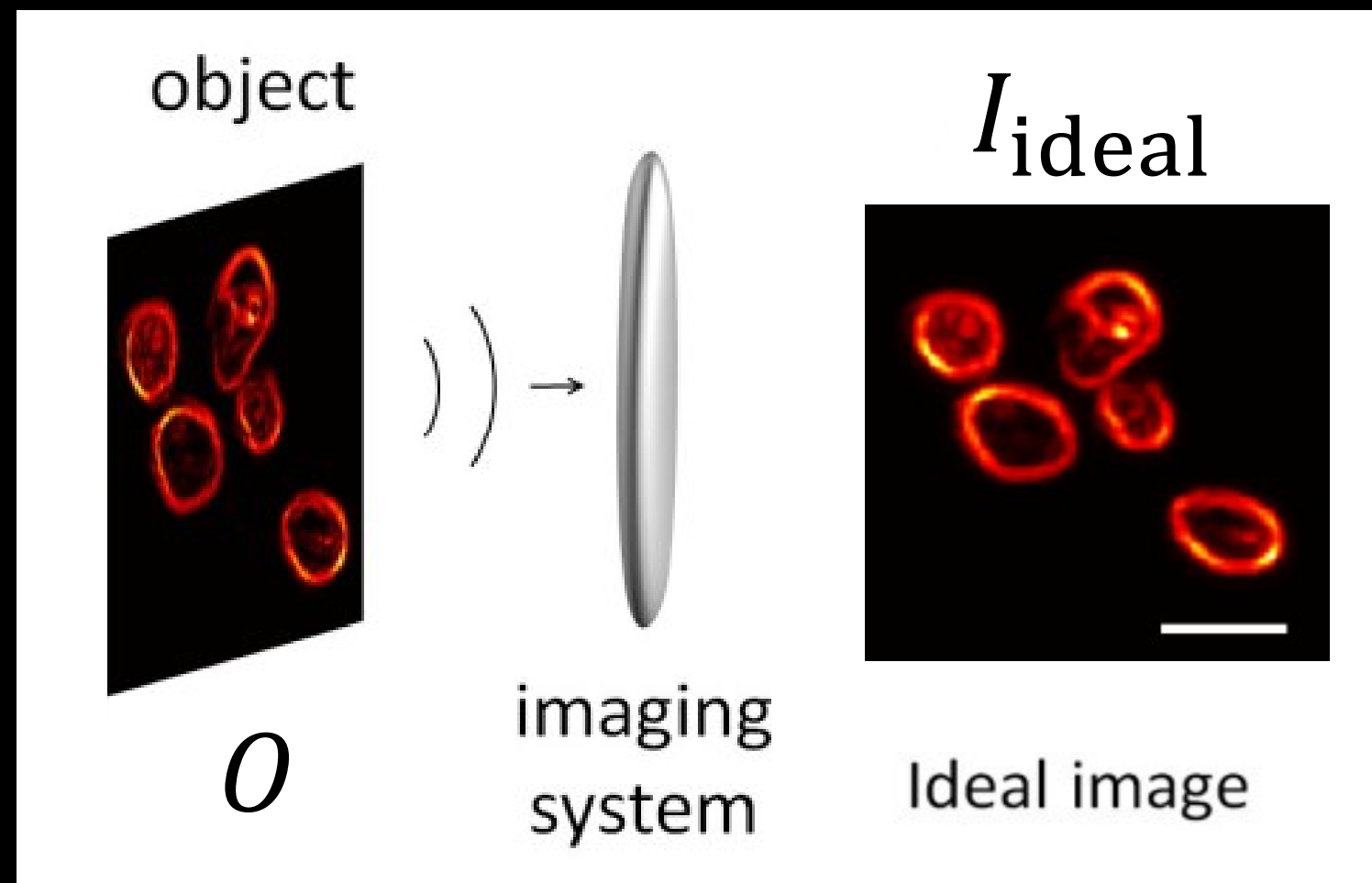




# Speckle-based fluorescence microscopy

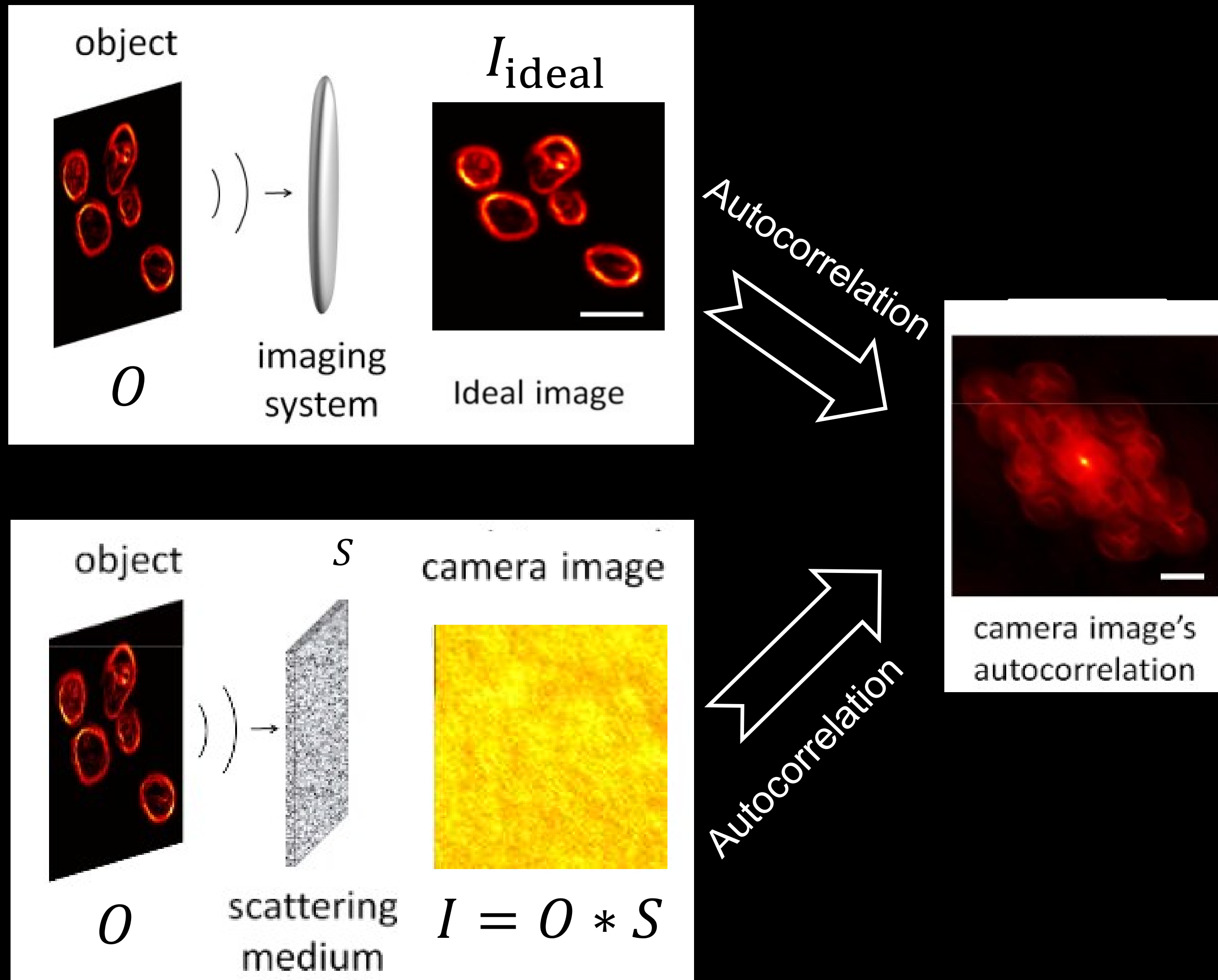


# Use the memory effect to image through scattering

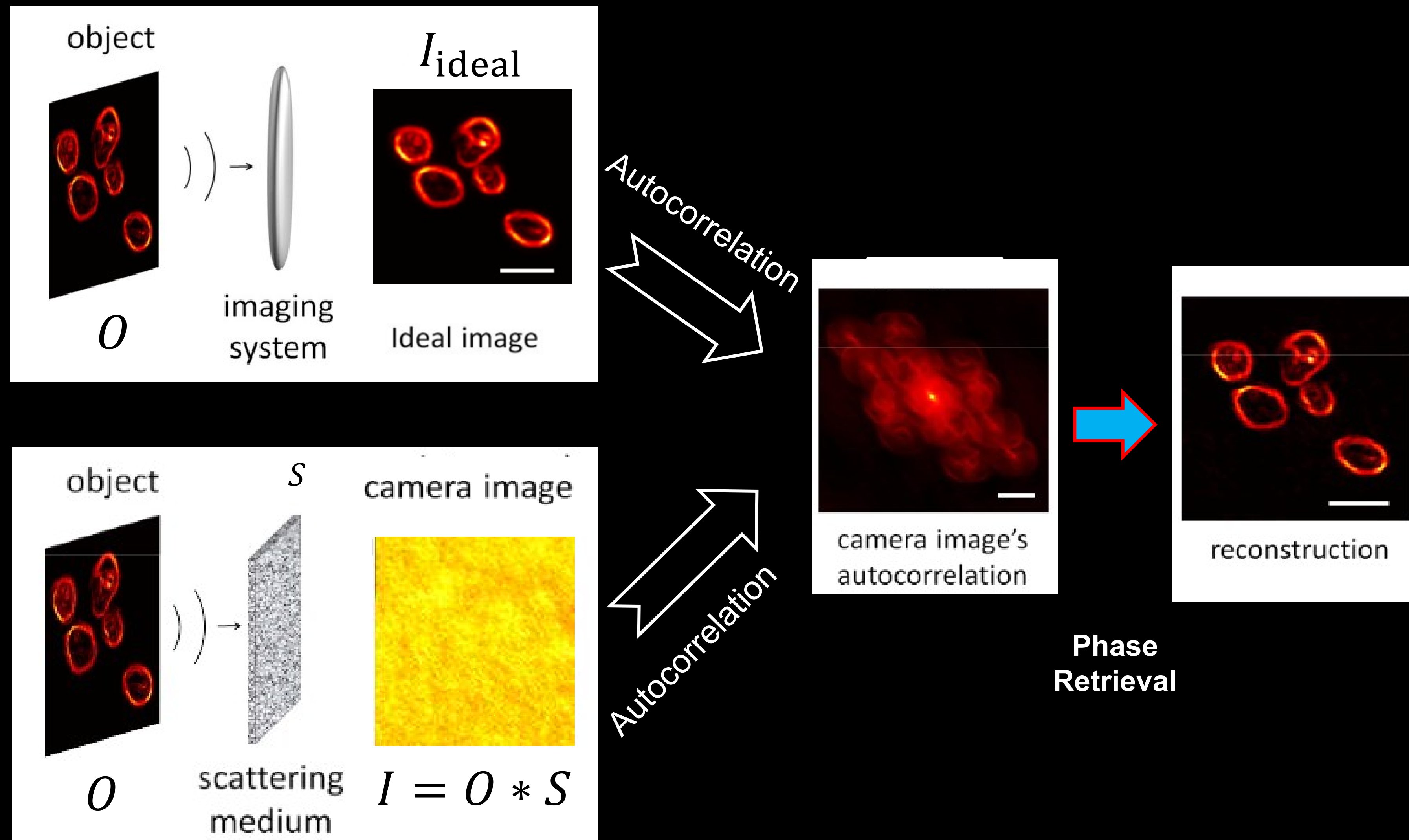




# Use the memory effect to image through scattering

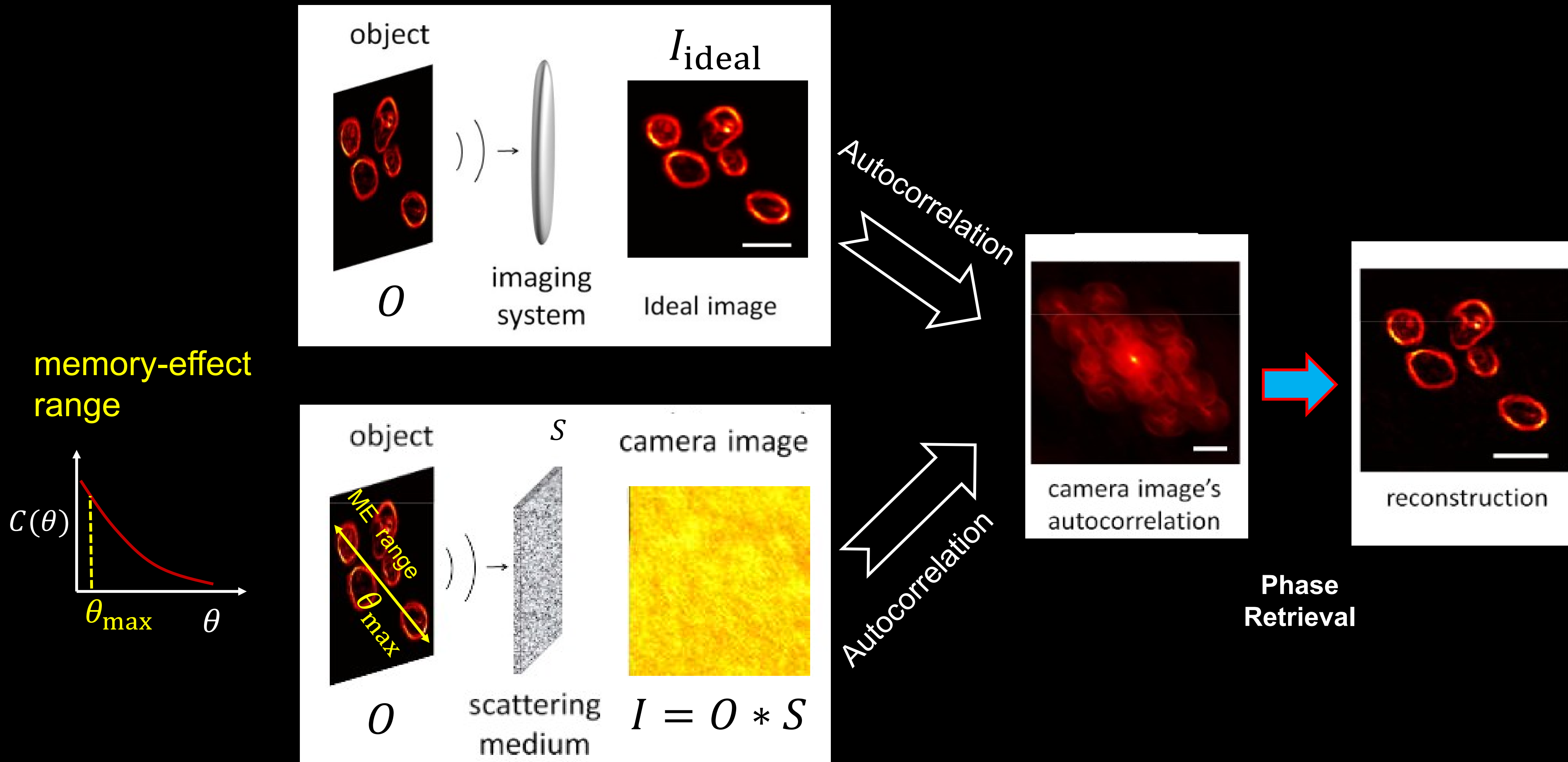


# Use the memory effect to image through scattering

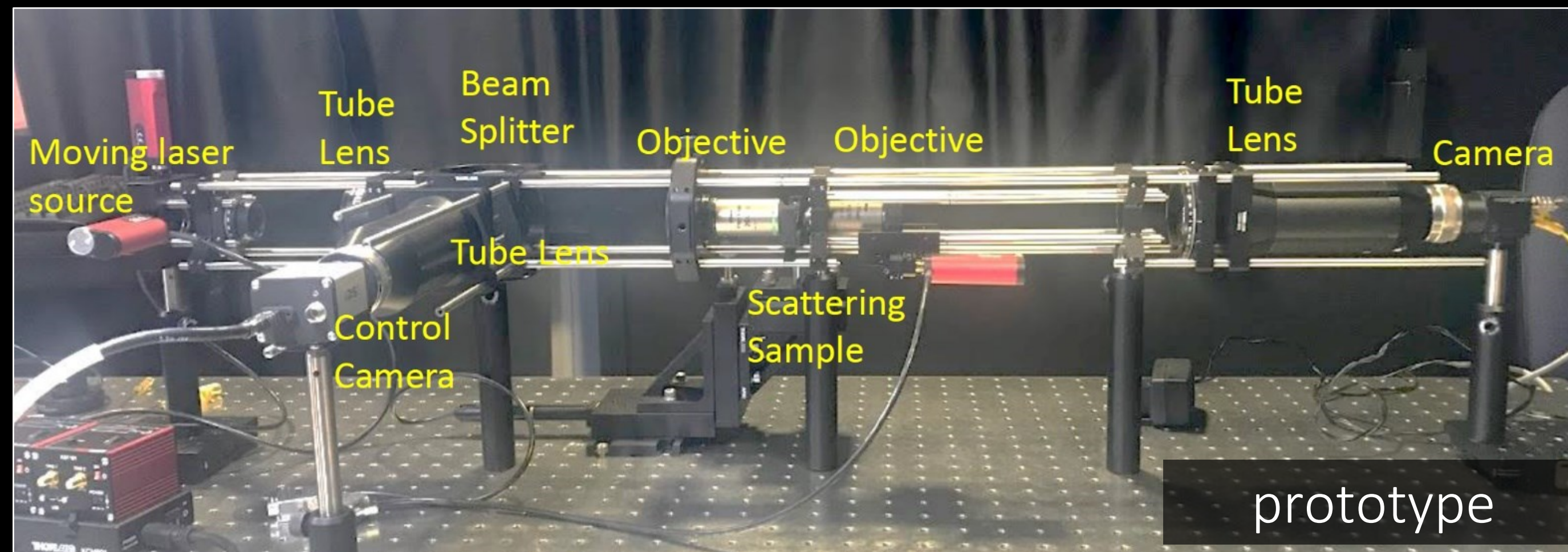
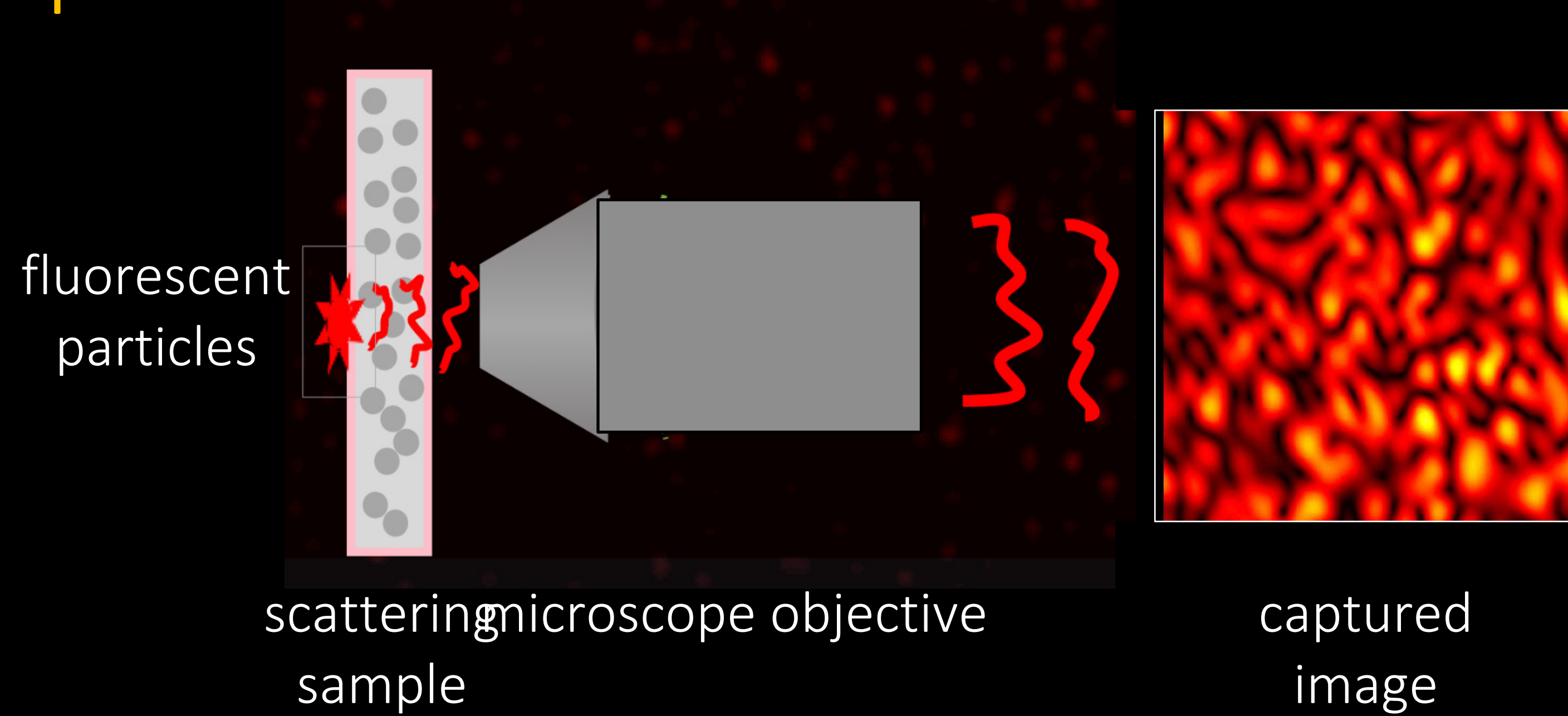




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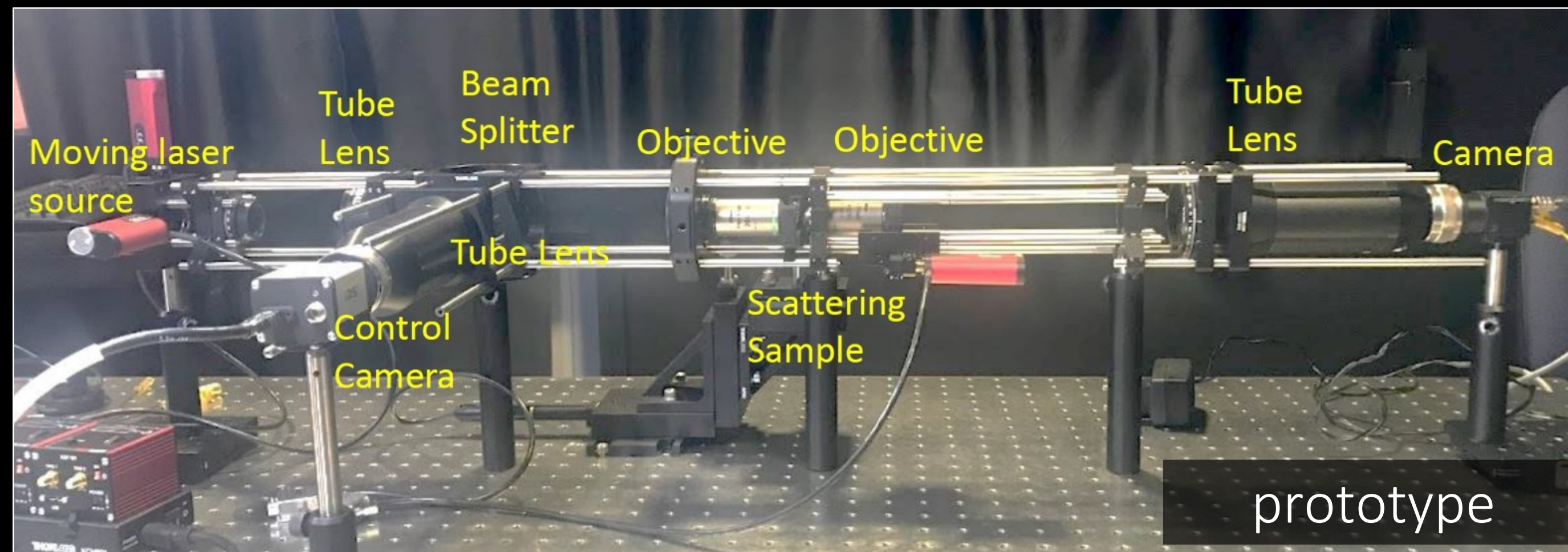
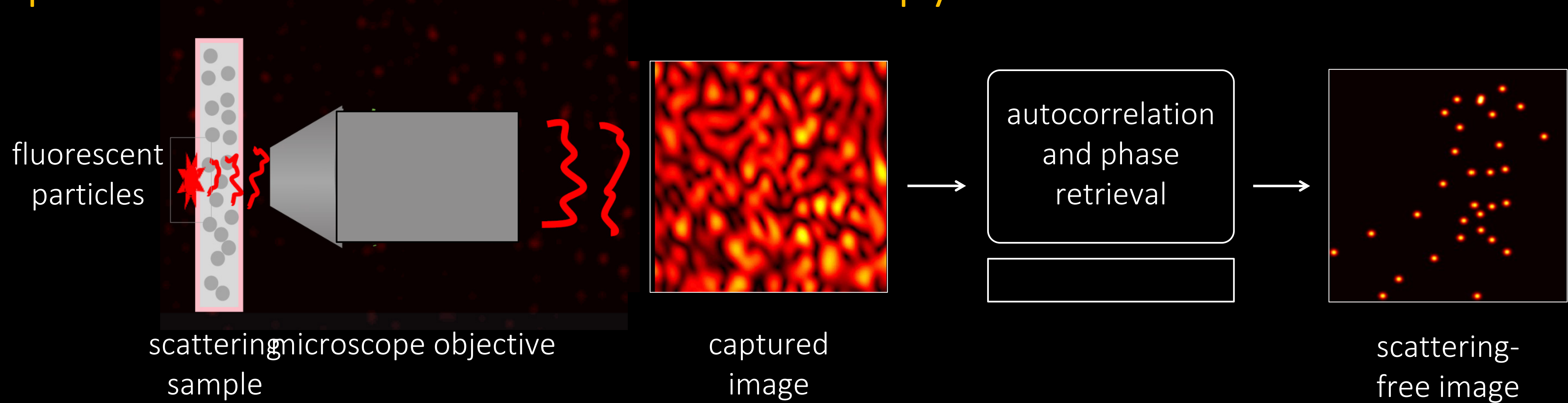
# Speckle-based fluorescence microscopy



[PIs: Gkioulekas, Levin]



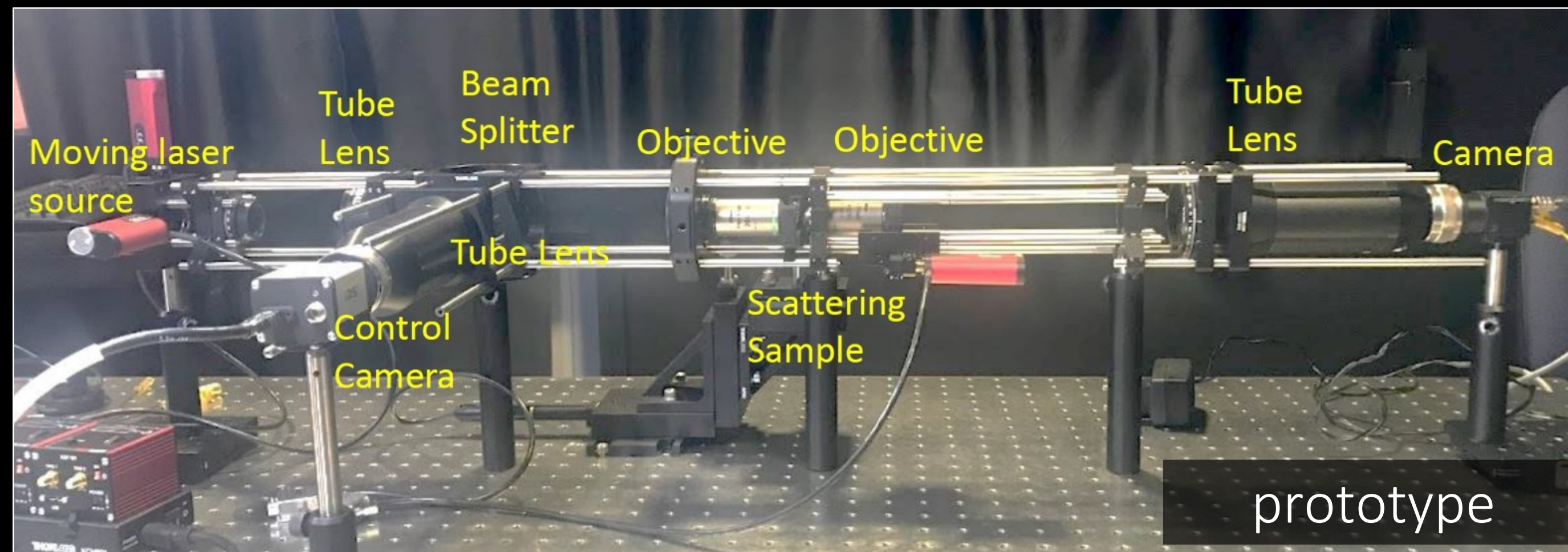
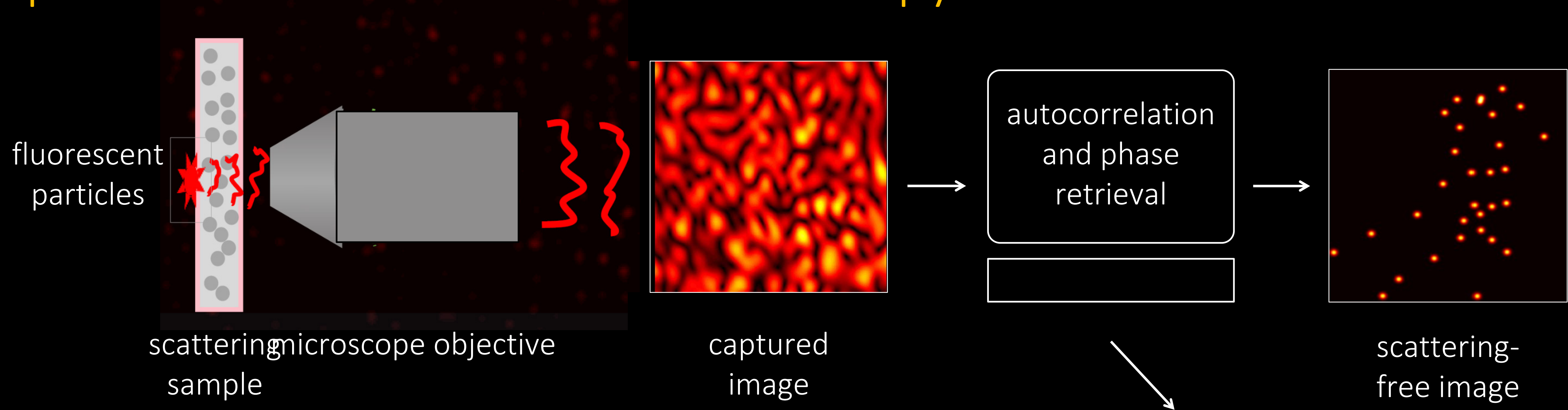
# Speckle-based fluorescence microscopy



[PIs: Gkioulekas, Levin]



# Speckle-based fluorescence microscopy



Performance strongly depends on:

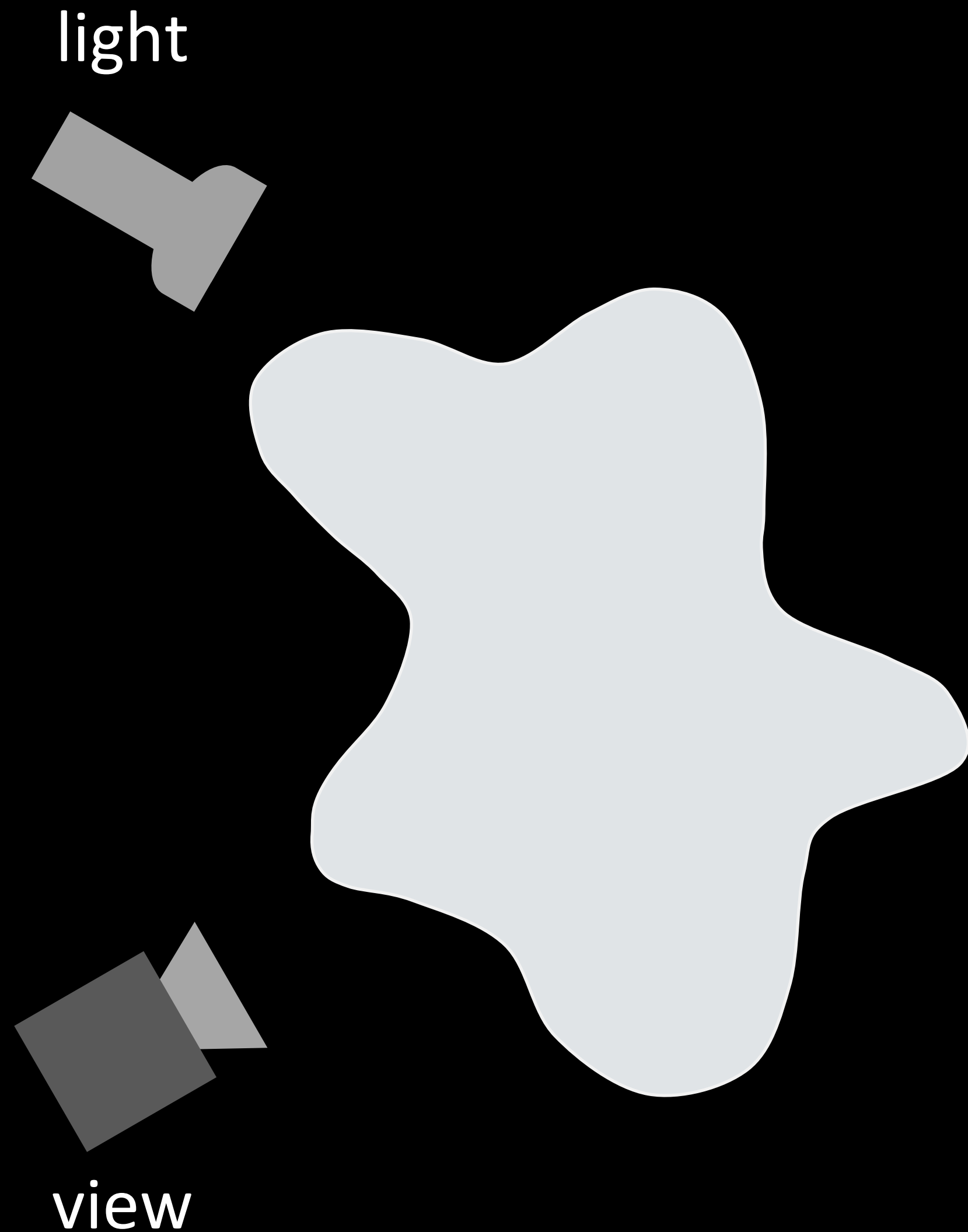
- speckle statistics
- image priors
- tissue parameters

[PIs: Gkioulekas, Levin]



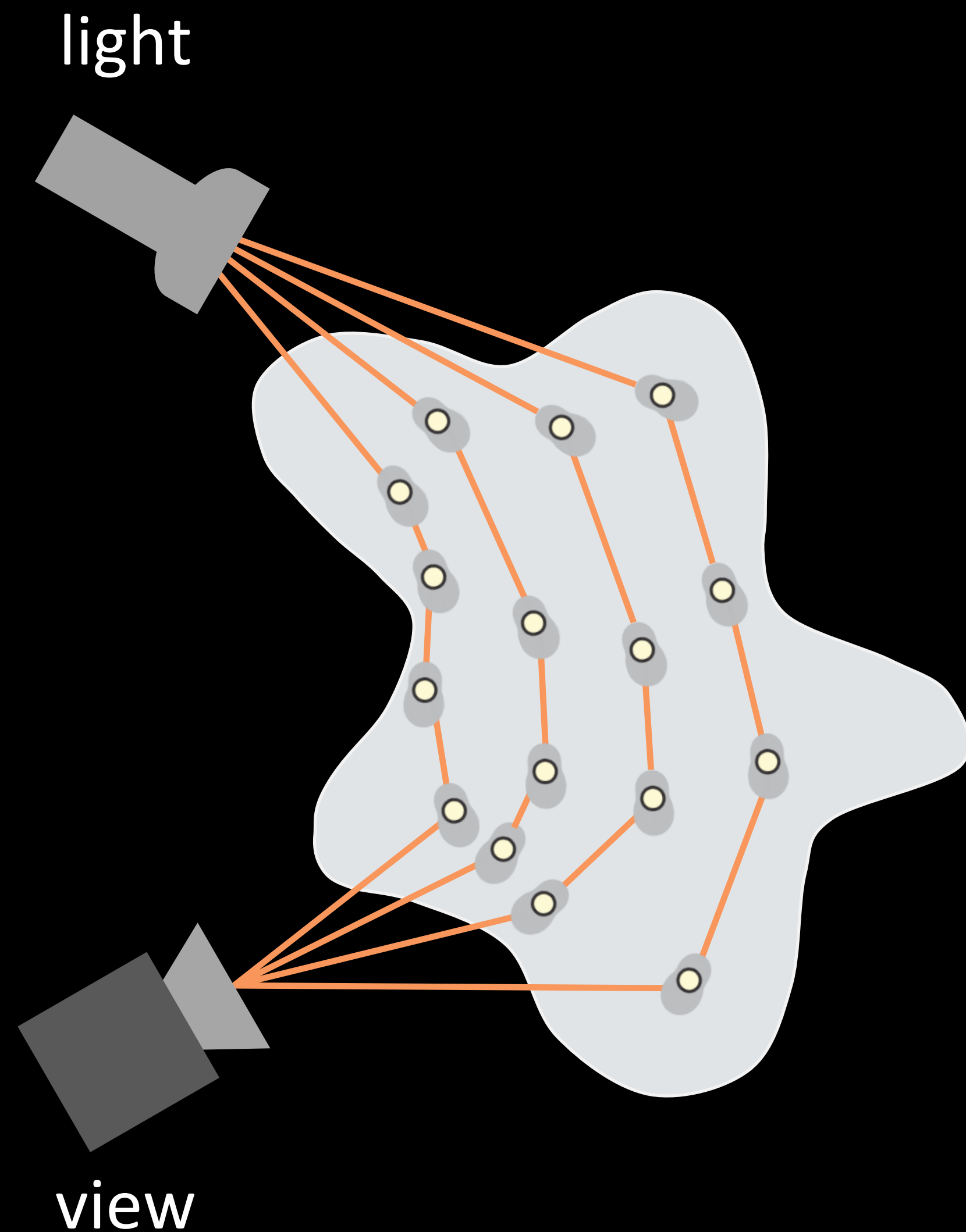
# Recap: Monte Carlo (volumetric) rendering

$$\text{Image} = \int_{\text{paths}} f(\text{path})$$



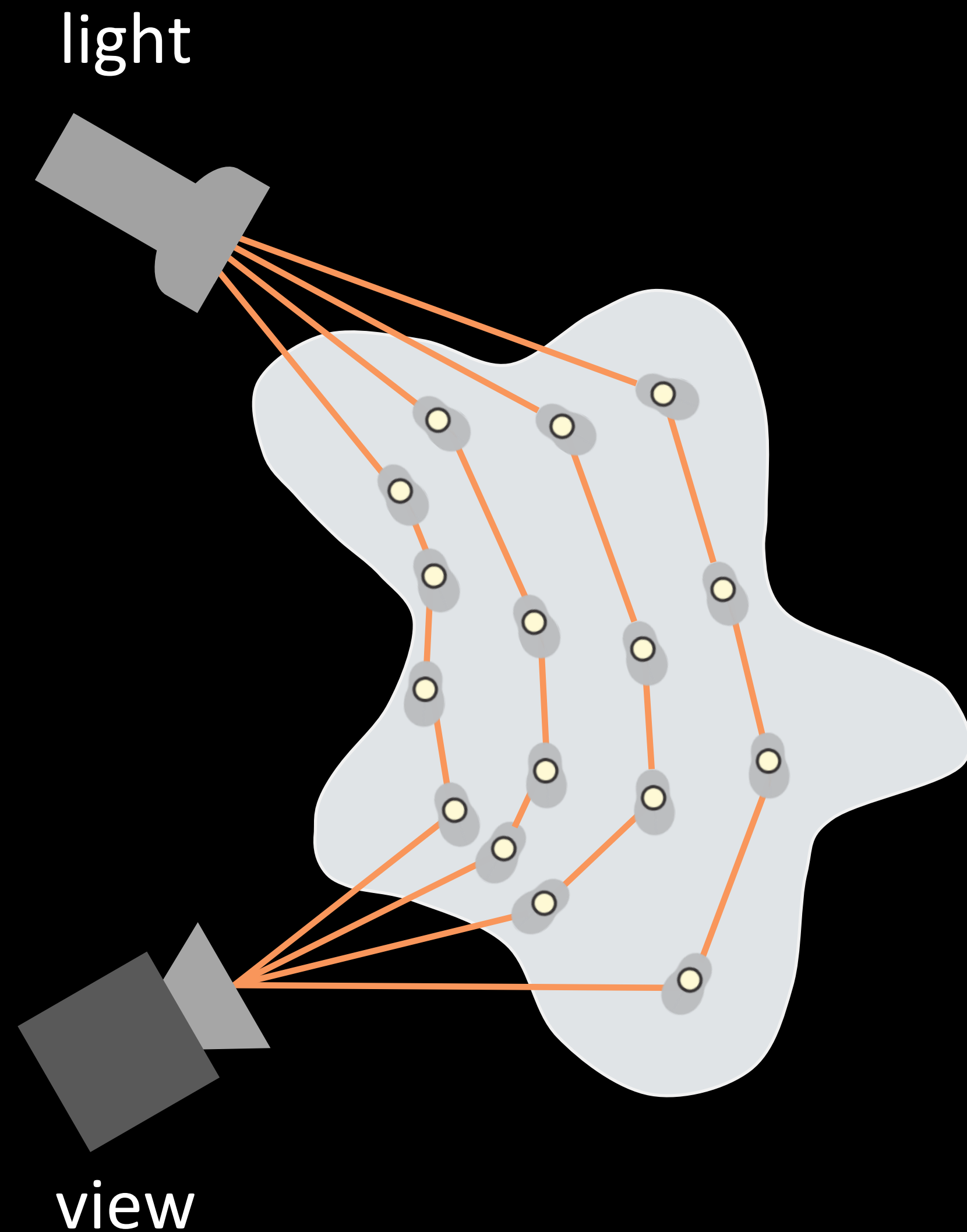
# Recap: Monte Carlo (volumetric) rendering

$$\text{Image} = \int_{\text{paths}} f(\text{path})$$





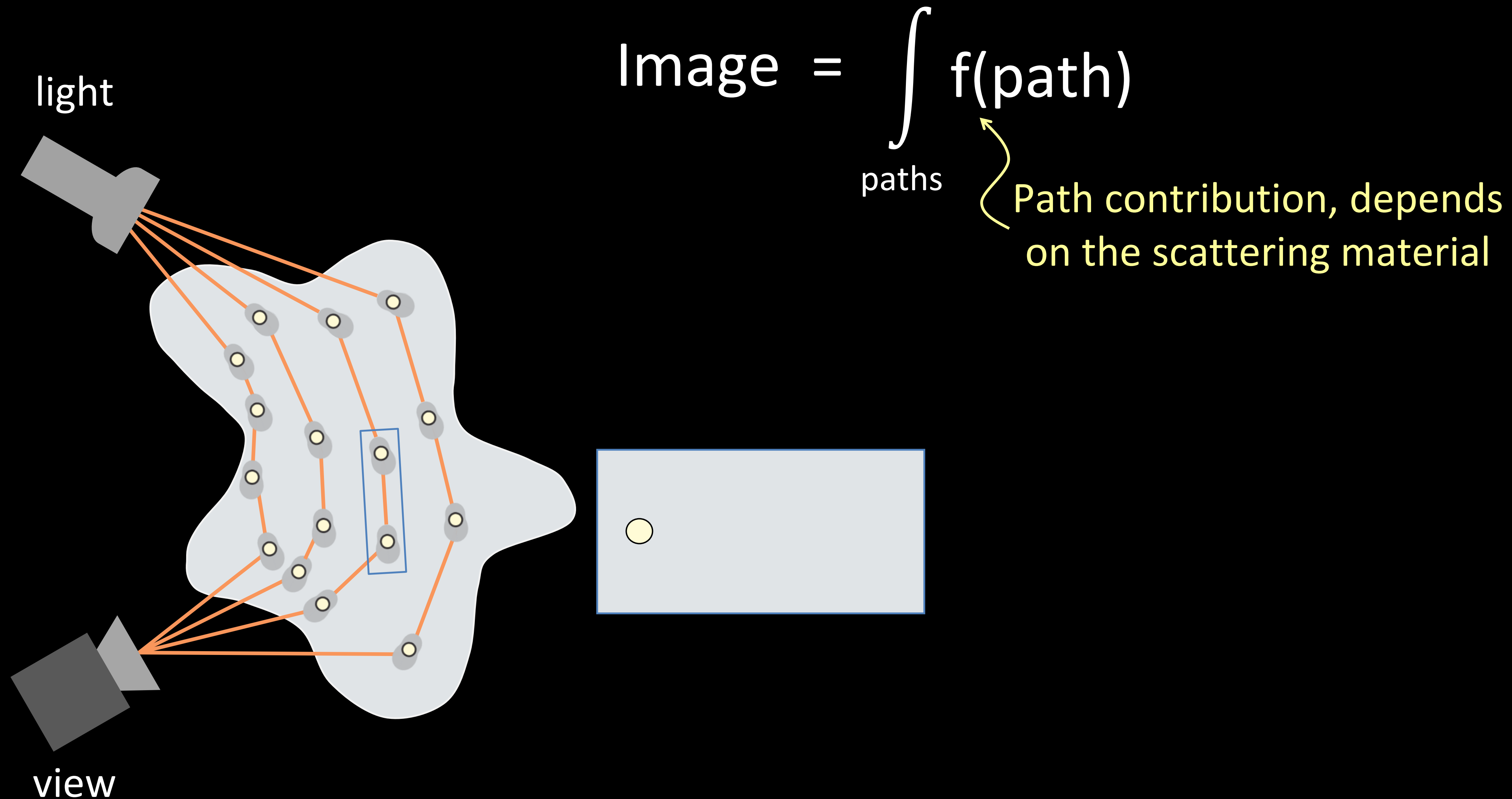
# Recap: Monte Carlo (volumetric) rendering



$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

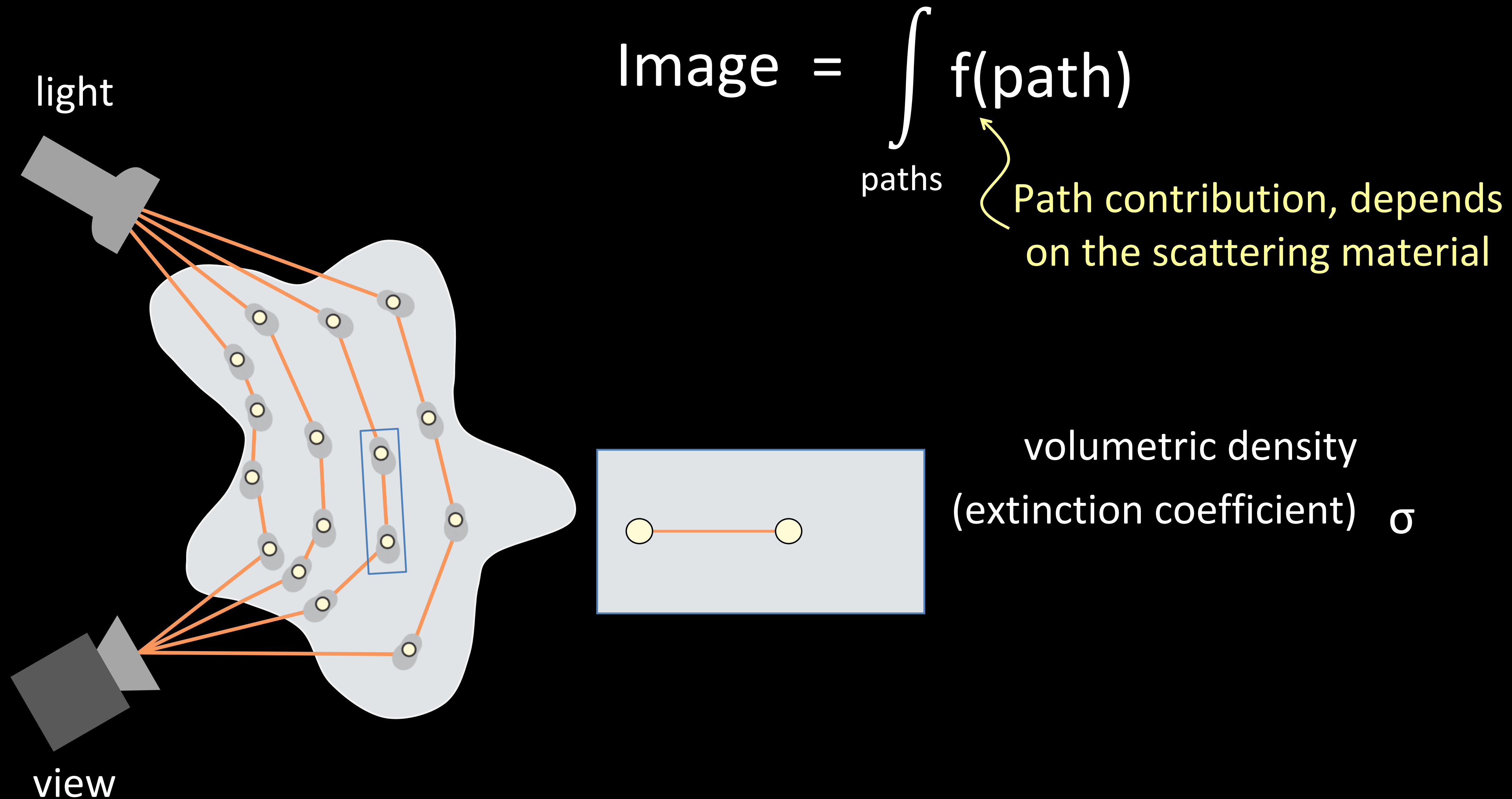
Path contribution, depends  
on the scattering material

# Recap: Monte Carlo (volumetric) rendering

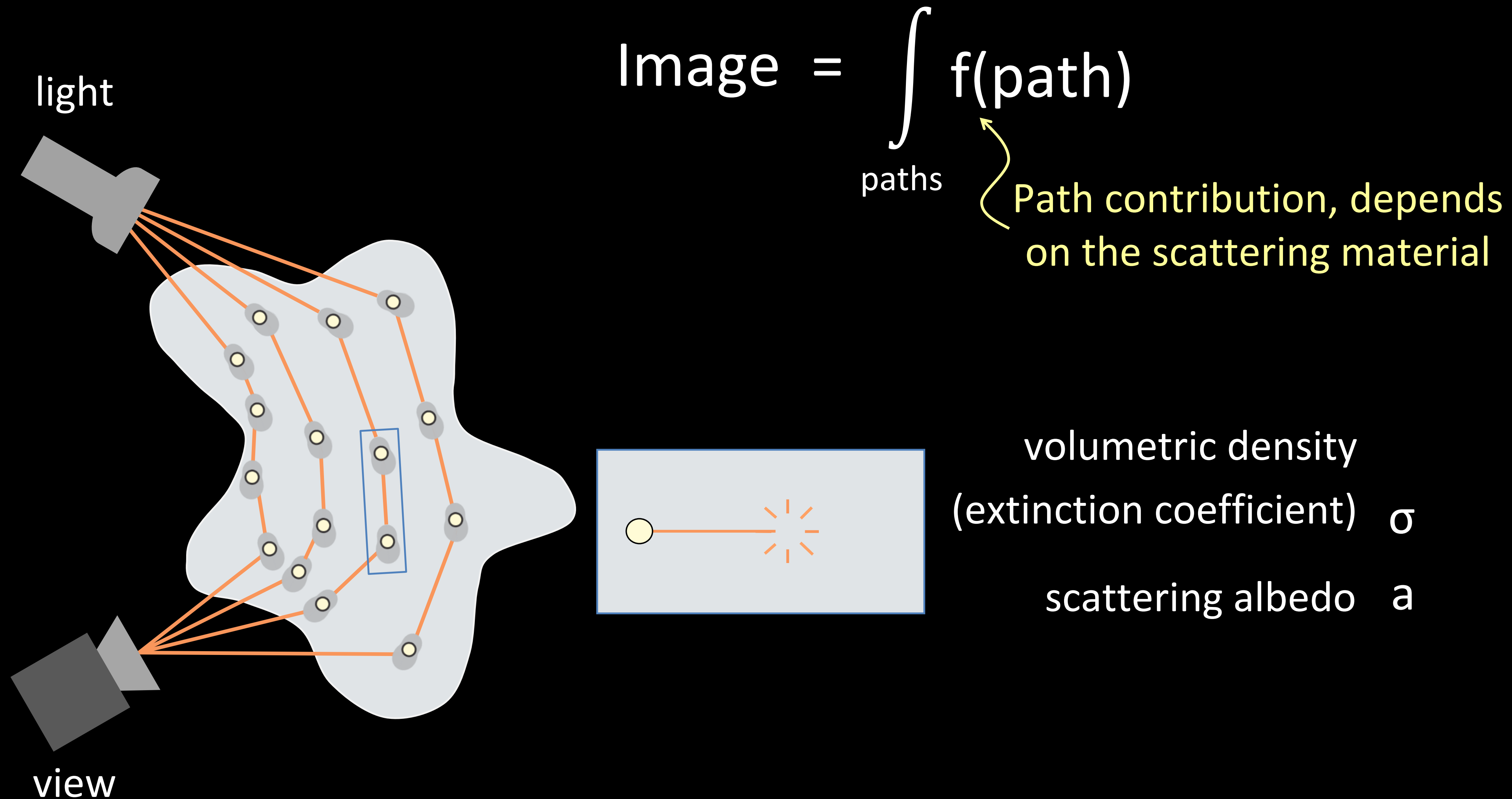




# Recap: Monte Carlo (volumetric) rendering

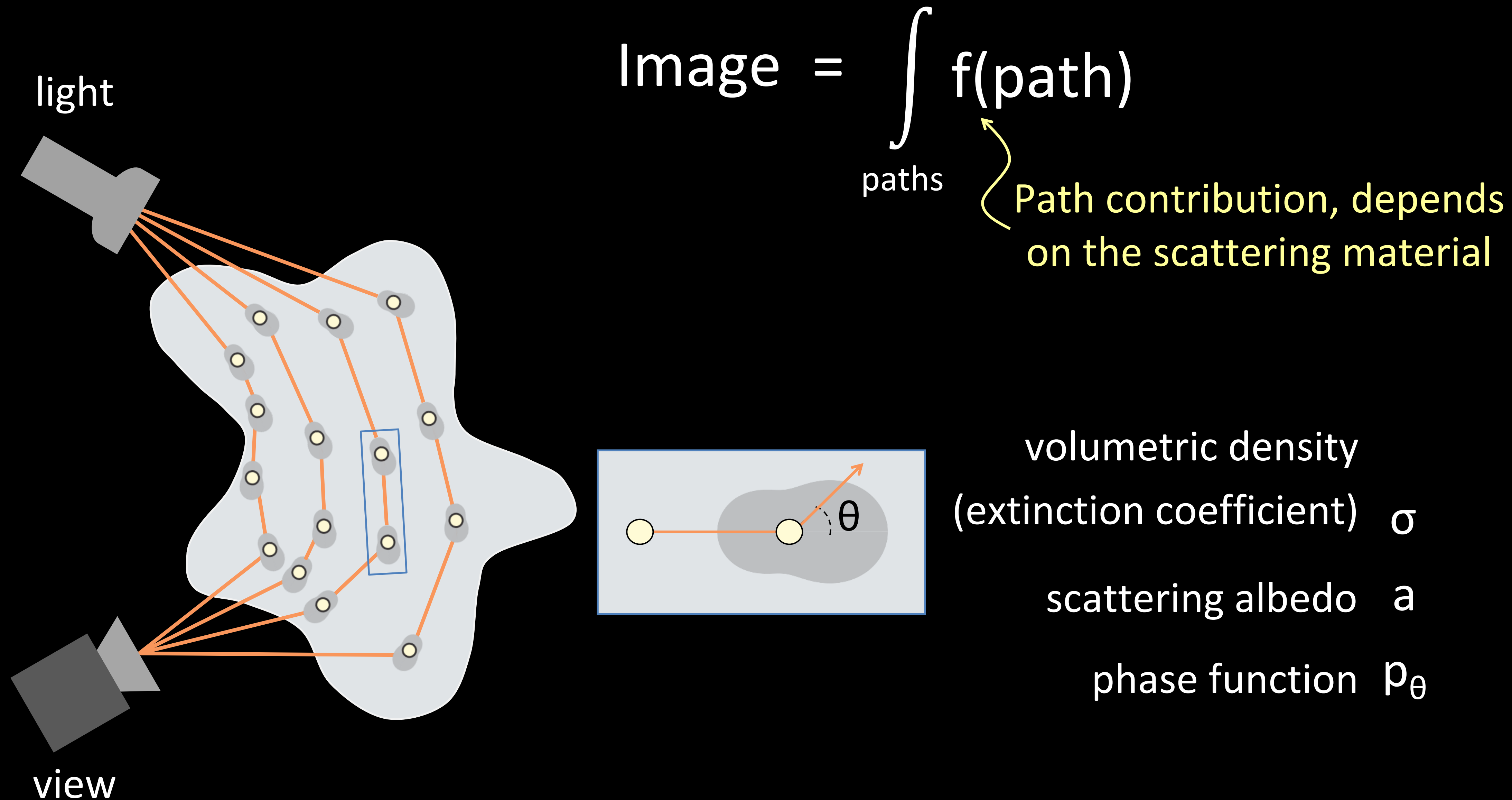


# Recap: Monte Carlo (volumetric) rendering

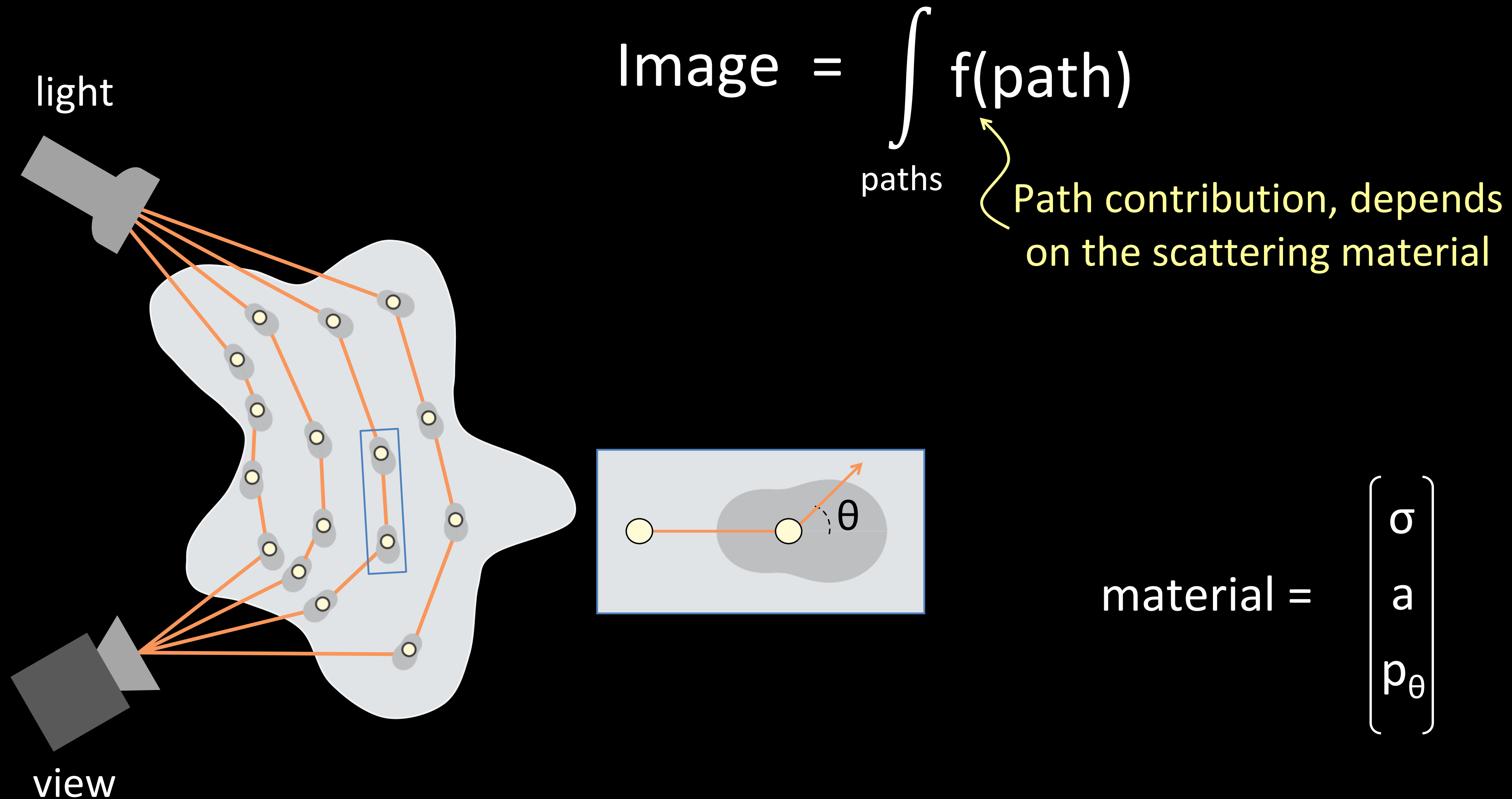




# Recap: Monte Carlo (volumetric) rendering

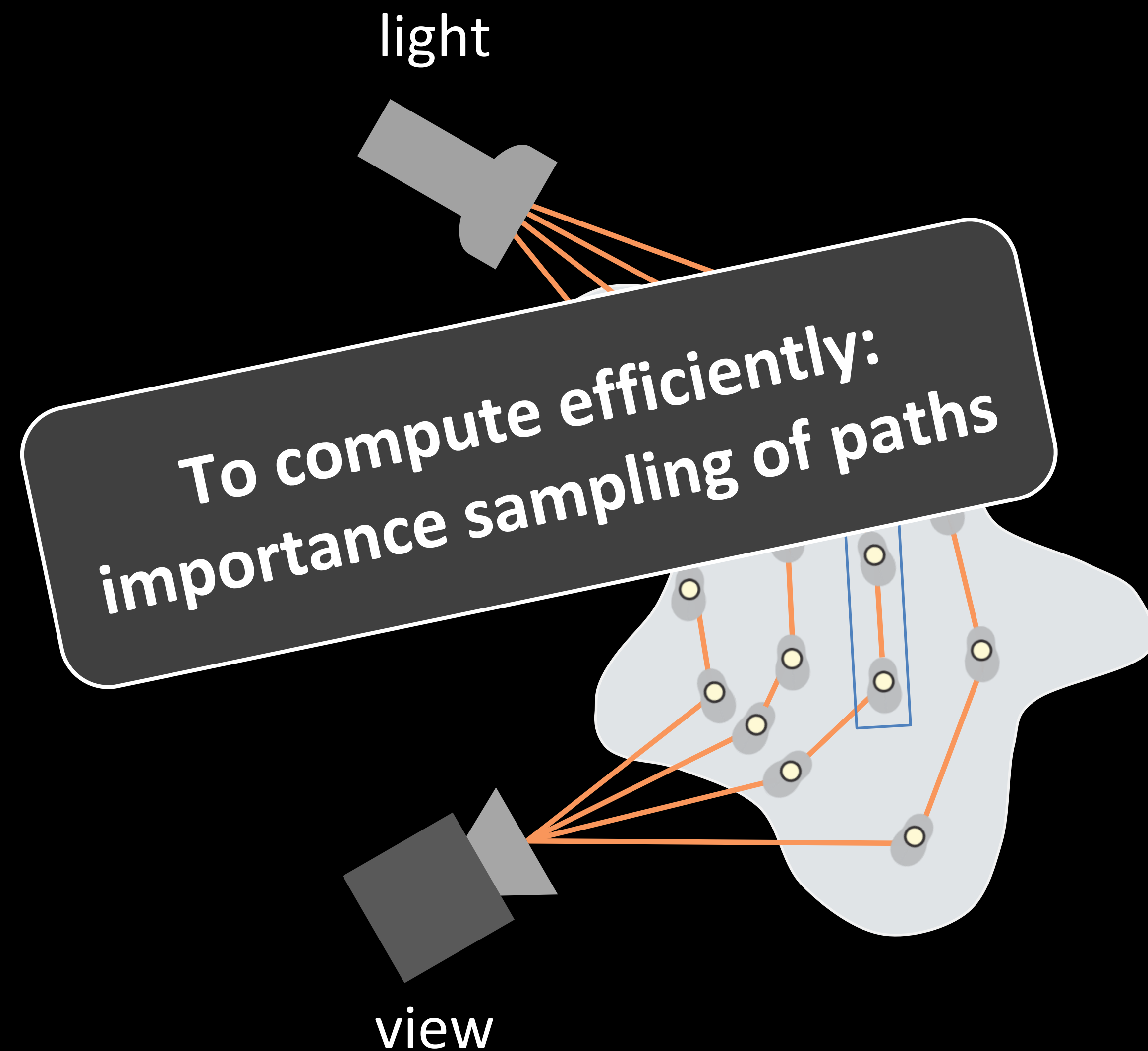


# Recap: Monte Carlo (volumetric) rendering



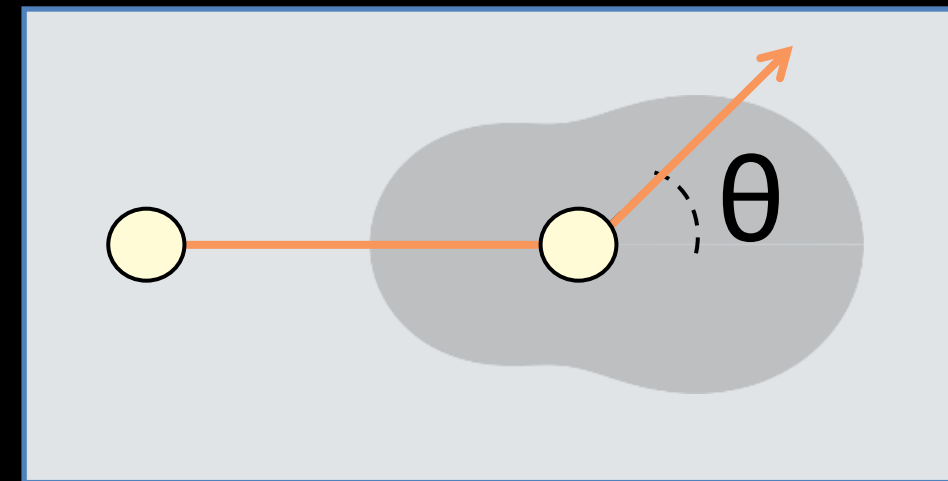


# Recap: Monte Carlo (volumetric) rendering



$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

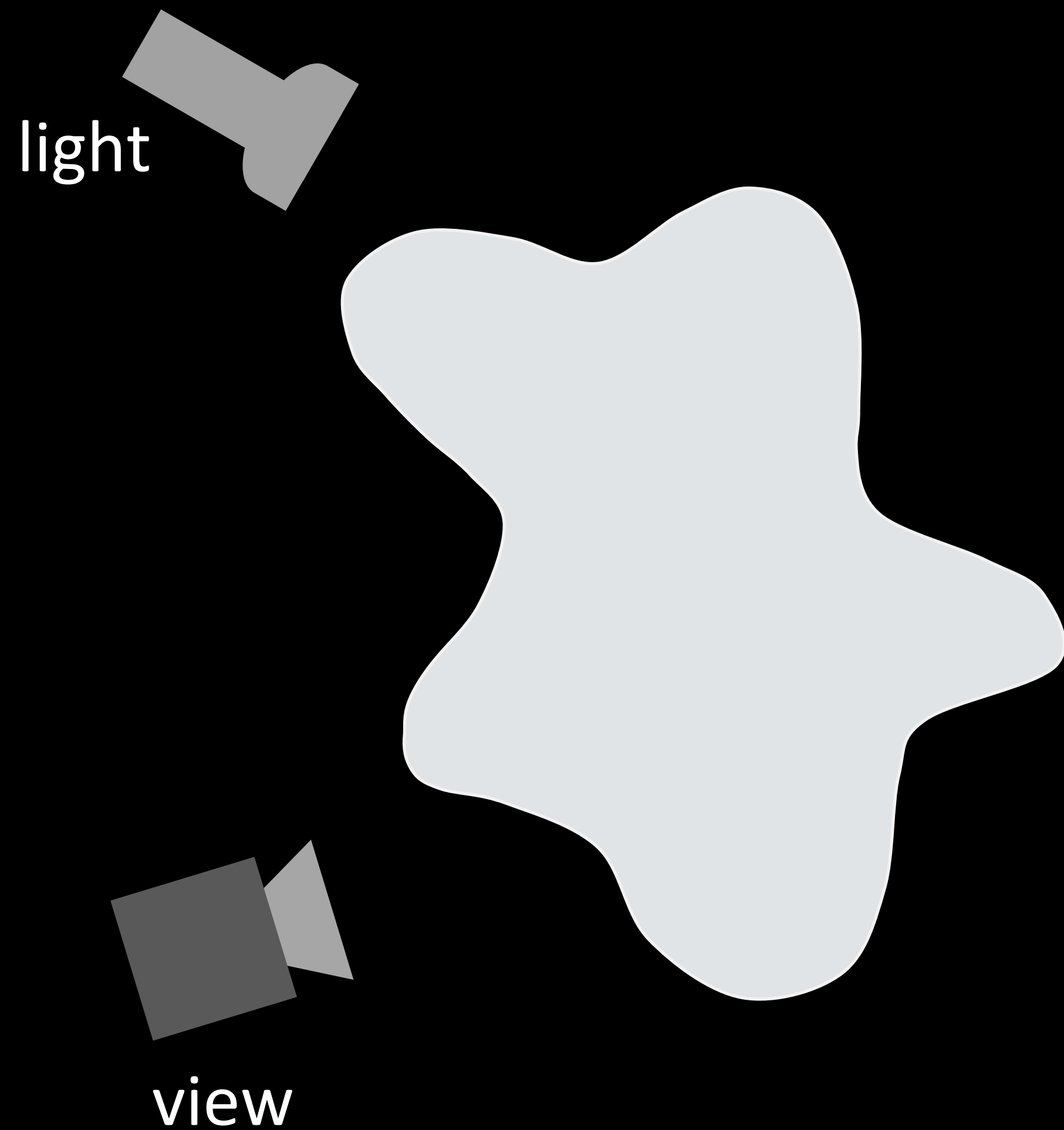
Path contribution, depends on the scattering material



$$\text{material} = \begin{bmatrix} \sigma \\ a \\ p_{\theta} \end{bmatrix}$$

# Memory effect: simulate *covariance* of speckle images

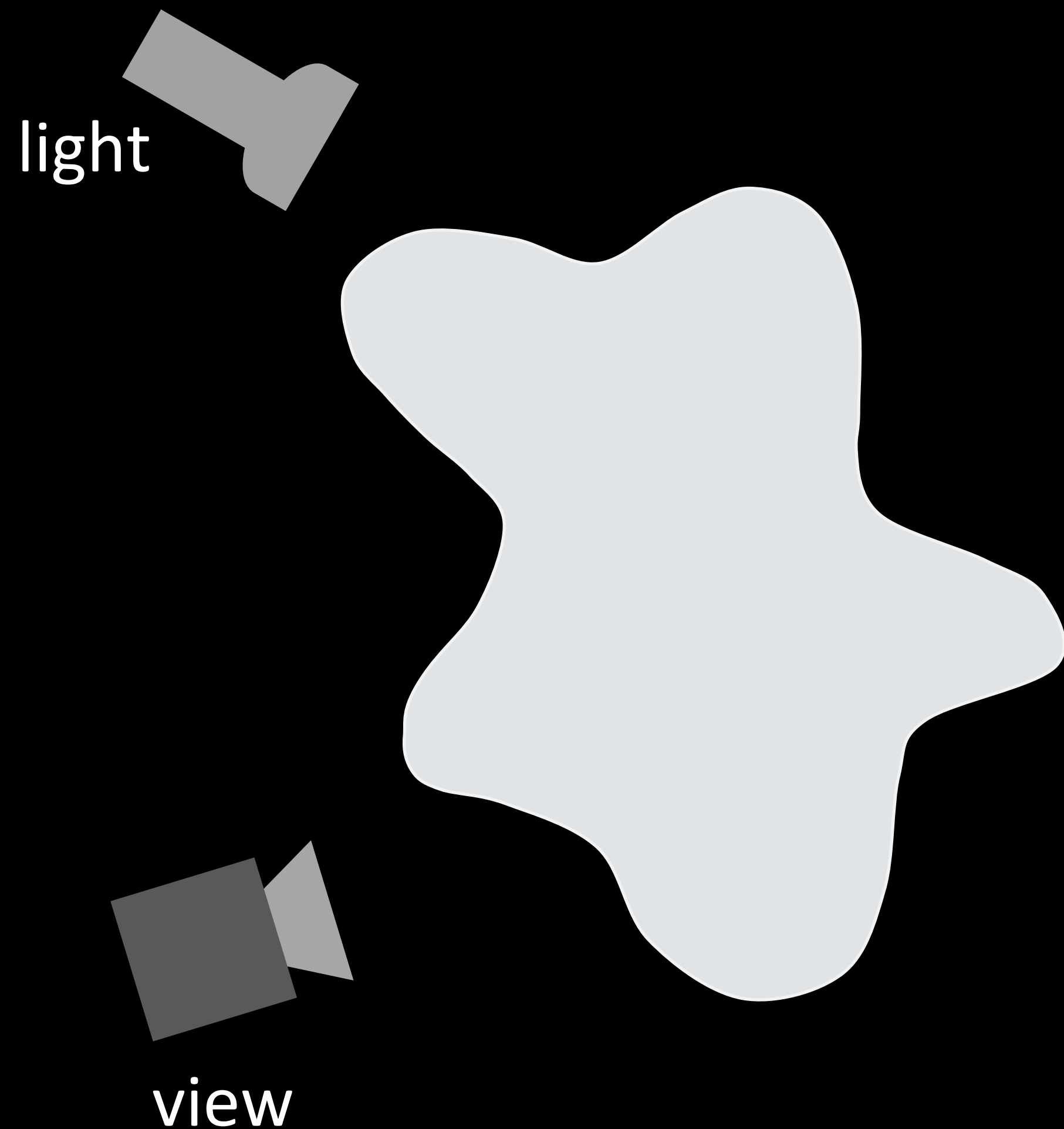
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$





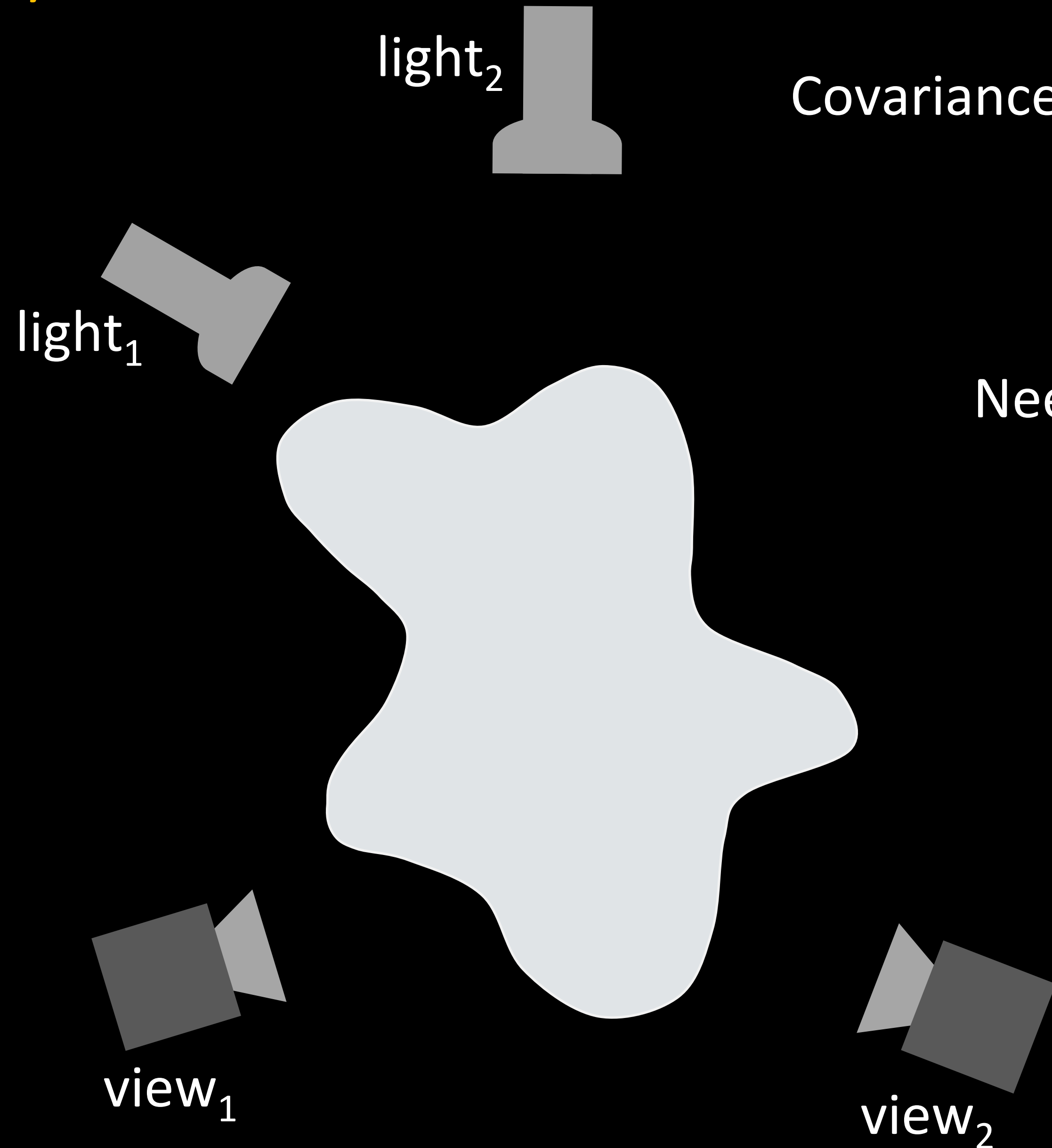
# Memory effect: simulate *covariance* of speckle images

$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



Need to consider products of ***pairs*** of paths

# Memory effect: simulate *covariance* of speckle images

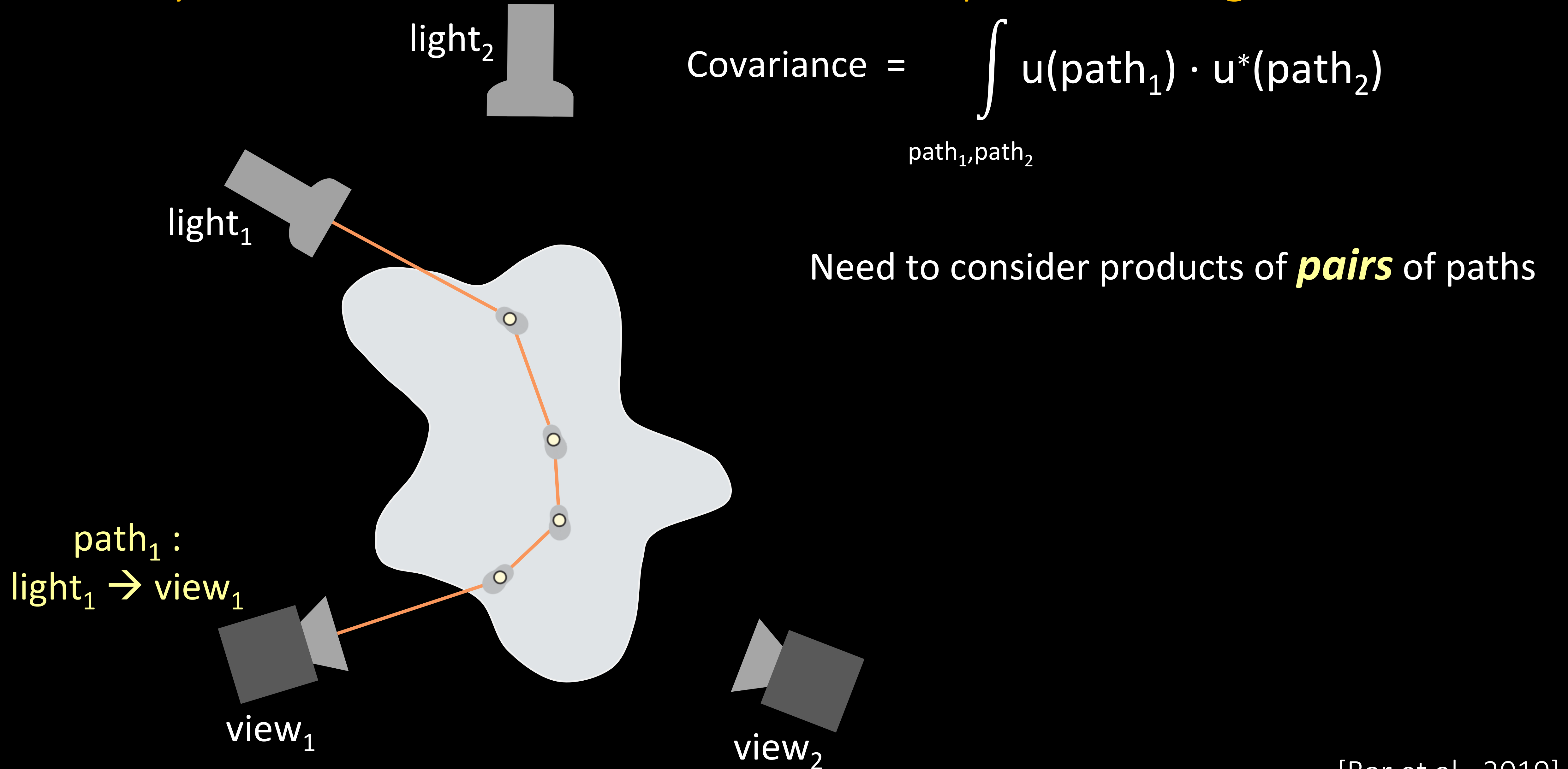


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

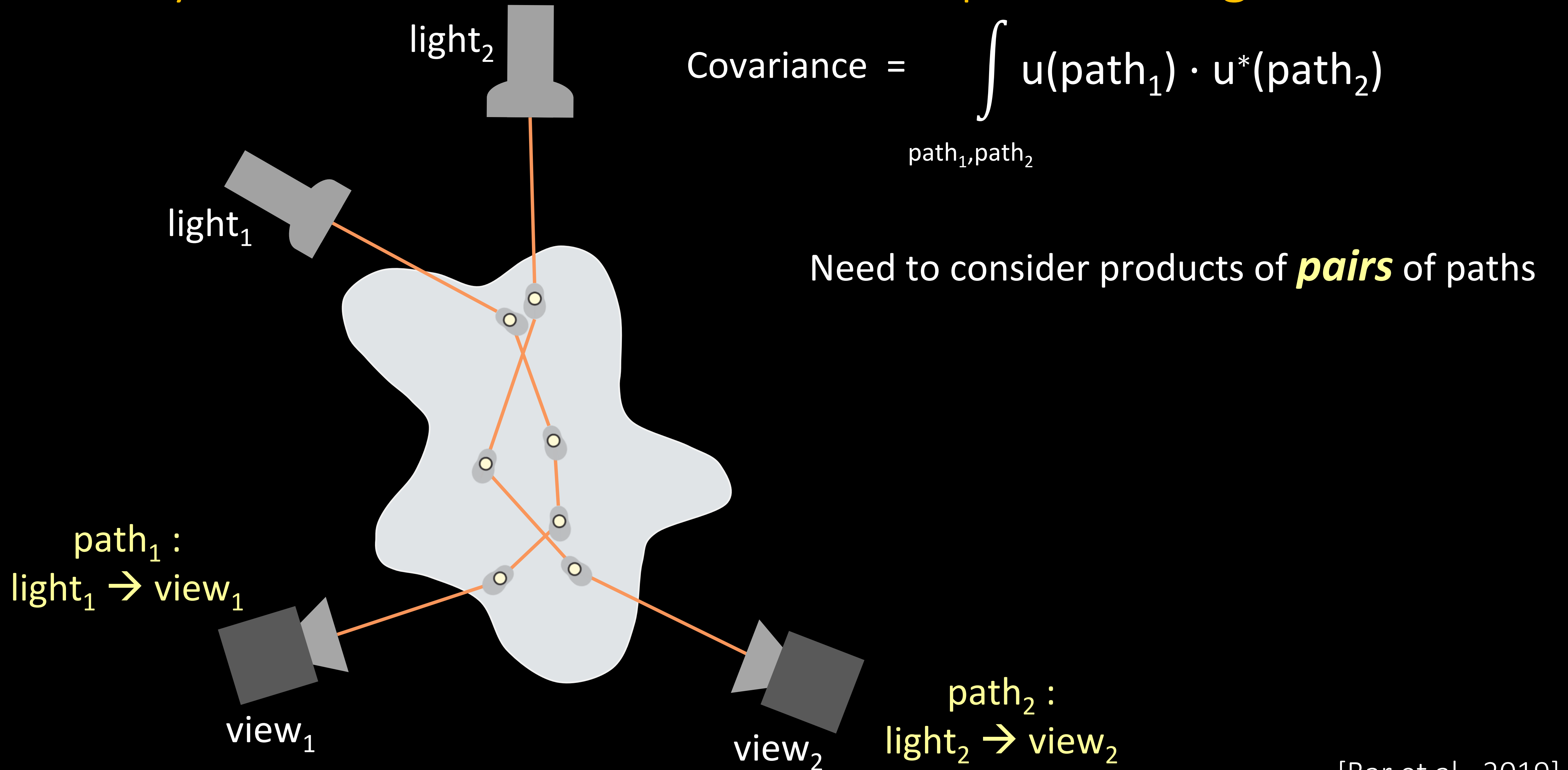
Need to consider products of *pairs* of paths



# Memory effect: simulate *covariance* of speckle images

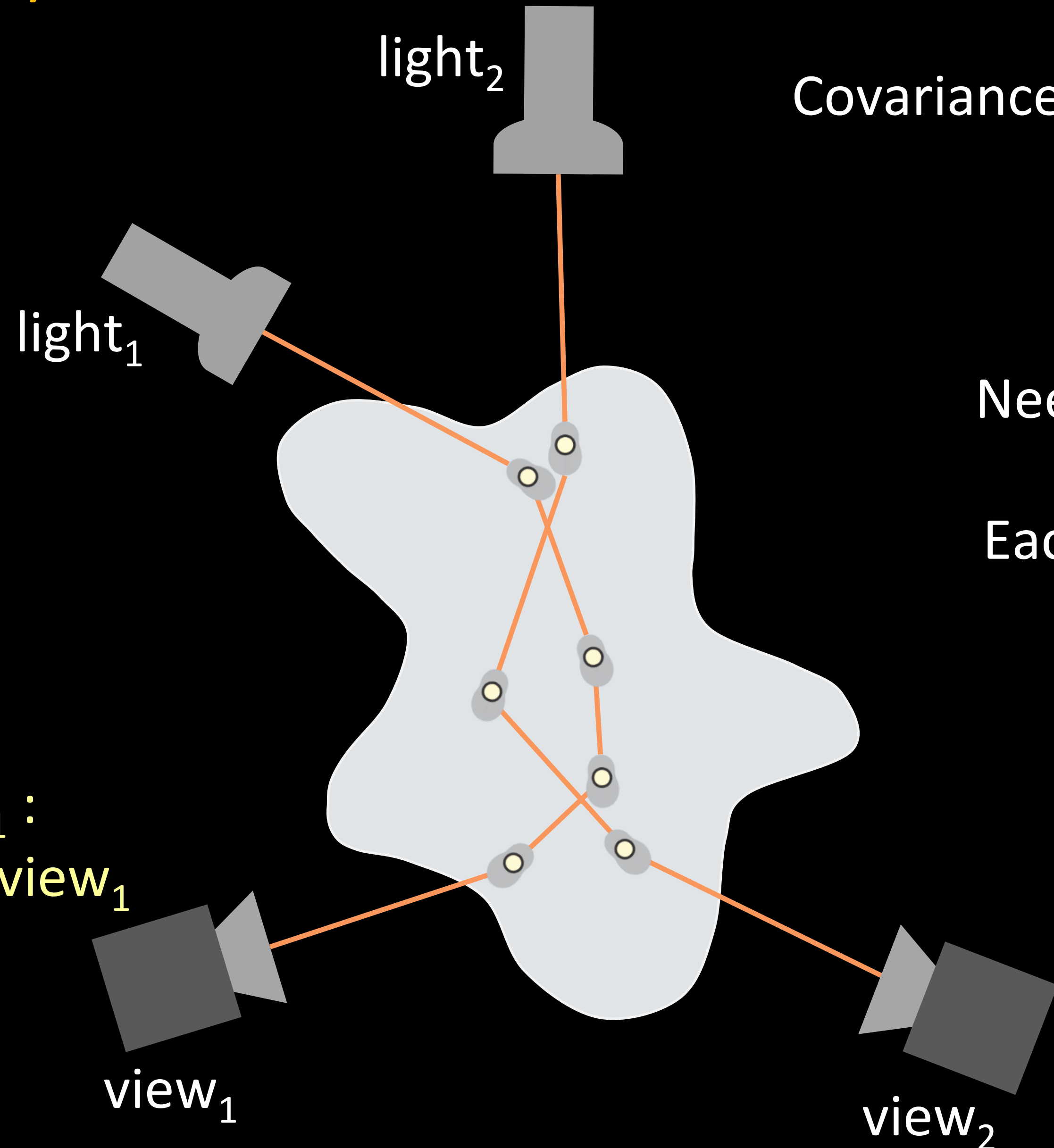


# Memory effect: simulate *covariance* of speckle images





# Memory effect: simulate *covariance* of speckle images



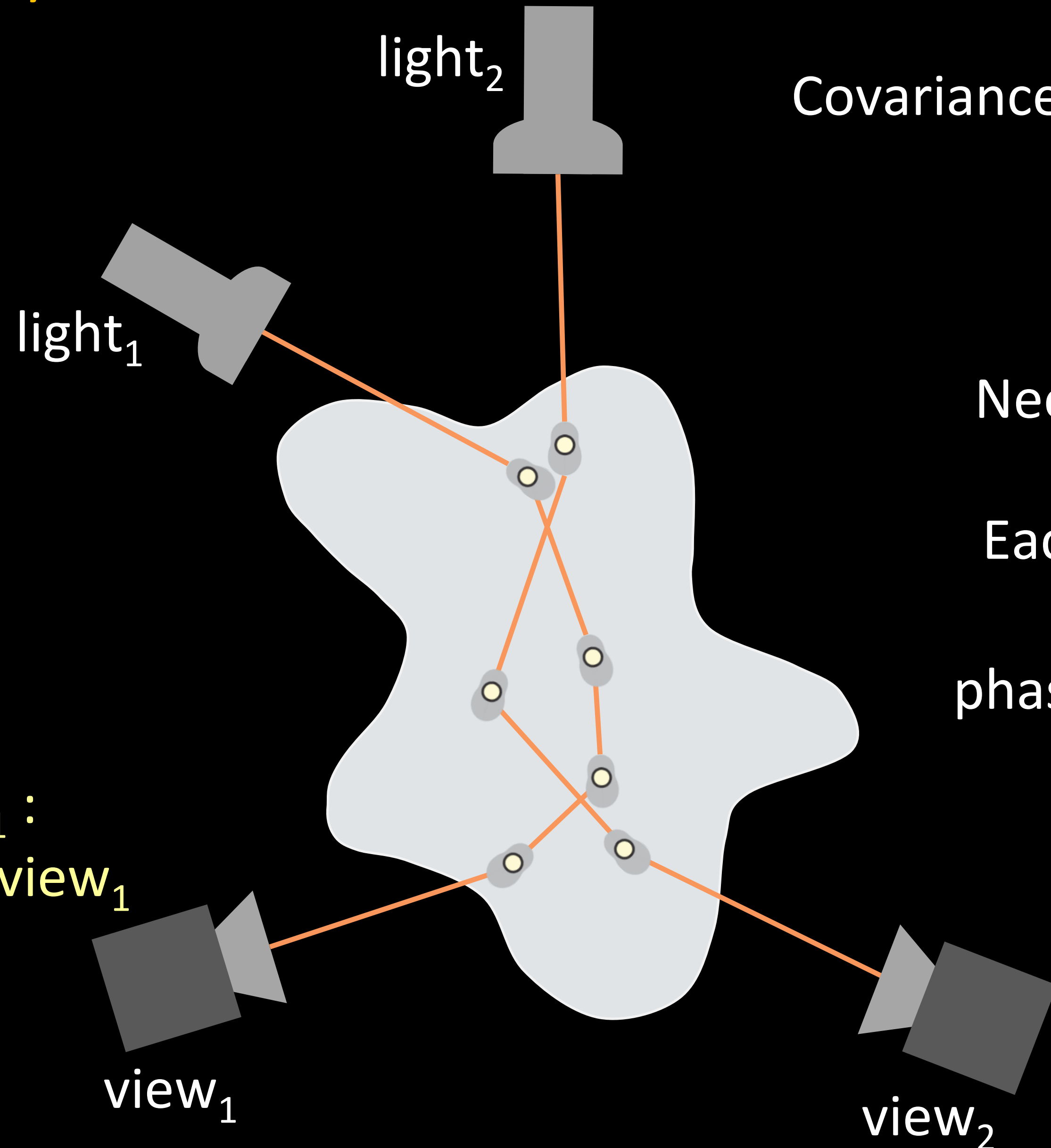
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

$$u = |u| e^{i \cdot \text{phase}}$$

Need to consider products of **pairs** of paths

Each path contributes a complex number  $u$

# Memory effect: simulate *covariance* of speckle images



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

$$u = |u| e^{i \cdot \text{phase}}$$

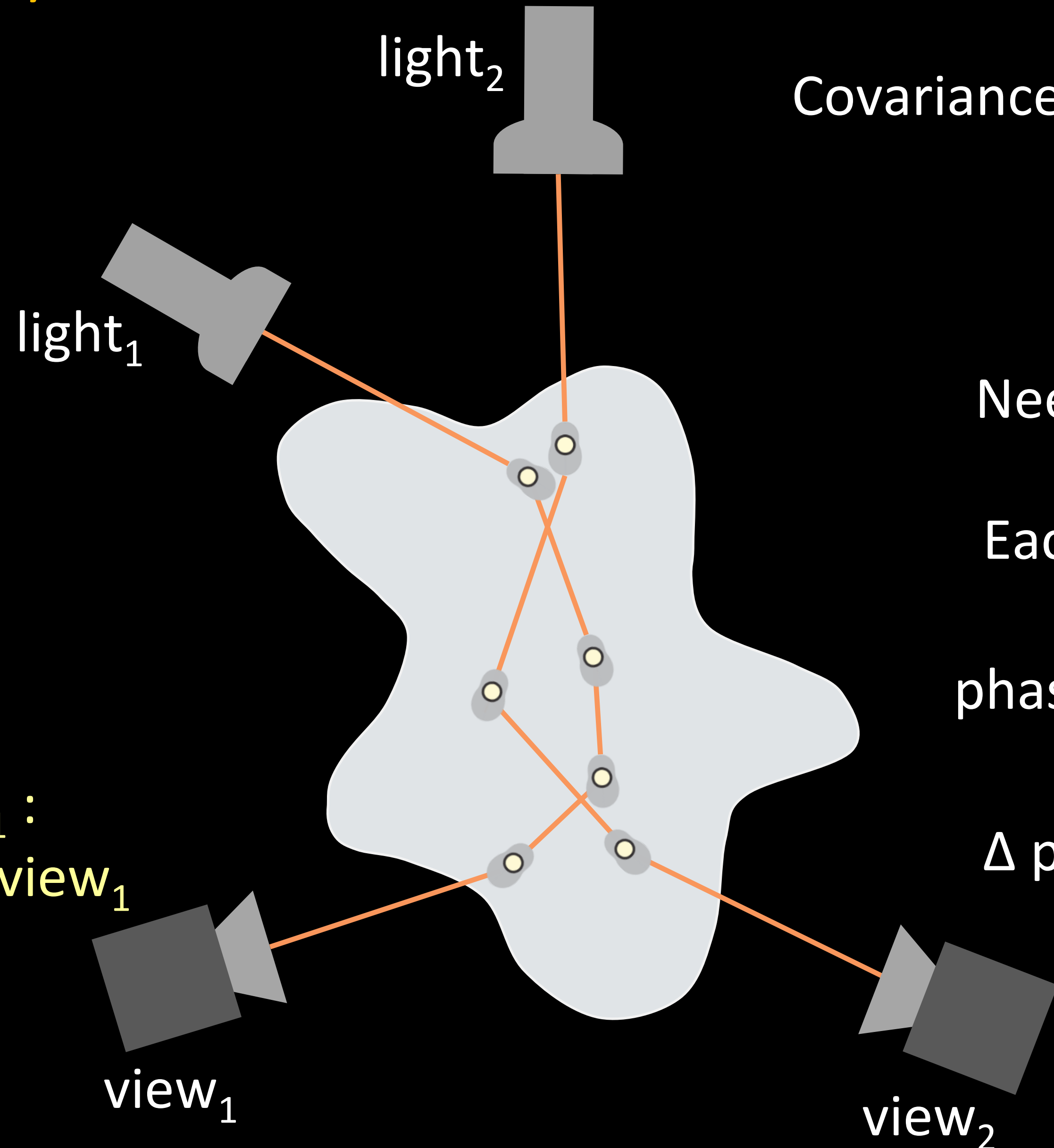
Need to consider products of ***pairs*** of paths

Each path contributes a complex number  $u$

phase  $\propto$  Length ( path )



# Memory effect: simulate *covariance* of speckle images



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

$$u = |u| e^{i \cdot \text{phase}}$$

Need to consider products of **pairs** of paths

Each path contributes a complex number  $u$

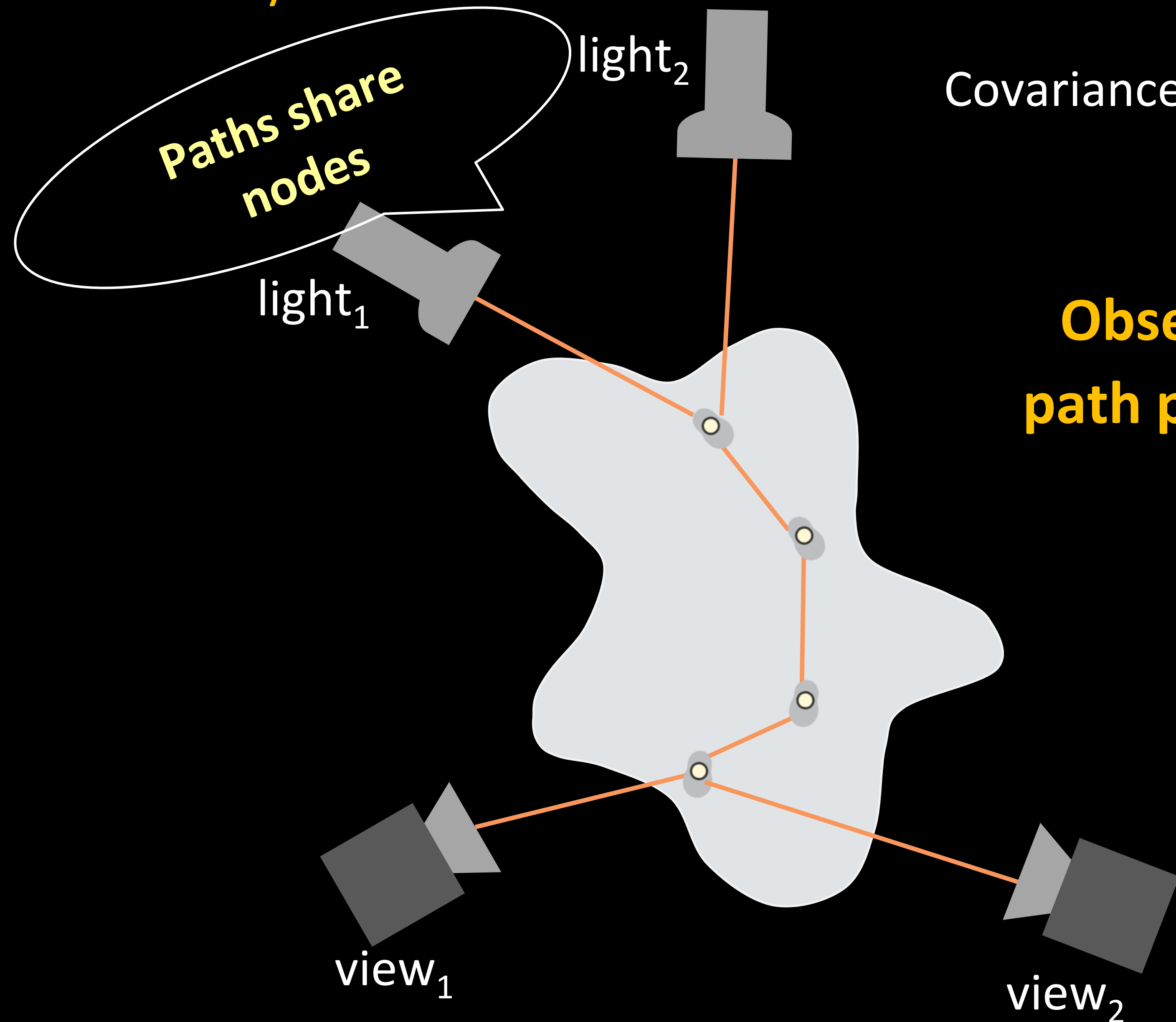
$\text{phase} \propto \text{Length ( path )}$

$\Delta \text{ phase} \propto \text{Length ( path}_1 \text{ )} - \text{Length ( path}_2 \text{ )}$

$\text{path}_1 :$   
 $\text{light}_1 \rightarrow \text{view}_1$

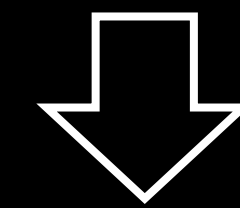
$\text{path}_2 :$   
 $\text{light}_2 \rightarrow \text{view}_2$

# Memory effect: simulate *covariance* of speckle images



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

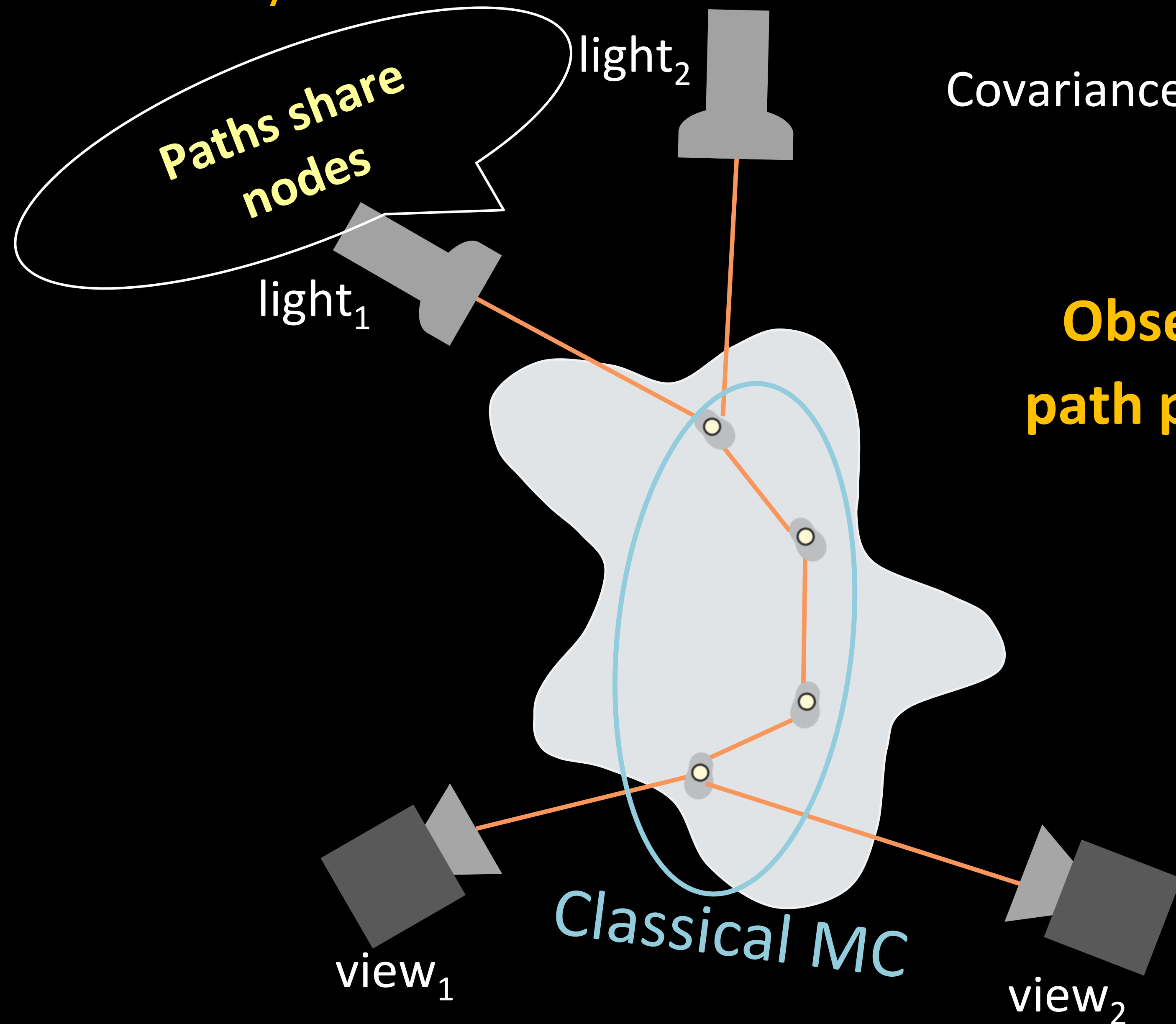
**Observation: need to consider only path pairs that share the *same* nodes (except start and end)**



**All other path pairs are averaged out in the integration**

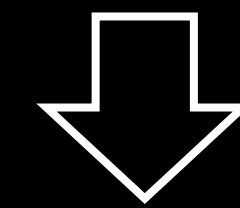


# Memory effect: simulate *covariance* of speckle images



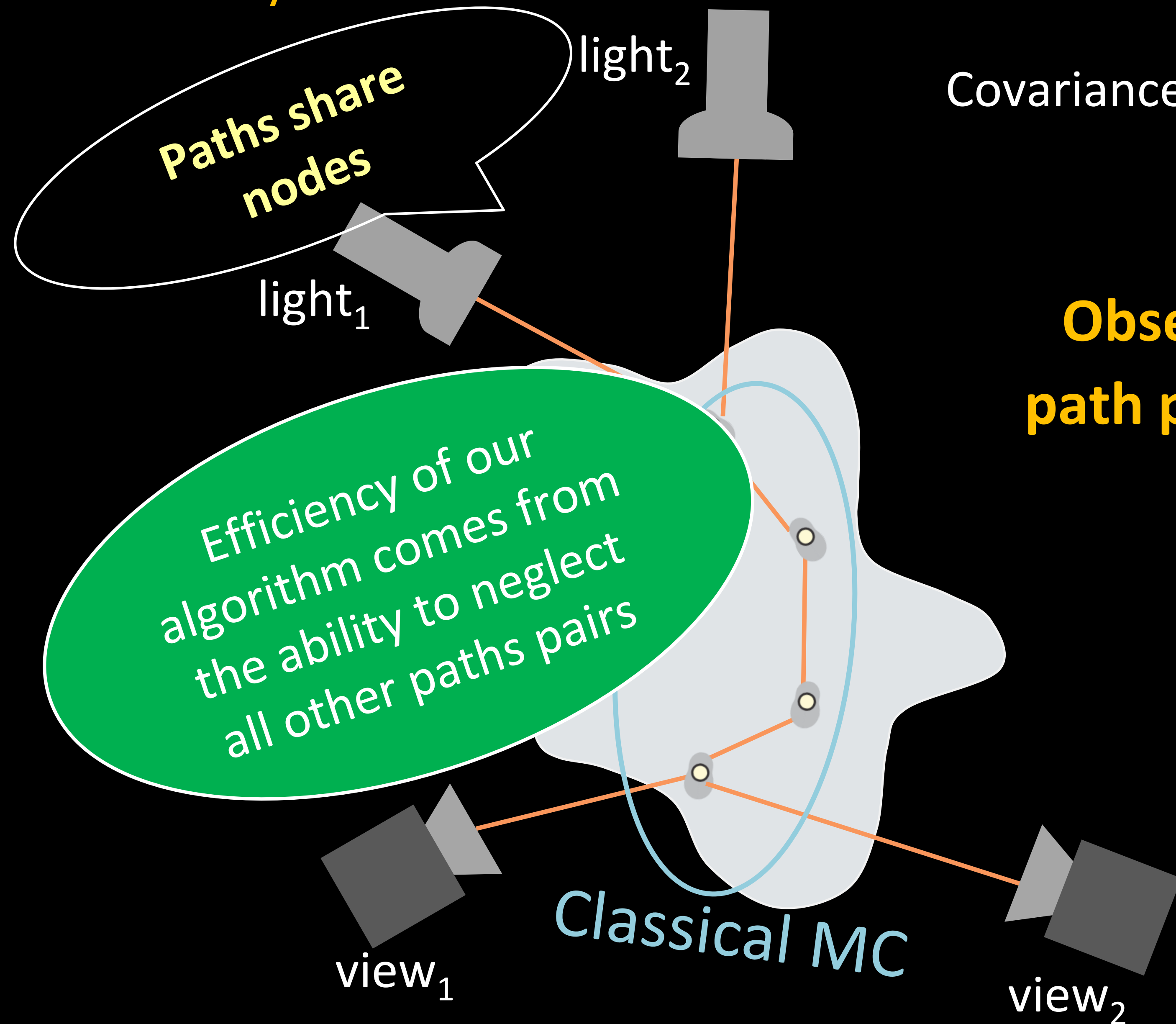
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

**Observation: need to consider only path pairs that share the *same* nodes (except start and end)**



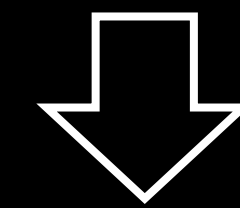
**All other path pairs are averaged out in the integration**

# Memory effect: simulate *covariance* of speckle images



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

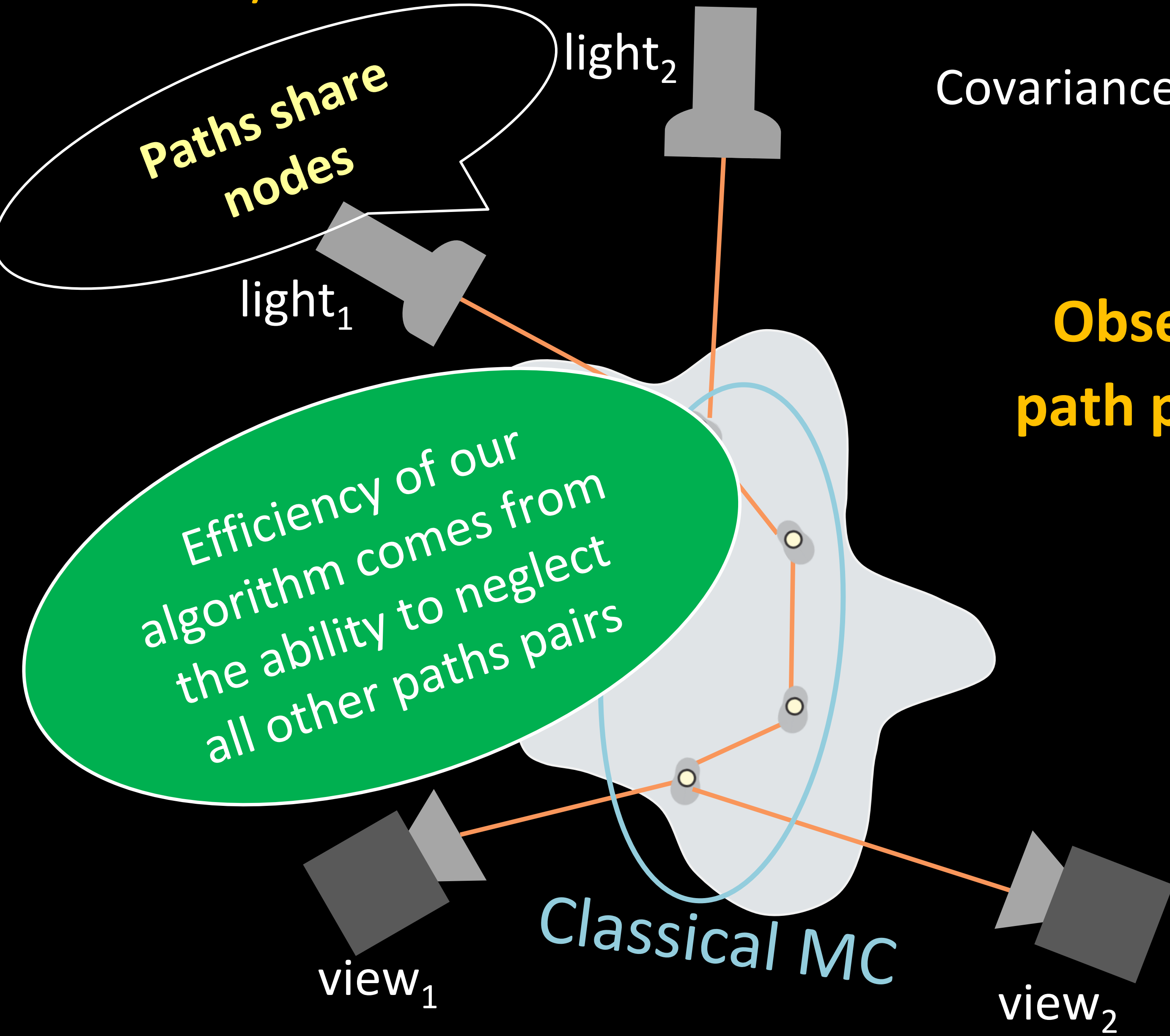
**Observation: need to consider only path pairs that share the *same* nodes (except start and end)**



**All other path pairs are averaged out in the integration**

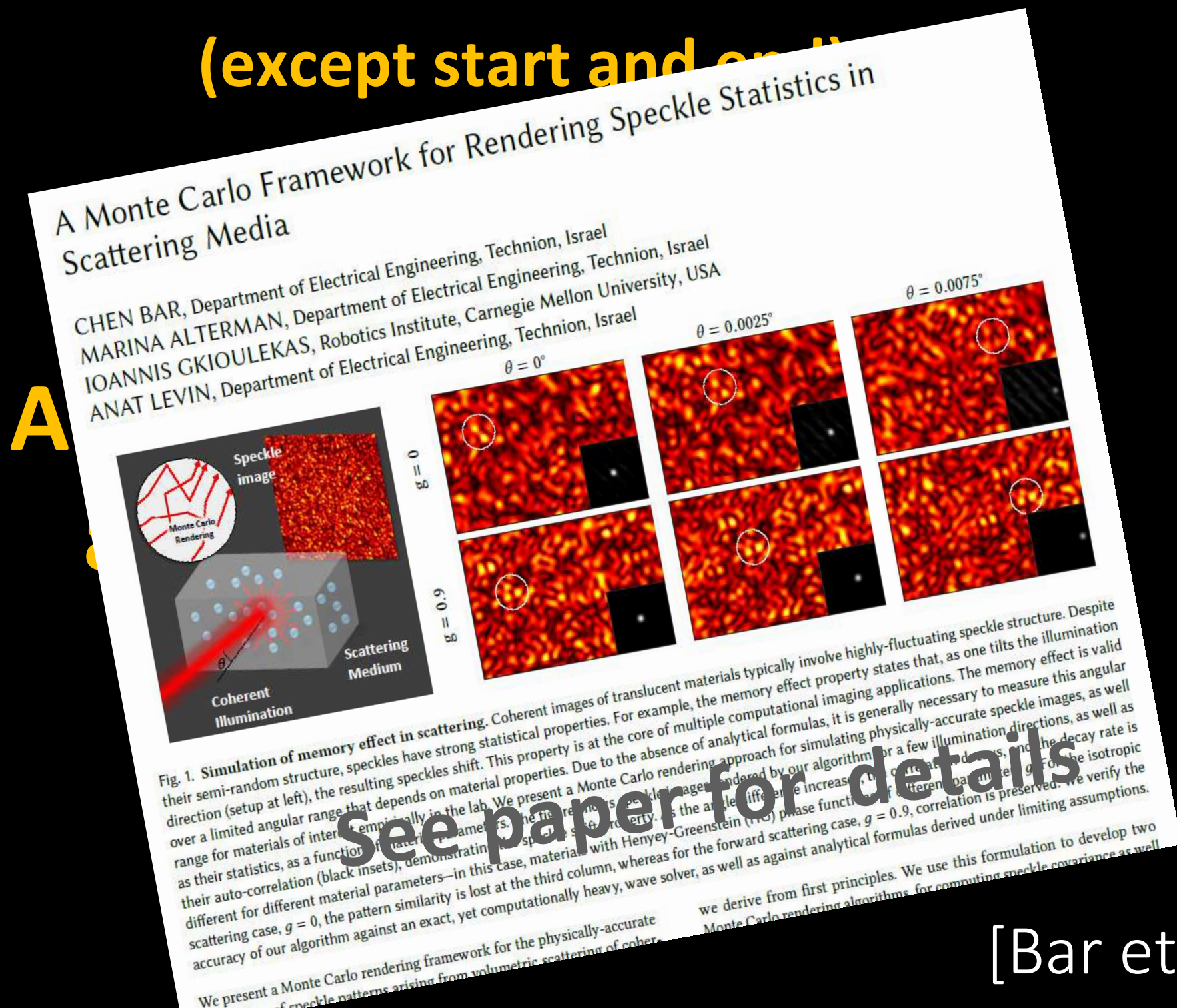


# Memory effect: simulate *covariance* of speckle images



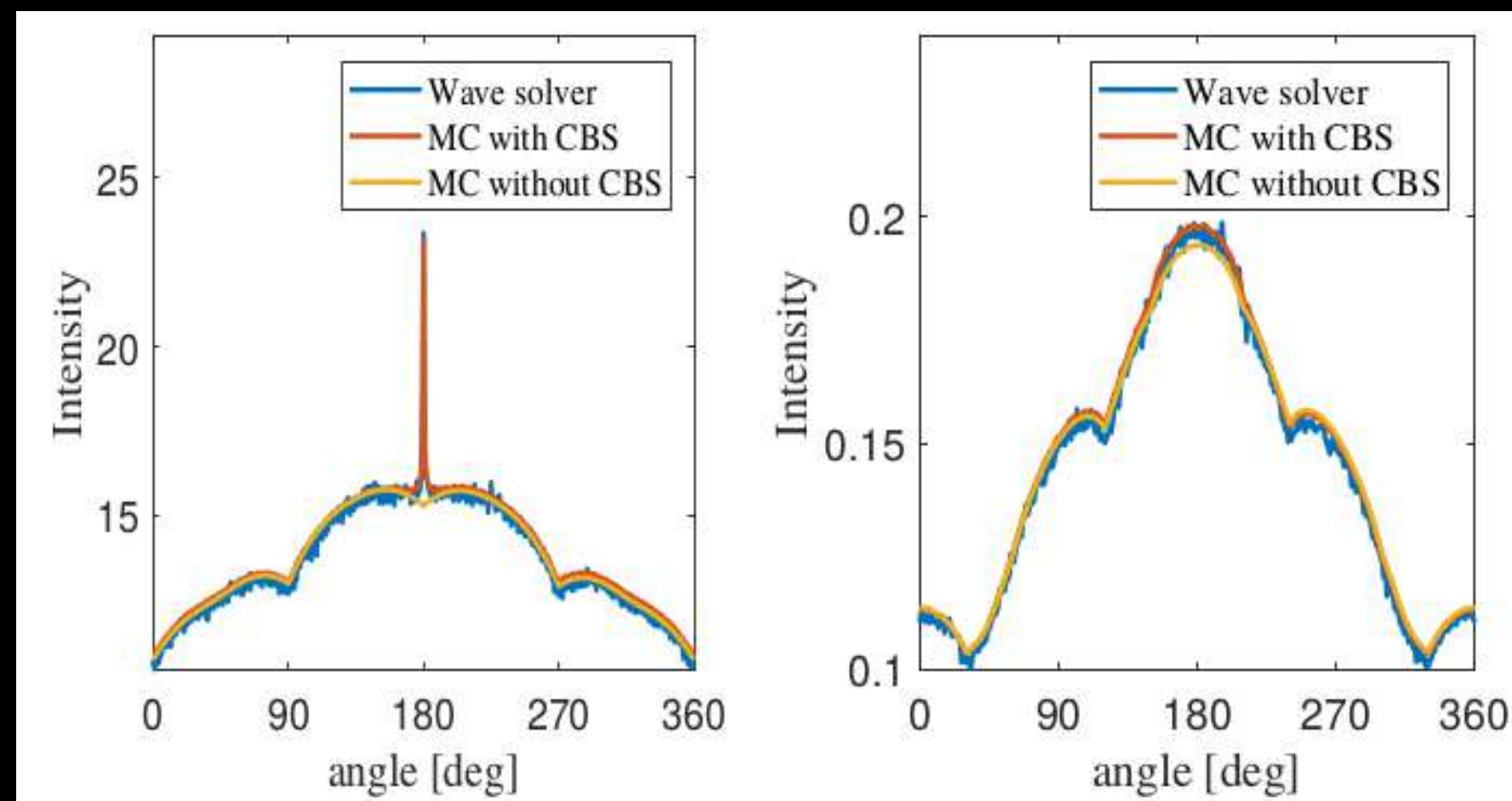
Covariance =  $\int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$

**Observation: need to consider only path pairs that share the *same* nodes (except start and end nodes)**

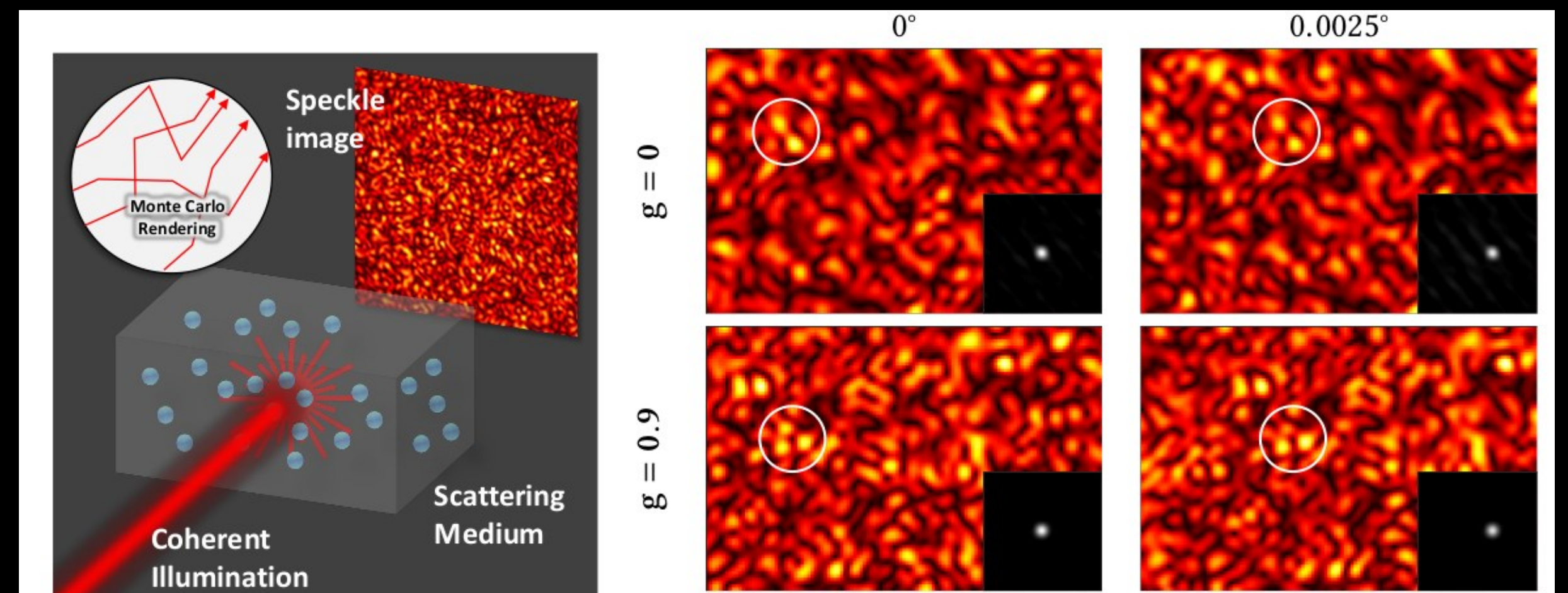




# Comparison with wave-equation solver and real measurements



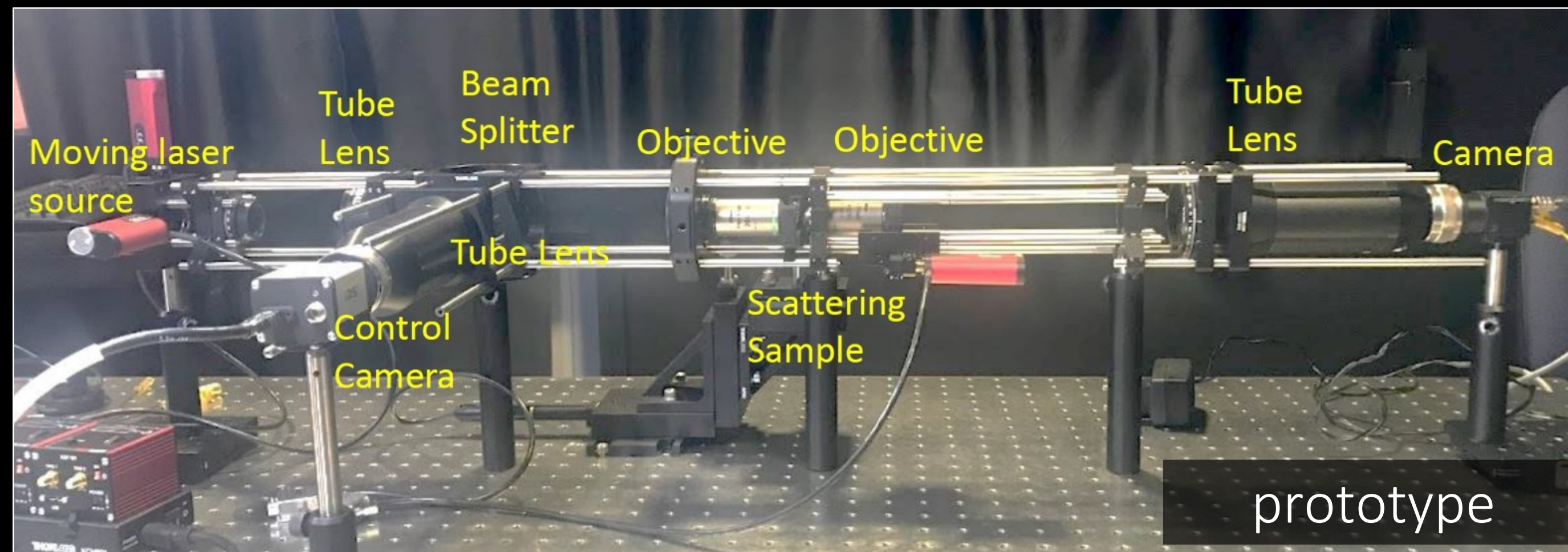
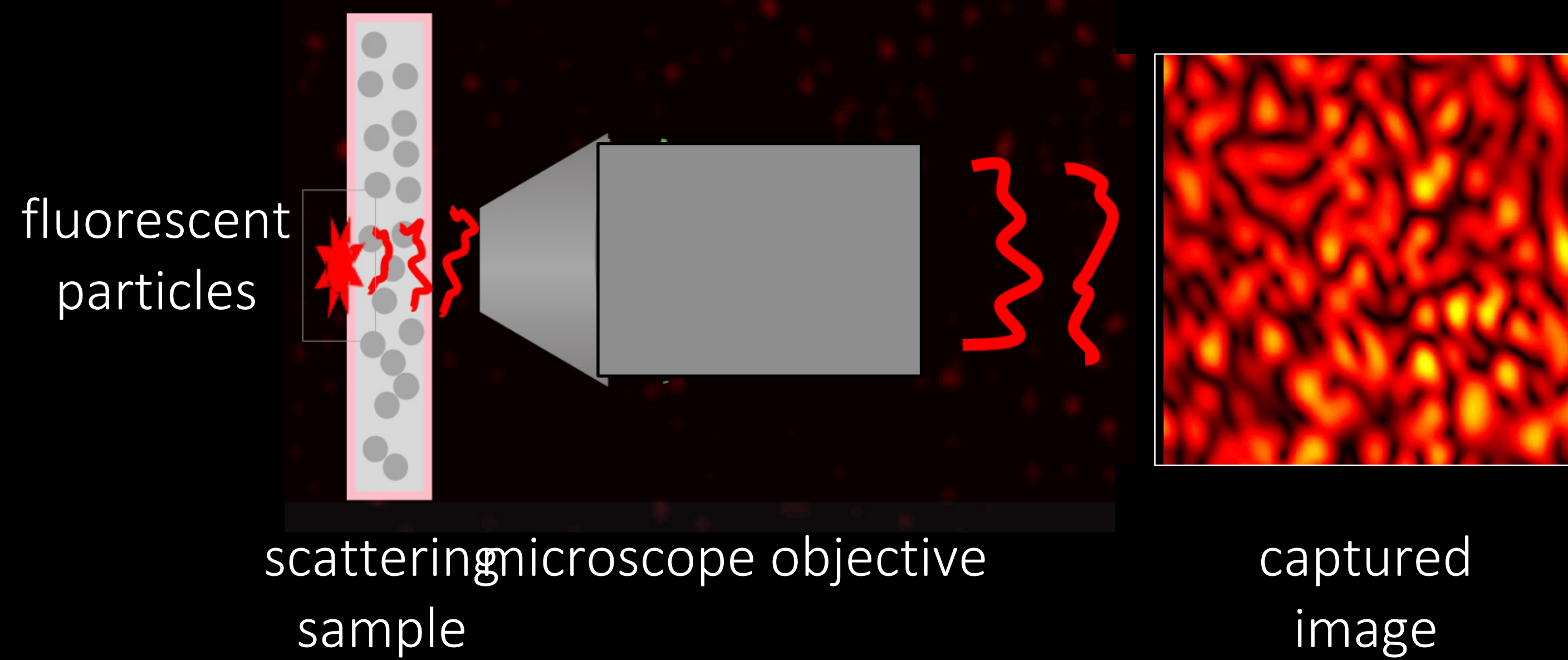
match wave equation solvers,  $10^5$ x faster



match real measurements of memory effect

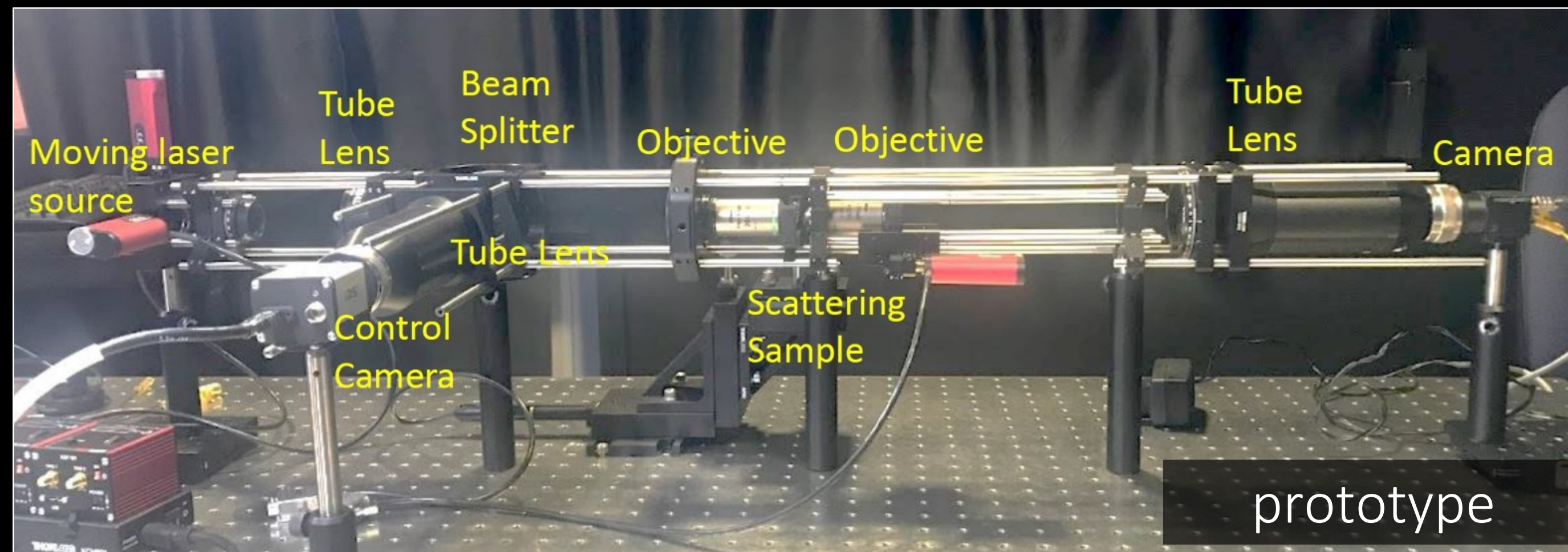
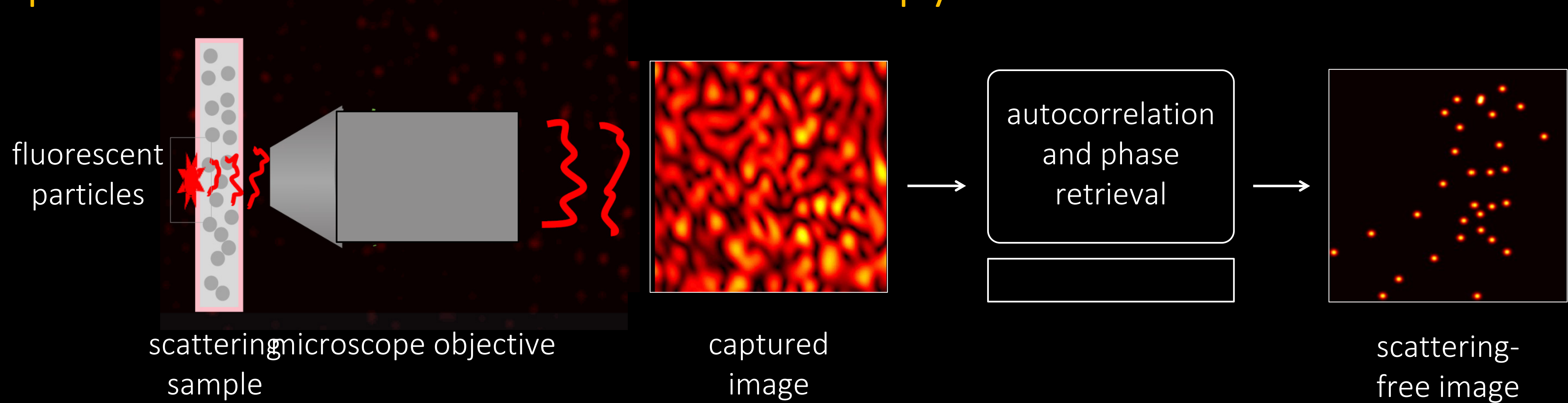


# Speckle-based fluorescence microscopy



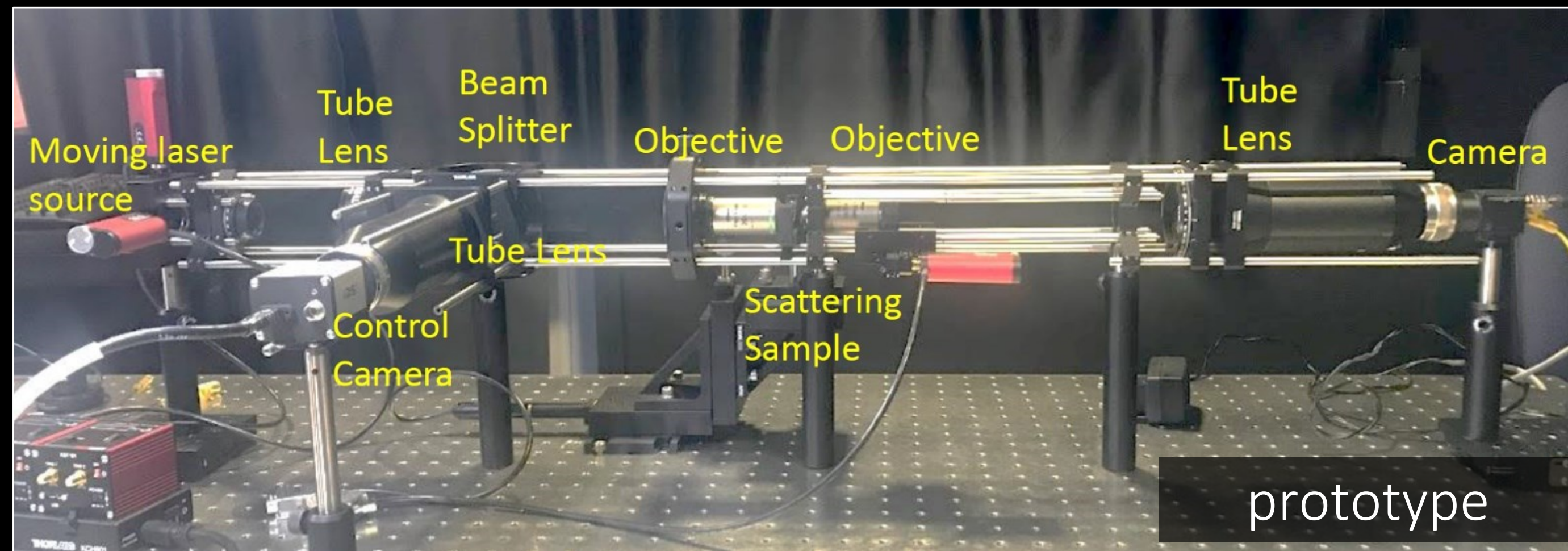
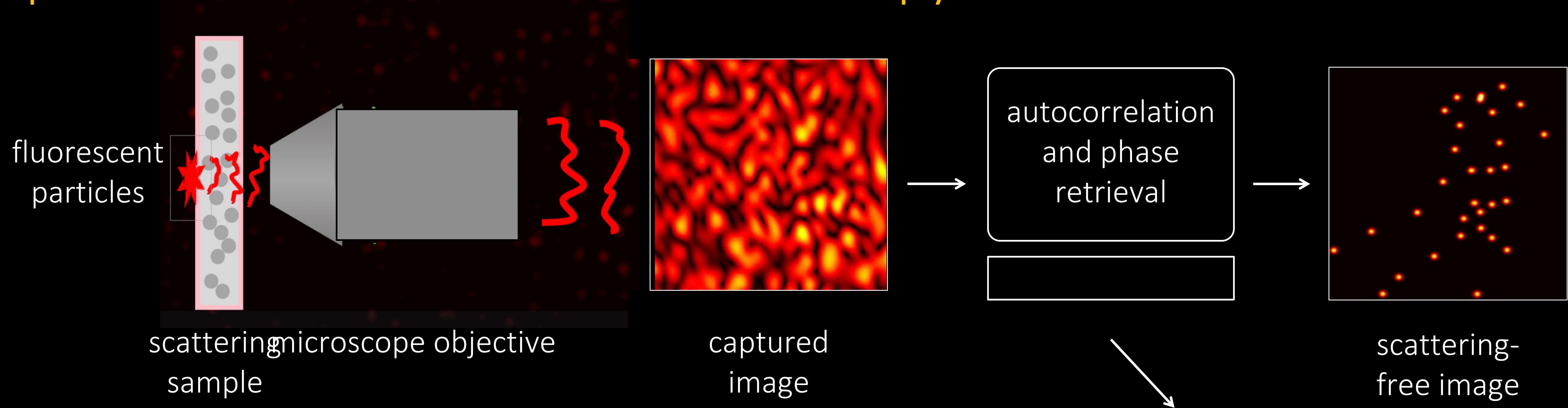


# Speckle-based fluorescence microscopy





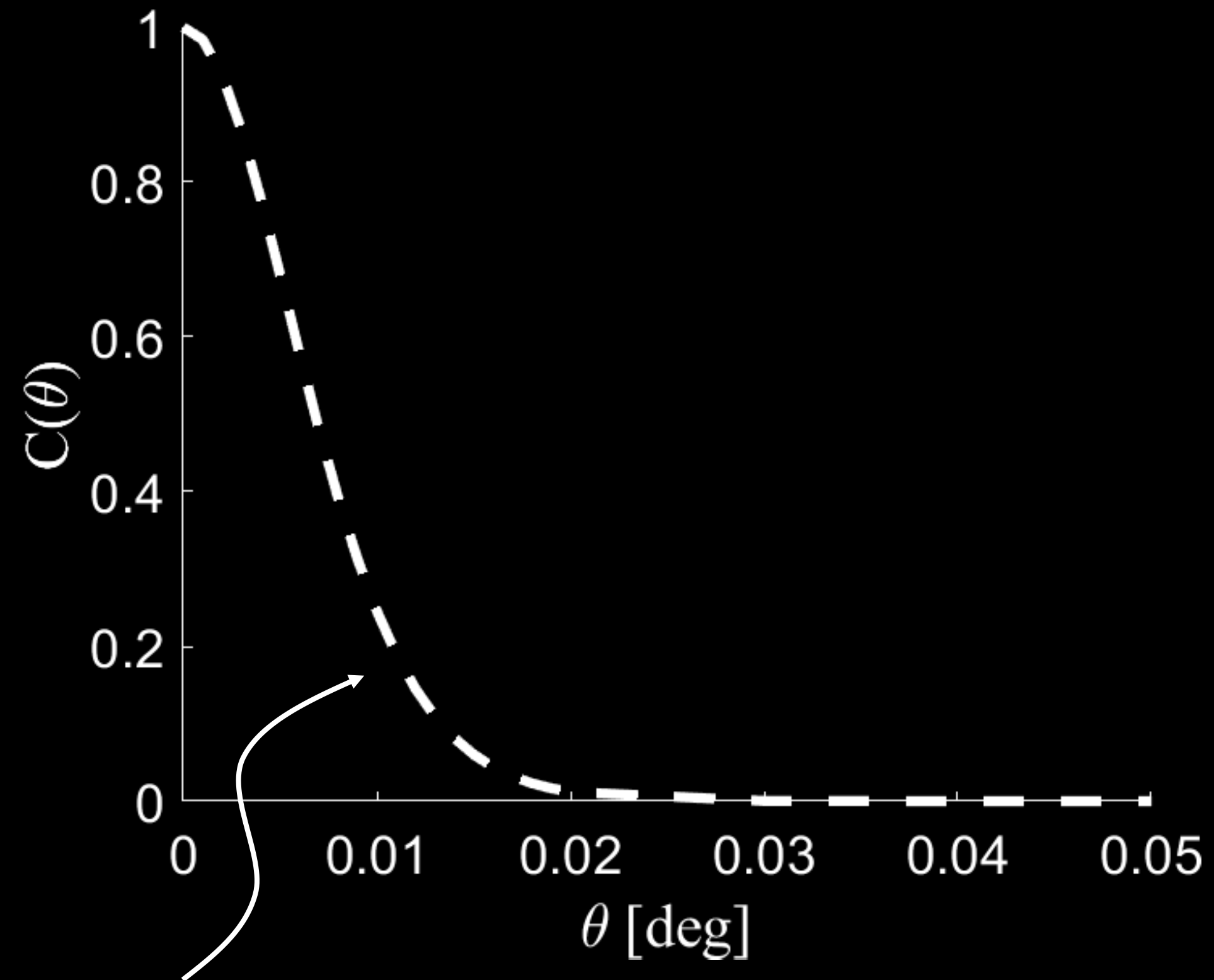
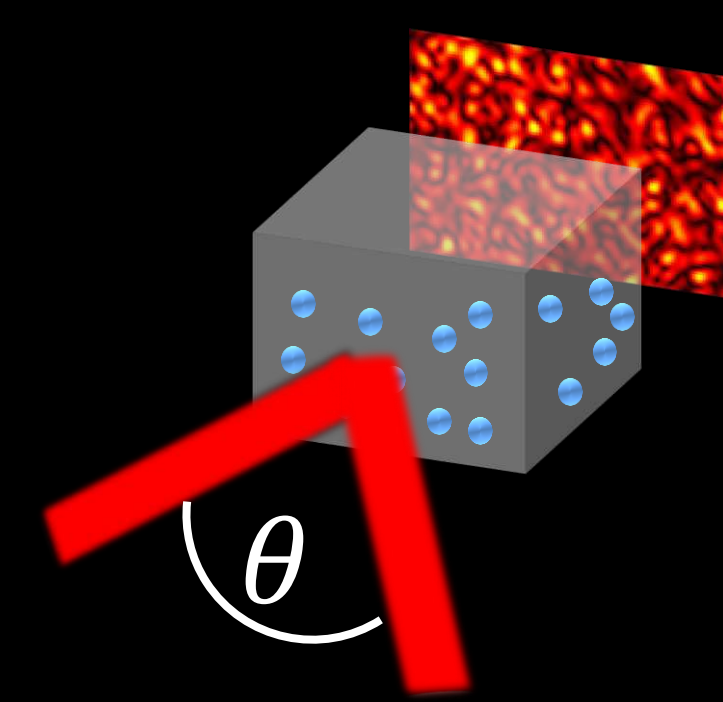
# Speckle-based fluorescence microscopy



Performance strongly depends on:

- speckle statistics
- image priors
- tissue parameters

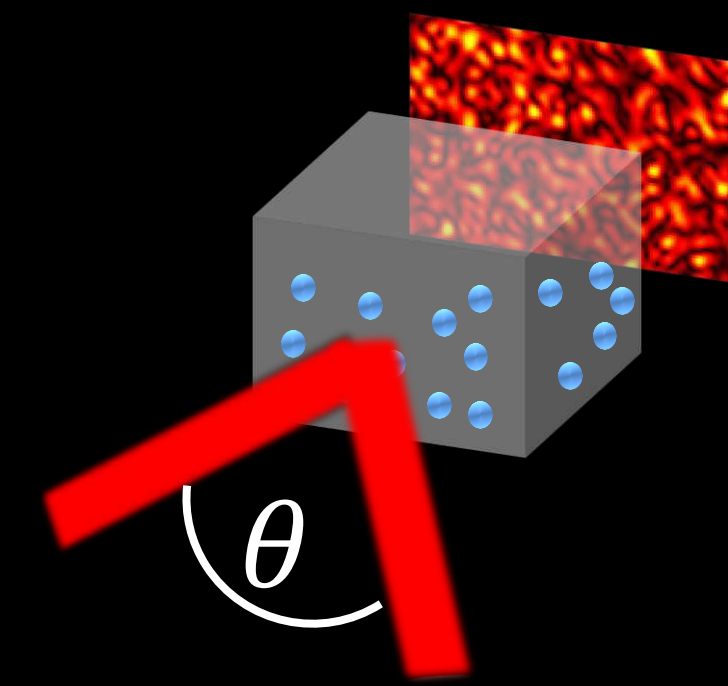
# Evaluate the memory effect



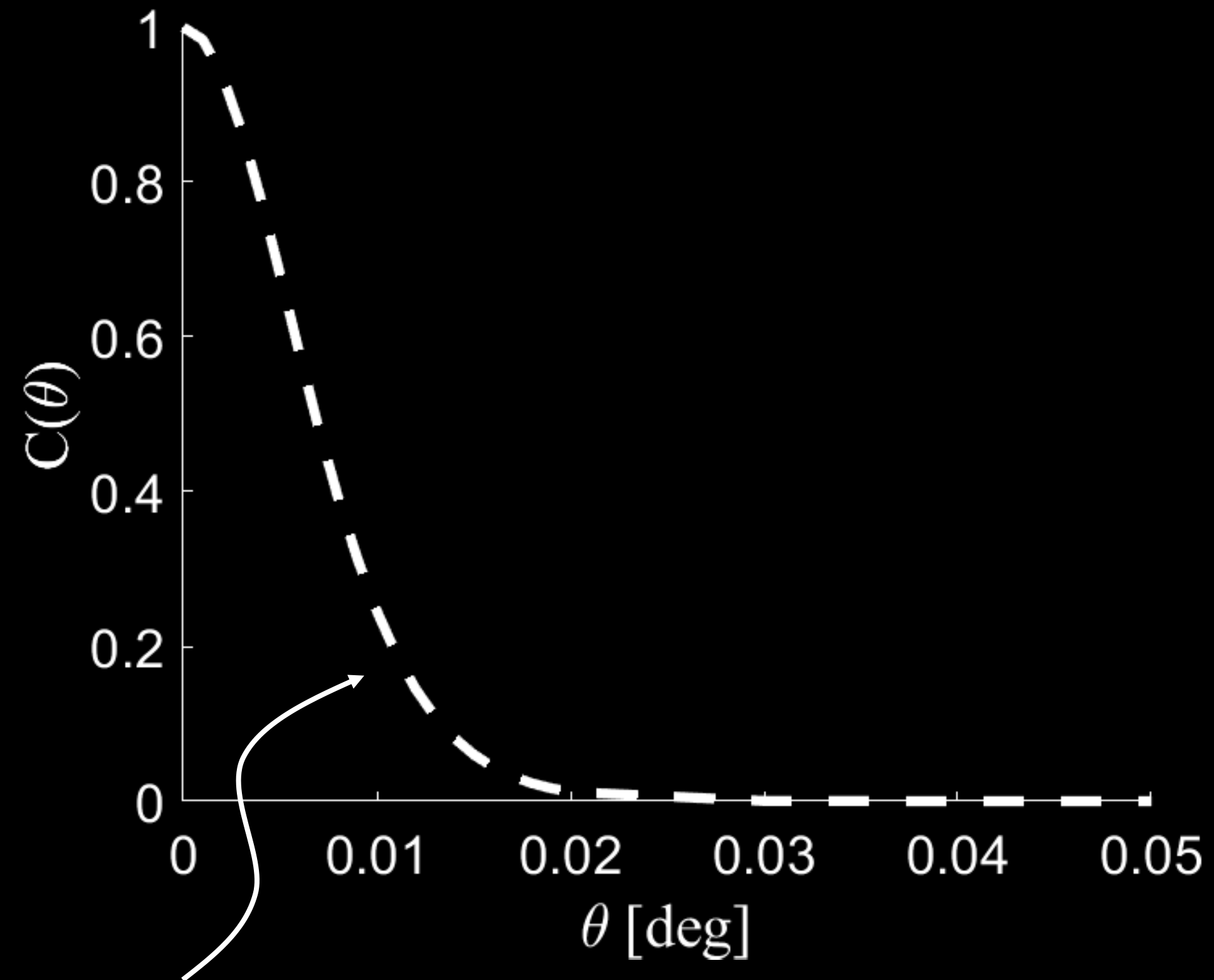
Analytical solution  
based on diffusion  
approximation



# Evaluate the memory effect

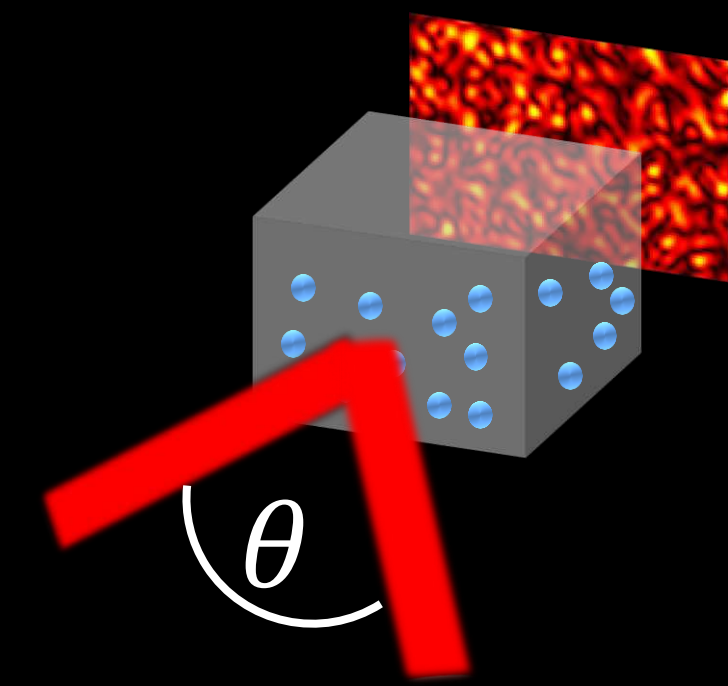


This approximation is correct for some materials and configurations



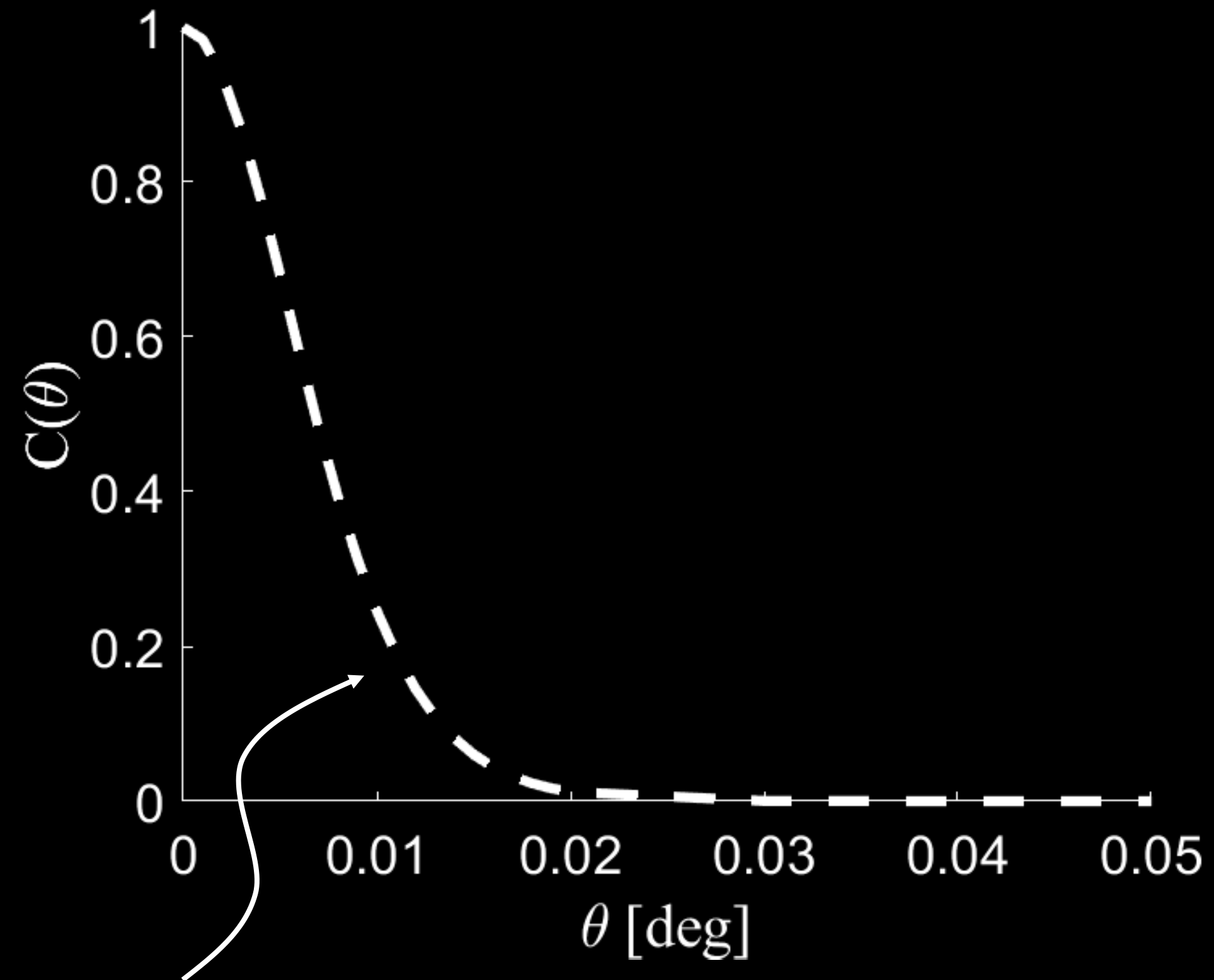
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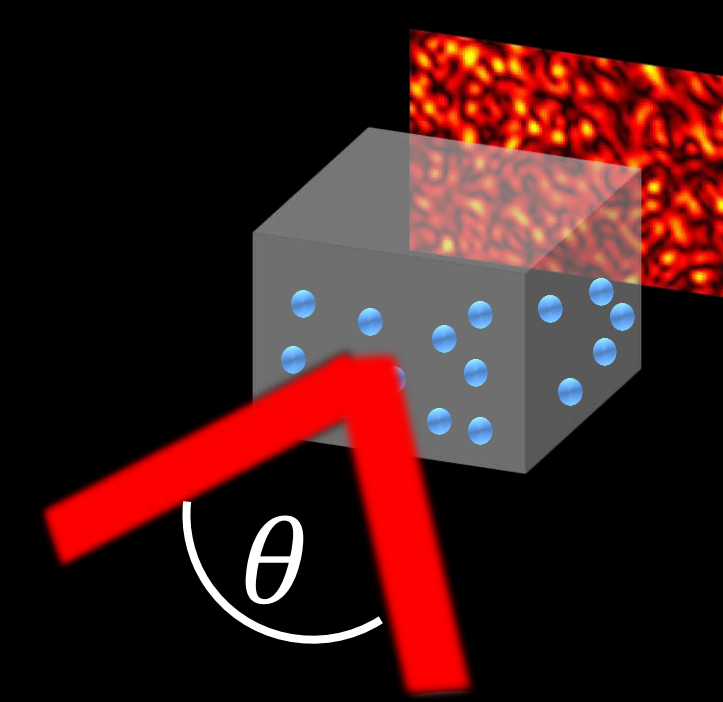
In practice ME extent is often wider



Analytical solution  
based on diffusion  
approximation



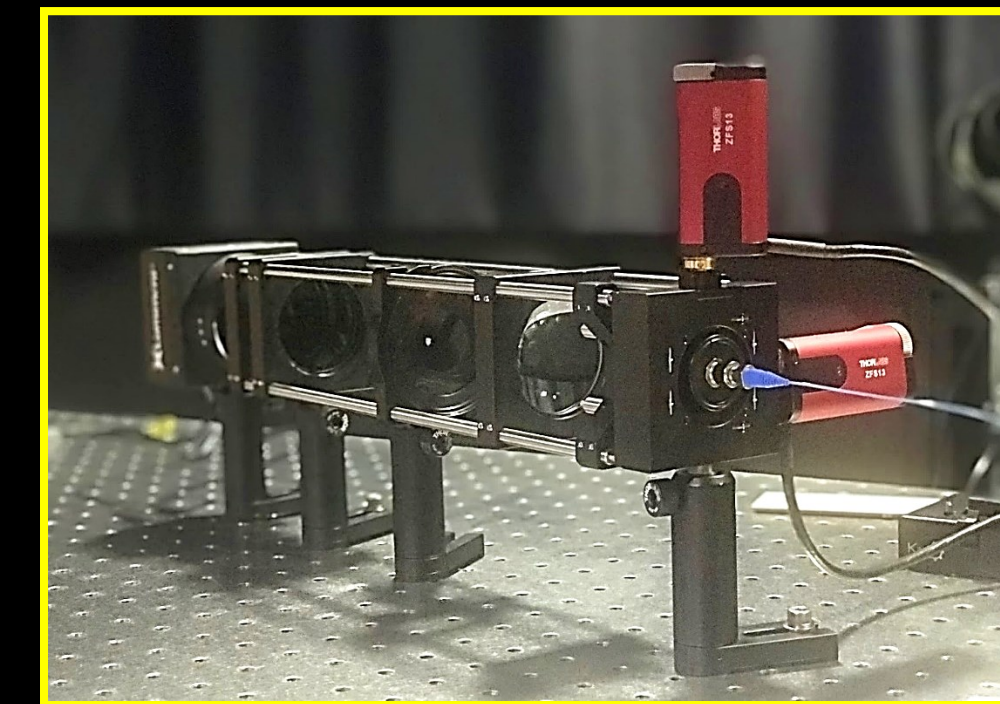
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In practice ME extent is often wider

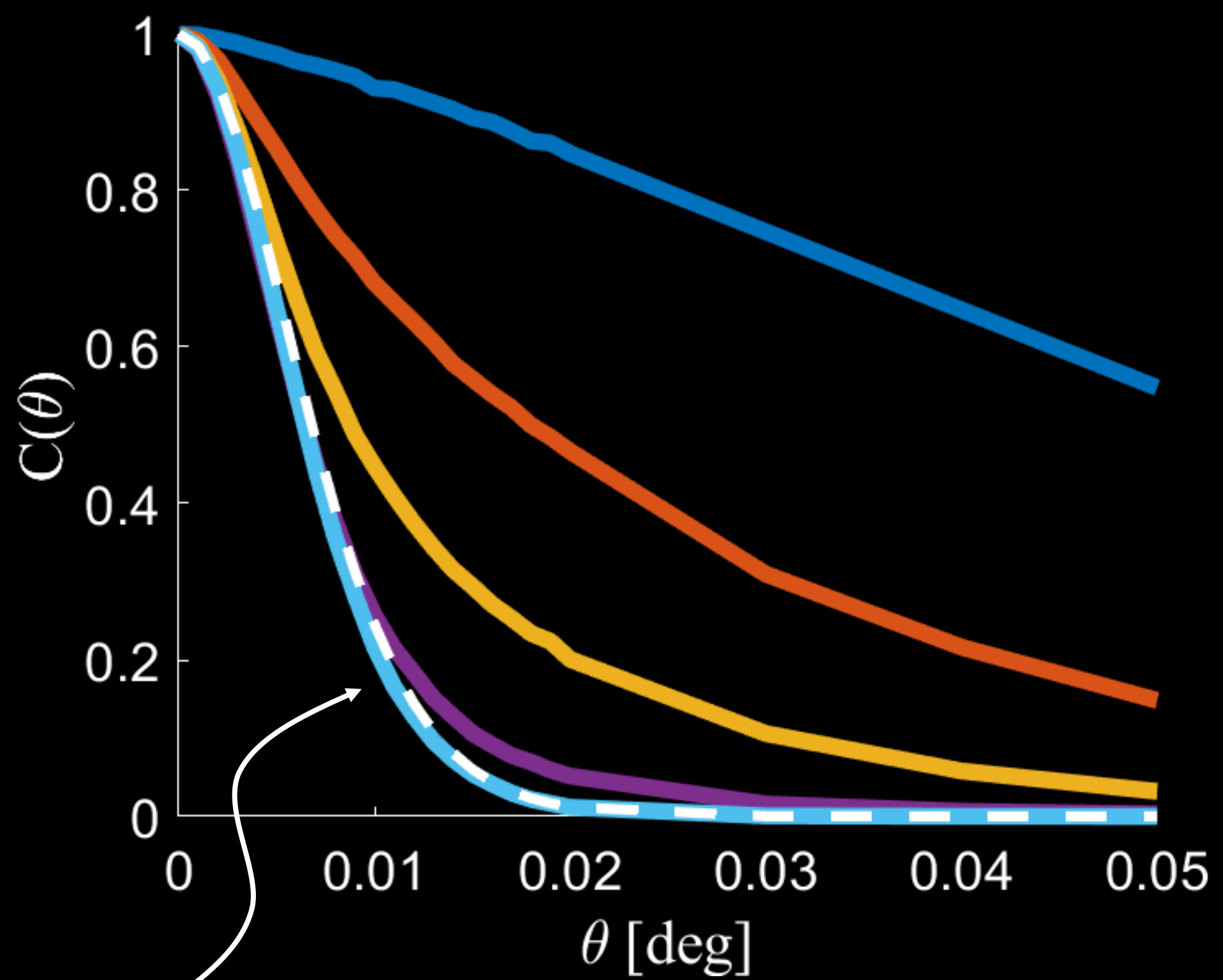
Currently: measured *empirically* in the lab



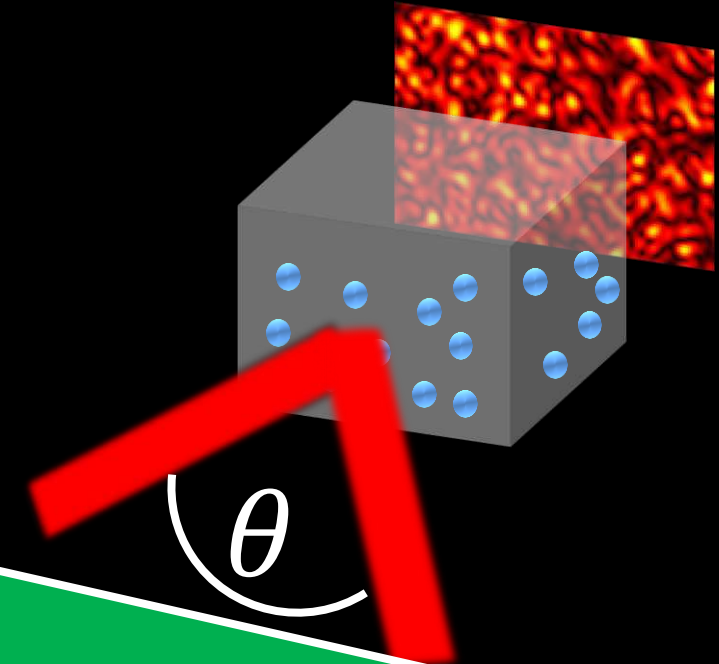
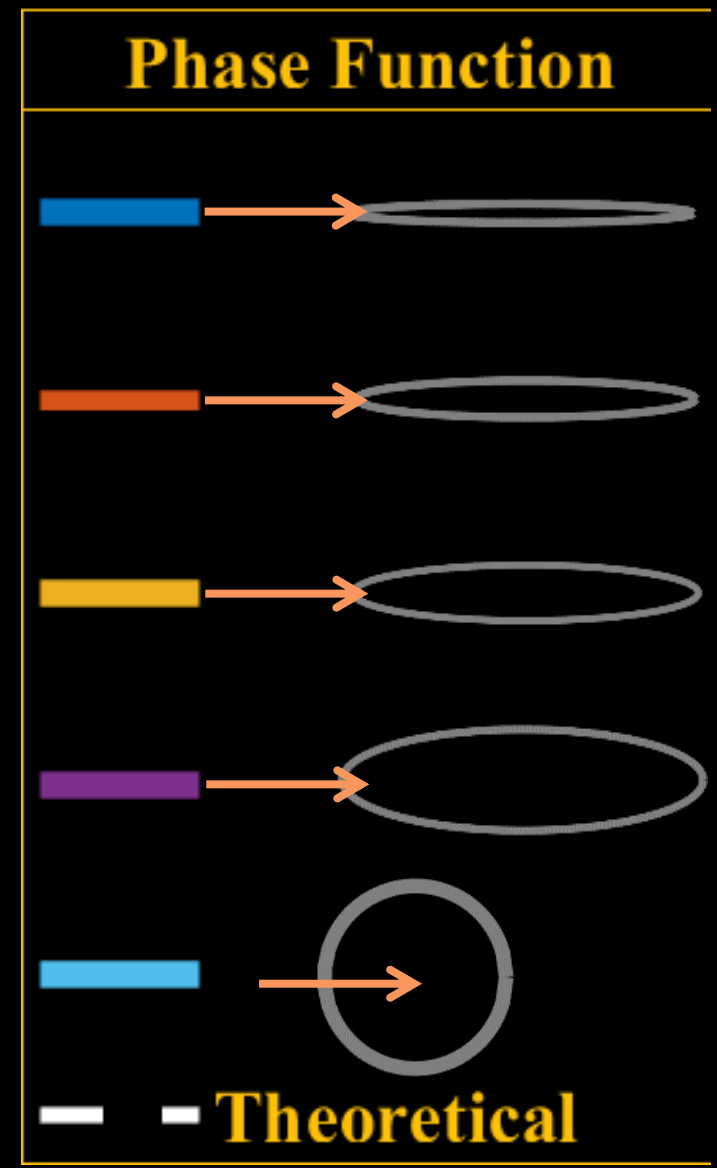
Analytical solution  
based on diffusion  
approximation

[Alterman et al., 2021]

# Evaluate the memory effect



Analytical solution based on diffusion approximation

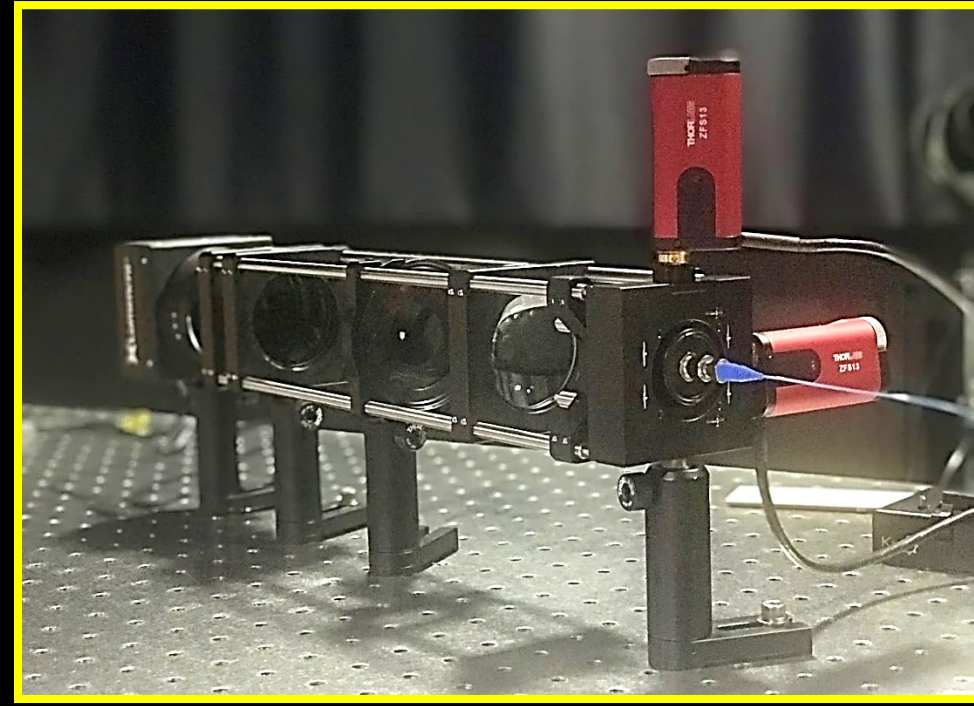


Now we can efficiently compute ME curves for all scattering parameters

This approach

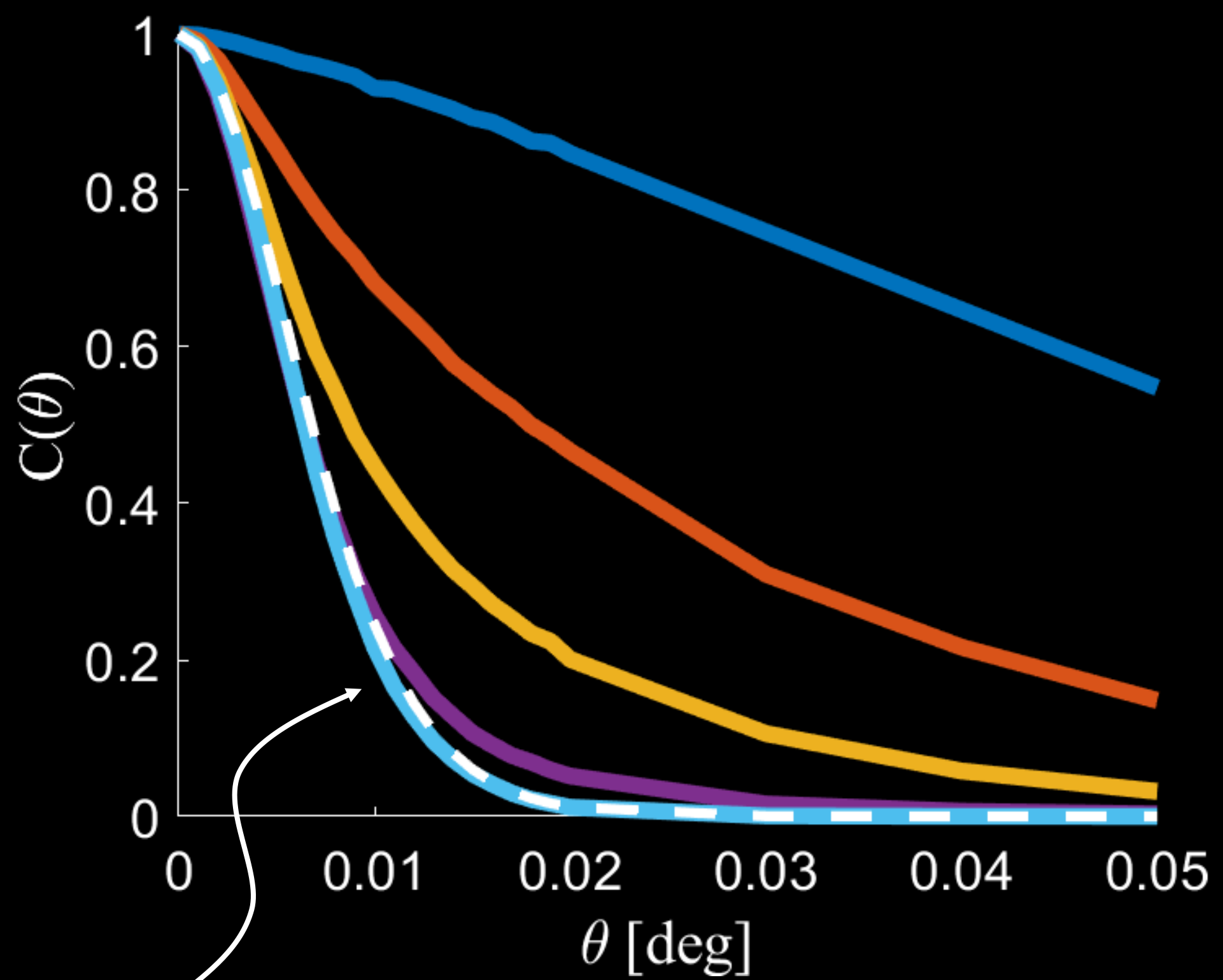
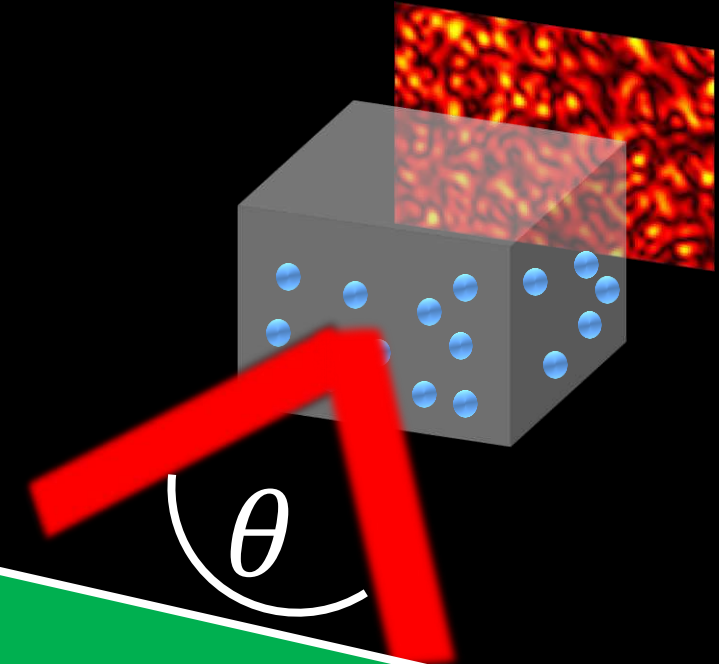
In practice ME extension

Currently: measured *empirically* in the lab

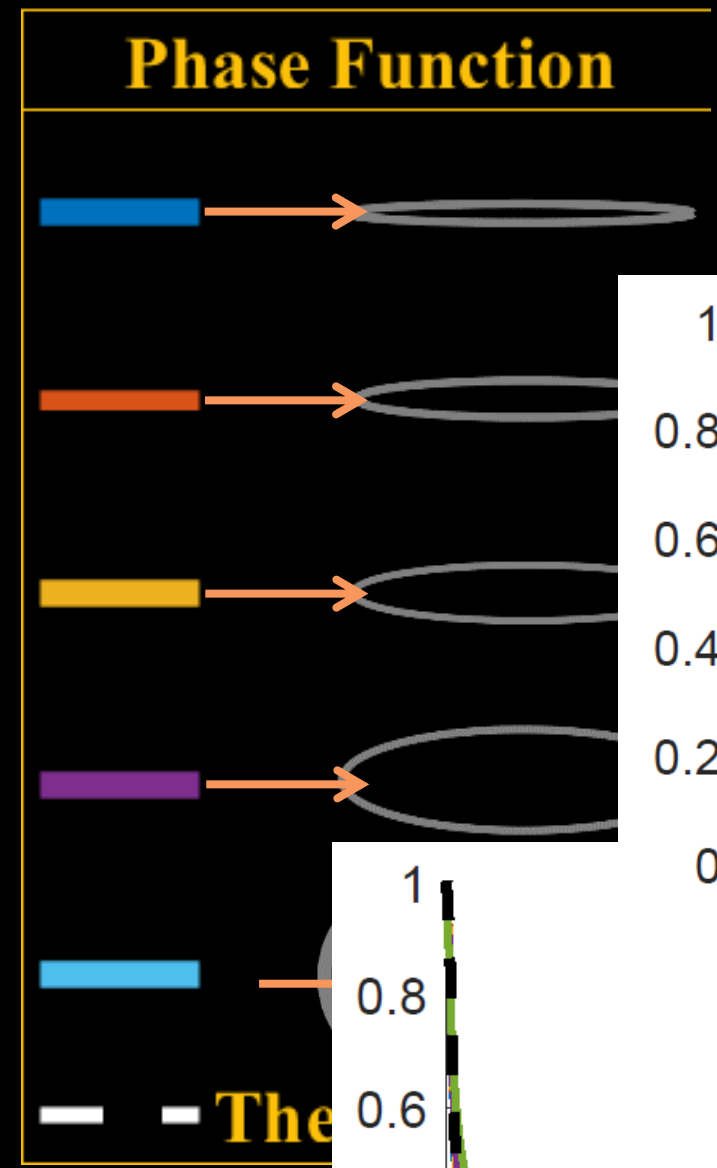




# Evaluate the memory effect

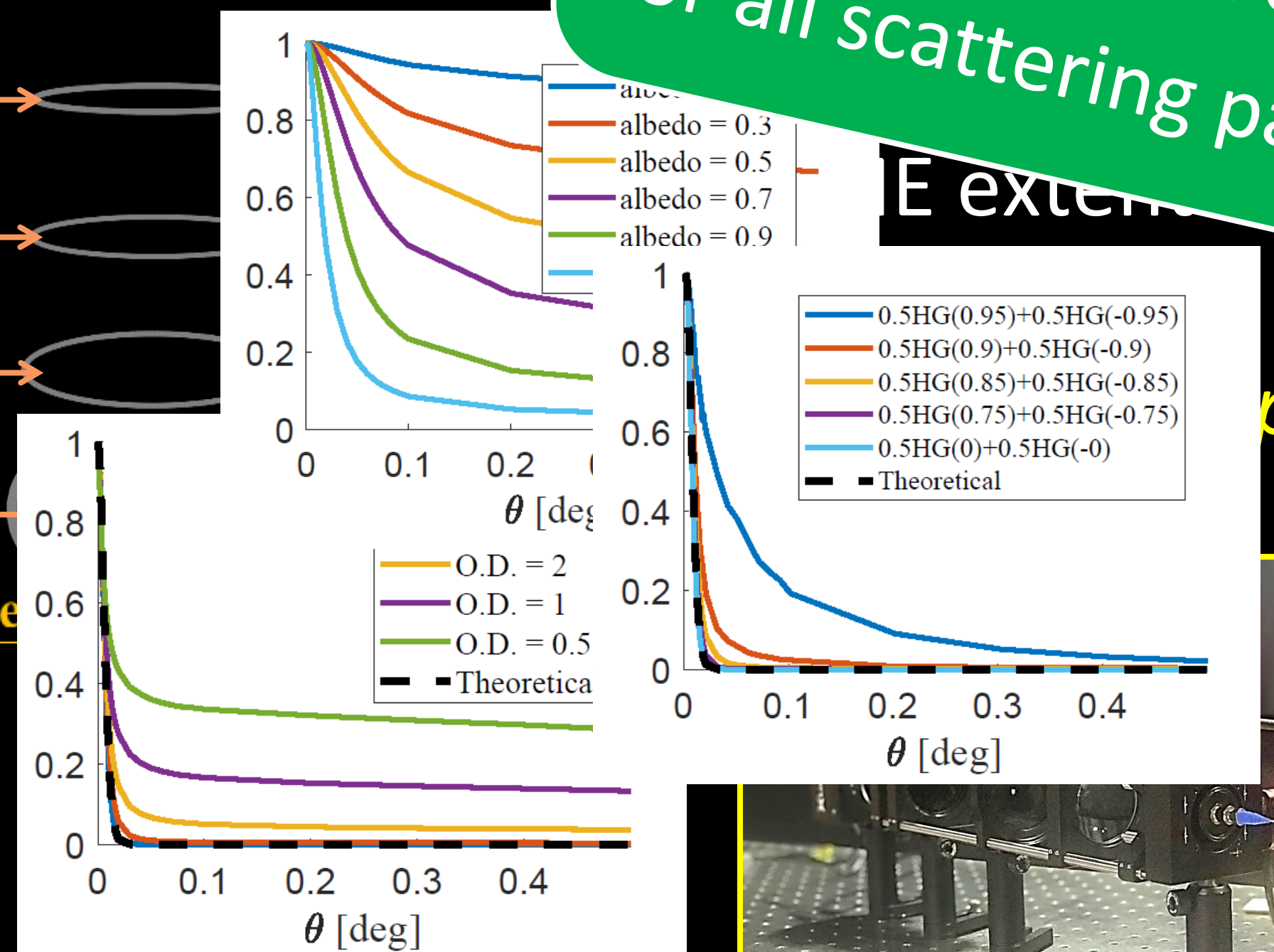


Analytical solution based on diffusion approximation

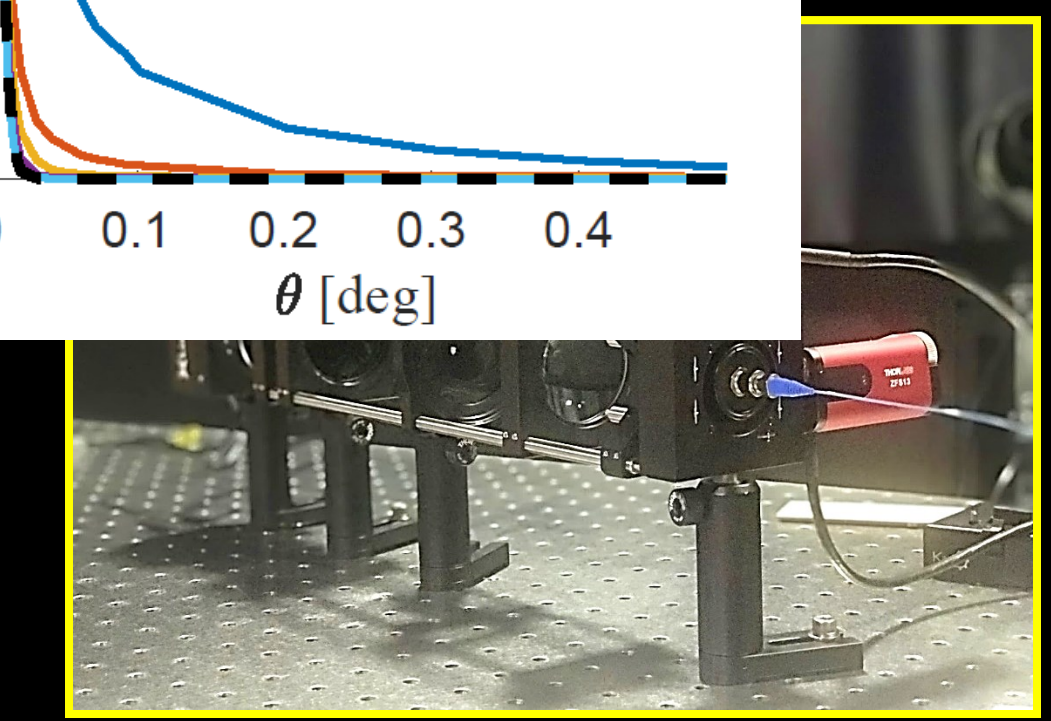


This approach

Now we can efficiently compute ME curves for all scattering parameters



empirically in the





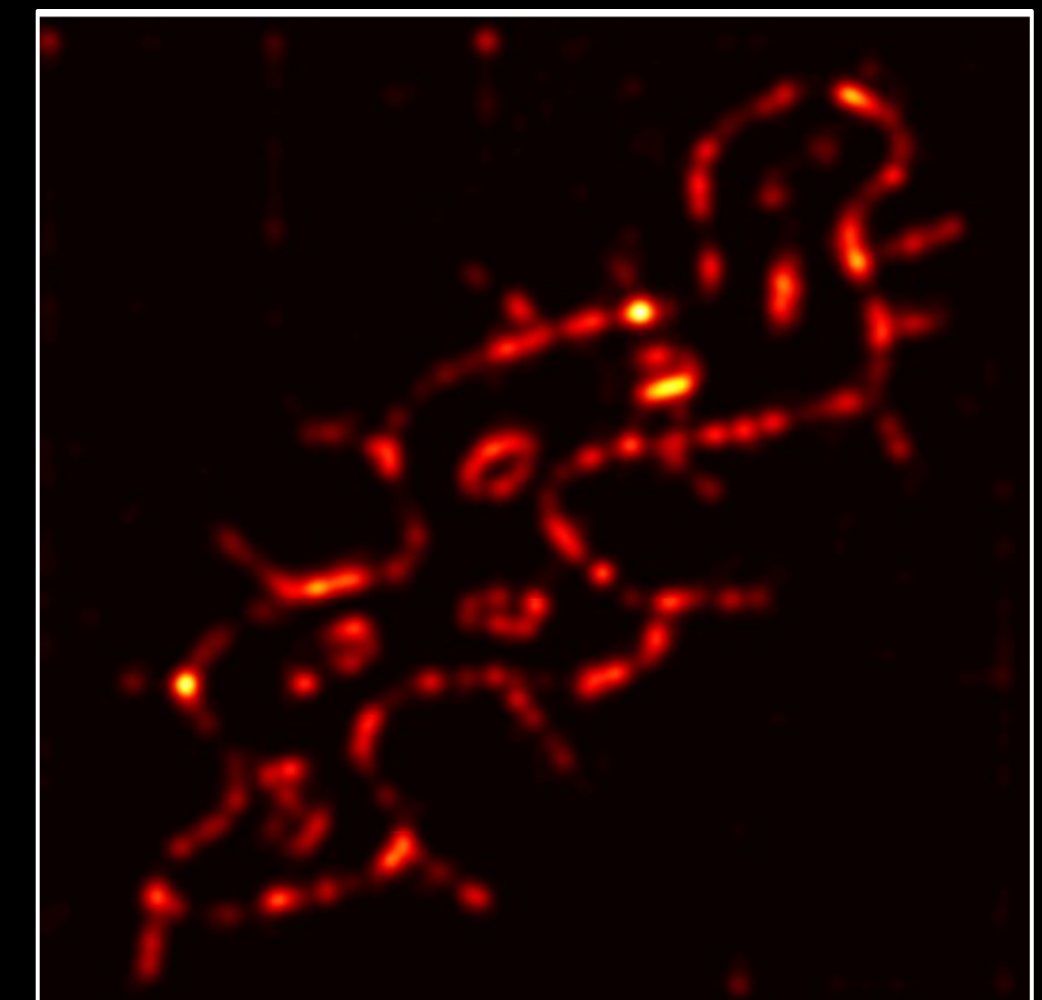
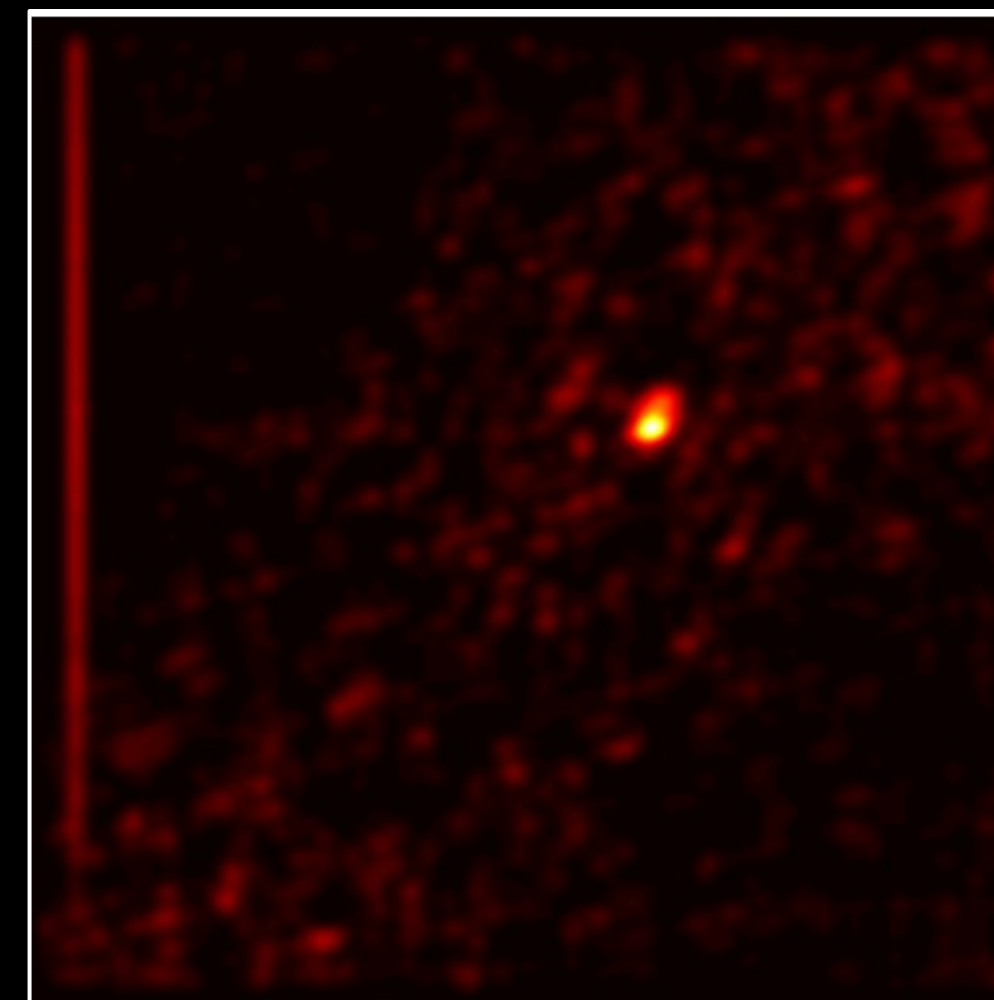
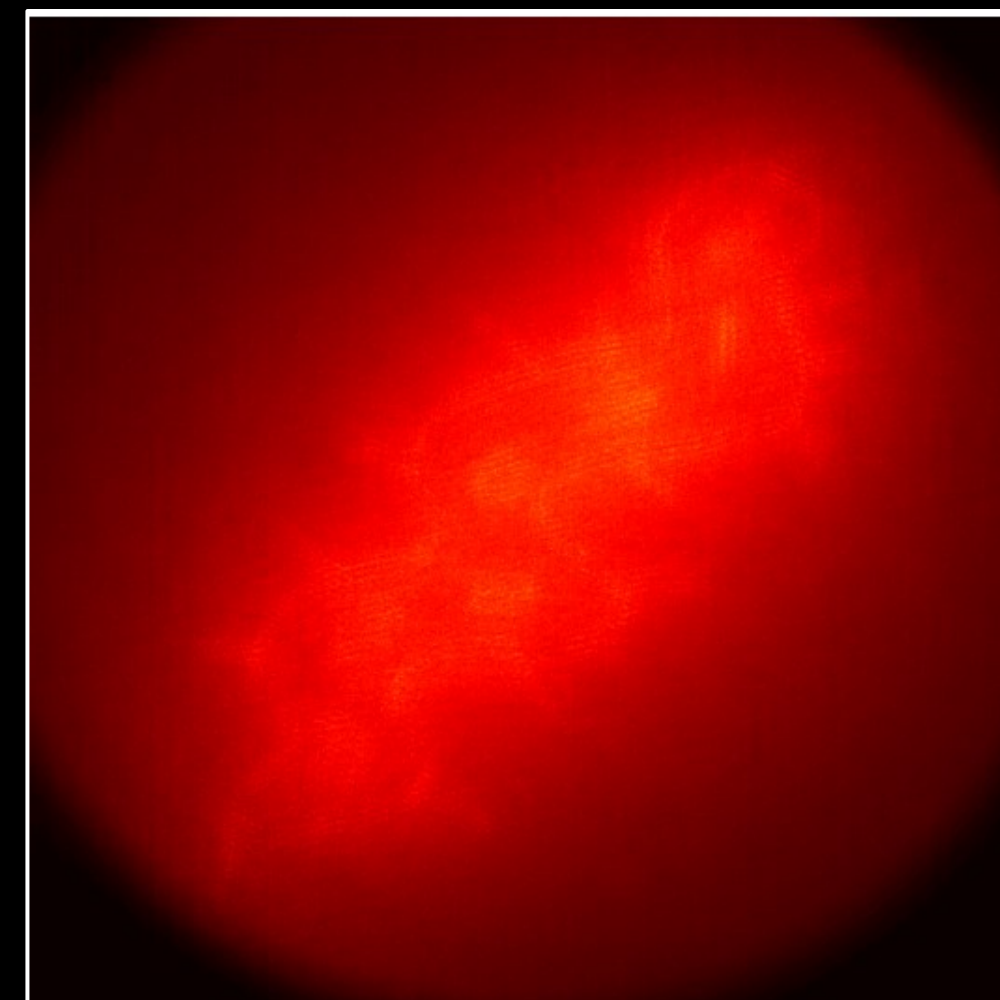
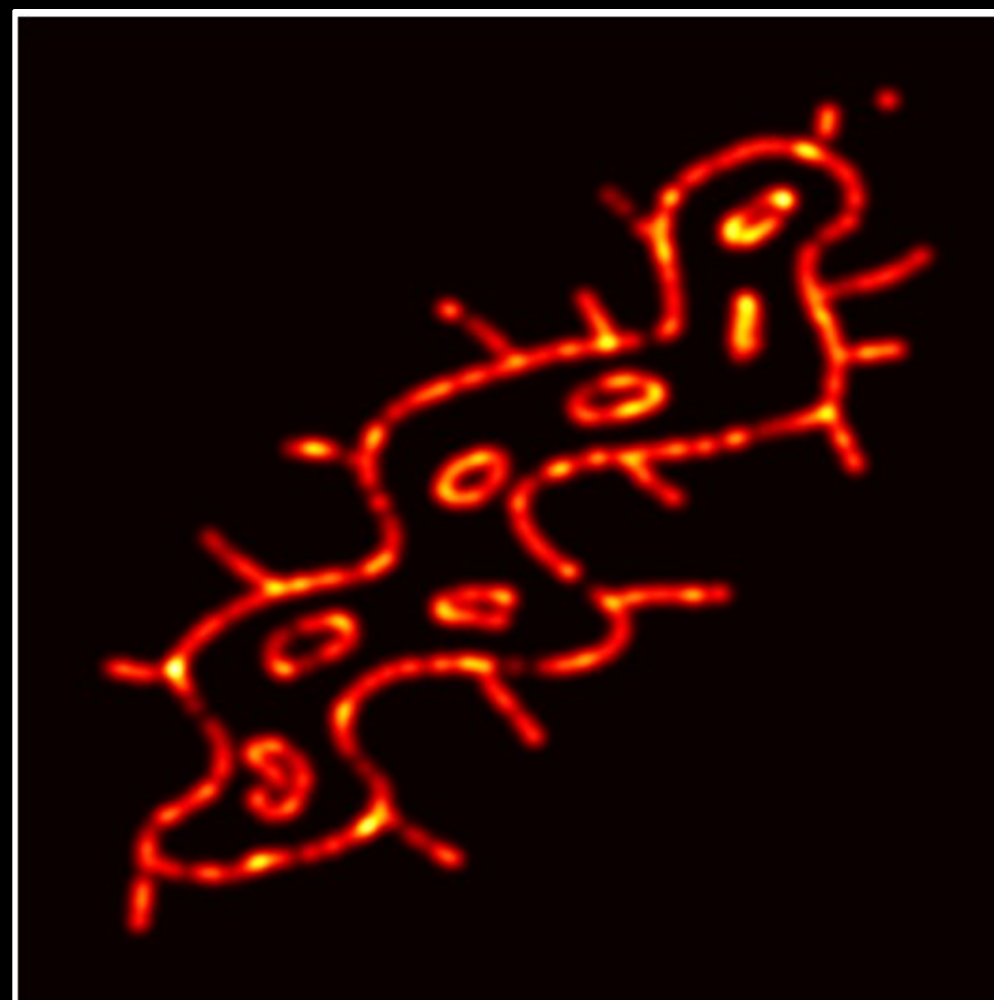
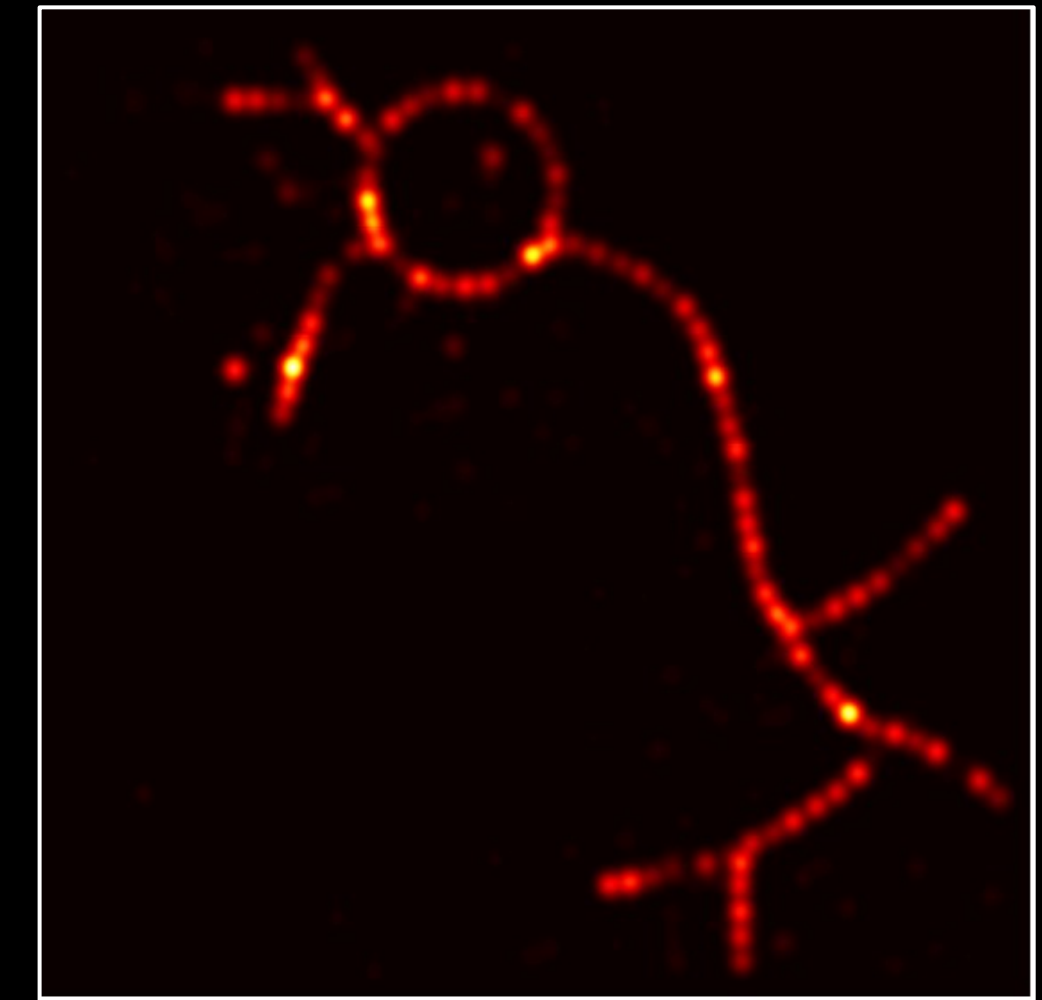
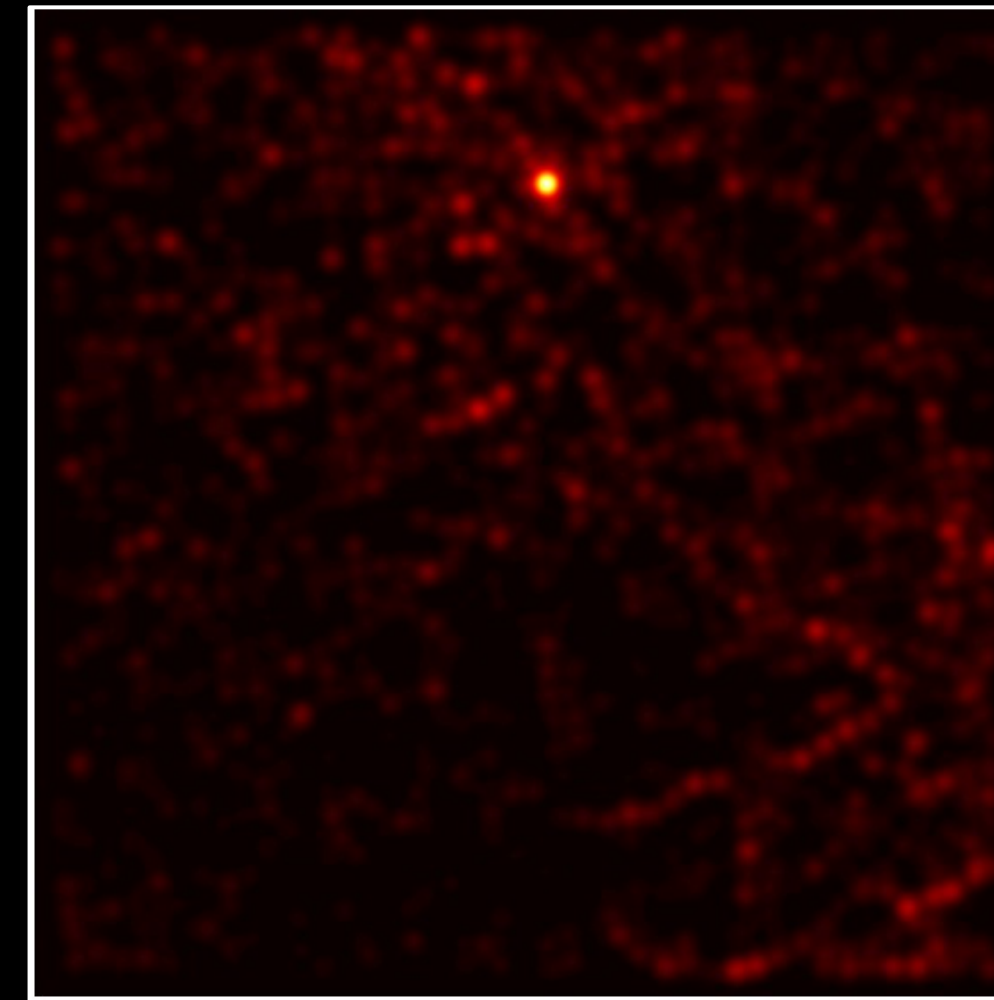
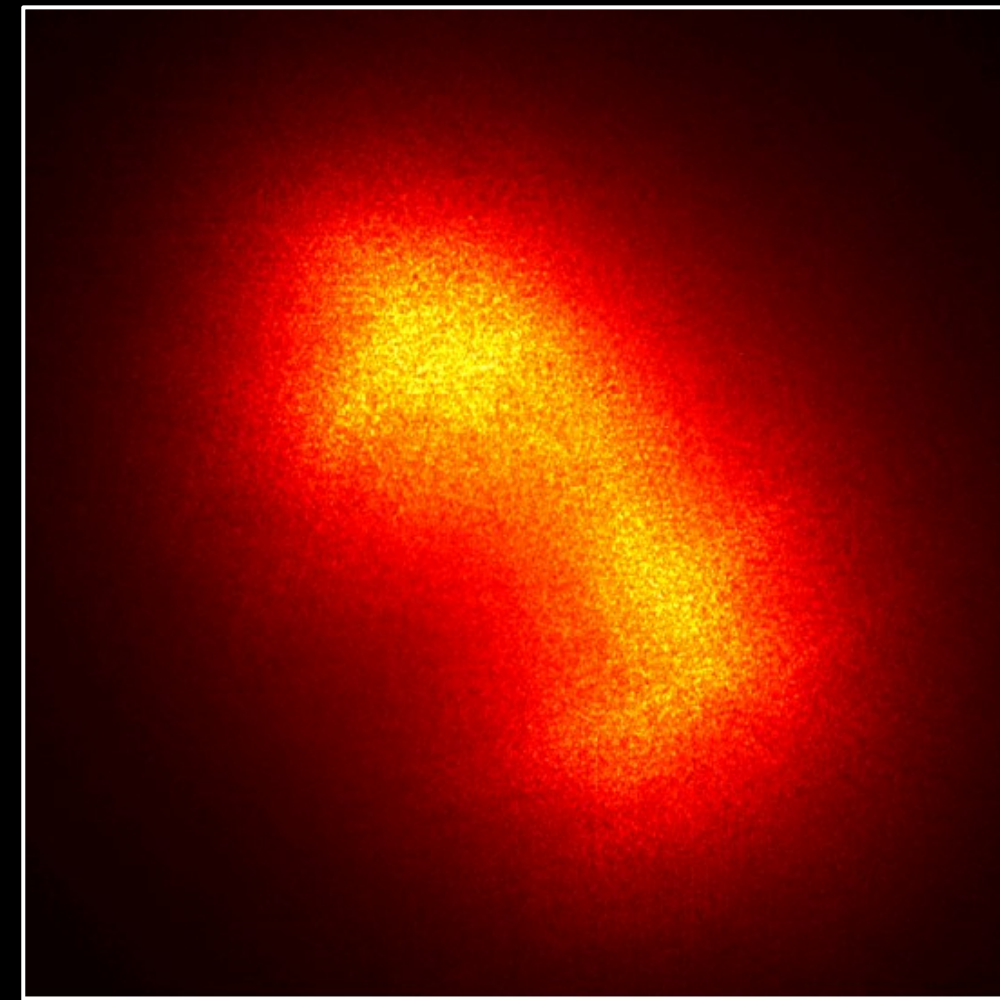
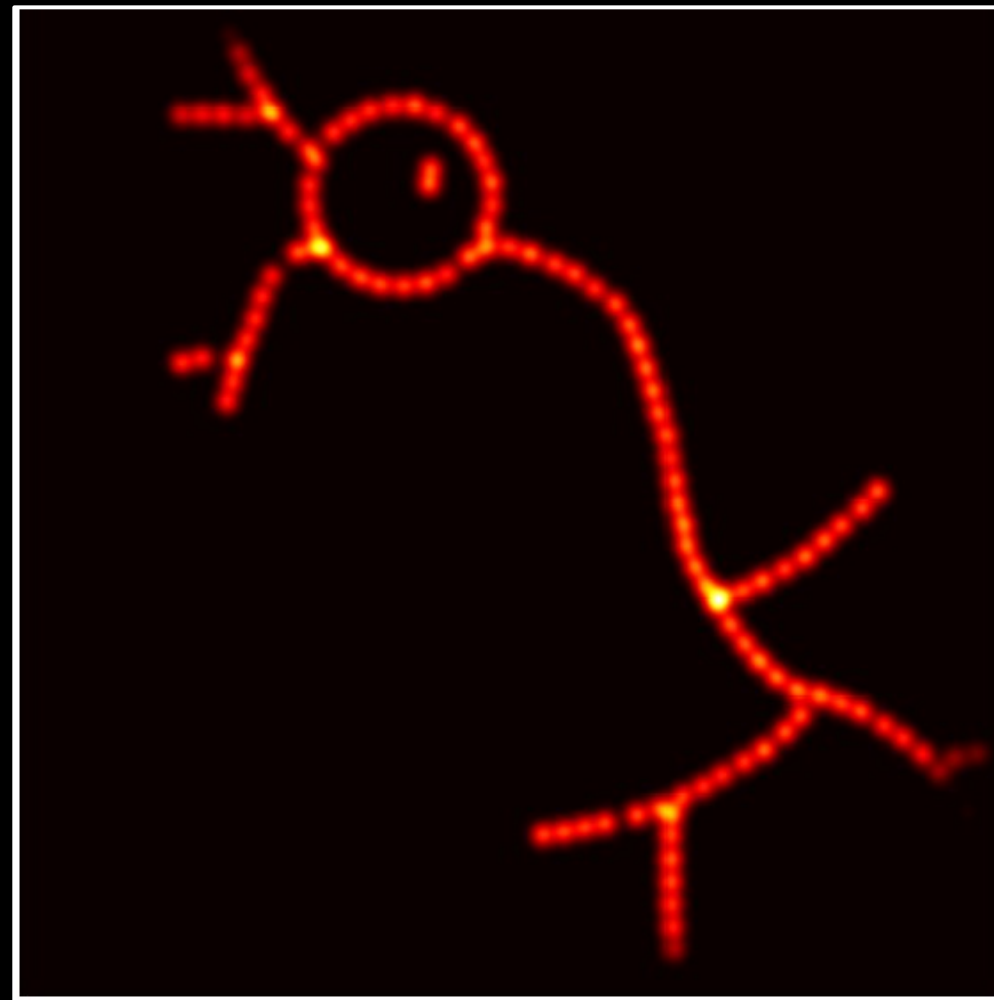
# Better algorithms for fluorescence microscopy

groundtruth

input image

prior algorithm

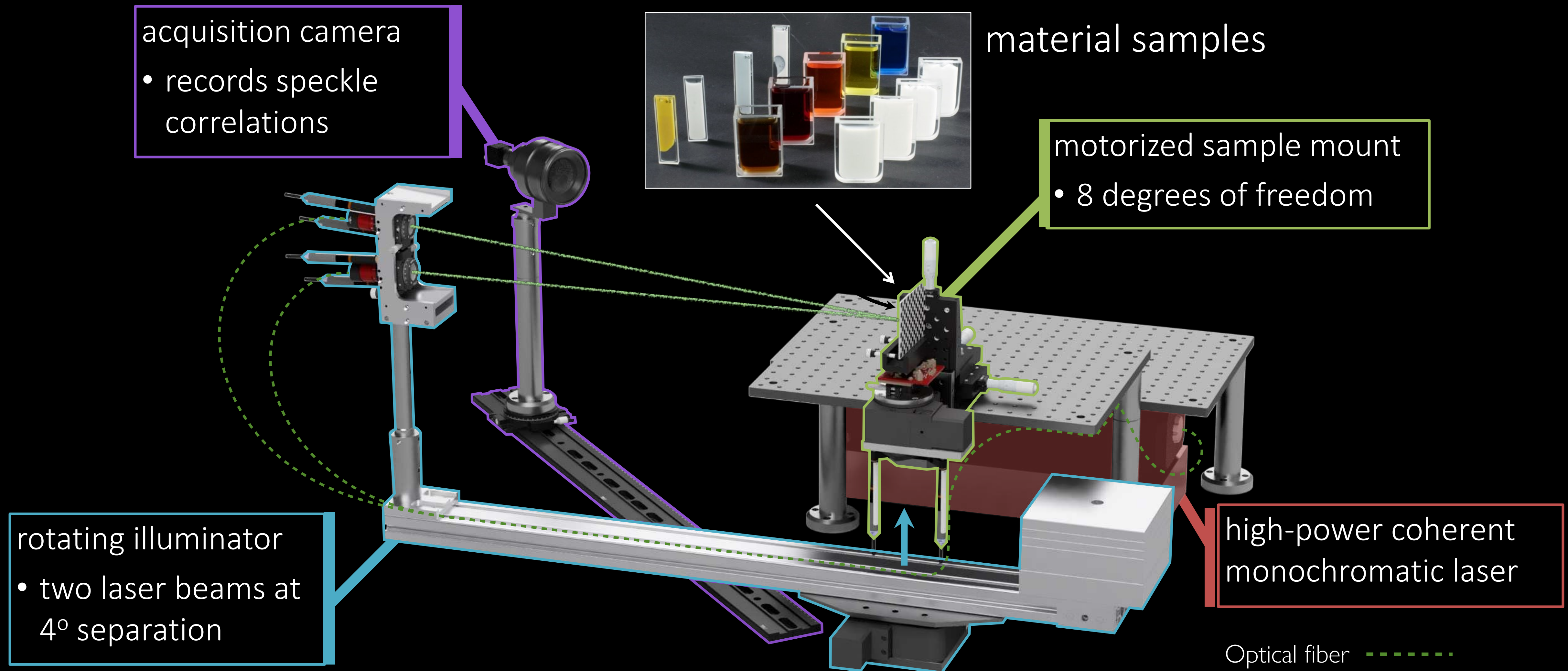
our algorithm





# Acquisition of scattering materials

Use differentiable speckle rendering to recover material parameters from speckle images

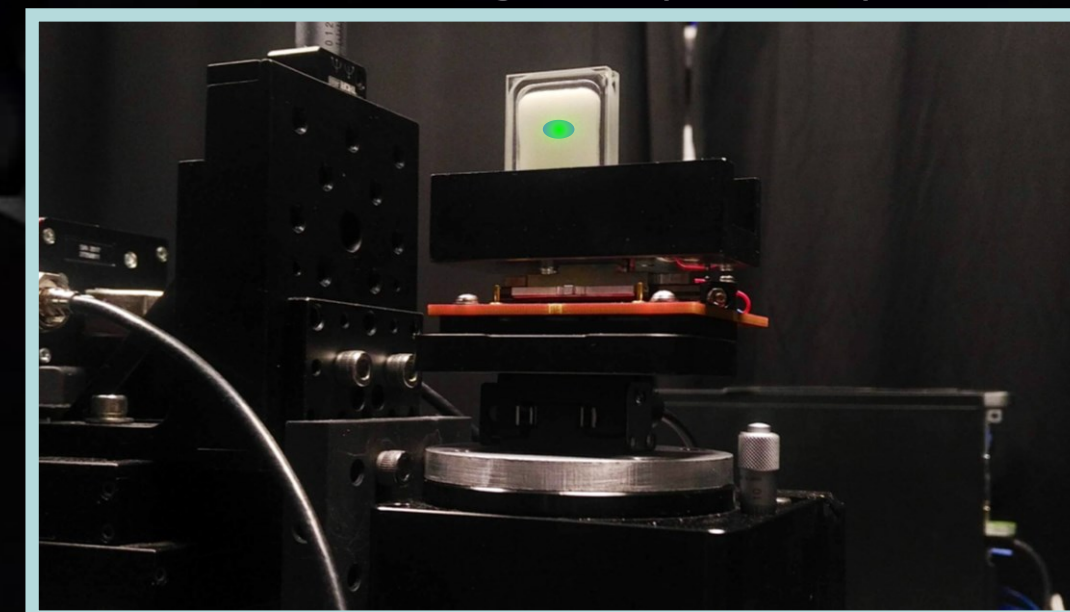




Acquisition

Speckle Video  
(Camera Feed)

Scattering Soap Sample





# Rendering wave optics is a very active area

## A Generic Framework for Physical Light Transport

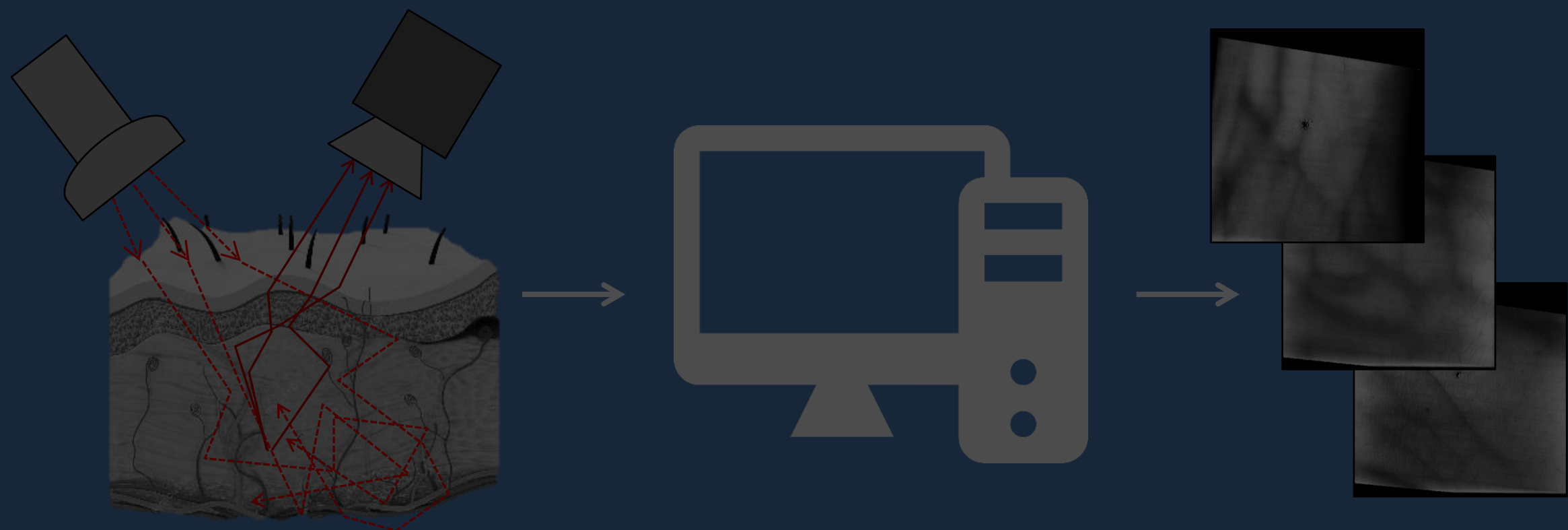
SHLOMI STEINBERG, University of California, Santa Barbara, USA

LING-QI YAN, University of California, Santa Barbara, USA



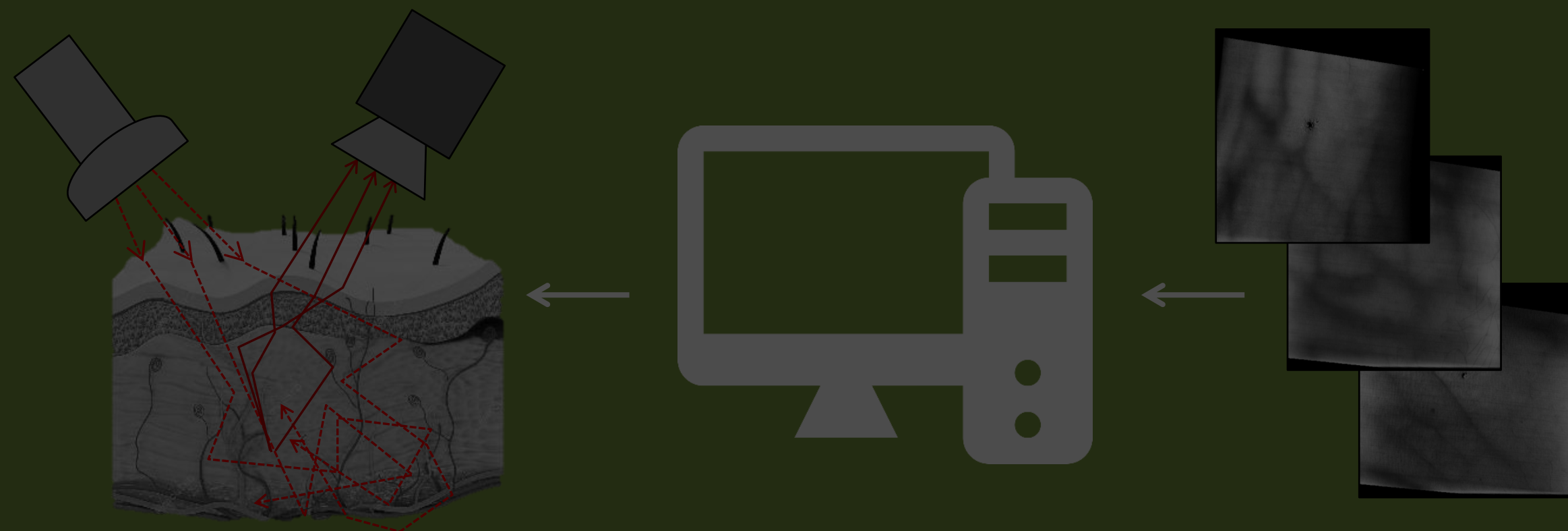
# Physics-based rendering and its applications to computational imaging

## forward rendering

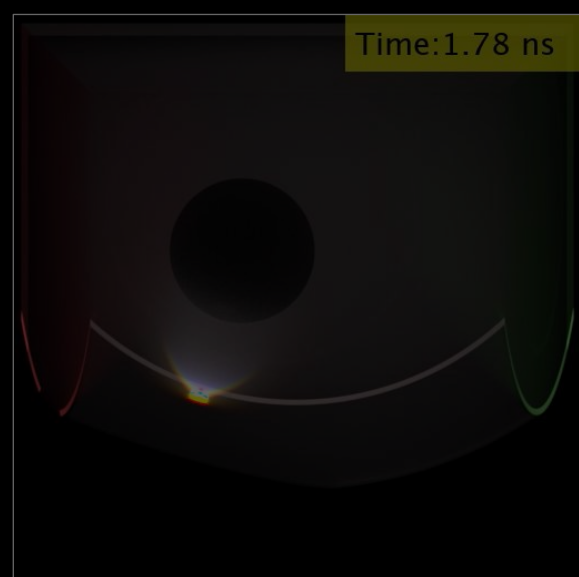


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

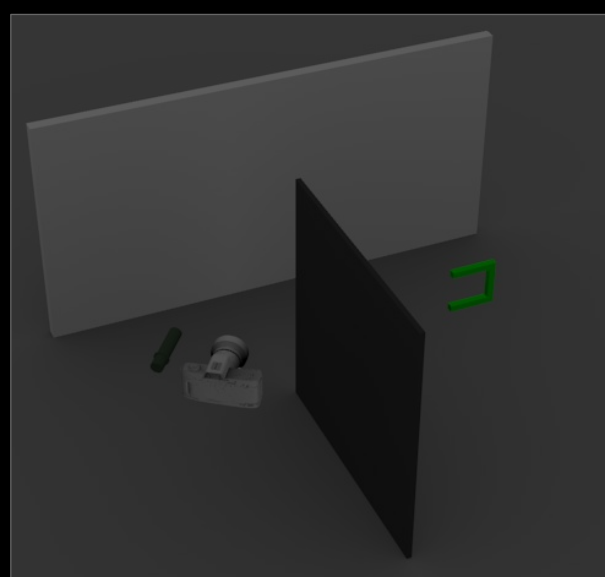
## inverse rendering



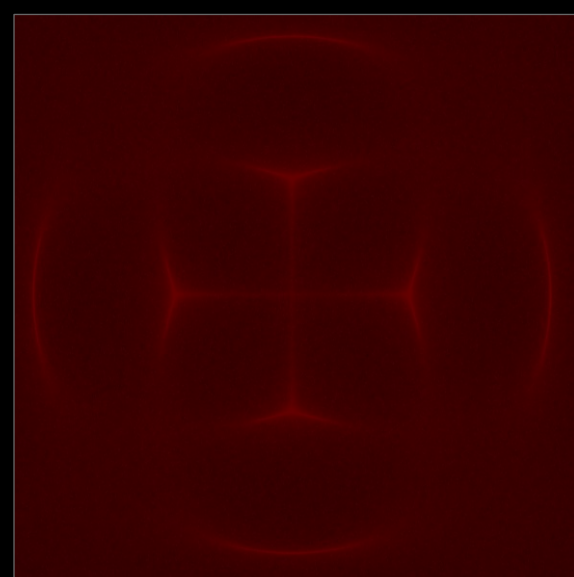
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



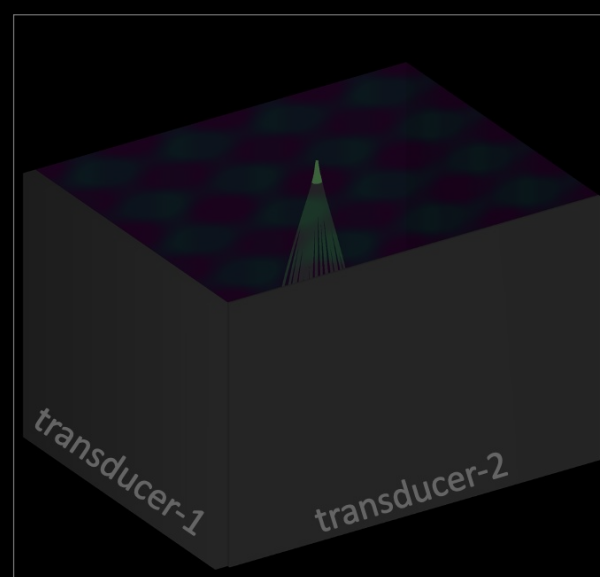
time-of-flight  
imaging



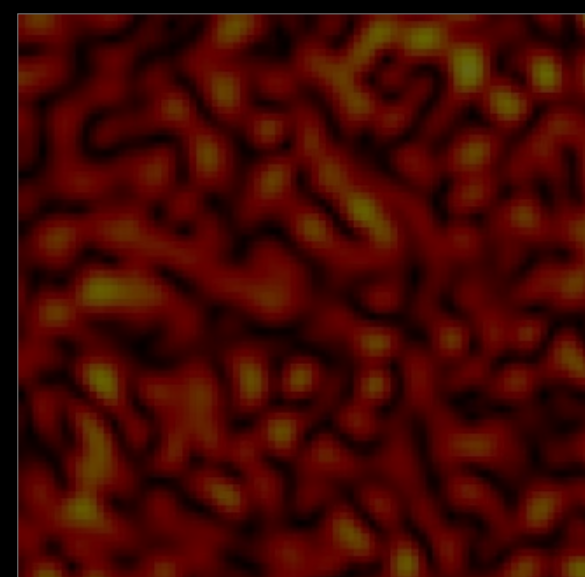
non-line-of-sight  
imaging



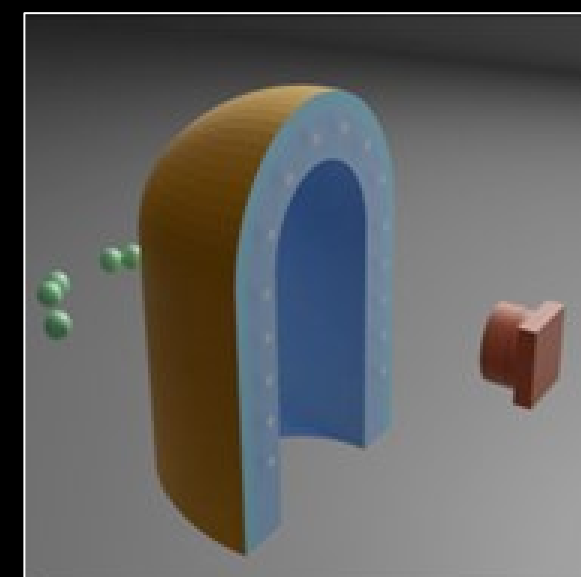
acousto-optic  
lensing



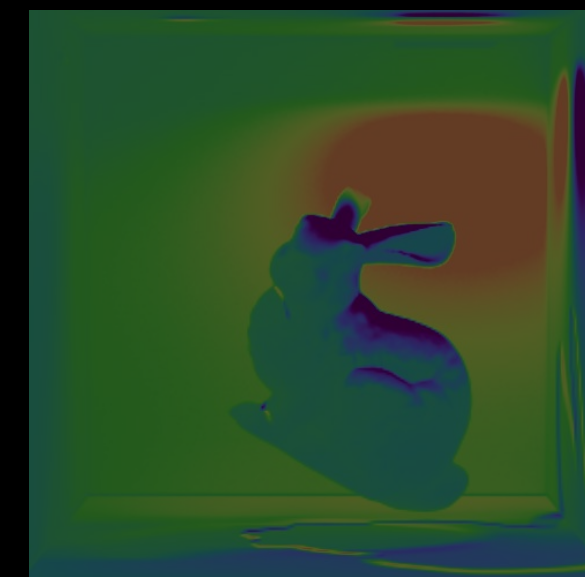
ultrafast light  
scanning



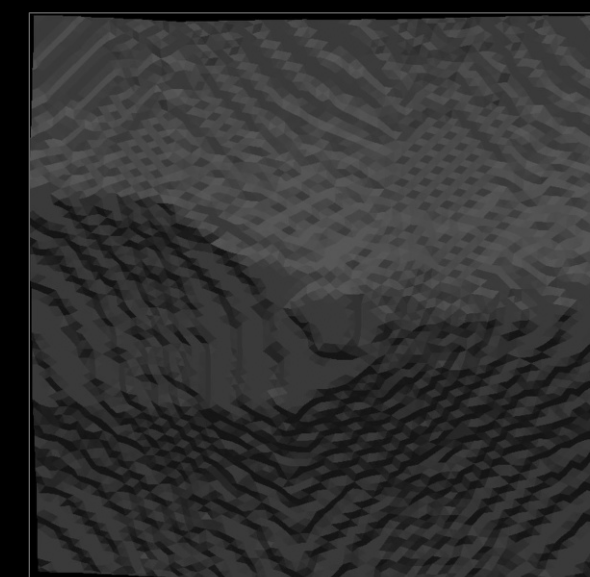
speckle  
imaging



tactile sensor  
design



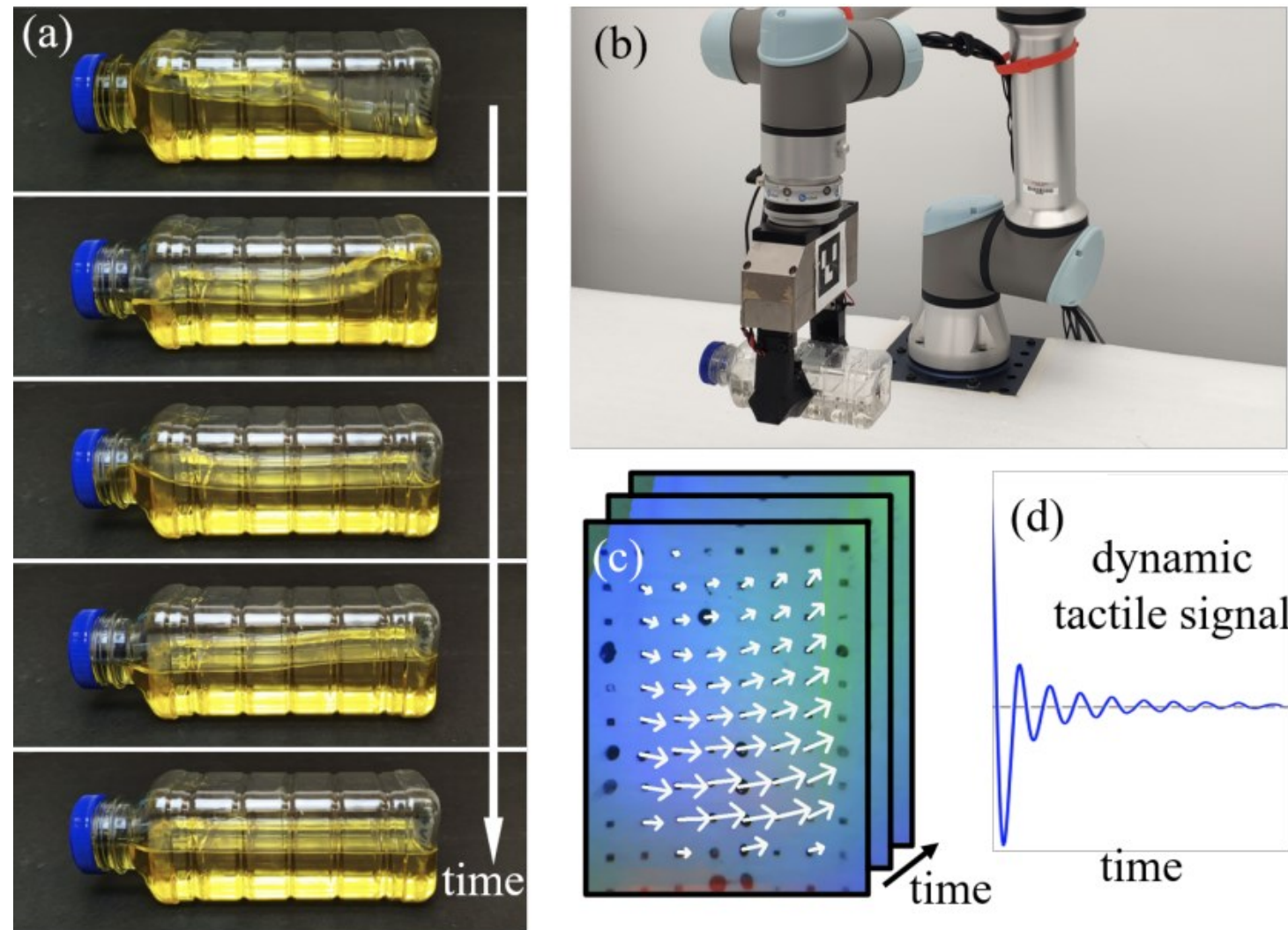
differentiable  
renderer



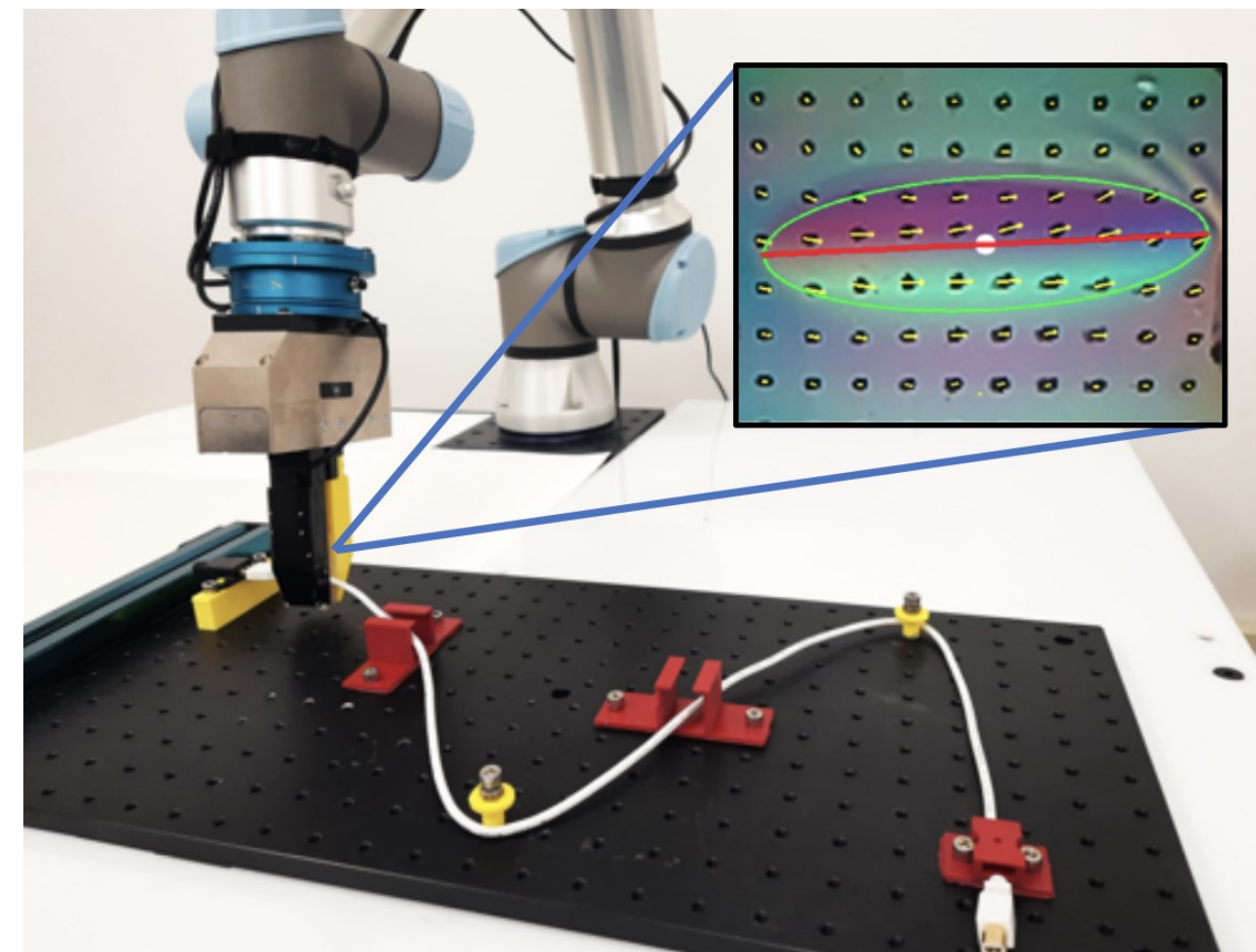
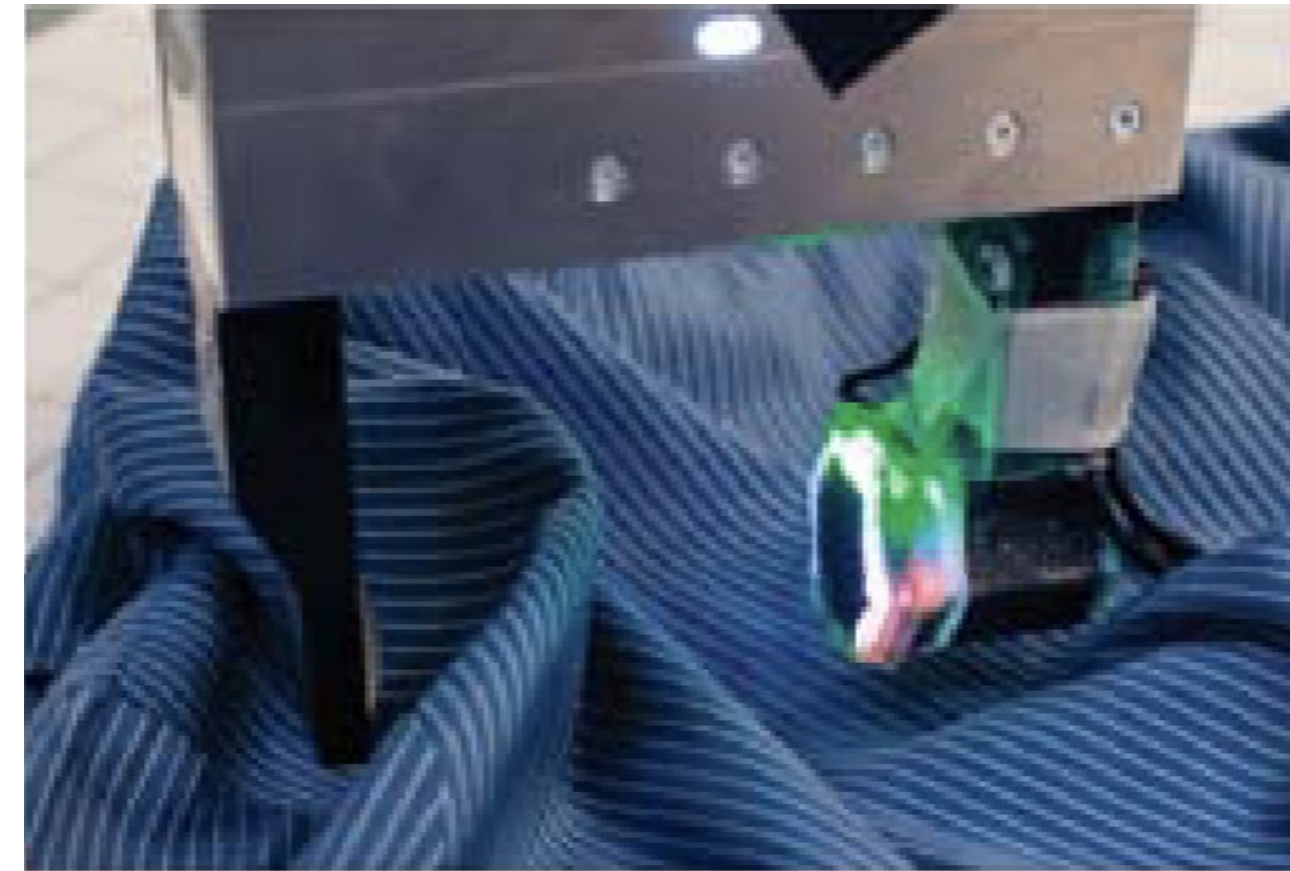
inverse  
problems



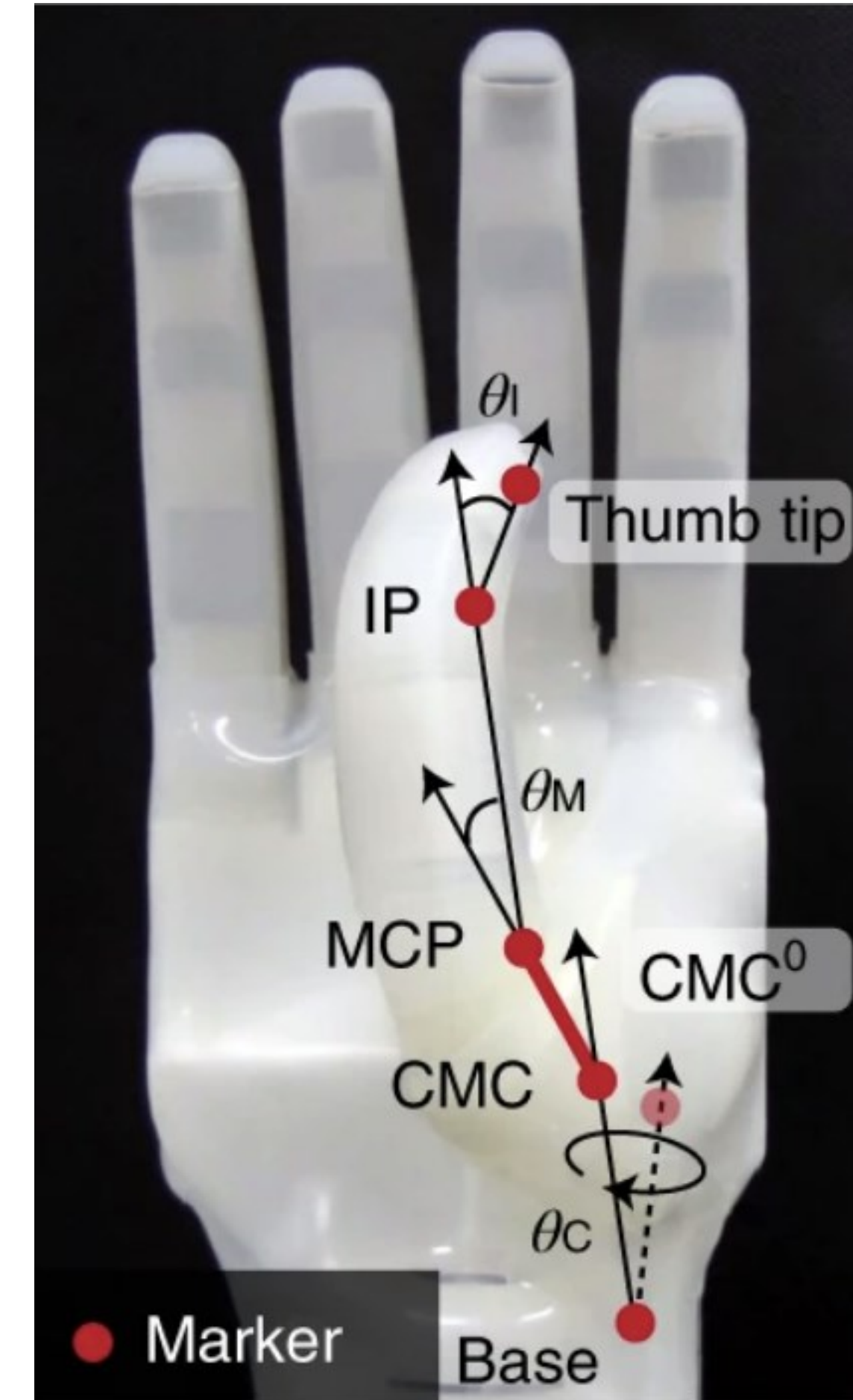
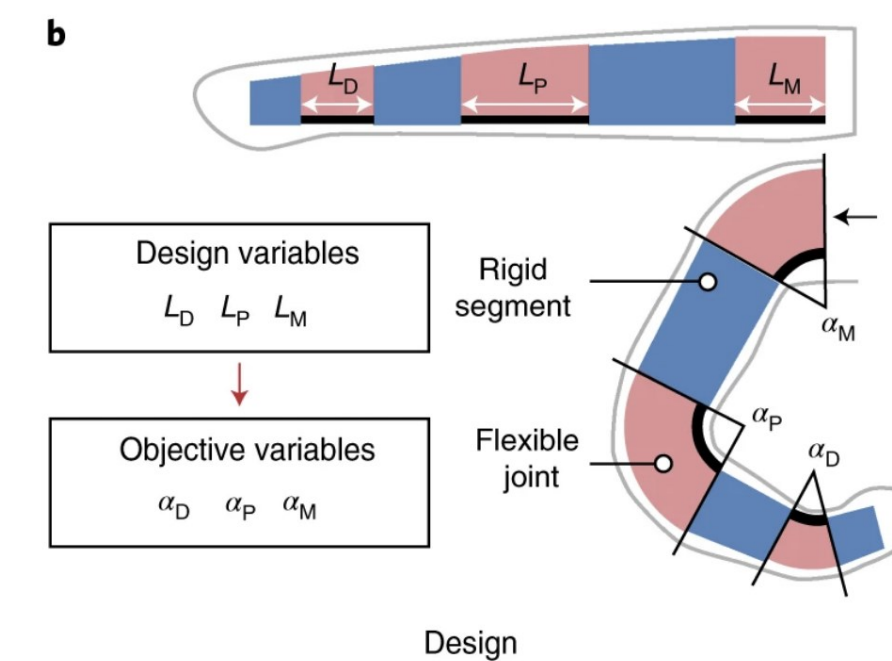
# Why tactile sensing?



Object perception  
[Huang et.al. 2022]

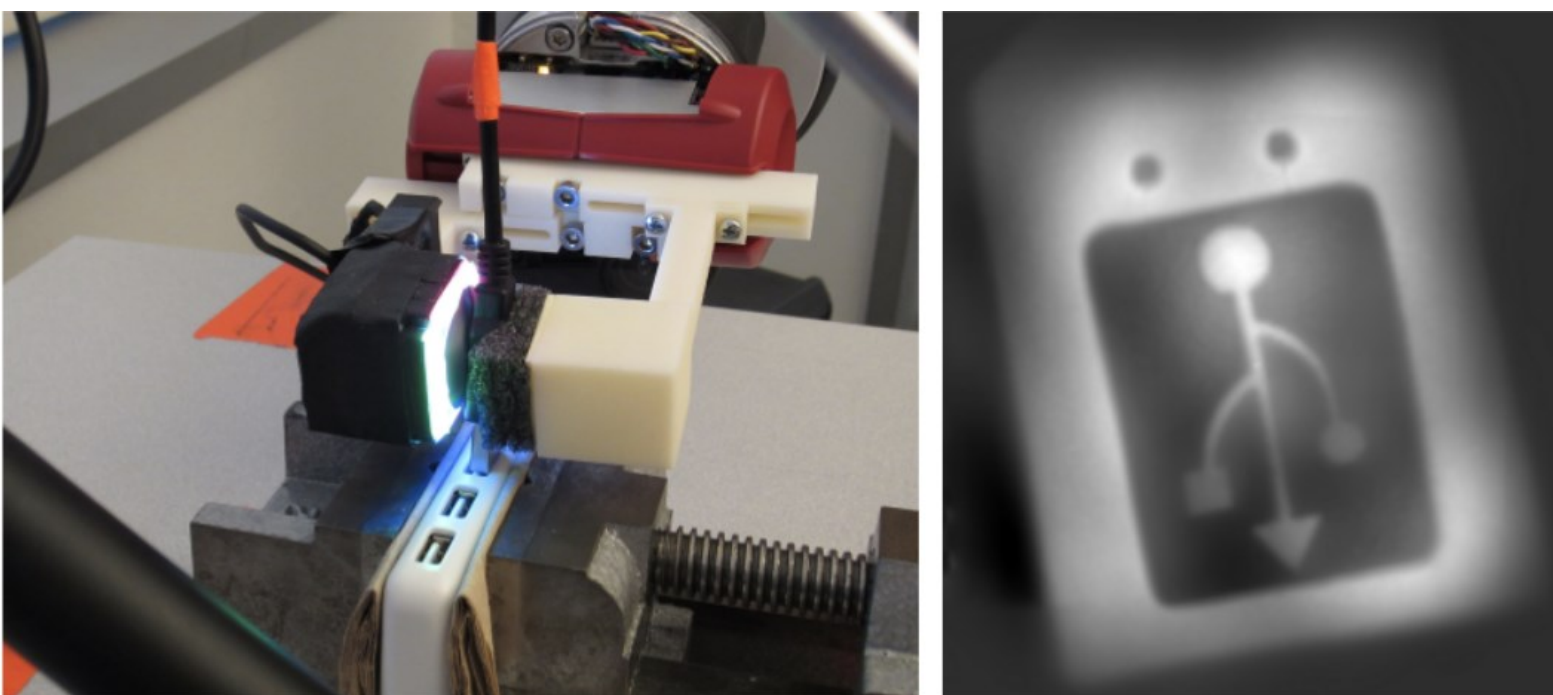


Robotic manipulation  
[Yuan et.al. 2018]  
[Wilson et.al. 2023]



Neuroprosthetics  
[Gu et.al. 2023]

Advanced manufacturing  
[Li et.al. 2014]

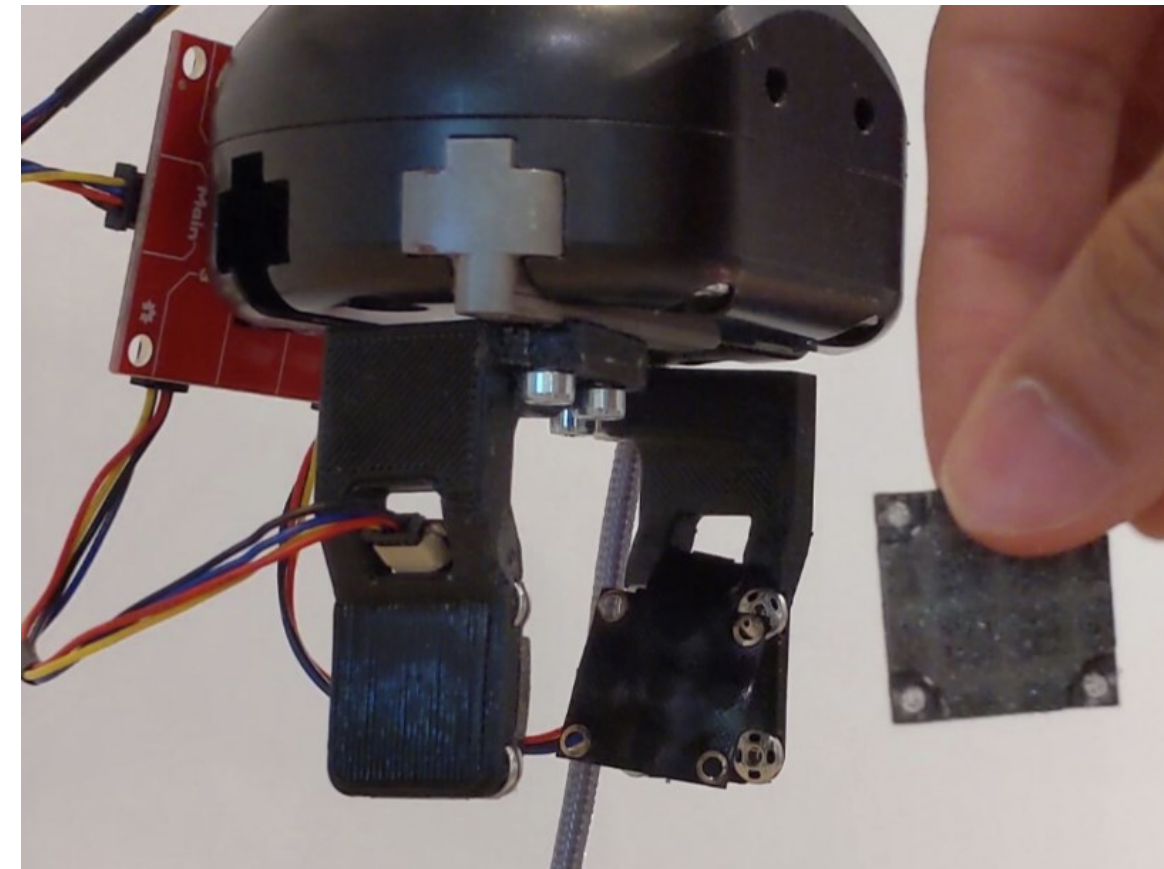




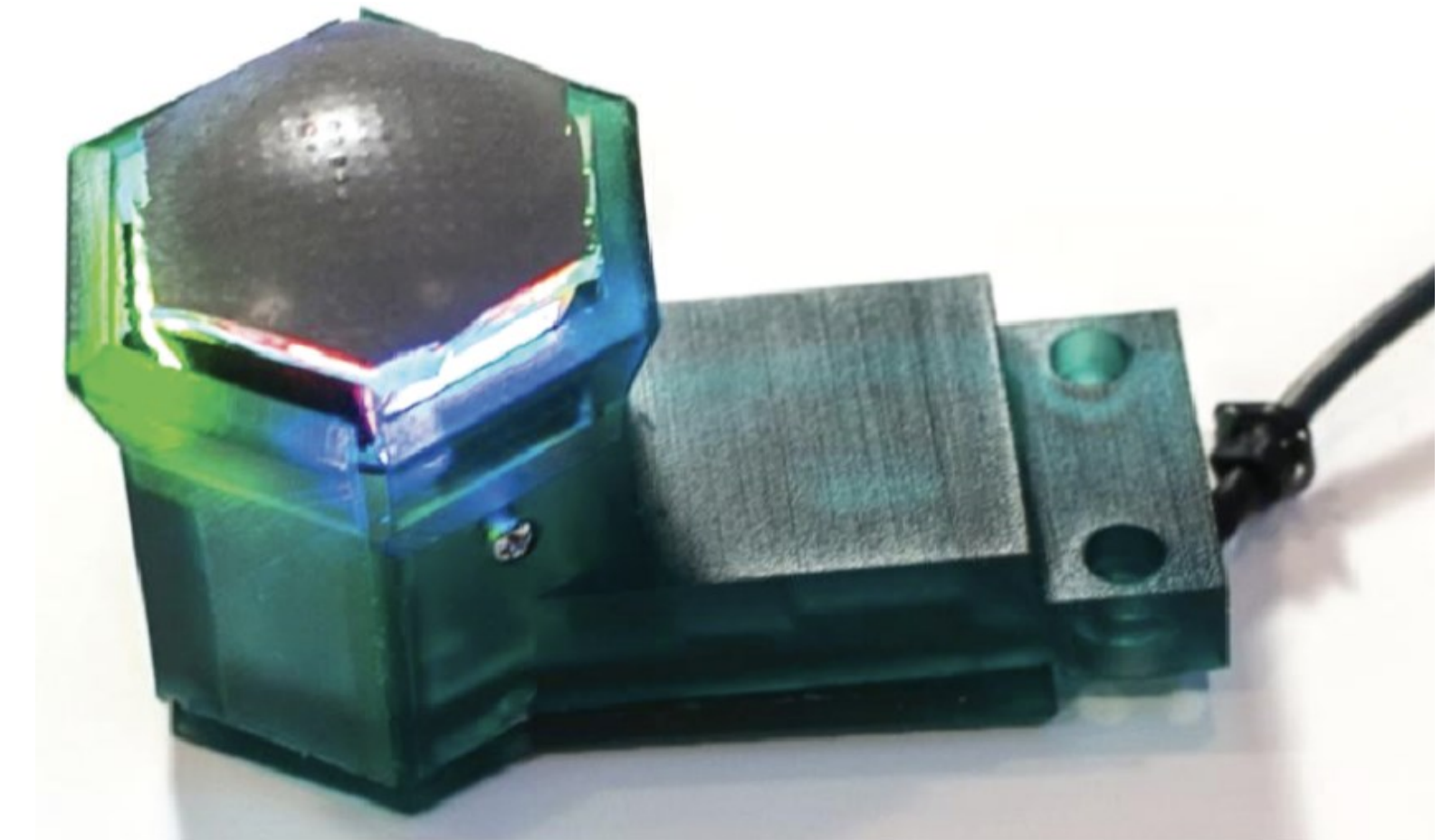
# Tactile sensors



BioTac  
ionically-conductive fluid based



ReSkin  
Magnetic field based



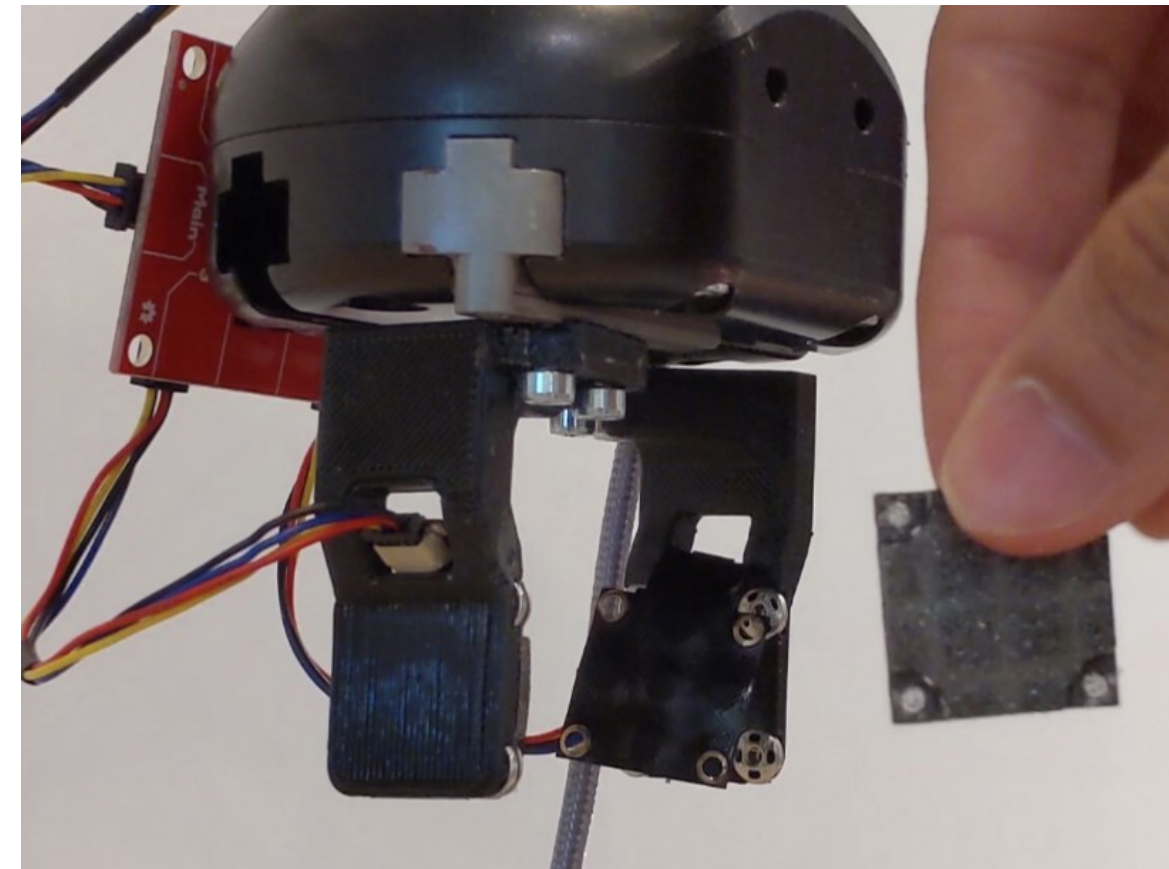
GelSight  
Vision-based



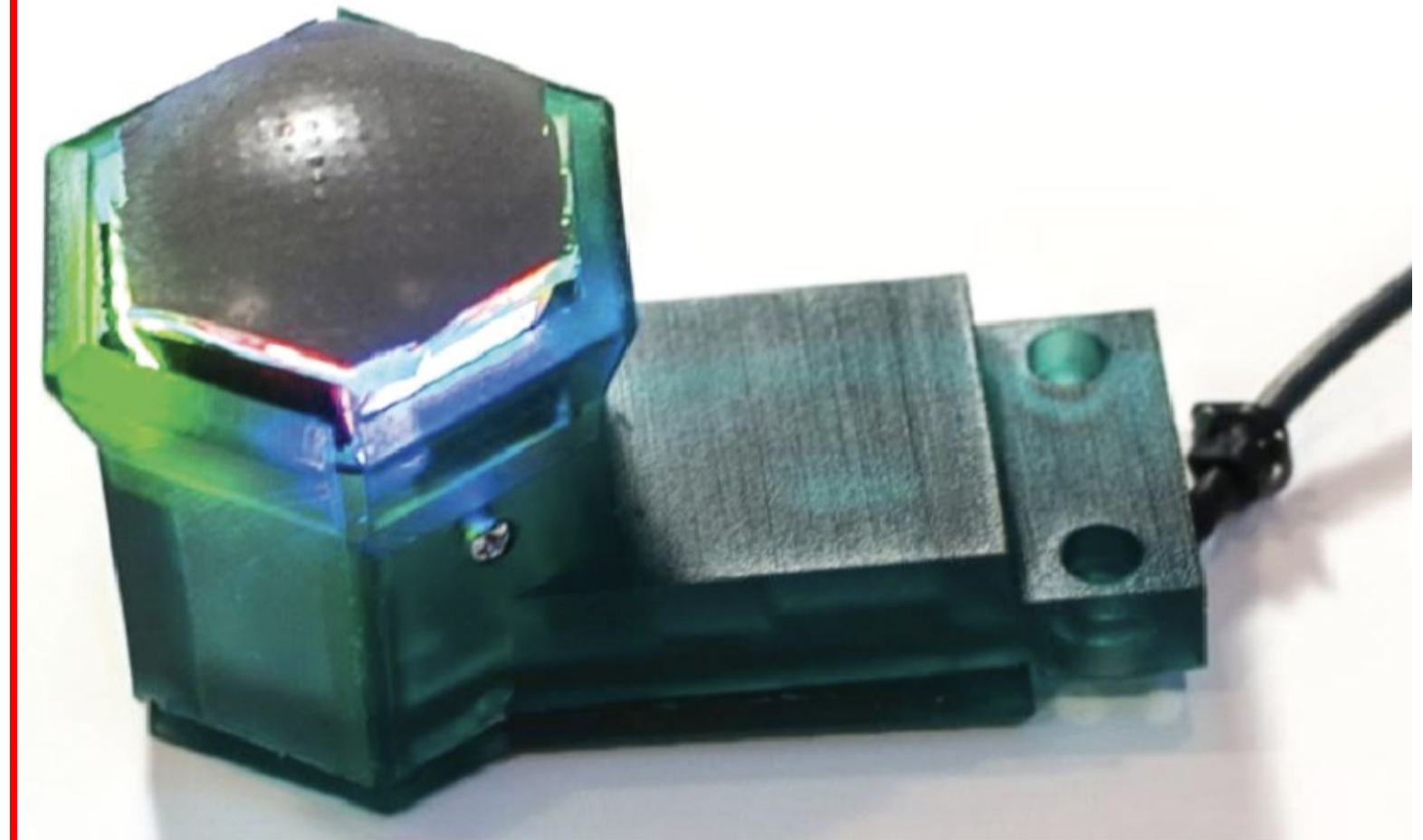
# Tactile sensors



BioTac  
ionically-conductive fluid based

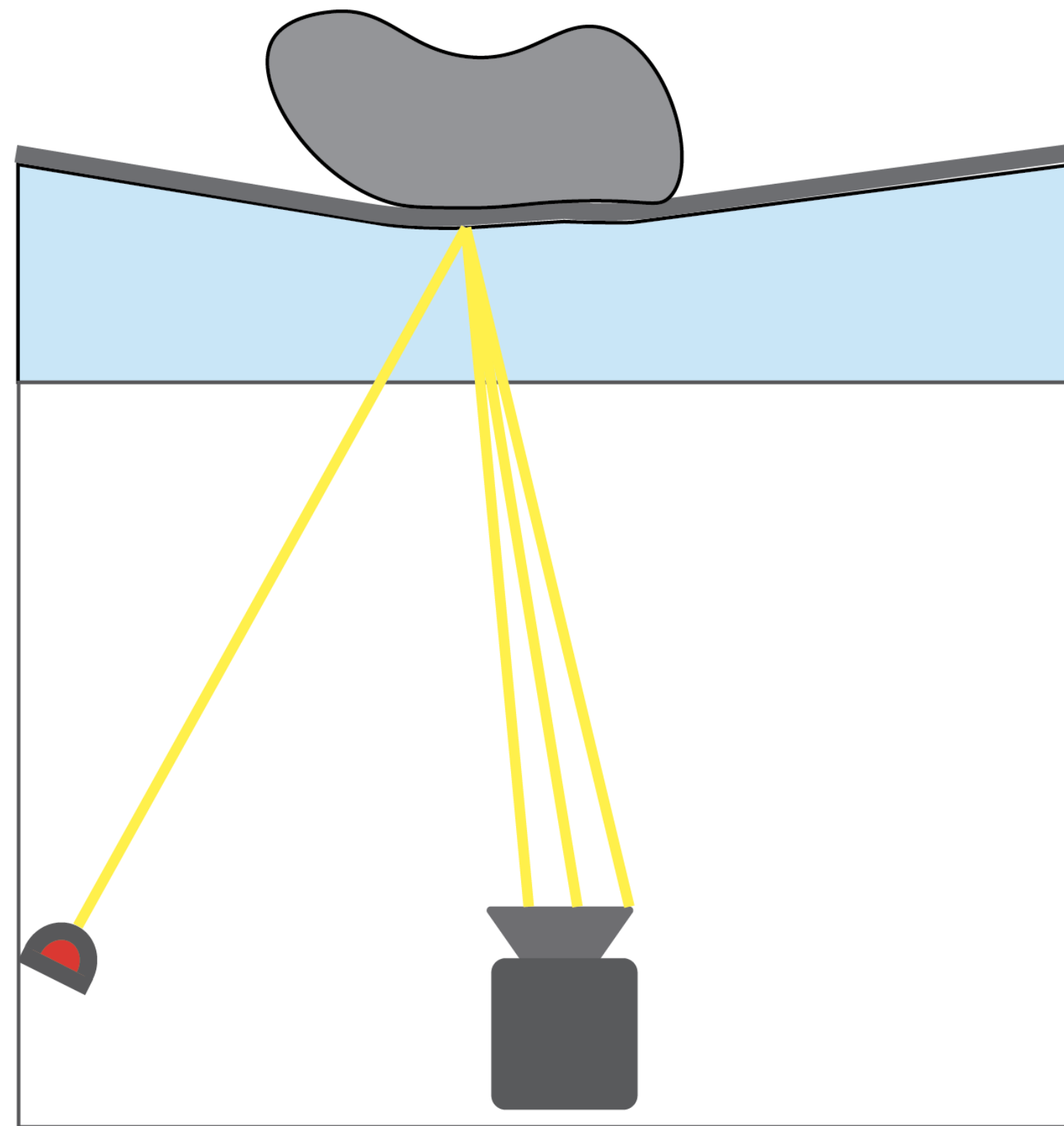


ReSkin  
Magnetic field based

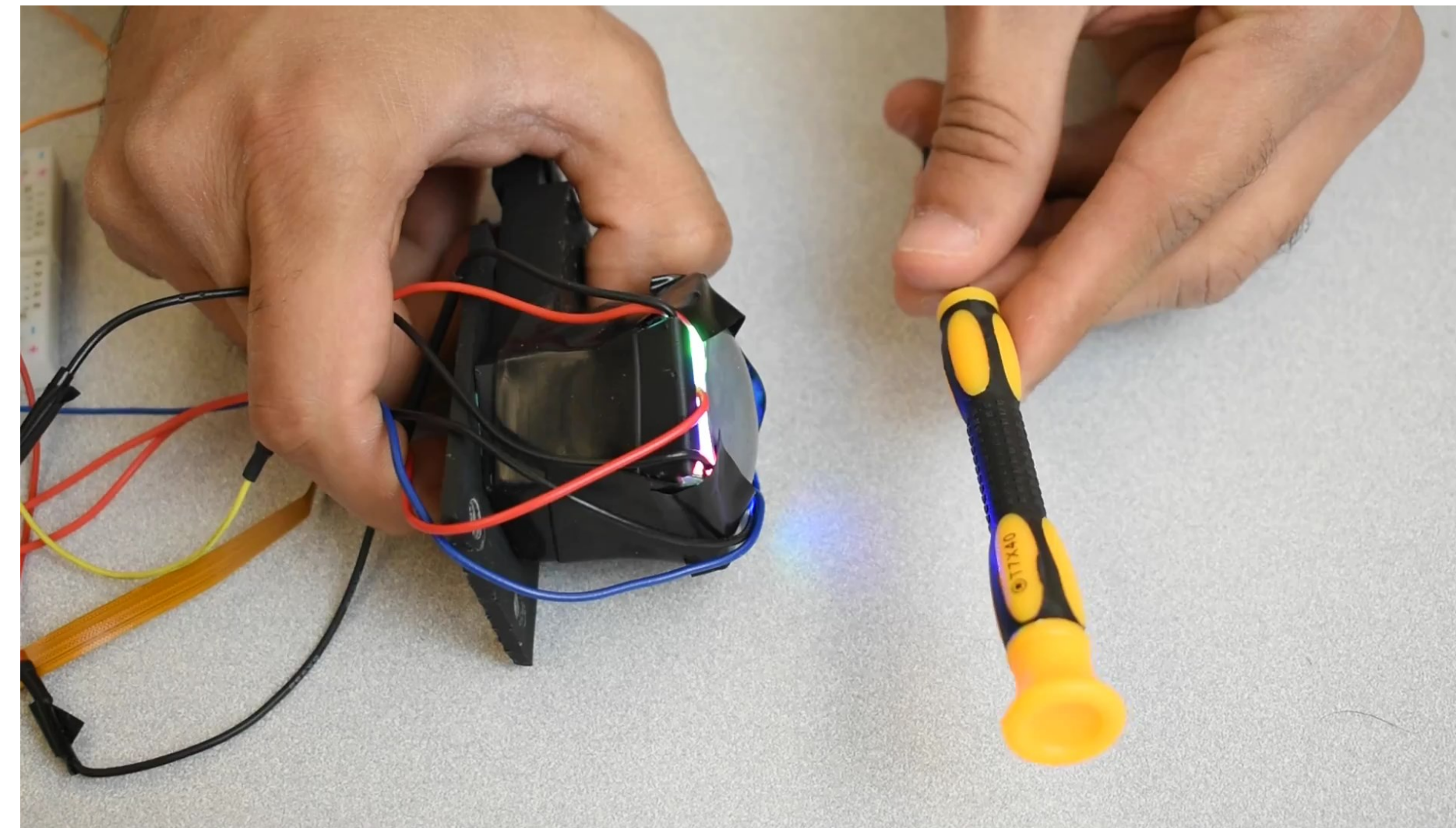


GelSight  
Vision-based

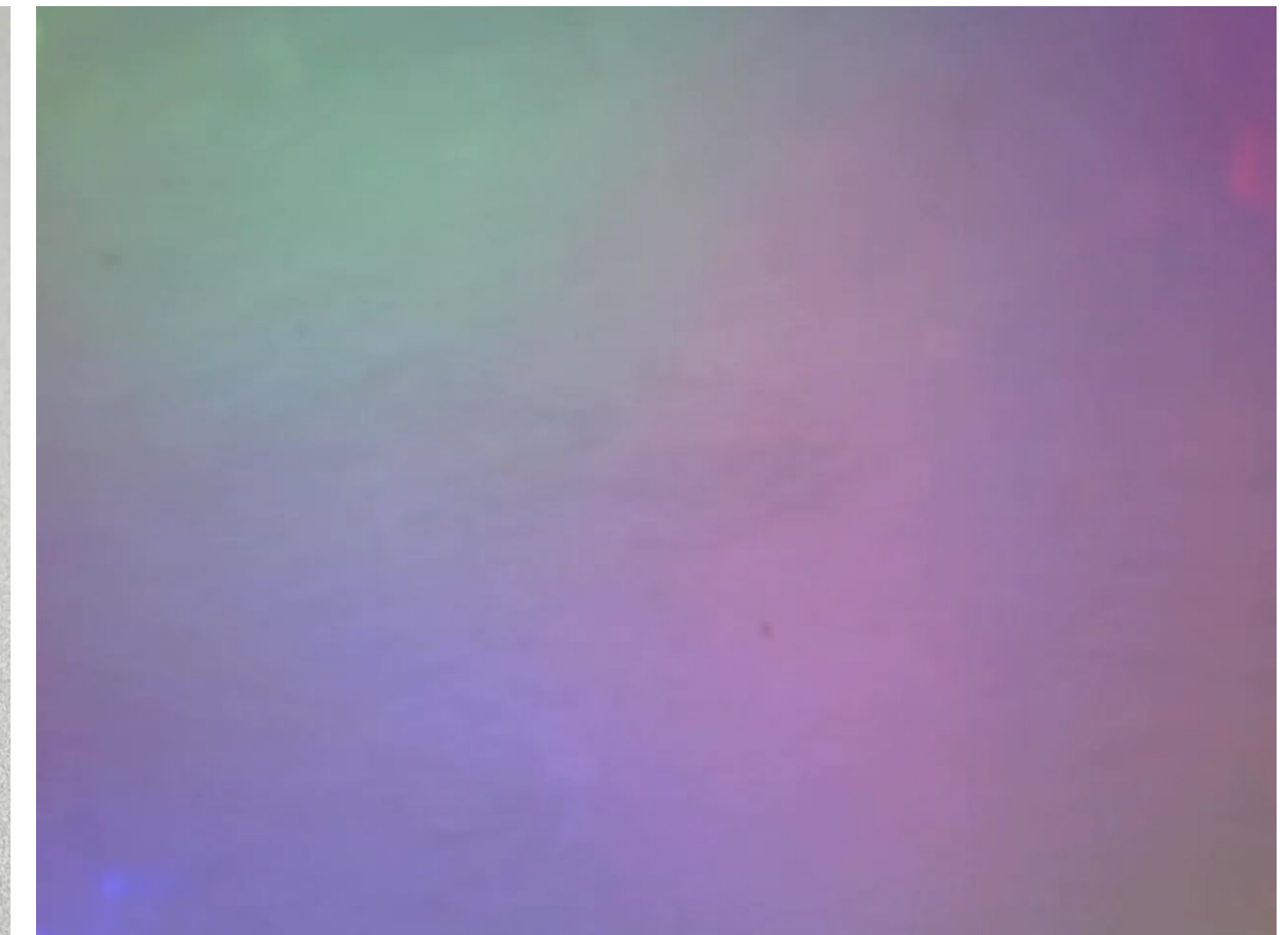
# Vision-based tactile sensors: working principle



Sensor schematic



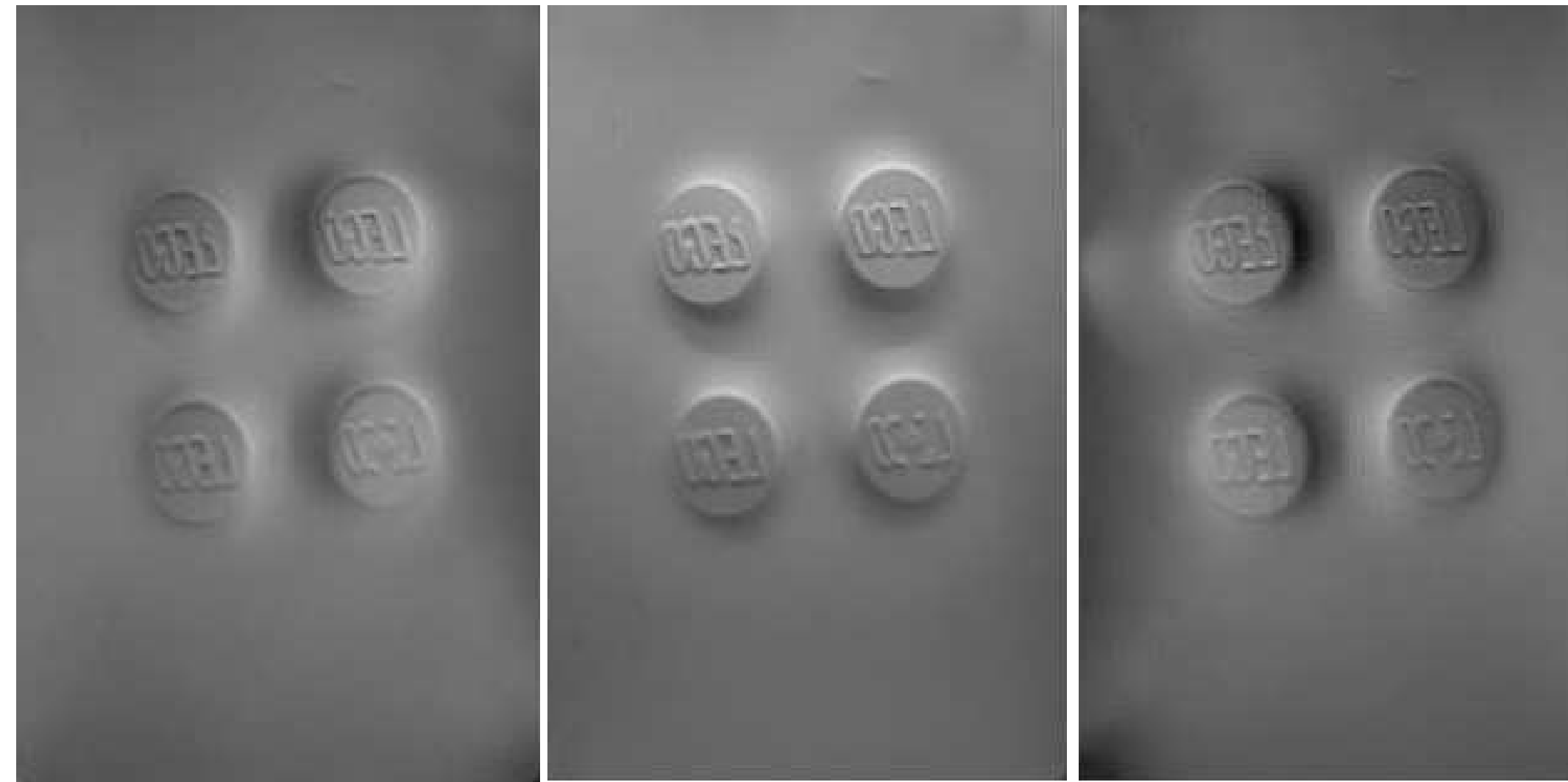
Live view



Sensor view



# Vision-based tactile sensors: photometric stereo



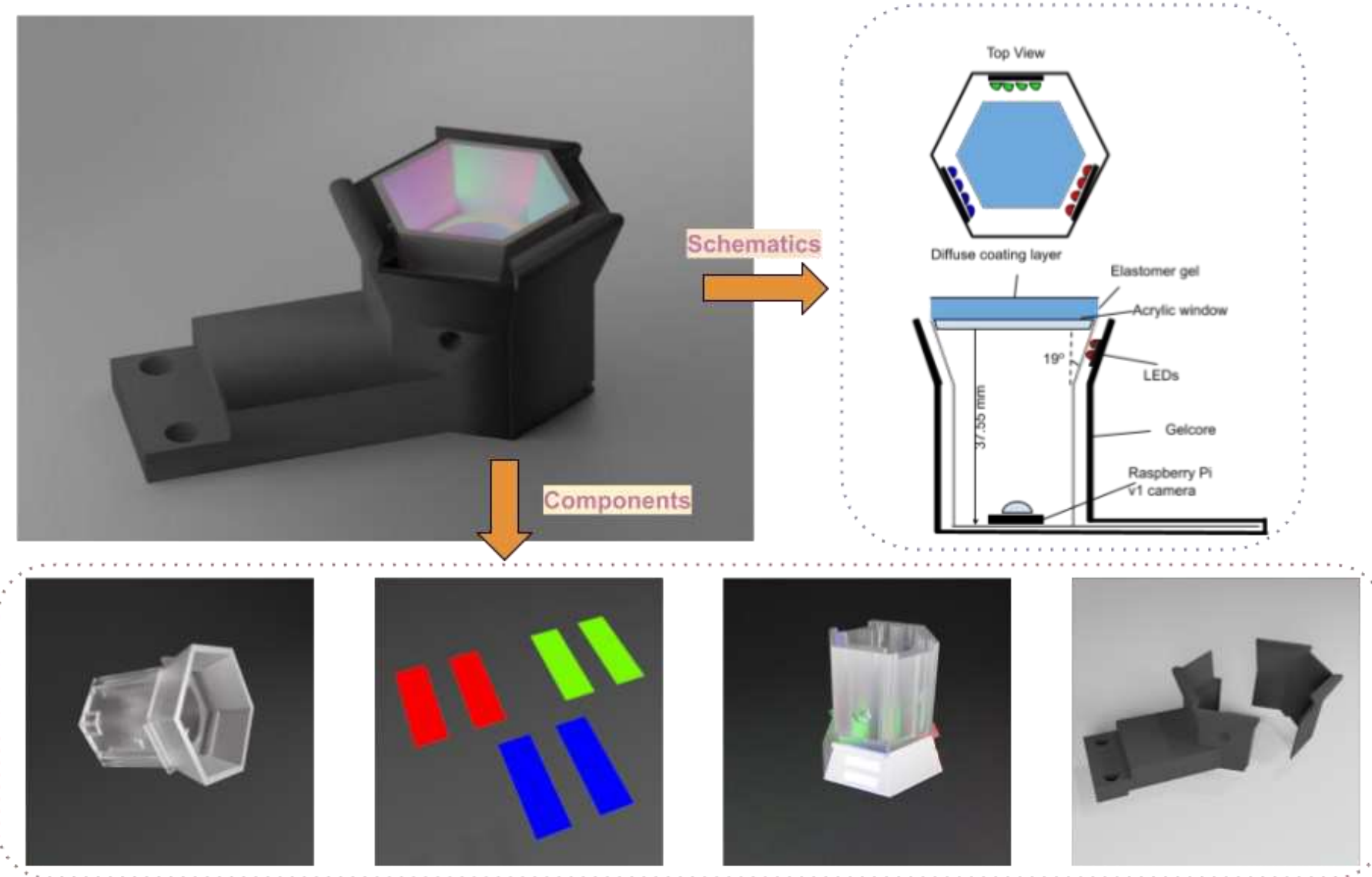
Camera Images



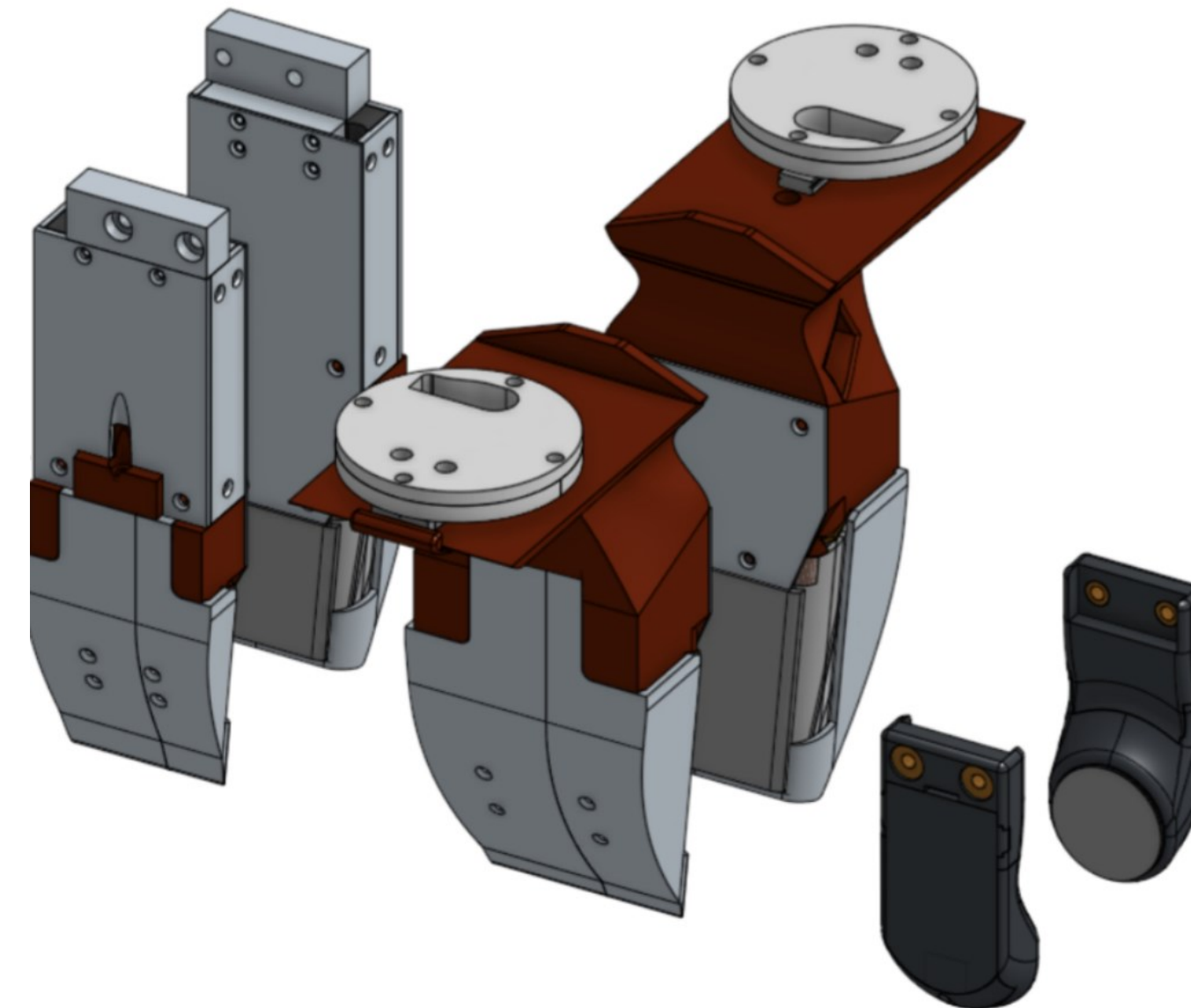
Normals

With photometric stereo, GelSight can encode surface normals as an RGB image.

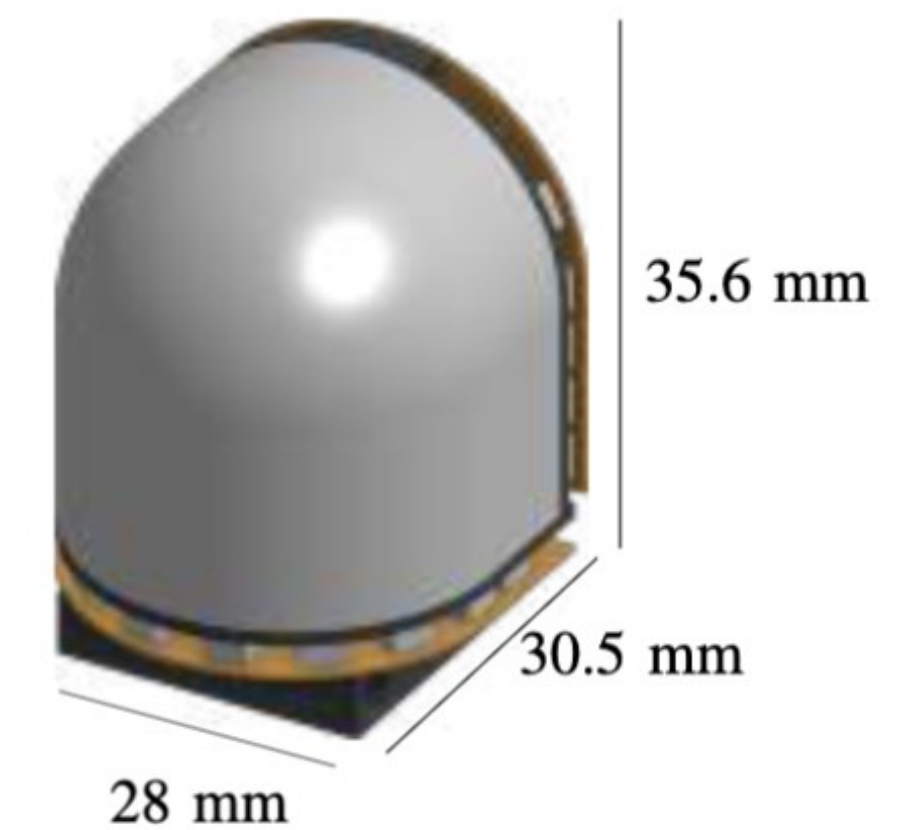
# Vision-based tactile sensors: design variants



FlatGel GelSight  
Dong et. al. 2017  
Agarwal et. al. 2021



GelSlim Family  
Donlon et. al. 2018  
Ma et. al. 2019  
Hogan et. al. 2020  
Taylor et. al. 2021



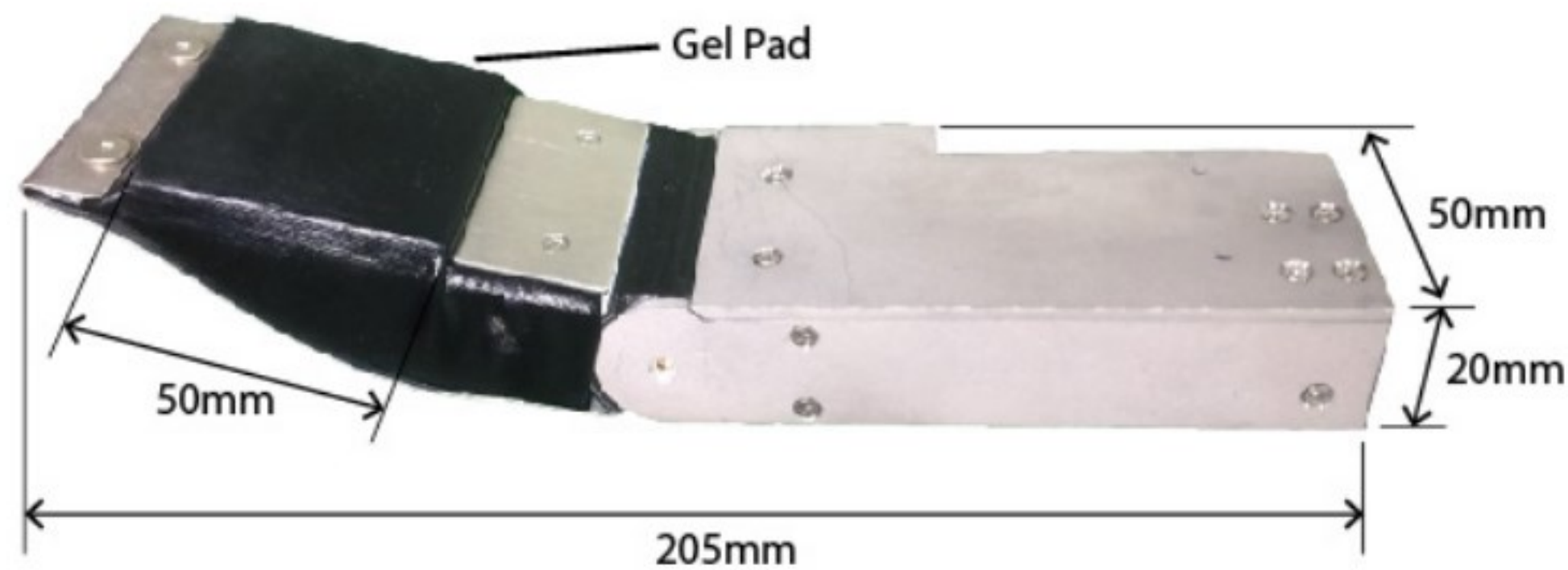
RoundTip GelSight  
Romero et. al. 2020



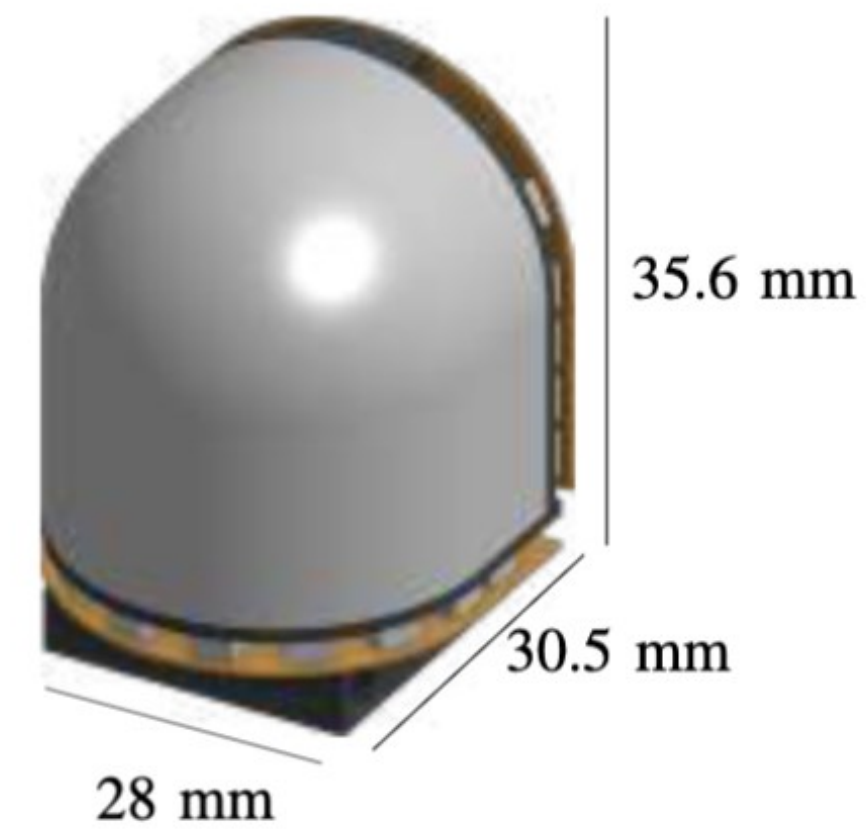
# Designing VBTS is hard

# Designing VBTS is hard

- Diversity of sensor shape and required form-factor



Flat sensing surface

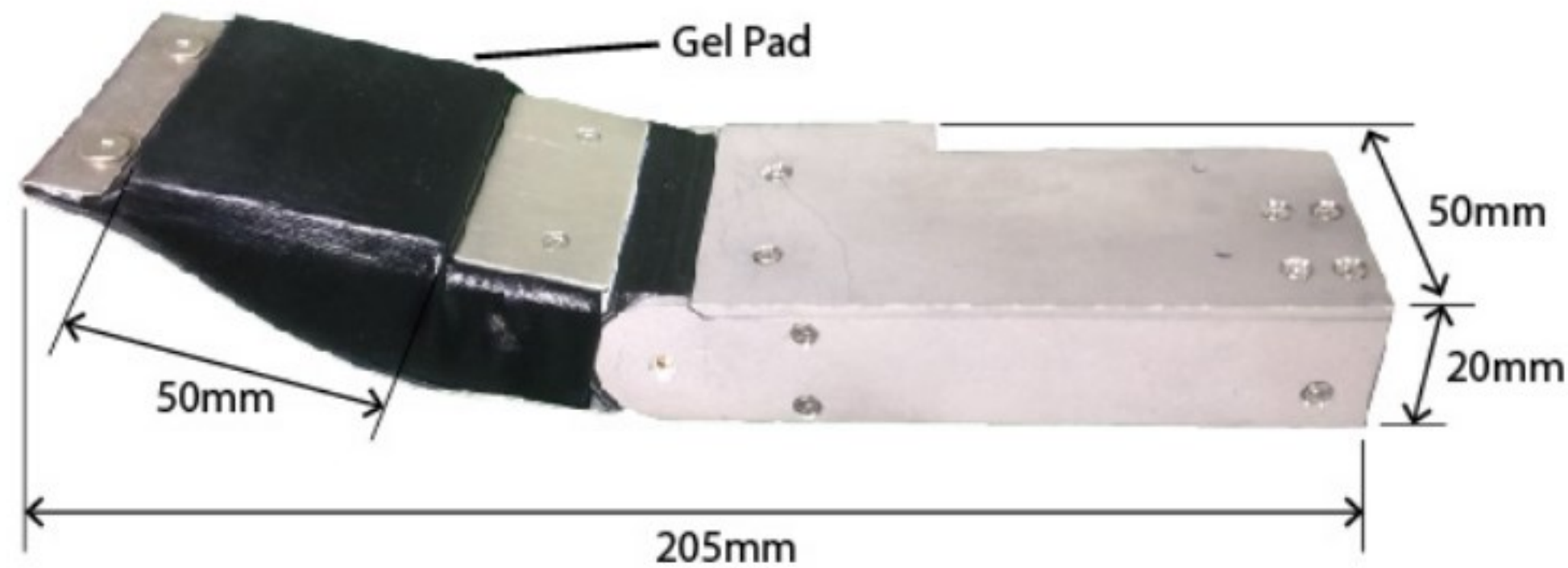


Curved sensing surface

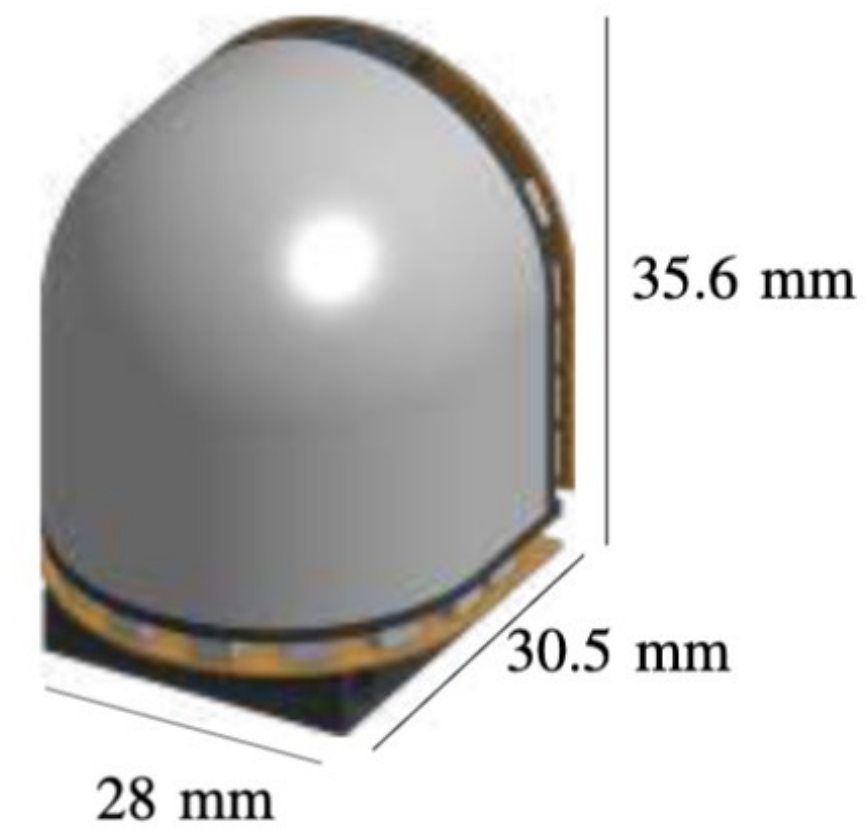


# Designing VBTS is hard

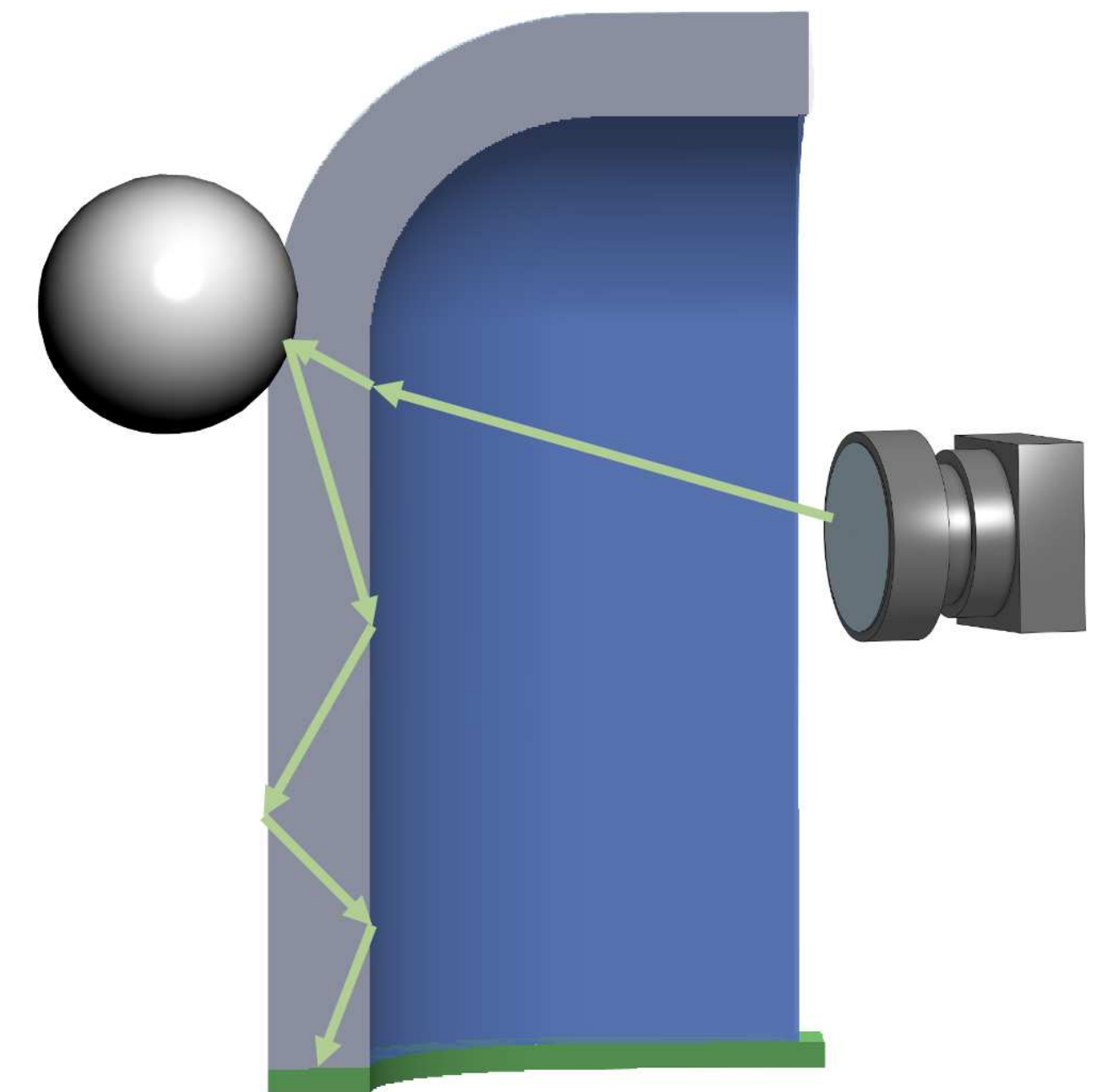
- Diversity of sensor shape and required form-factor
- Complex light interaction



Flat sensing surface



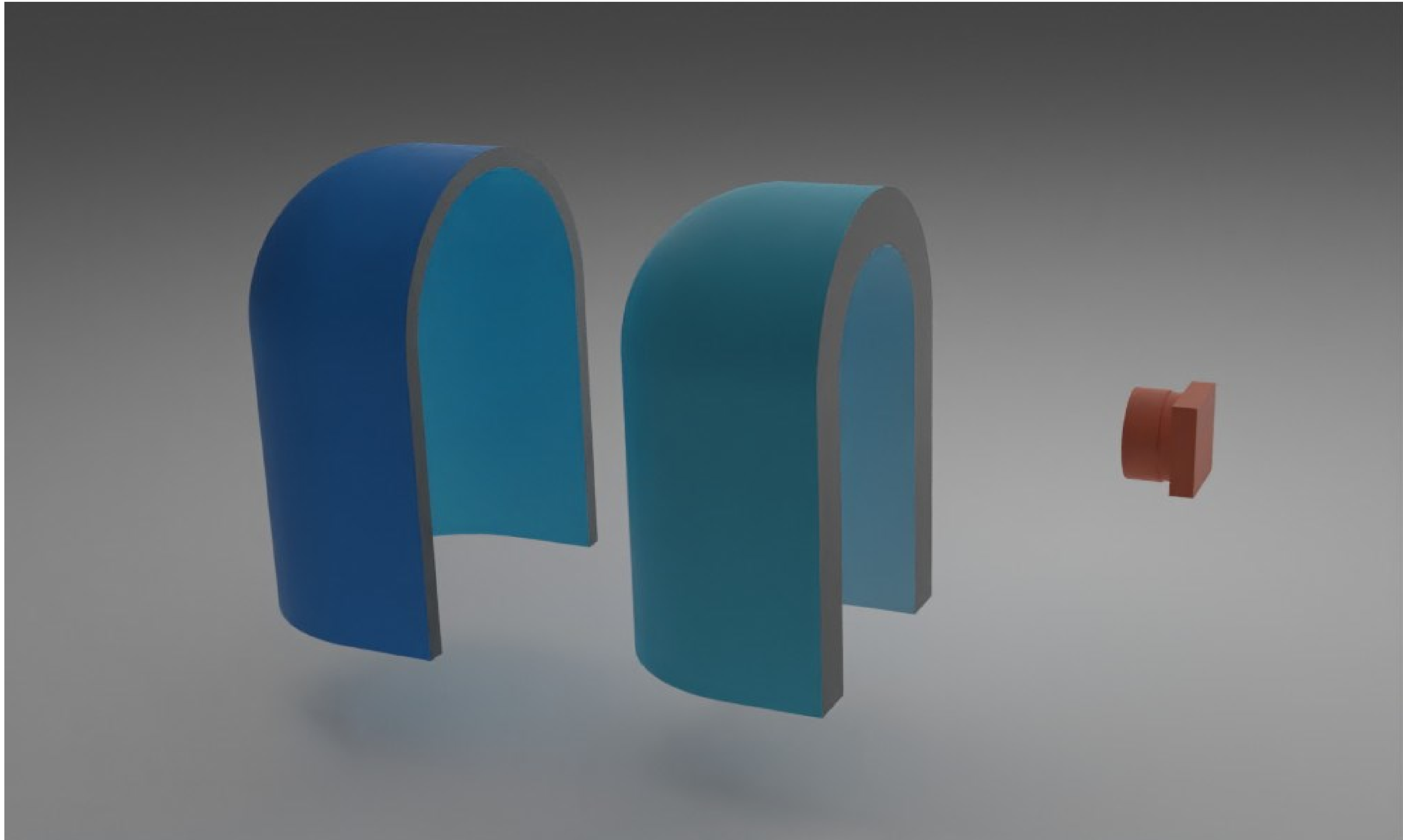
Curved sensing surface



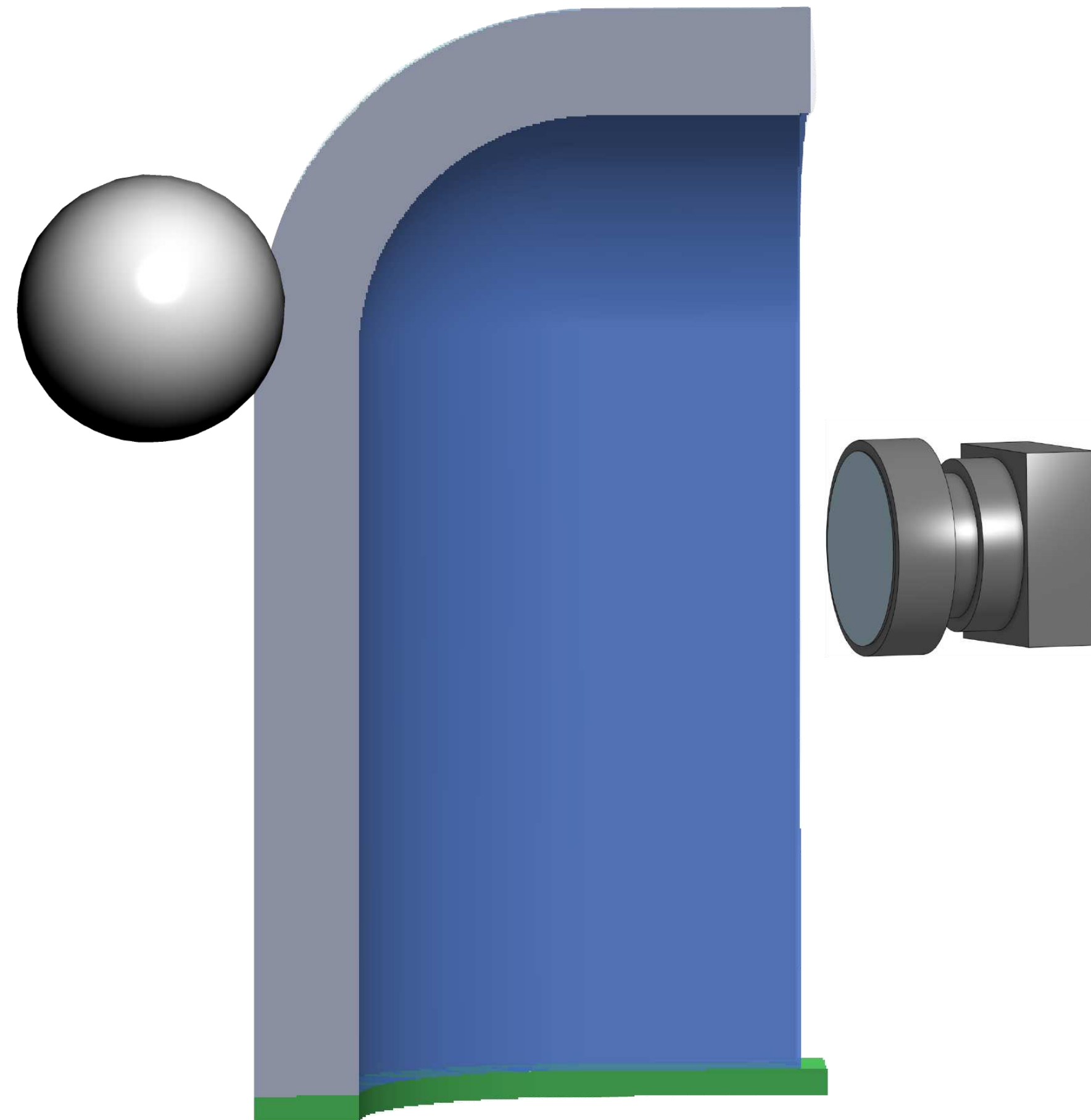
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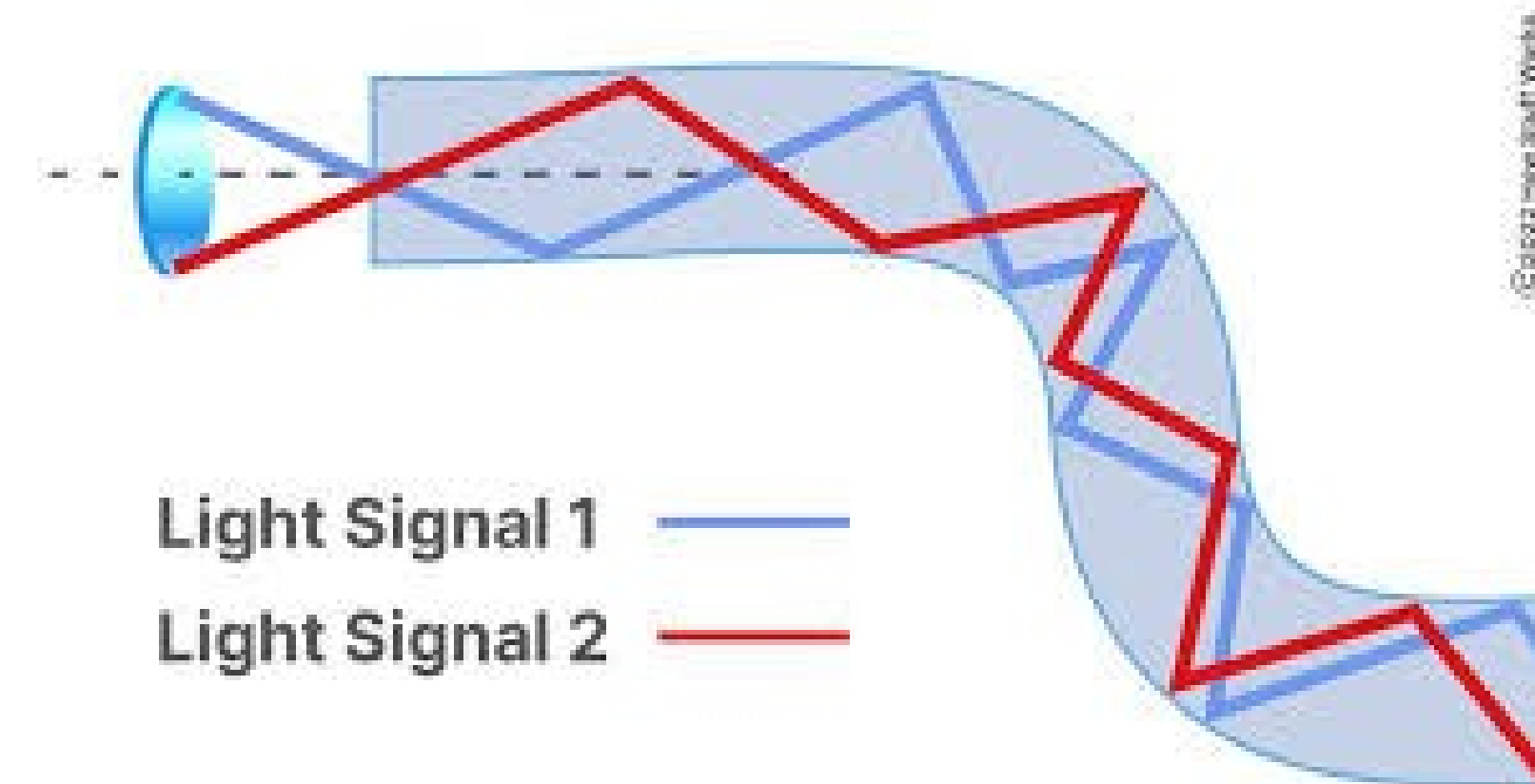
# Curved tactile sensor



# Curved tactile sensor: working principle



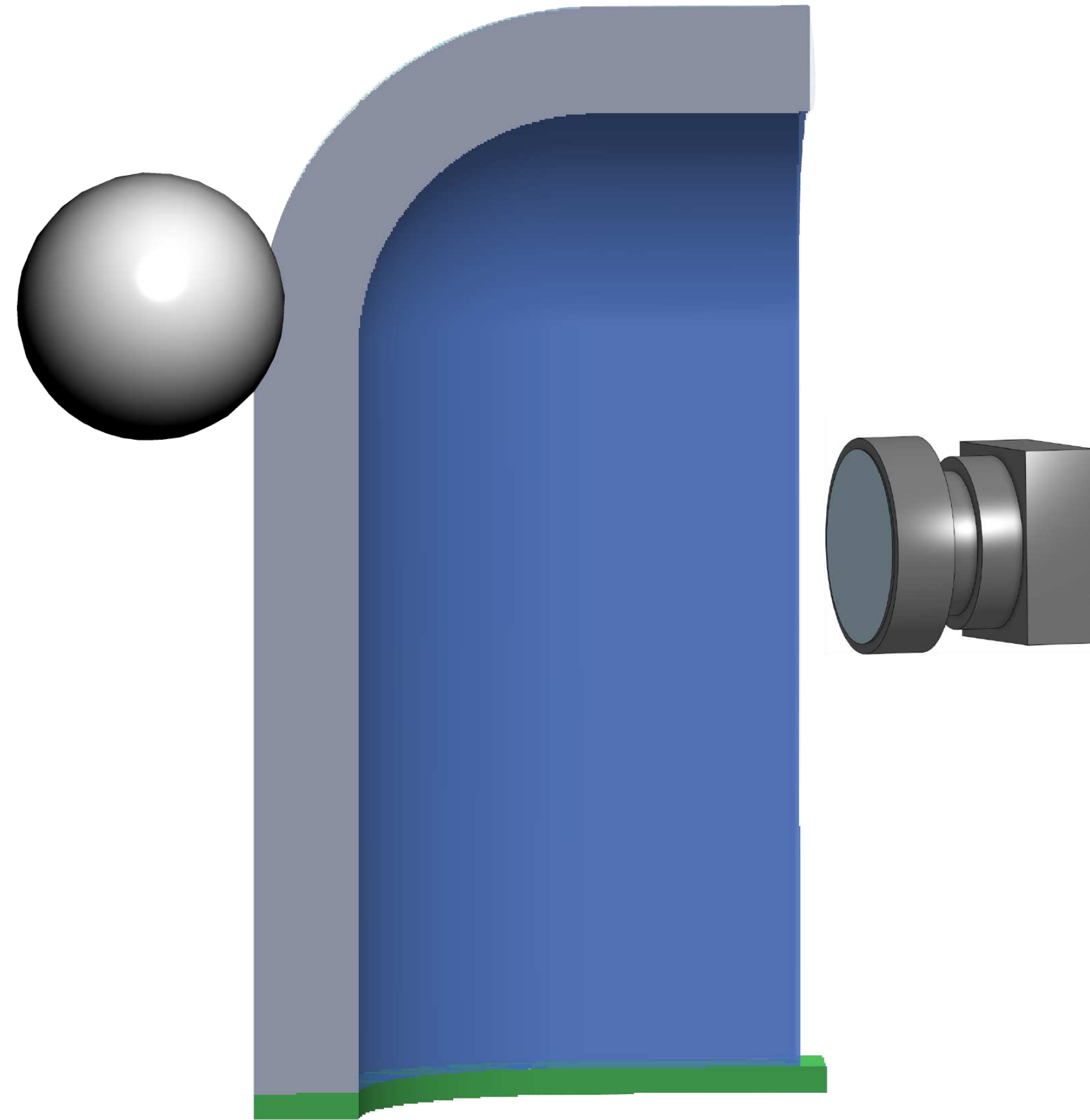
Section View



Light Piping

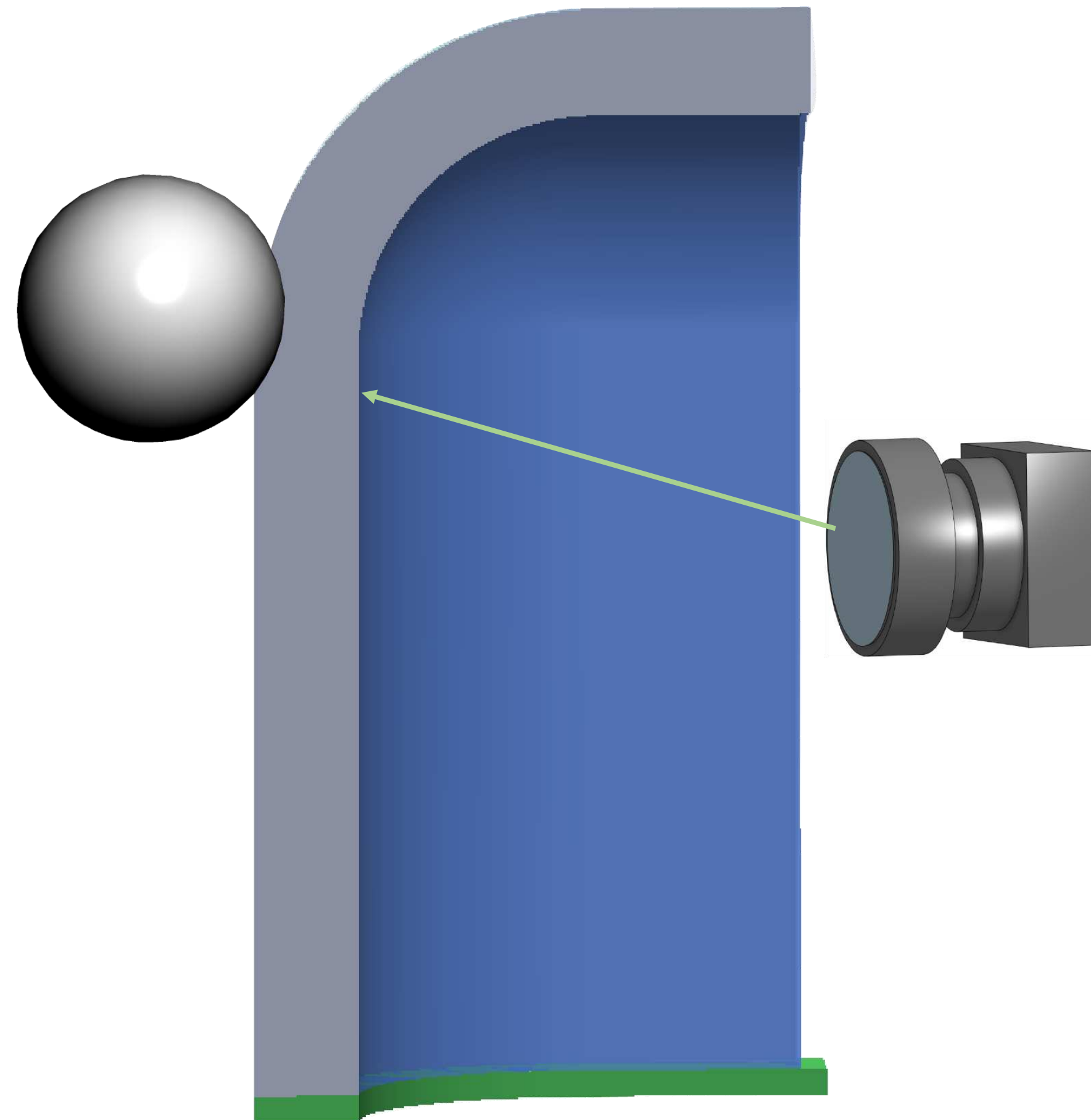


# Curved tactile sensor: working principle

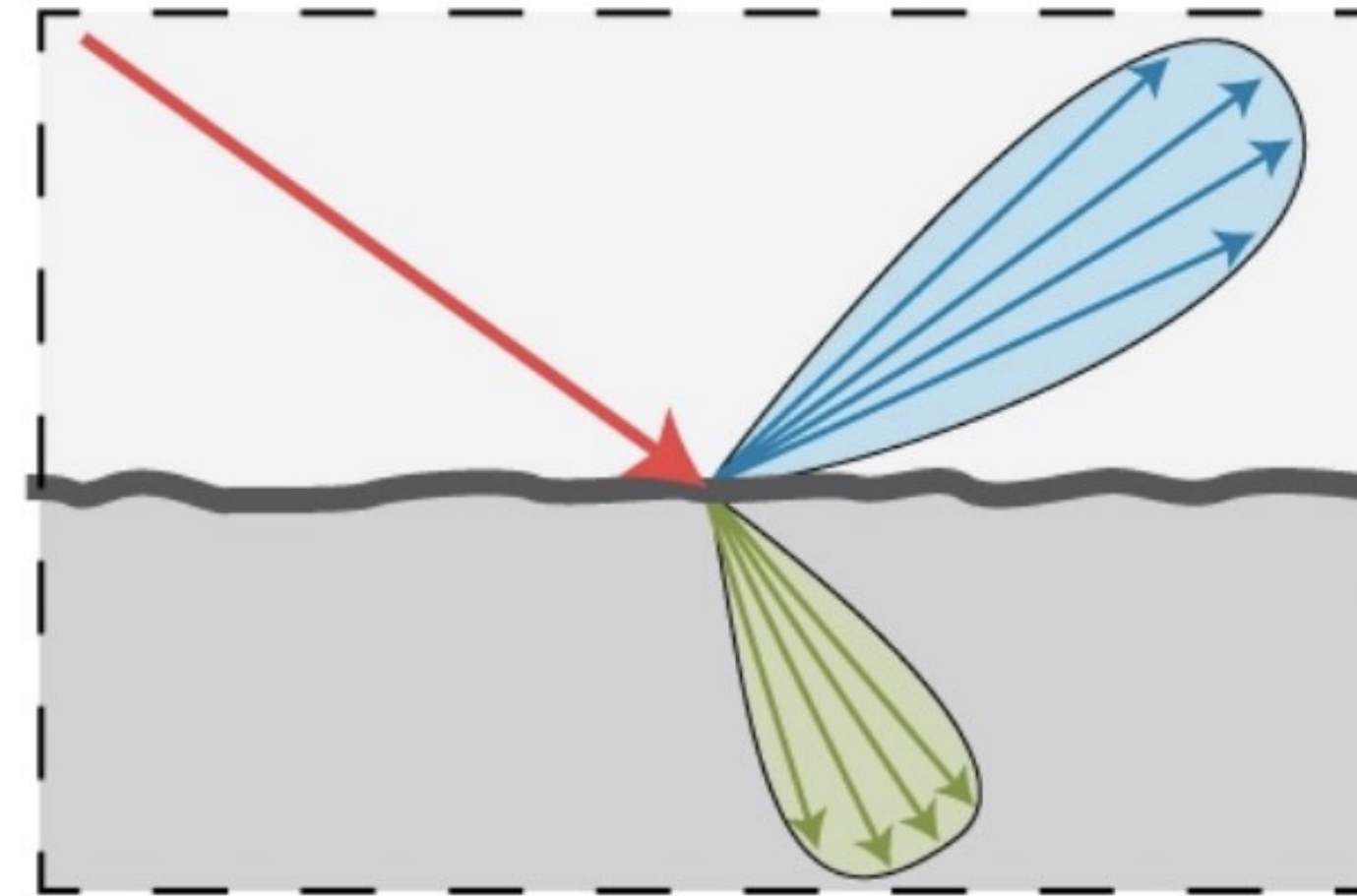


Section View

# Curved tactile sensor: working principle



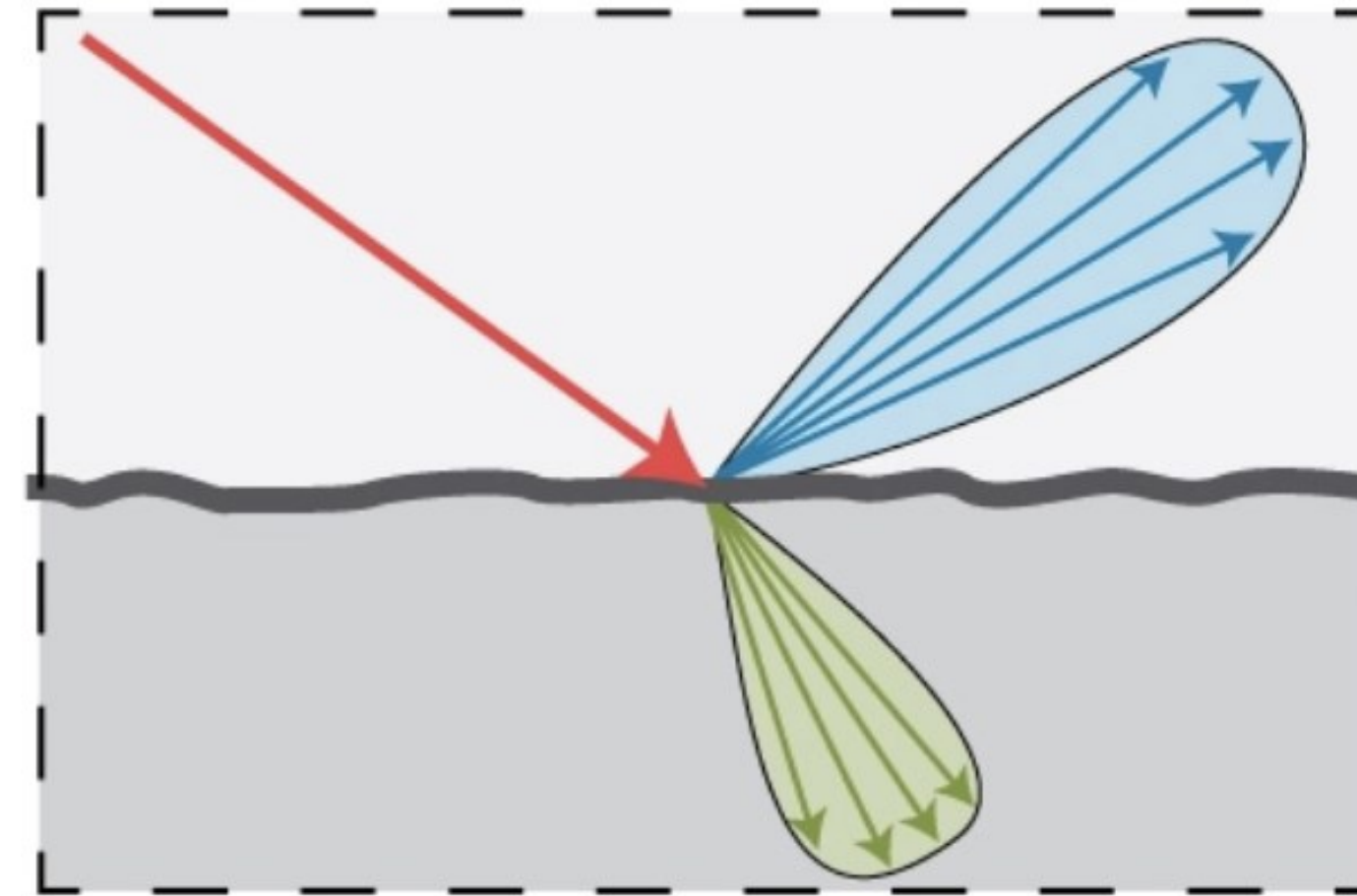
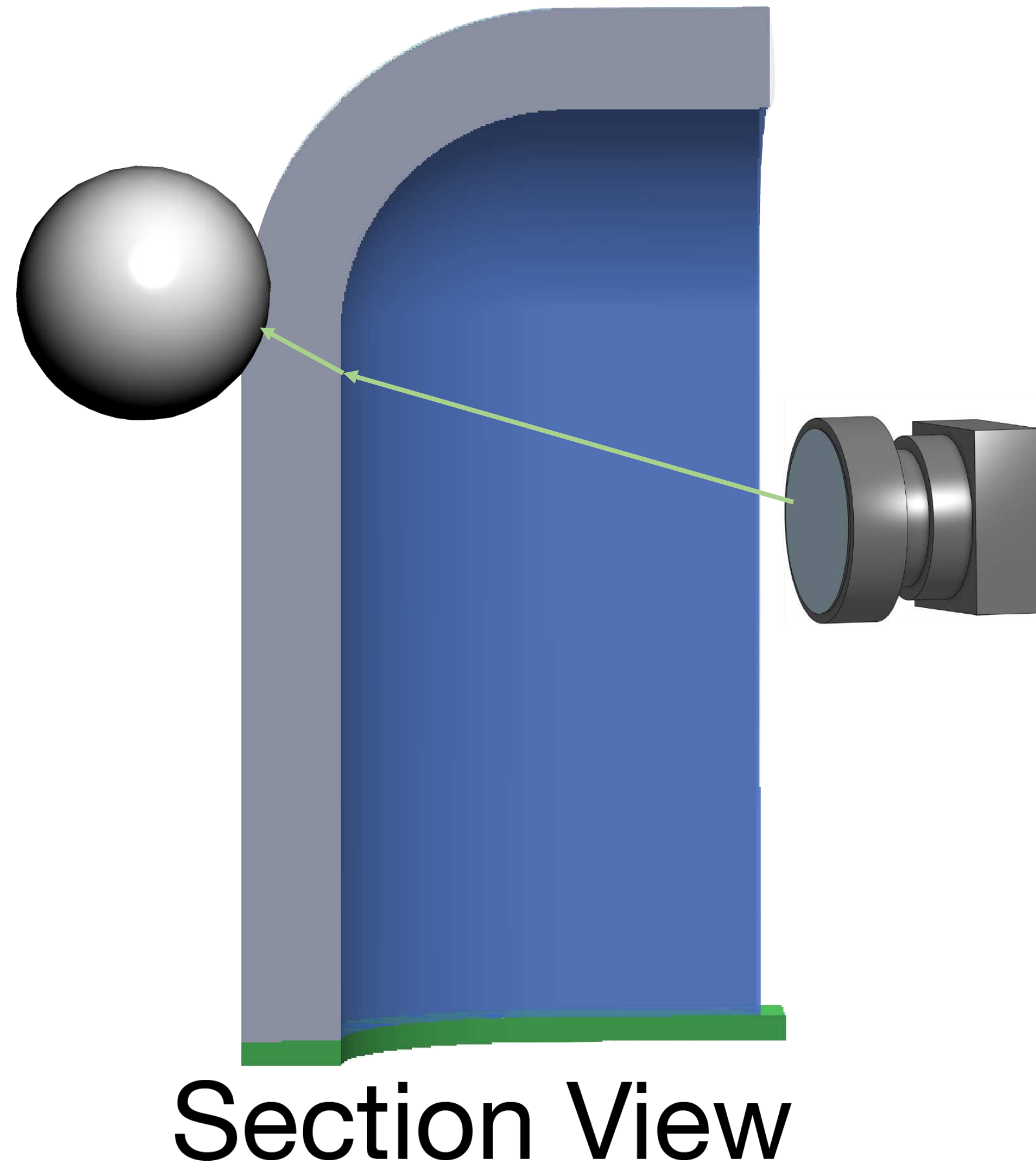
Section View



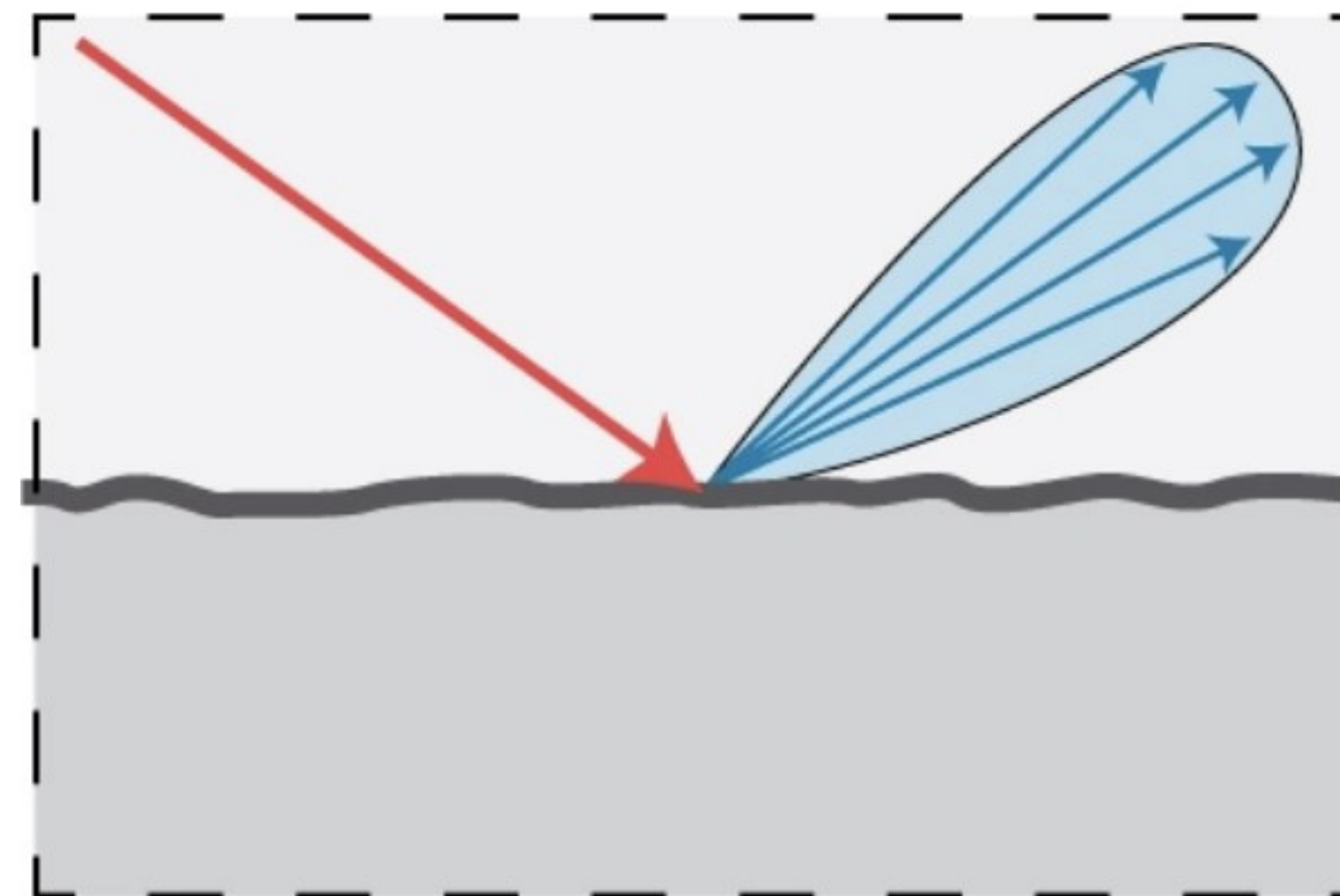
Refraction at rough interface



# Curved tactile sensor: working principle

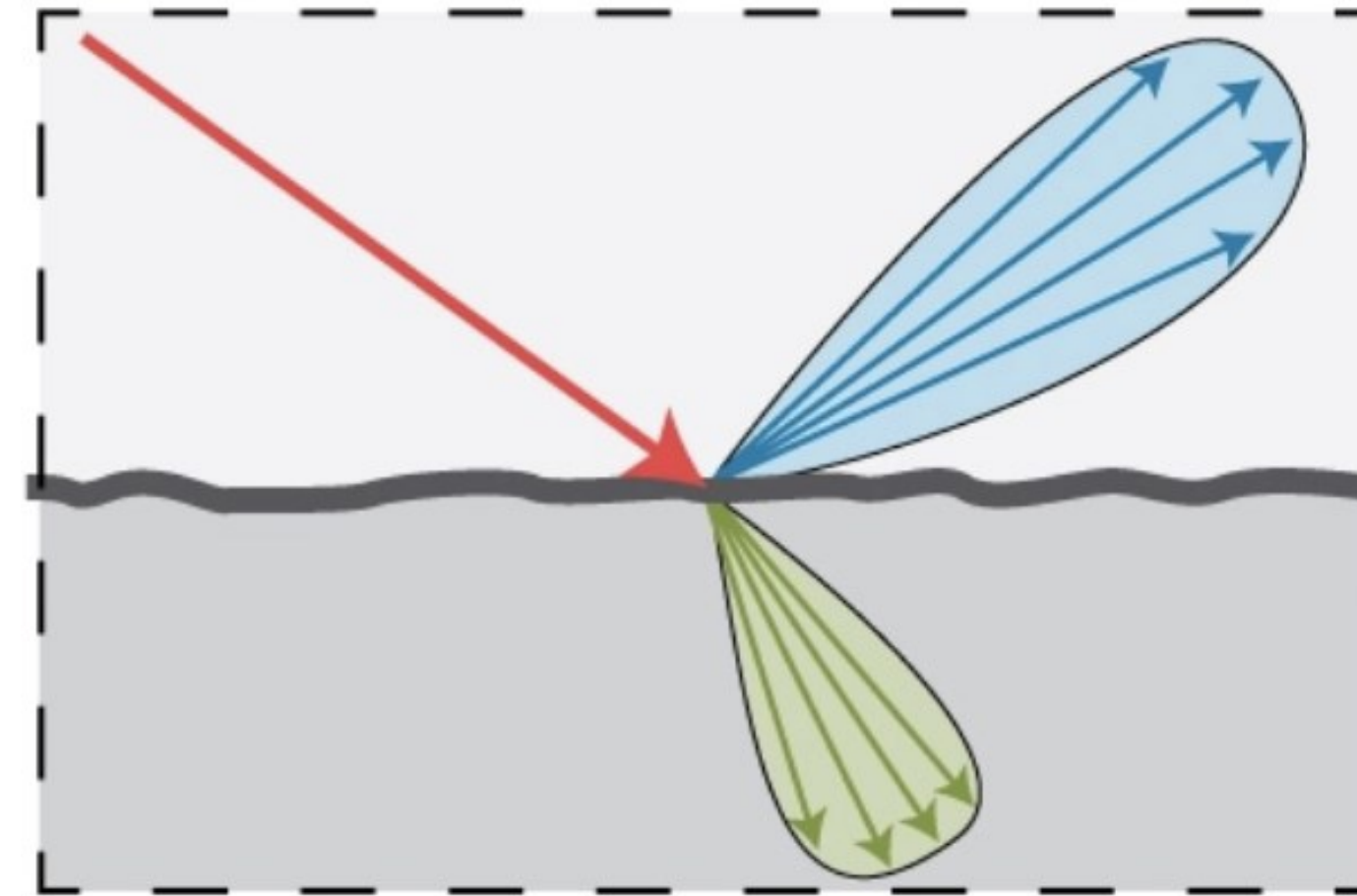
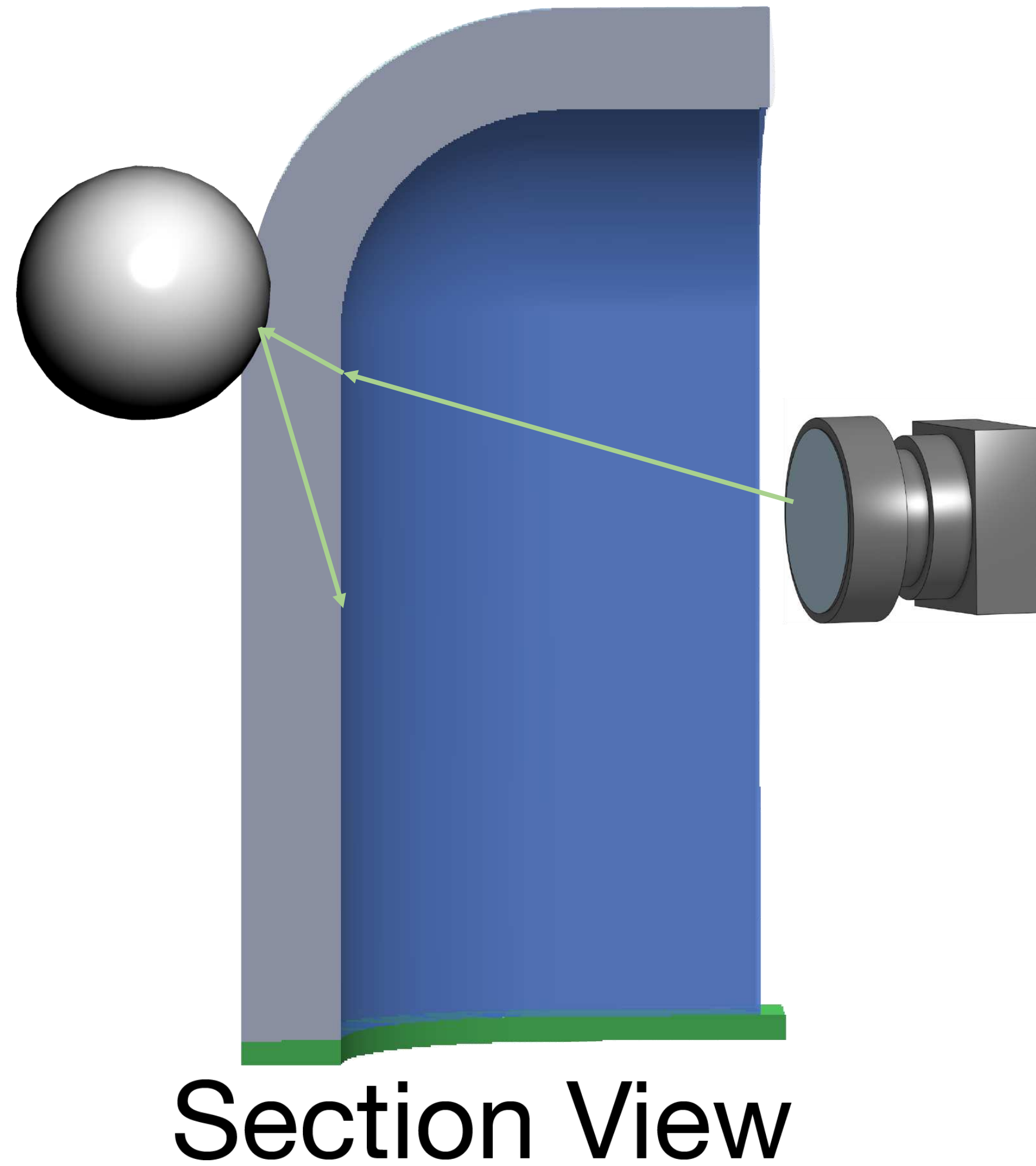


Refraction at rough interface

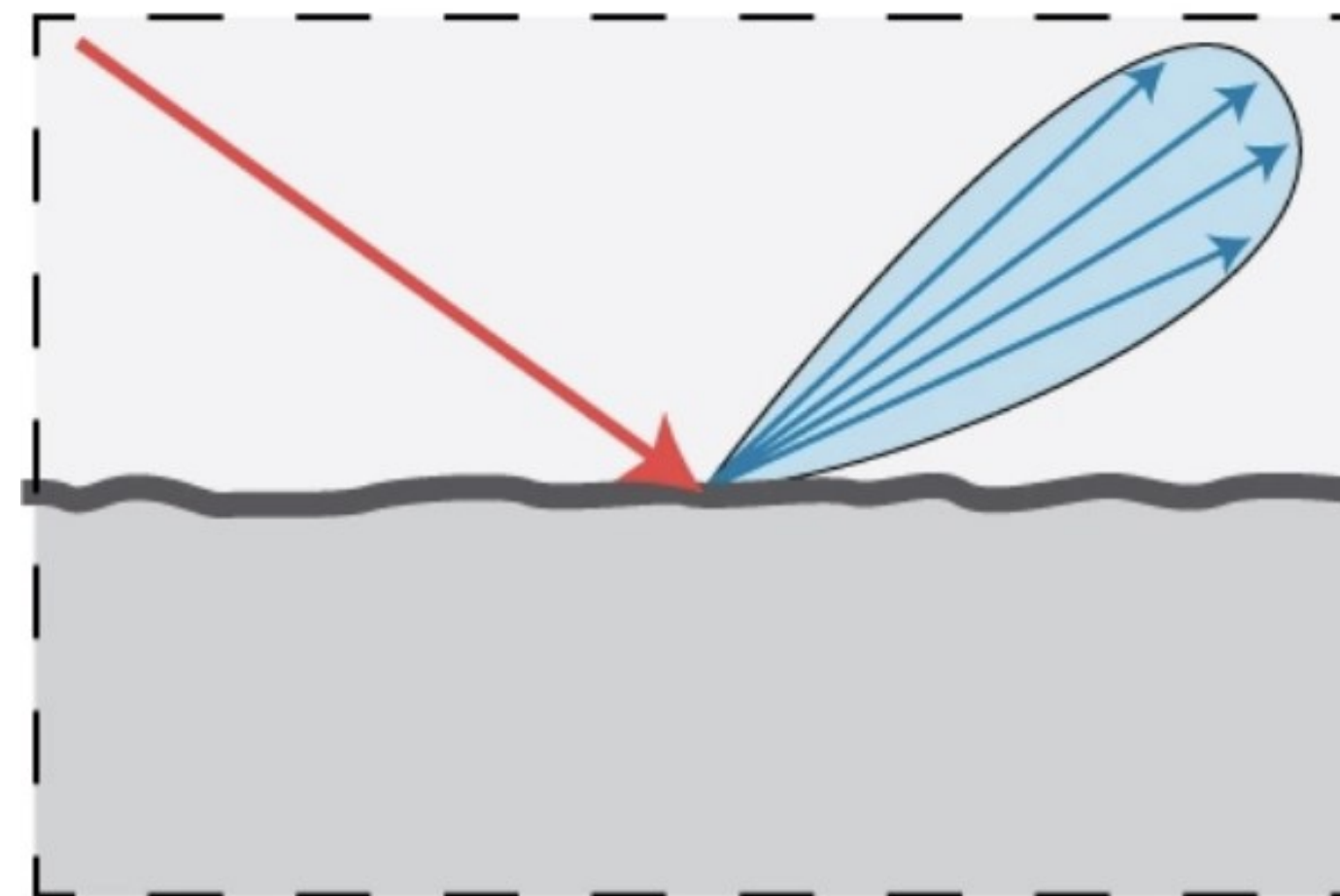


Reflection at glossy surfaces

# Curved tactile sensor: working principle



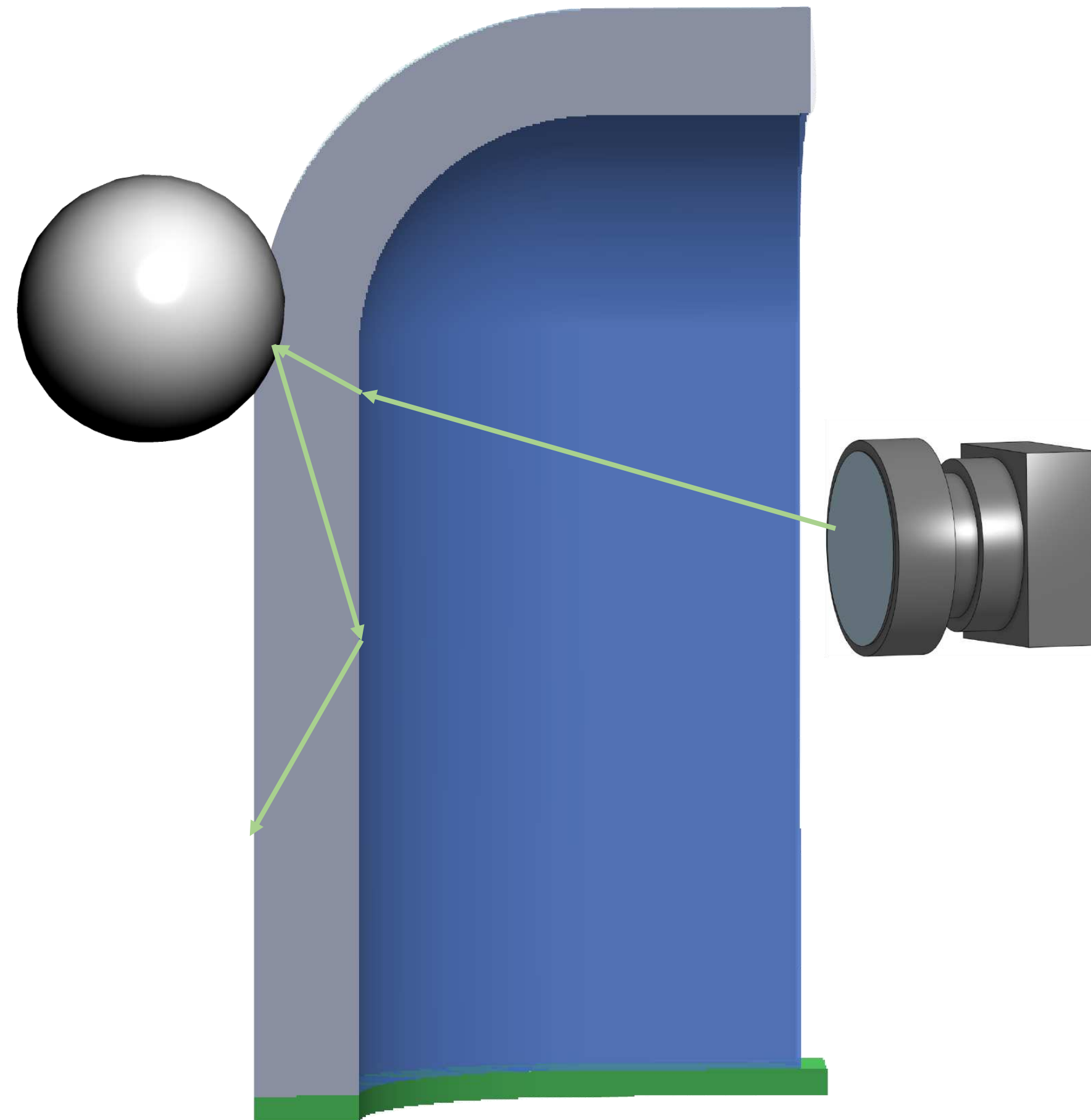
Refraction at rough interface



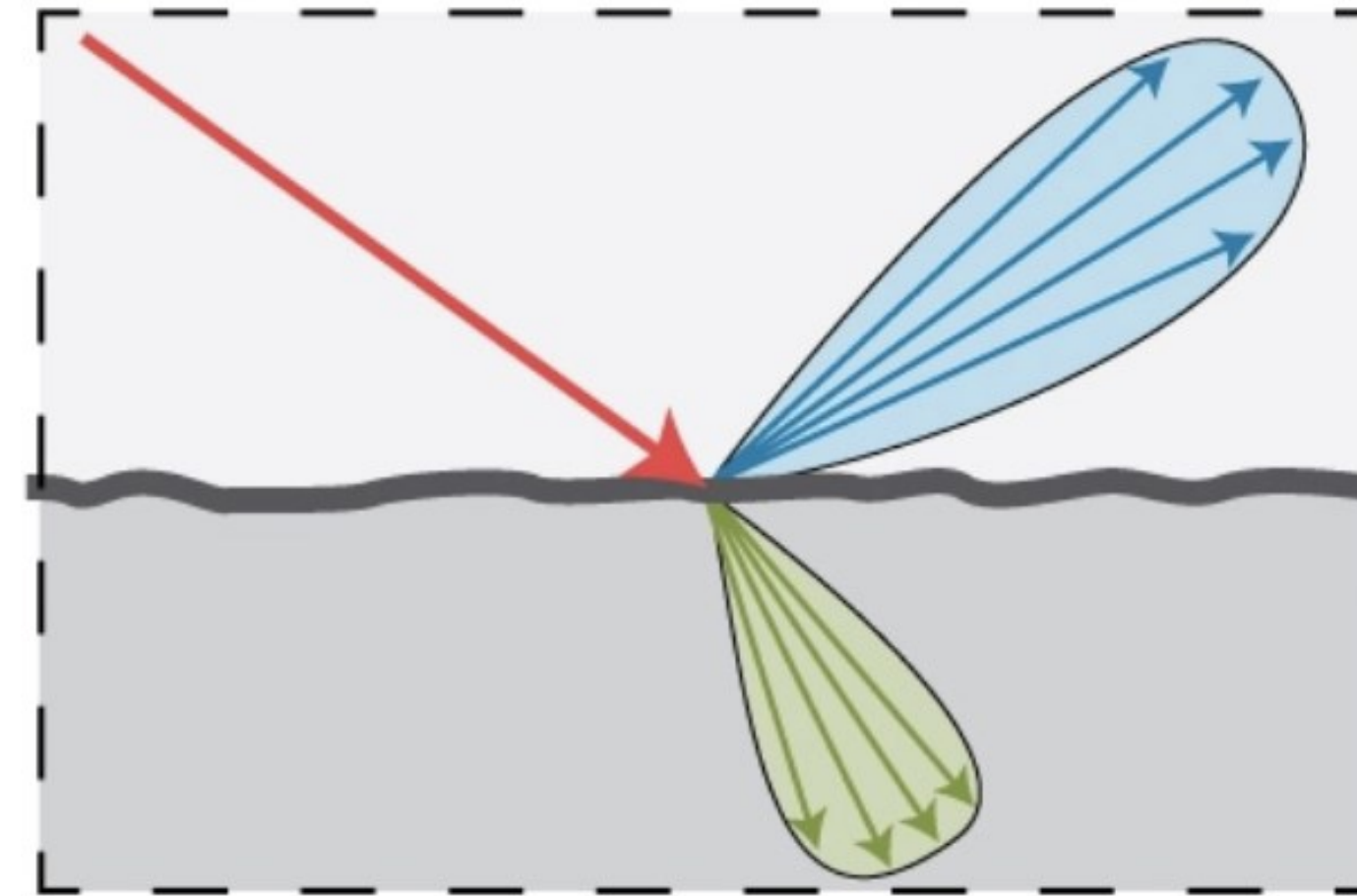
Reflection at glossy surfaces



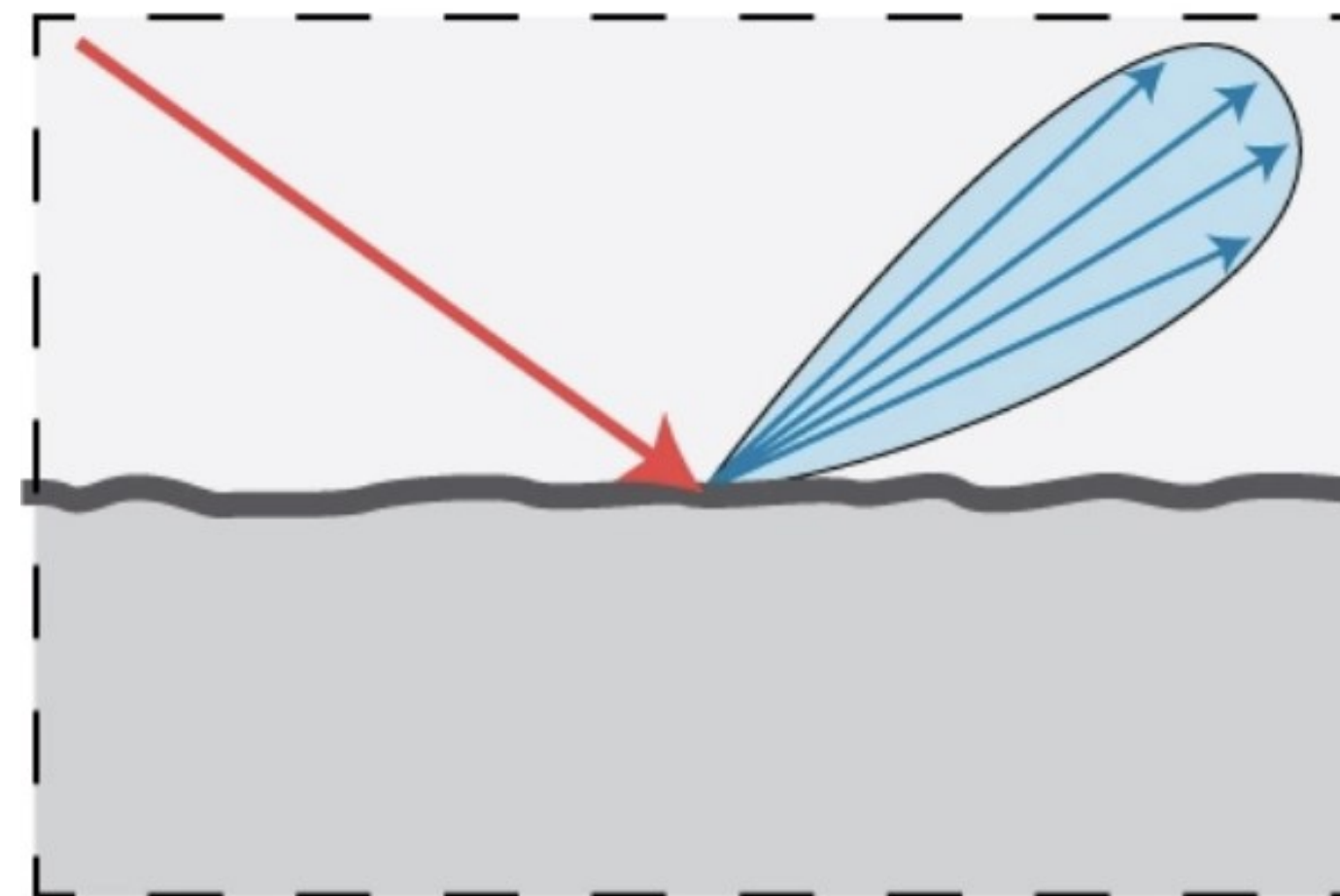
# Curved tactile sensor: working principle



Section View

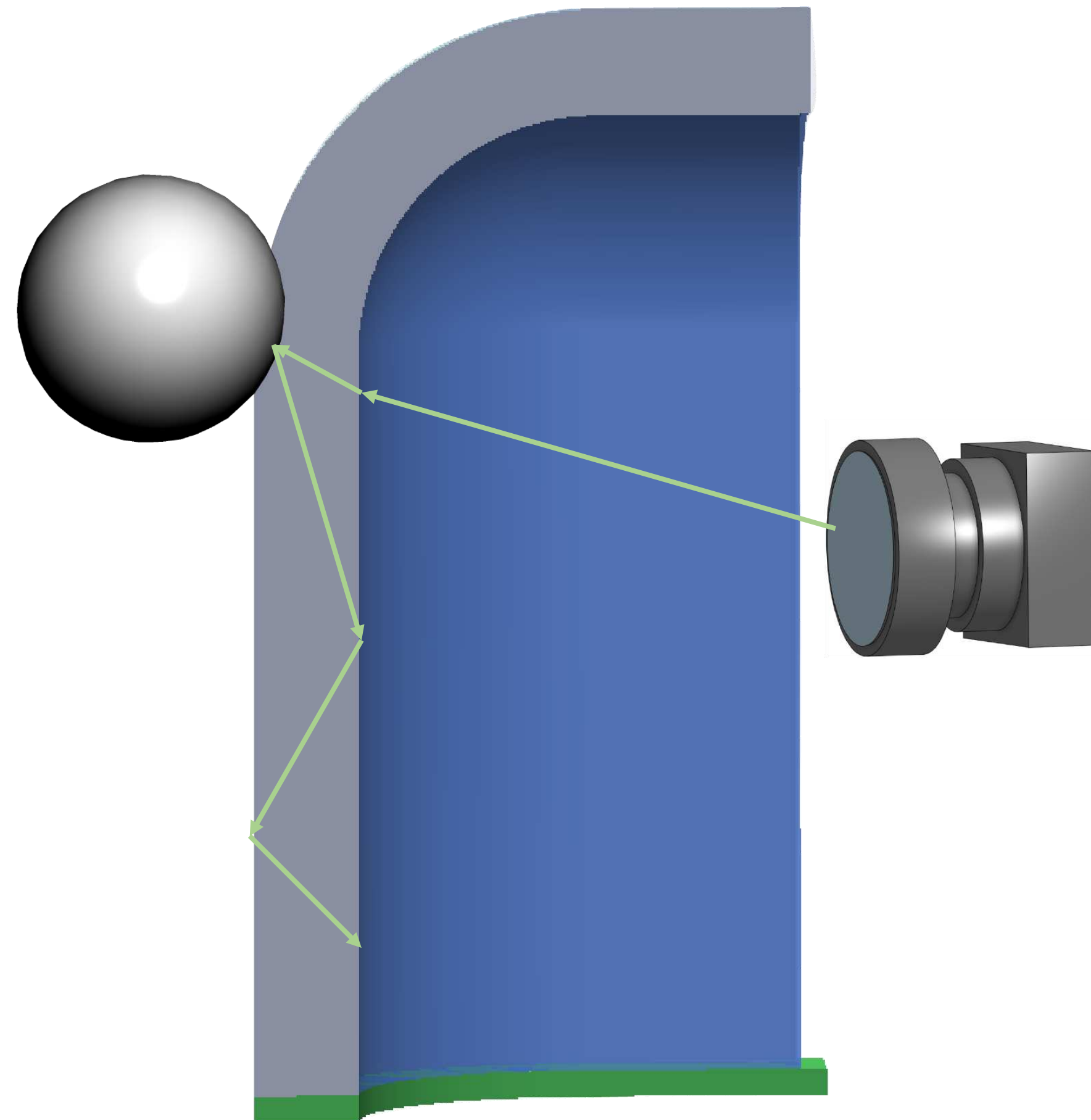


Refraction at rough interface

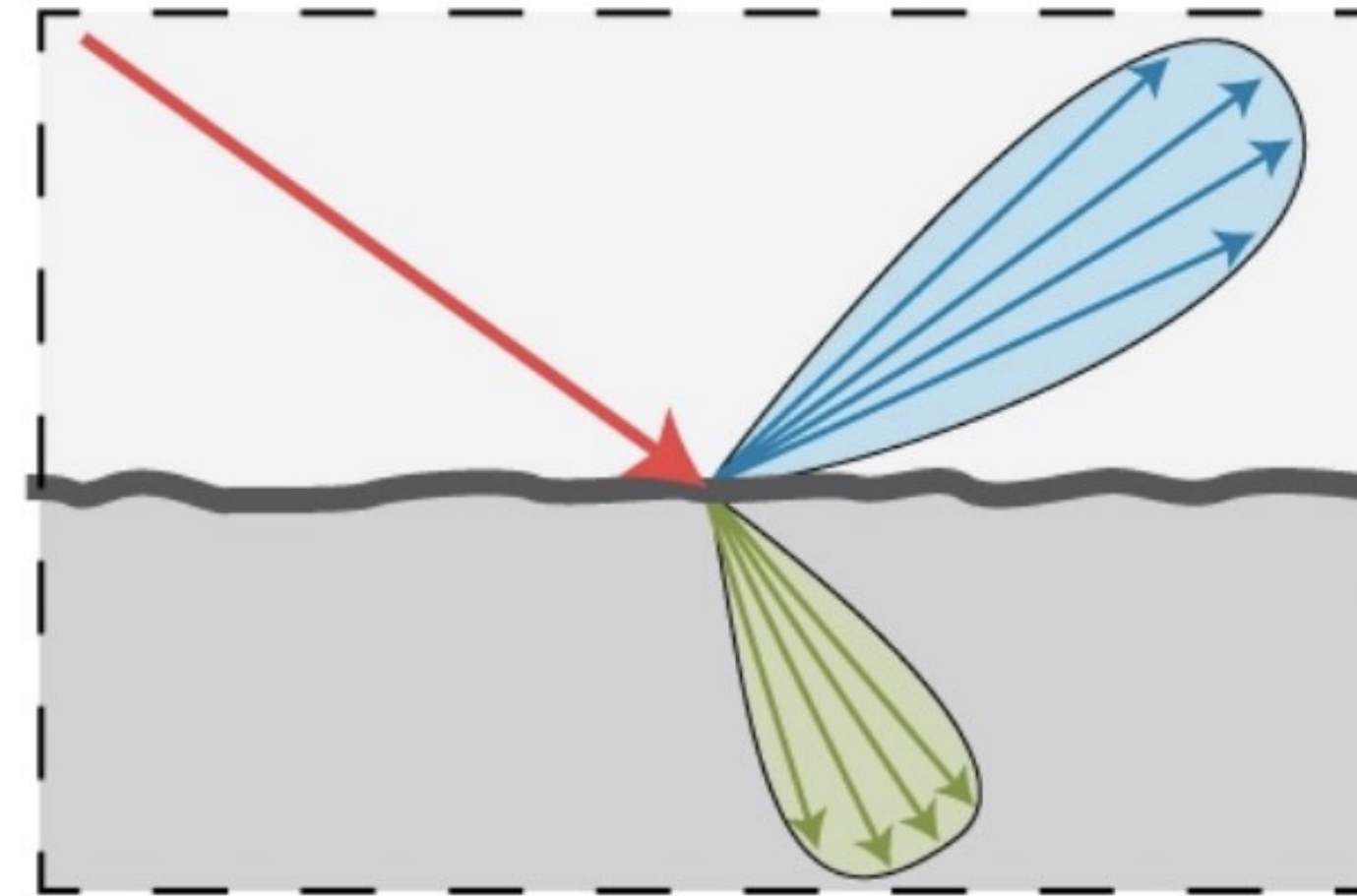


Reflection at glossy surfaces

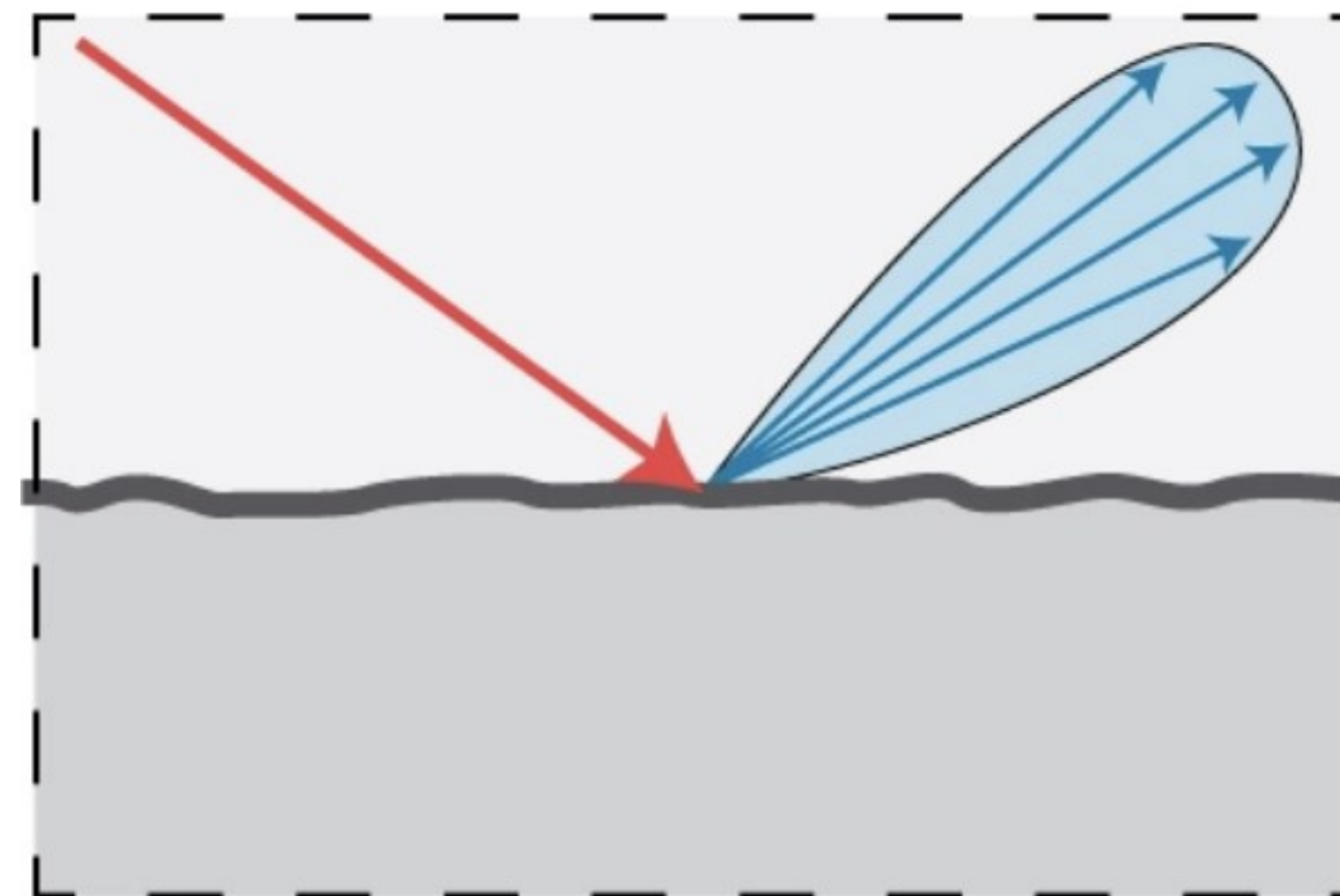
# Curved tactile sensor: working principle



Section View



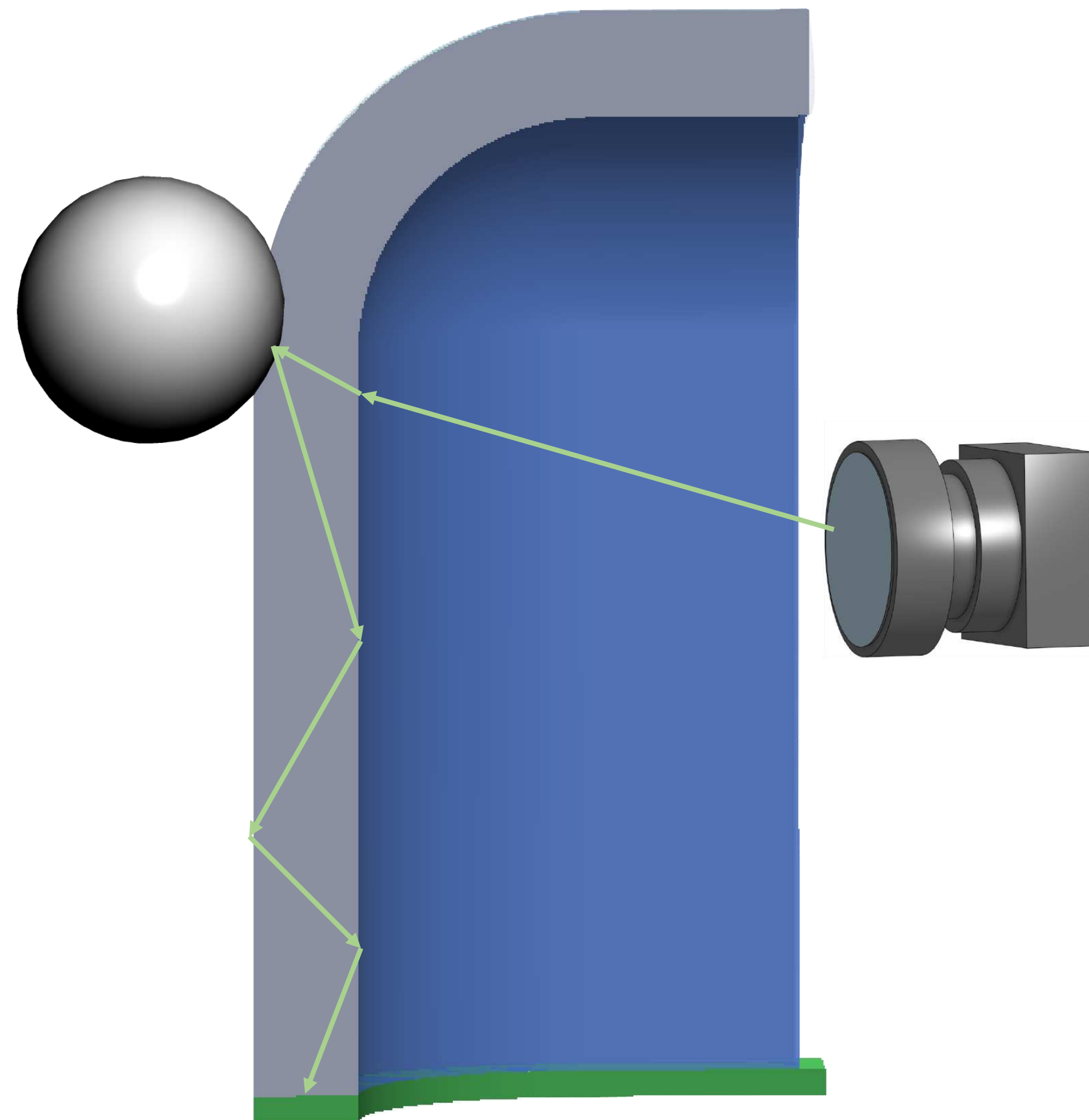
Refraction at rough interface



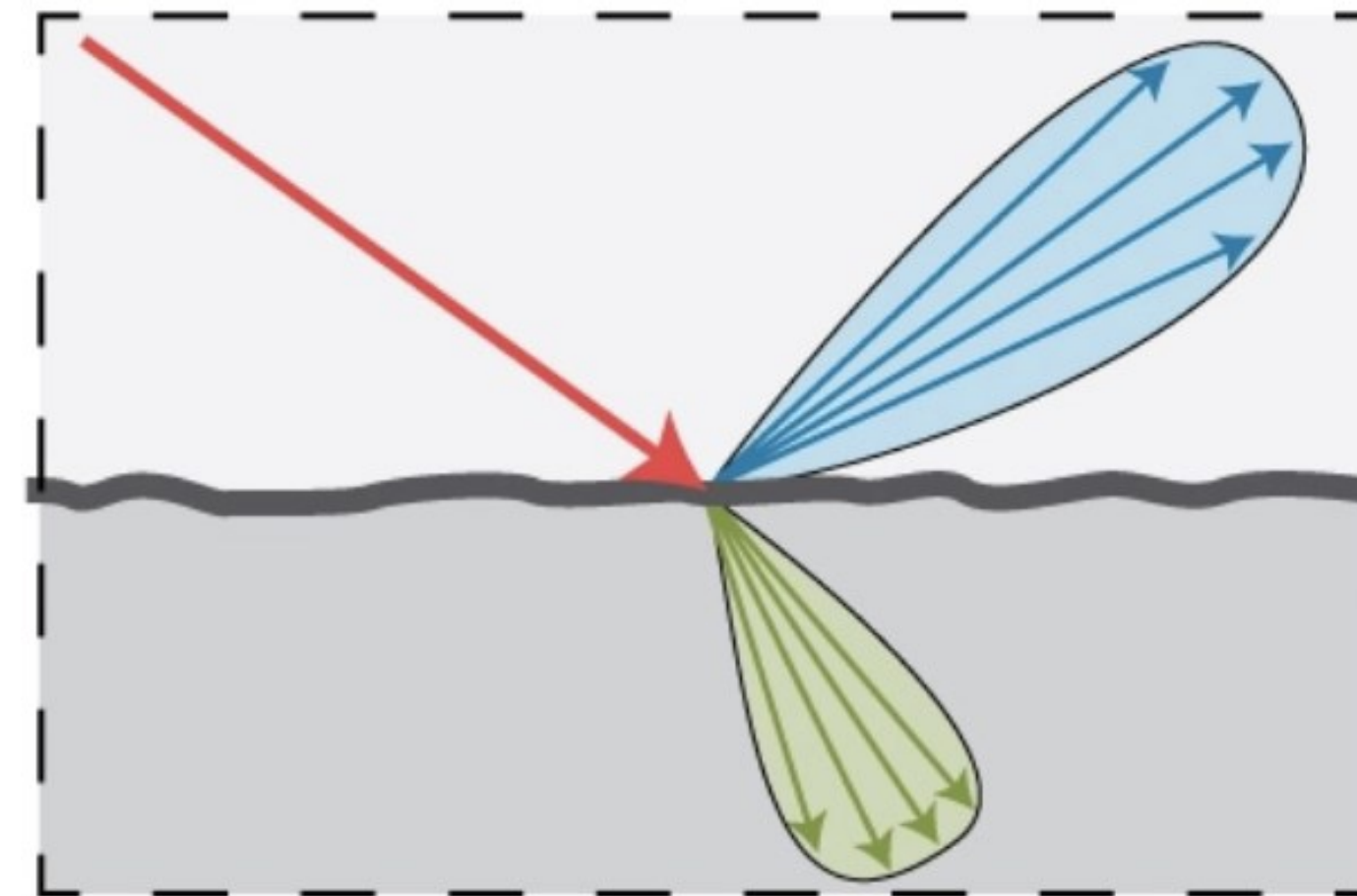
Reflection at glossy surfaces



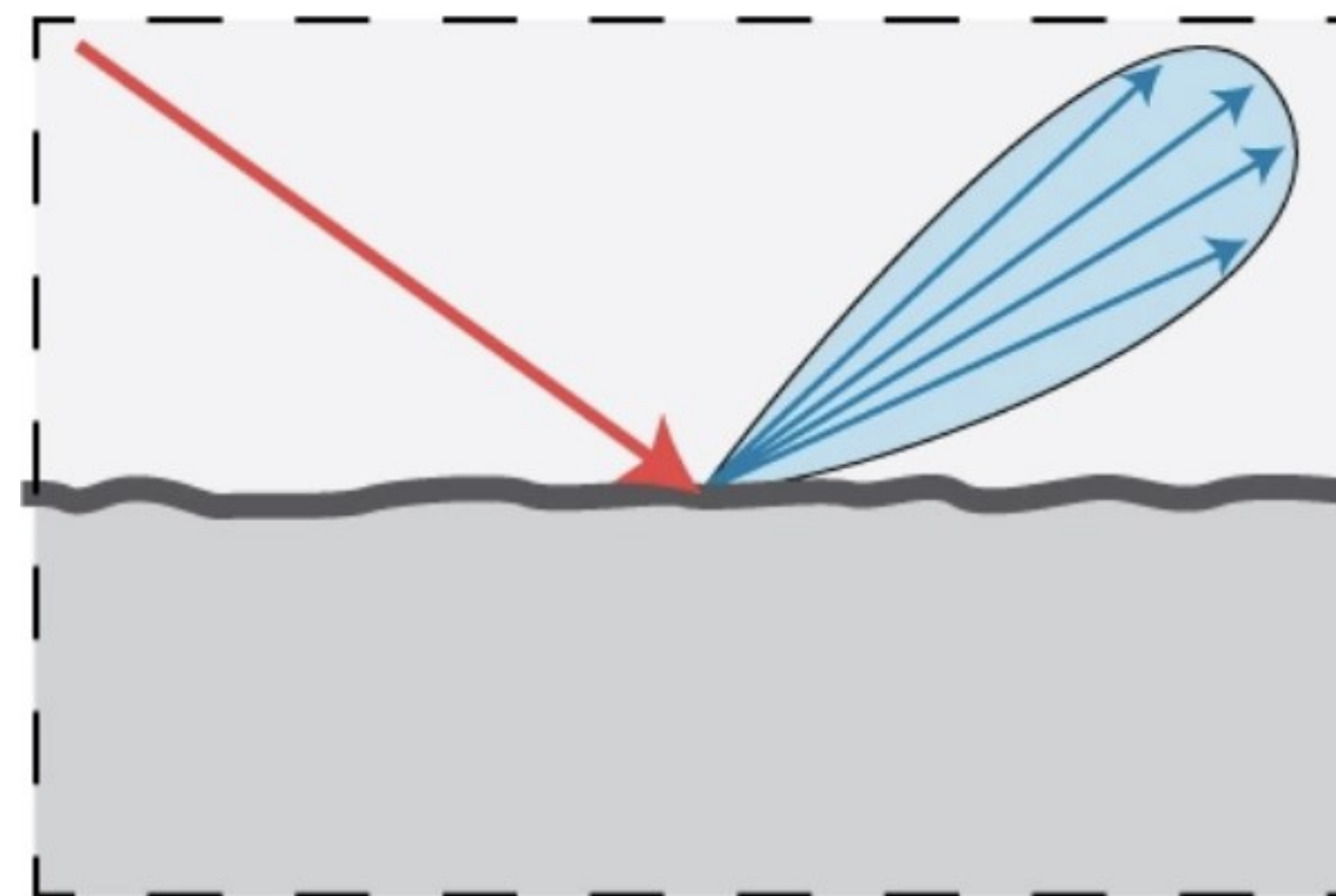
# Curved tactile sensor: working principle



Section View



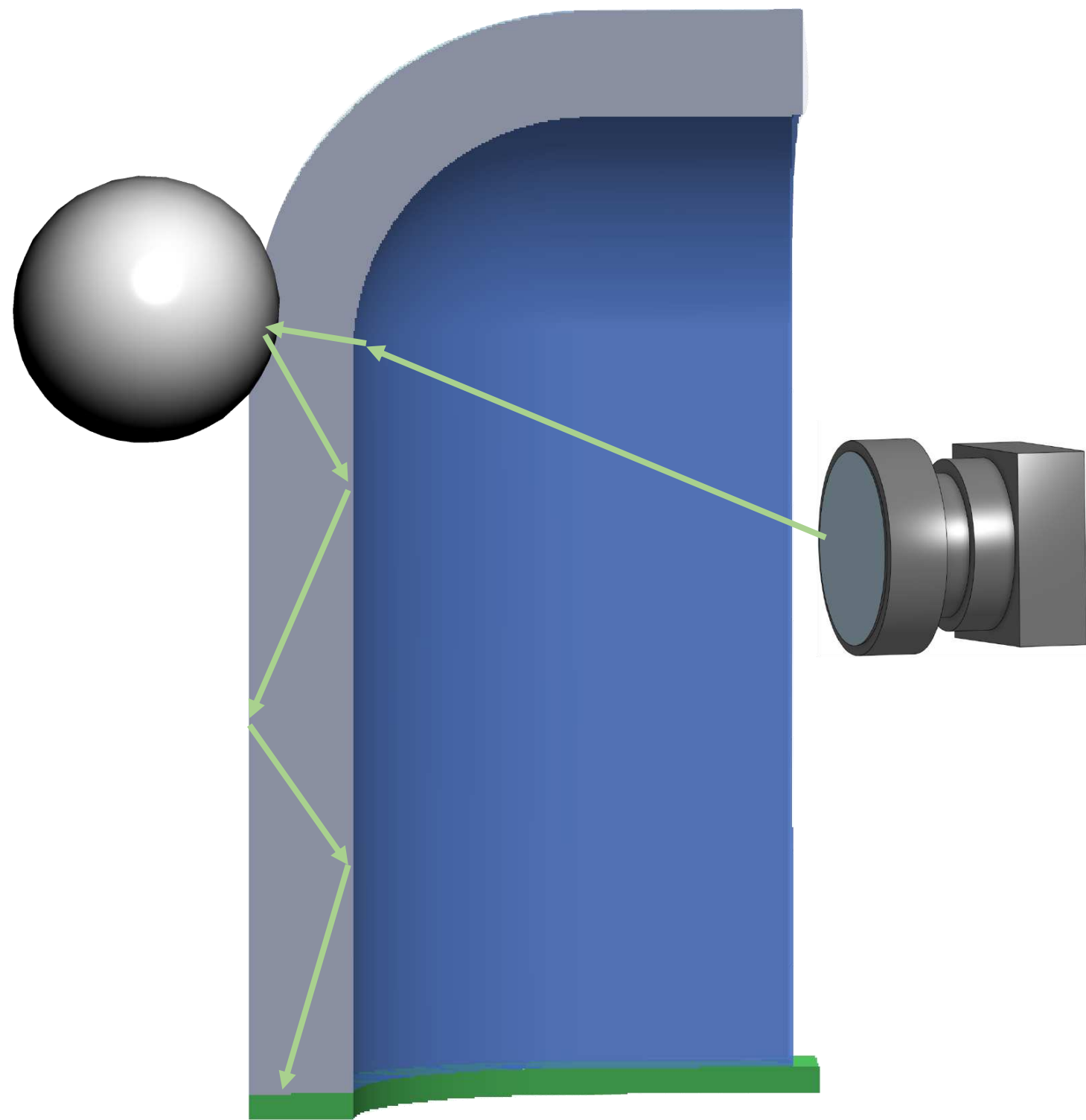
Refraction at rough interface



Reflection at glossy surfaces

# Sensor design framework: optical simulation

- Sensor geometry
- Material specifications
- Placement of camera and lights



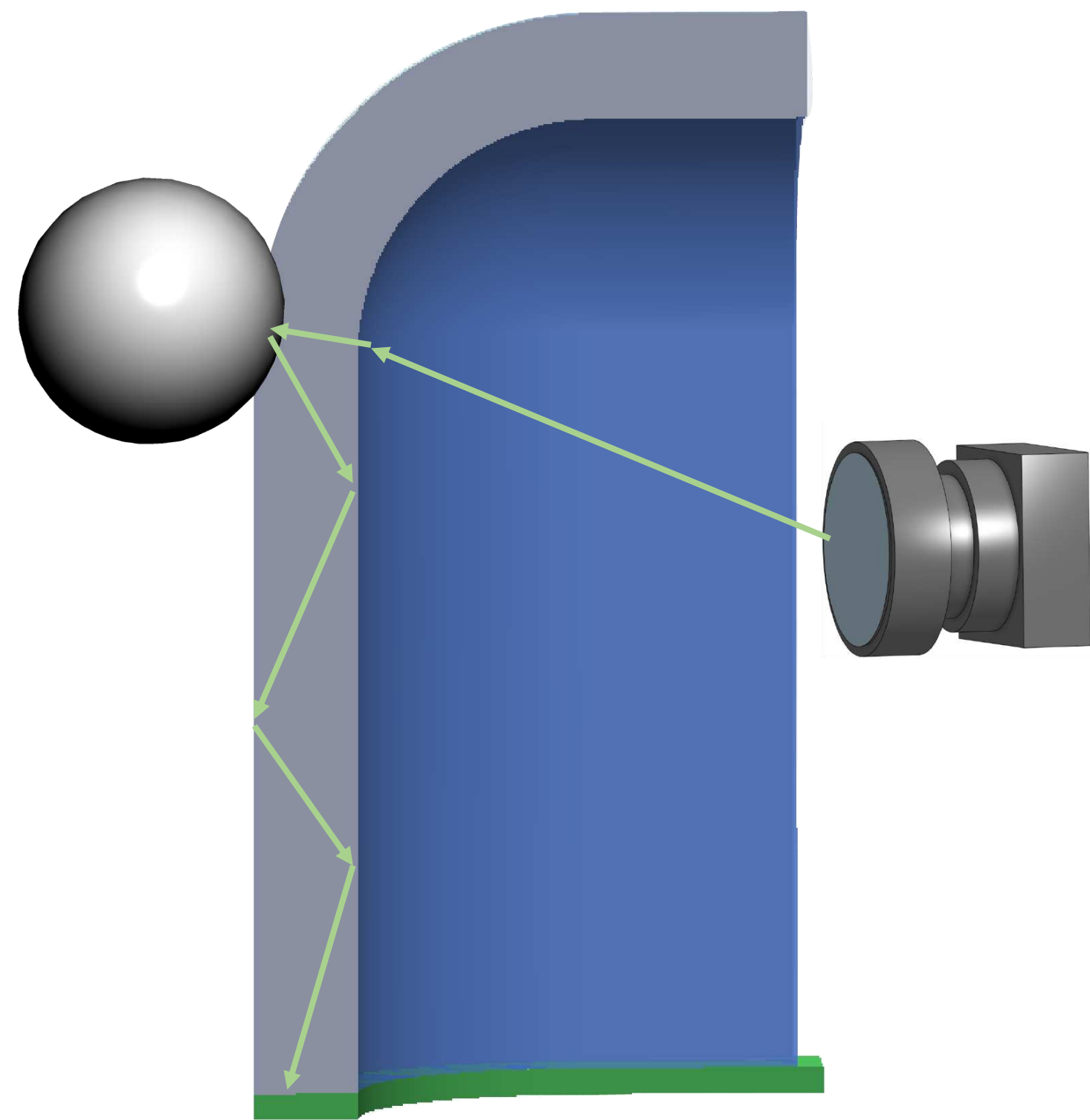
Section view

Tactile image [Agarwal et al., 2023]

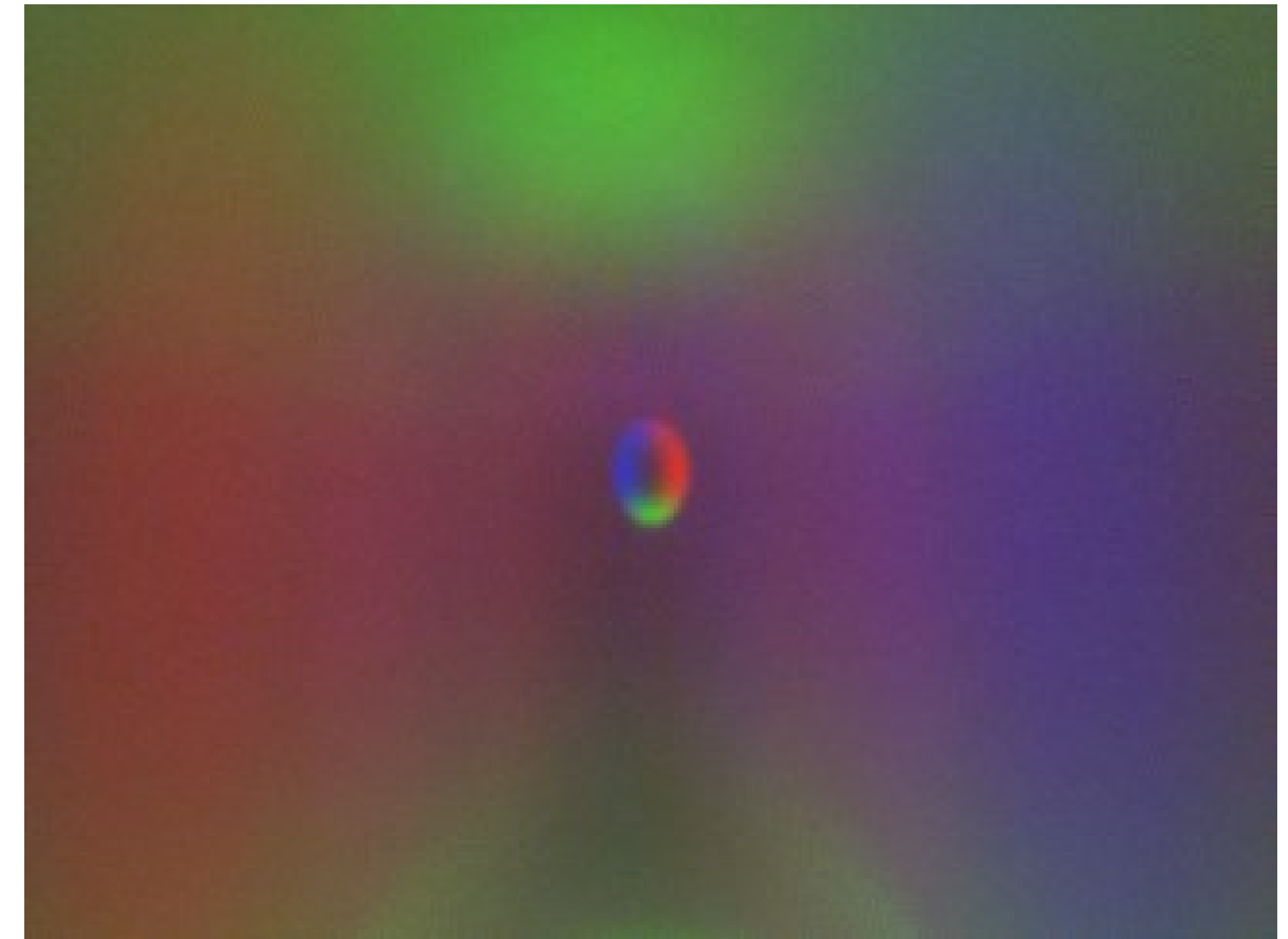


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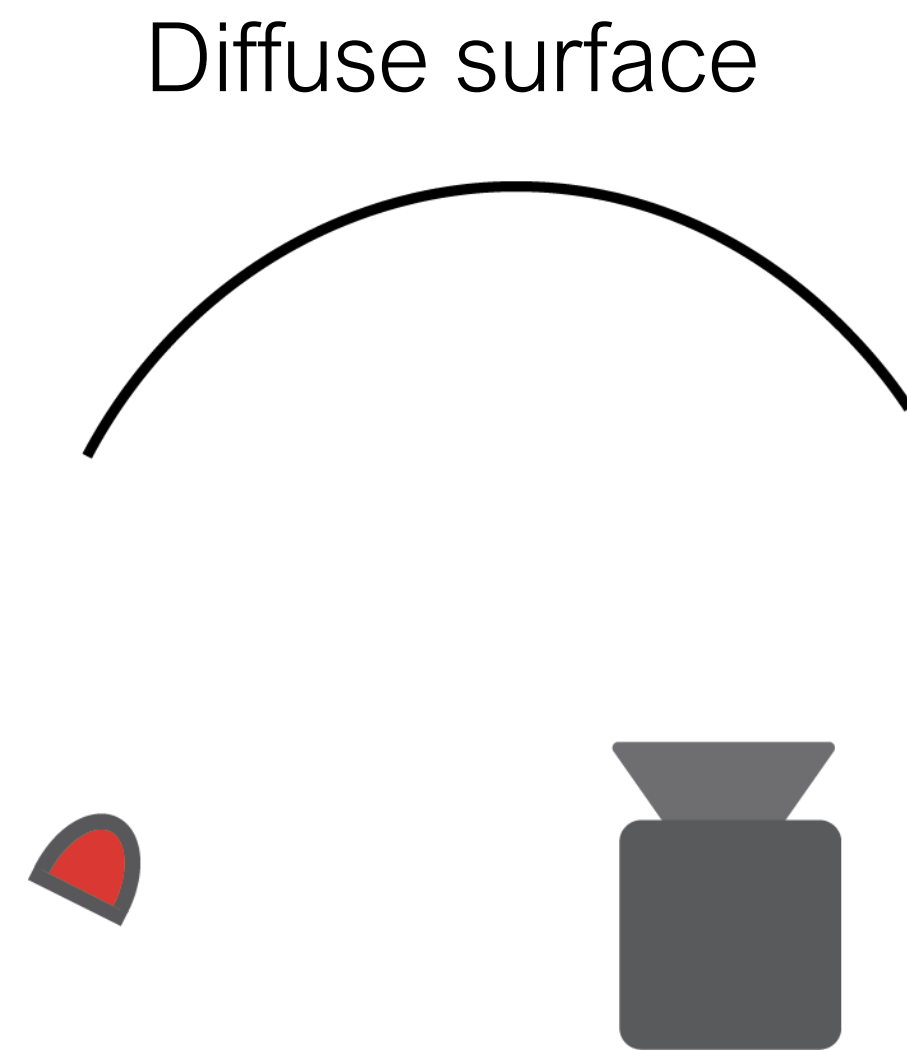
Section view



Tactile image [Agarwal et al., 2023]

# Sensor design framework: optical simulation

- What makes simulation challenging



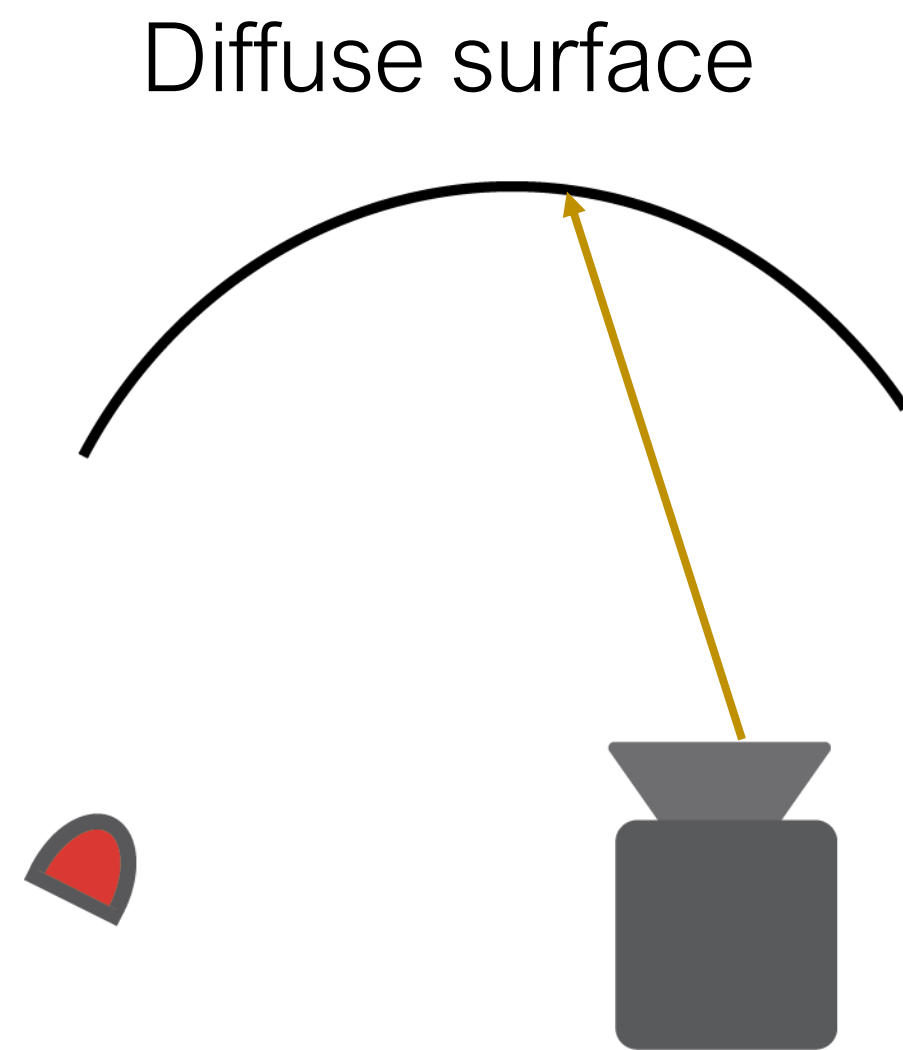
(Relatively) easy cases

Hard case



# Sensor design framework: optical simulation

- What makes simulation challenging

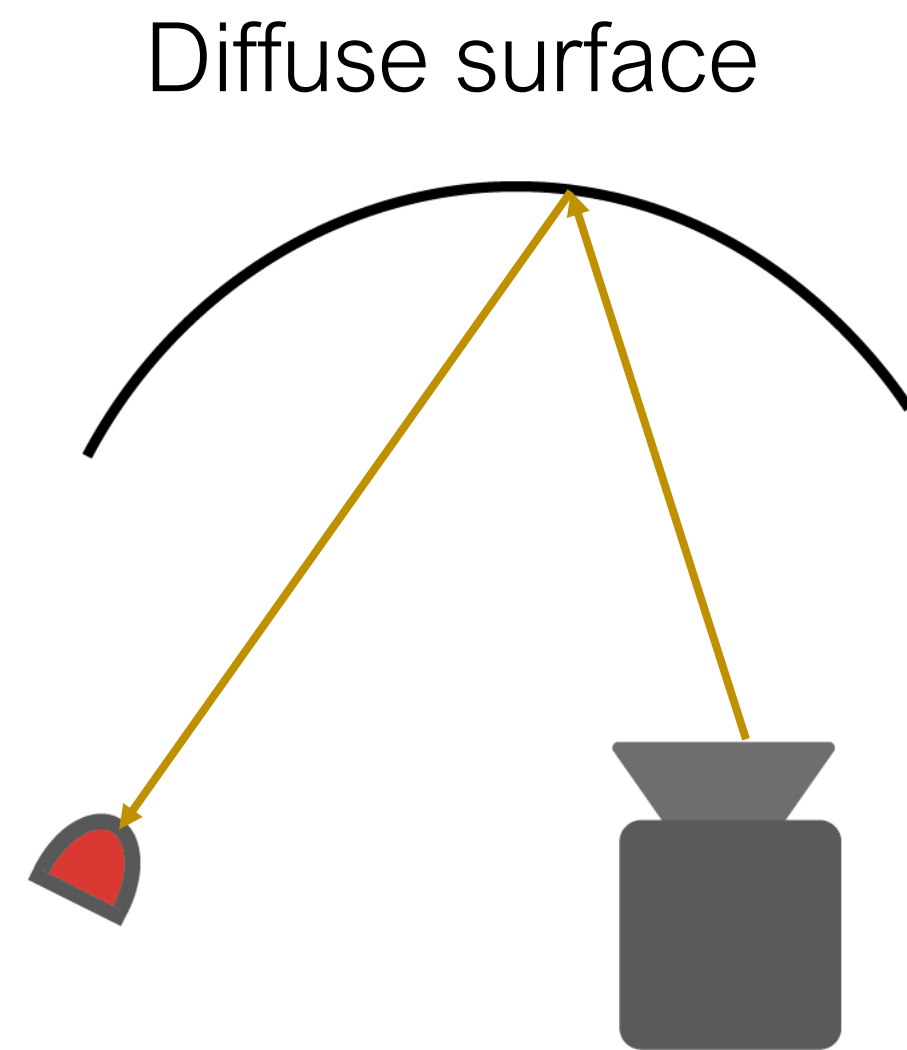


(Relatively) easy cases

Hard case

# Sensor design framework: optical simulation

- What makes simulation challenging



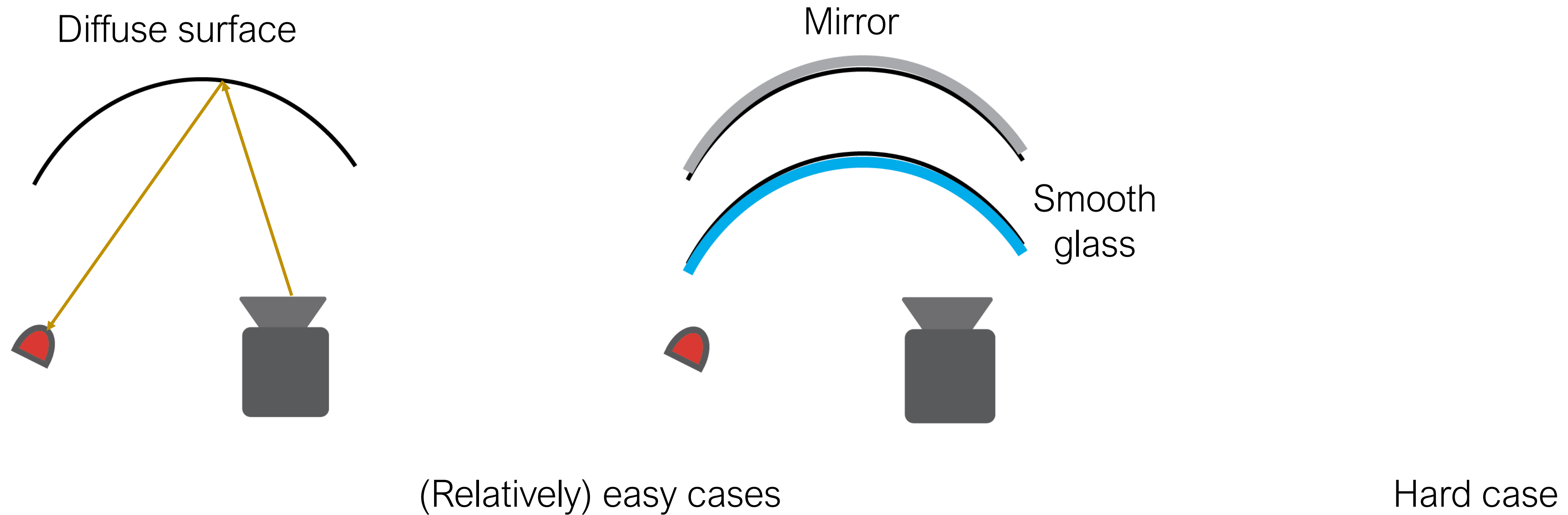
(Relatively) easy cases

Hard case



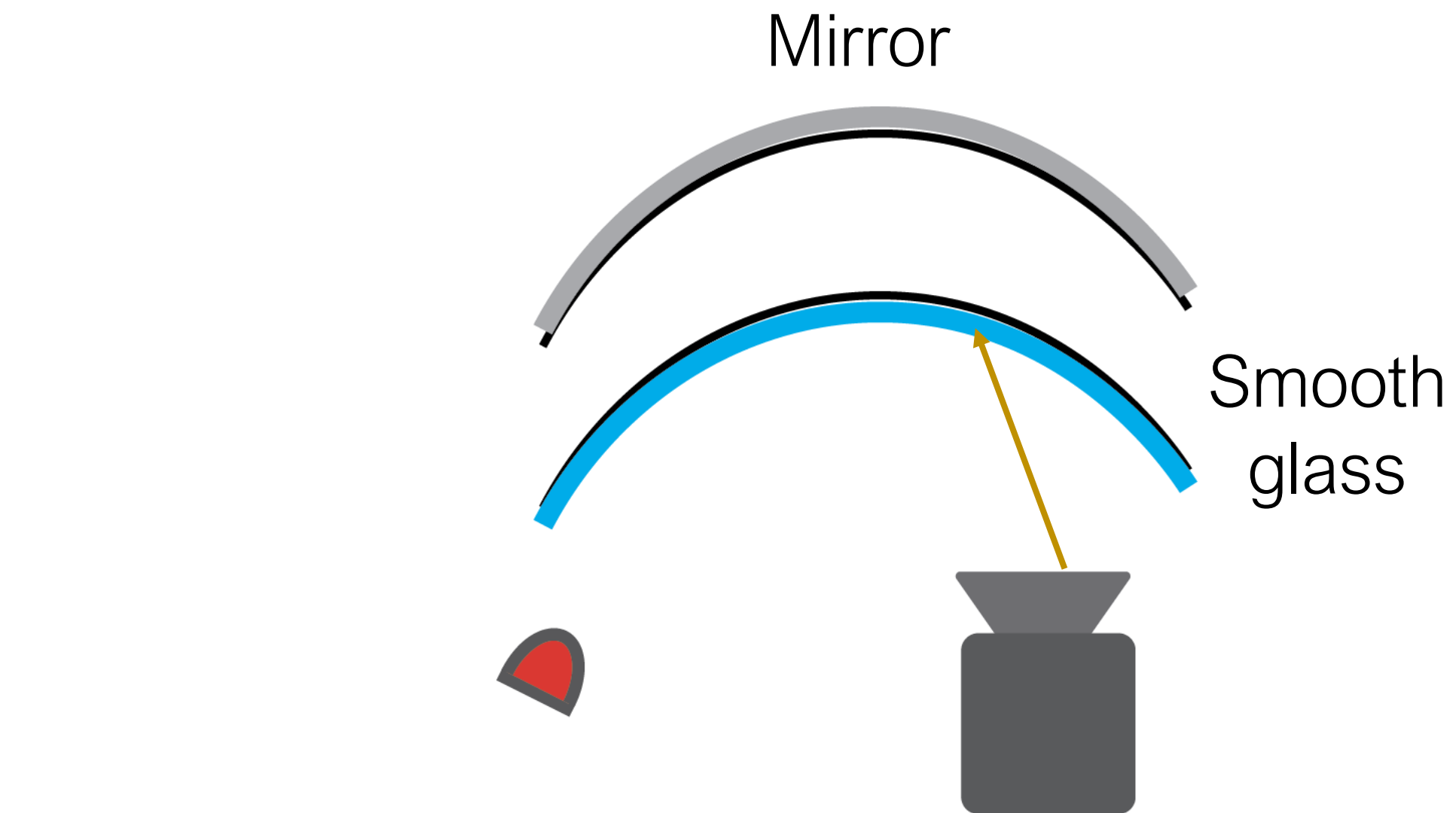
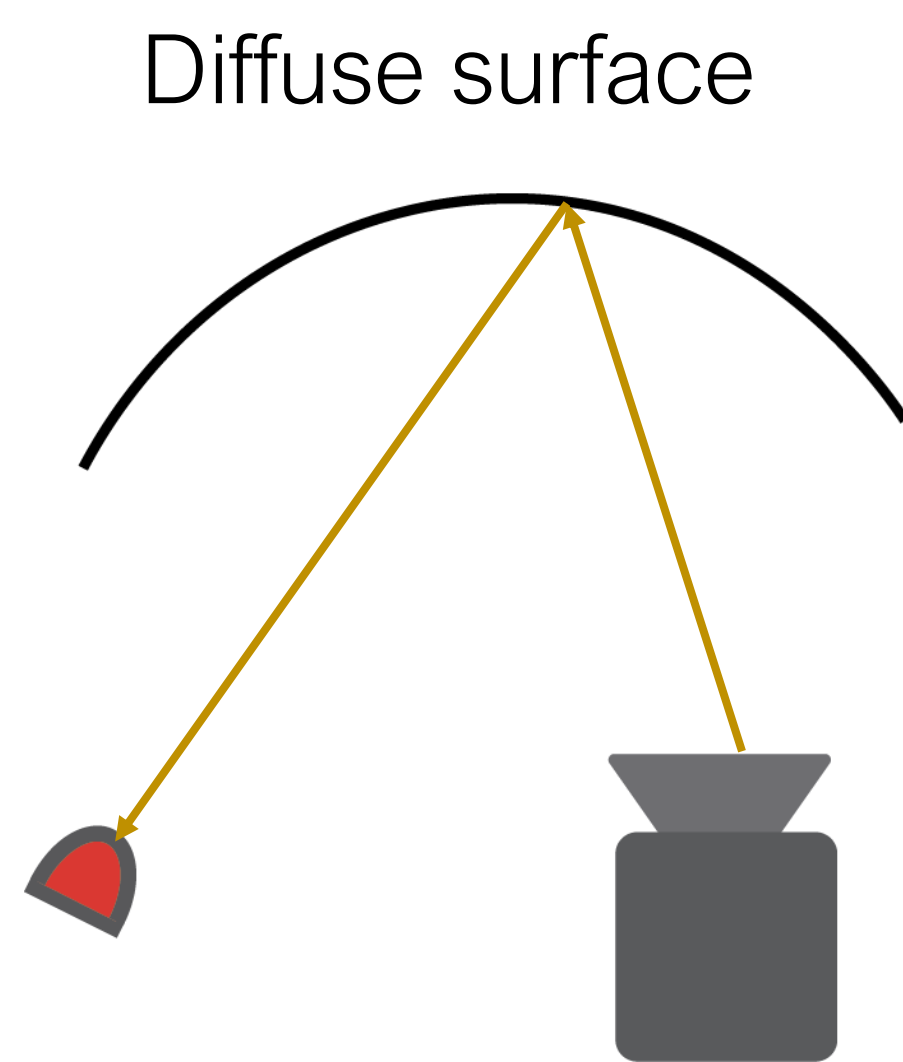
# Sensor design framework: optical simulation

- What makes simulation challenging



# Sensor design framework: optical simulation

- What makes simulation challenging



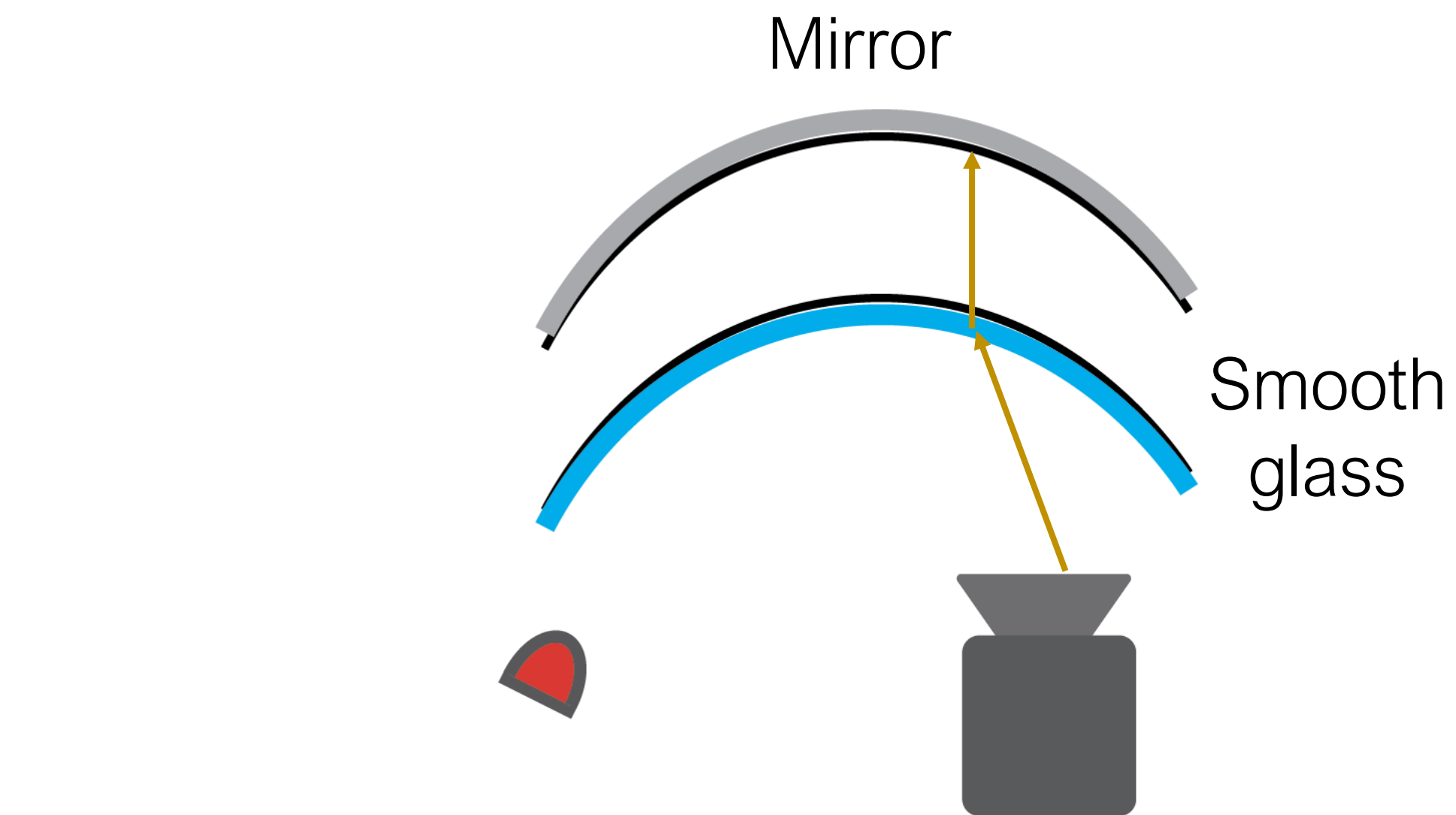
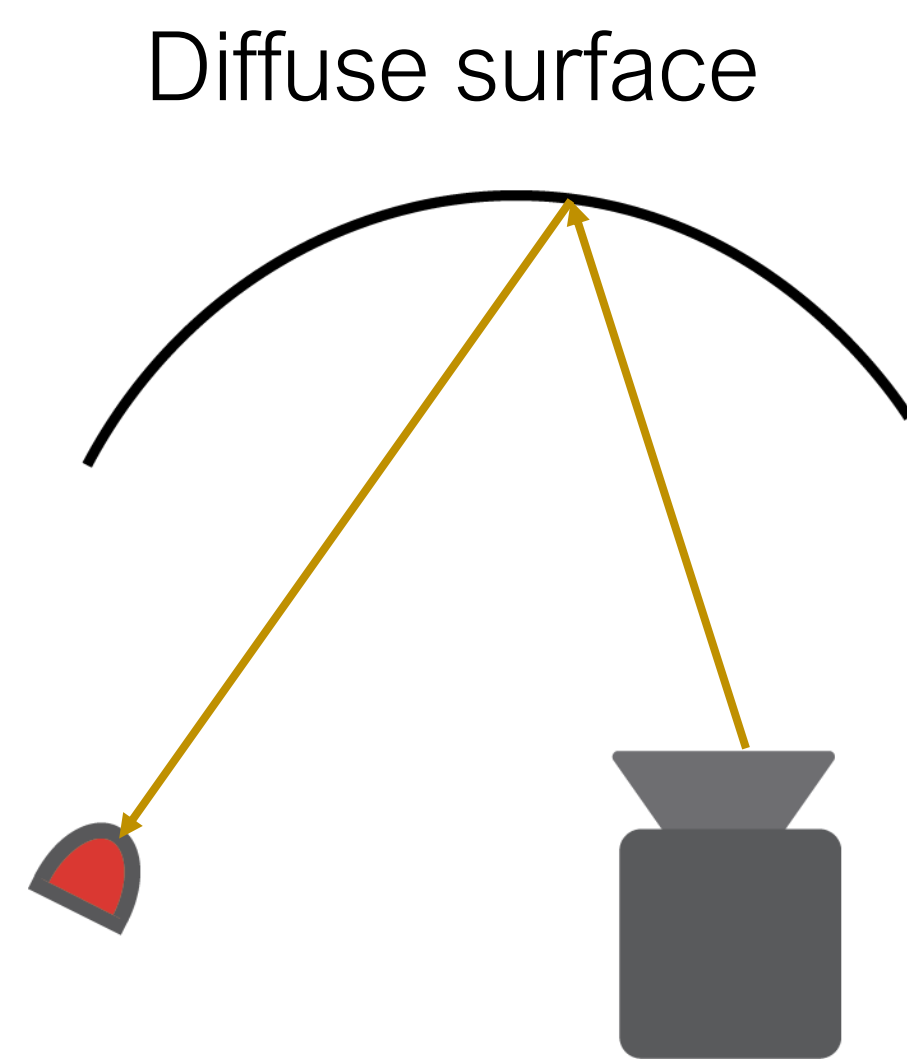
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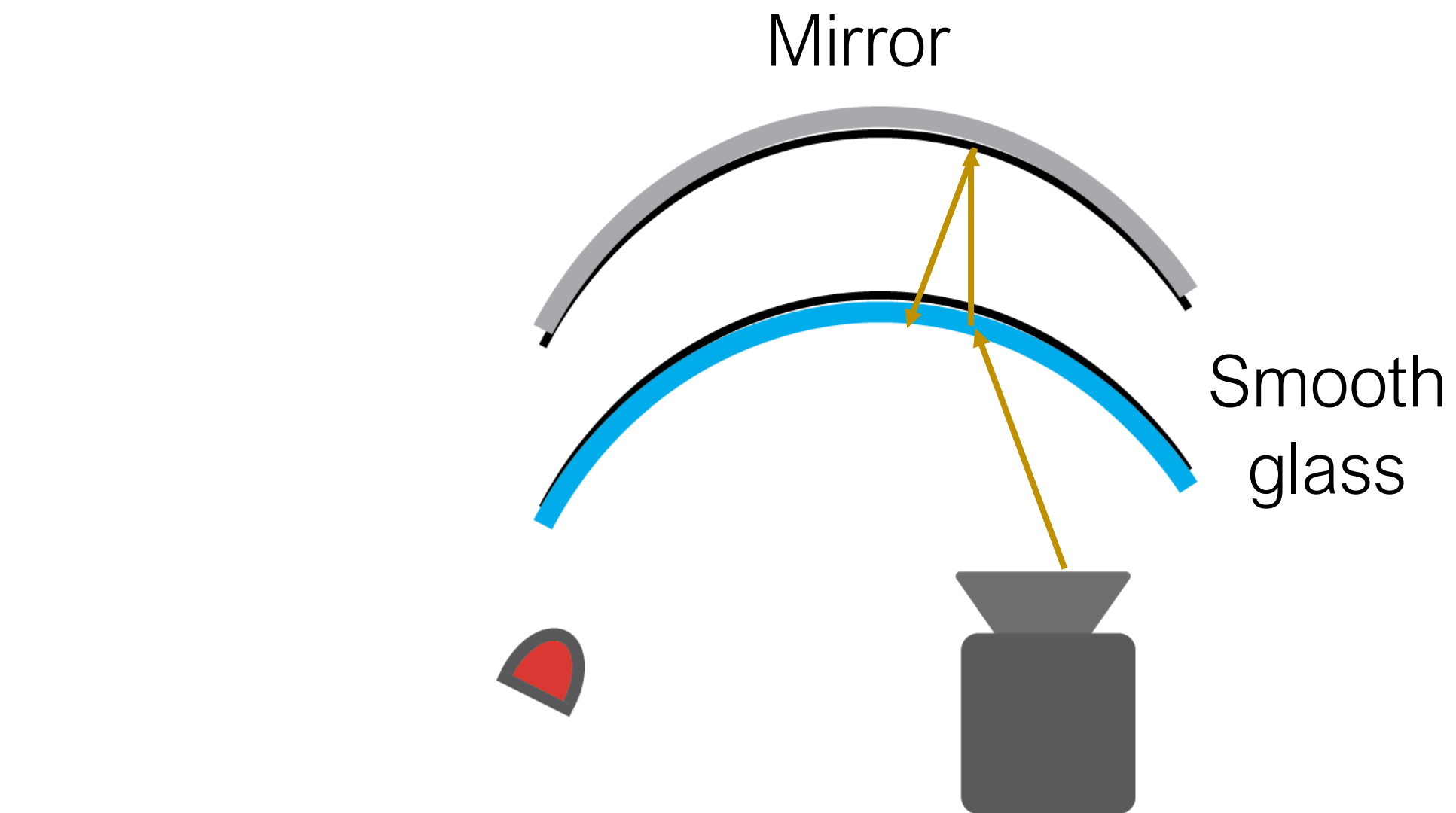
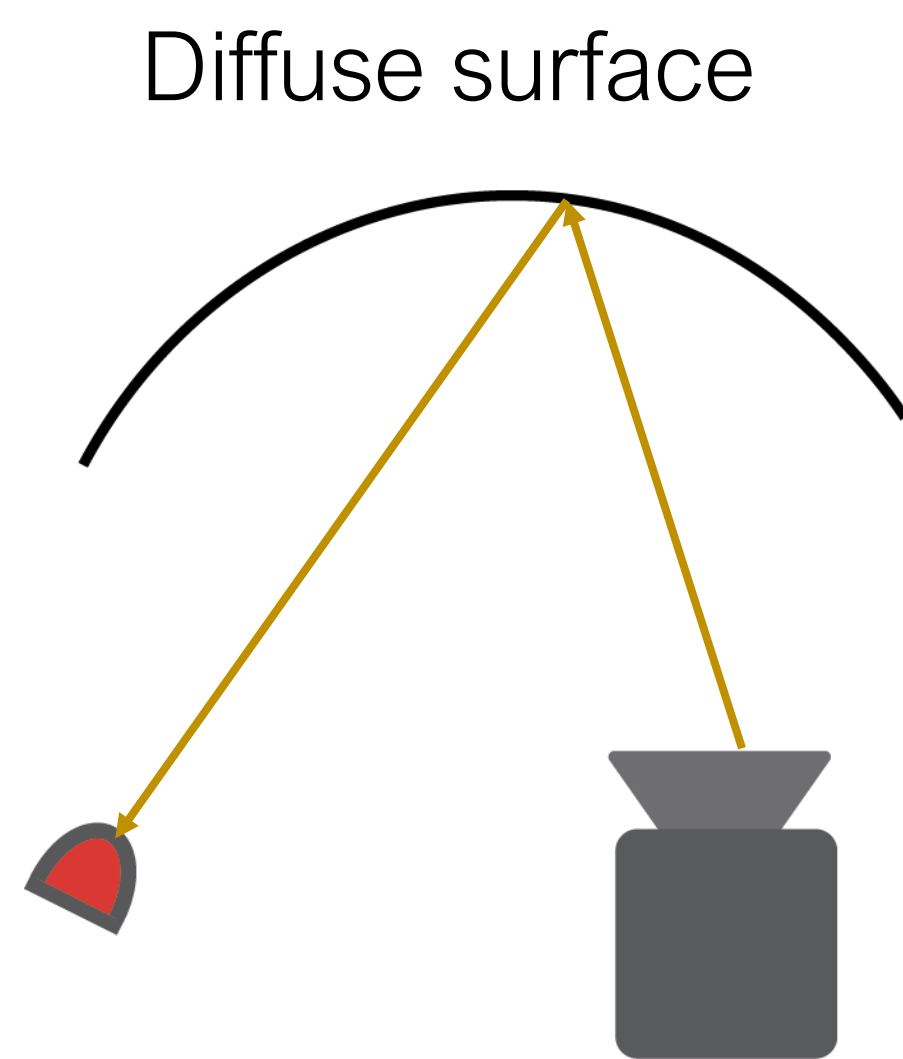


(Relatively) easy cases

Hard case

# Sensor design framework: optical simulation

- What makes simulation challenging



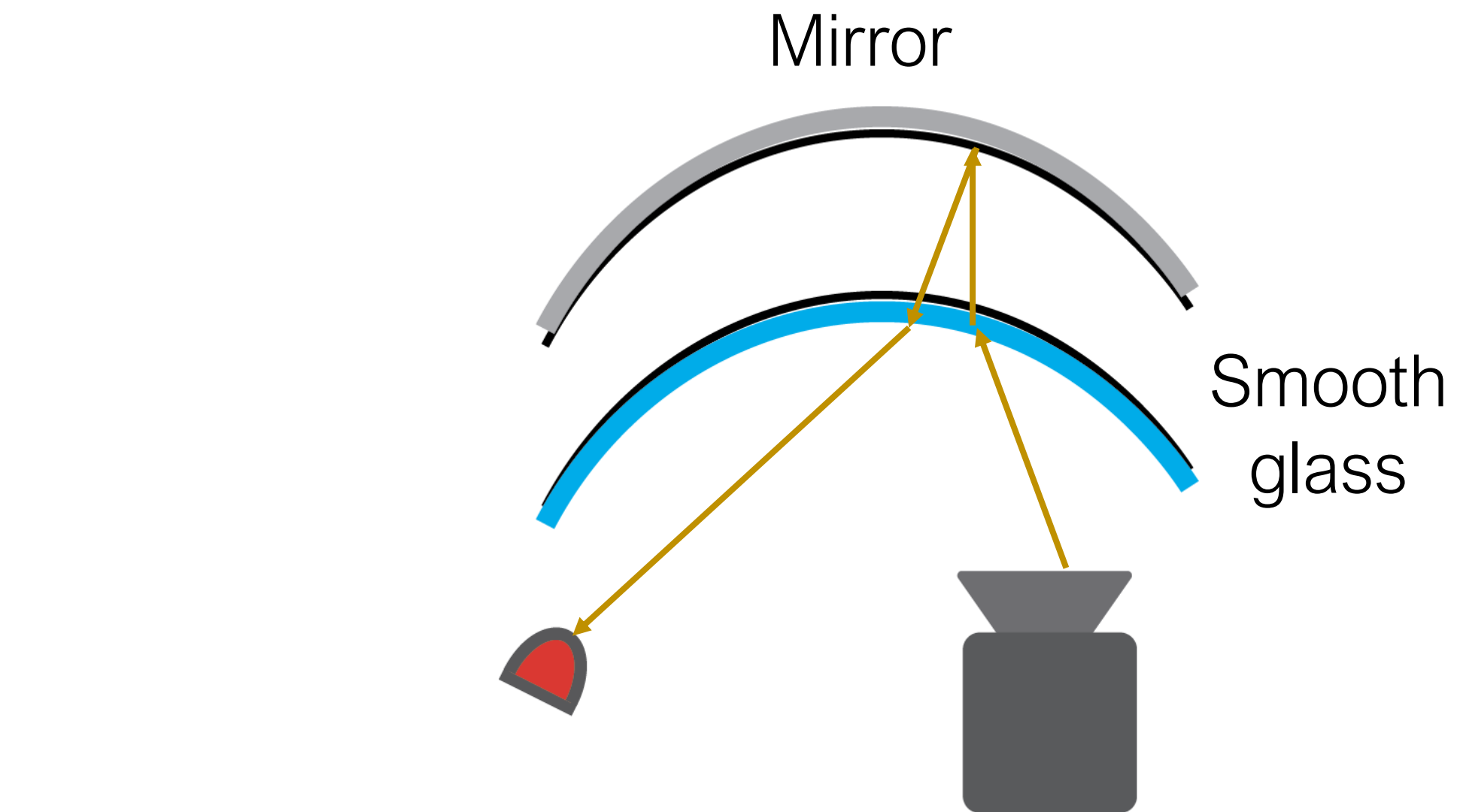
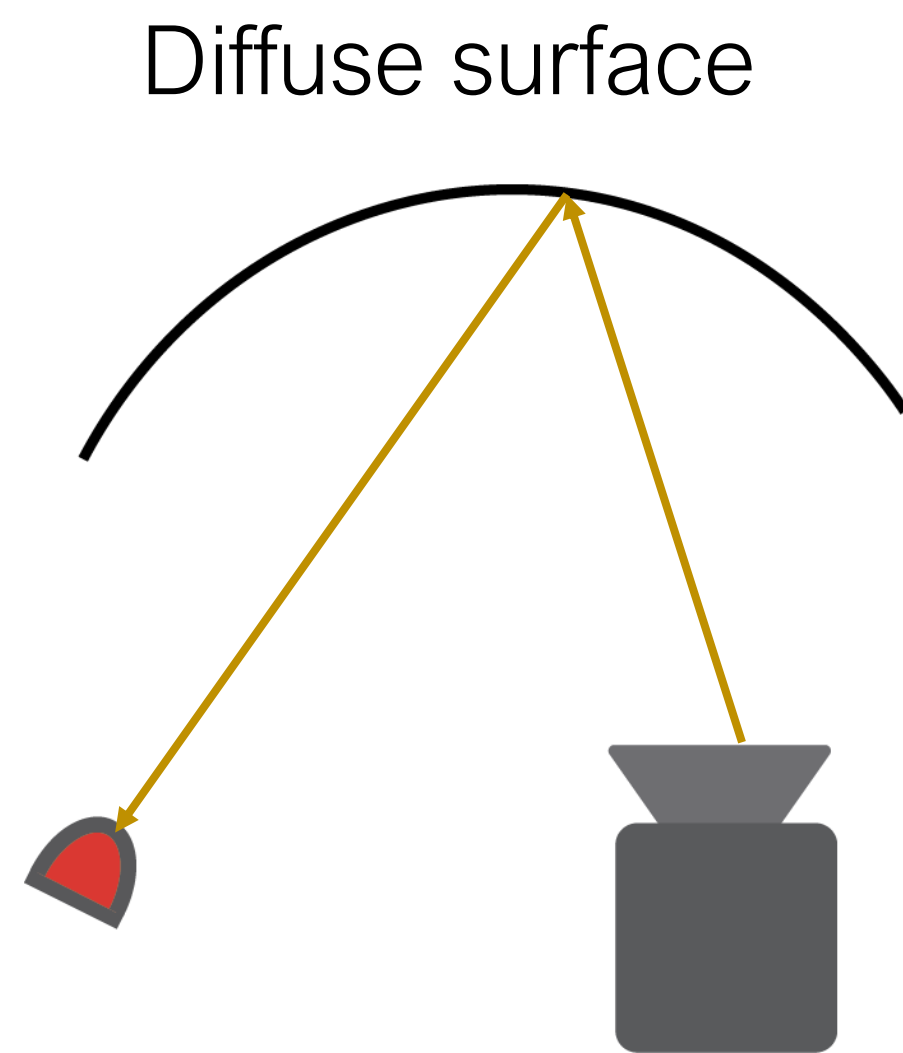
(Relatively) easy cases

Hard case



# Sensor design framework: optical simulation

- What makes simulation challenging

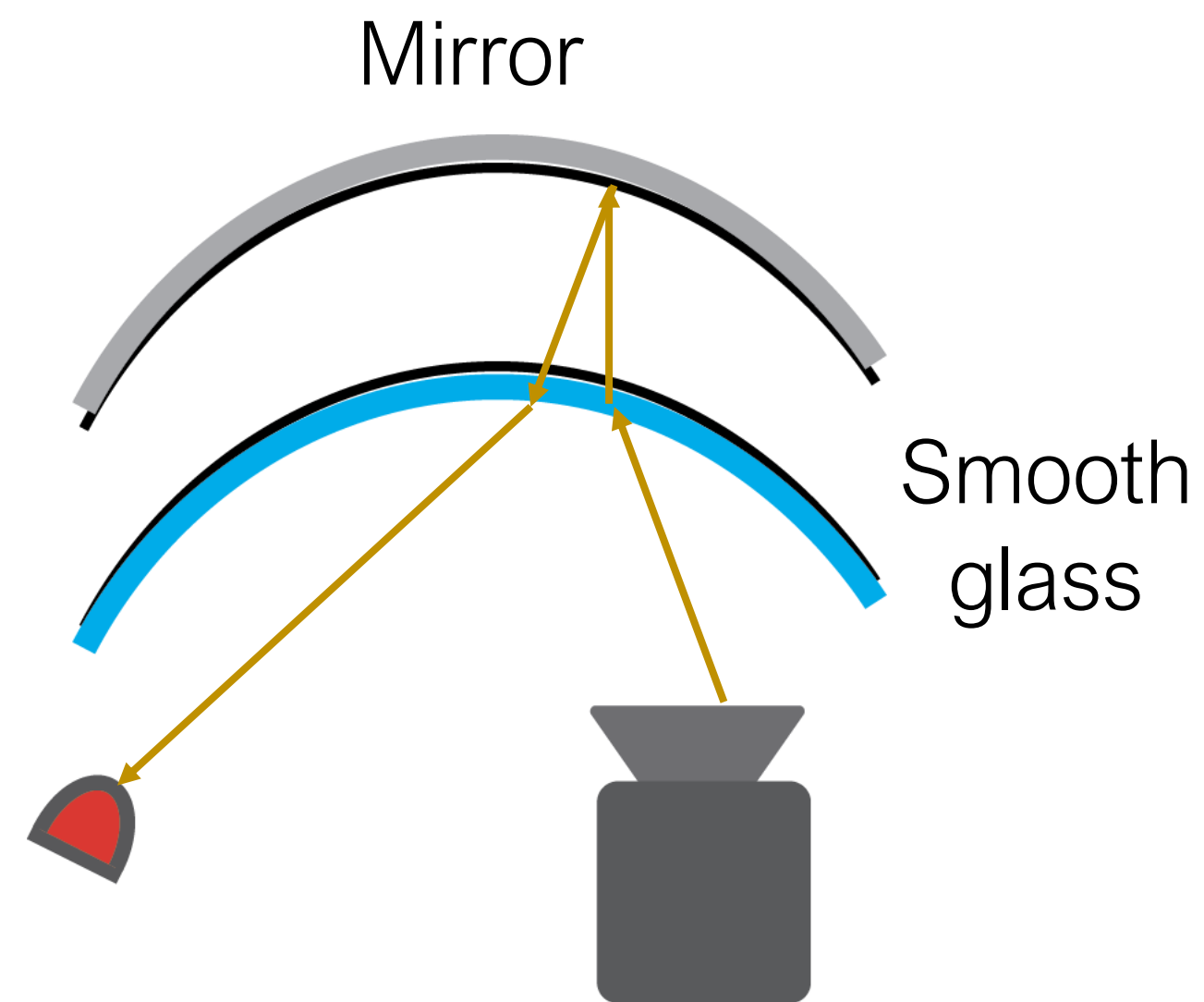
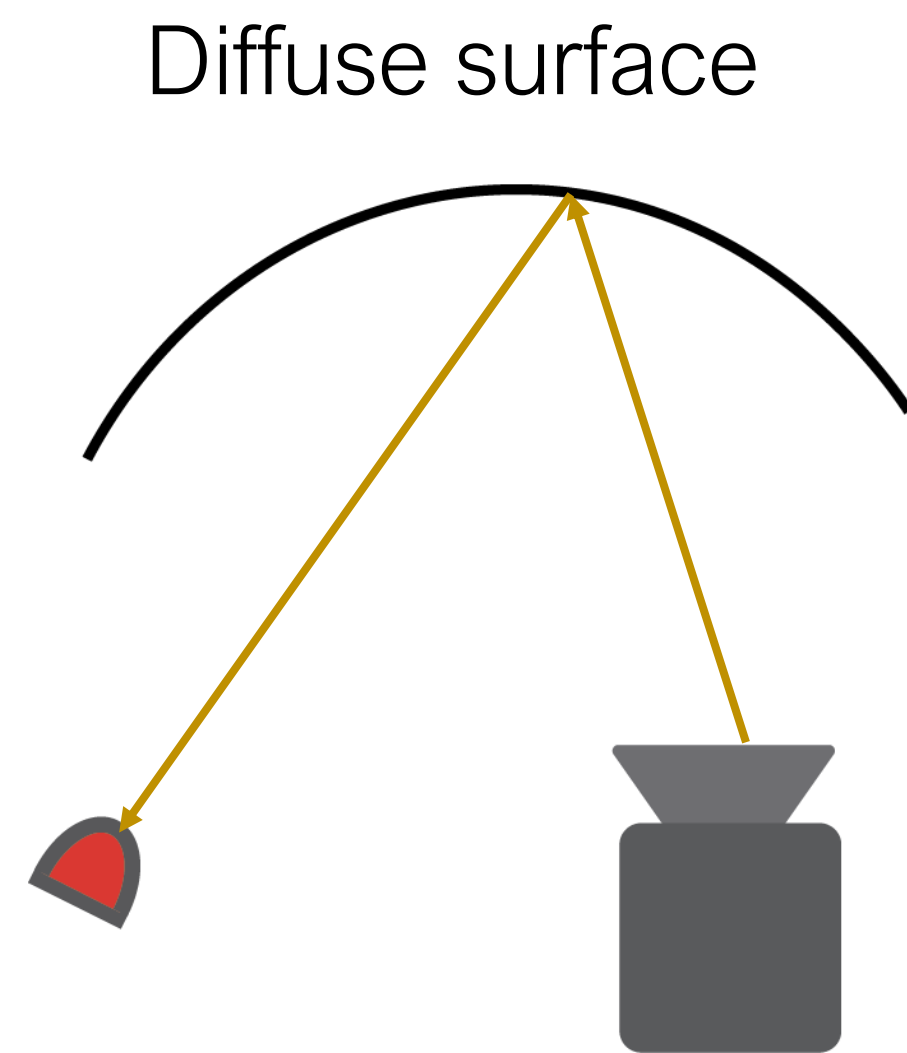


(Relatively) easy cases

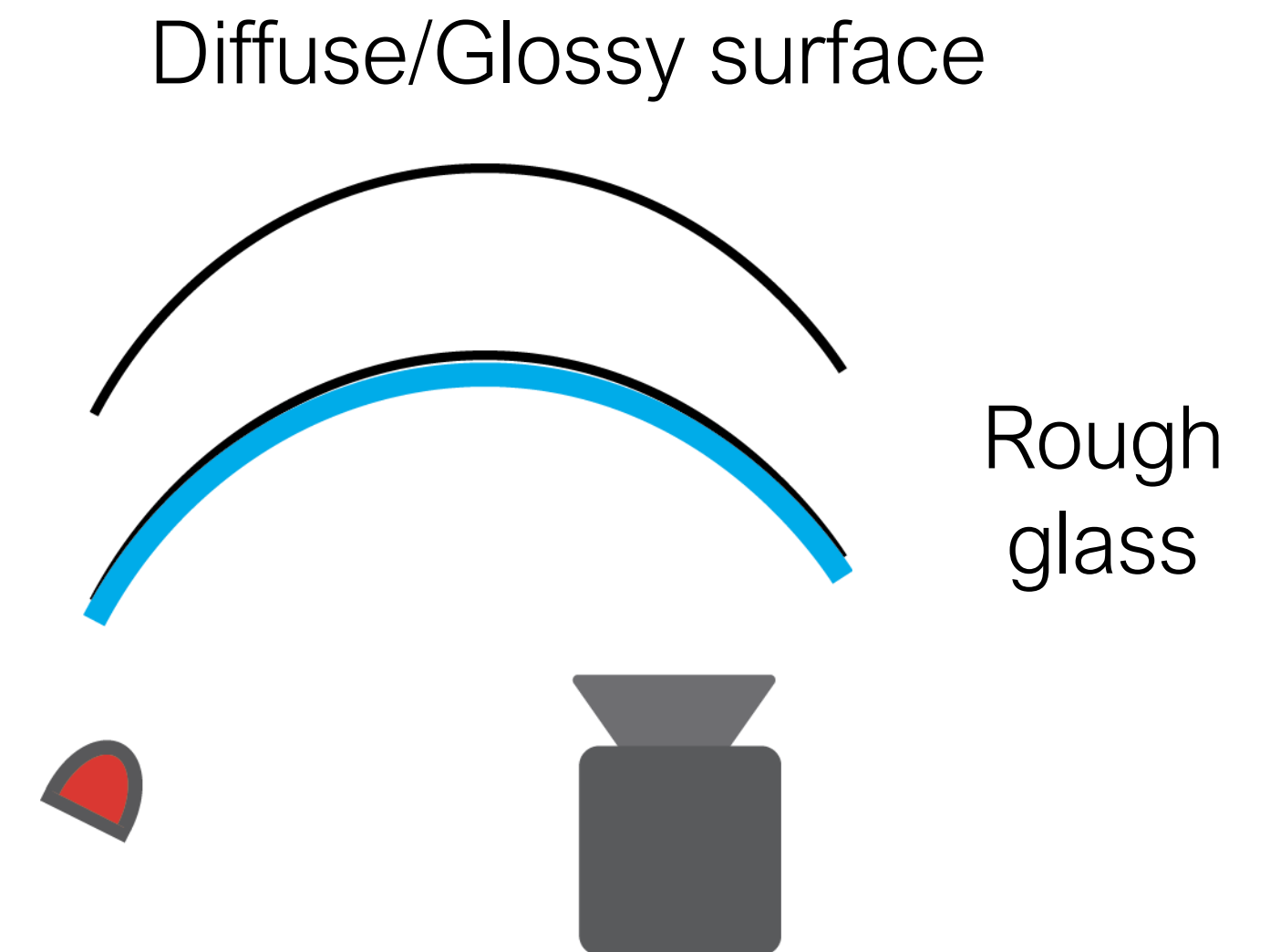
Hard case

# Sensor design framework: optical simulation

- What makes simulation challenging



(Relatively) easy cases

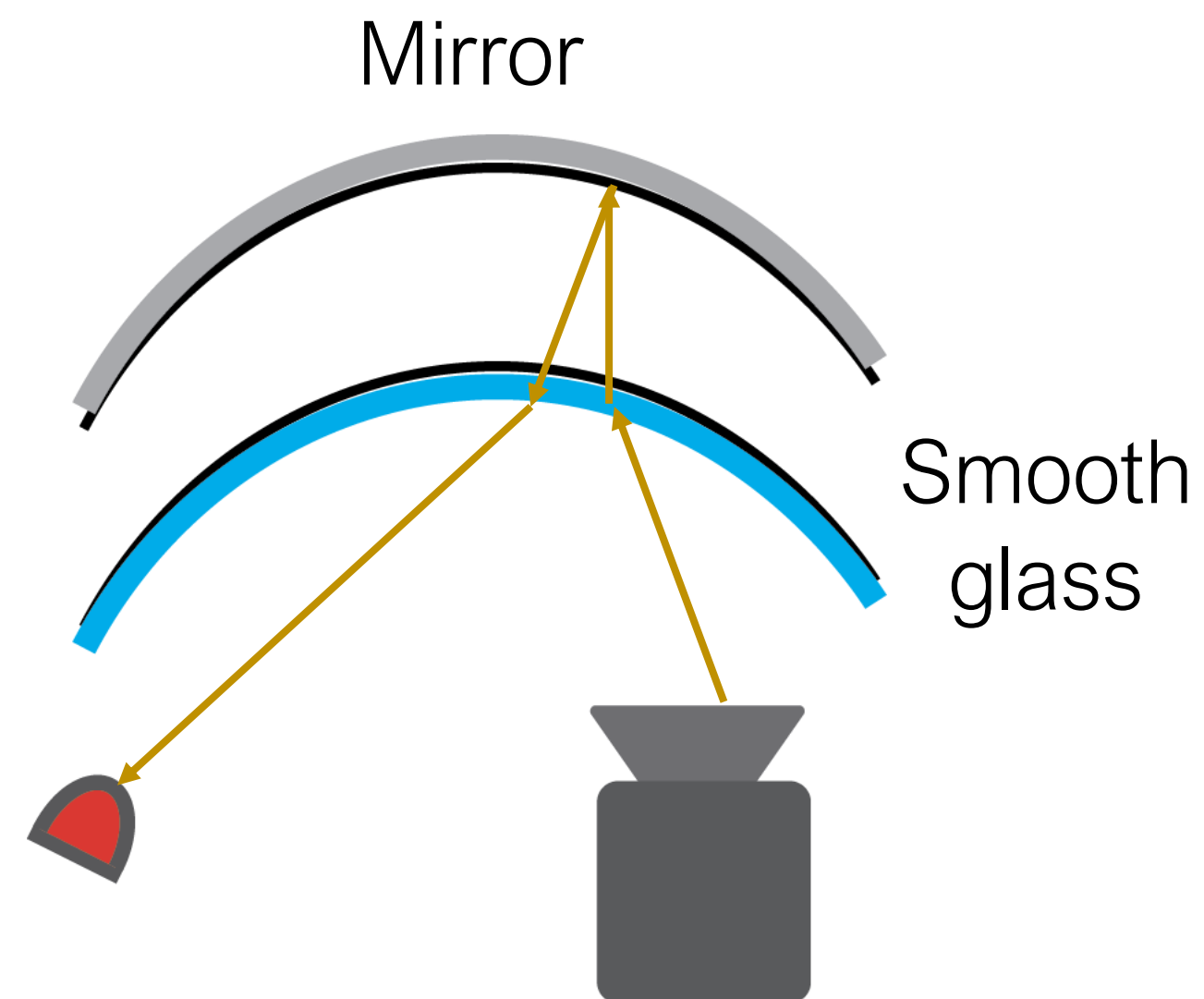
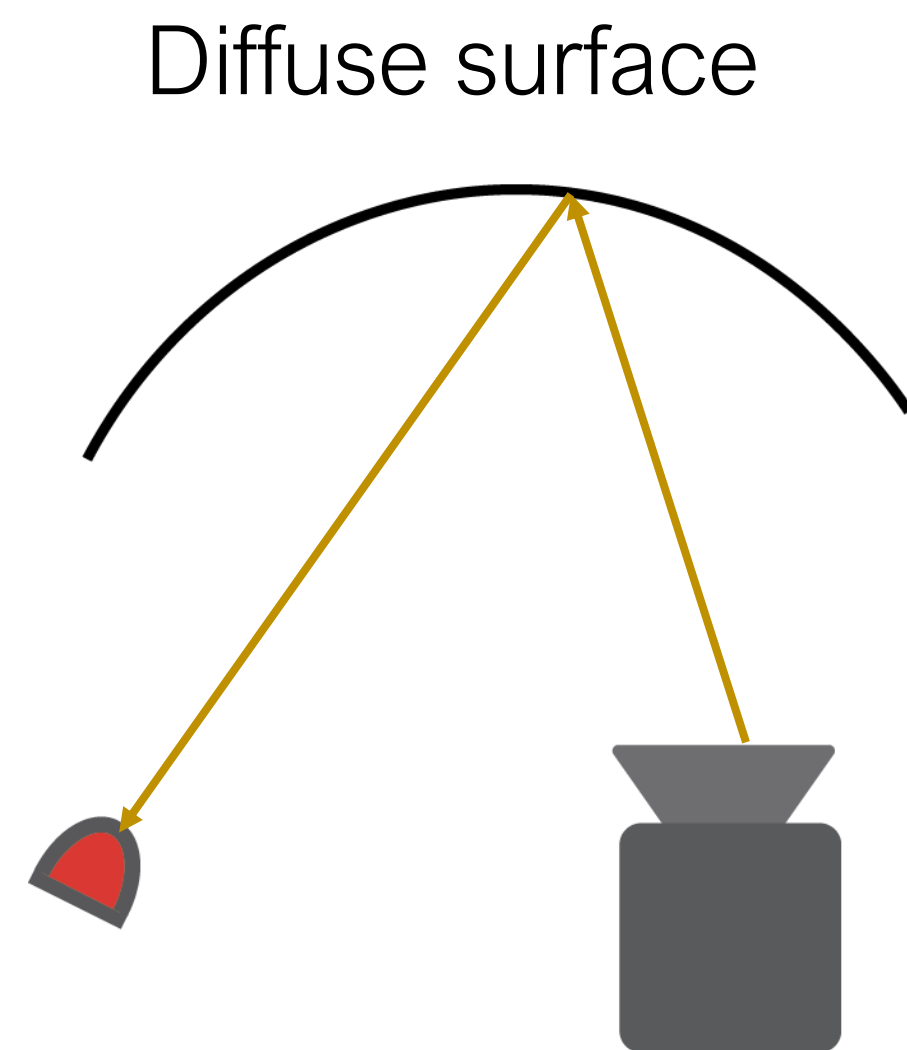


Hard case

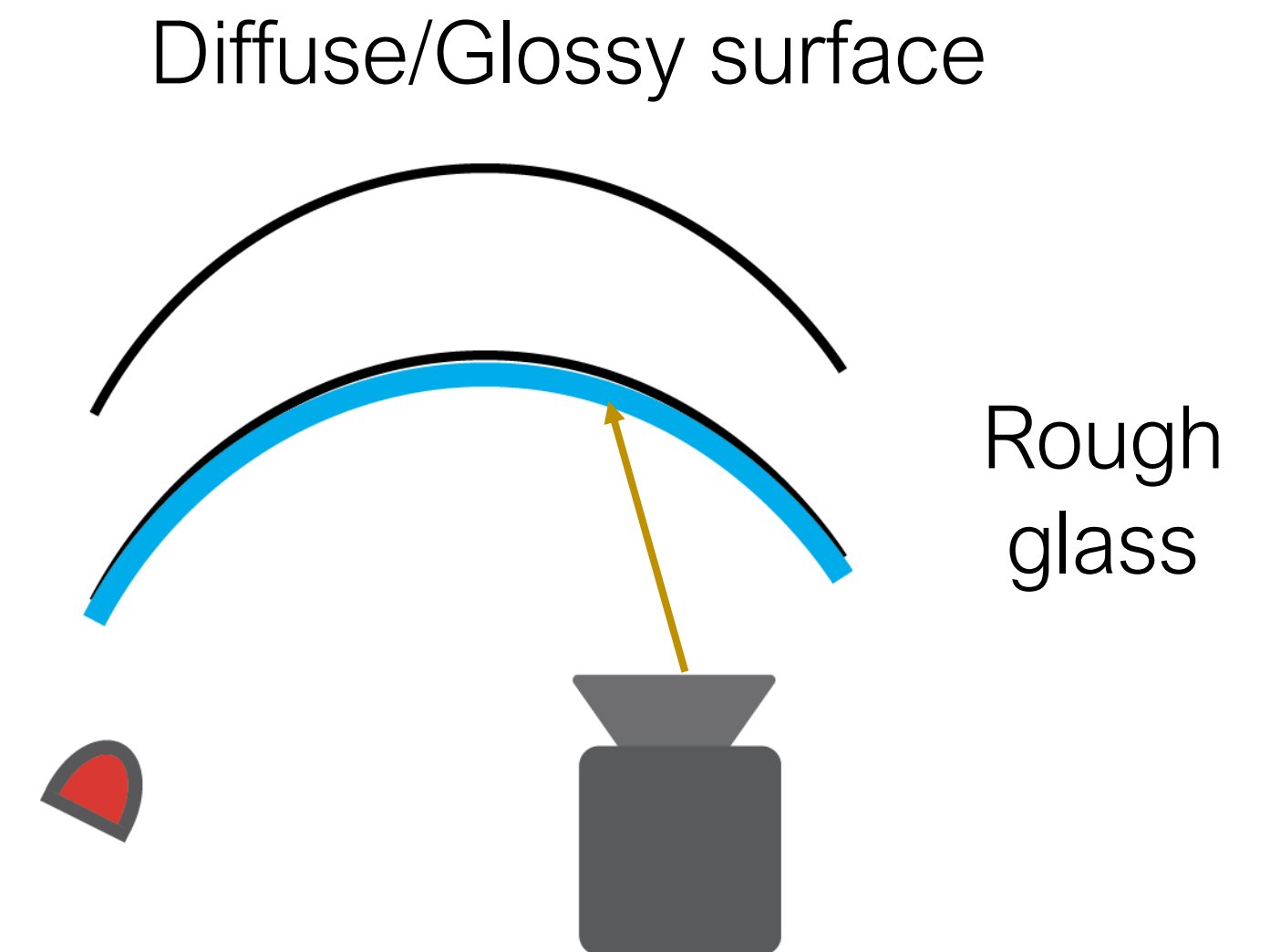


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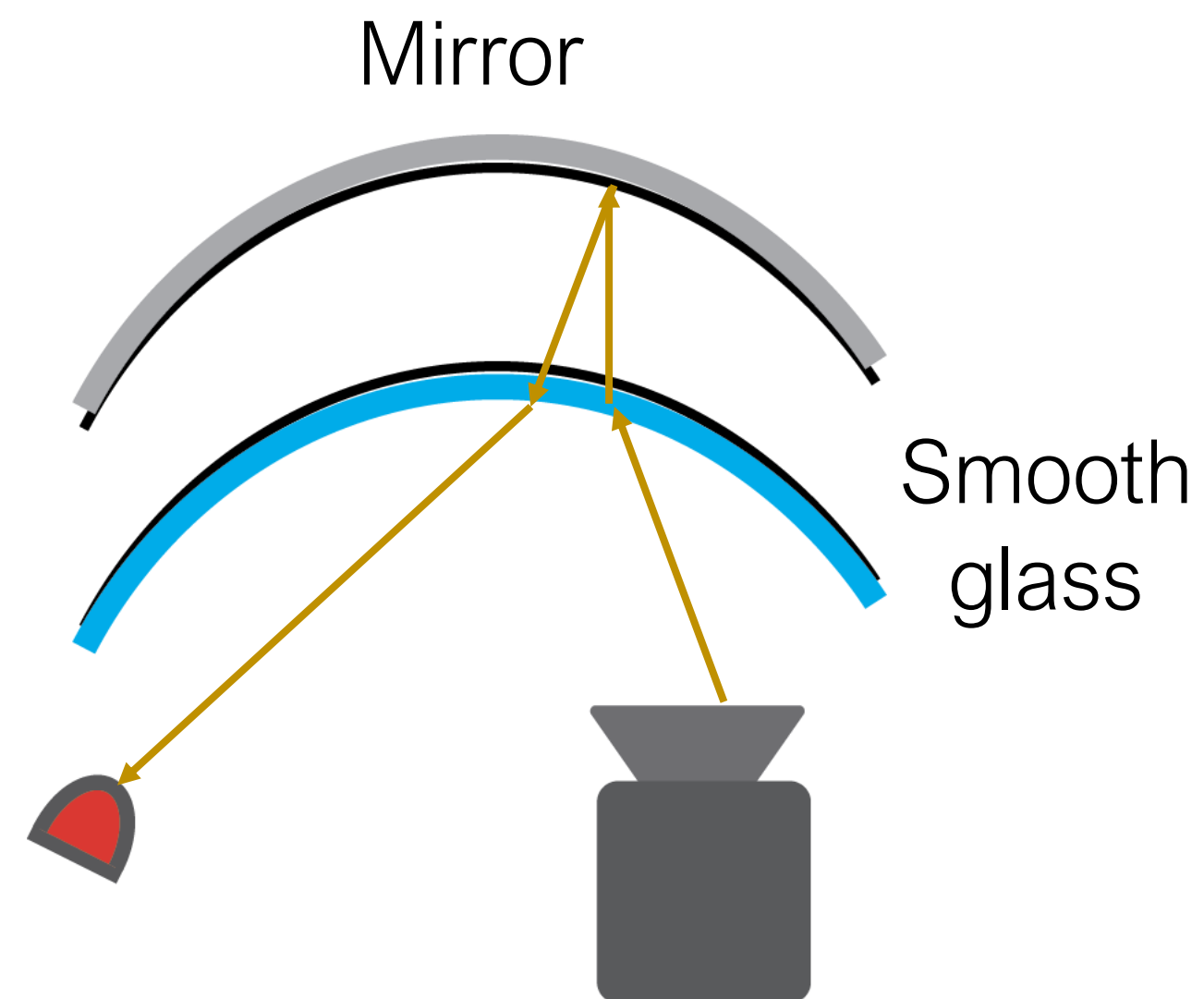
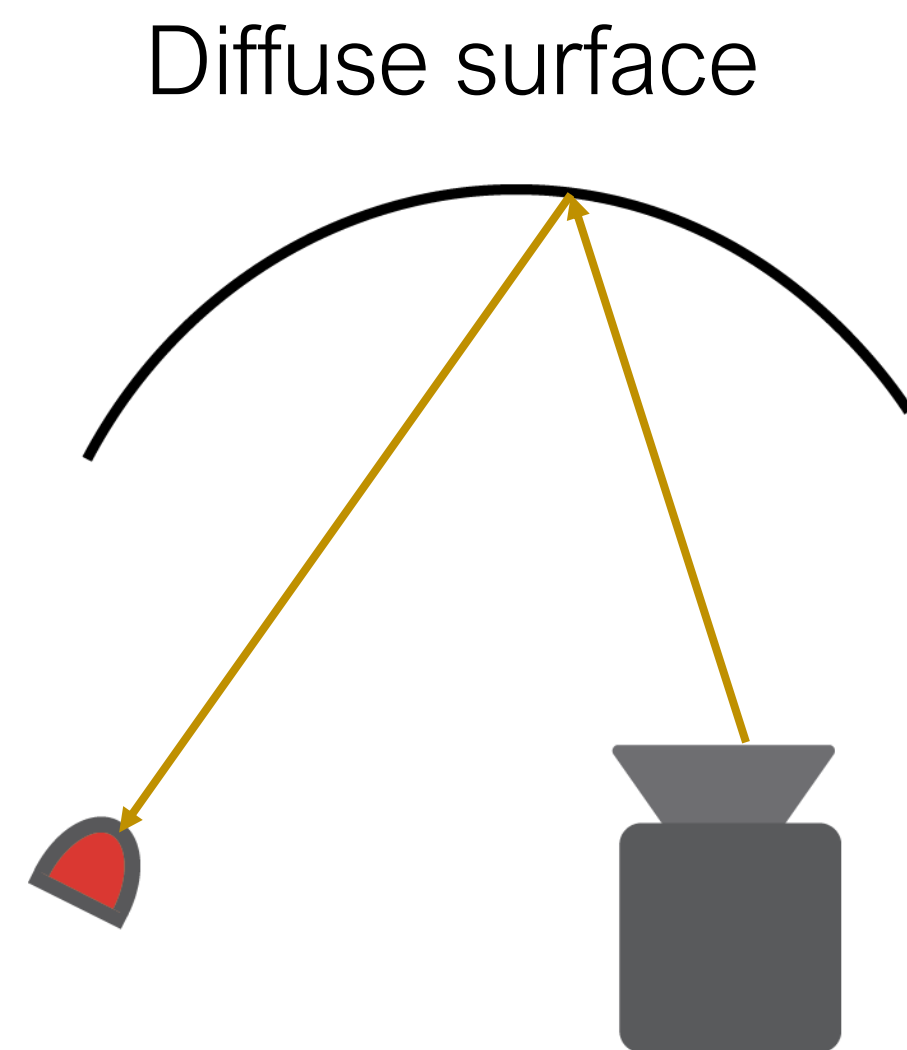
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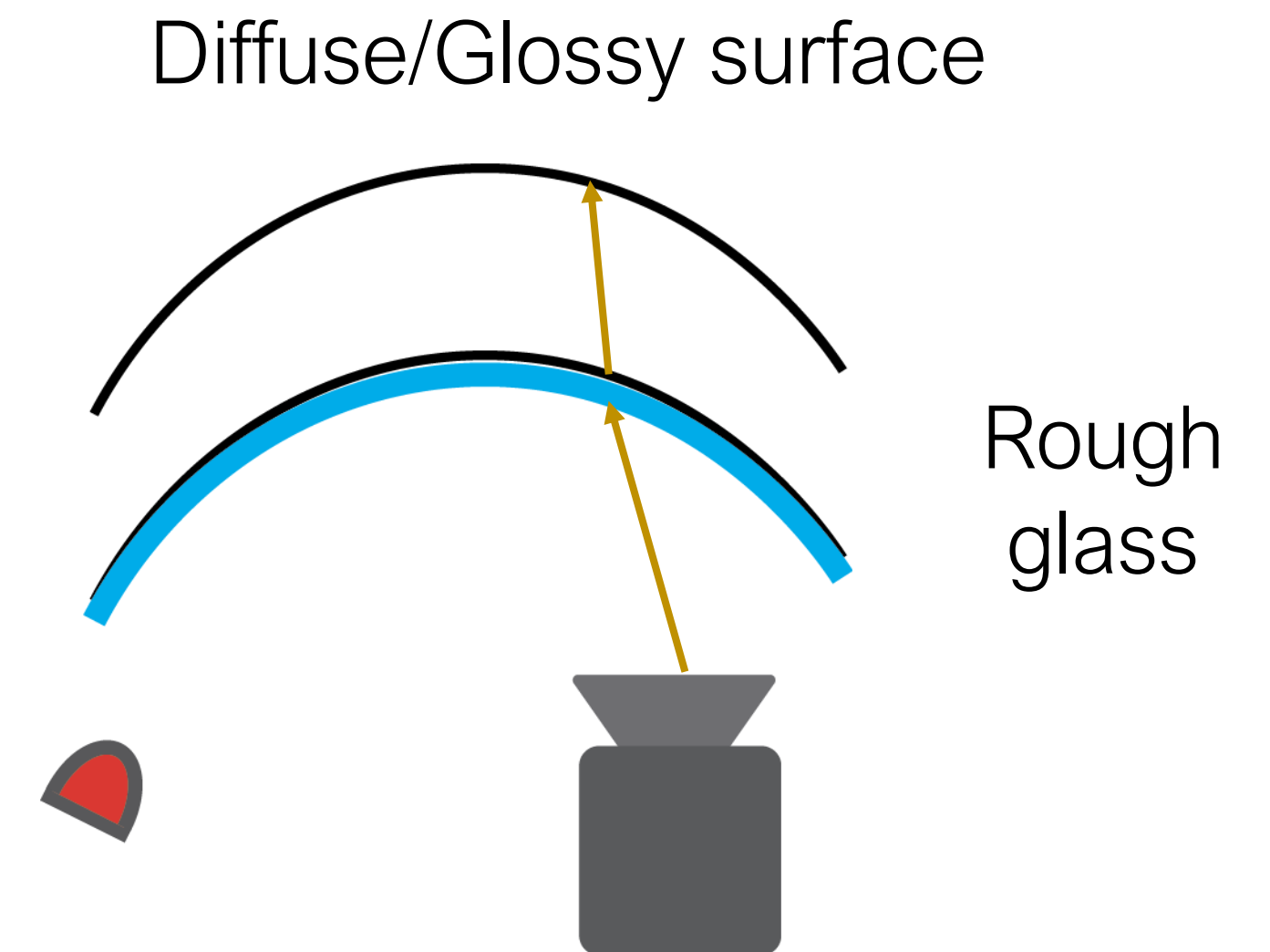
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# Sensor design framework: optical simulation

- What makes simulation challenging



(Relatively) easy cases

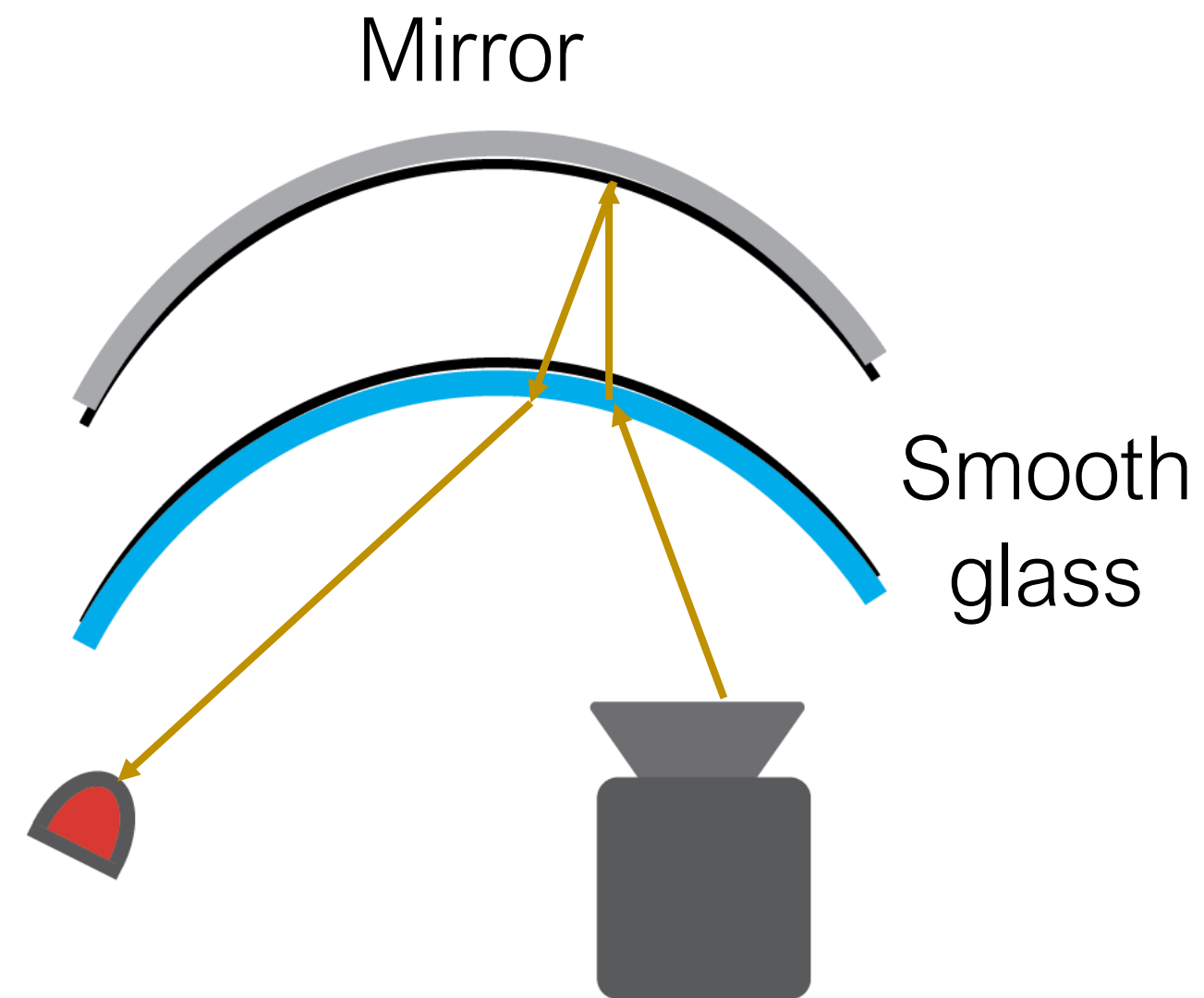
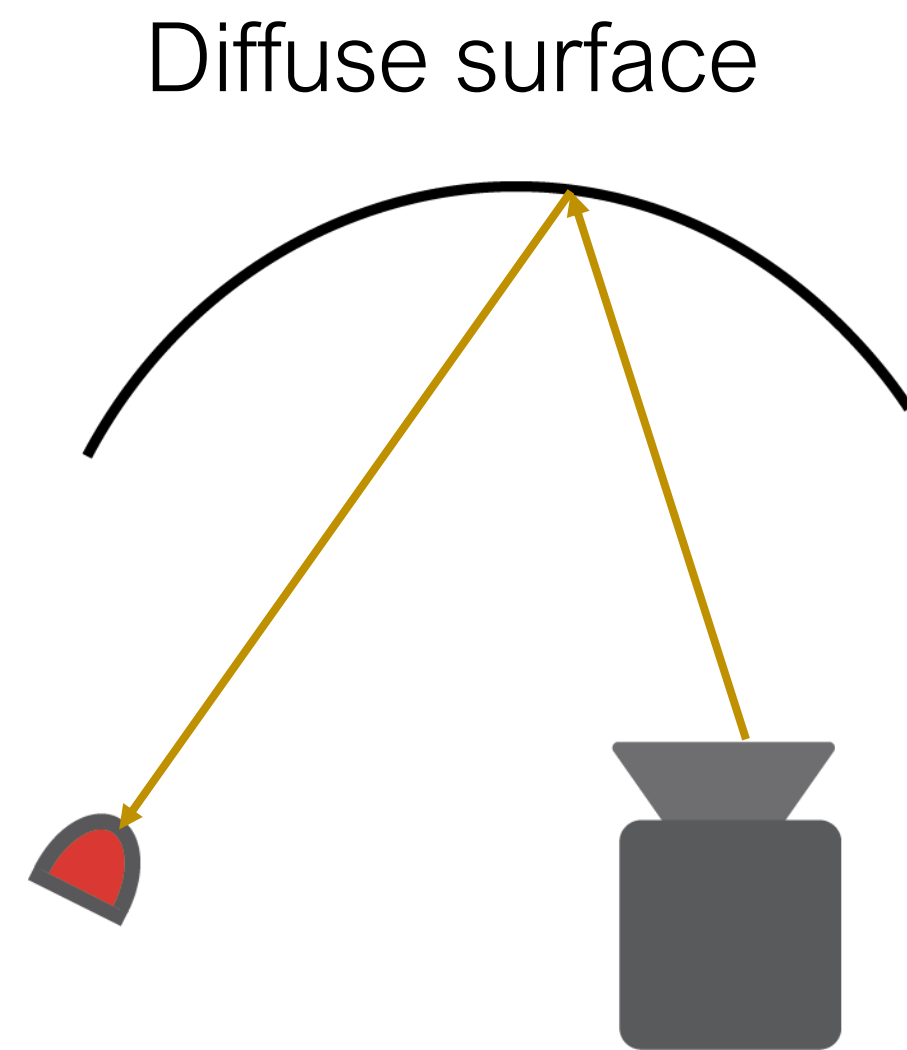


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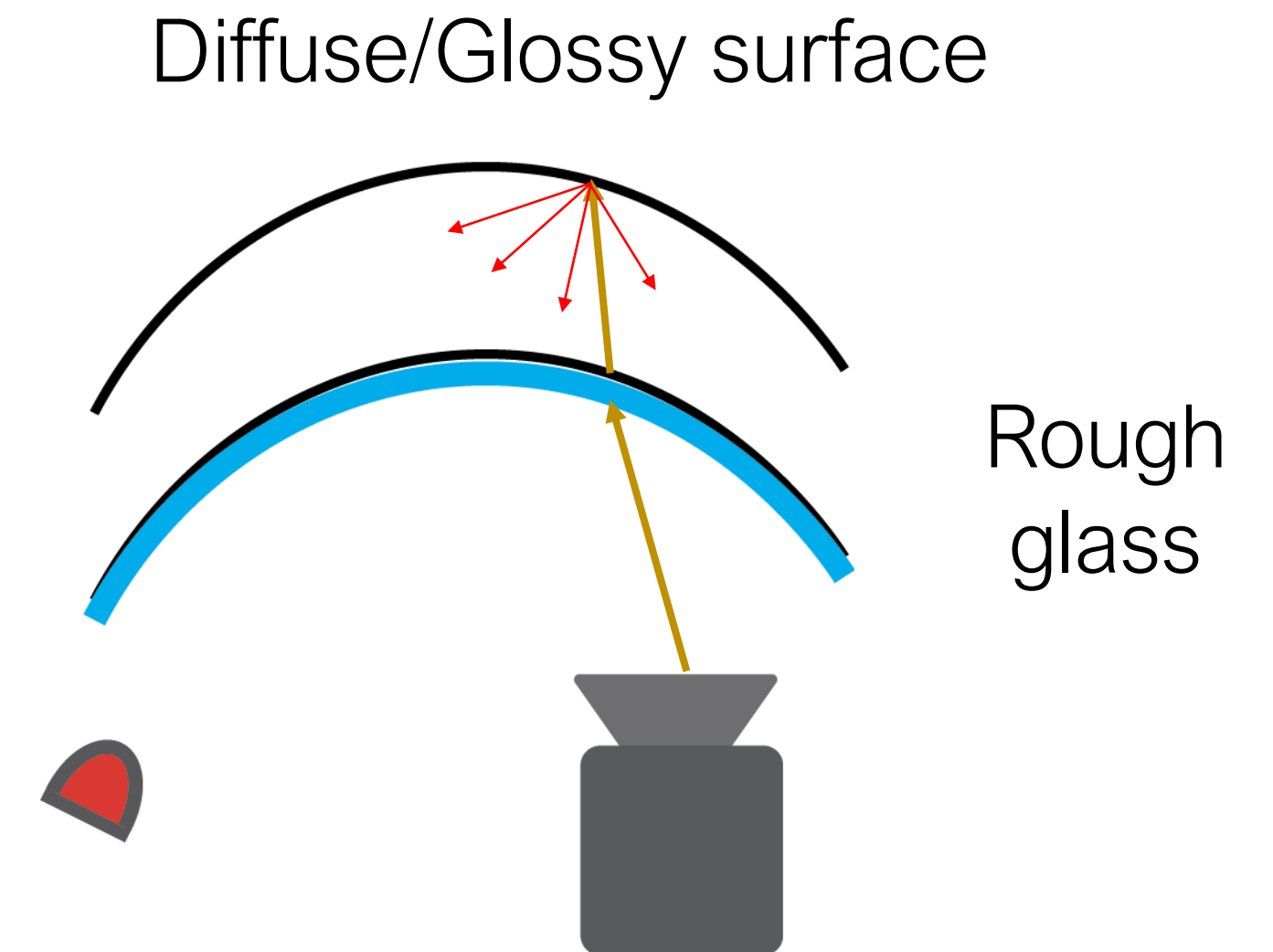


# Sensor design framework: optical simulation

- What makes simulation challenging



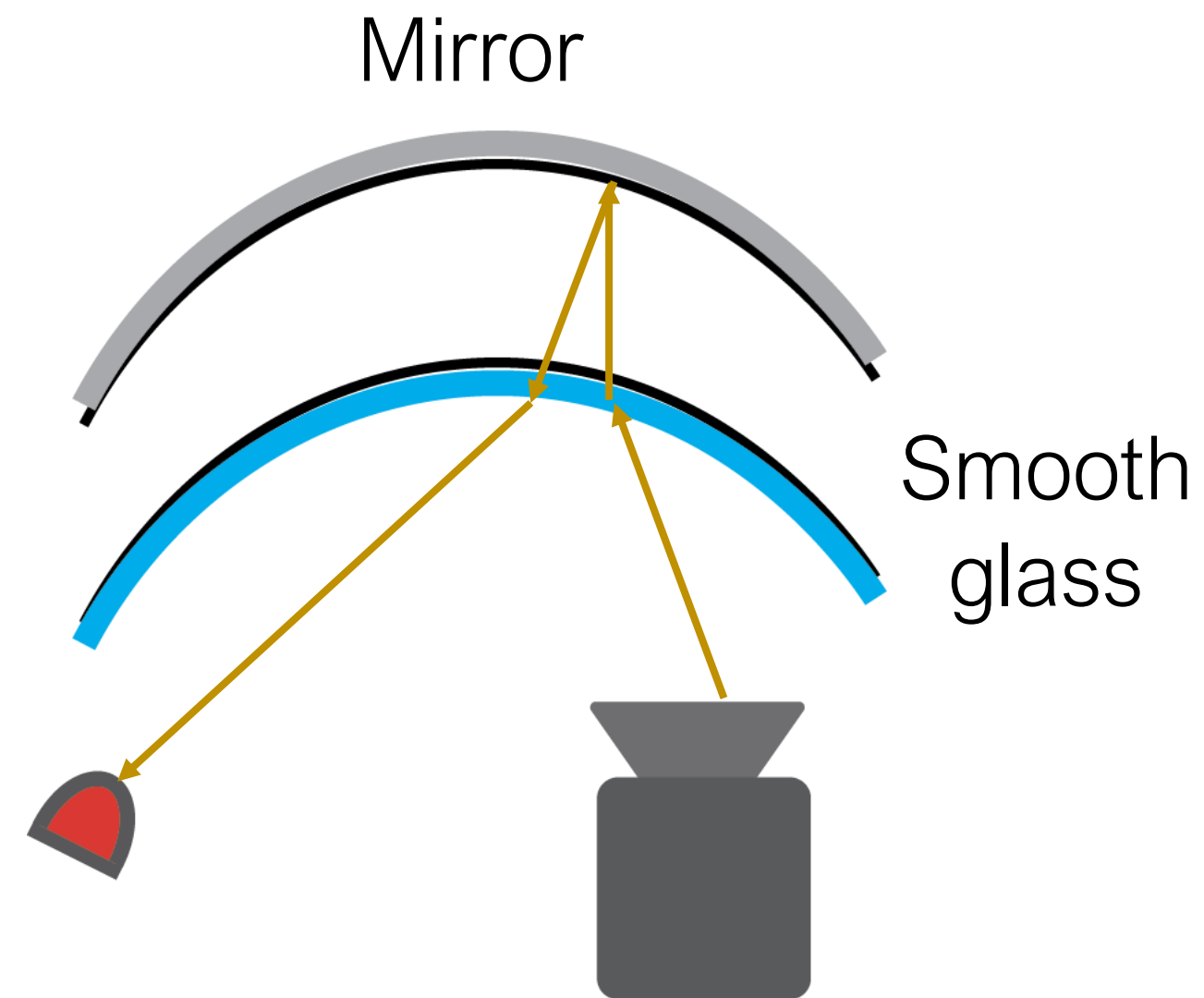
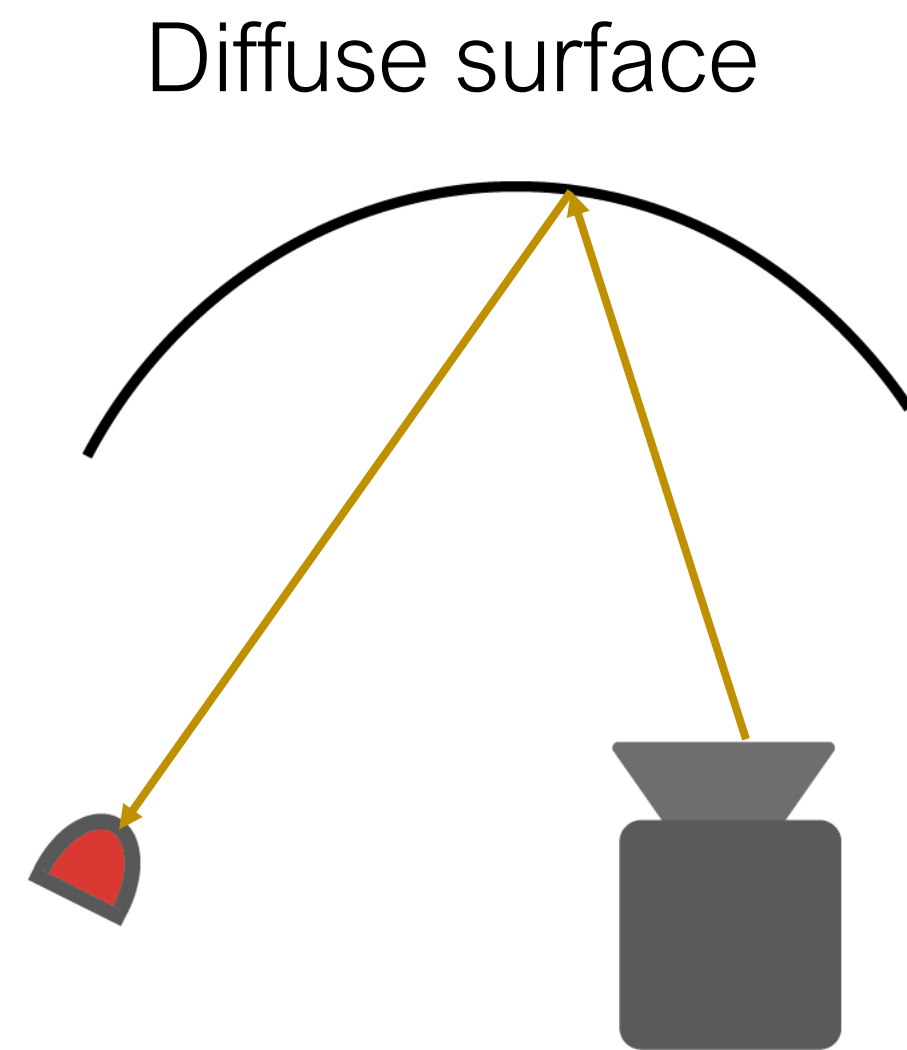
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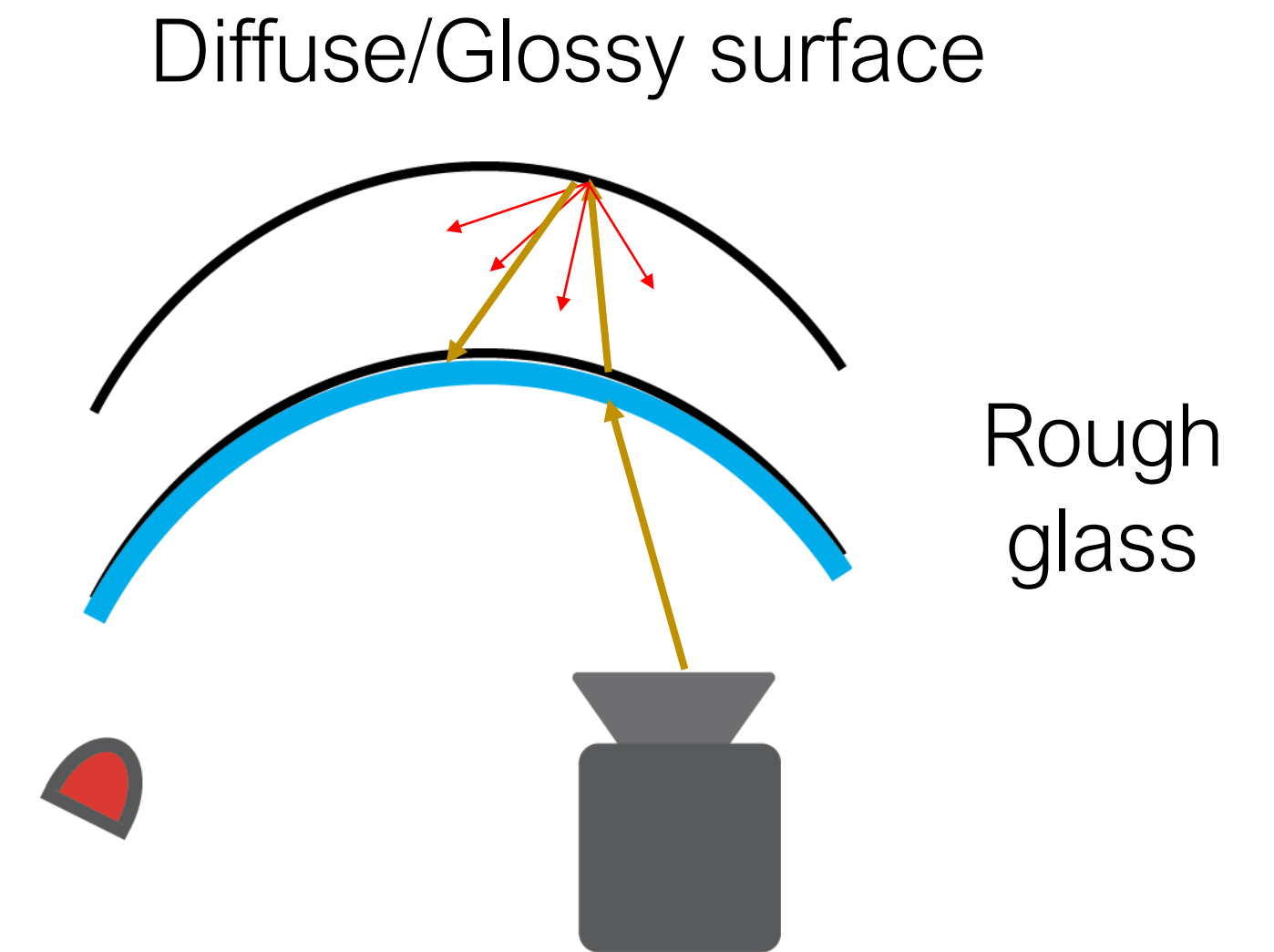
Hard case

# Sensor design framework: optical simulation

- What makes simulation challenging



(Relatively) easy cases

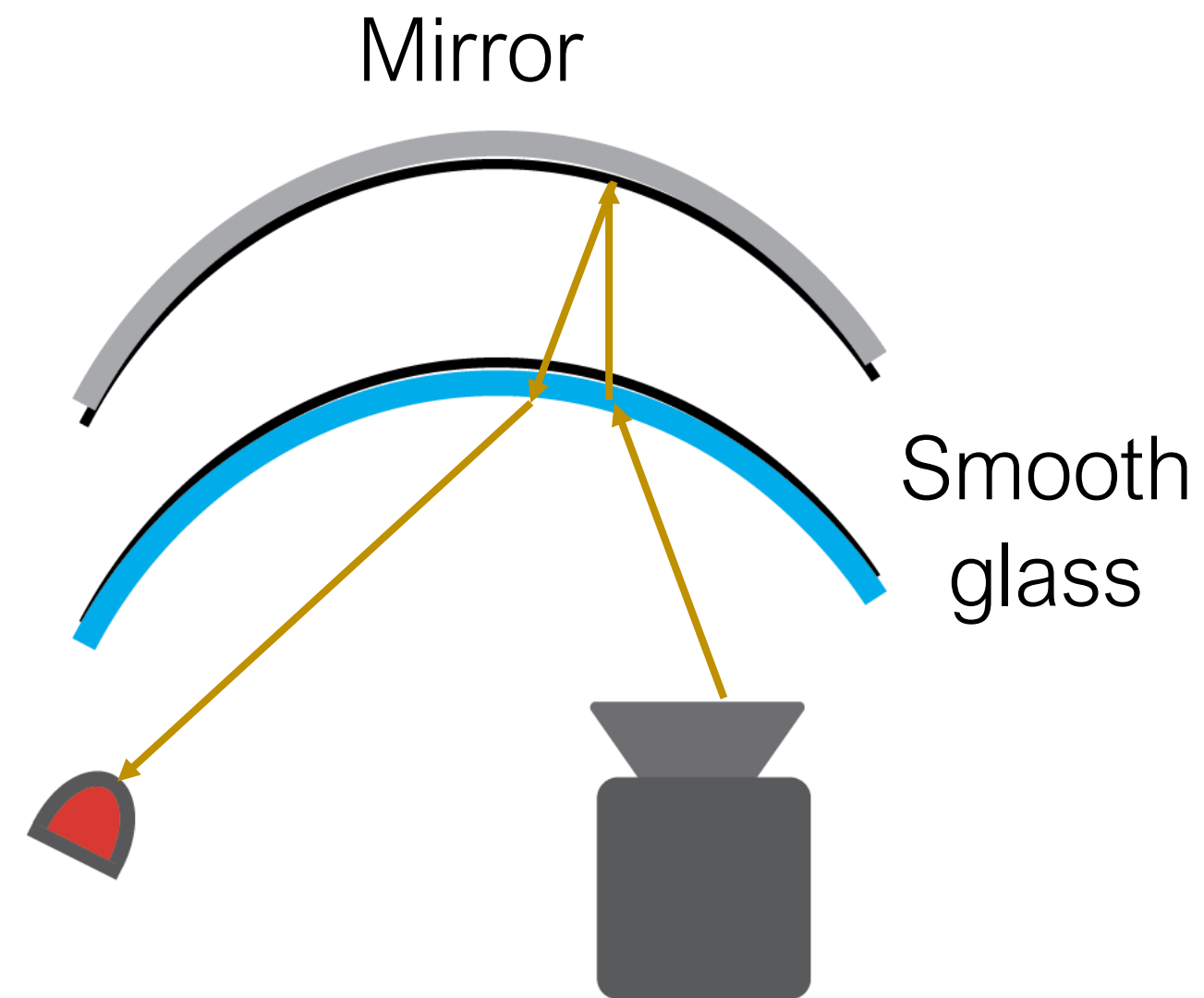
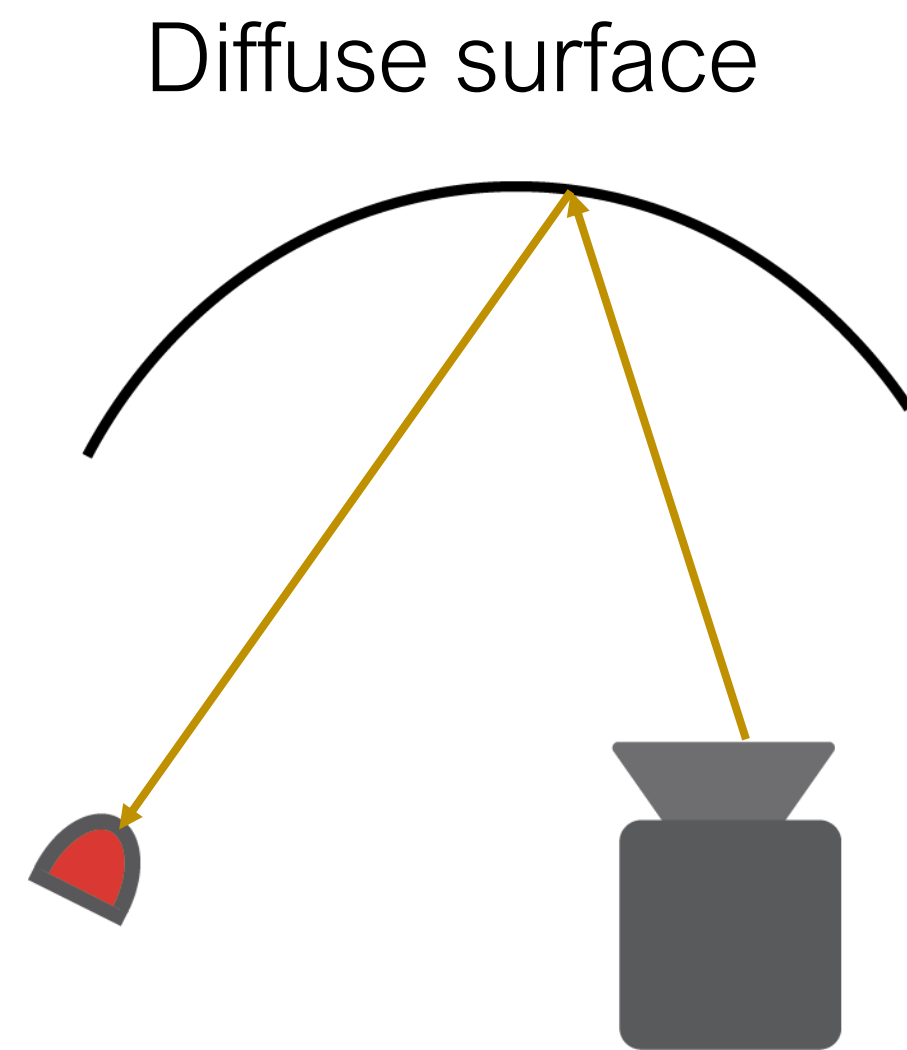


Hard case

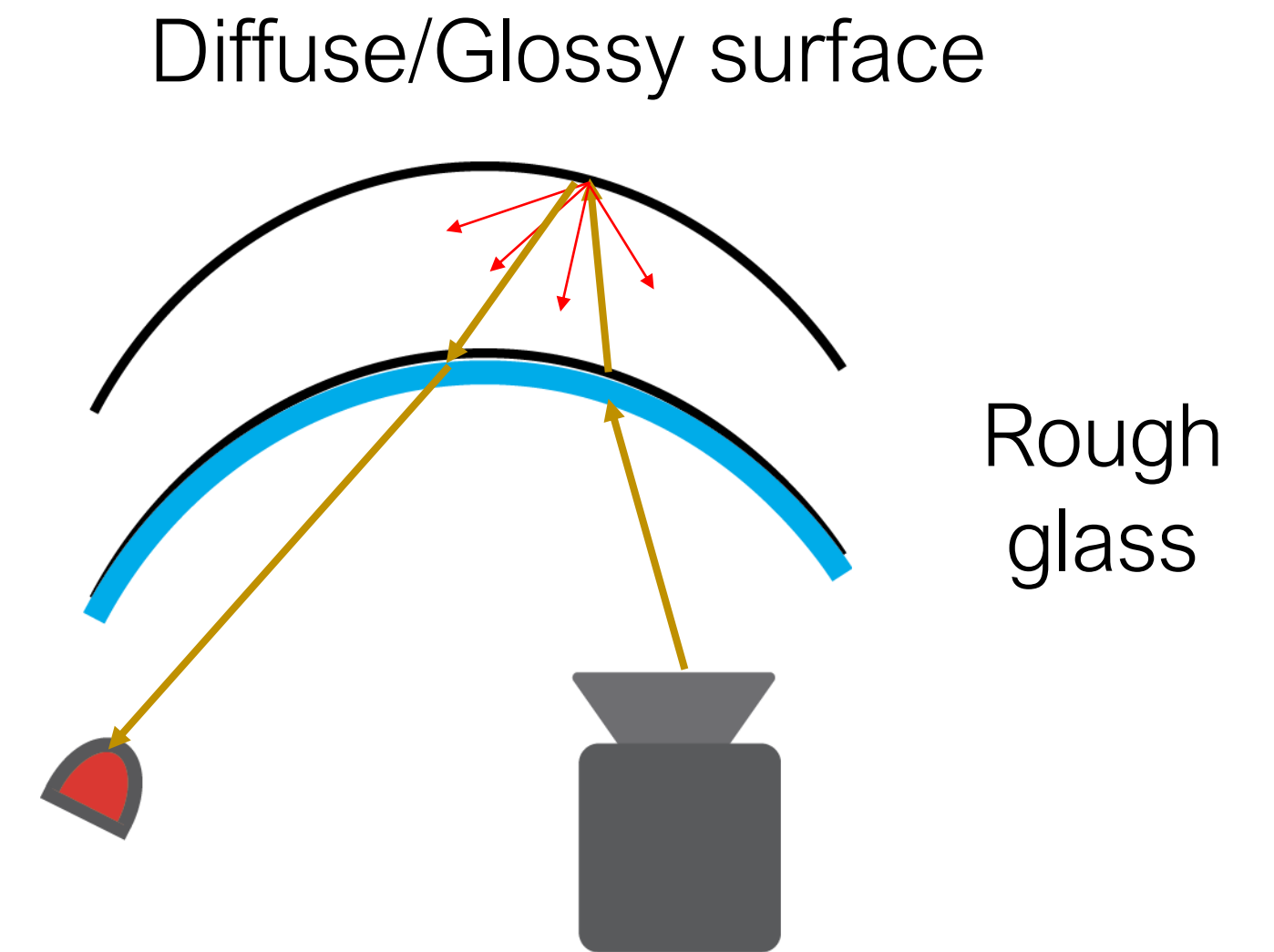


# Sensor design framework: optical simulation

- What makes simulation challenging



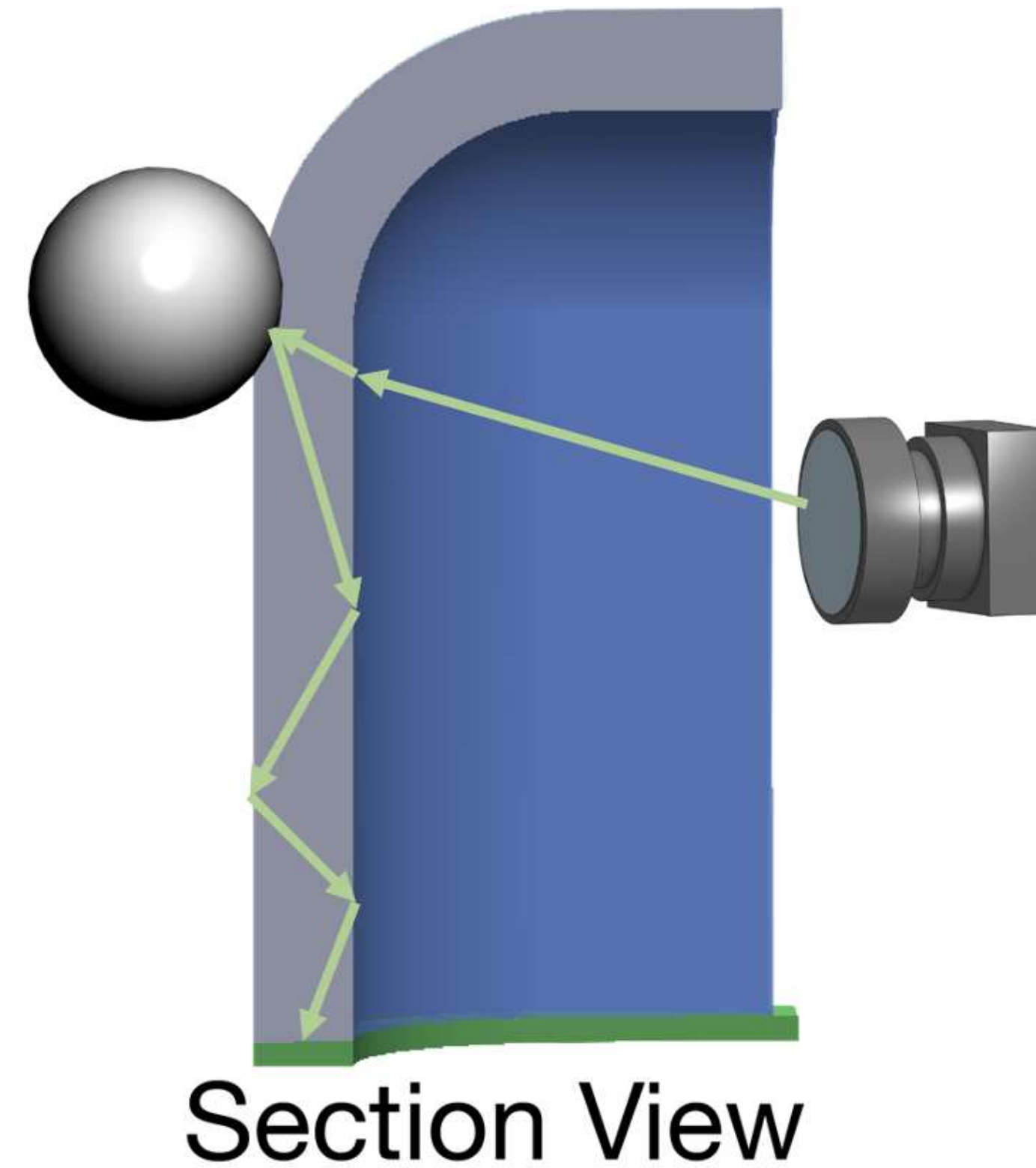
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Hard case


# Sensor design framework: optical simulation

- What makes simulation challenging
  - SDS light paths
  - Indirect illumination






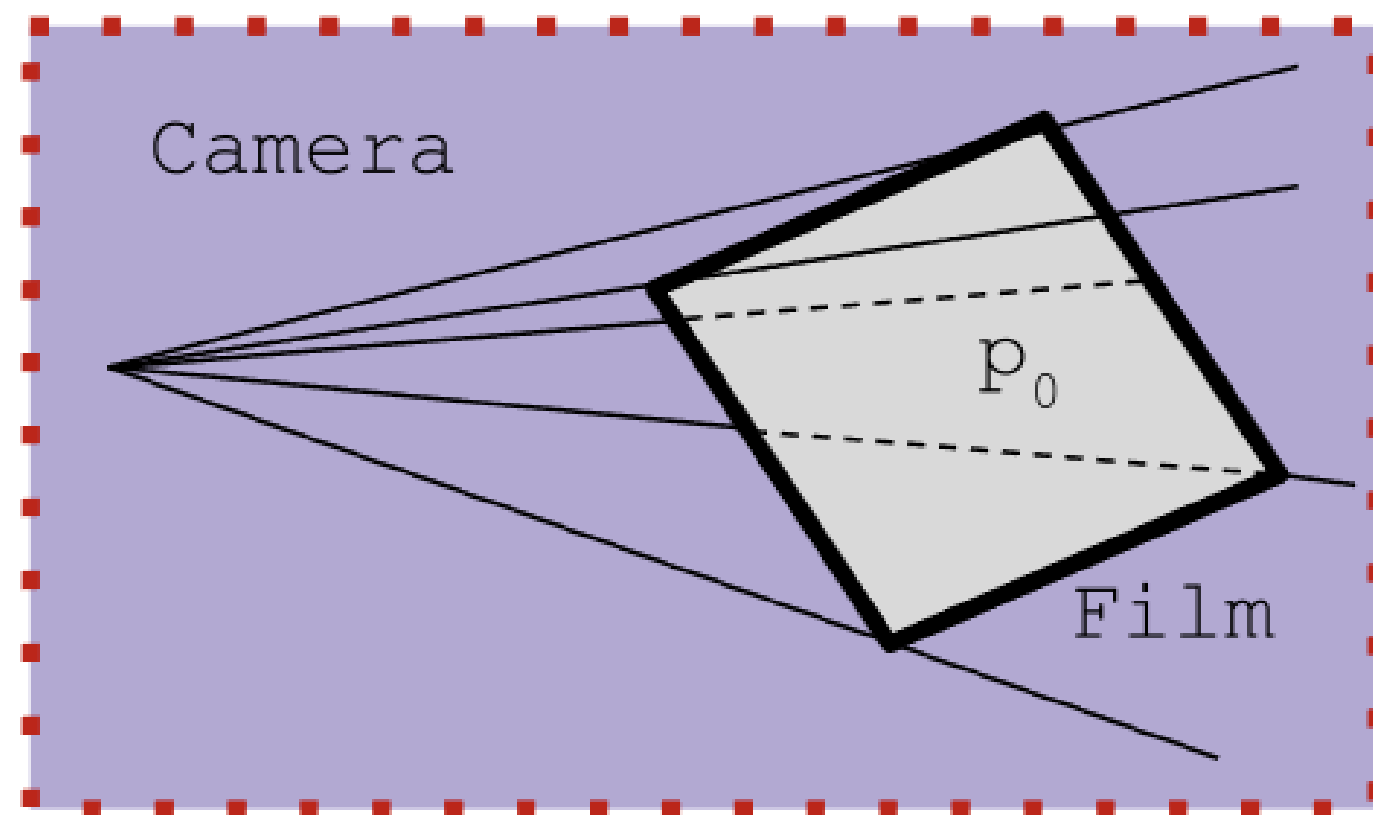
# Sensor design framework: optical simulation

- What makes simulation challenging
  - SDS light paths
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- Require Markov Chain Monte Carlo rendering techniques
  - Key idea  Slowly mutate paths to generate useful paths




# Sensor design framework: optical simulation

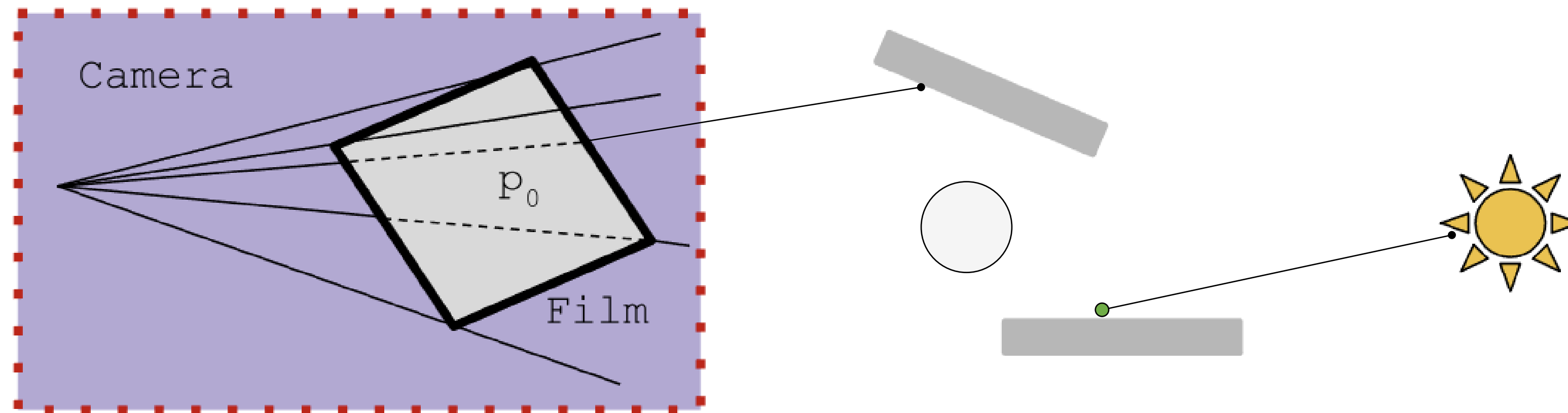
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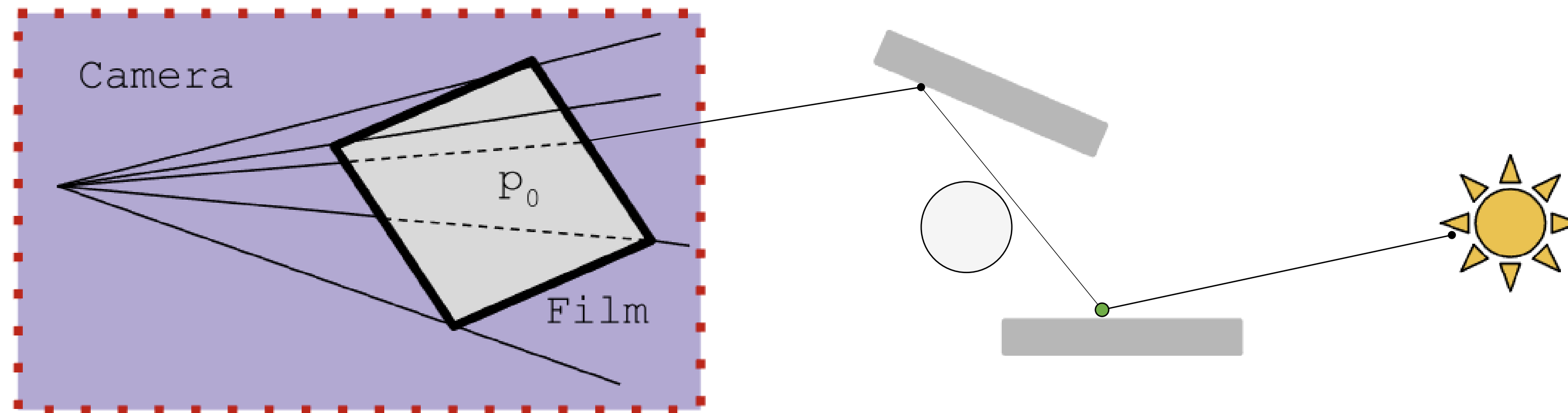
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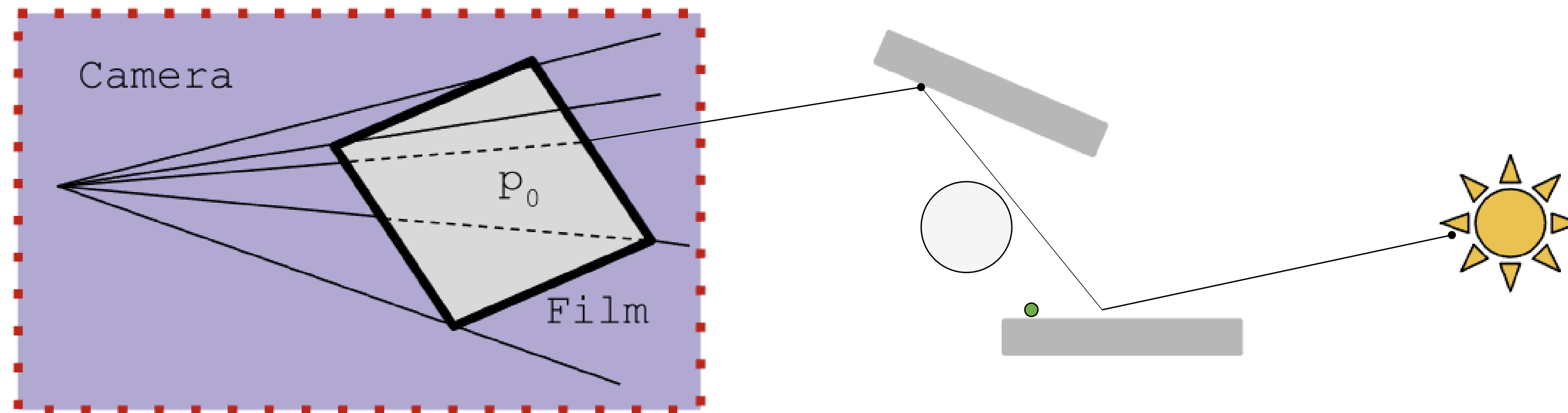
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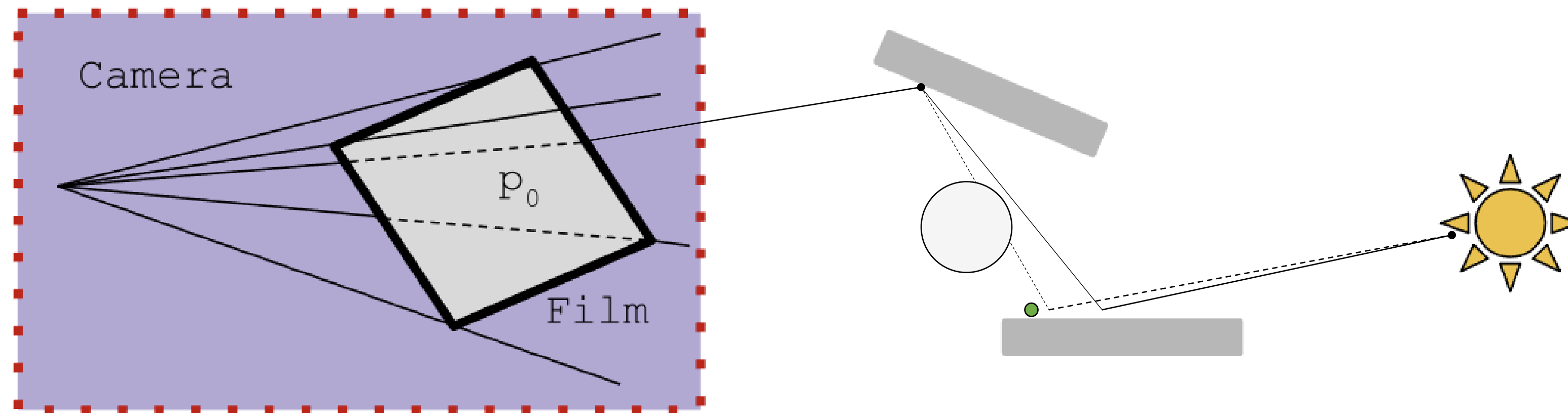
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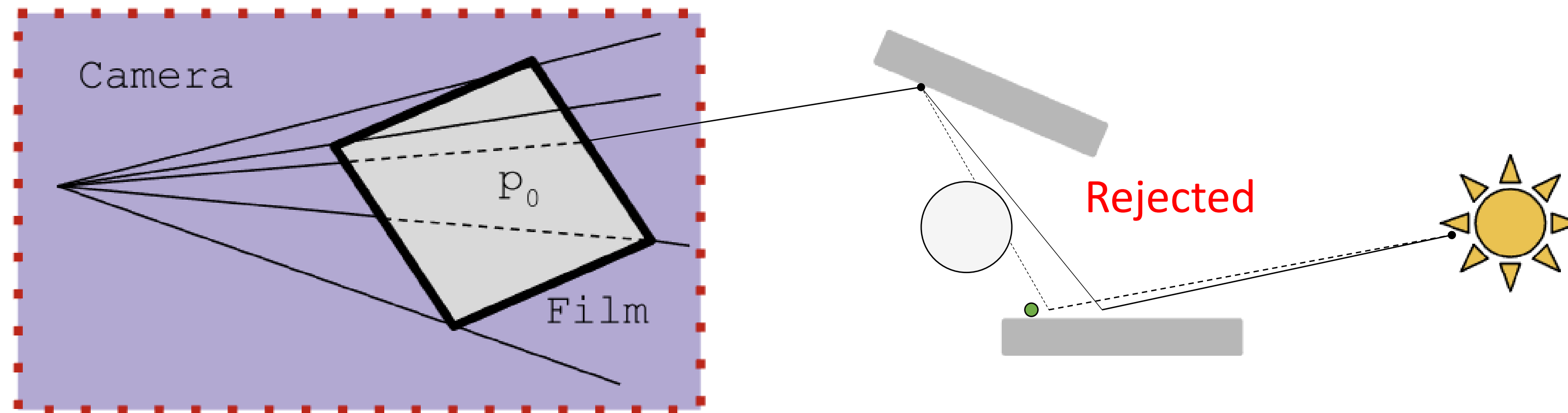
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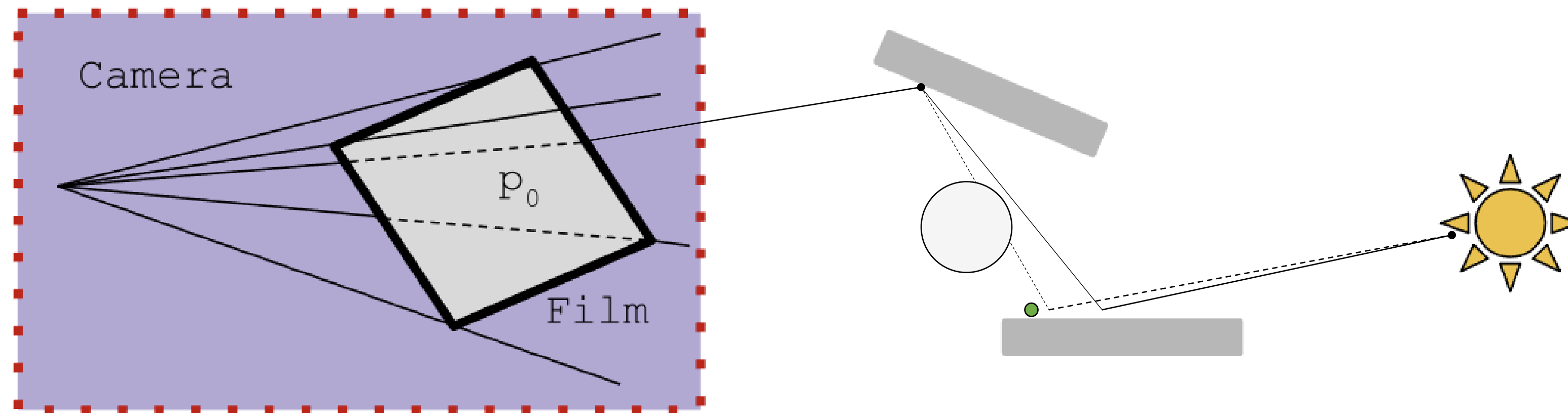
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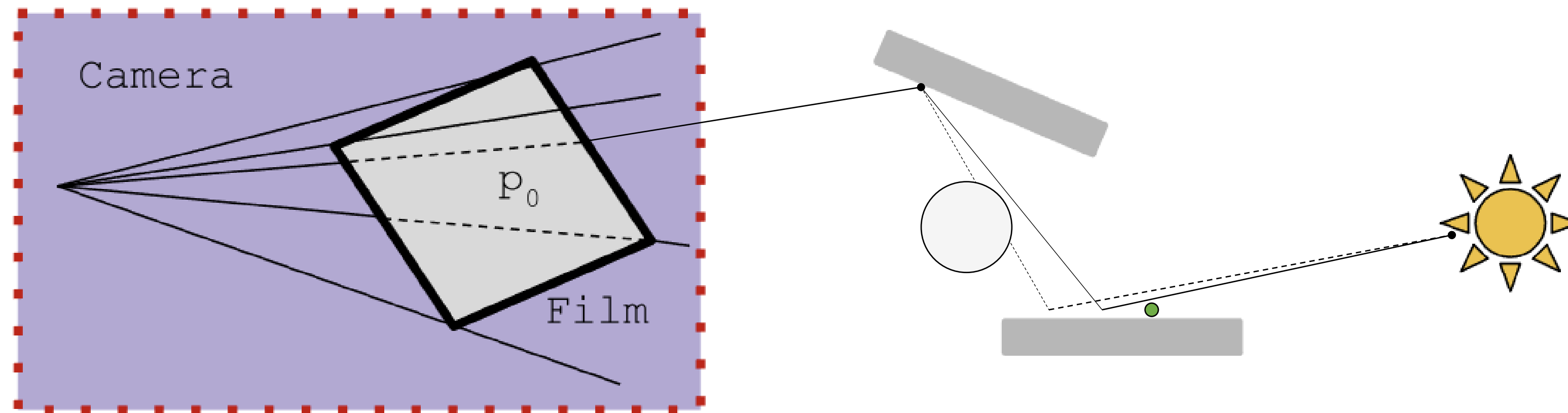
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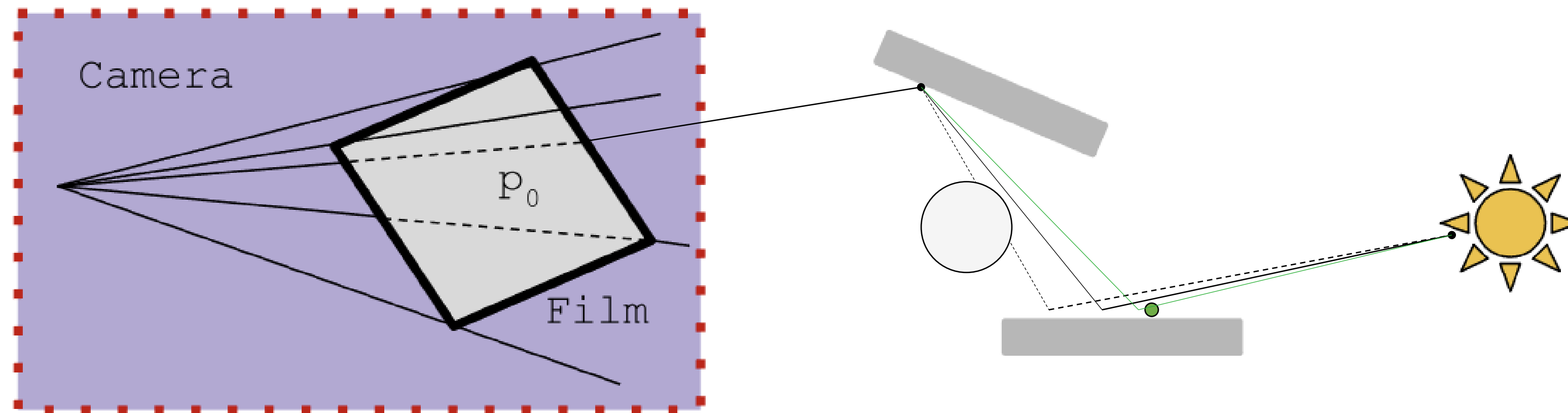
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  - Key idea  Slowly mutate paths to generate useful paths




# Sensor design framework: optical simulation

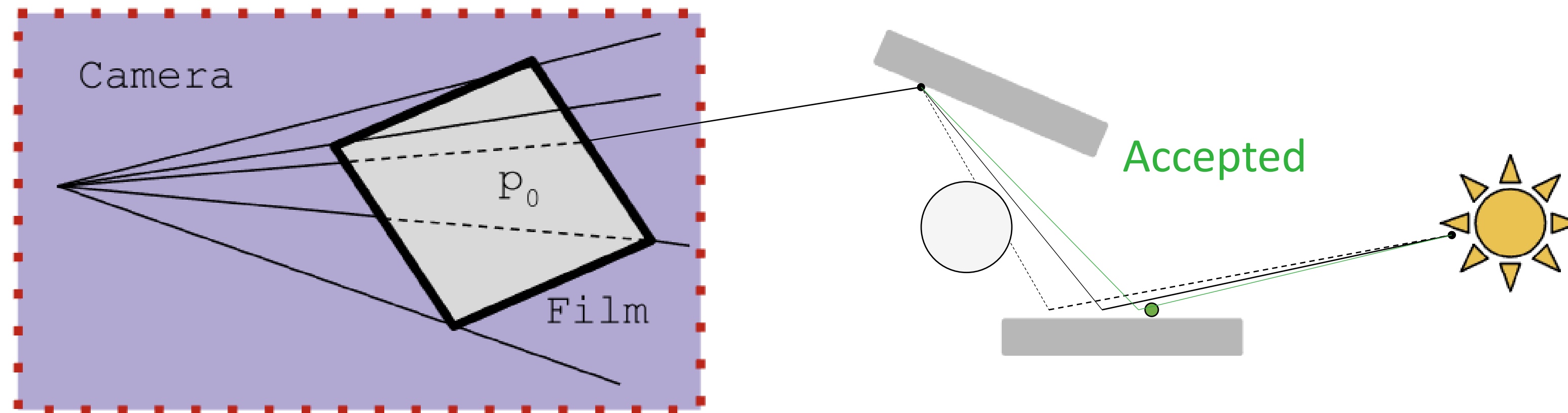
- What makes simulation challenging
  - SDS light paths
  - Indirect illumination
- Require Markov Chain Monte Carlo rendering techniques
  - Key idea  Slowly mutate paths to generate useful paths





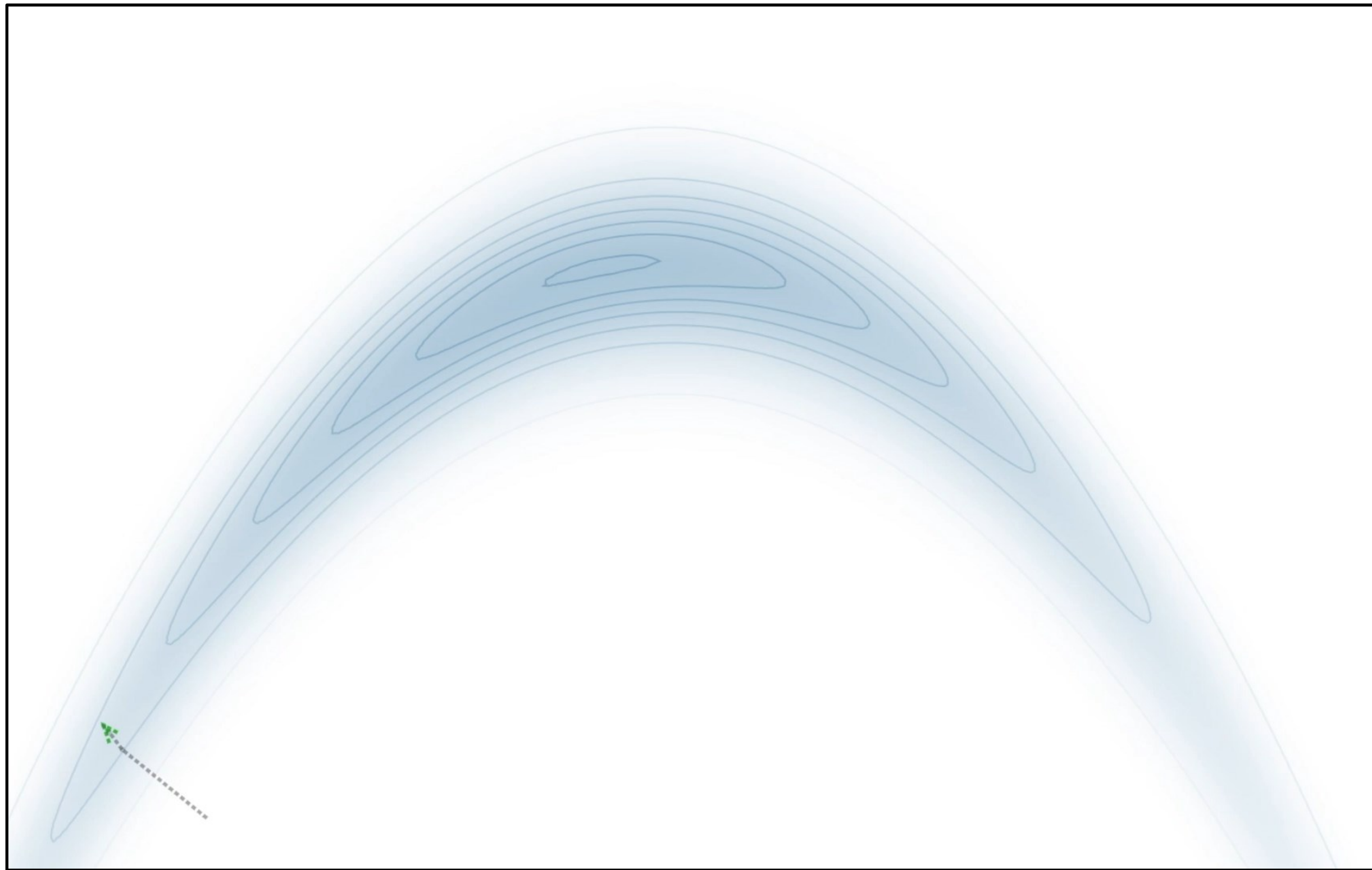
# Sensor design framework: optical simulation

- What makes simulation challenging
  - SDS light paths
  - Indirect illumination
- Require Markov Chain Monte Carlo rendering techniques
  - Key idea  Slowly mutate paths to generate useful paths



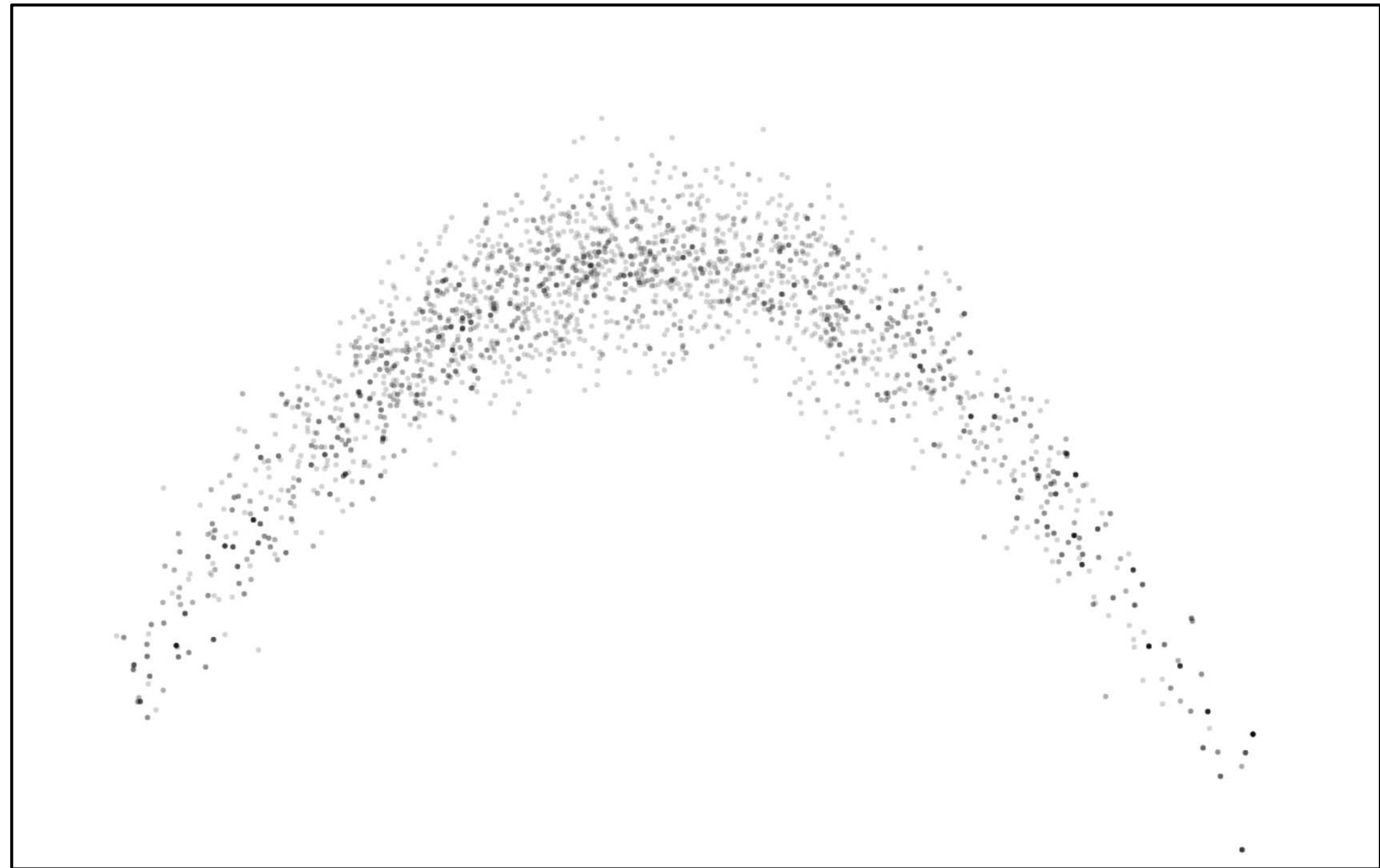
# Optimization:

- Stochastic Gradient Descent (SGD)



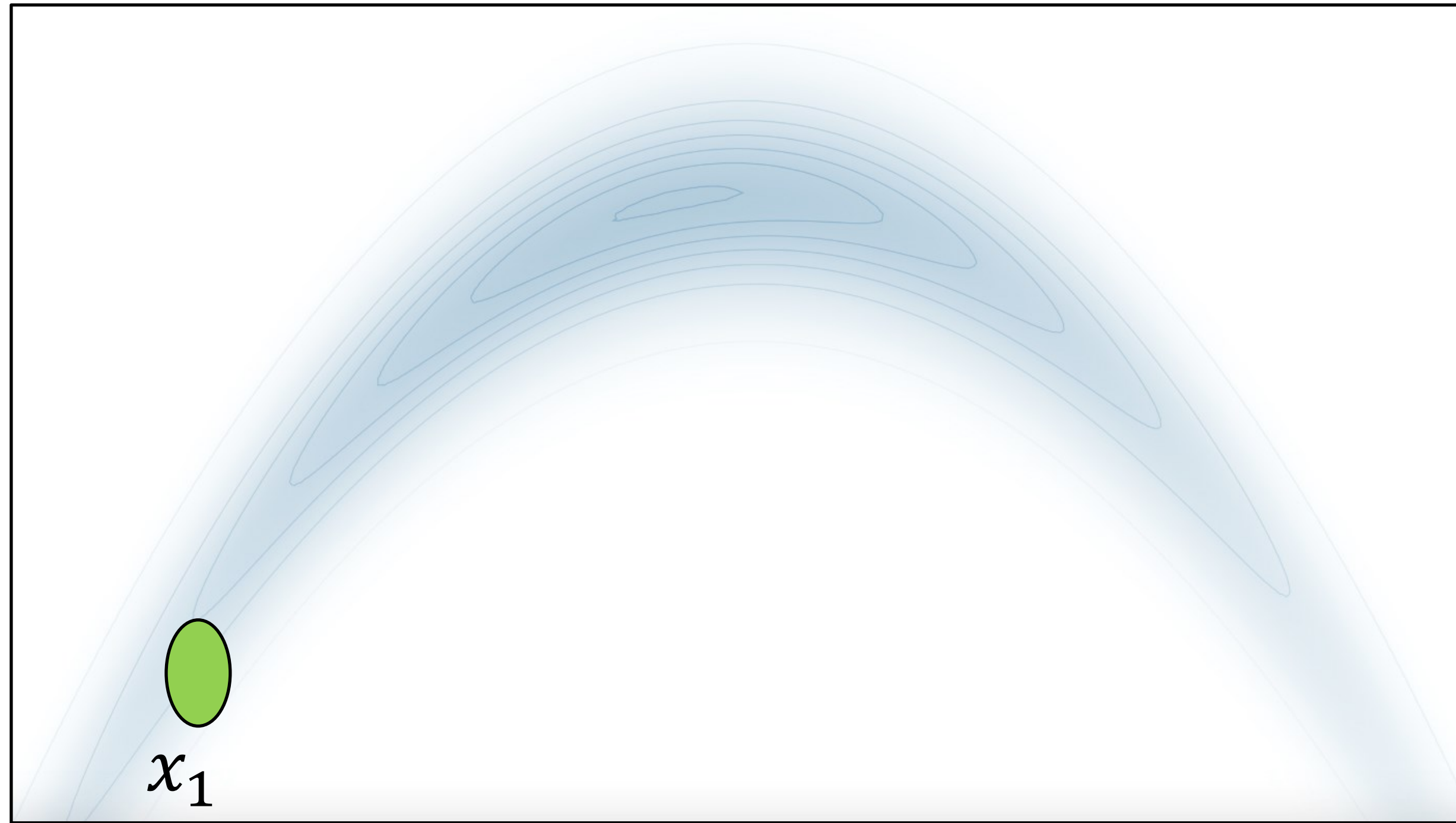
# MCMC sampling:

- Langevin Monte Carlo (LMC)





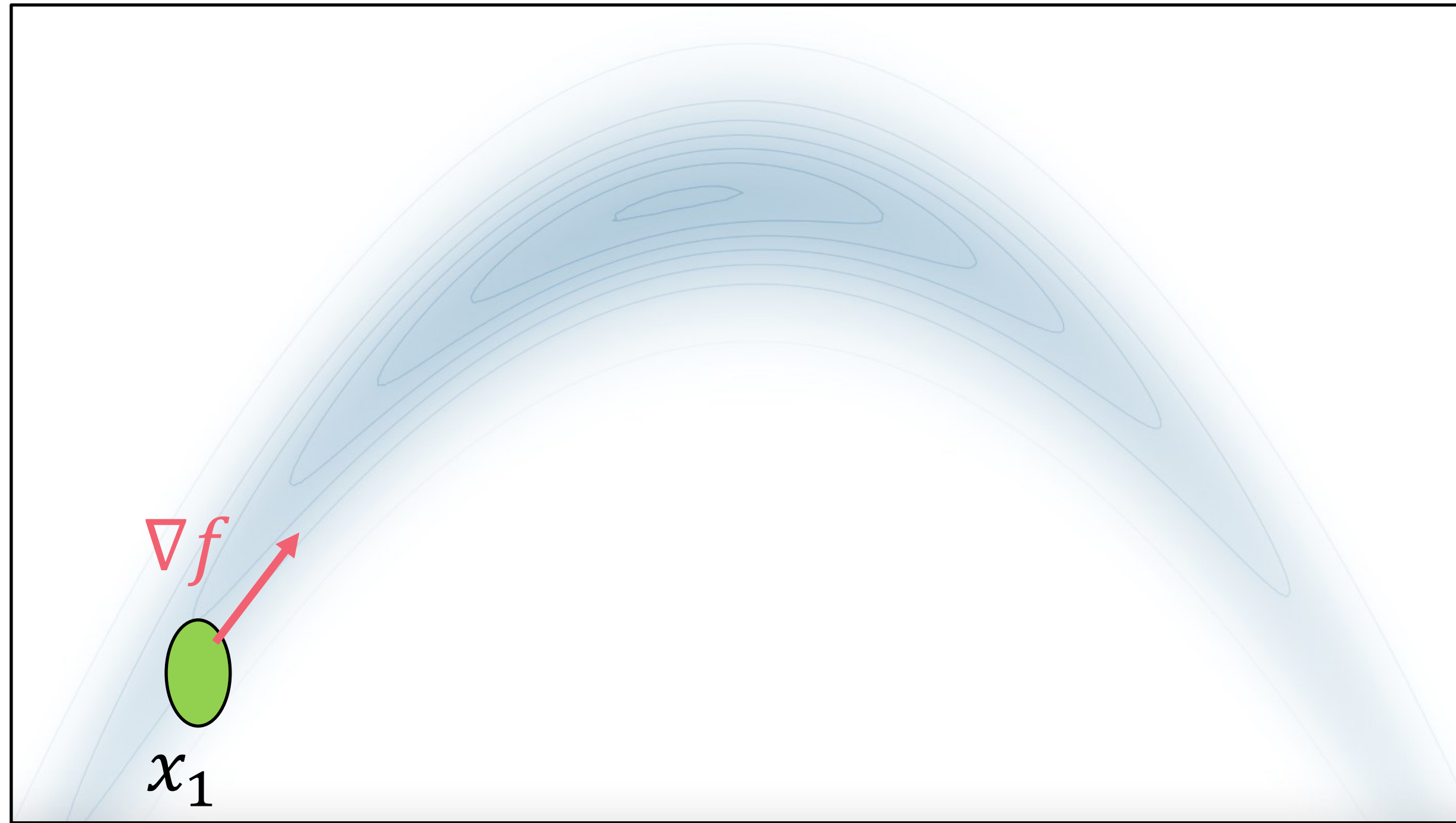
# GD OVERVIEW



Optimization problem:  
$$\max_x f(\mathbf{x})$$

Gradient descent/ascent:  
$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$

# GD OVERVIEW

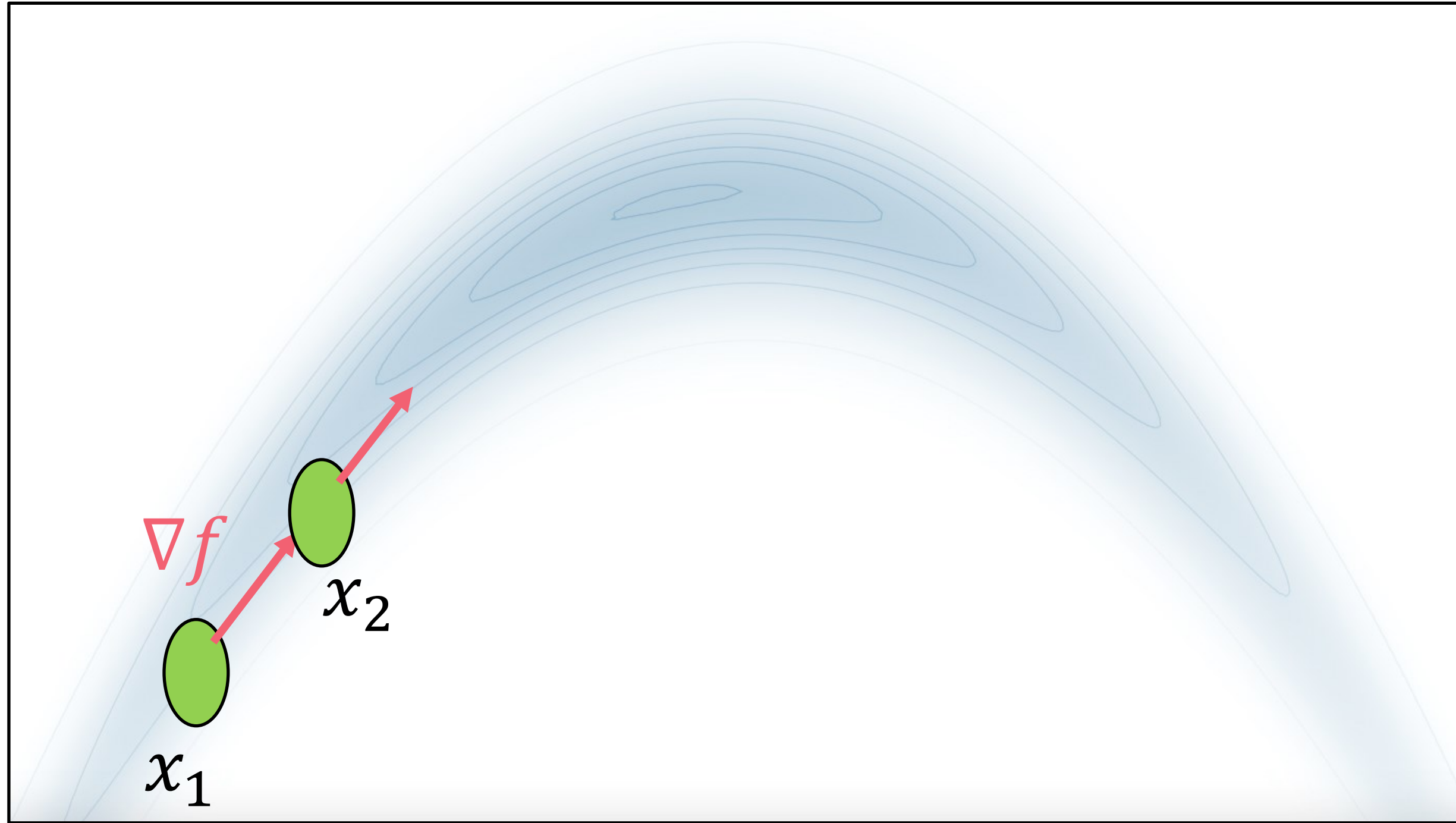


Optimization problem:  
$$\max_x f(x)$$

Gradient descent/ascent:  
$$x_t = x_{t-1} + s_{t-1} \nabla f(x_{t-1})$$



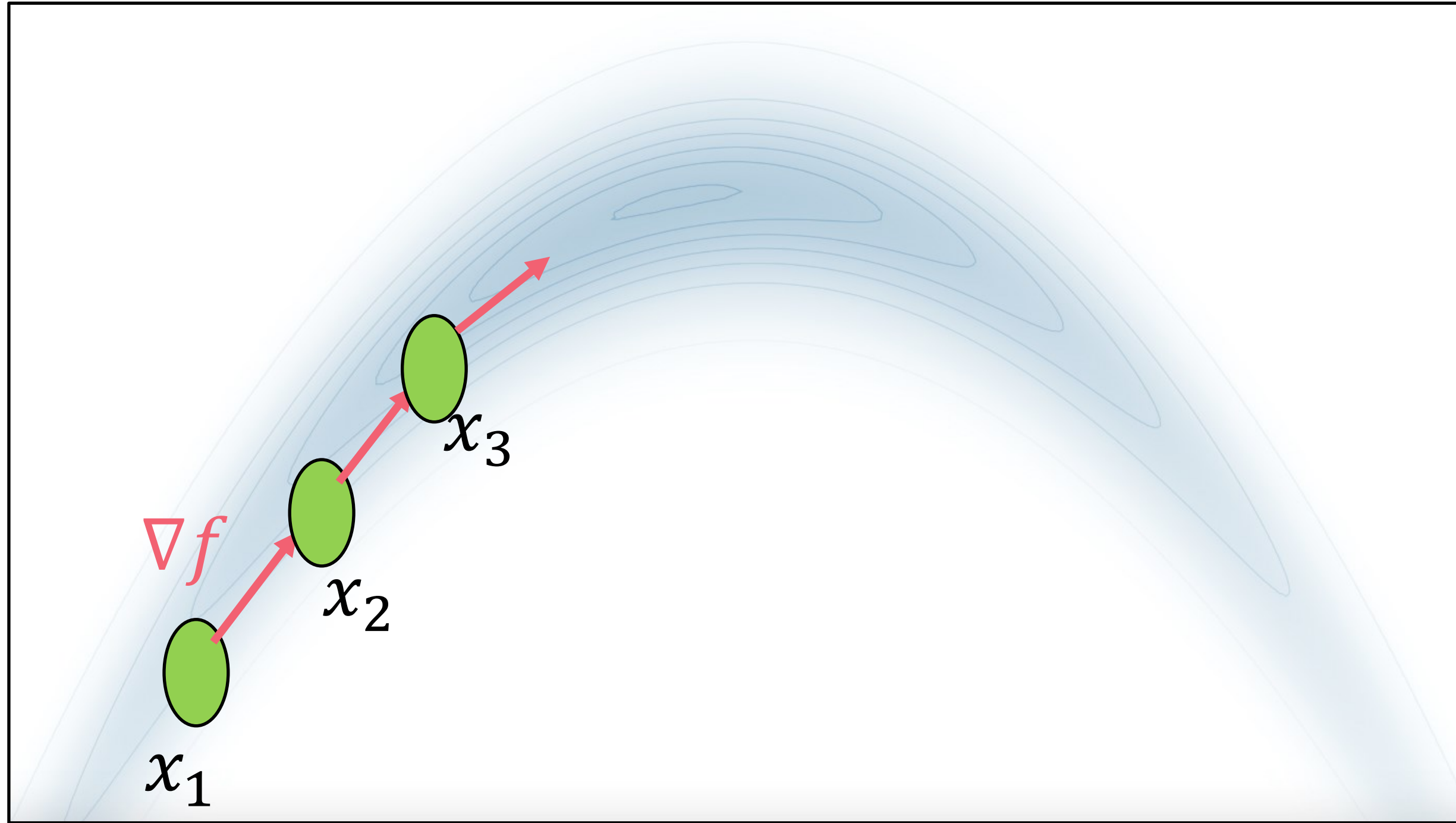
# GD OVERVIEW



Optimization problem:  
$$\max_x f(x)$$

Gradient descent/ascent:  
$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$

# GD OVERVIEW

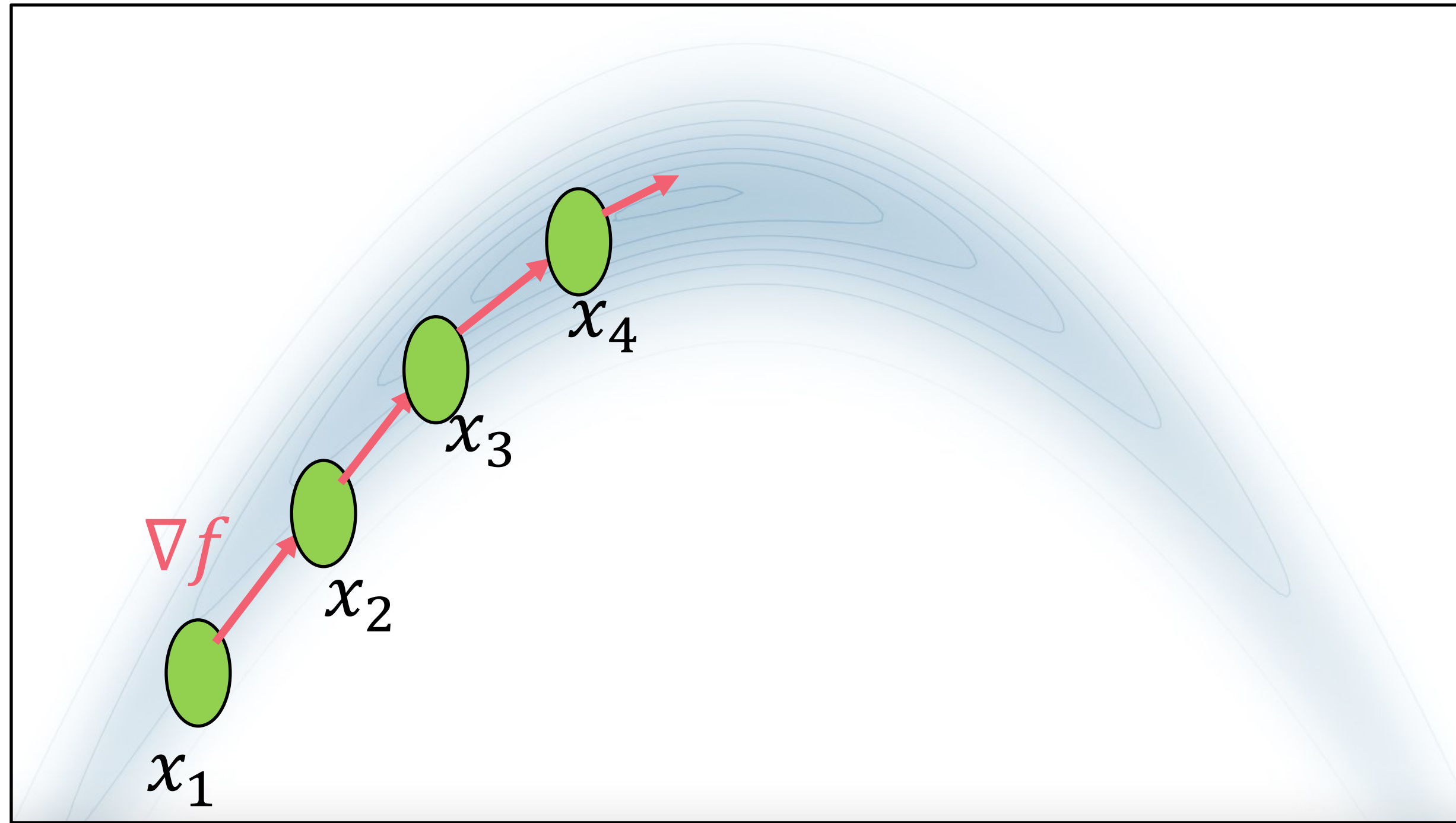


Optimization problem:  
$$\max_x f(x)$$

Gradient descent/ascent:  
$$x_t = x_{t-1} + s_{t-1} \nabla f(x_{t-1})$$



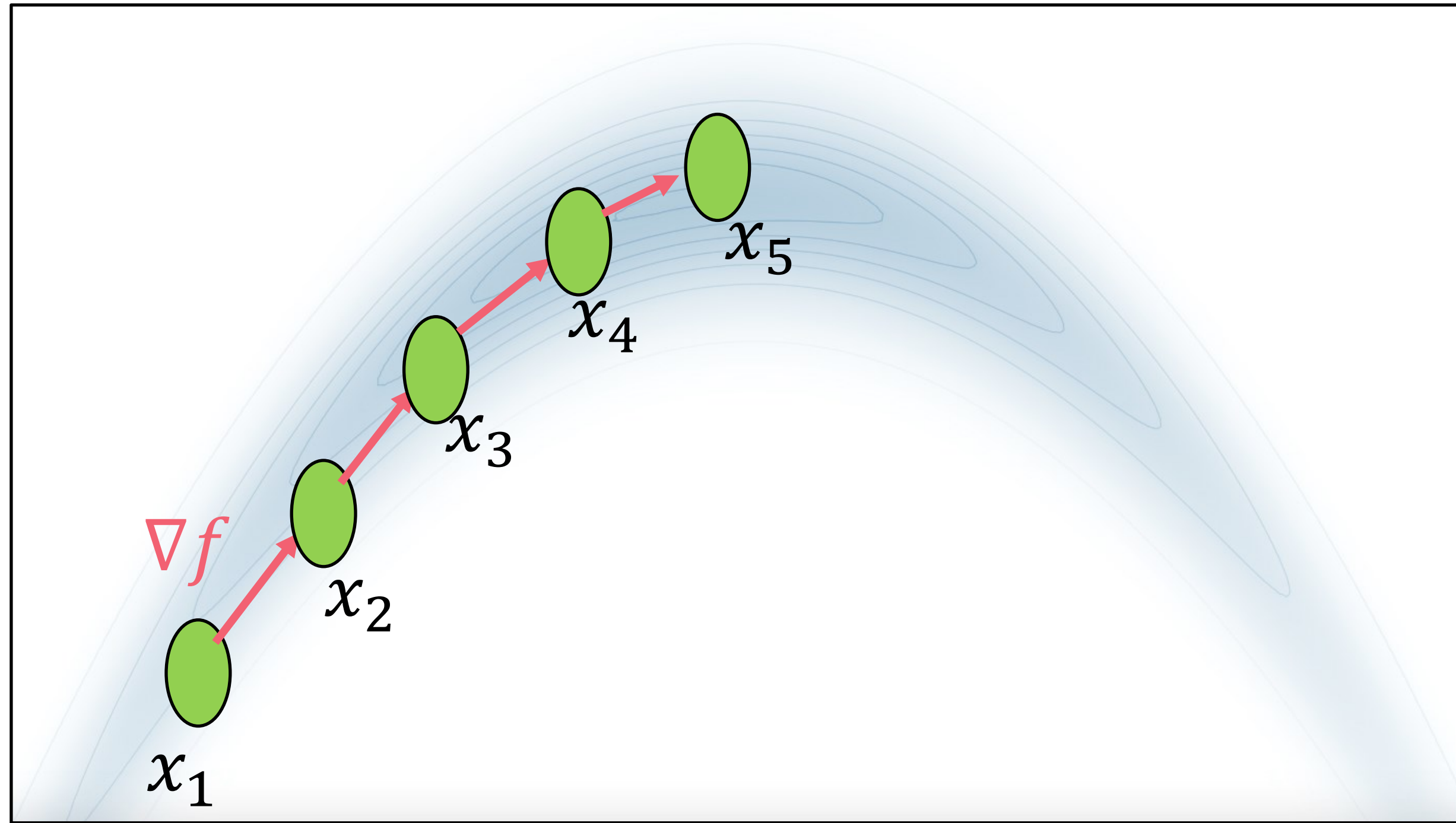
# GD OVERVIEW



Optimization problem:  
$$\max_x f(\mathbf{x})$$

Gradient descent/ascent:  
$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$

# GD OVERVIEW

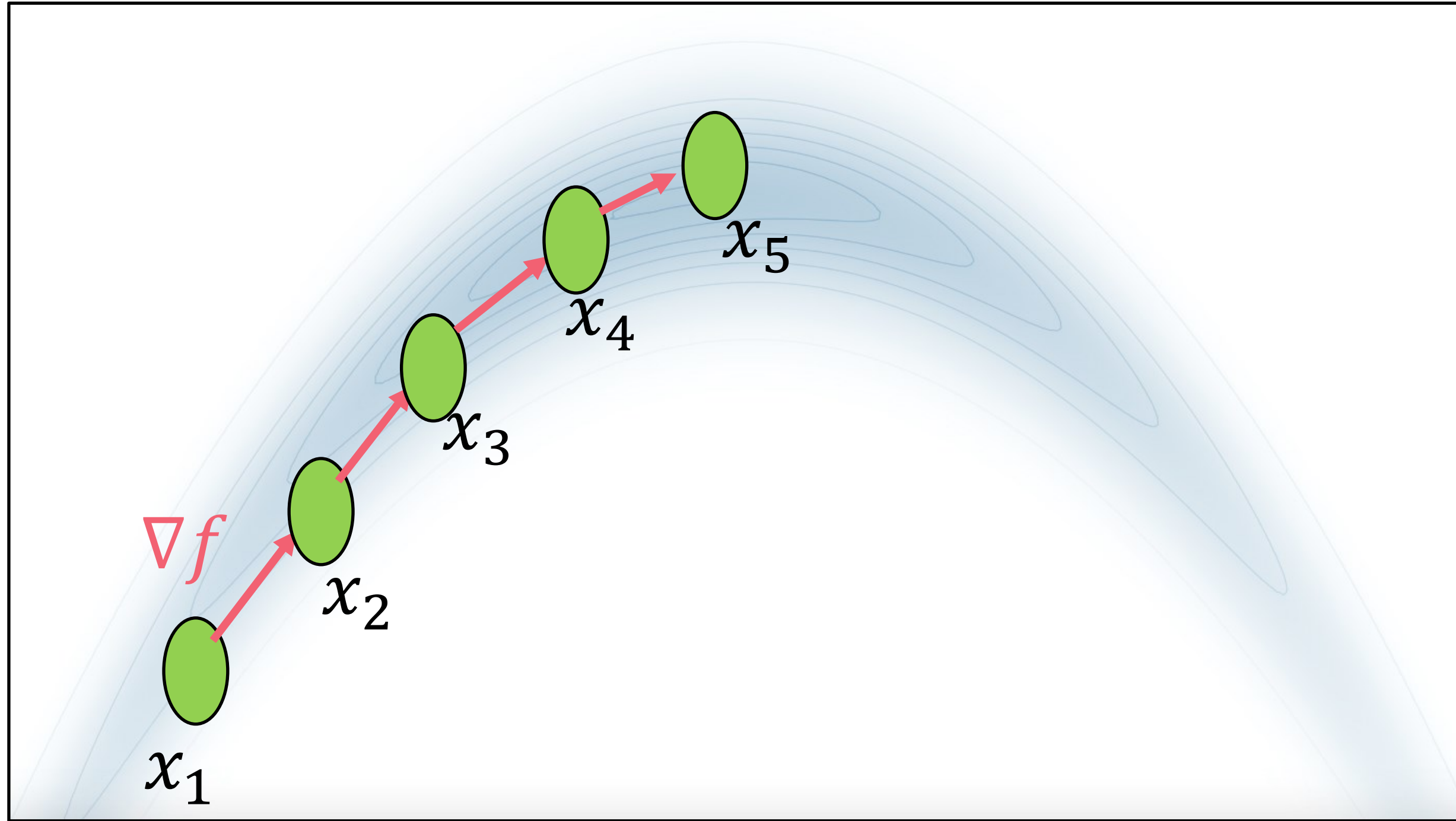


Optimization problem:  
$$\max_x f(\mathbf{x})$$

Gradient descent/ascent:  
$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$



# GD OVERVIEW



Optimization problem:

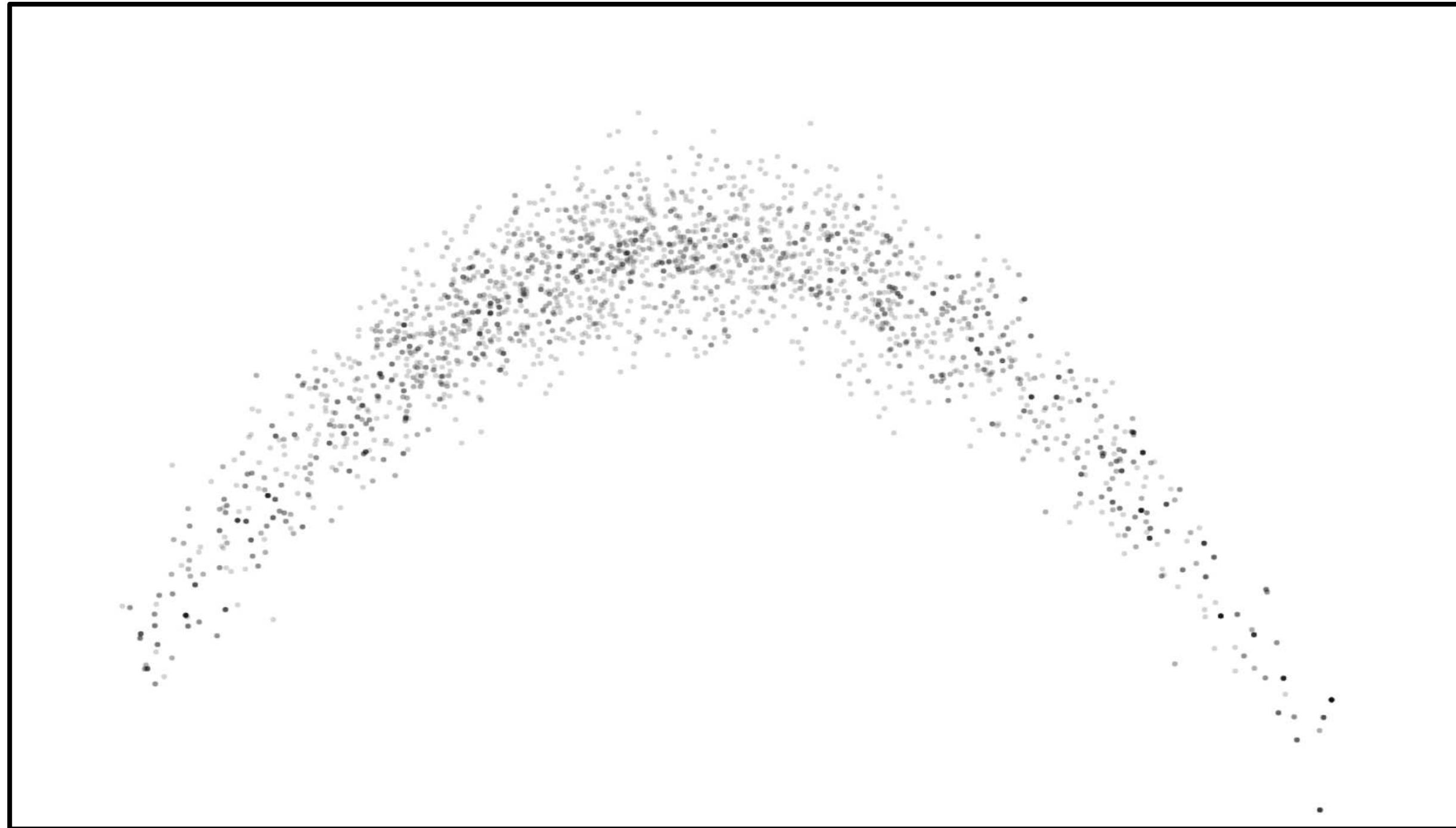
$$\max_{\boldsymbol{x}} f(\boldsymbol{x})$$

## Gradient descent/ascent:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$

scalar step size

# LMC OVERVIEW



Sampling problem:

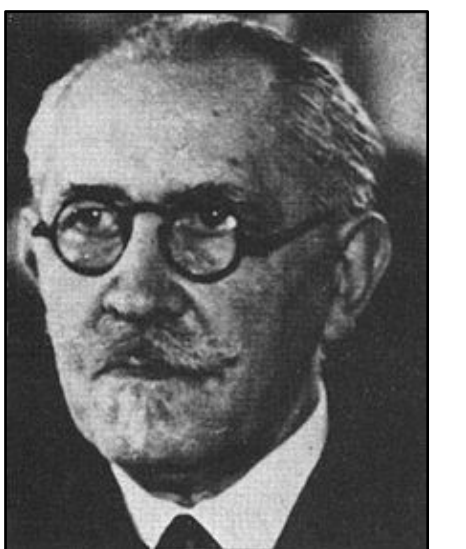
$$\mathbf{x}_t \sim f$$

Langevin MC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \mathbf{I})$$

Apply Metropolis Hastings to accept/reject

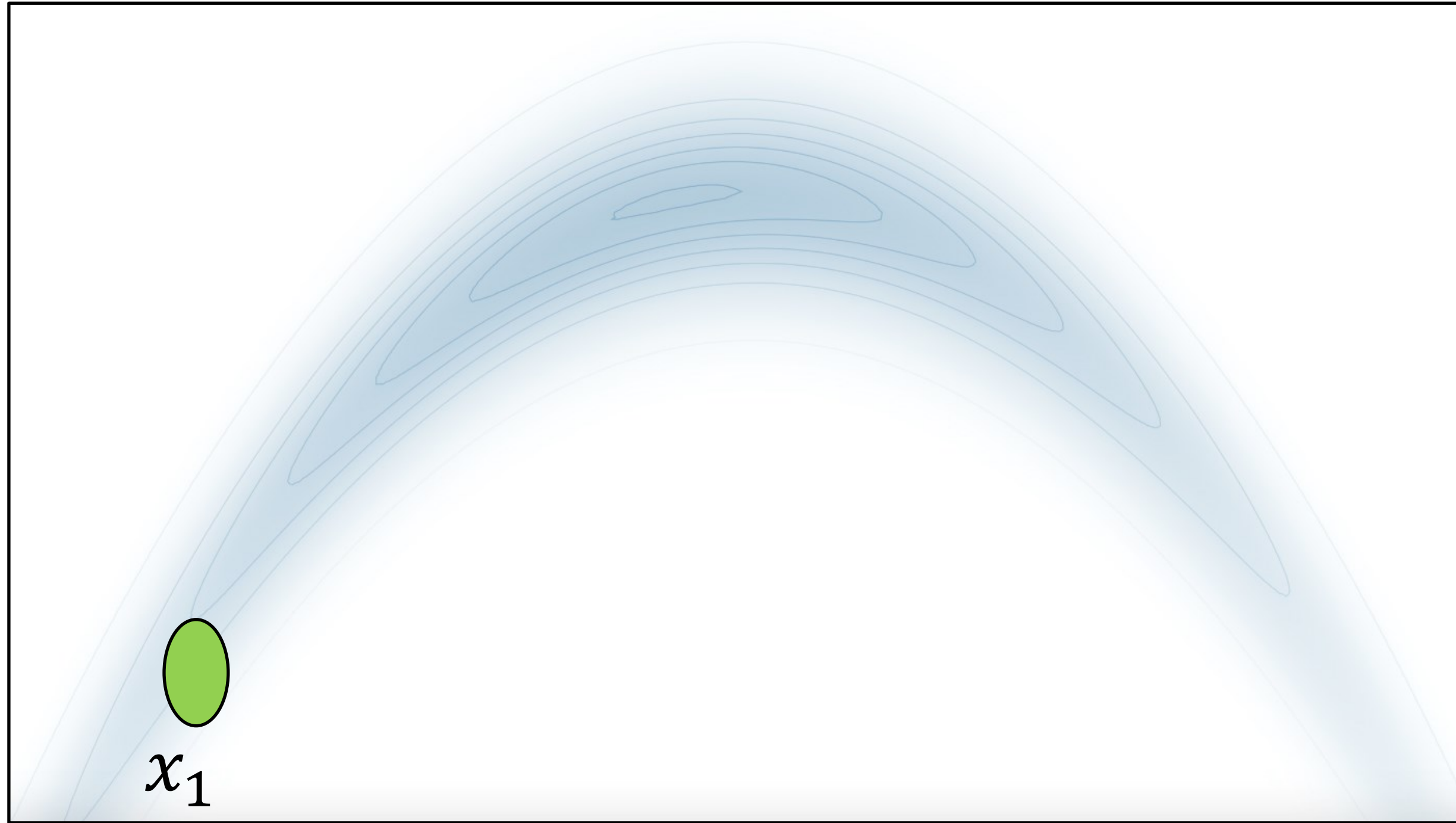
Paul Langevin



[Luan et al., 2020]



# LMC OVERVIEW



Sampling problem:

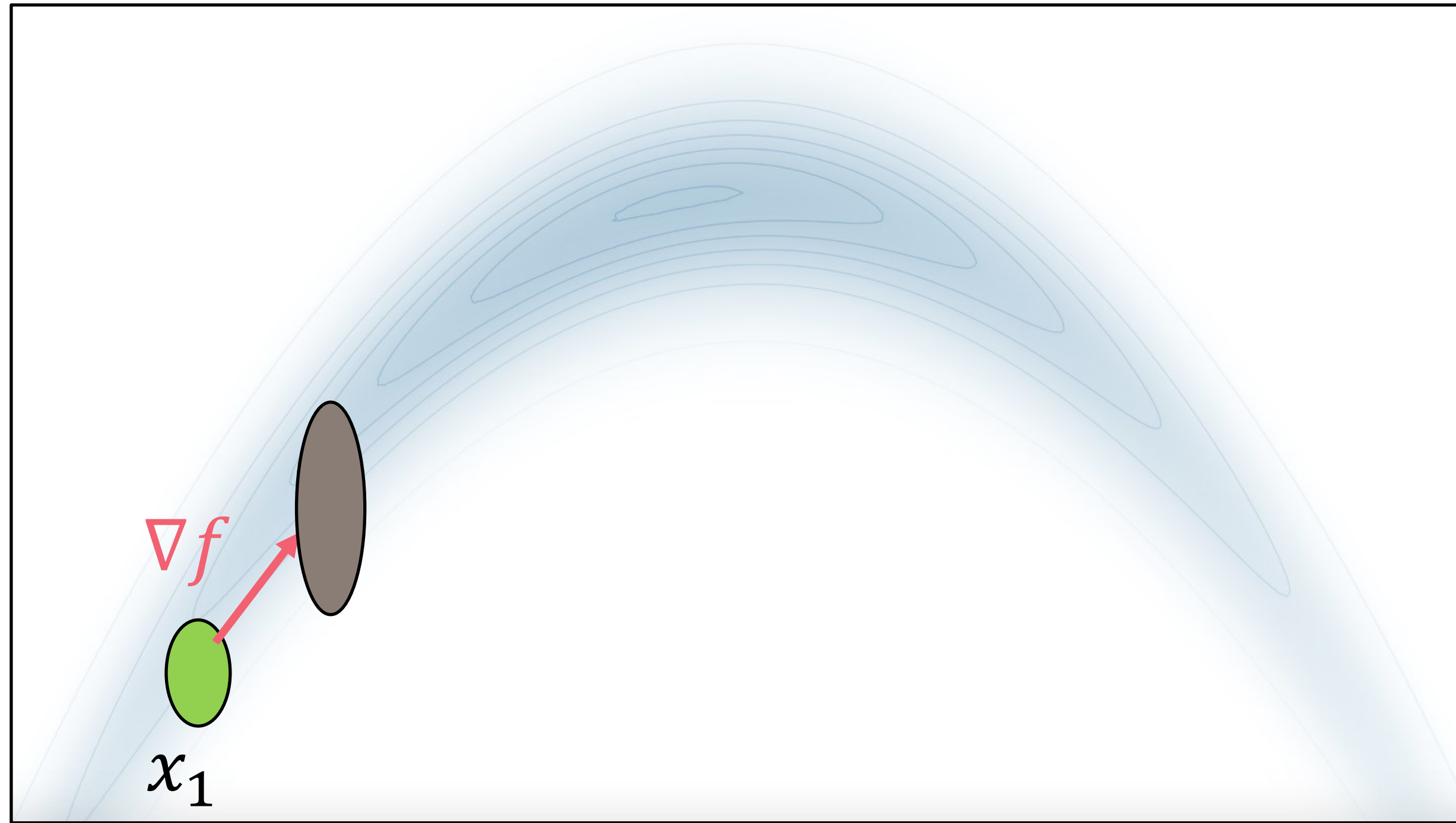
$$\mathbf{x}_t \sim f$$

Langevin MC:

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Apply Metropolis Hastings to accept/reject

# LMC OVERVIEW



Sampling problem:

$$\boldsymbol{x}_t \sim f$$

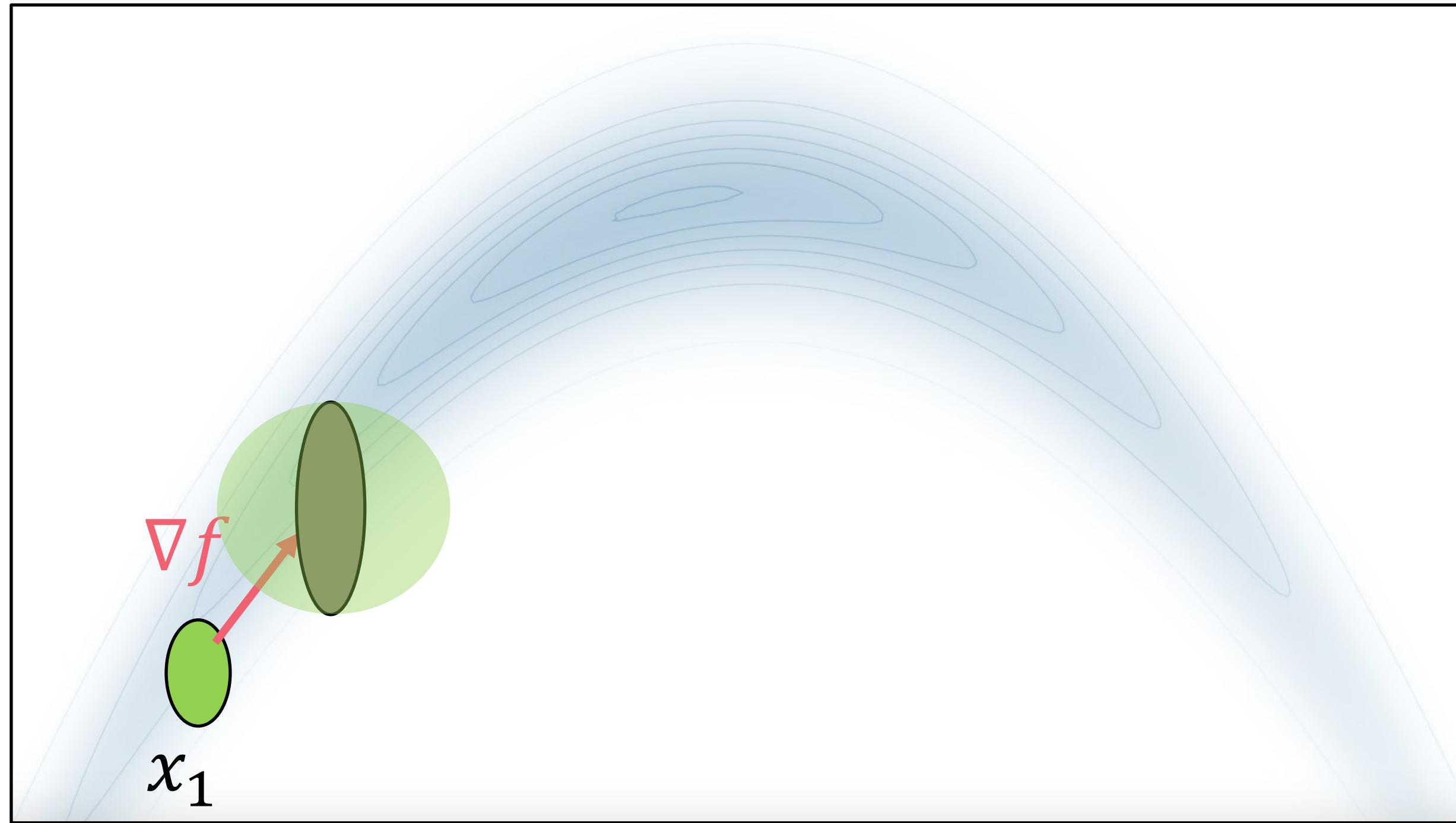
Langevin MC:

$$\boldsymbol{x}_t = \boldsymbol{x}_{t-1} + s_{t-1} \nabla f(\boldsymbol{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \boldsymbol{I})$$

Apply Metropolis Hastings to accept/reject



# LMC OVERVIEW



Sampling problem:

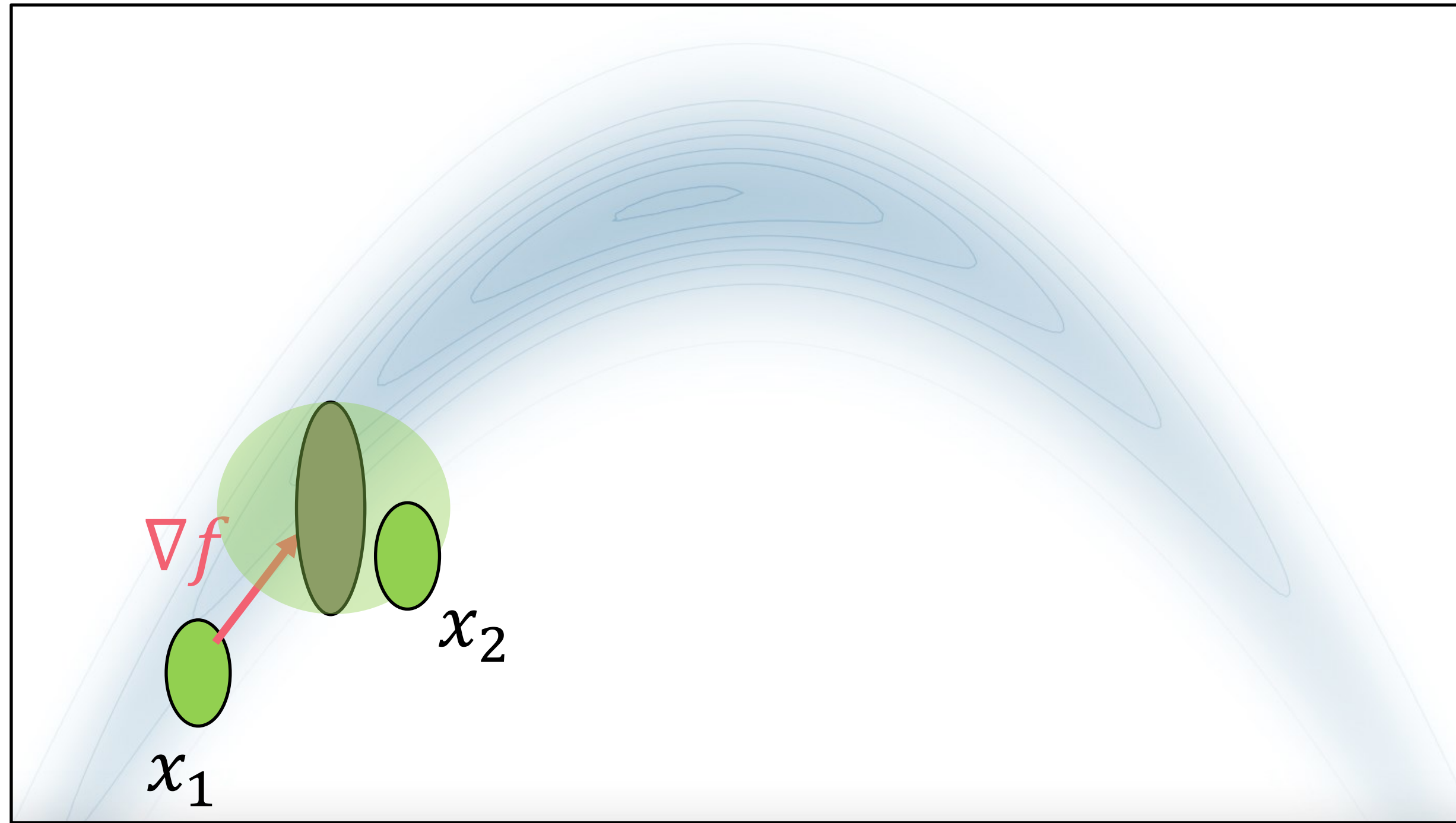
$$\mathbf{x}_t \sim f$$

Langevin MC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \mathbf{I})$$

Apply Metropolis Hastings to accept/reject

# LMC OVERVIEW



Sampling problem:

$$\boldsymbol{x}_t \sim f$$

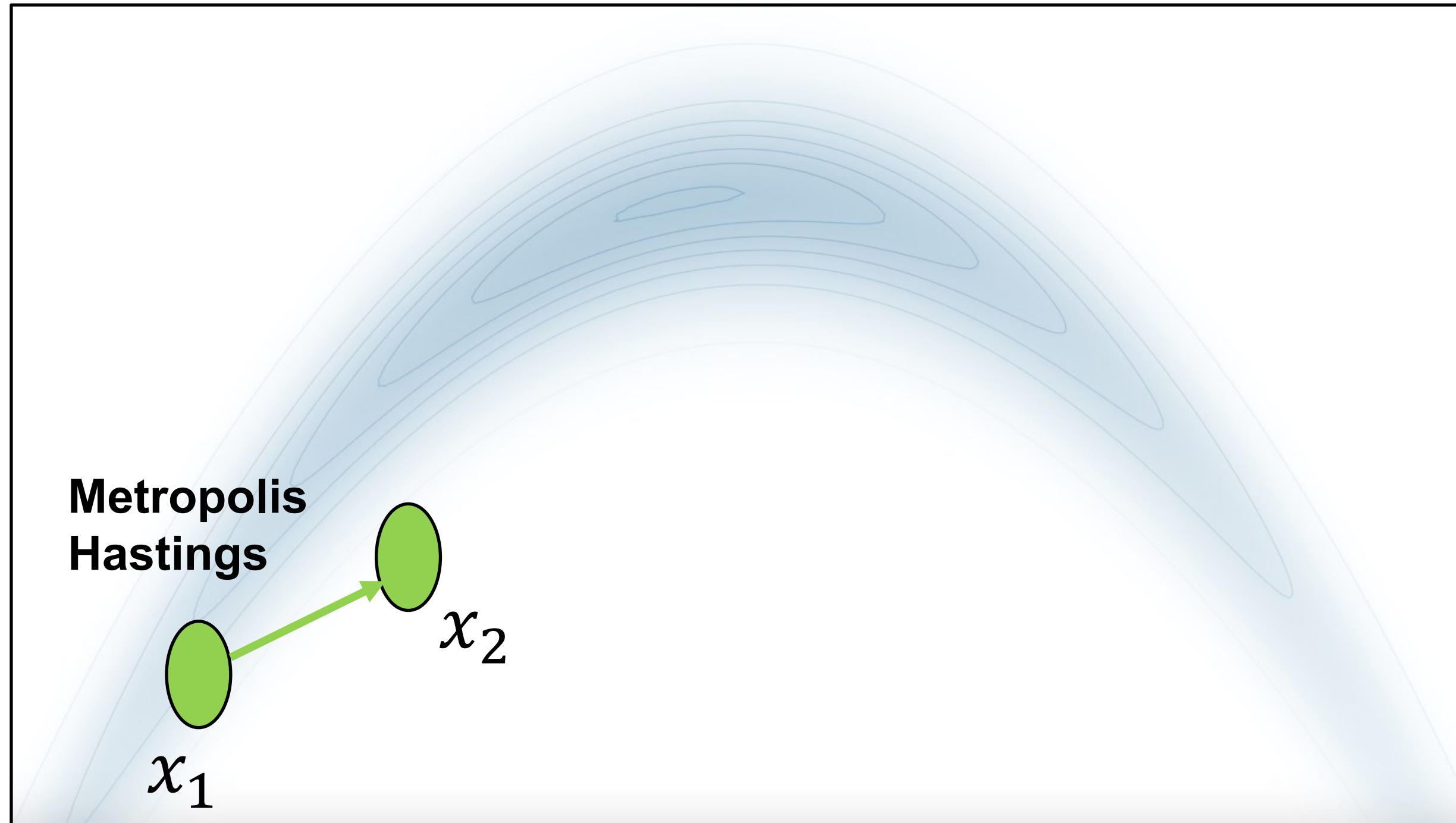
Langevin MC:

$$\boldsymbol{x}_t = \boldsymbol{x}_{t-1} + s_{t-1} \nabla f(\boldsymbol{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \boldsymbol{I})$$

Apply Metropolis Hastings to accept/reject



# LMC OVERVIEW



Sampling problem:

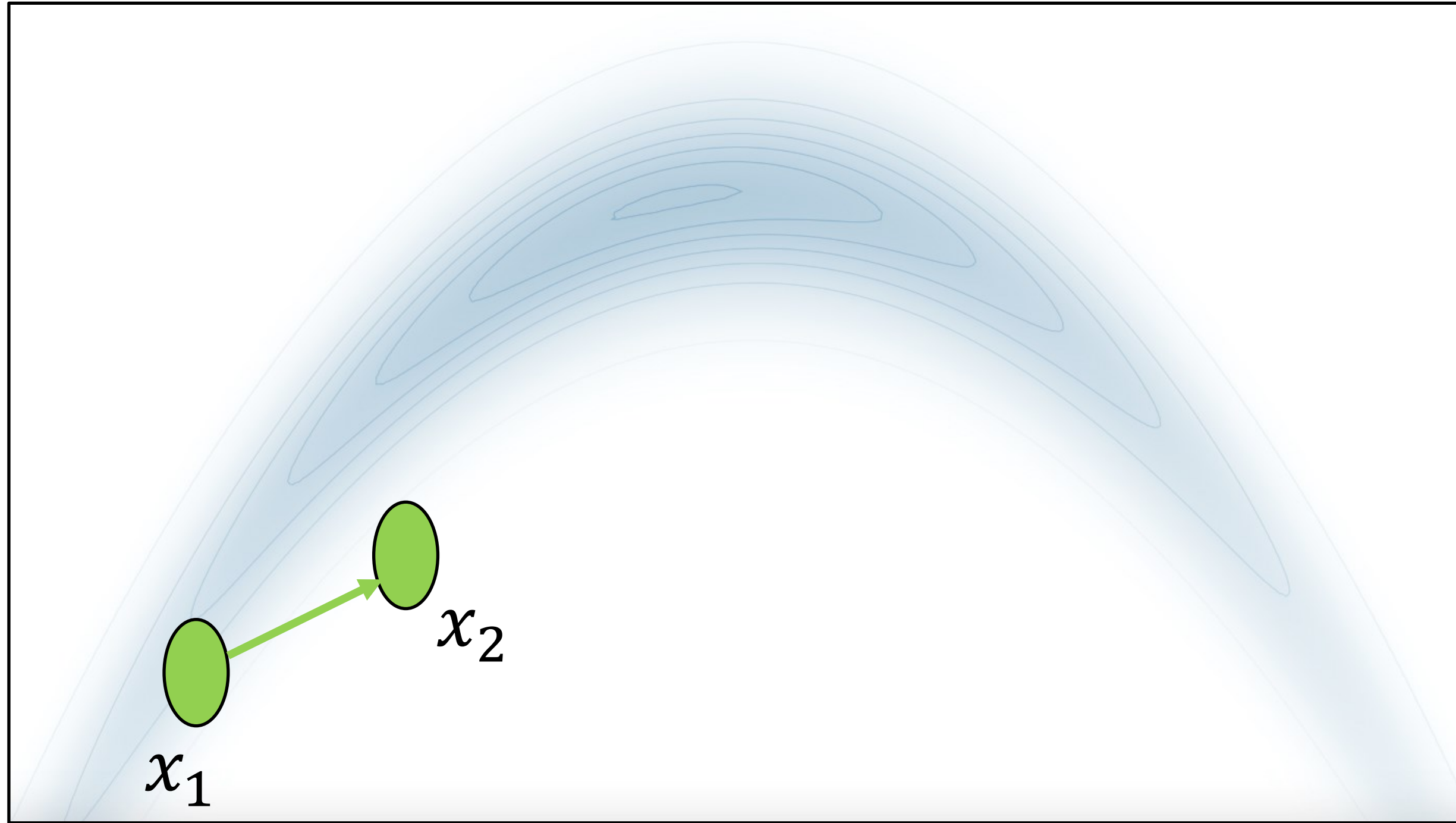
$$\mathbf{x}_t \sim f$$

Langevin MC:

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Apply Metropolis Hastings to accept/reject

# LMC OVERVIEW



Sampling problem:

$$\mathbf{x}_t \sim f$$

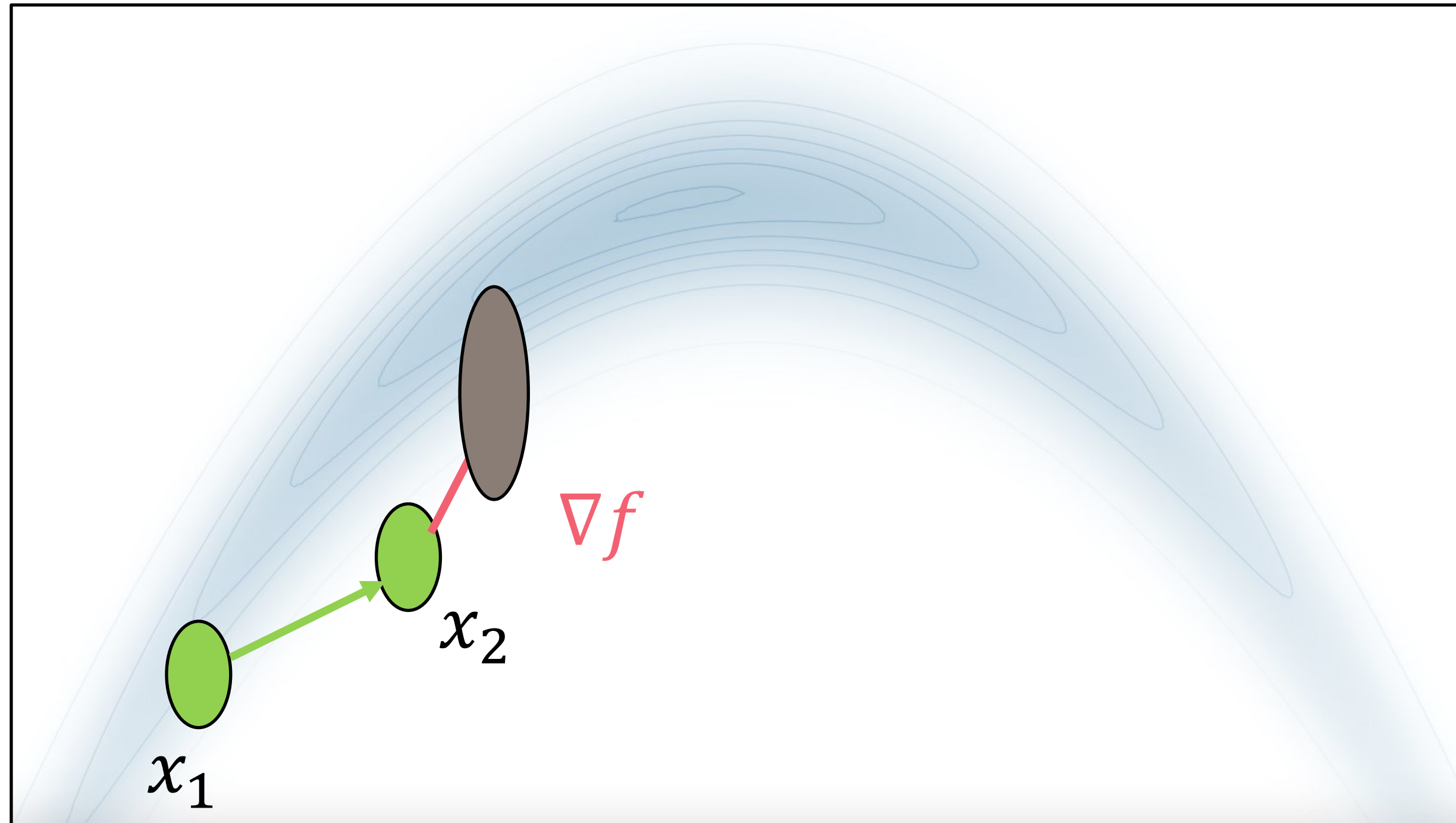
Langevin MC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \mathbf{I})$$

Apply Metropolis Hastings to accept/reject



# LMC OVERVIEW



Sampling problem:

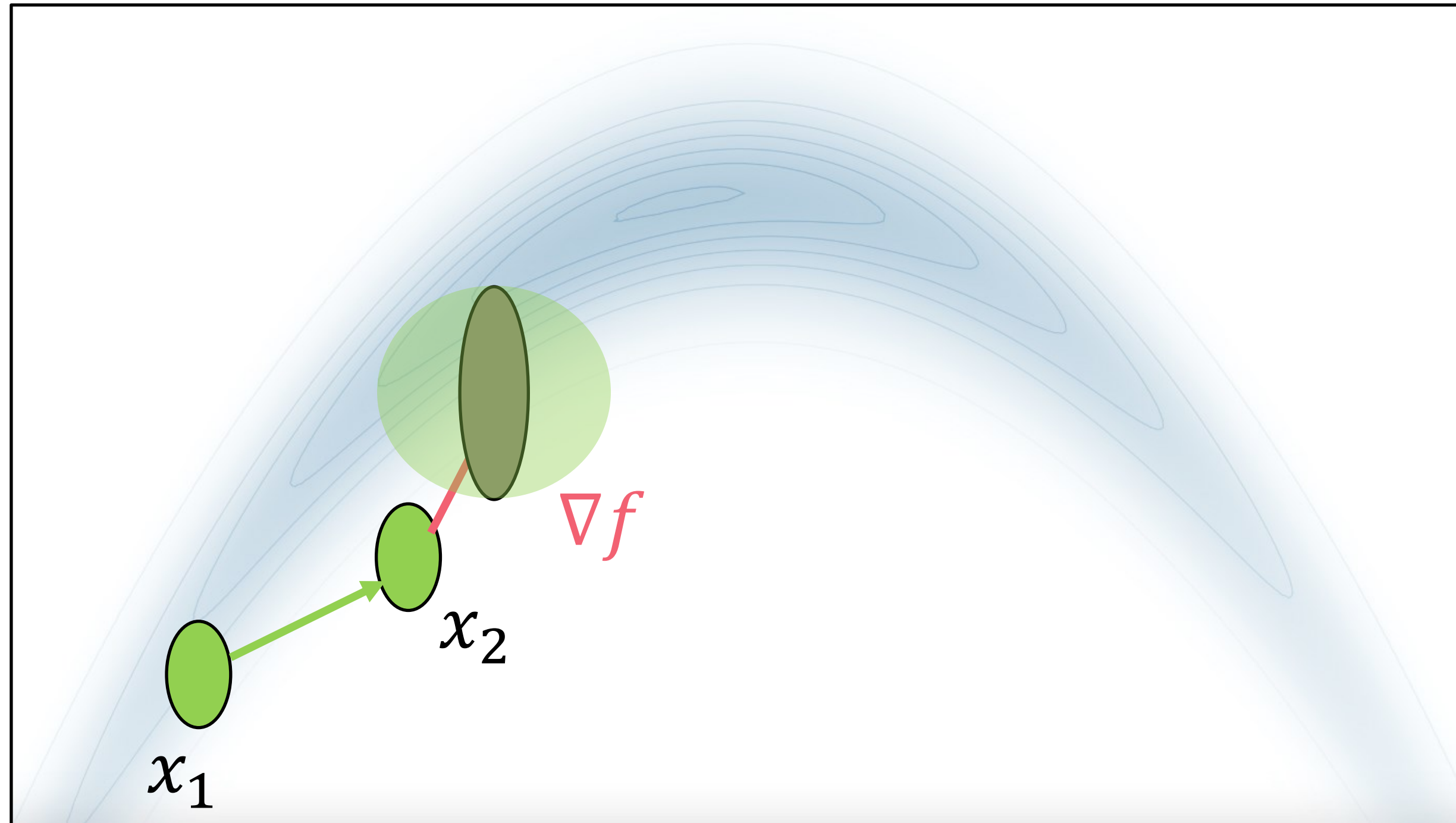
$$\mathbf{x}_t \sim f$$

Langevin MC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \mathbf{I})$$

Apply Metropolis Hastings to accept/reject

# LMC OVERVIEW



Sampling problem:

$$\boldsymbol{x}_t \sim f$$

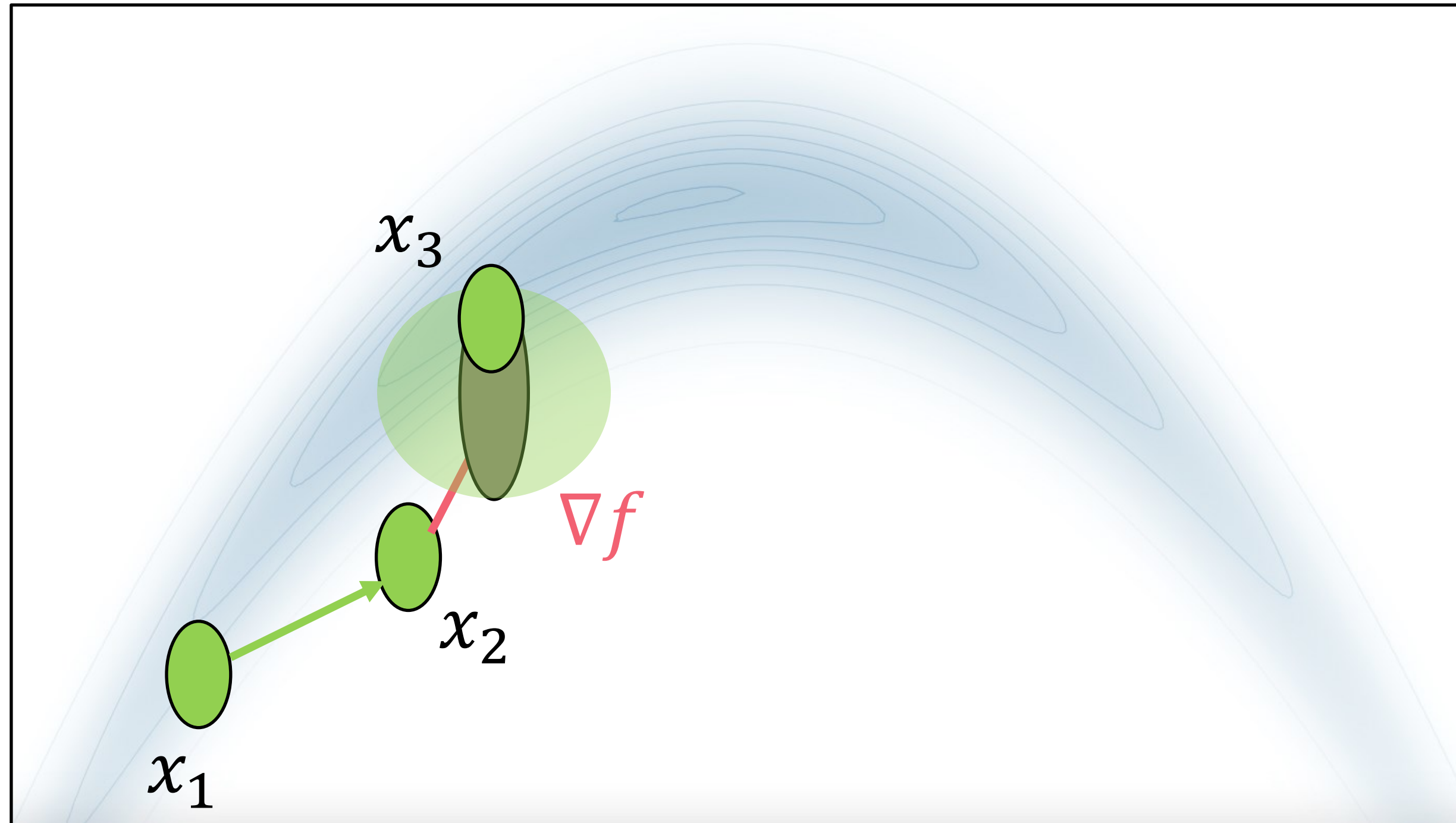
Langevin MC:

$$\boldsymbol{x}_t = \boldsymbol{x}_{t-1} + s_{t-1} \nabla f(\boldsymbol{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \boldsymbol{I})$$

Apply Metropolis Hastings to accept/reject



# LMC OVERVIEW



Sampling problem:

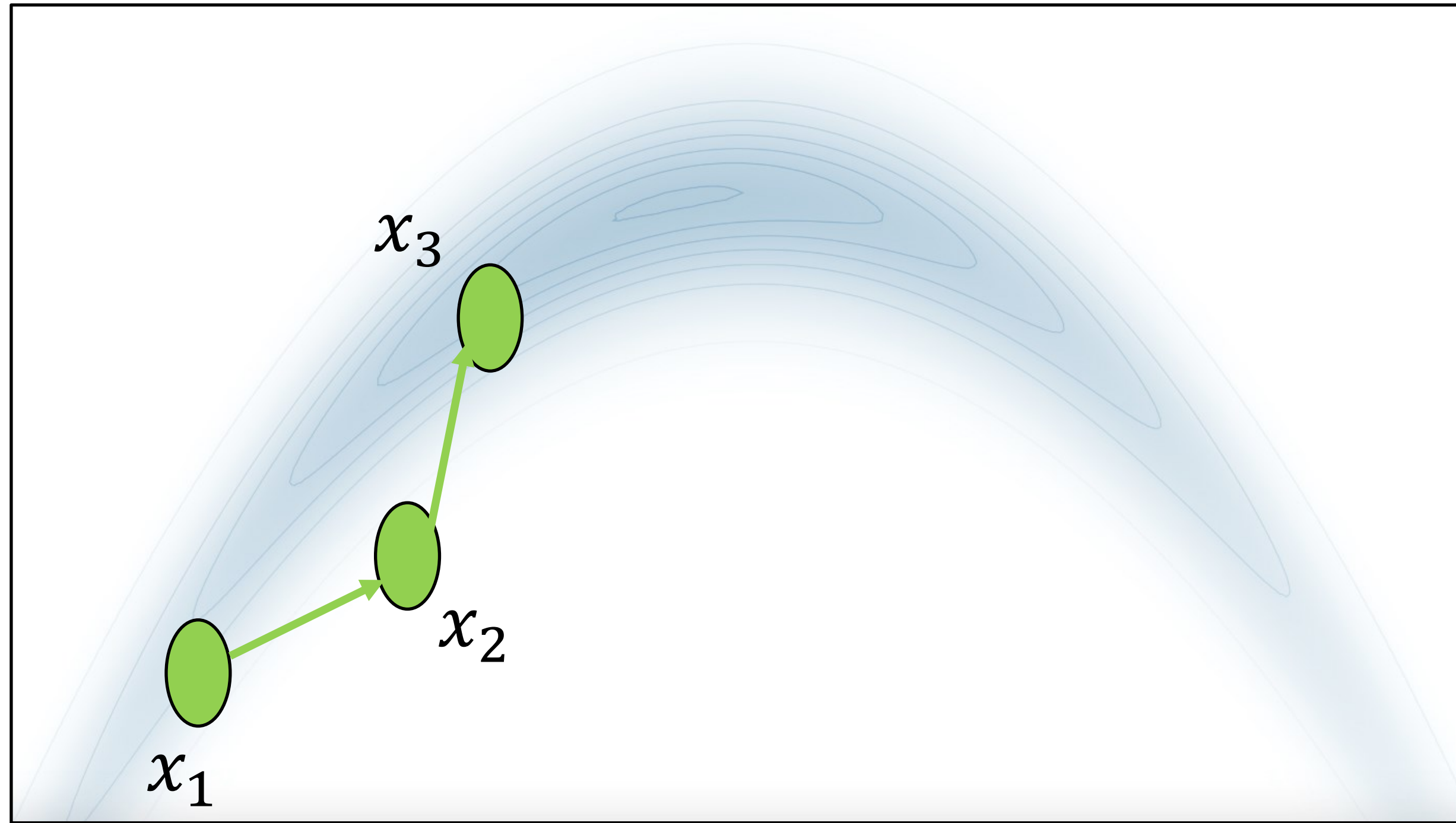
$$\mathbf{x}_t \sim f$$

Langevin MC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \mathbf{I})$$

Apply Metropolis Hastings to accept/reject

# LMC OVERVIEW



Sampling problem:

$$\mathbf{x}_t \sim f$$

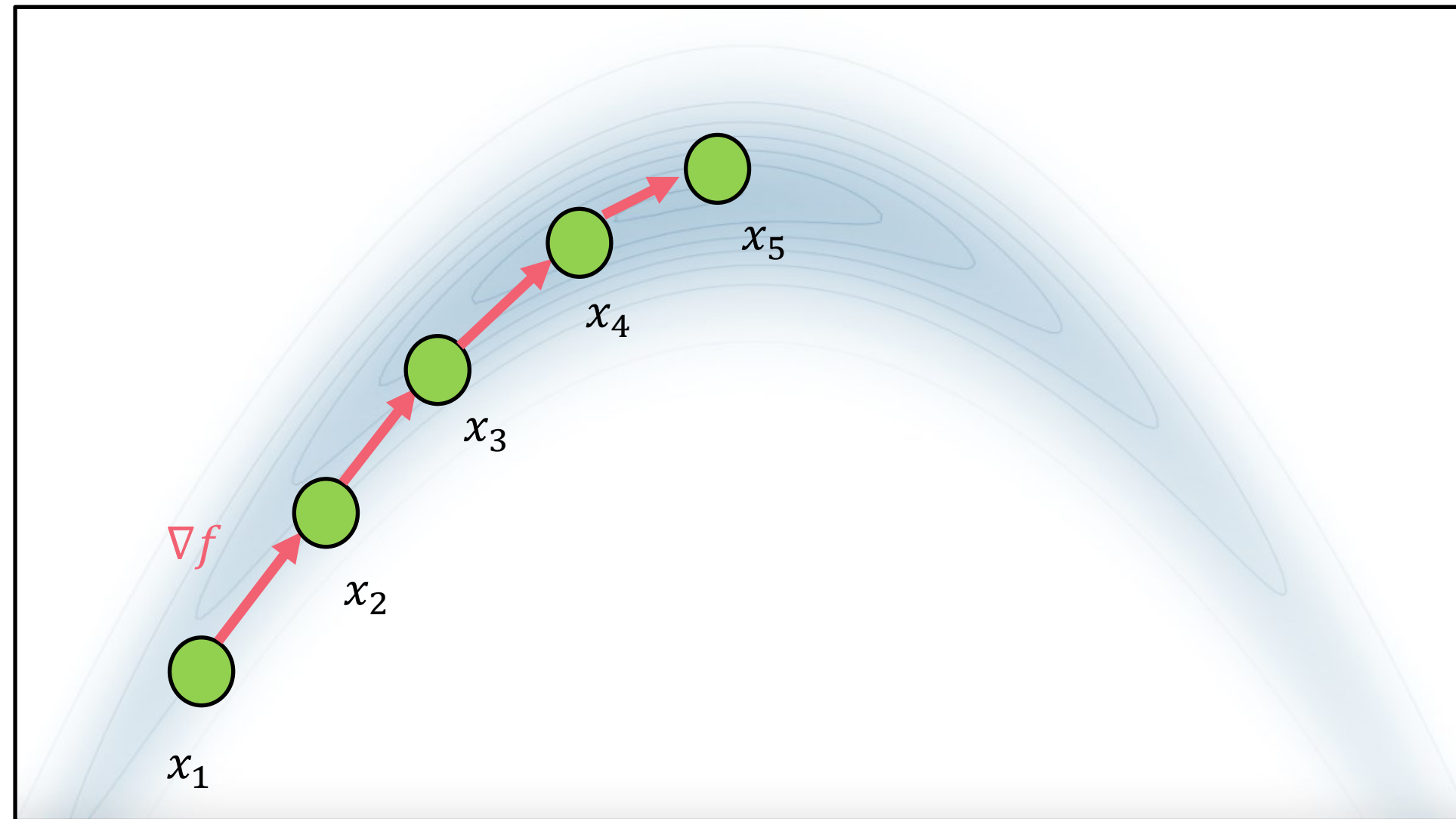
Langevin MC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 \mathbf{I})$$

Apply Metropolis Hastings to accept/reject



# GD VS. LMC

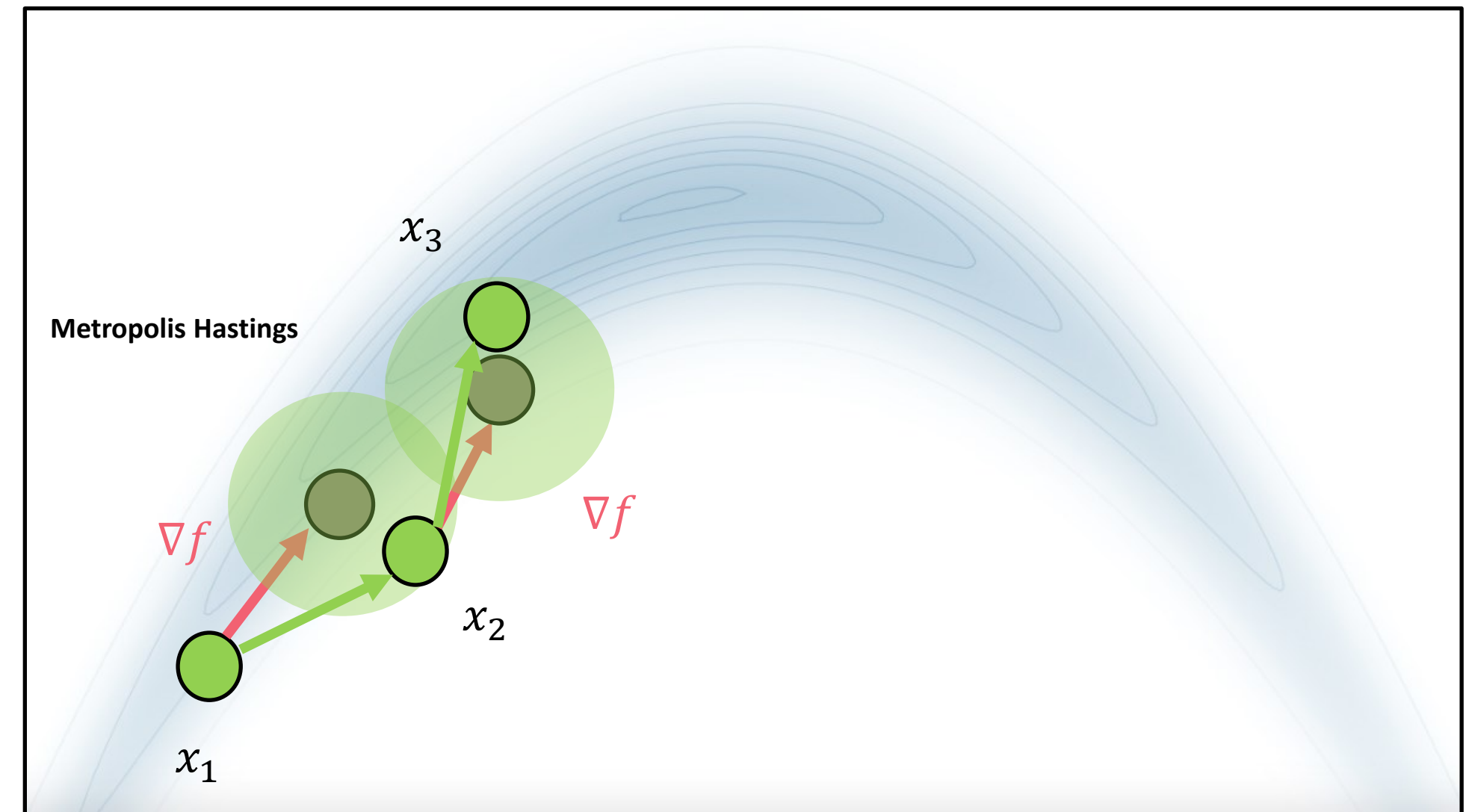


Optimization:

$$\max_{\mathbf{x}} f(\mathbf{x})$$

GD:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$



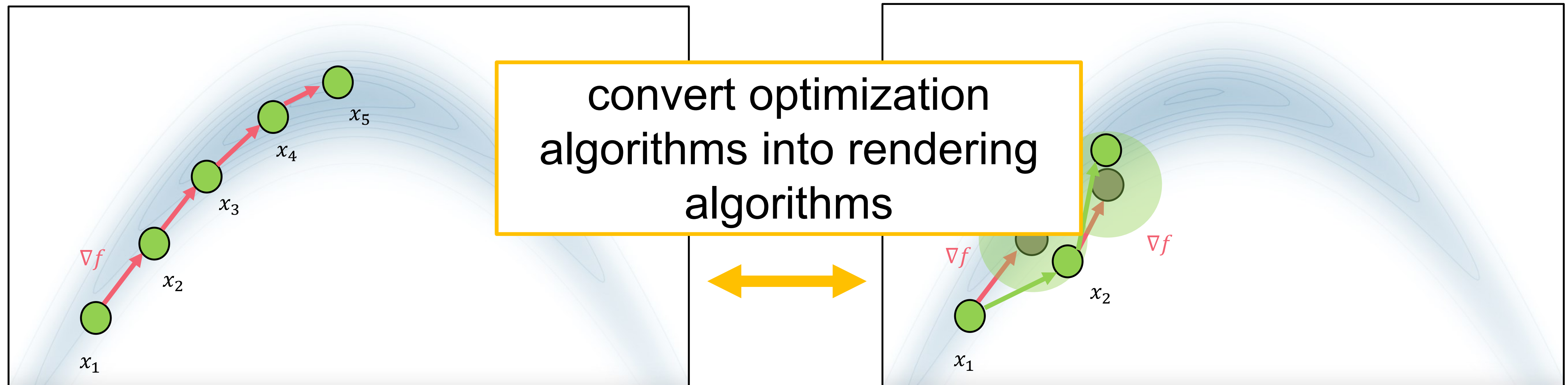
Sampling:

$$\mathbf{x}_t \sim f$$

LMC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 I)$$

# GD VS. LMC



Optimization:

$$\max_{\mathbf{x}} f(\mathbf{x})$$

GD:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1})$$

Sampling:

$$\mathbf{x}_t \sim f$$

LMC:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + s_{t-1} \nabla f(\mathbf{x}_{t-1}) + \frac{1}{s_{t-1}} N(0, \sigma^2 I)$$

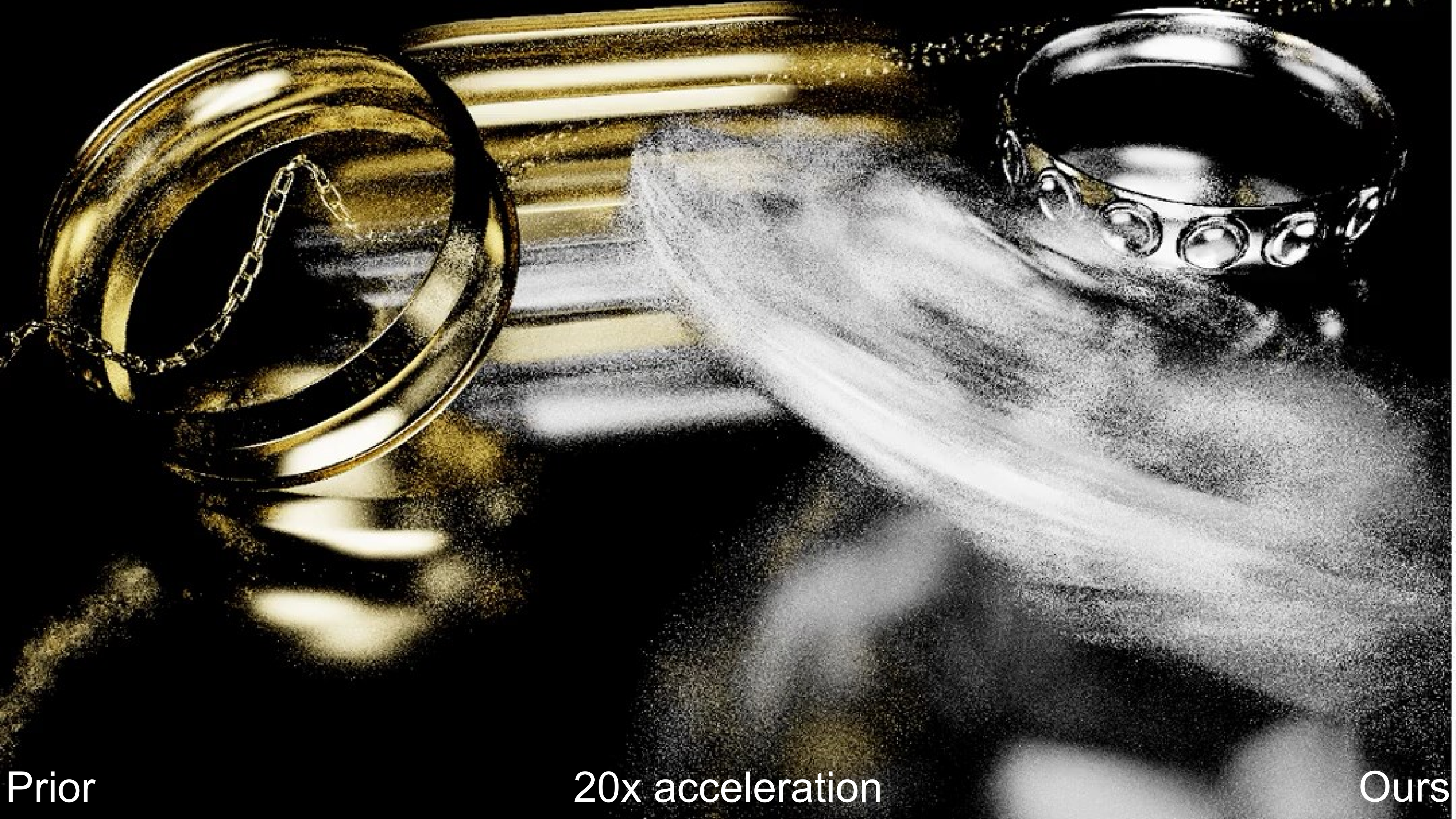


Prior

10x acceleration

Ours



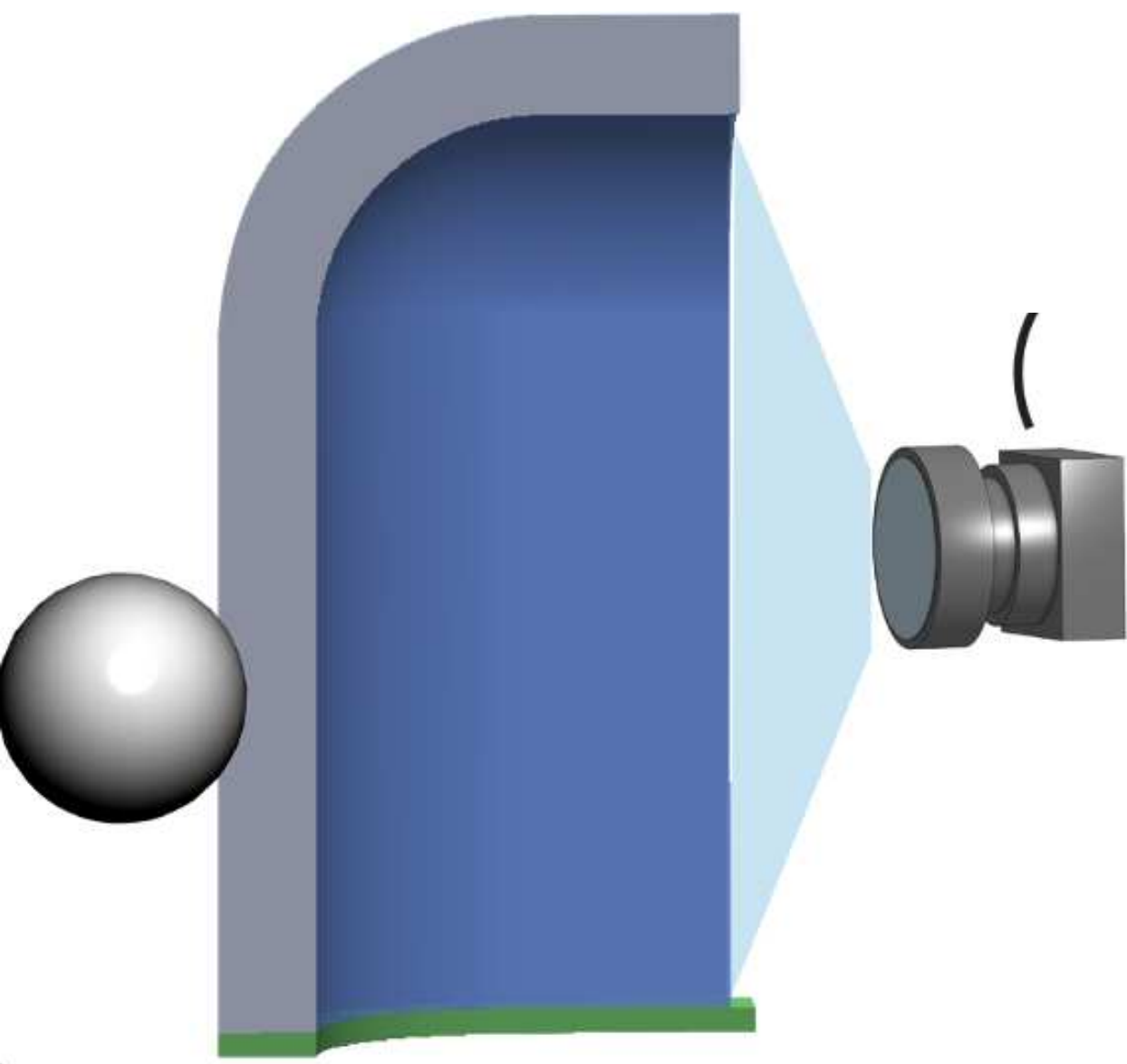


Prior

20x acceleration

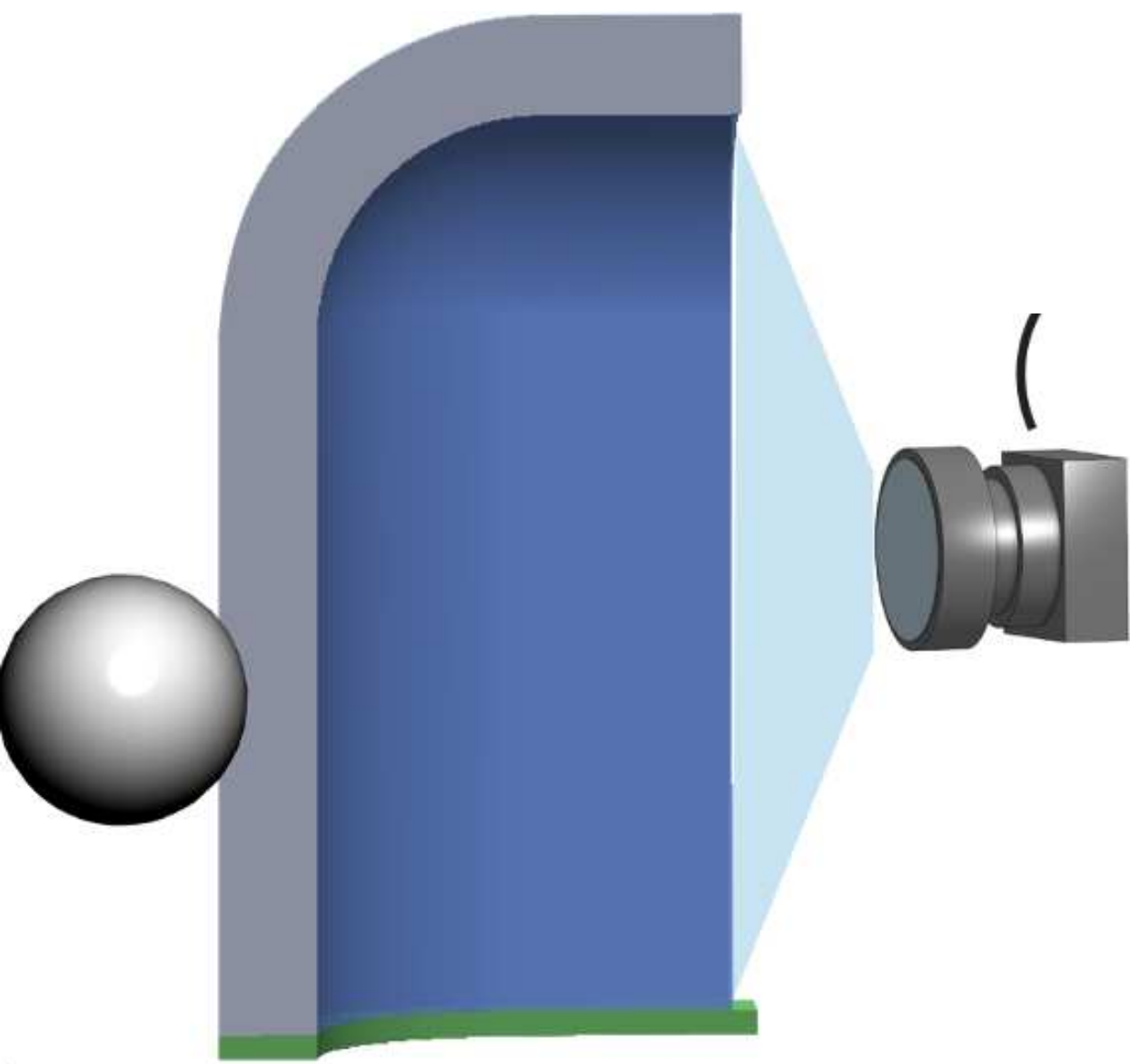
Ours

# Evaluating optical simulation framework



Real-world prototype

# Evaluating optical simulation framework



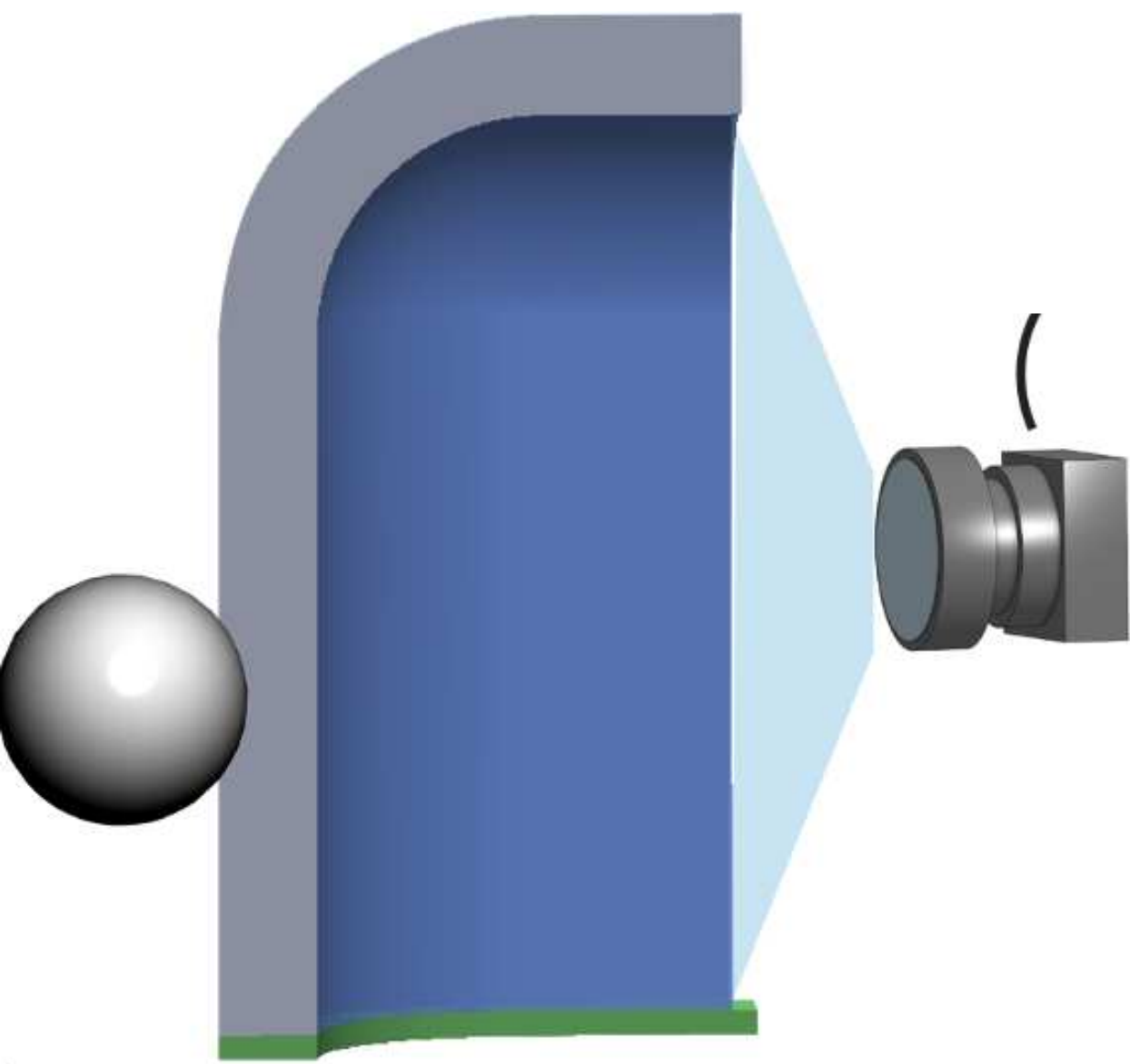
Real-world prototype



Rasterization



# Evaluating optical simulation framework



Real-world prototype



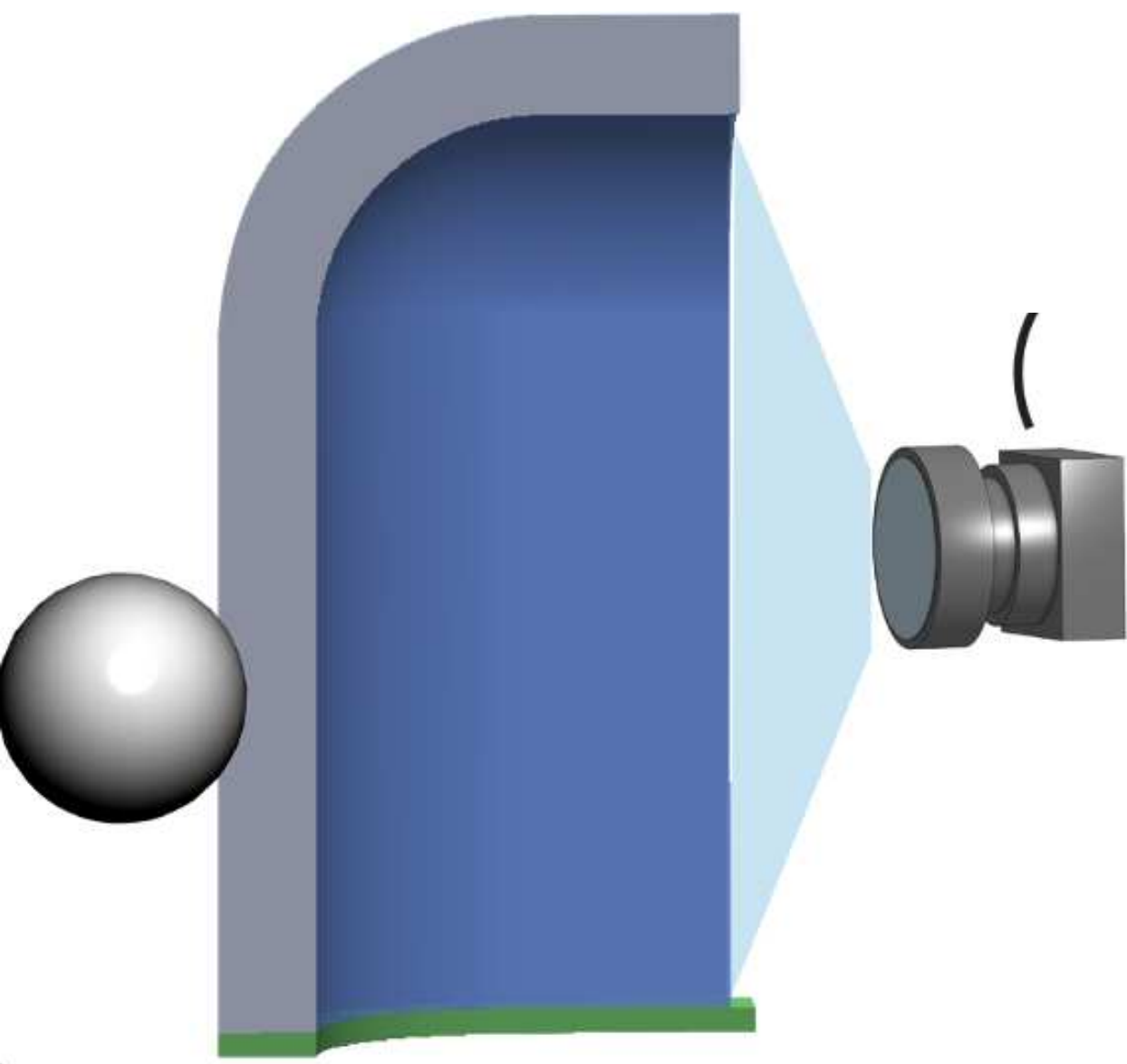
Rasterization



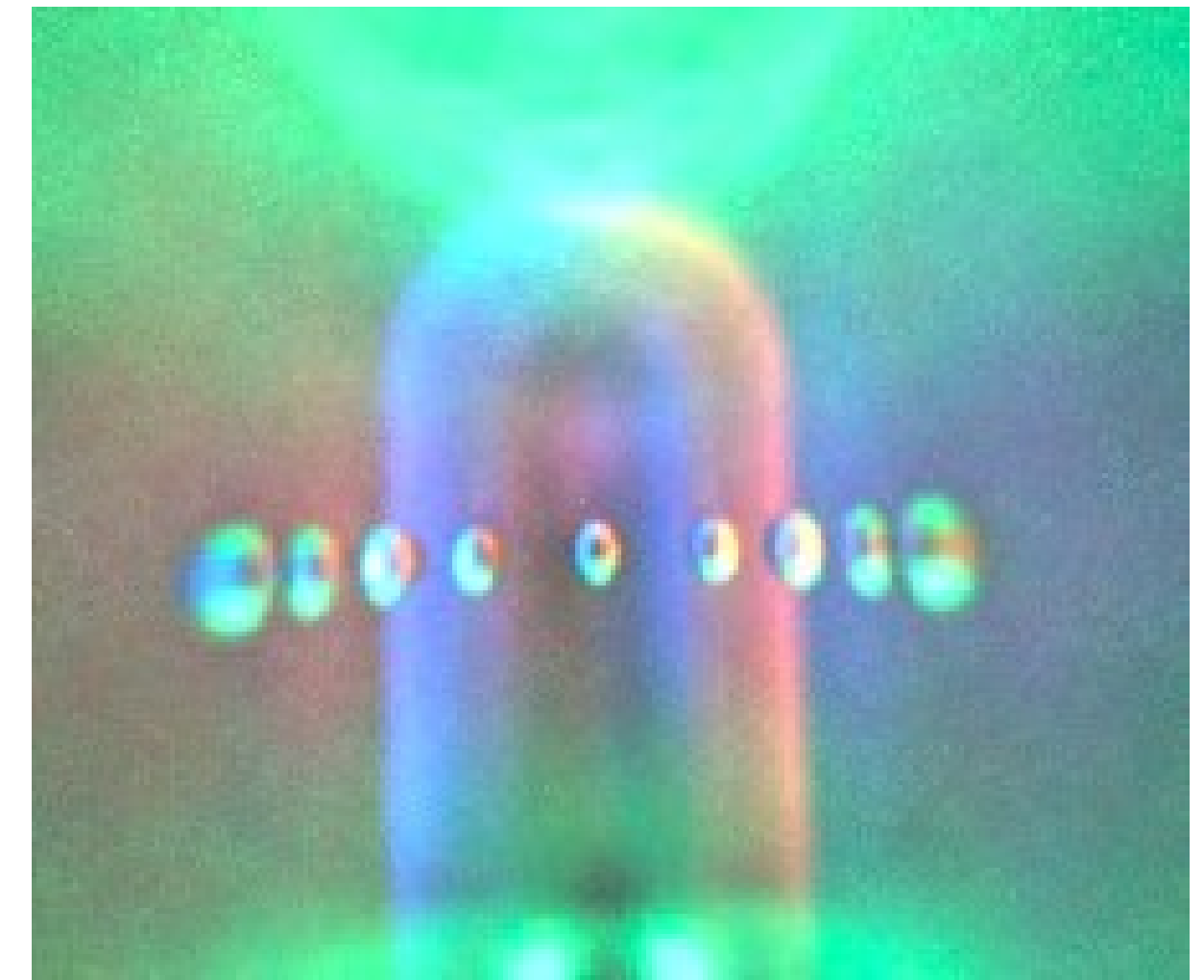
Path tracing

[Agarwal et al., 2023]

# Evaluating optical simulation framework



Real-world prototype



Our simulation result



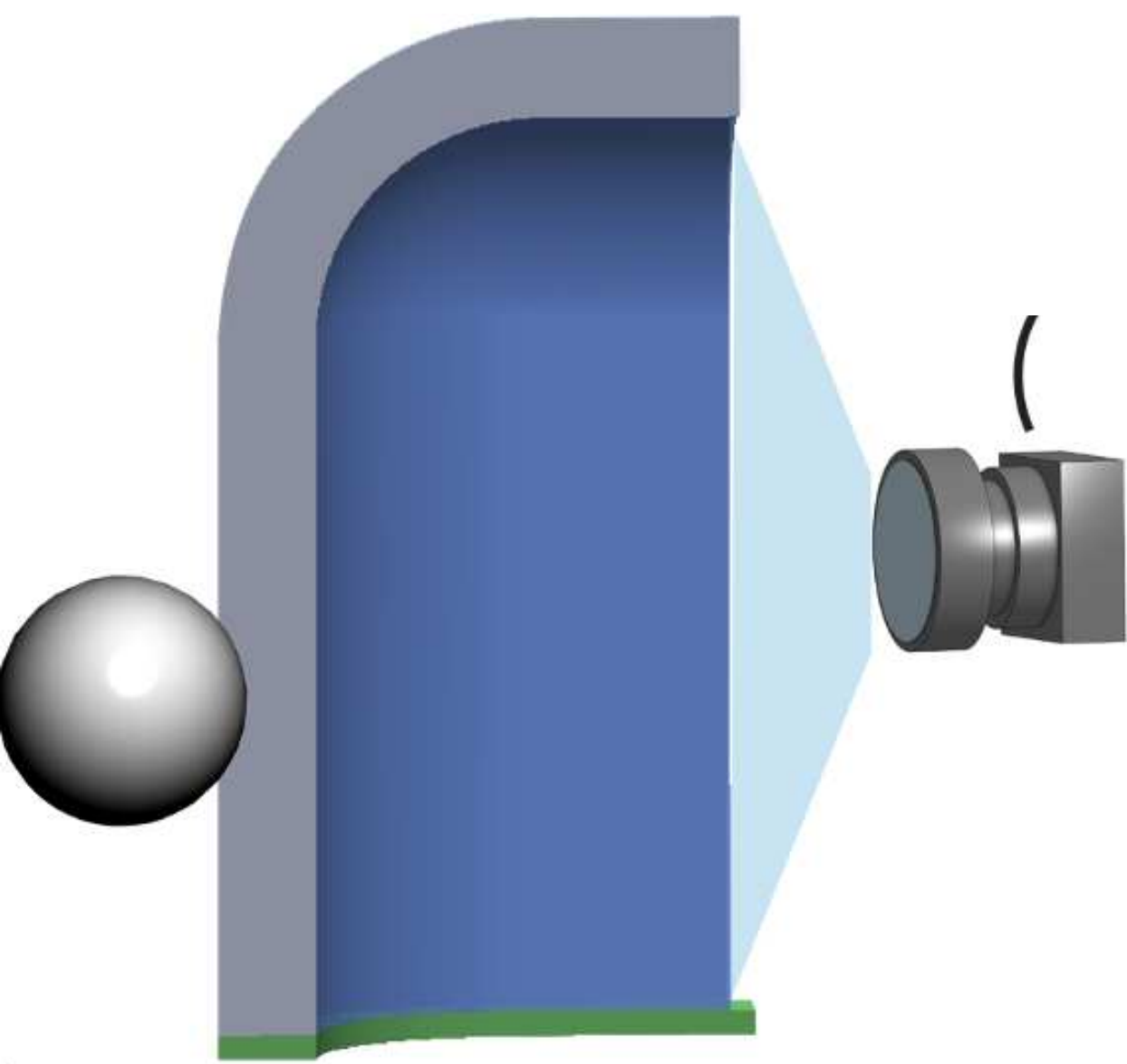
Rasterization



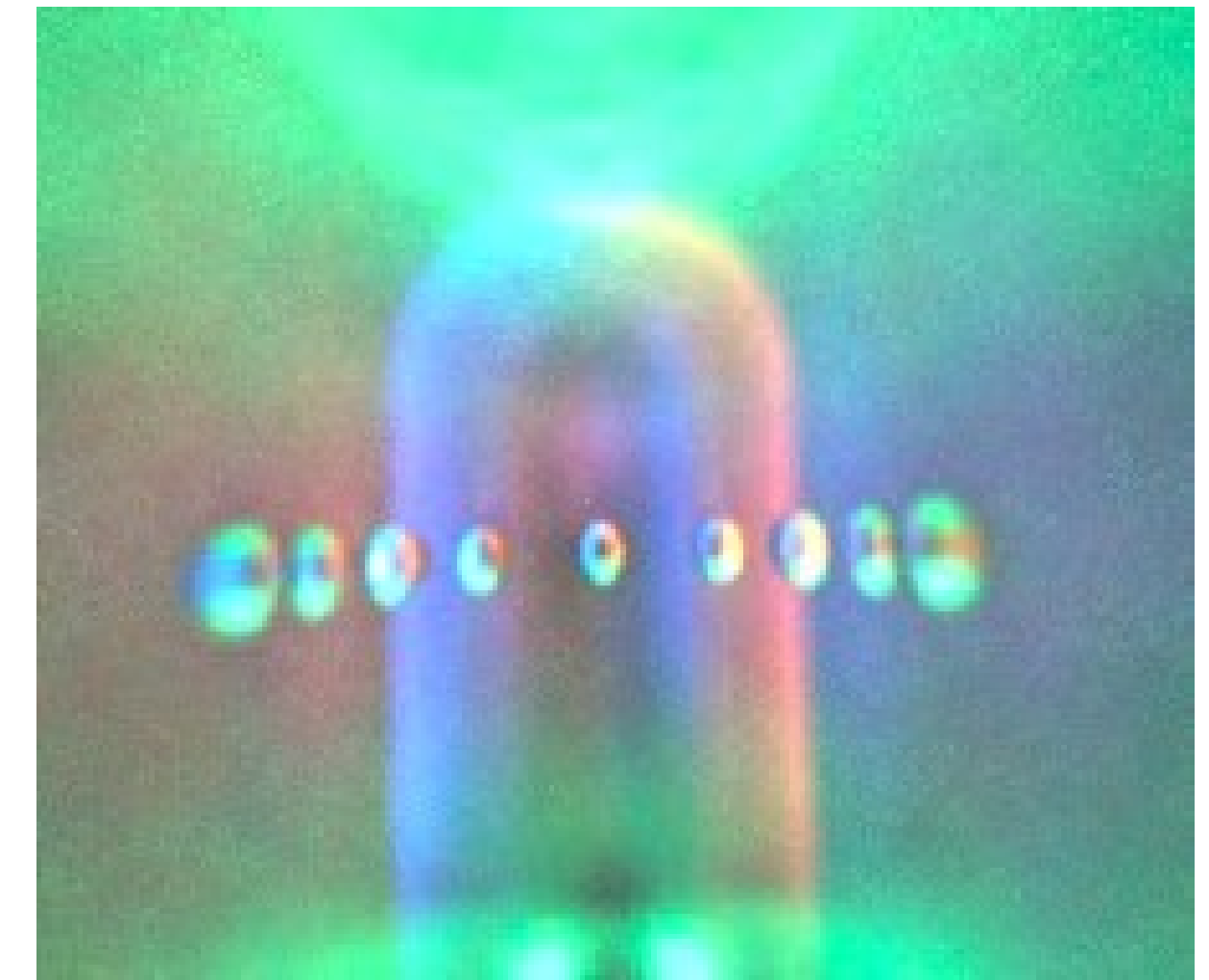
Path tracing

[Agarwal et al., 2023]

# Evaluating optical simulation framework



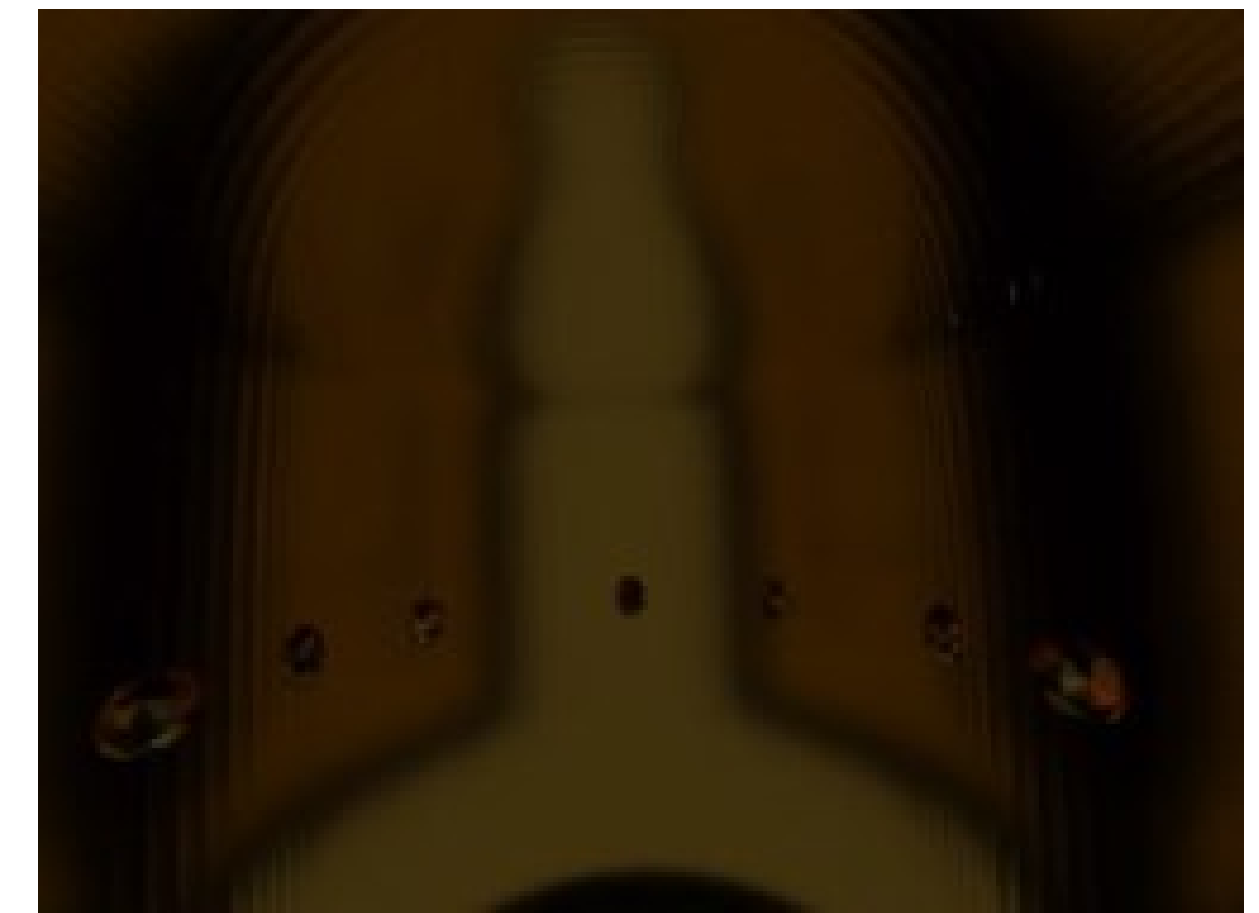
Real-world prototype



Our simulation result



Rasterization

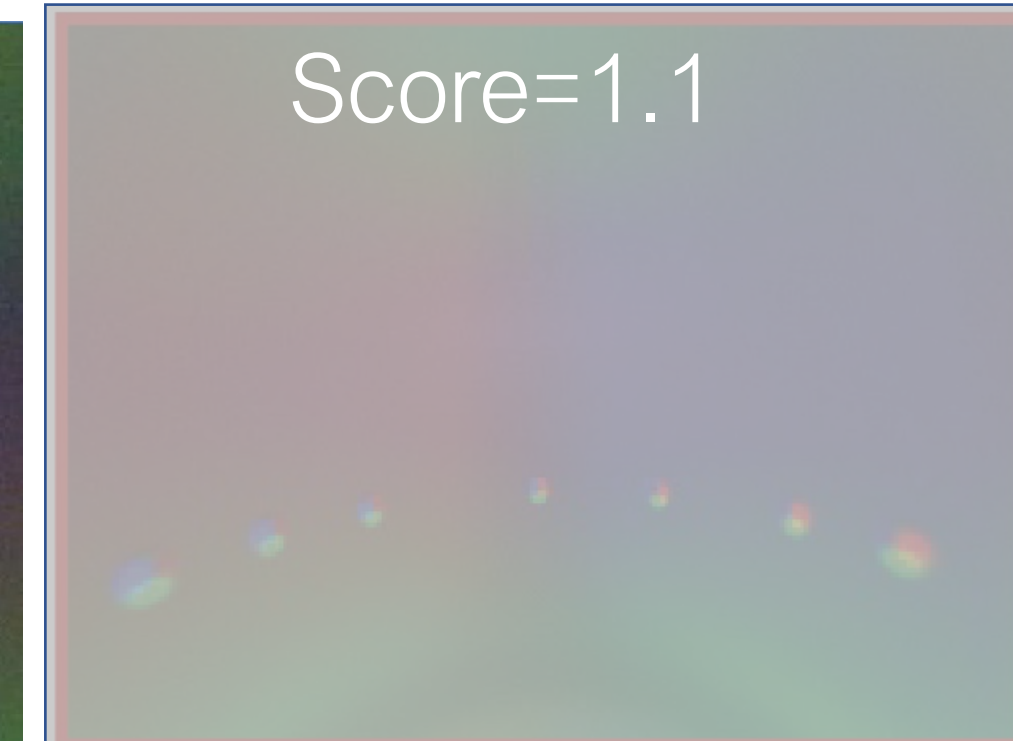
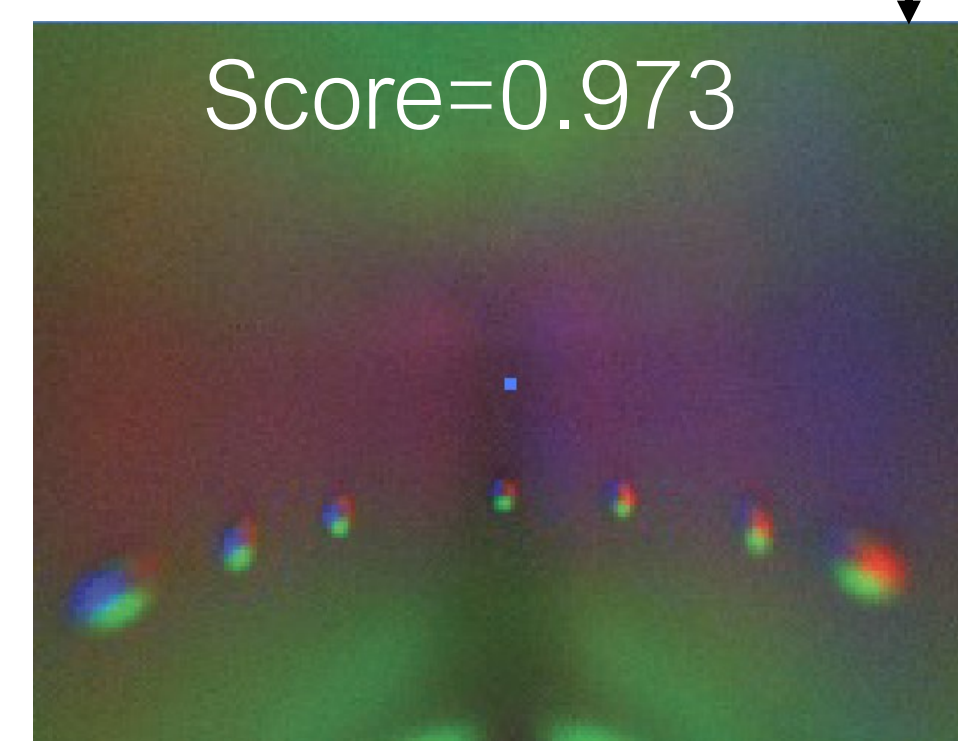
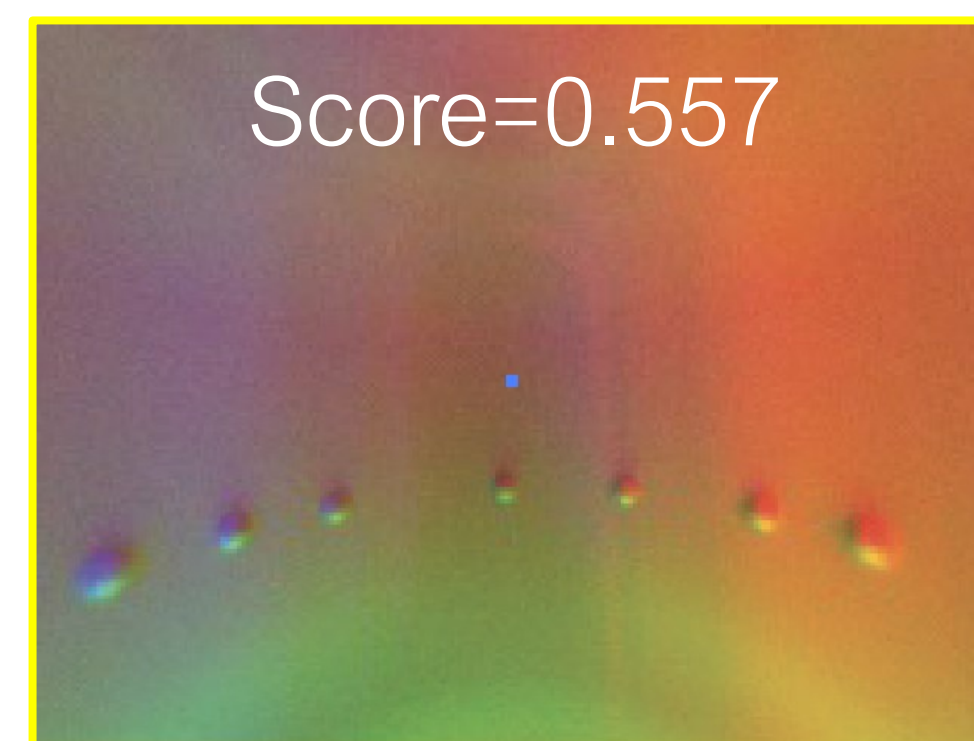
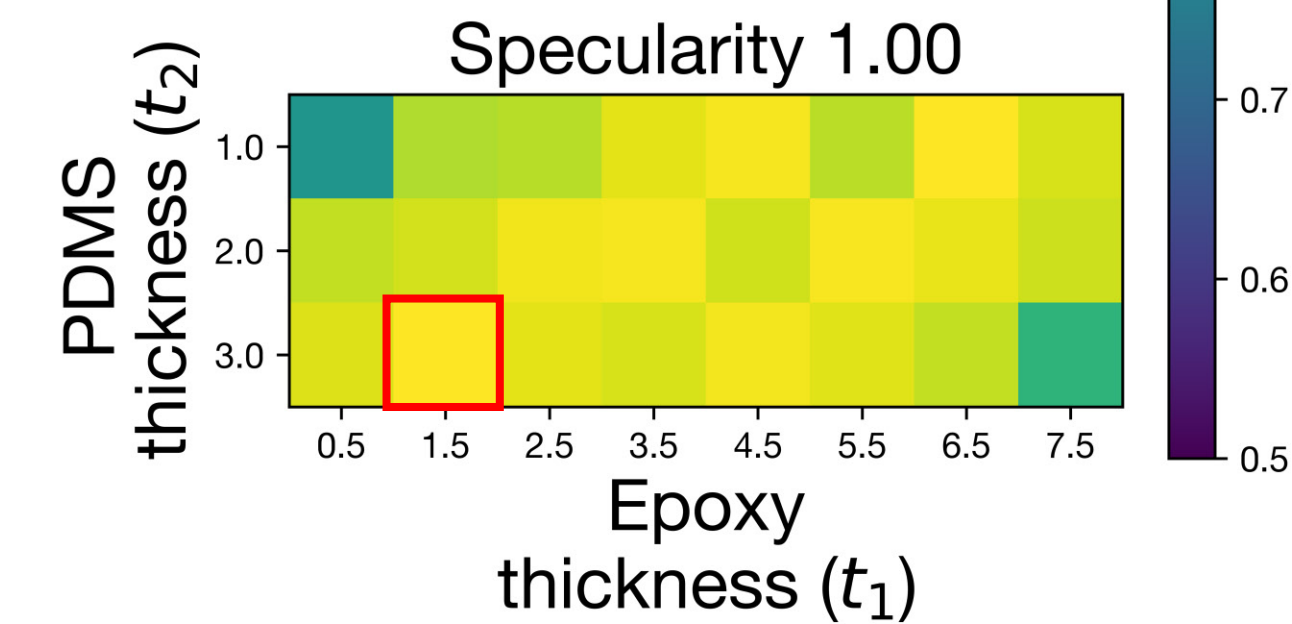
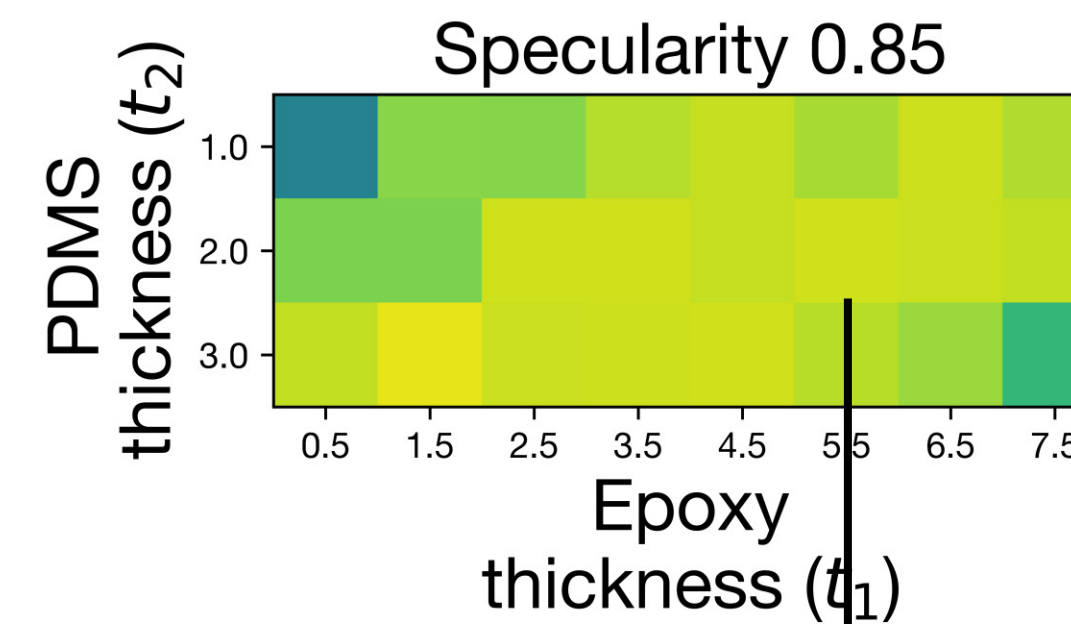
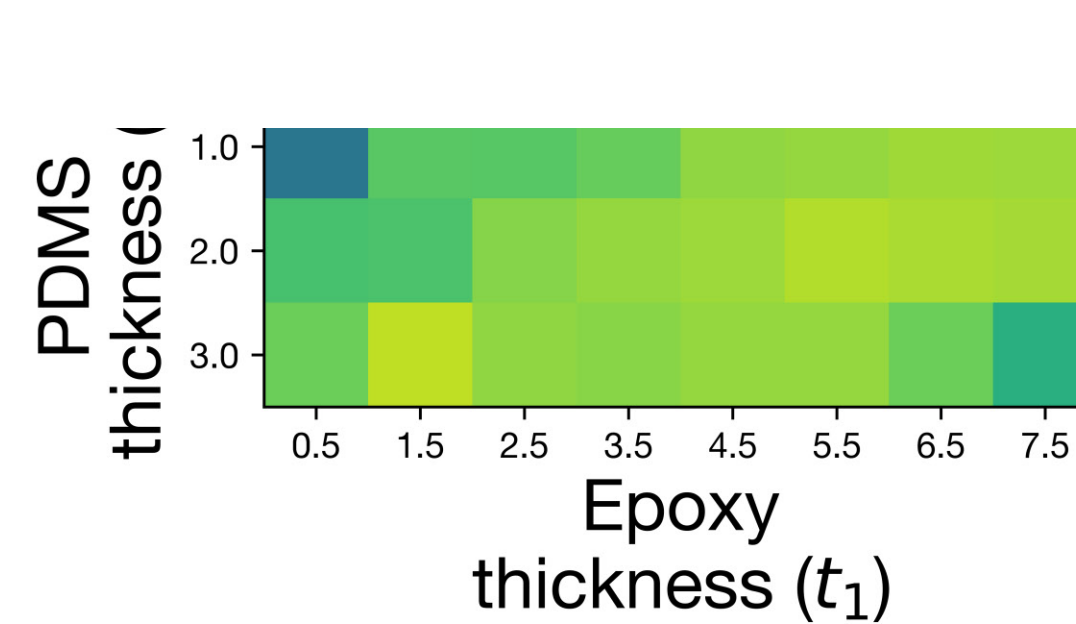
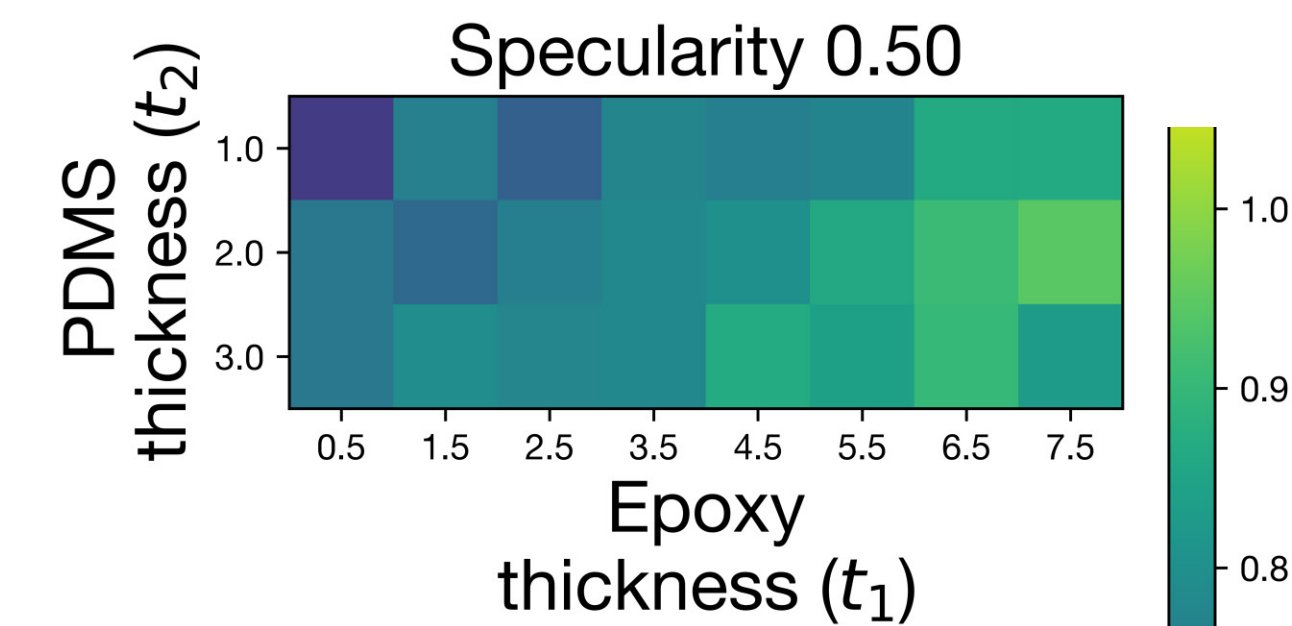
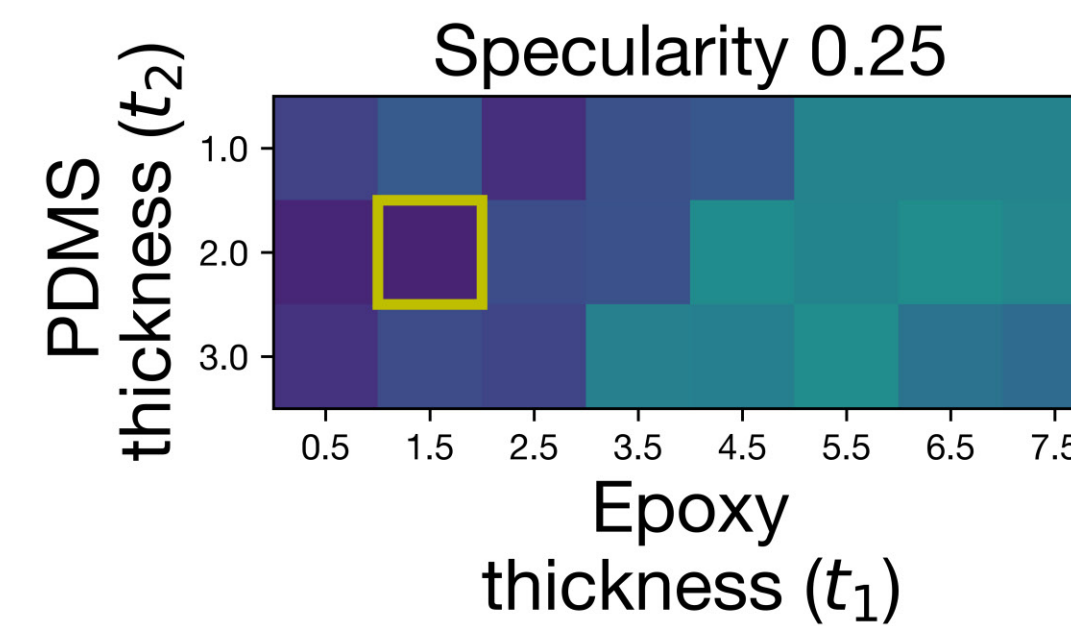
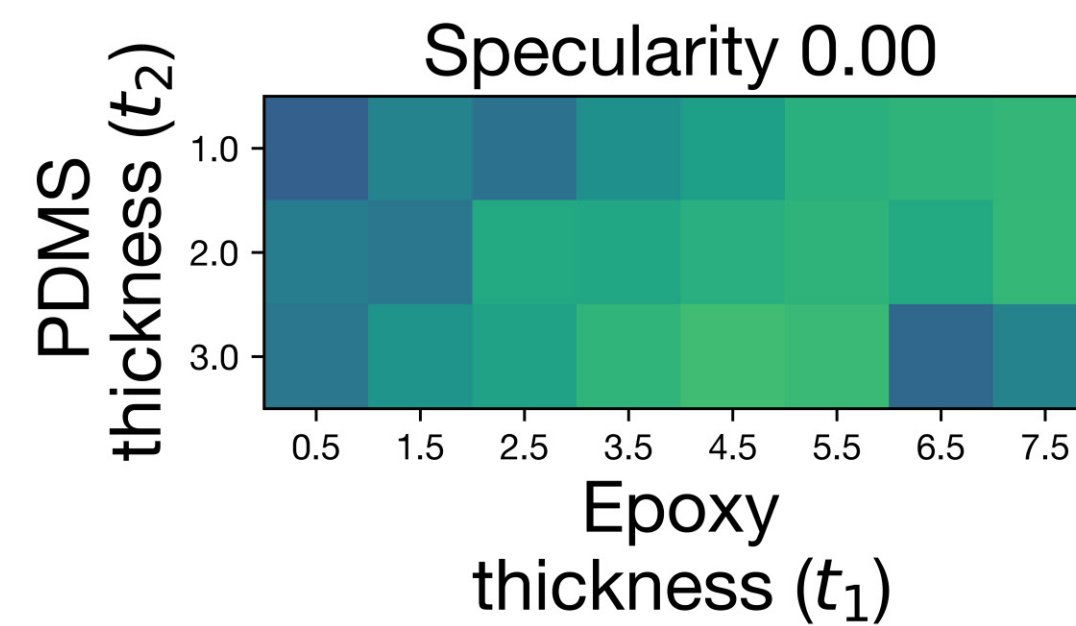
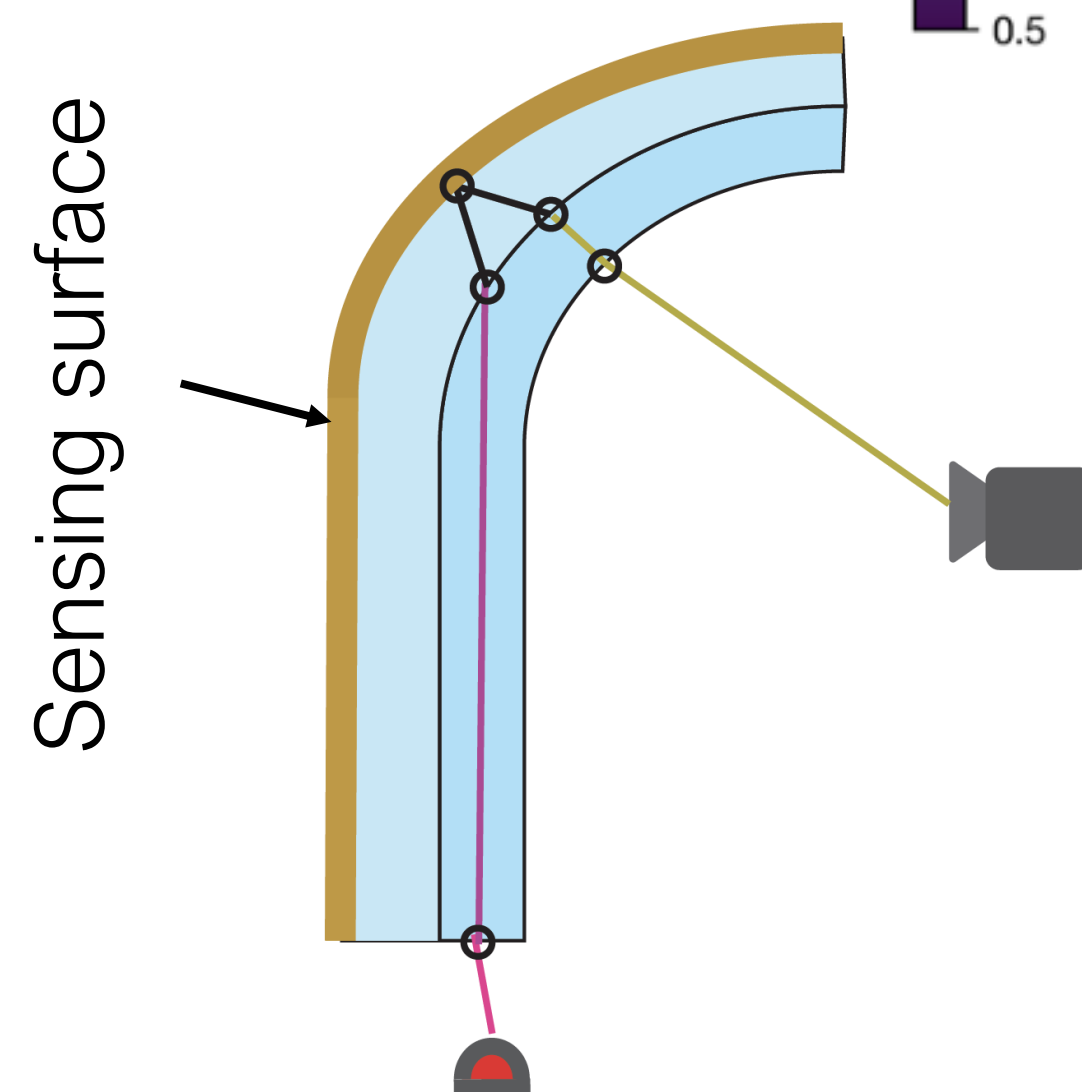


Path tracing

[Agarwal et al., 2023]

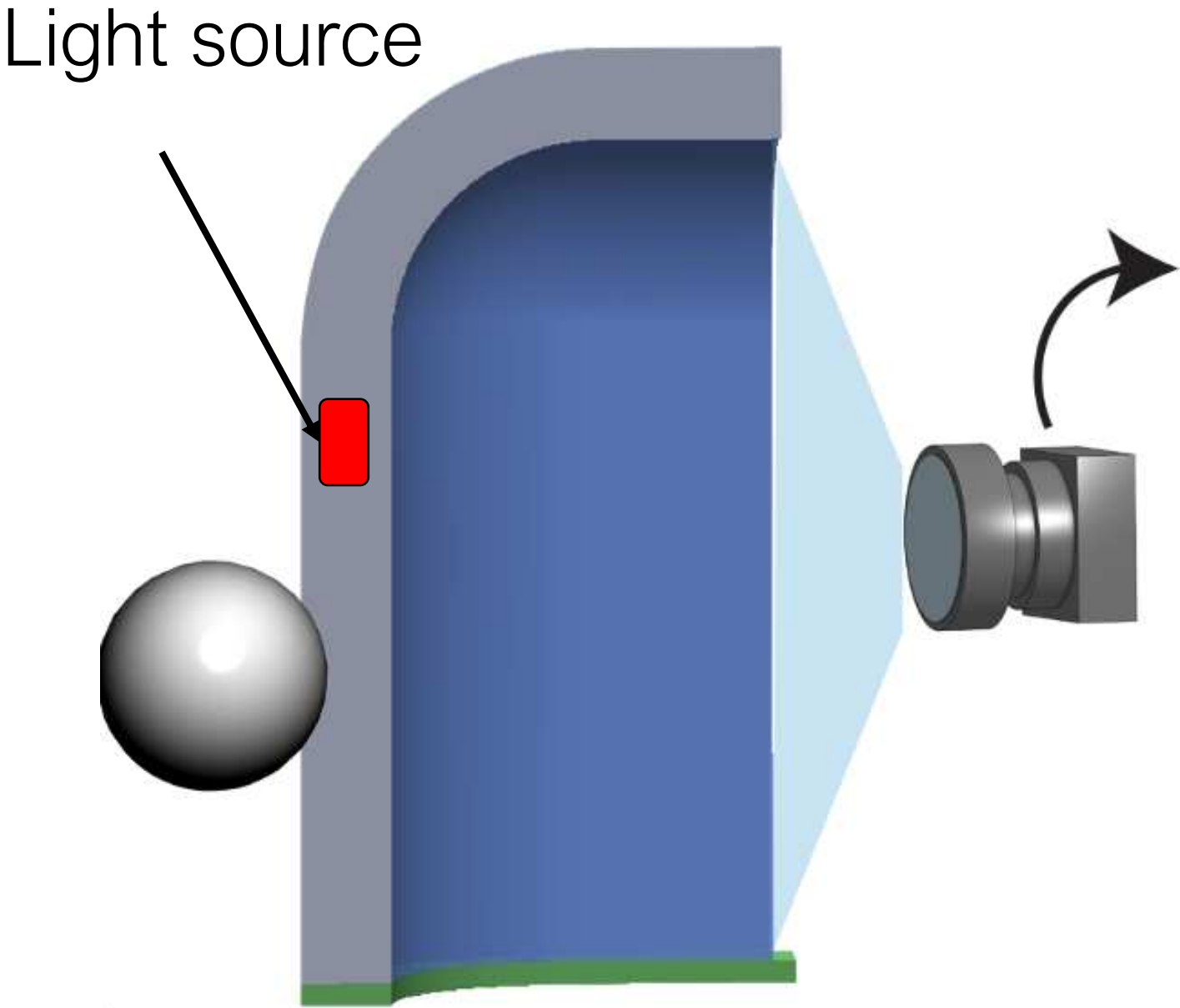


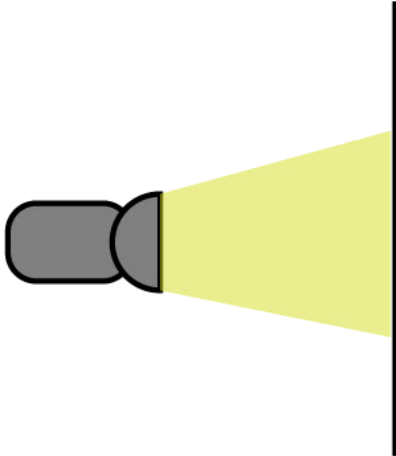
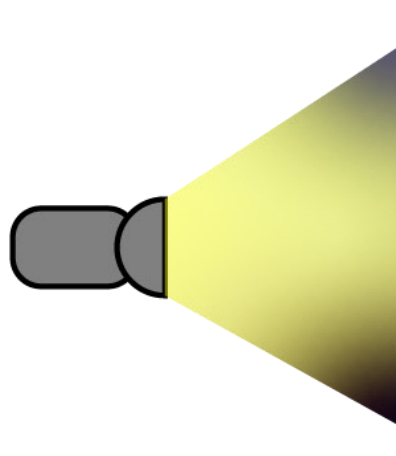
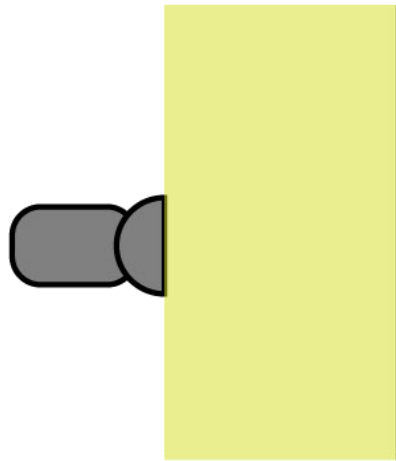
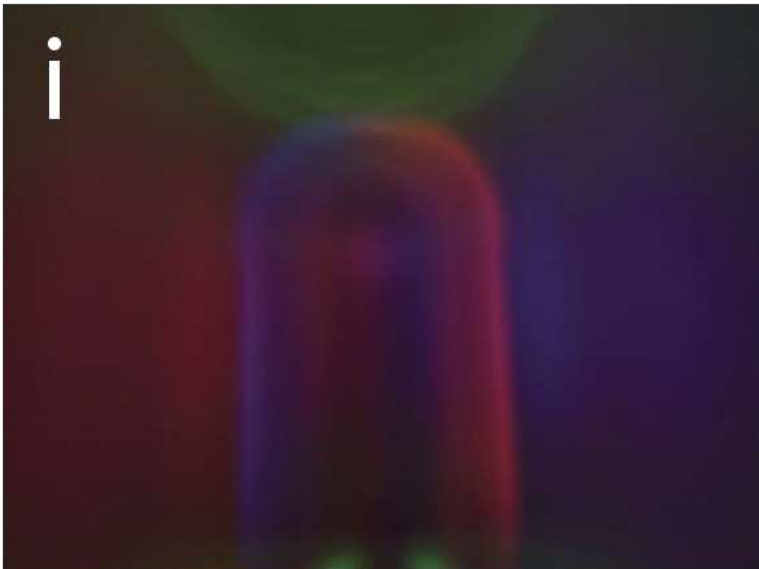
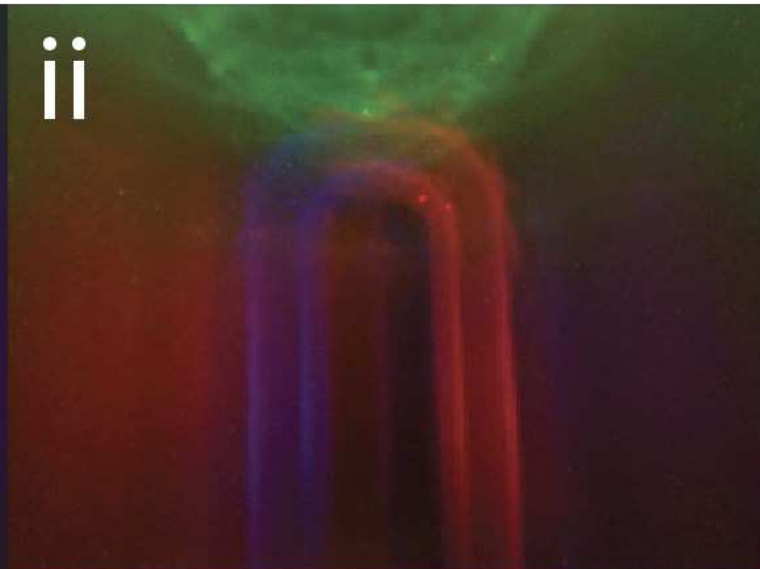


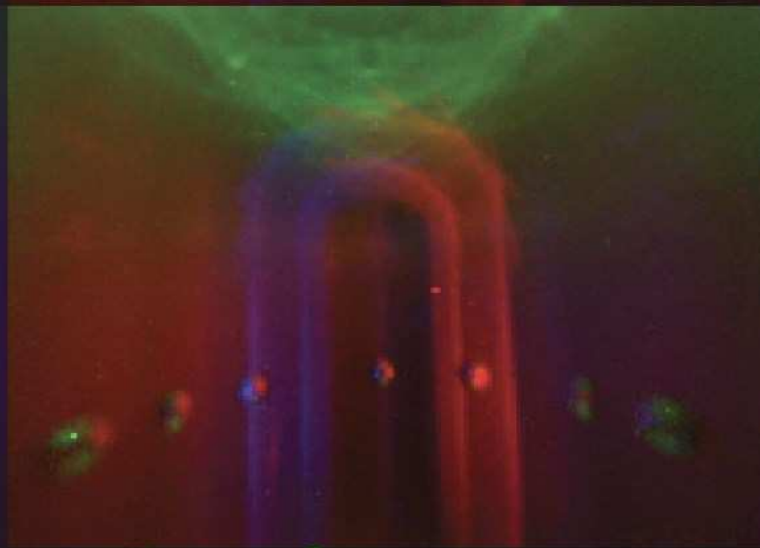
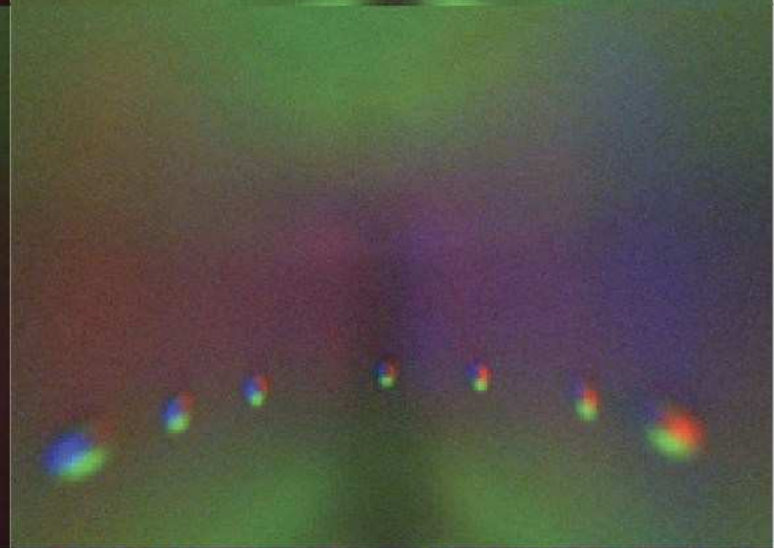
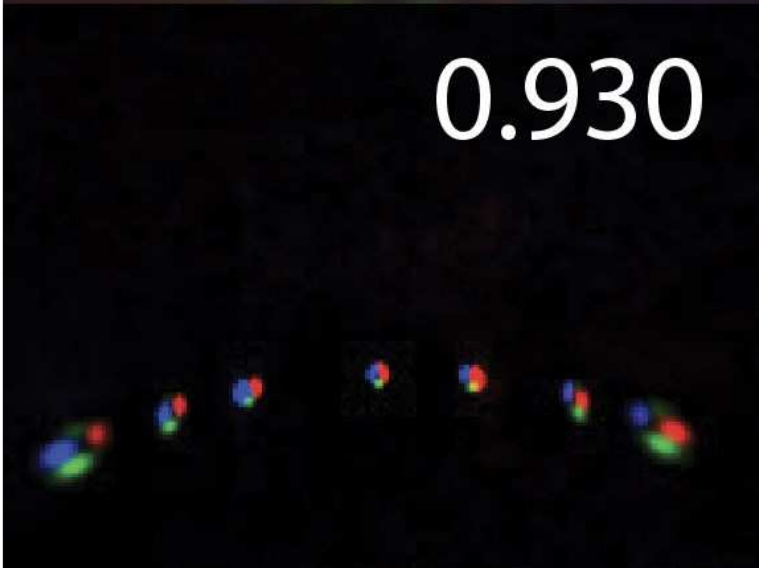
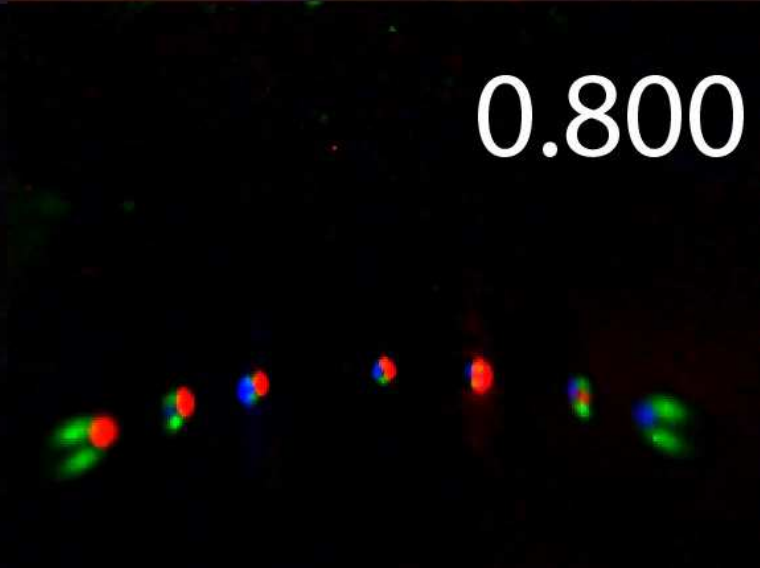
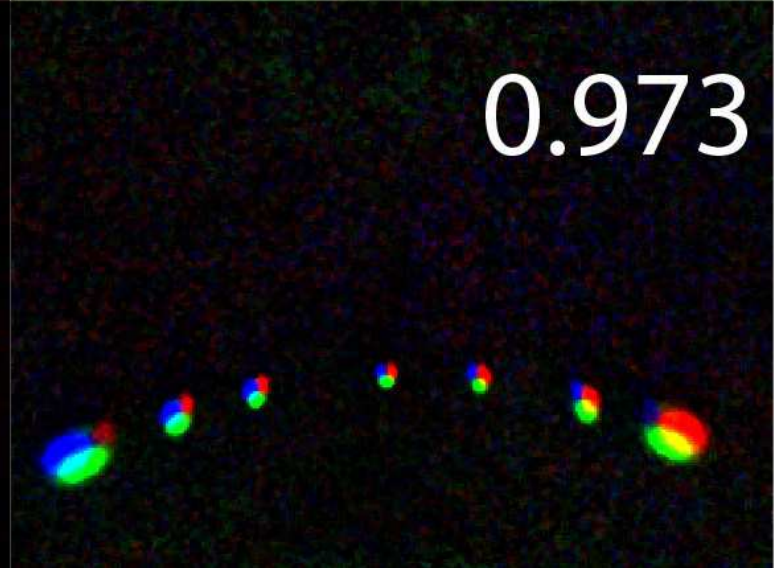
# Curved sensor material design





# Curved sensor illumination design

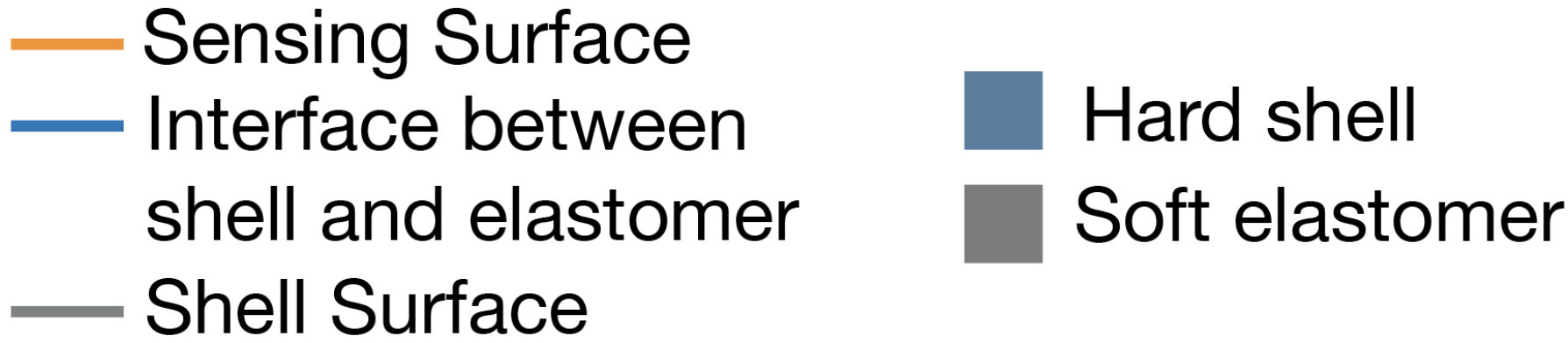


	Spot Light	IES Light	Area Light
Light Type			
Background Image			
Indenter Image			
Color Signal	 0.930	 0.800	 0.973



# Curved sensor shape design

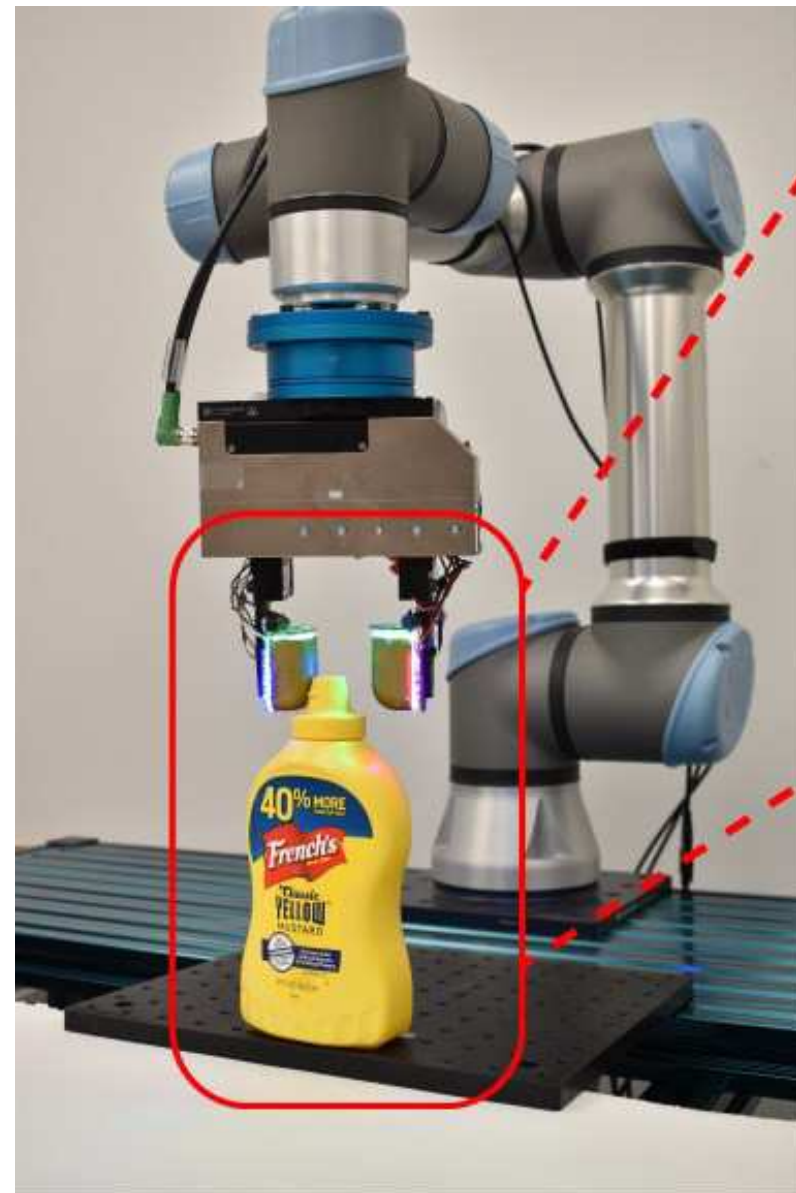
- Used gradient-free optimization, CMA-ES, for optimizing the sensor shape
- Optimized sensor design is 35% better than initial design
- Optimized sensor design outperforms human-expert design in 3D shape reconstruction



	2D Curve	CAD Model	Simulated Image
Initial Design			
Human Expert Design			
Optimized Design			



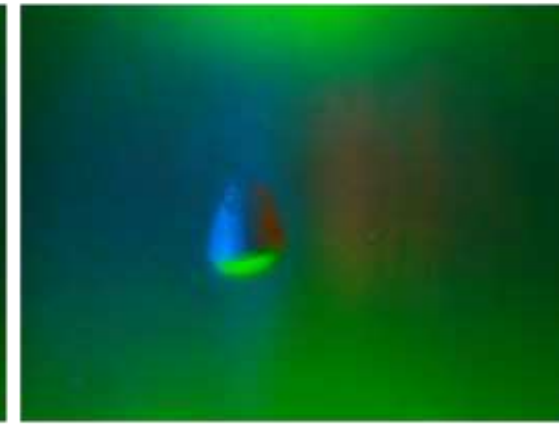
# Results: Robotic grasping



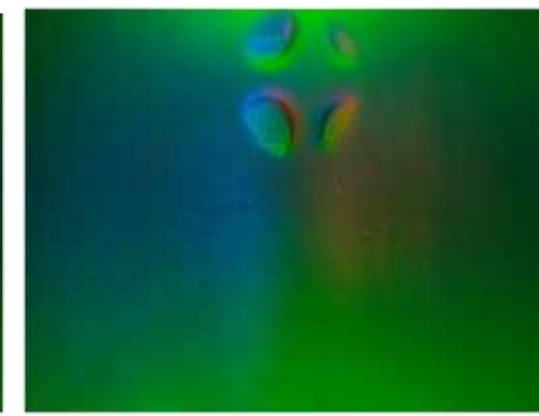
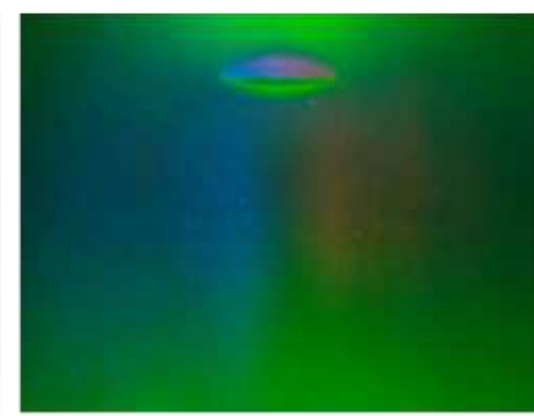
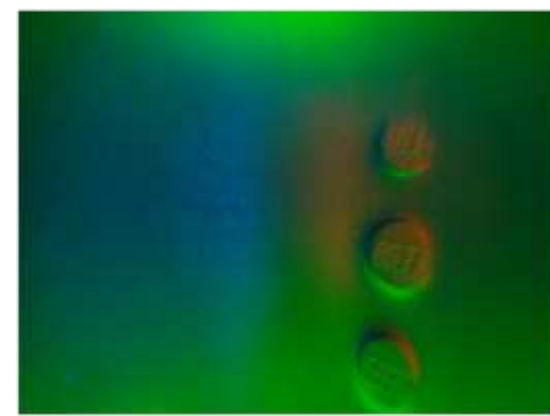
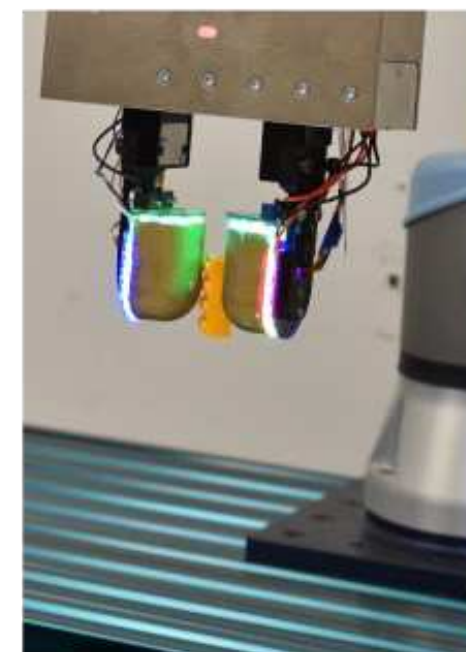
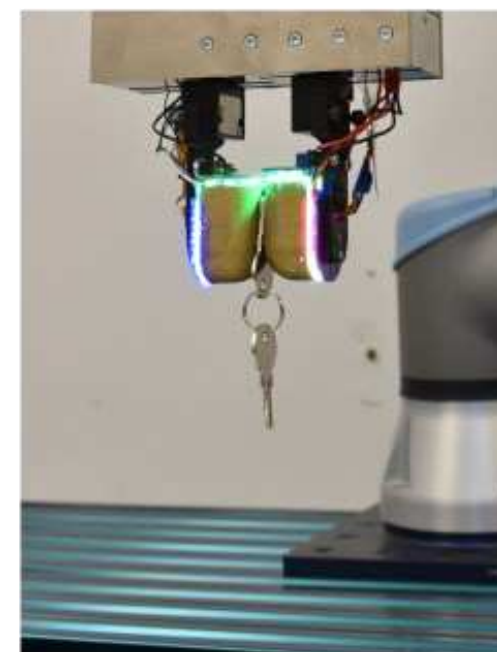
Robotic arm with  
optimized tactile sensor



Common Objects



Tactile Images

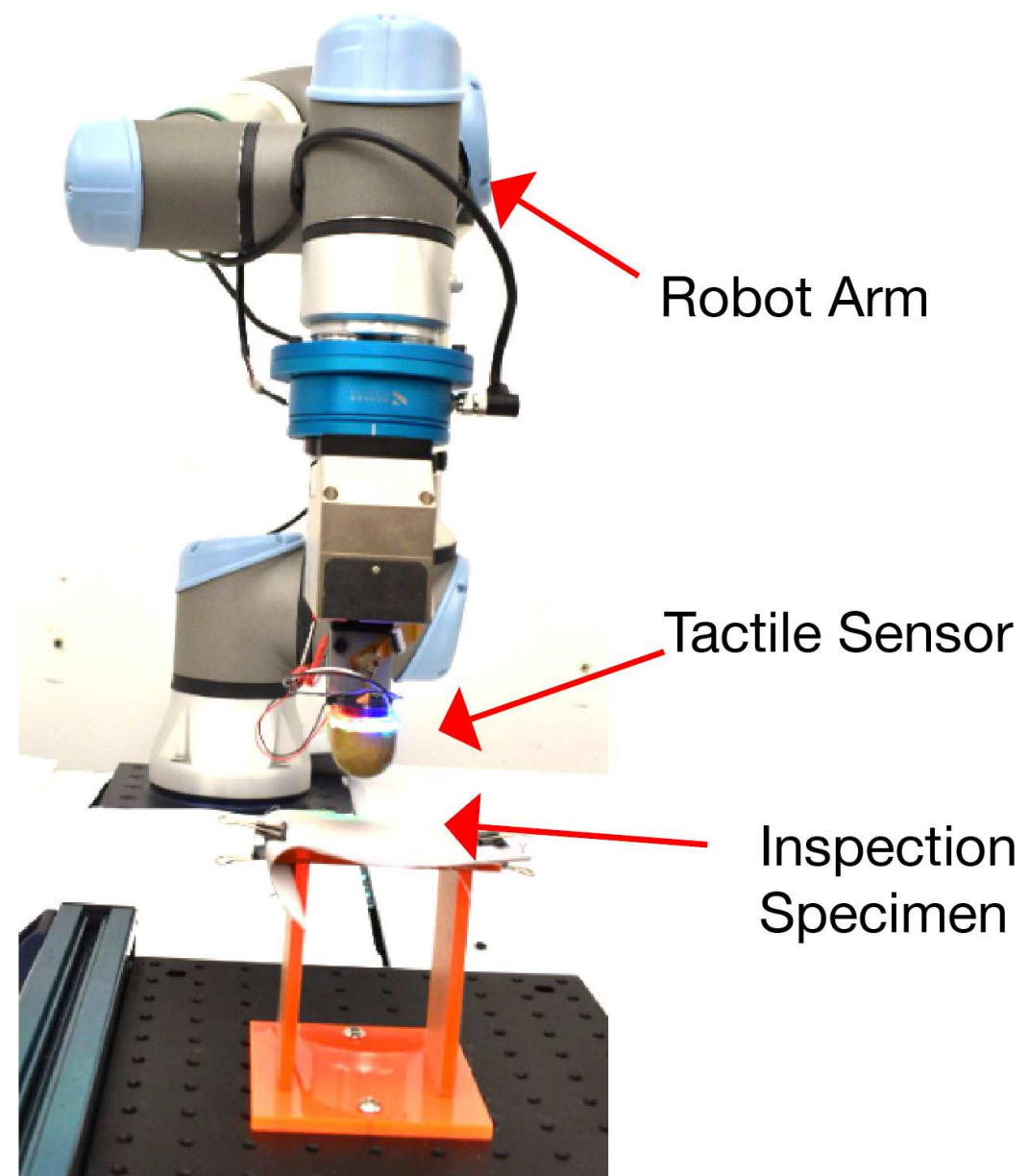


Tactile Images



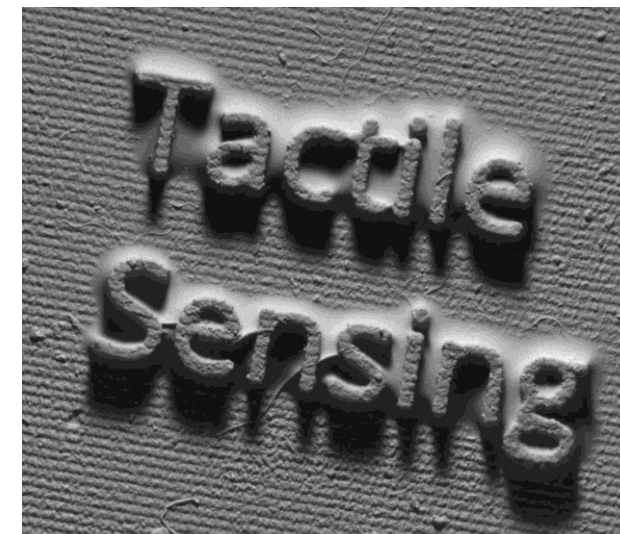
# Results: surface inspection

## Experiment 1: Detection accuracy vs text size



Experiment setup

Text description:  
Tactile  
Sensing

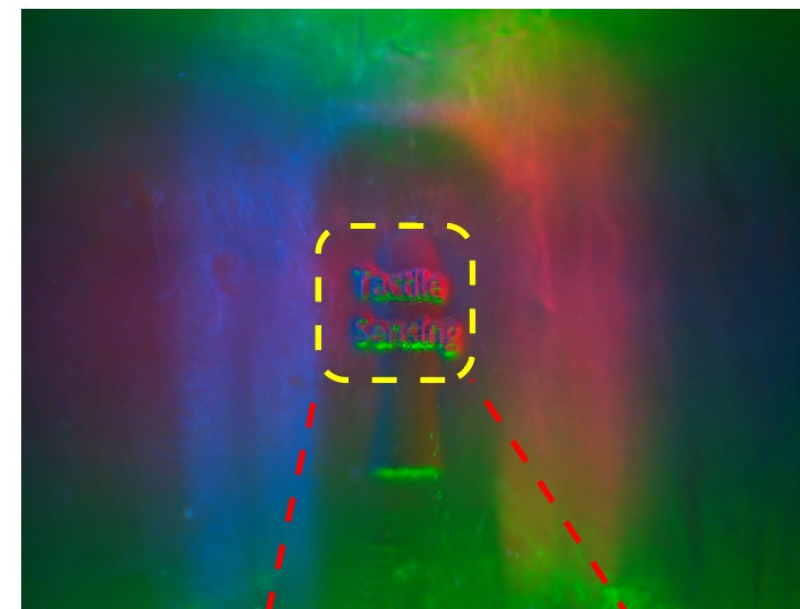


Text size = 1.5 mm



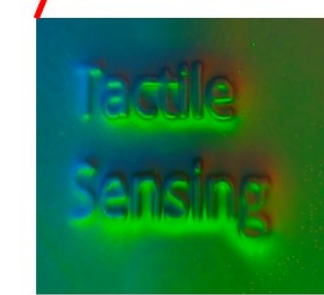
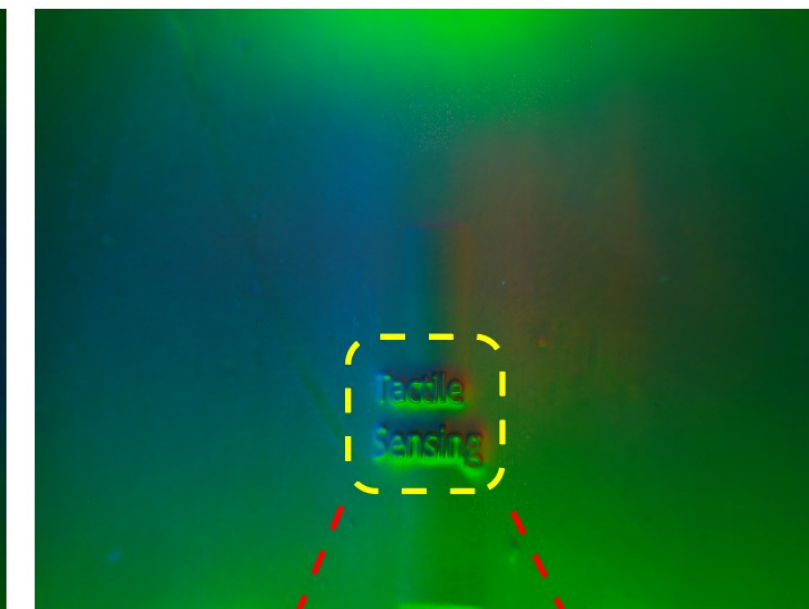
Text size = 1.0 mm

Human expert  
design

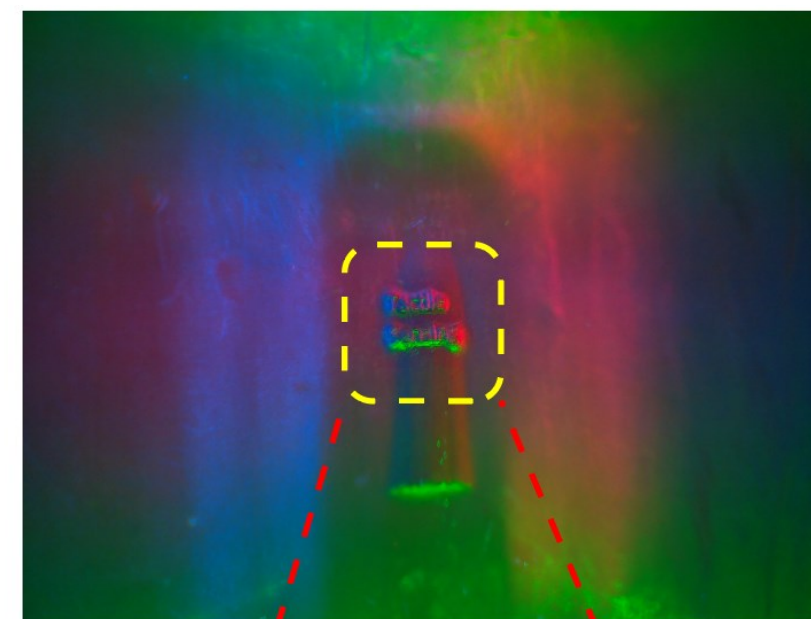


“Tatle Sending”

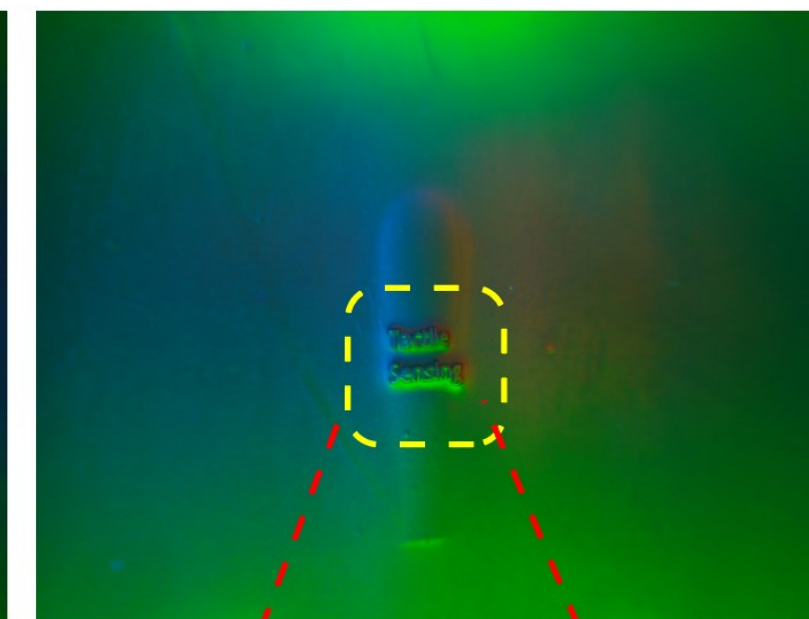
Optimized design



“Tacolle Sending”



“\_”

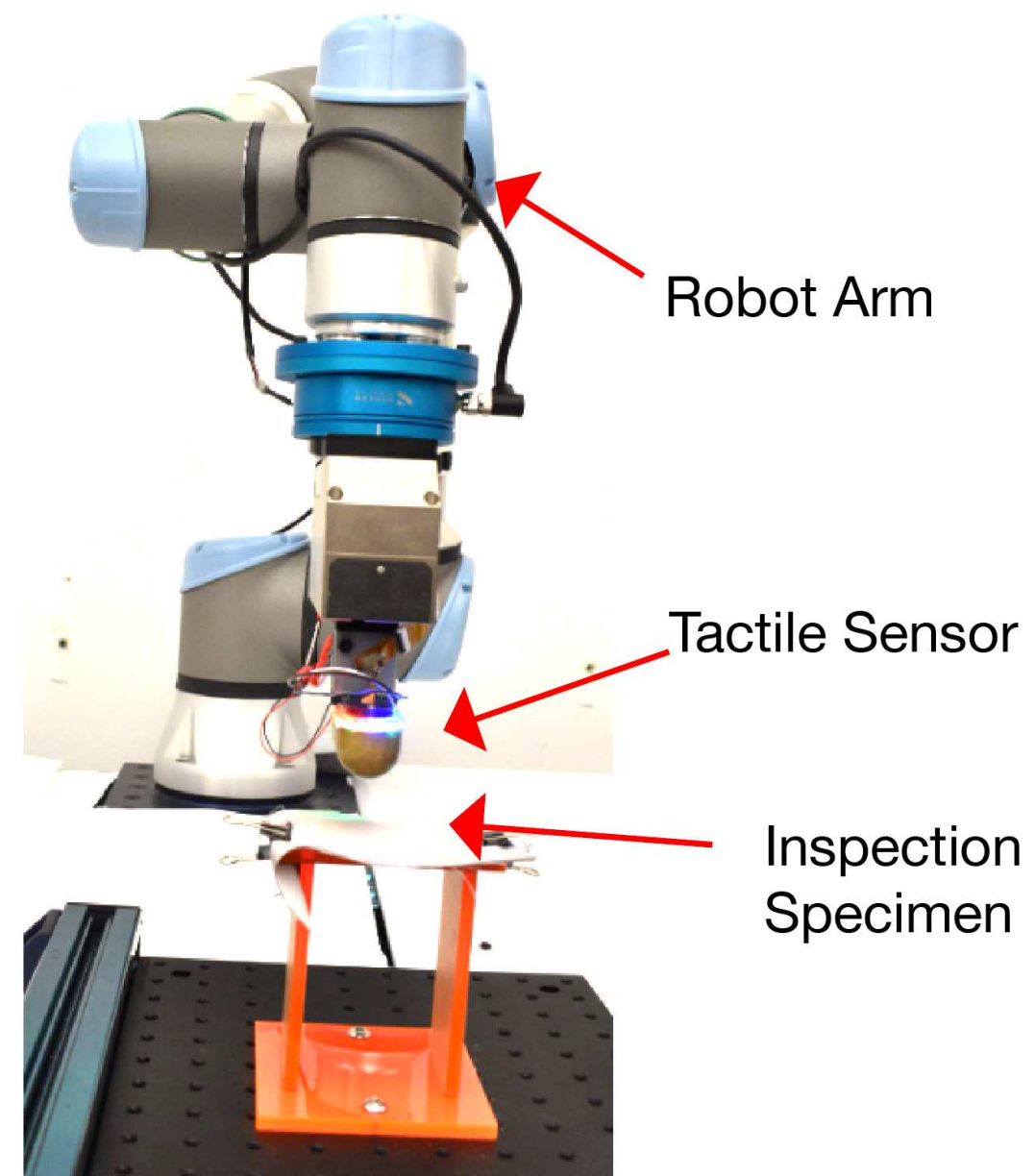


“Tacolle Sensing”

[Agarwal et al., 2023]



# Results: surface inspection

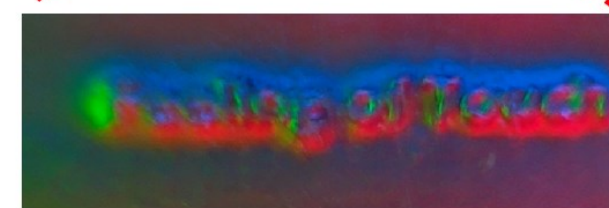
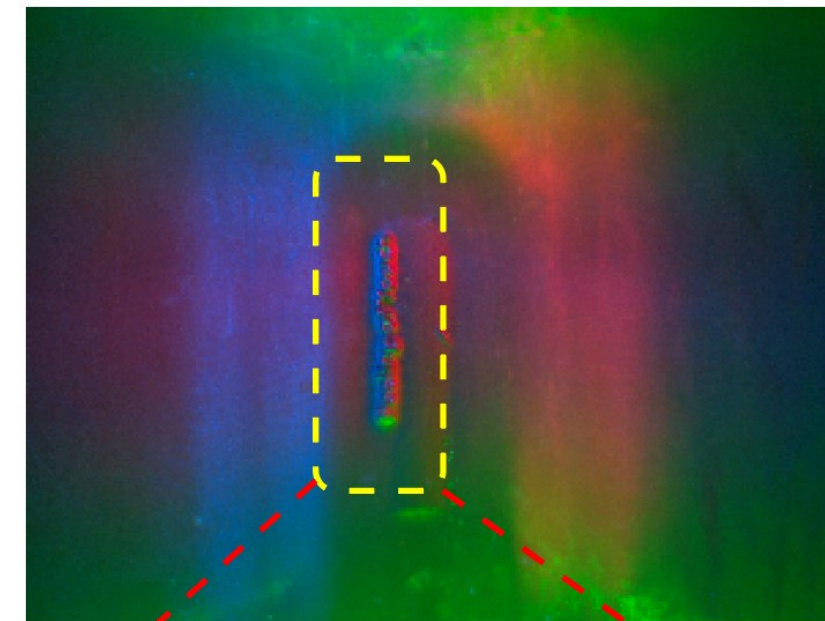


Experiment setup

Text description: Feeling of Touch

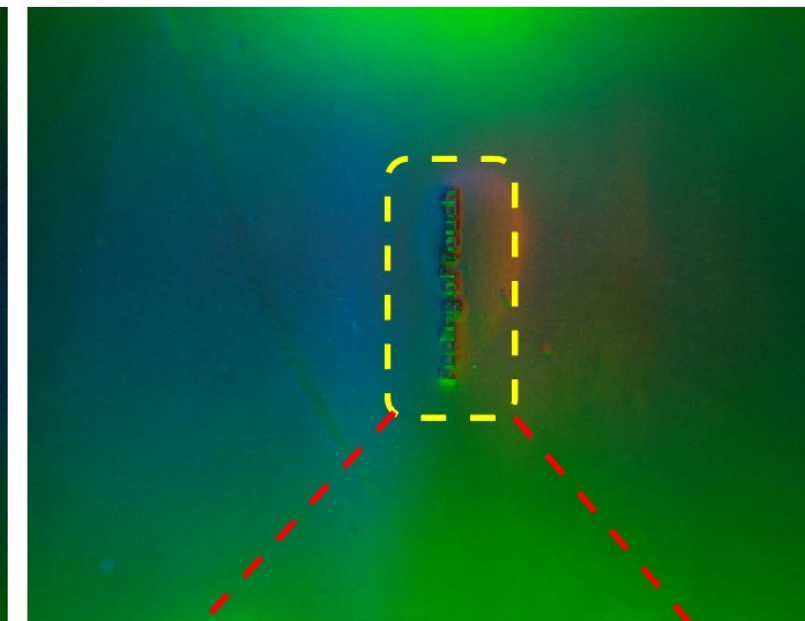
## Experiment 2: Detection accuracy vs contact location

Human expert design

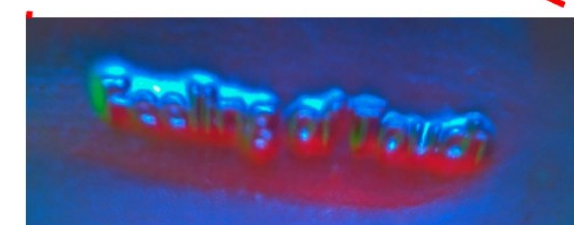
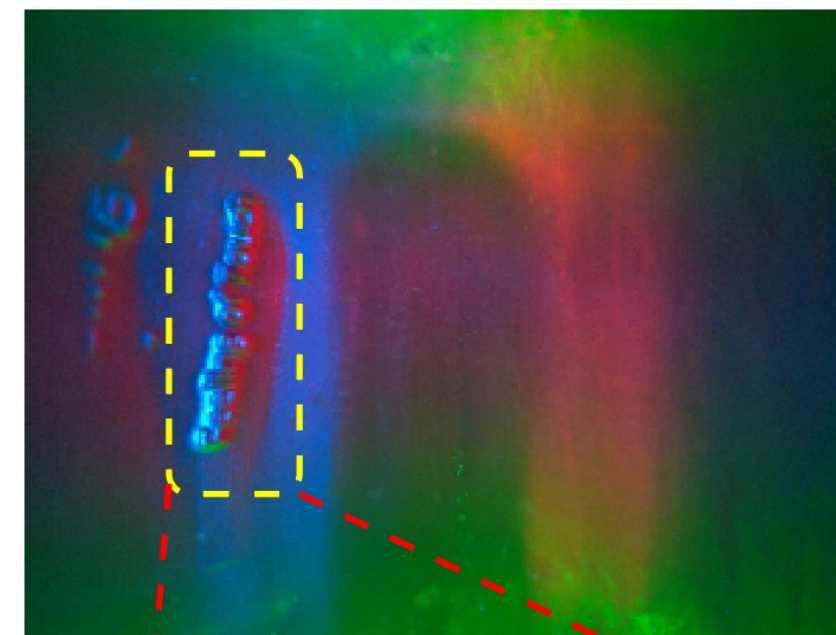


“\_”

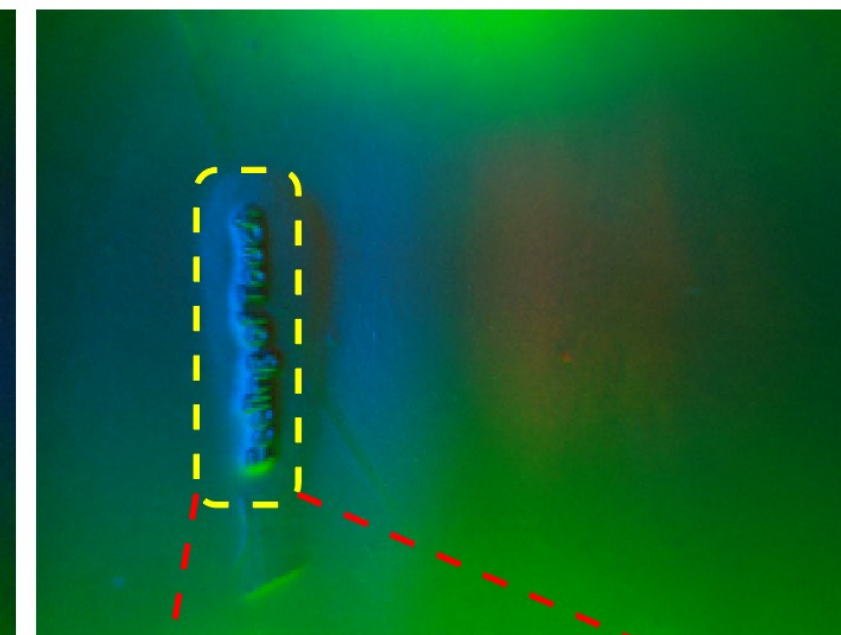
Optimized design



“Feeling of Touch”



“\_”

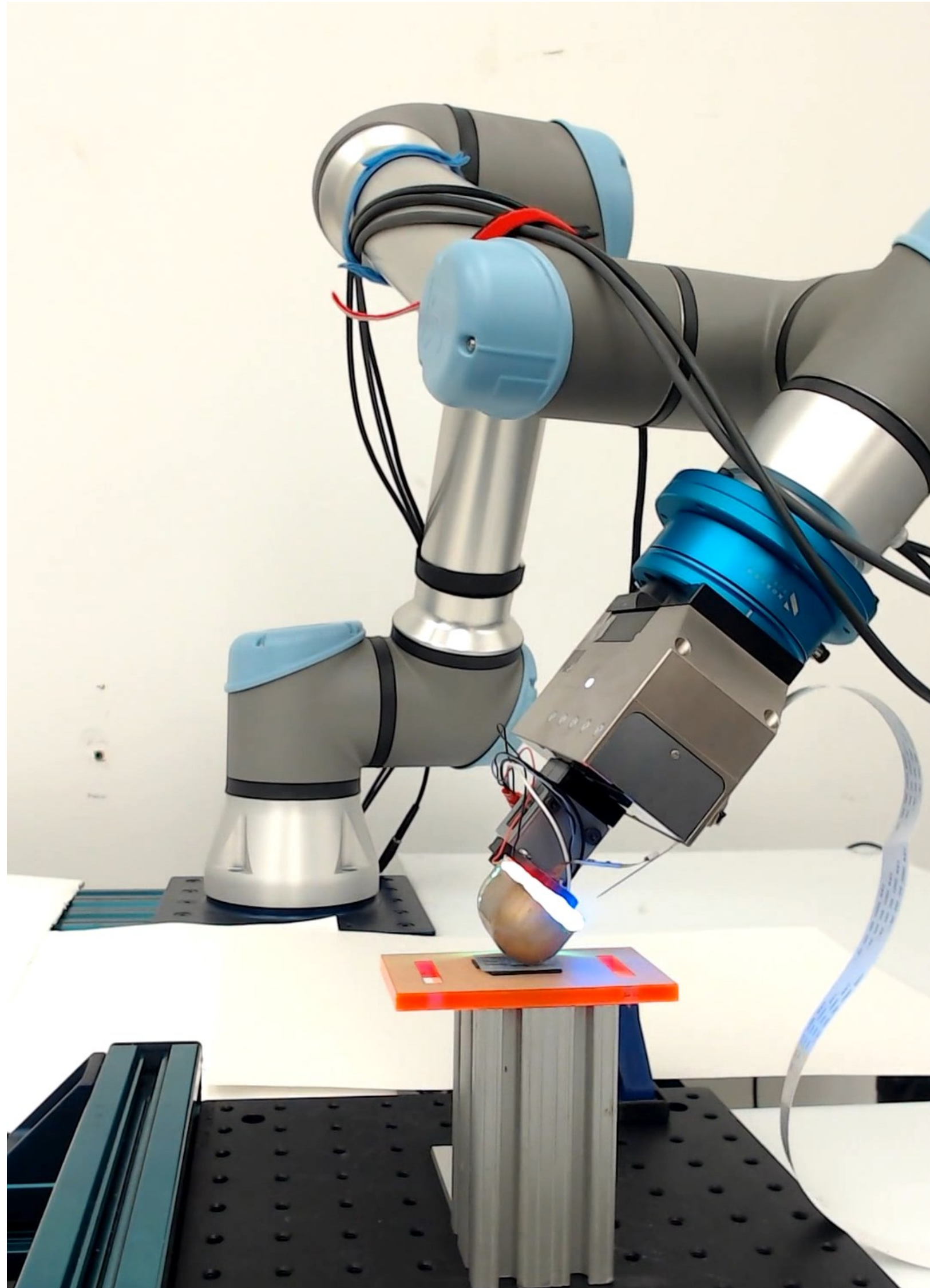


“Peeling of Touch”

[Agarwal et al., 2023]



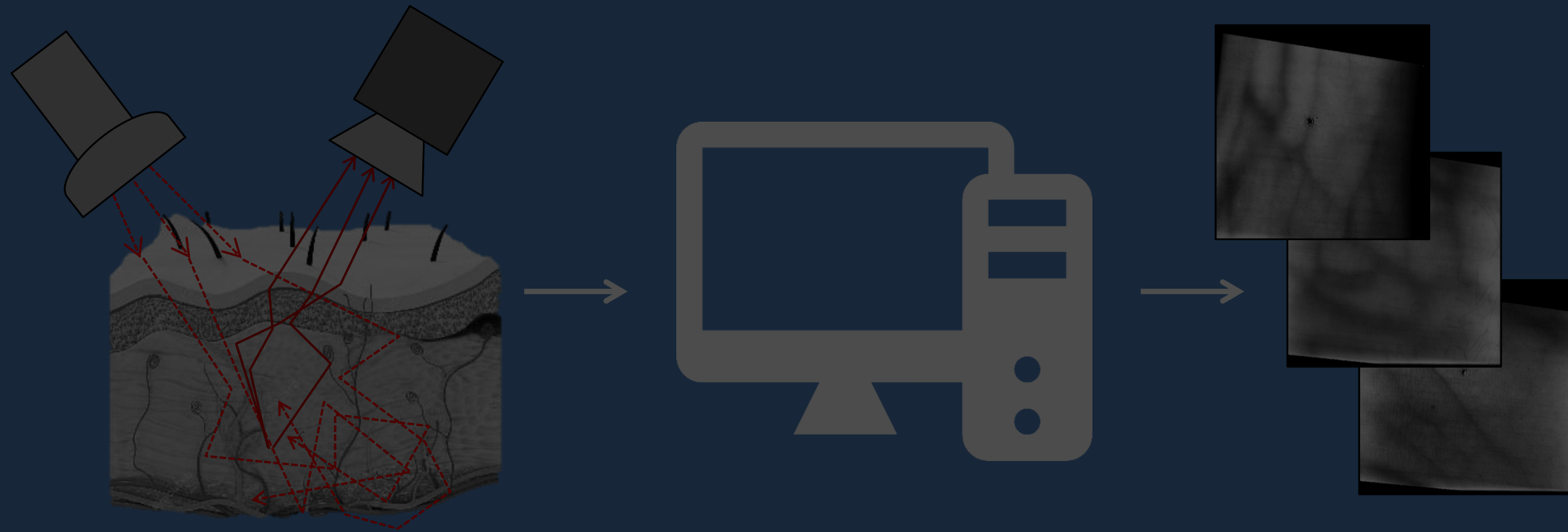
# Results: surface inspection





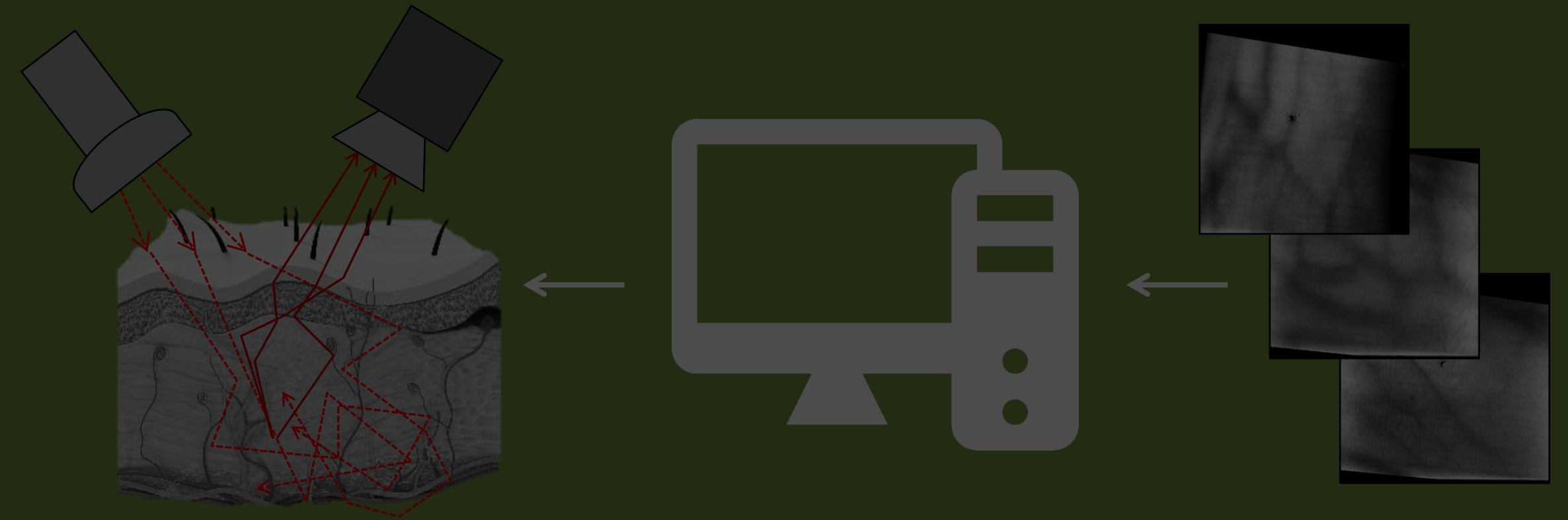
# Physics-based rendering and its applications to computational imaging

## forward rendering

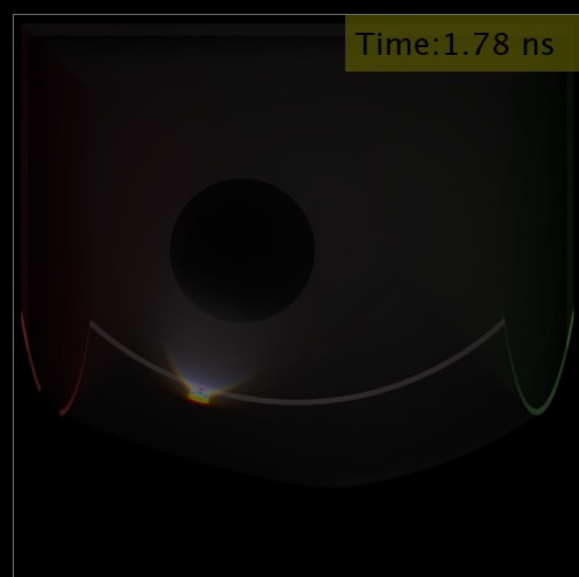


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

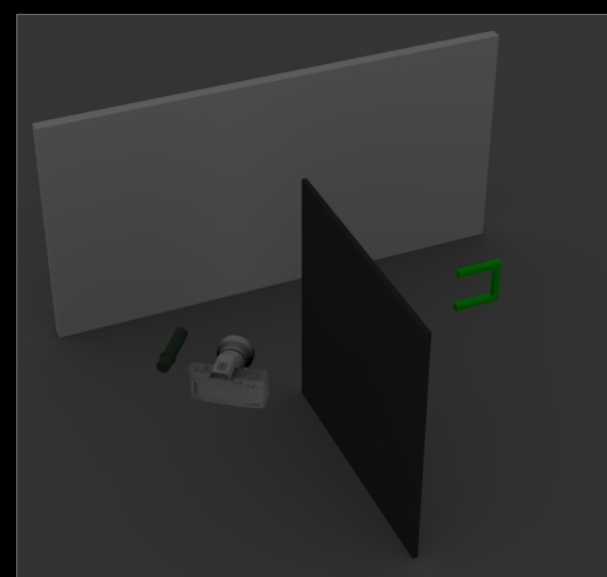
## inverse rendering



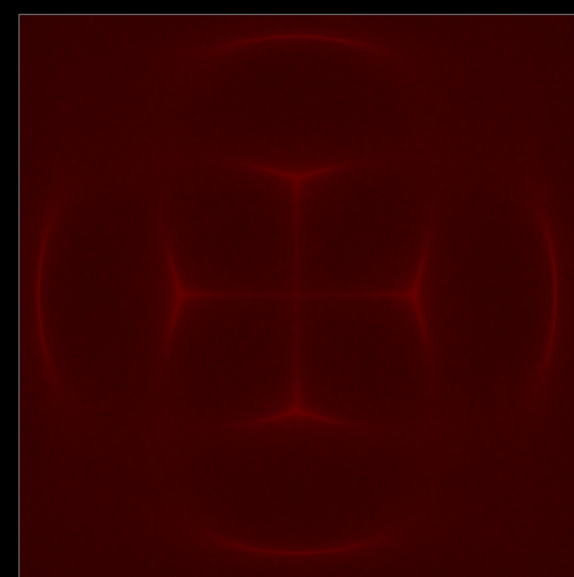
- accurate and efficient differentiable simulation
- tractably solve general inverse problems



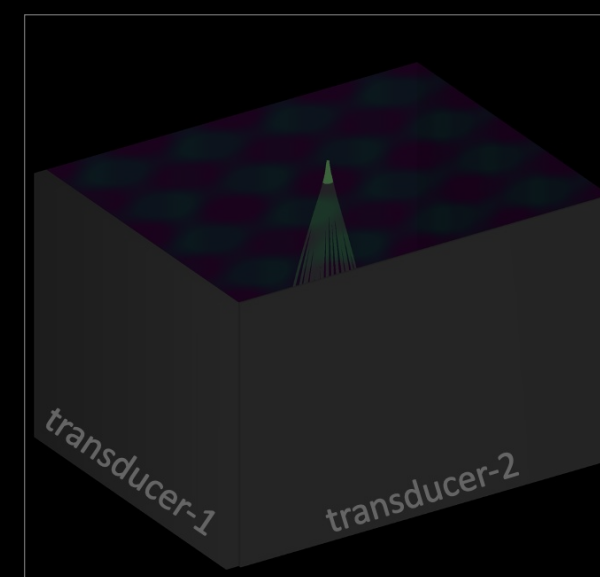
time-of-flight  
imaging



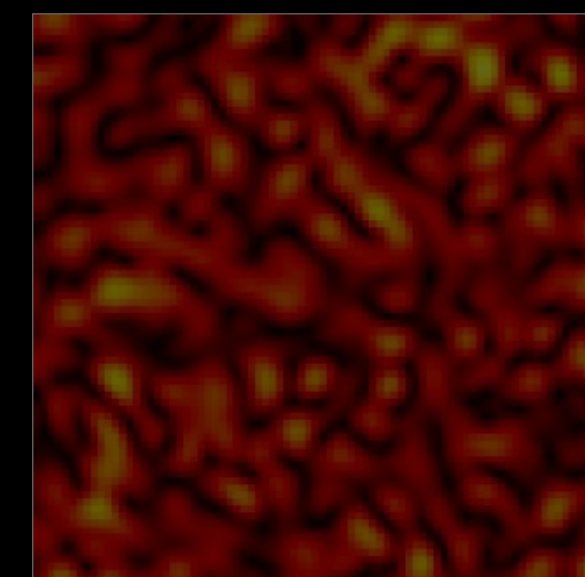
non-line-of-sight  
imaging



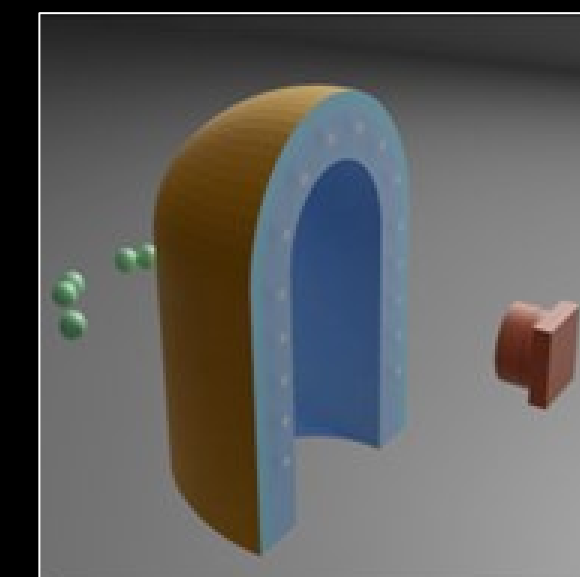
acousto-optic  
lensing



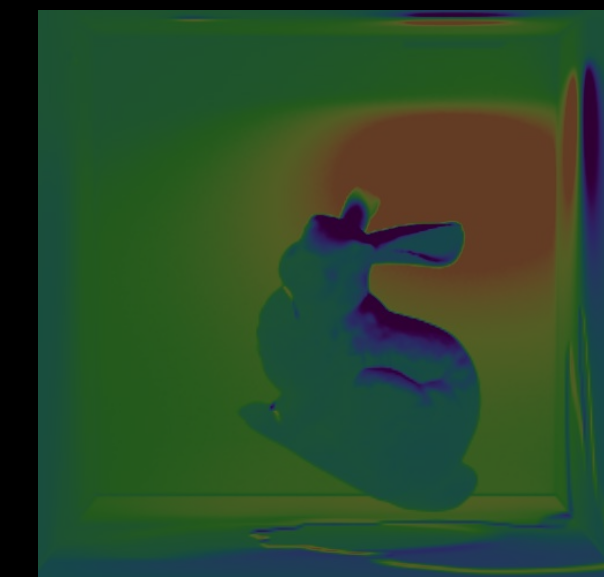
ultrafast light  
scanning



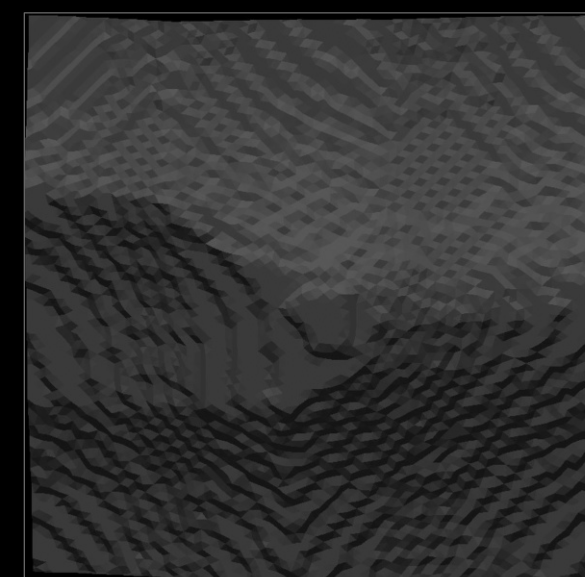
speckle  
imaging



tactile sensor  
design



differentiable  
renderer

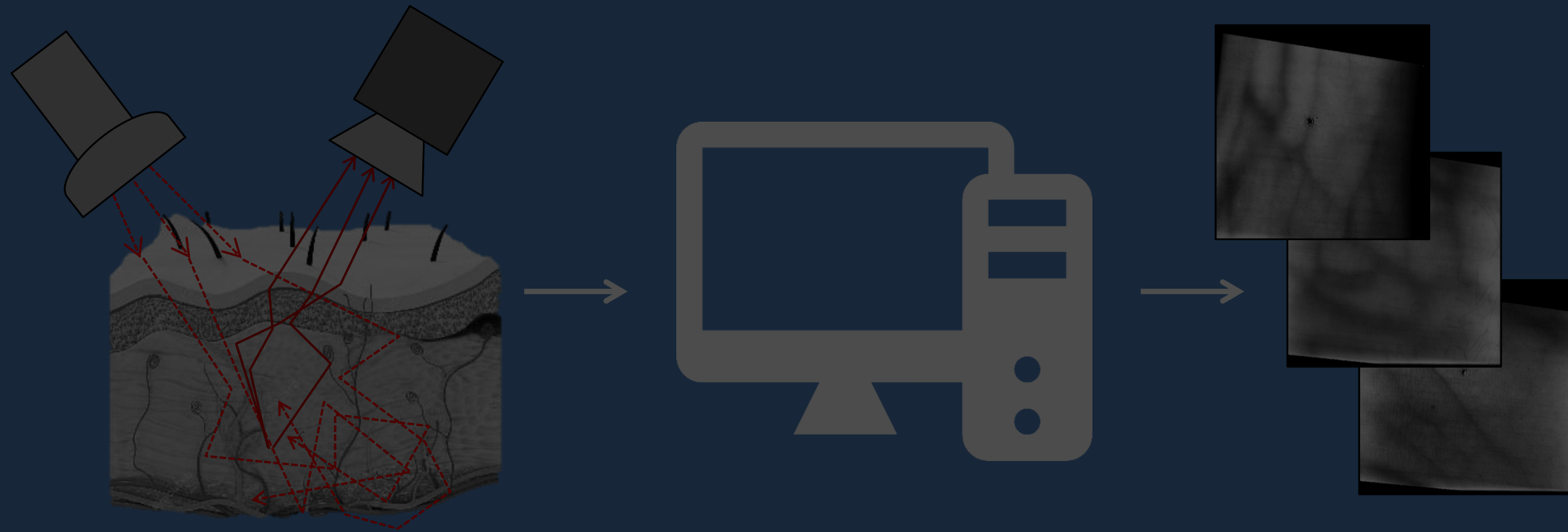


inverse  
problems



# Physics-based rendering and its applications to computational imaging

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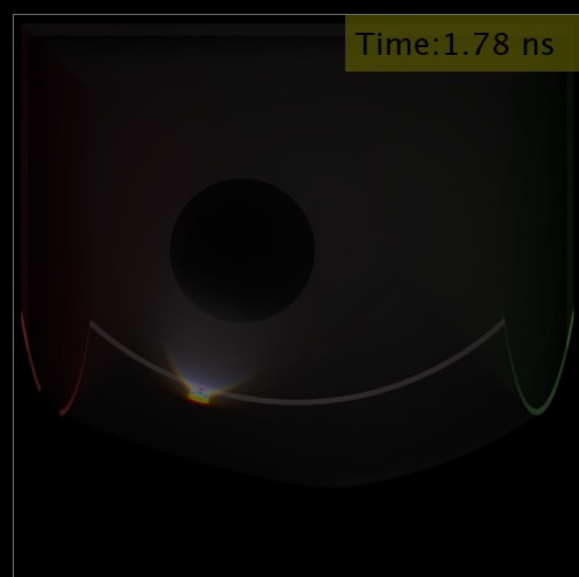


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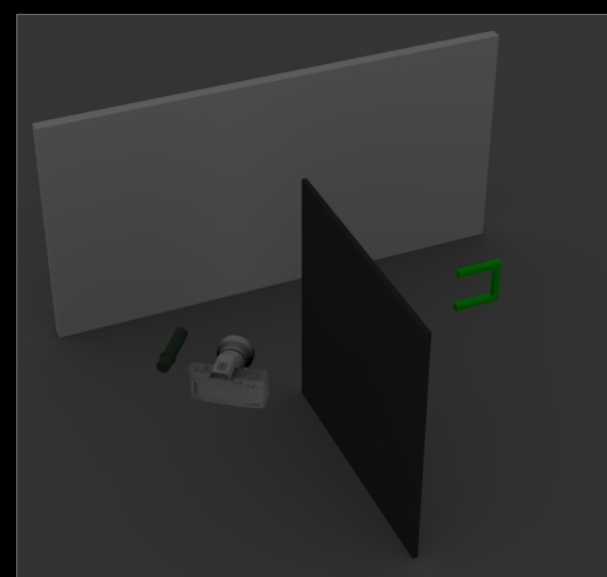
## inverse rendering



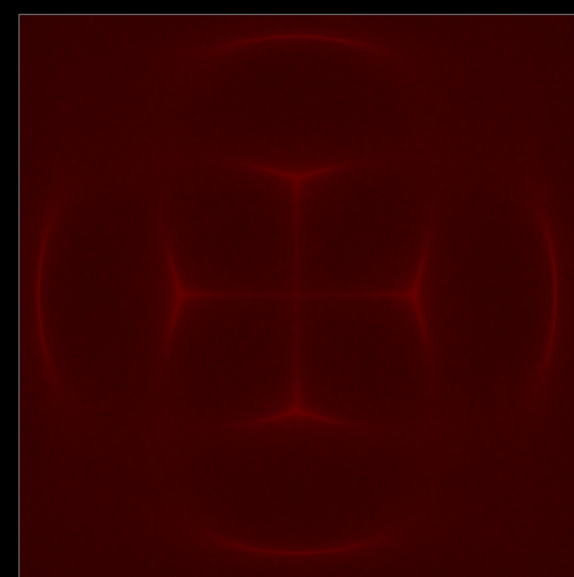
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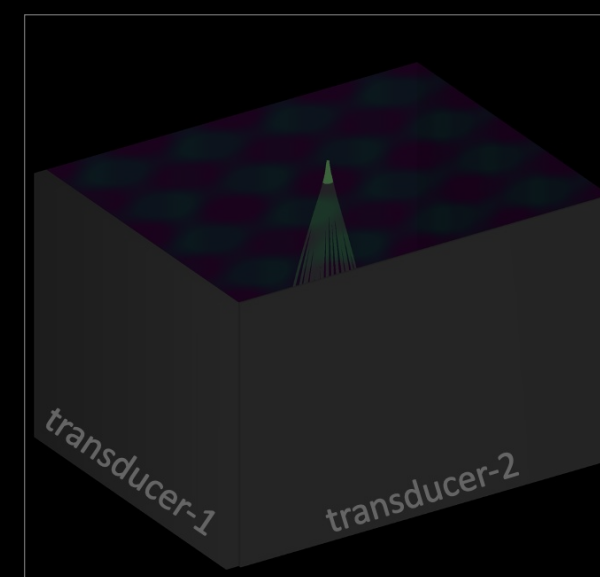
time-of-flight  
imaging



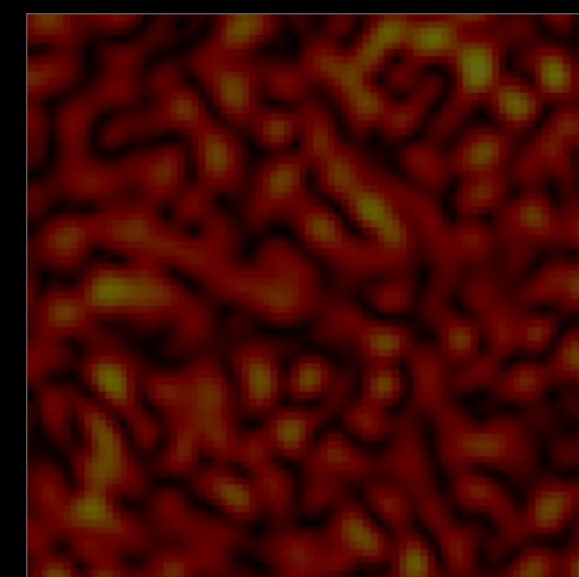
non-line-of-sight  
imaging



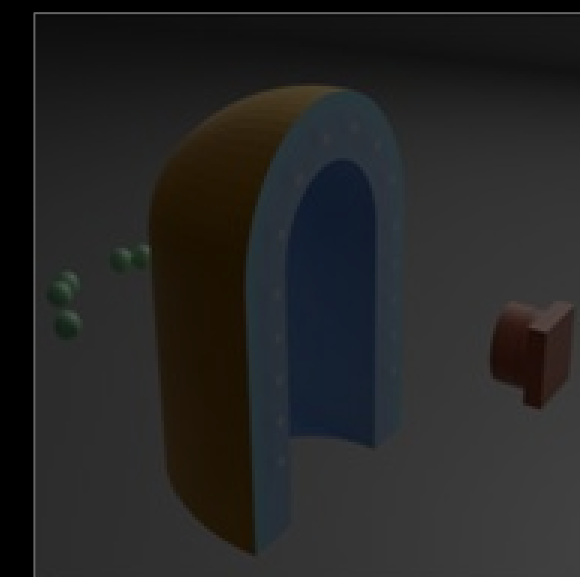
acousto-optic  
lensing



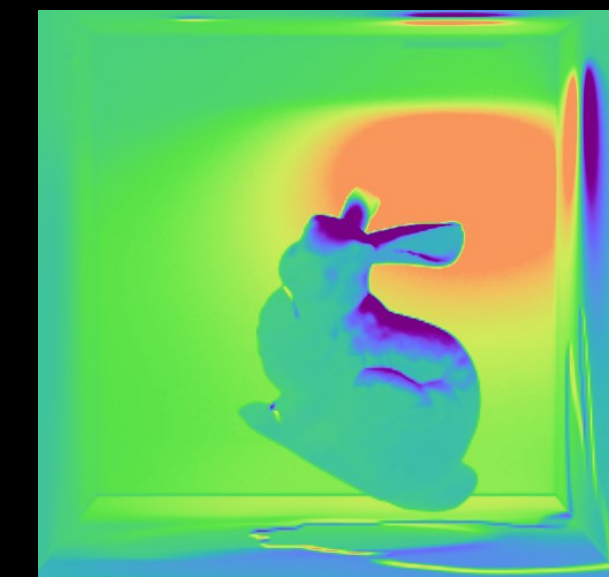
ultrafast light  
scanning



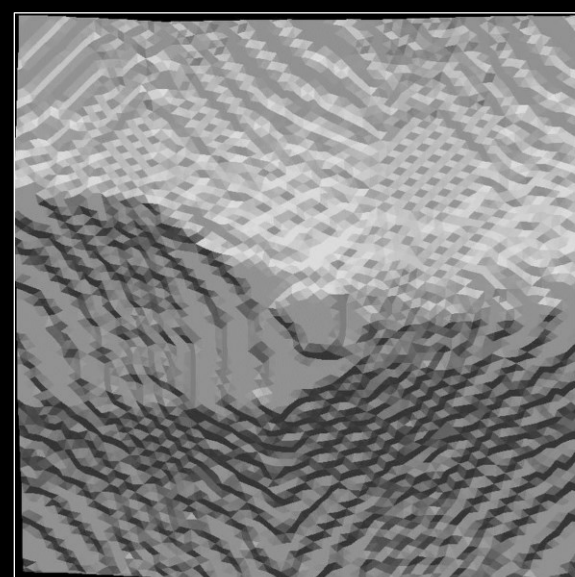
speckle  
imaging



tactile sensor  
design



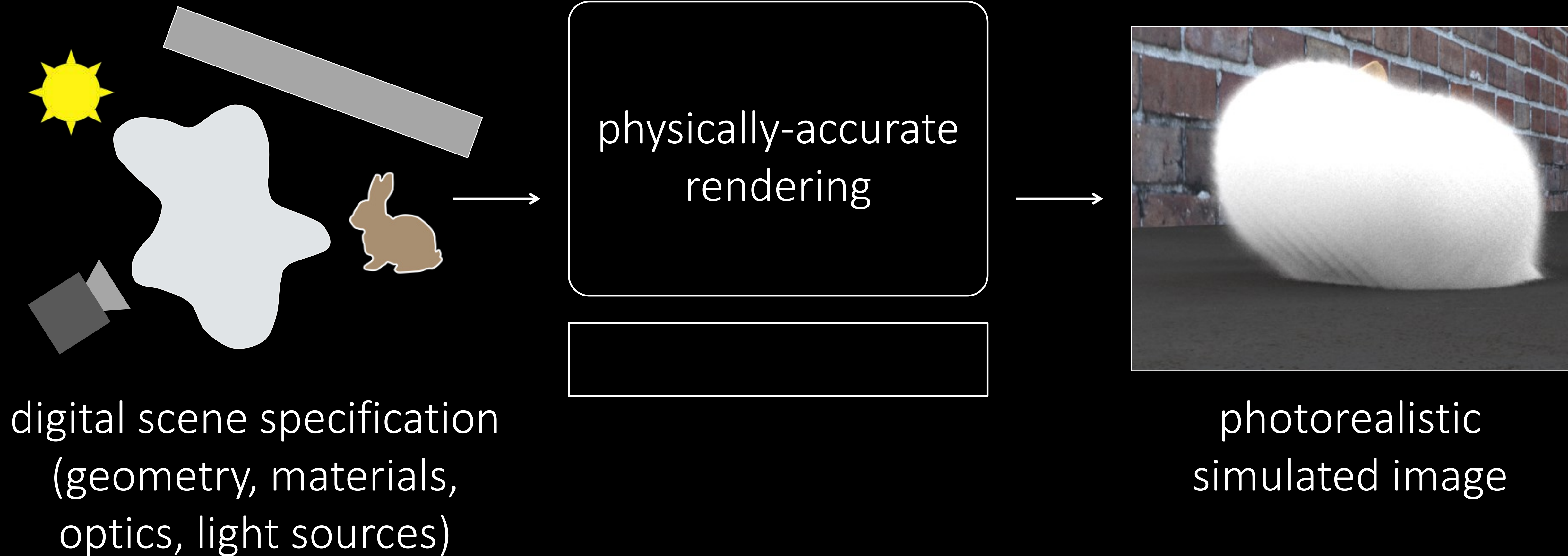
differentiable  
renderer



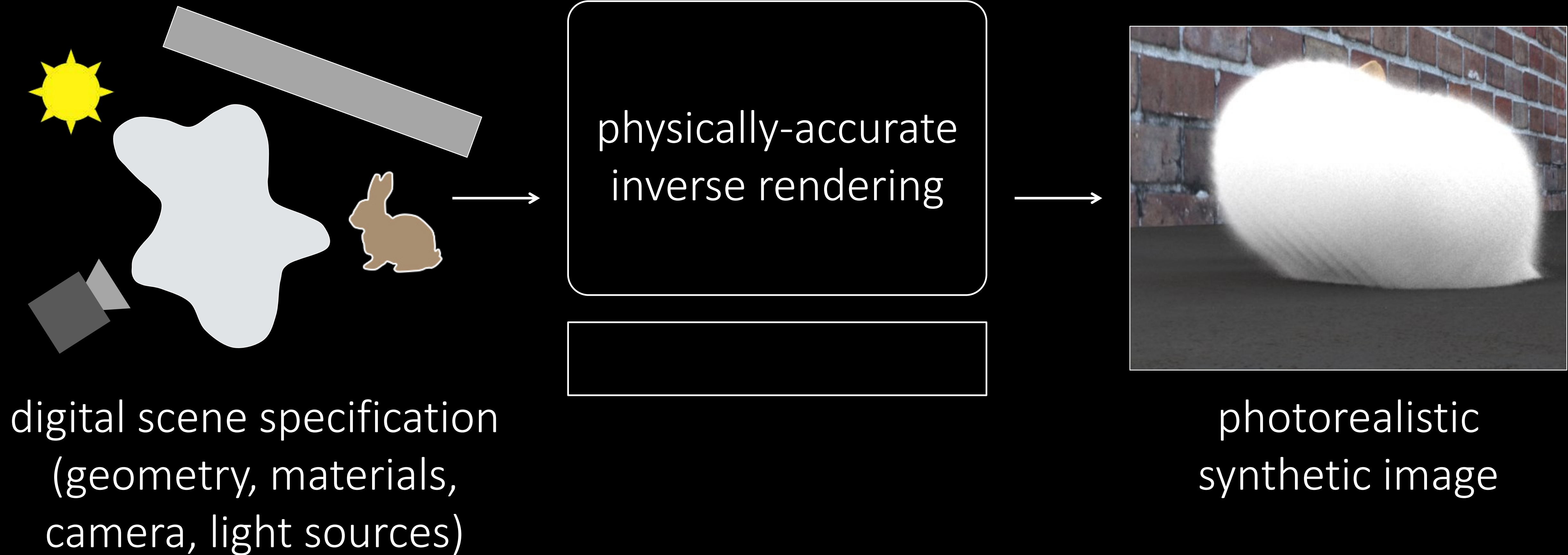
inverse  
problems



# Forward rendering

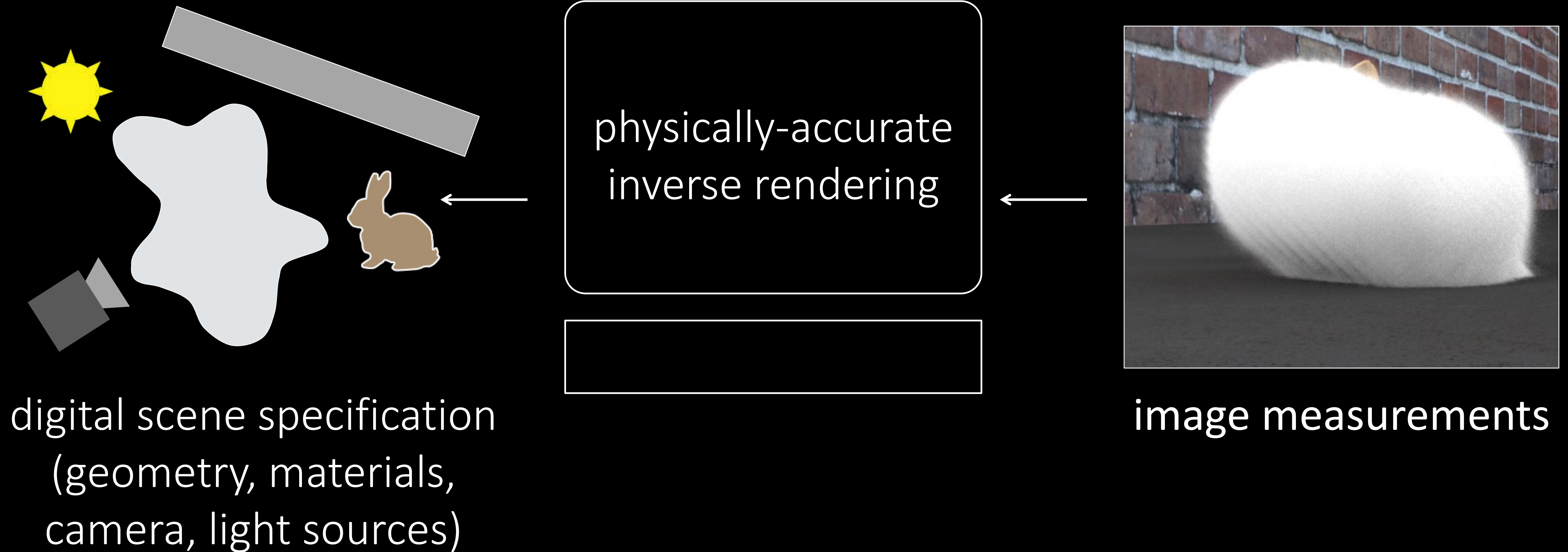


# Inverse rendering

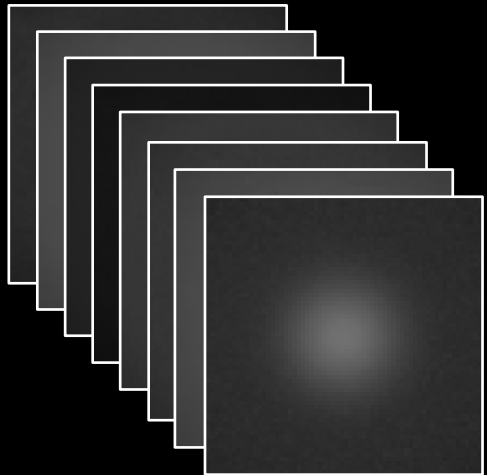




# Inverse rendering



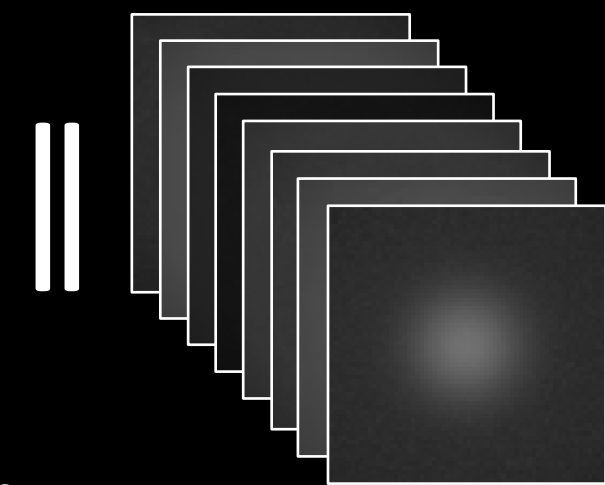
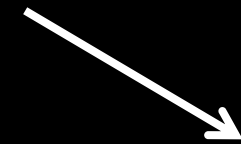
# Analysis by synthesis (a.k.a. inverse rendering)

$$\min_{\substack{\text{unknowns } m \\ \text{(tissue properties)}}} \| \text{stack of images} - \text{image}(m) \|^2$$




# Analysis by synthesis (a.k.a. inverse rendering)

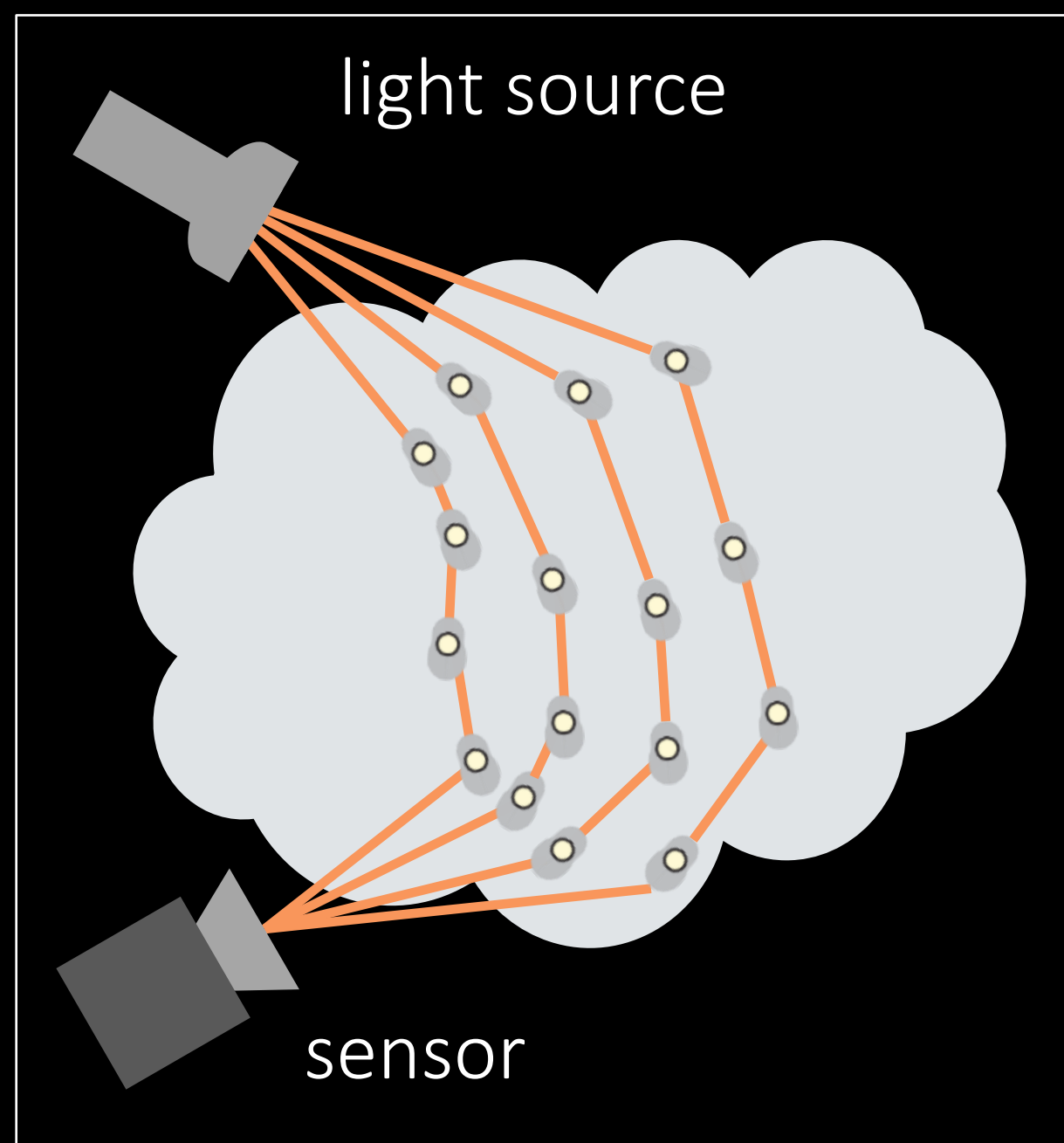
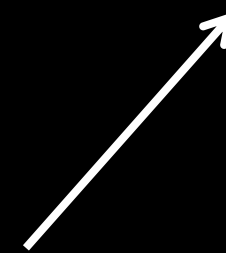
captured  
measurements



min  
unknowns  $m$   
(tissue properties)

$\|$

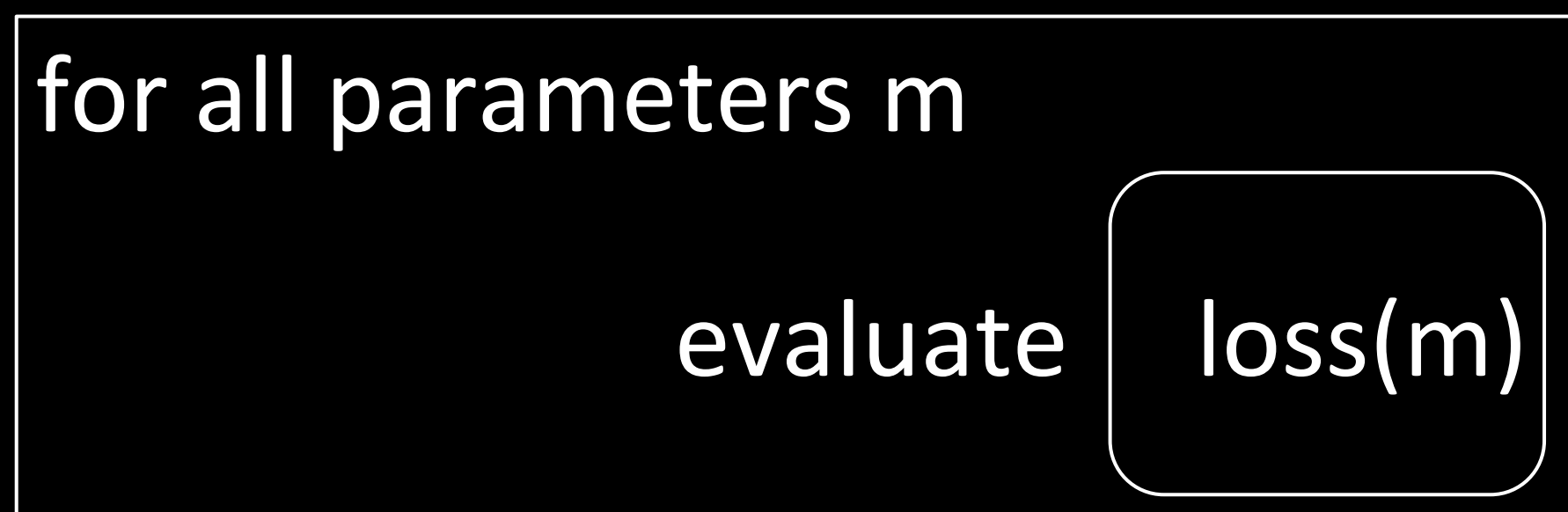
$- \text{image}(m) \|^2$



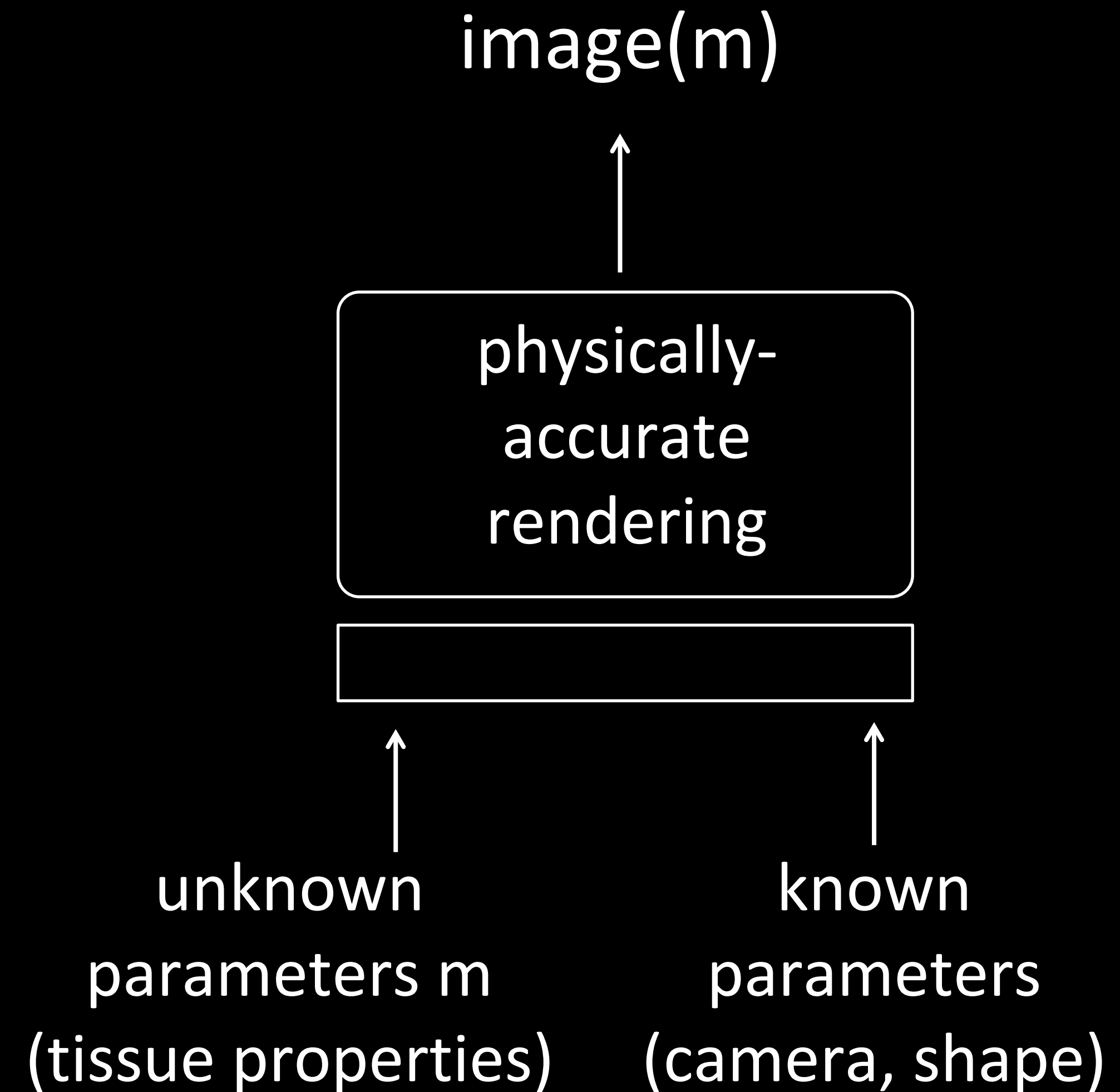
# Analysis by synthesis (a.k.a. inverse rendering)

$$\min_{\text{unknowns } m \text{ (tissue properties)}} \| \text{stack of images} - \text{image}(m) \|^2$$

solve with exhaustive search



← computed with rendering





# Analysis by synthesis (a.k.a. inverse rendering)

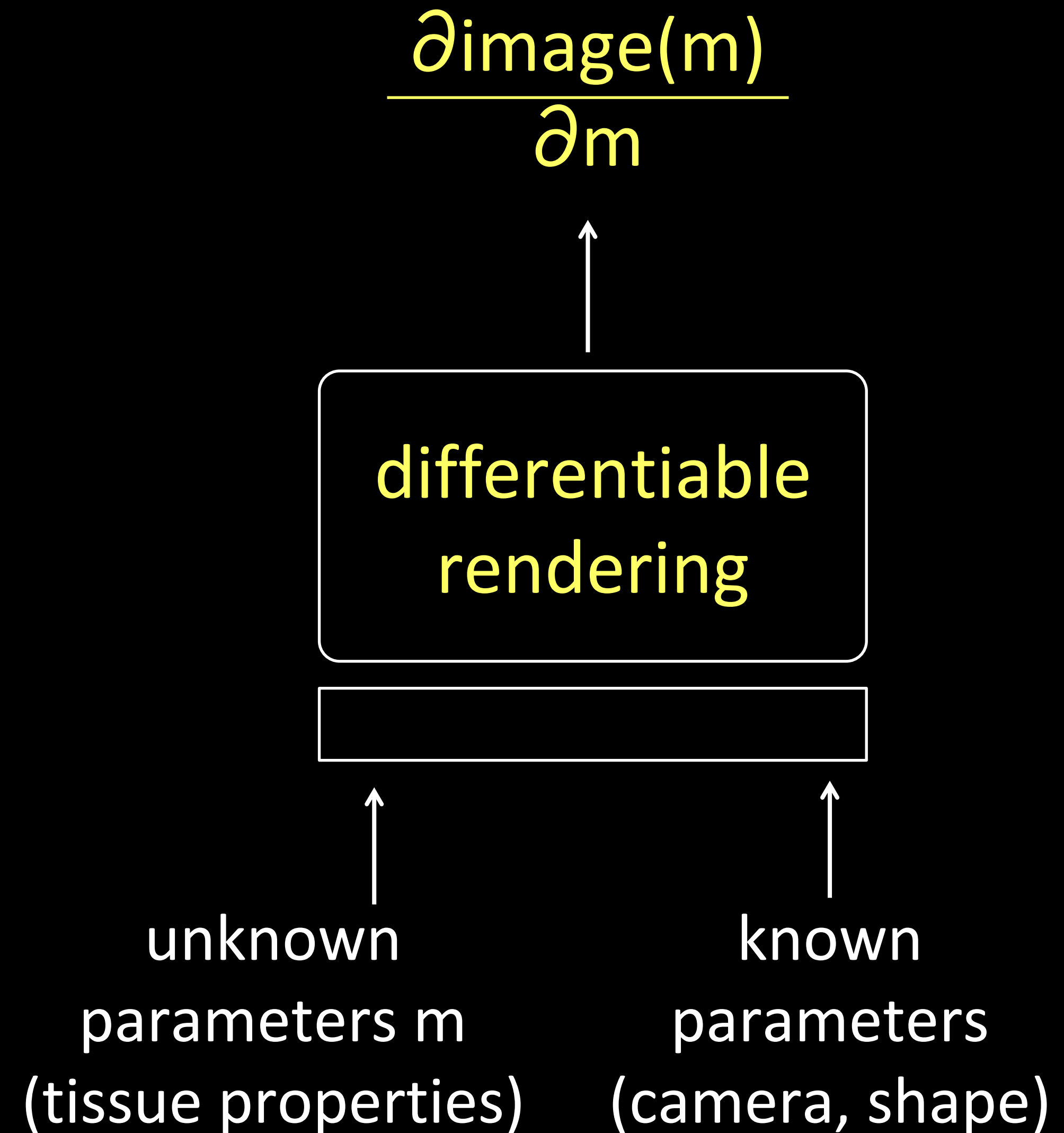
$$\min_{\text{unknowns } m \text{ (tissue properties)}} \| \text{stack of images} - \text{image}(m) \|^2$$

solve with gradient descent

while (not converged)


update  $m$  with  $\frac{\partial \text{loss}(m)}{\partial m}$

← computed with  
differentiable  
rendering



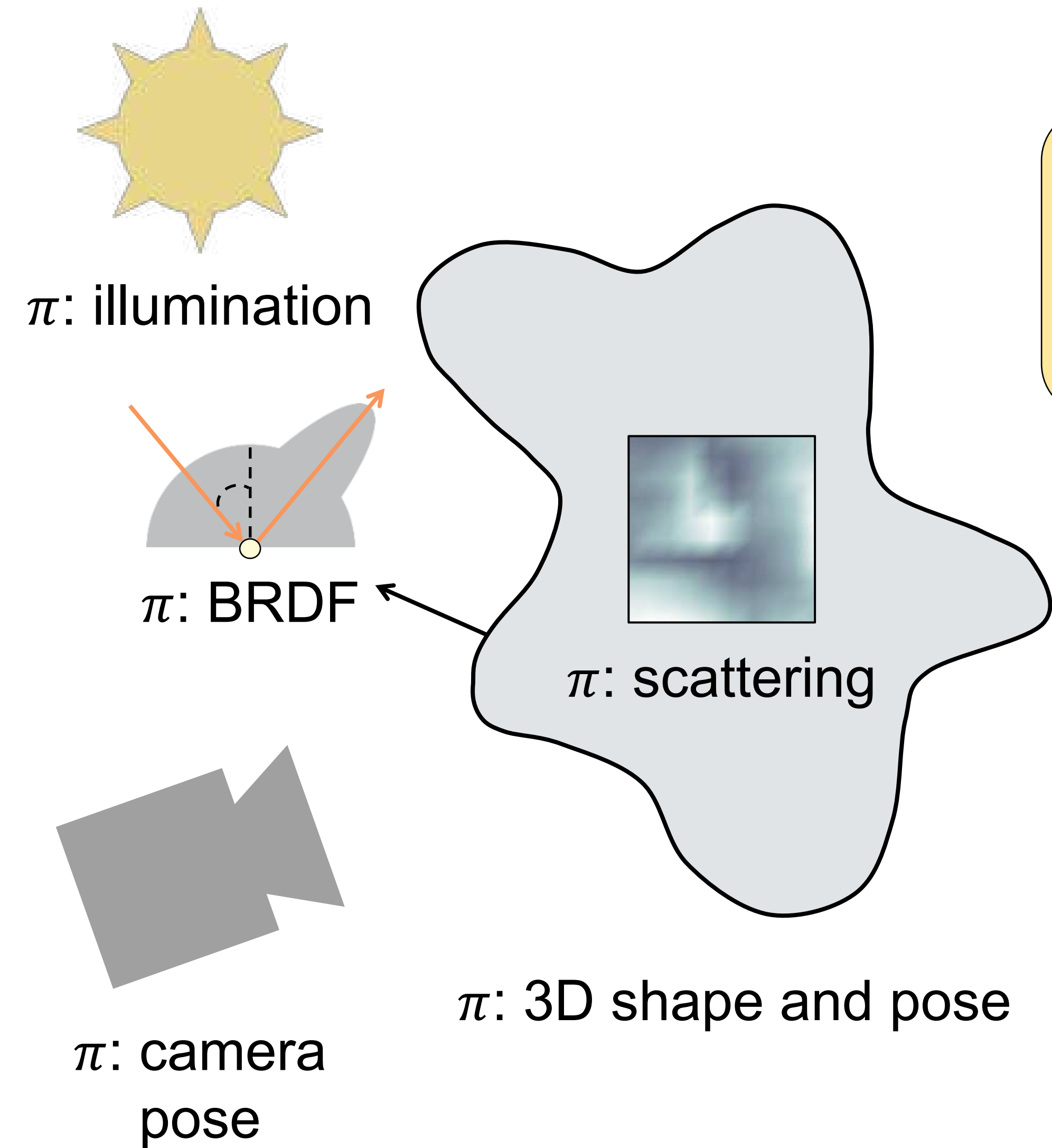
# Analysis by synthesis (a.k.a. inverse rendering)

Analysis-by-synthesis optimization:

$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{img}, \text{render} \left( \begin{matrix} \text{scene} \\ \text{unknowns } \pi \end{matrix} \right) \right]$$




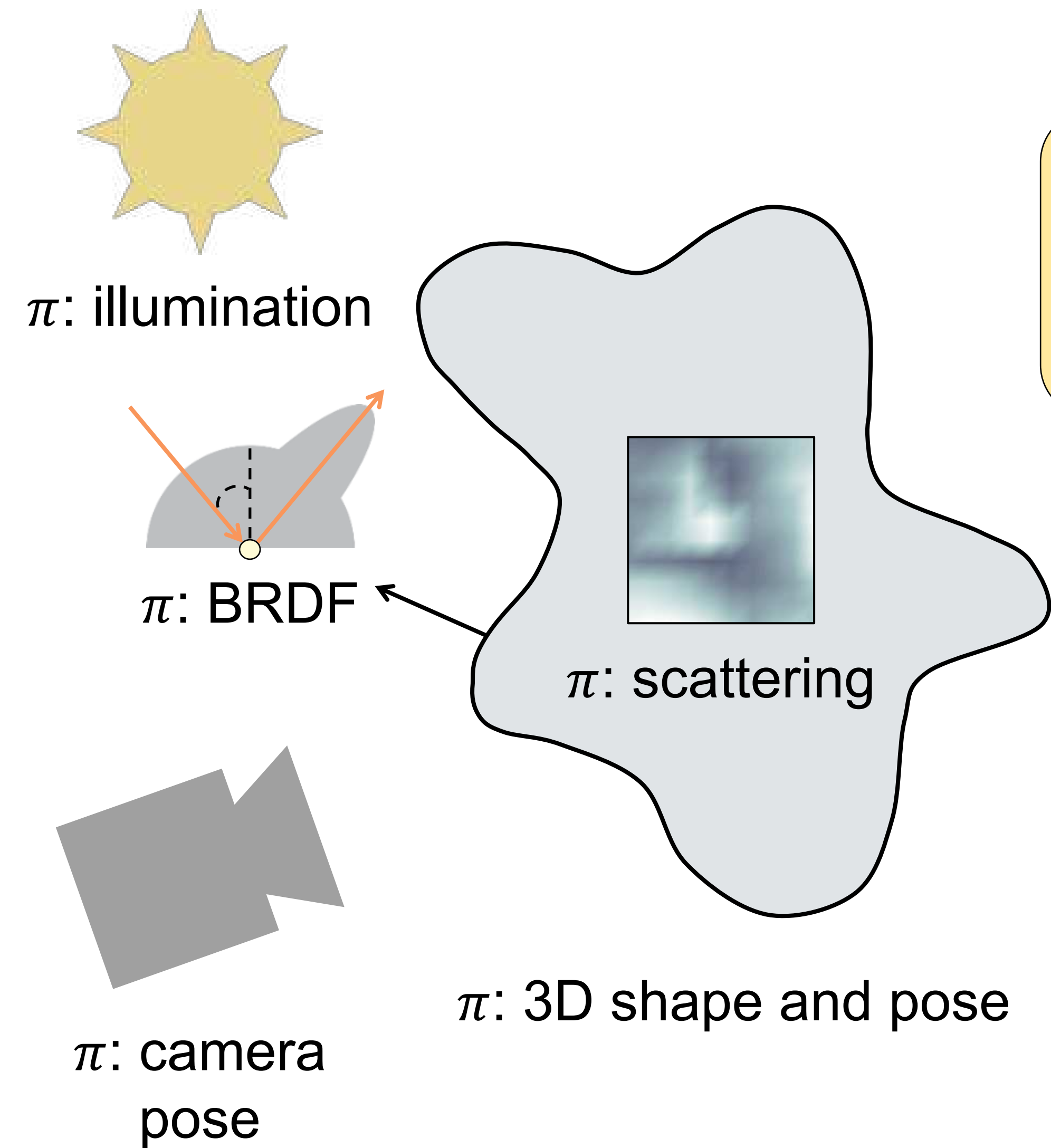
# Analysis by synthesis (a.k.a. inverse rendering)



Analysis-by-synthesis optimization:

$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{image of pumpkin}, \text{render} \left( \begin{matrix} \text{scene} \\ \text{unknowns } \pi \end{matrix} \right) \right]$$

# Analysis by synthesis (a.k.a. inverse rendering)



Analysis-by-synthesis optimization:

$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{scene}, \text{render}(\text{unknowns } \pi) \right]$$


Stochastic gradient descent (e.g., Adam):

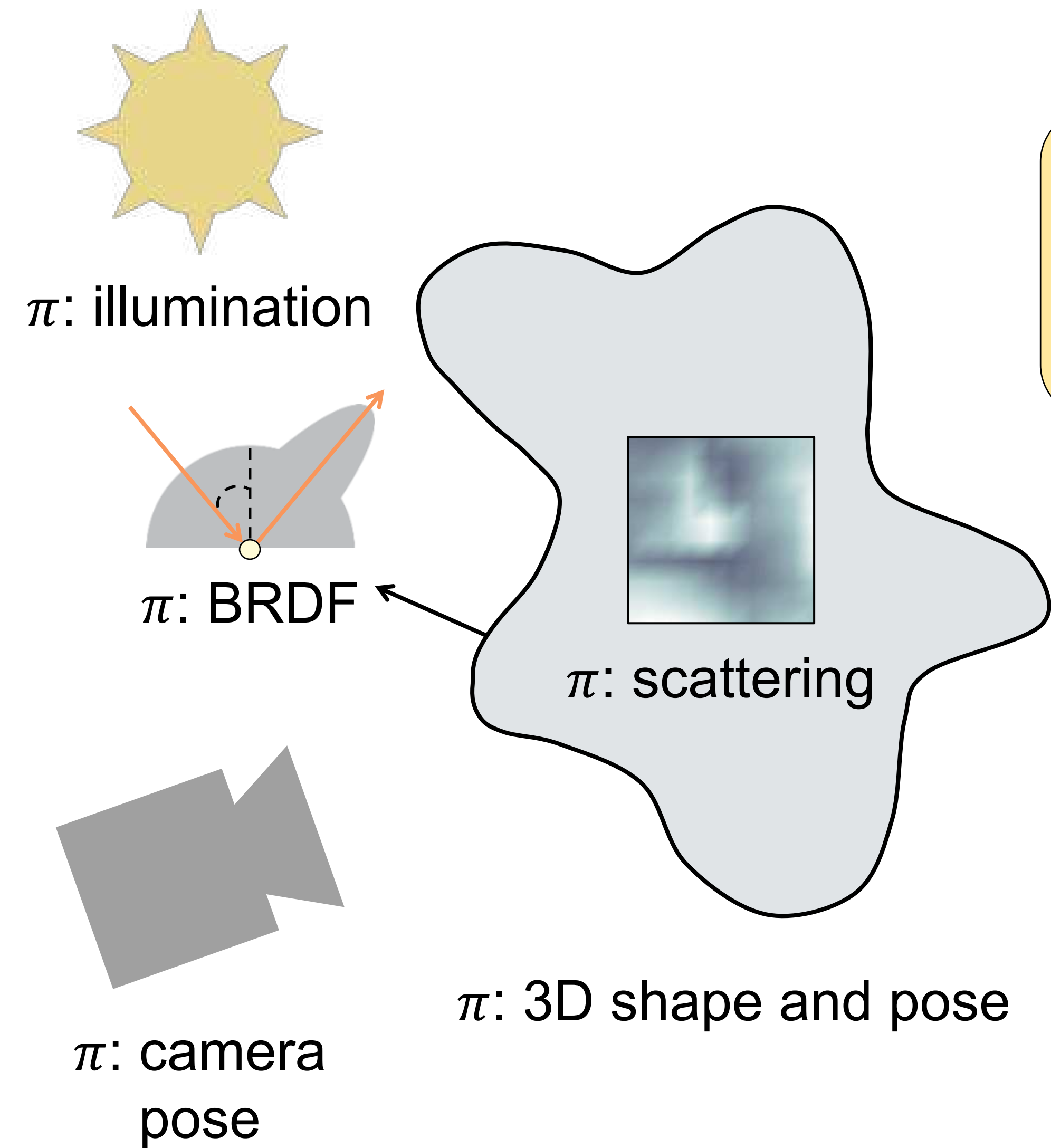
initialize  $\pi \leftarrow \pi_0$

while (not converged)

update  $\pi \leftarrow \pi + \eta \cdot \frac{d\text{loss}(\pi)}{d\pi}$



# Analysis by synthesis (a.k.a. inverse rendering)



Analysis-by-synthesis optimization:

$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{reference image}, \text{render} \left( \begin{matrix} \text{scene} \\ \text{unknowns } \pi \end{matrix} \right) \right]$$

Stochastic gradient descent (e.g., Adam):

initialize  $\pi \leftarrow \pi_0$

while (not converged)

update  $\pi \leftarrow \pi + \eta \cdot \frac{d\text{loss}(\pi)}{d\pi}$

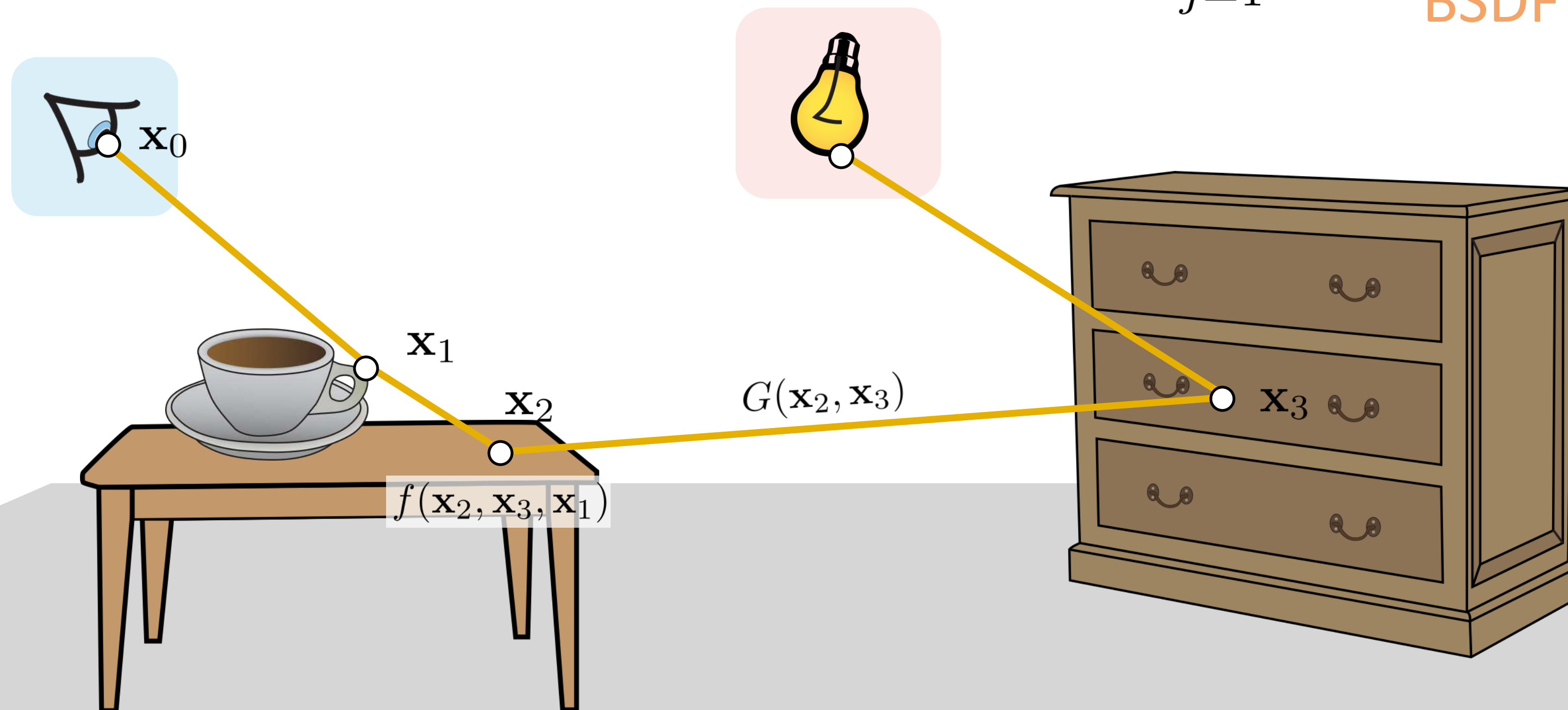
Differentiable  
rendering

# How do we differentiate light transport?

$$\text{image } I = \int_{\mathcal{P}} \overset{\text{sensor weight}}{W_e(\mathbf{x}_0, \mathbf{x}_1)} \overset{\text{source weight}}{L_e(\mathbf{x}_k, \mathbf{x}_{k-1})} \overset{\text{light path}}{T(\bar{\mathbf{x}})} d\bar{\mathbf{x}} \quad \text{path throughput}$$

space of all light paths

$$T(\bar{\mathbf{x}}) = G(\mathbf{x}_0, \mathbf{x}_1) \prod_{j=1}^{k-1} \overset{\text{BSDF}}{f(\mathbf{x}_j, \mathbf{x}_{j+1}, \mathbf{x}_{j-1})} \overset{\text{geometry}}{G(\mathbf{x}_j, \mathbf{x}_{j+1})}$$



# REMINDER (?) FROM CALCULUS



# Reminder from calculus

$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx \stackrel{?}{=}$$

# Reminder from calculus

$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx \stackrel{?}{=}$$

# Reminder from calculus

## Differentiation under the integral sign

Also known as the Leibniz integral rule

$$\frac{d}{d\pi} \int_{a(\pi)}^b f(x, \pi) dx = ?$$



# Reminder from calculus

## Differentiation under the integral sign

Also known as the Leibniz integral rule

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) \mathrm{d}x &= \int_{a(\pi)}^{b(\pi)} \frac{\mathrm{d}}{\mathrm{d}\pi} f(x, \pi) \mathrm{d}x \\ &+ f(b(\pi), \pi) \frac{\mathrm{d}b(\pi)}{\mathrm{d}\pi} - f(a(\pi), \pi) \frac{\mathrm{d}a(\pi)}{\mathrm{d}\pi} \\ &+ \sum_i \left( f(c_i(\pi)^-, \pi) - f(c_i(\pi)^+, \pi) \right) \frac{\mathrm{d}c_i(\pi)}{\mathrm{d}\pi} \end{aligned}$$

# Reminder from calculus

## Differentiation under the integral sign

Also known as the Leibniz integral rule

$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx = \int_{a(\pi)}^{b(\pi)} \frac{d}{d\pi} f(x, \pi) dx$$

Move derivative  
inside integral

$$+ f(b(\pi), \pi) \frac{db(\pi)}{d\pi} - f(a(\pi), \pi) \frac{da(\pi)}{d\pi}$$

$$+ \sum_i (f(c_i(\pi)^-, \pi) - f(c_i(\pi)^+, \pi)) \frac{dc_i(\pi)}{d\pi}$$

# Reminder from calculus

**Differentiation under the integral sign**

Also known as the Leibniz integral rule

$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx = \int_{a(\pi)}^{b(\pi)} \frac{d}{d\pi} f(x, \pi) dx$$

Move derivative  
inside integral

Account for changes in  
integration limits

$$+ f(b(\pi), \pi) \frac{db(\pi)}{d\pi} - f(a(\pi), \pi) \frac{da(\pi)}{d\pi}$$

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$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx = \int_{a(\pi)}^{b(\pi)} \frac{d}{d\pi} f(x, \pi) dx$$

Move derivative inside integral

Account for changes in integration limits

$$+ f(b(\pi), \pi) \frac{db(\pi)}{d\pi} - f(a(\pi), \pi) \frac{da(\pi)}{d\pi}$$

Account for discontinuities of integrand that depend on  $\pi$

$$+ \sum_i \left( f(c_i(\pi)^-, \pi) - f(c_i(\pi)^+, \pi) \right) \frac{dc_i(\pi)}{d\pi}$$

# A simple example

$$f(x, \pi) = \begin{cases} 0 & \text{if } x < 2\pi \\ 1 & \text{if } x \geq 2\pi \end{cases}$$

$$\frac{\mathrm{d}}{\mathrm{d}\pi} \int_0^{4\pi} f(x, \pi) \mathrm{d}x =$$

# A simple example

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$$f(x, \pi) = \begin{cases} 0 & \text{if } x < 2\pi \\ 1 & \text{if } x \geq 2\pi \end{cases}$$

$$\frac{d}{d\pi} \int_0^{4\pi} f(x, \pi) dx = \int_0^{2\pi} \frac{d}{d\pi} 0 dx + \int_{2\pi}^{4\pi} \frac{d}{d\pi} 1 dx \quad \begin{array}{l} \text{Move derivative} \\ \text{inside integral} \end{array}$$

# A simple example

$$f(x, \pi) = \begin{cases} 0 & \text{if } x < 2\pi \\ 1 & \text{if } x \geq 2\pi \end{cases}$$

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Account for changes in integration limits

$$+ 1 \frac{d(4\pi)}{d\pi} - 0 \frac{d0}{d\pi}$$

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Account for changes in  
integration limits

$$+ 1 \frac{d(4\pi)}{d\pi} - 0 \frac{d0}{d\pi}$$

Account for discontinuities of  
integrand that depend on  $\pi$

$$+ (0 - 1) \frac{d(2\pi)}{d\pi}$$



# Leibniz integral rule

**Differentiation under the integral sign**

Also known as the Leibniz integral rule

$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx$$

$$= \int_{a(\pi)}^{b(\pi)} \frac{d}{d\pi} f(x, \pi) dx$$

Move derivative  
inside integral

Account for changes in  
integration limits

$$+ f(b(\pi), \pi) \frac{db(\pi)}{d\pi} - f(a(\pi), \pi) \frac{da(\pi)}{d\pi}$$

Account for discontinuities of  
integrand that depend on  $\pi$

$$+ \sum_i \left( f(c_i(\pi)^-, \pi) - f(c_i(\pi)^+, \pi) \right) \frac{dc_i(\pi)}{d\pi}$$

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Move derivative  
inside integral

Account for changes in  
integration limits

$$+ f(b(\pi), \pi) \frac{db(\pi)}{d\pi} - f(a(\pi), \pi) \frac{da(\pi)}{d\pi}$$

Account for discontinuities of  
integrand that depend on  $\pi$

$$+ \sum_i \left( f(c_i(\pi)^-, \pi) - f(c_i(\pi)^+, \pi) \right) \frac{dc_i(\pi)}{d\pi}$$

# Leibniz integral rule

**Differentiation under the integral sign**

Also known as the Leibniz integral rule

$$\frac{d}{d\pi} \int_{a(\pi)}^{b(\pi)} f(x, \pi) dx$$

=

Interior integral

$$\int_{a(\pi)}^{b(\pi)} \frac{d}{d\pi} f(x, \pi) dx$$

Move derivative  
inside integral

Account for changes in  
integration limits

Boundary terms

$$+ f(b(\pi), \pi) \frac{db(\pi)}{d\pi} - f(a(\pi), \pi) \frac{da(\pi)}{d\pi}$$

Account for discontinuities of  
integrand that depend on  $\pi$

$$+ \sum_i \left( f(c_i(\pi)^-, \pi) - f(c_i(\pi)^+, \pi) \right) \frac{dc_i(\pi)}{d\pi}$$



# Reynolds transport theorem

$$\frac{d}{d\pi} \int_{\Omega(\pi)} f(x, \pi) dA(x) \stackrel{?}{=}$$

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**Reynolds transport theorem [1903]**

Generalization of the Leibniz rule



# Reynolds transport theorem

$$\frac{d}{d\pi} \int_{\Omega(\pi)} f(x, \pi) dA(x) = \int_{\Omega(\pi)} \frac{df(x, \pi)}{d\pi} dA(x) + \int_{\partial\Omega(\pi)} g(x, \pi) dl(x)$$

**Reynolds transport theorem [1903]**

Generalization of the Leibniz rule

# Reynolds transport theorem

$$\frac{d}{d\pi} \int_{\Omega(\pi)} f(x, \pi) dA(x) = \int_{\Omega(\pi)} \frac{df(x, \pi)}{d\pi} dA(x) + \int_{\partial\Omega(\pi)} g(x, \pi) dl(x)$$

Boundary domain

**Reynolds transport theorem [1903]**

Generalization of the Leibniz rule

||  
discontinuity points  $\cup$  boundary of domain  $\Omega$   
(if they depend on  $\pi$ )

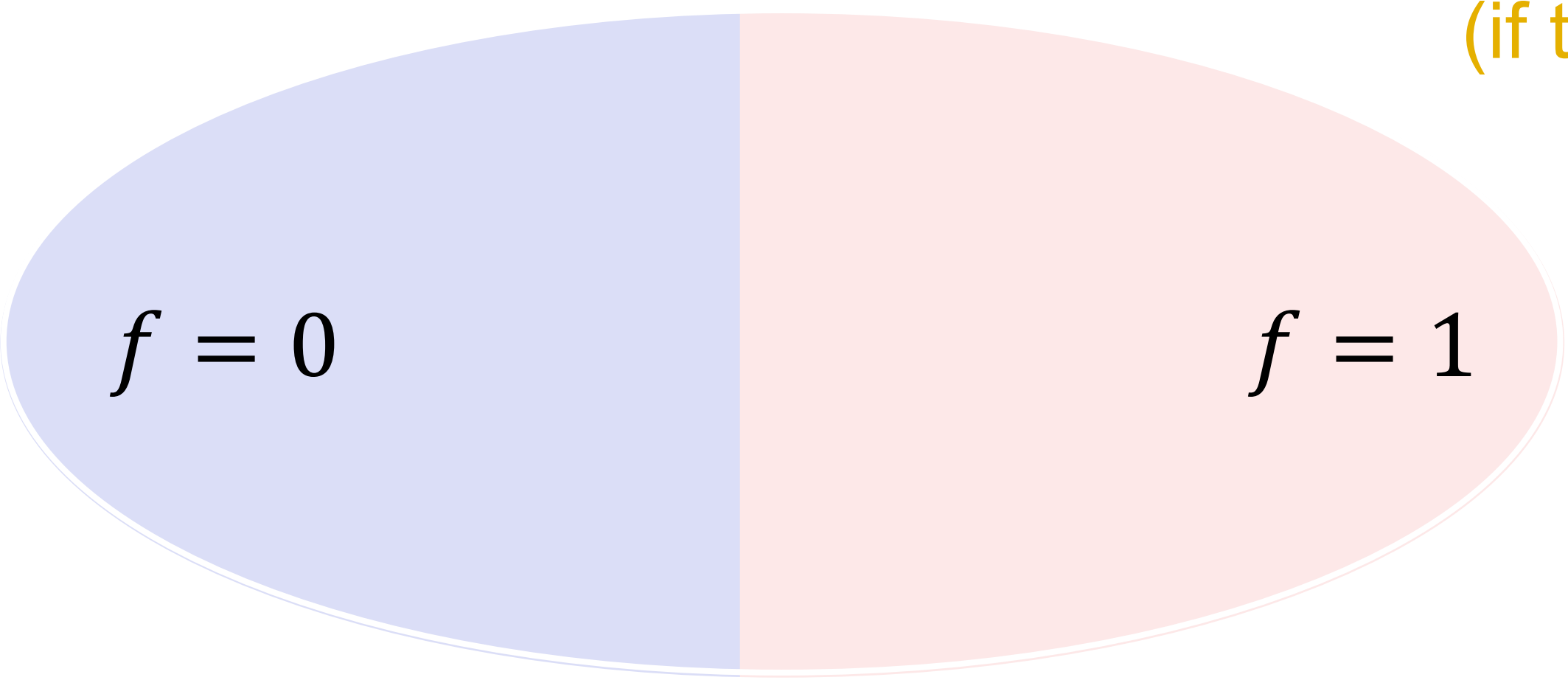
# Reynolds transport theorem

$$\frac{d}{d\pi} \int_{\Omega(\pi)} f(x, \pi) dA(x) = \int_{\Omega(\pi)} \frac{df(x, \pi)}{d\pi} dA(x) + \int_{\partial\Omega(\pi)} g(x, \pi) dl(x)$$

Boundary domain

**Reynolds transport theorem [1903]**  
Generalization of the Leibniz rule

discontinuity points  $\cup$  boundary of domain  $\Omega$   
(if they depend on  $\pi$ )



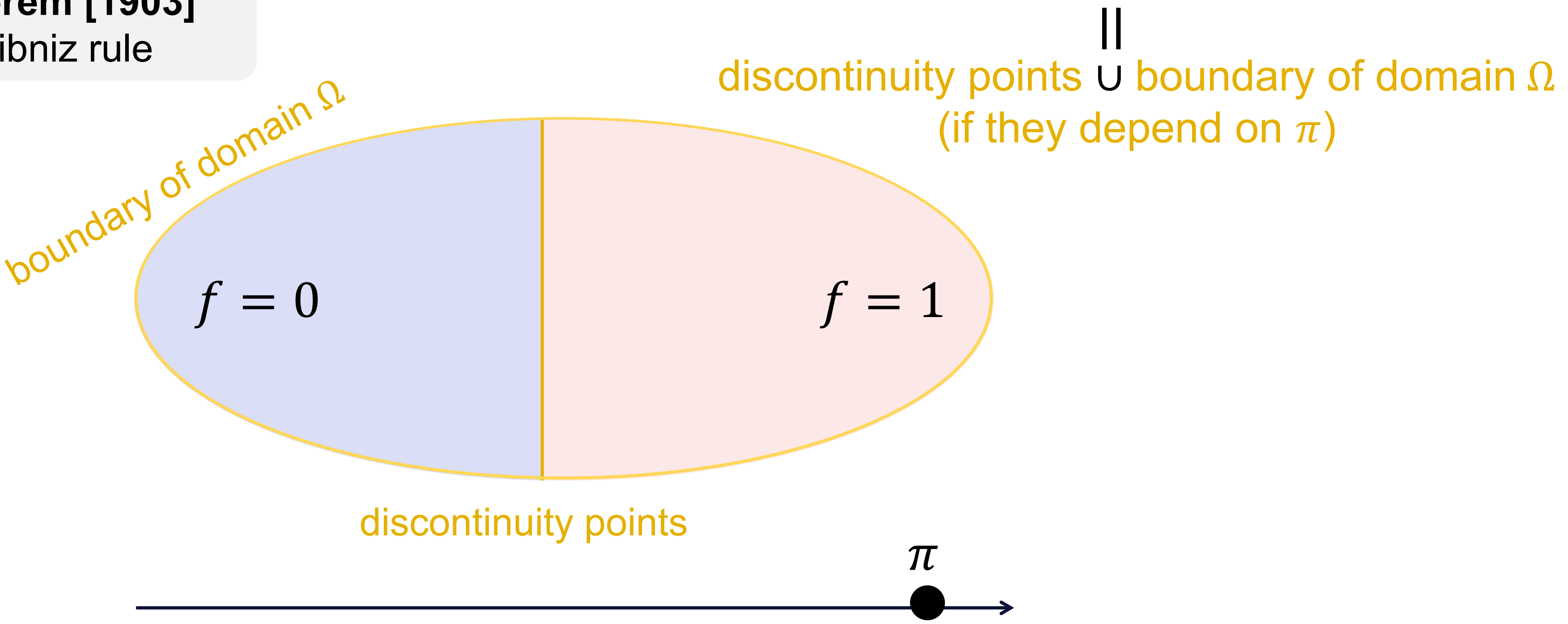


# Reynolds transport theorem

$$\frac{d}{d\pi} \int_{\Omega(\pi)} f(x, \pi) dA(x) = \int_{\Omega(\pi)} \frac{df(x, \pi)}{d\pi} dA(x) + \int_{\partial\Omega(\pi)} g(x, \pi) dl(x)$$

Boundary domain

**Reynolds transport theorem [1903]**  
Generalization of the Leibniz rule



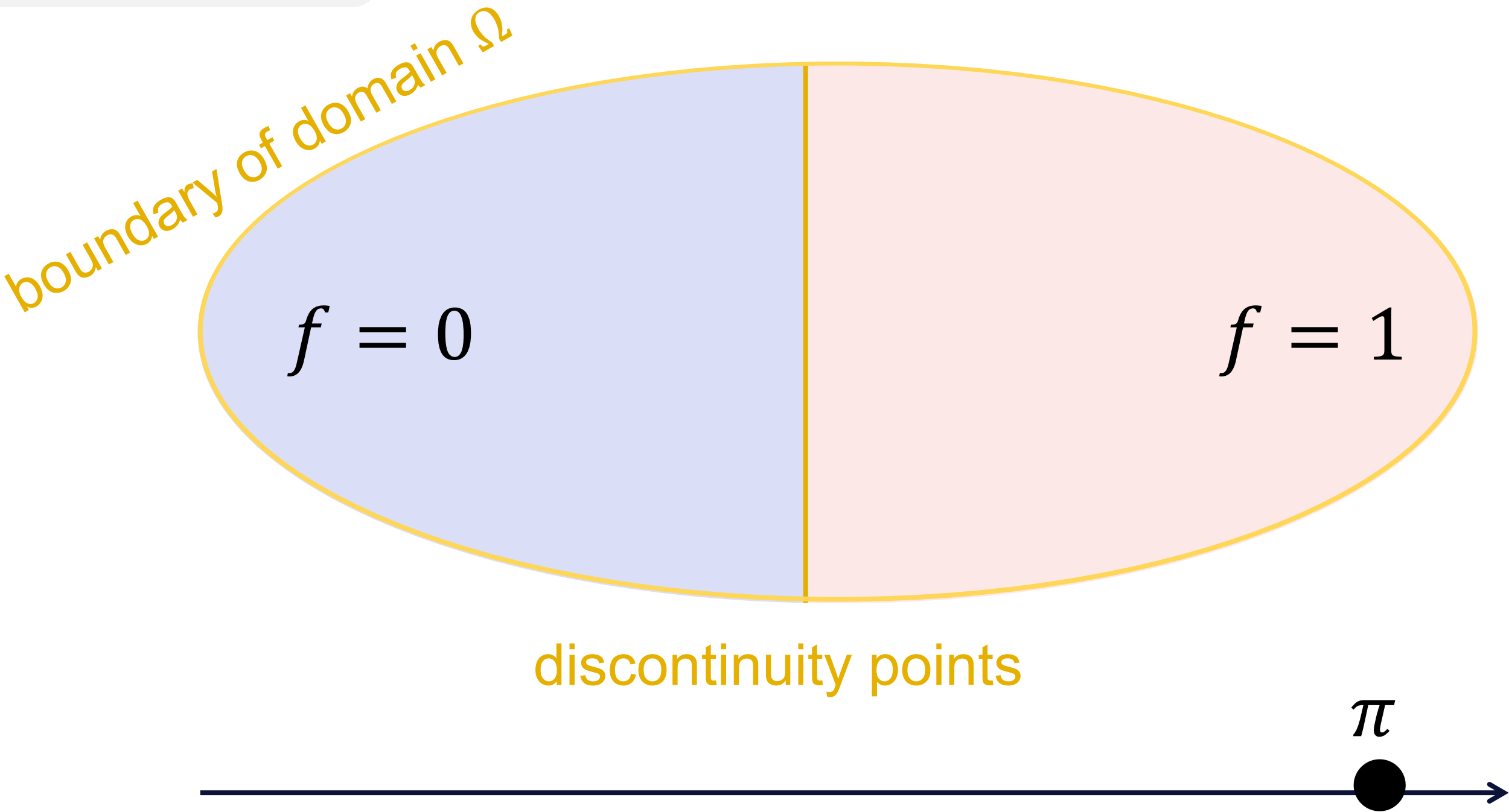
# Reynolds transport theorem

$$\frac{d}{d\pi} \int_{\Omega(\pi)} f(x, \pi) dA(x) = \int_{\Omega(\pi)} \frac{df(x, \pi)}{d\pi} dA(x) + \int_{\partial\Omega(\pi)} g(x, \pi) dl(x)$$

**Reynolds transport theorem [1903]**  
Generalization of the Leibniz rule

Interior integral

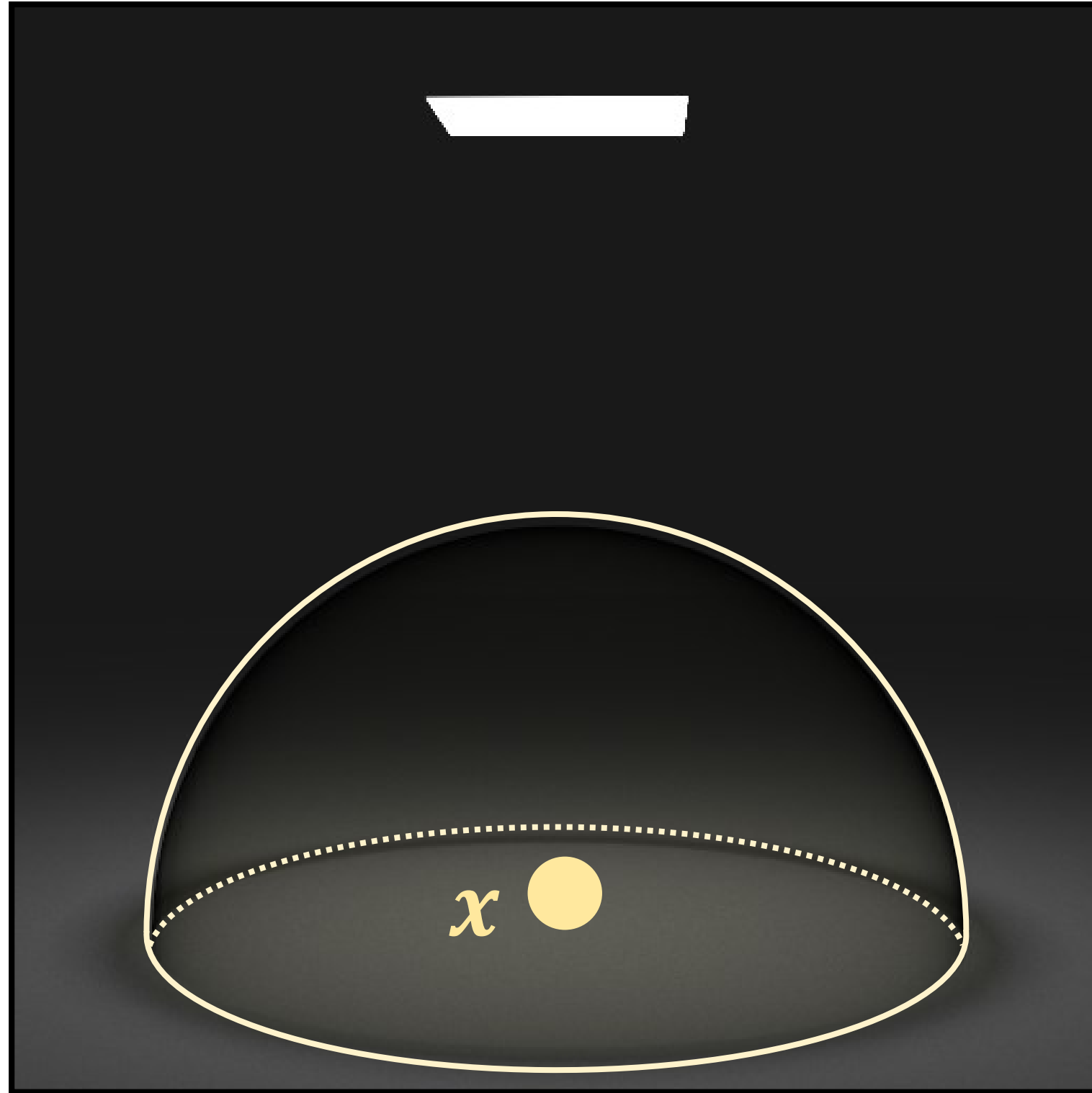
Boundary integral



# **DIFFERENTIATING DIRECT ILLUMINATION**



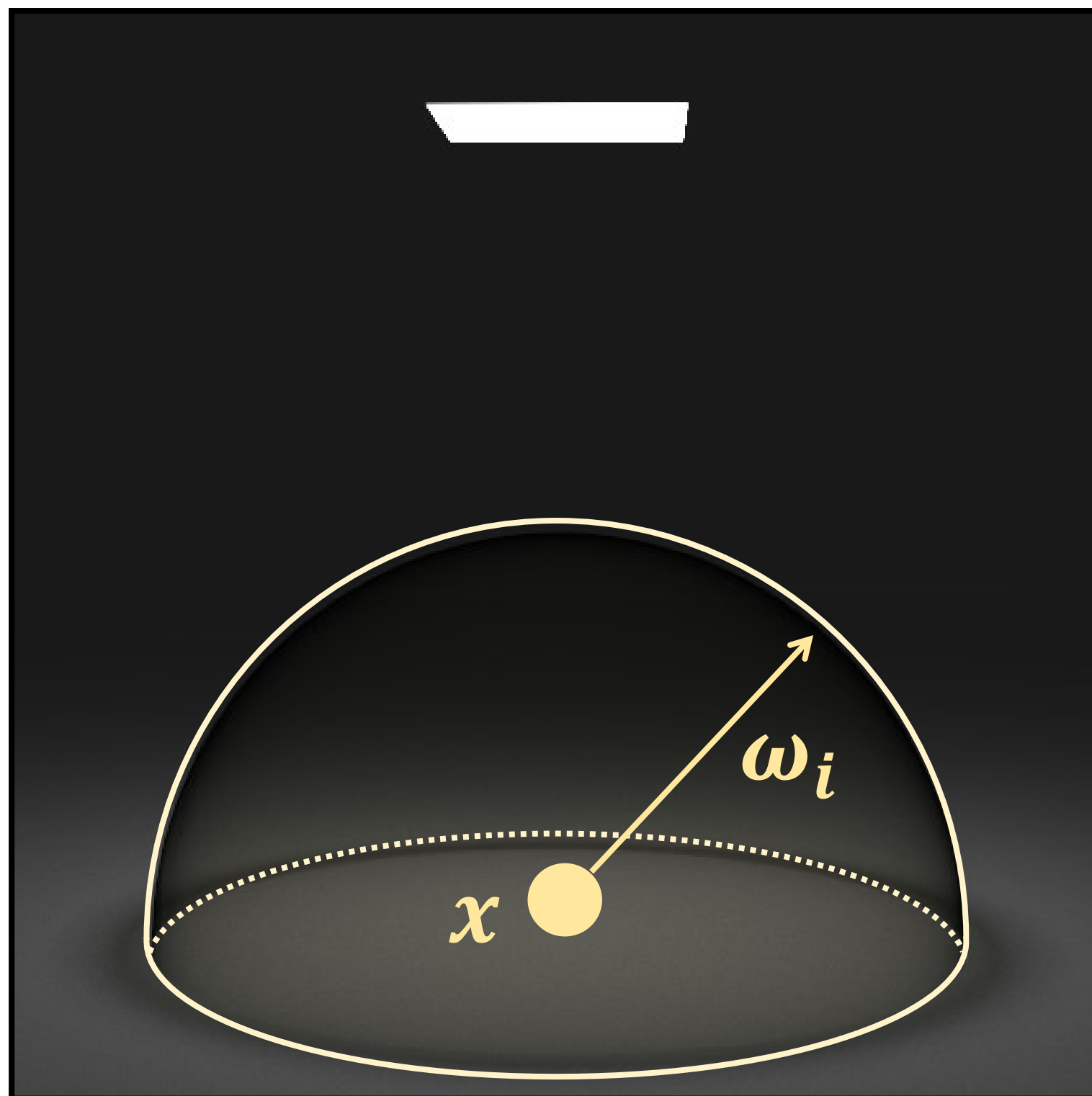
# Direct illumination integral



Radiance from  $x$ :

$$I = \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$

# Direct illumination integral

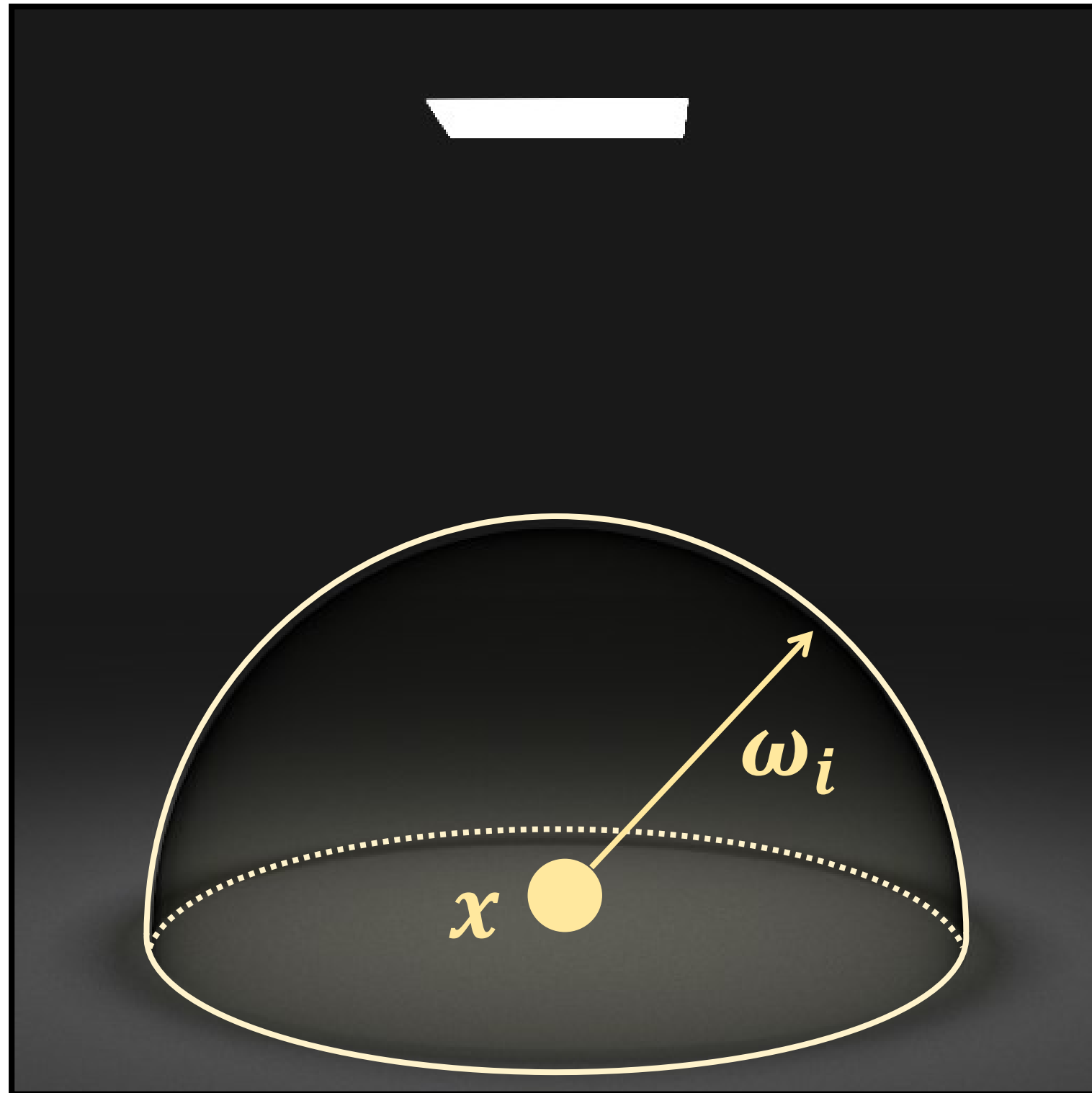


Radiance from  $x$ :

$$I = \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$

Unit hemisphere

# Direct illumination integral



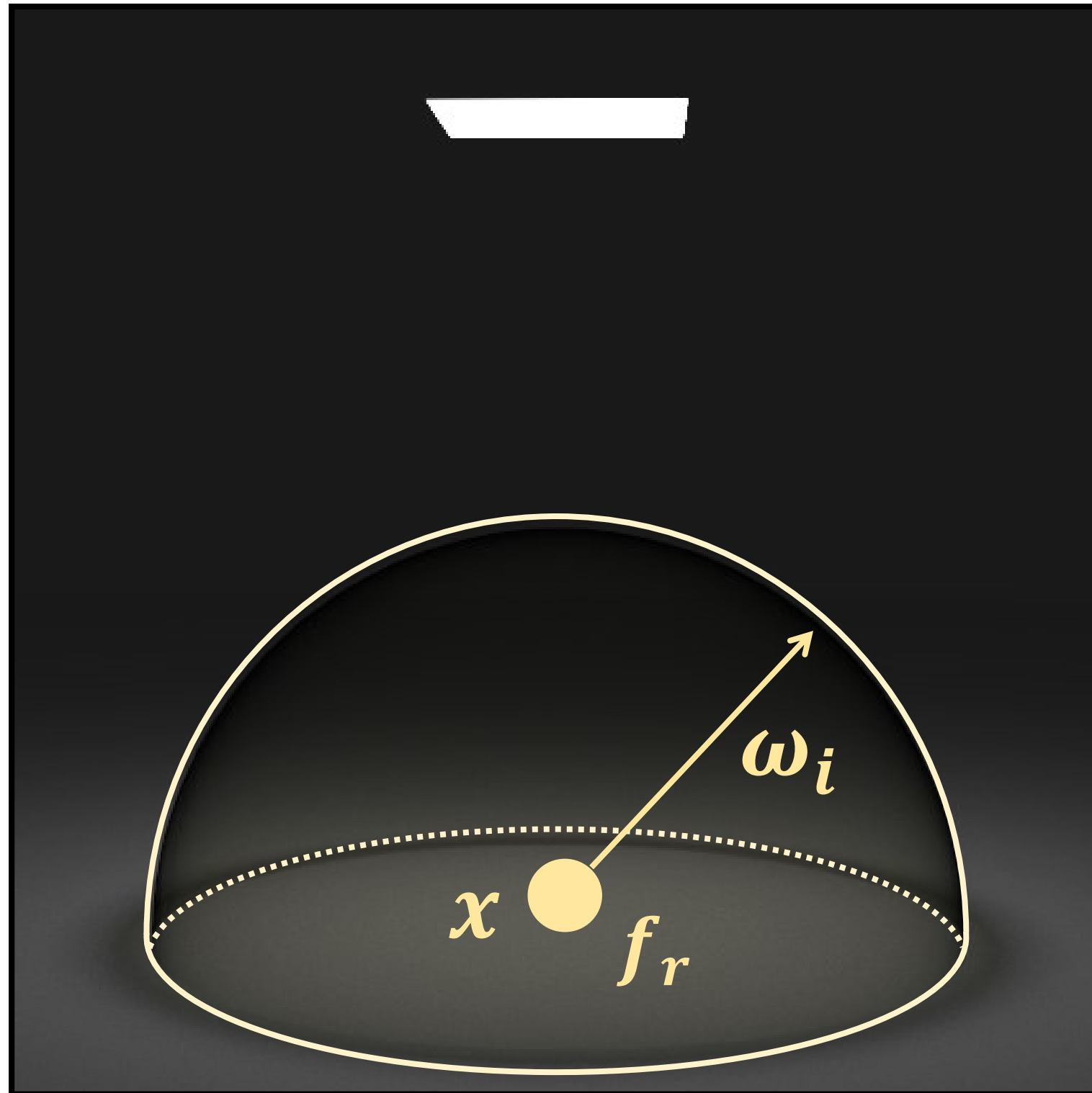
Radiance from  $x$ :

$$I = \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) \overset{\text{Incident radiance}}{L_i(\omega_i)} (n \cdot \omega_i) d\sigma(\omega_i)$$

Unit hemisphere



# Direct illumination integral

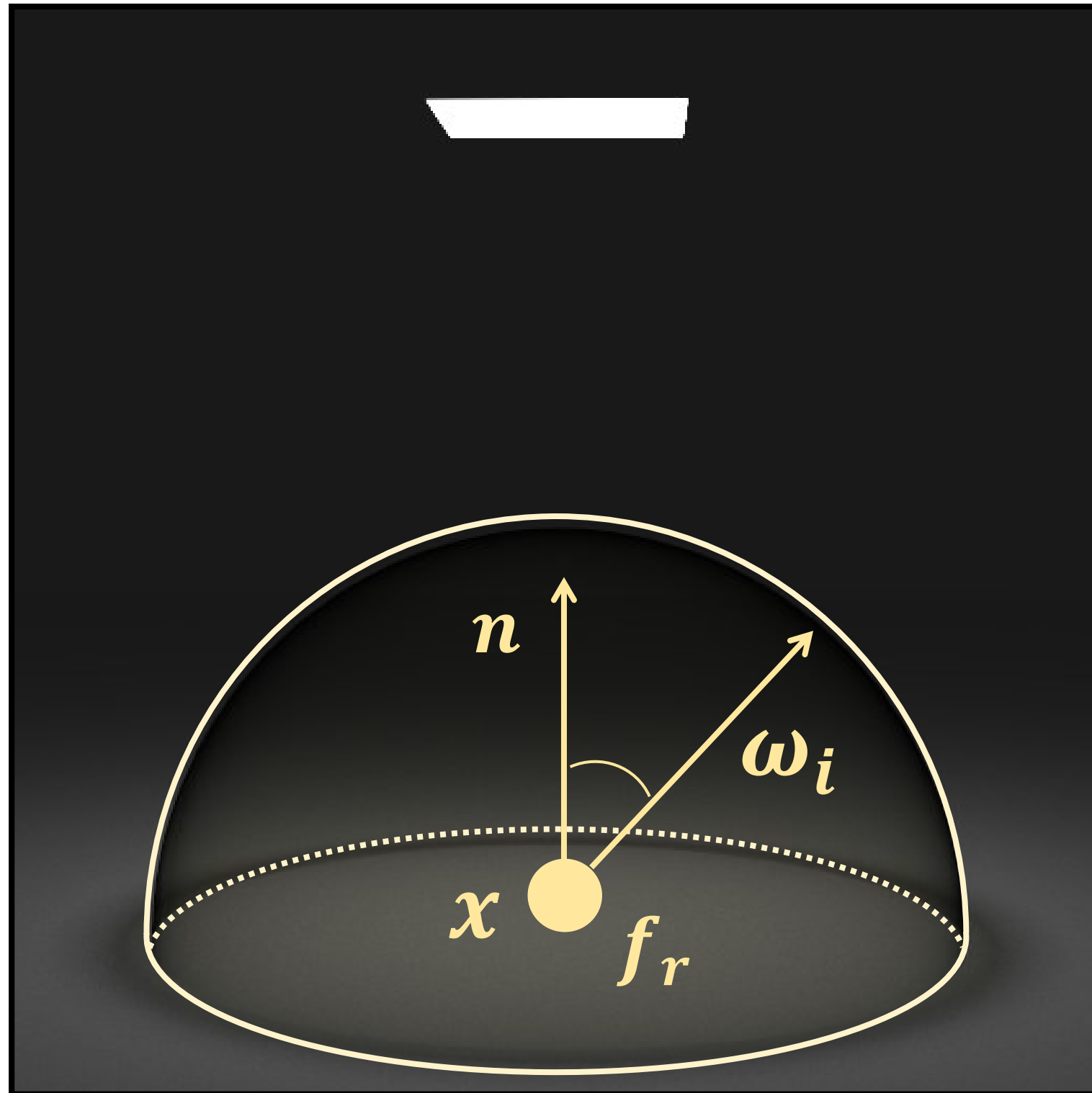


Radiance from  $x$ :

$$I = \int_{\mathbb{H}^2} \overset{\text{Reflectance (BRDF)}}{f_r(\omega_i, \omega_o)} \overset{\text{Incident radiance}}{L_i(\omega_i)} (n \cdot \omega_i) d\sigma(\omega_i)$$

Unit hemisphere

# Direct illumination integral

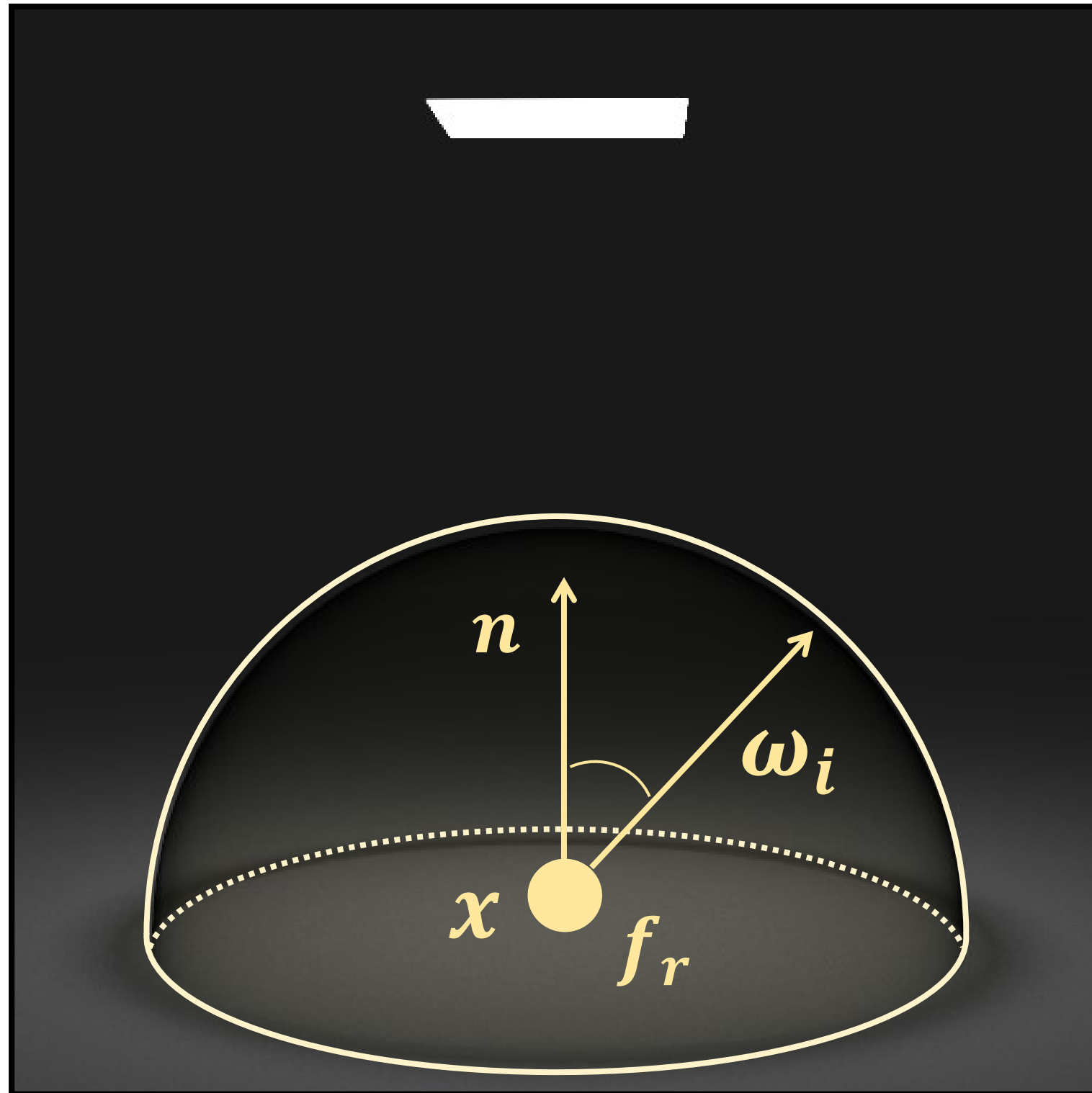


Radiance from  $x$ :

$$I = \int_{\mathbb{H}^2} \overset{\text{Reflectance (BRDF)}}{f_r(\omega_i, \omega_o)} \overset{\text{Incident radiance}}{L_i(\omega_i)} \overset{\text{Shading wrt normal } n}{(n \cdot \omega_i)} d\sigma(\omega_i)$$

Unit hemisphere

# Direct illumination integral



Radiance from  $x$ :

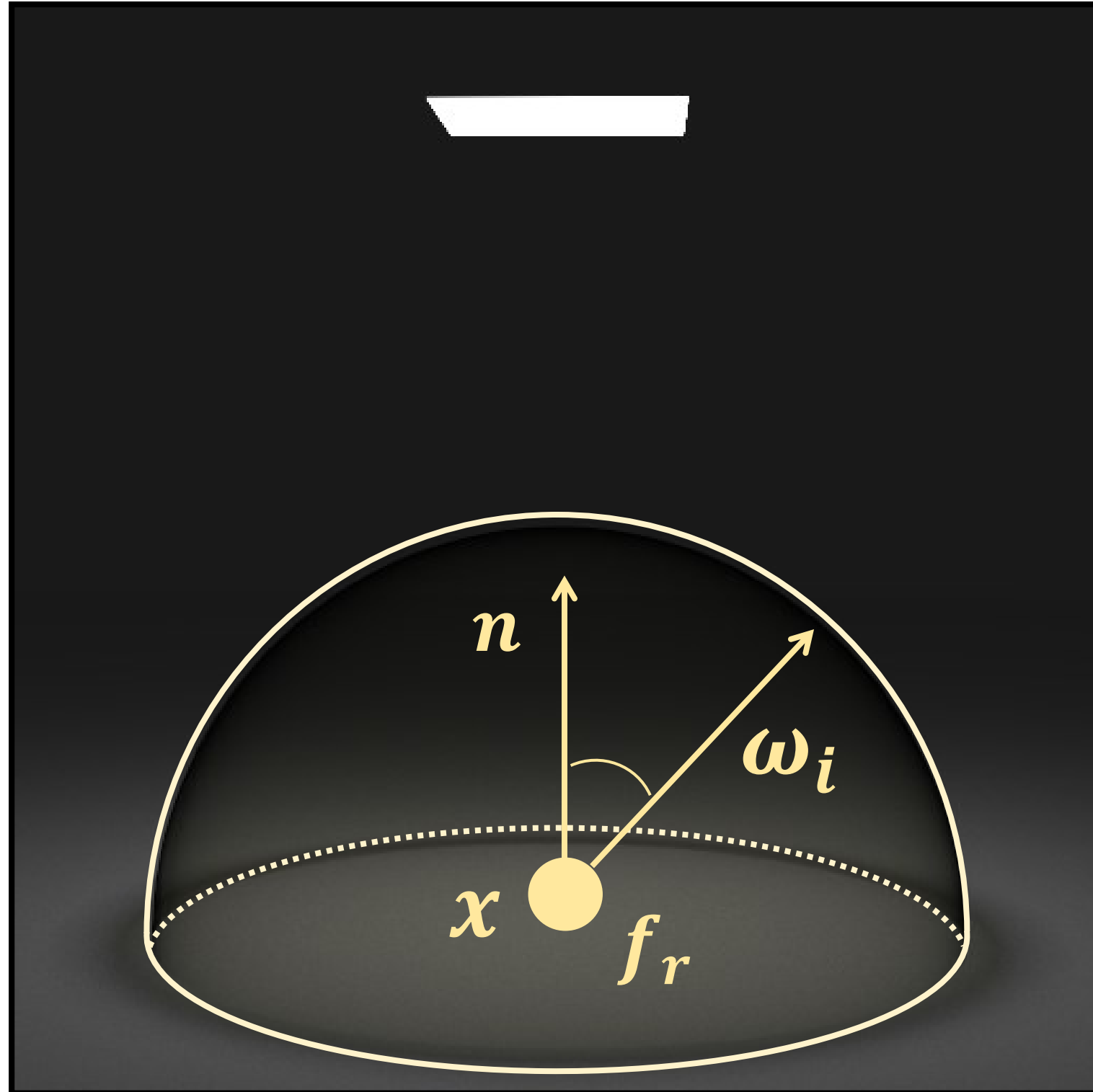
$$I = \int_{\mathbb{H}^2} \overset{\text{Reflectance (BRDF)}}{f_r(\omega_i, \omega_o)} \overset{\text{Incident radiance}}{L_i(\omega_i)} \overset{\text{Shading wrt normal } n}{(n \cdot \omega_i)} d\sigma(\omega_i)$$

Unit hemisphere

Monte Carlo rendering:



# Direct illumination integral



Radiance from  $x$ :

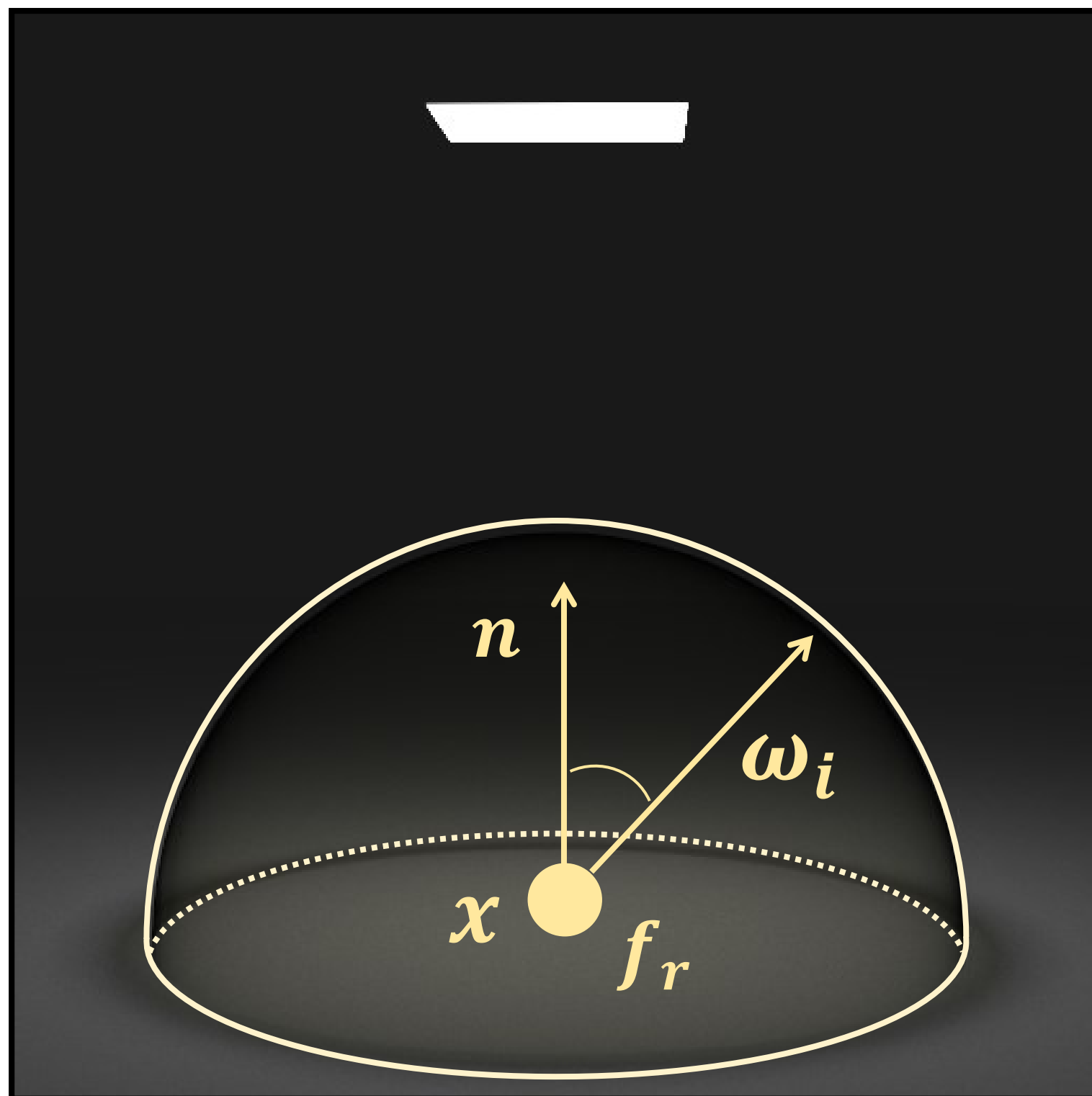
$$I = \int_{\mathbb{H}^2} \overset{\text{Reflectance (BRDF)}}{f_r(\omega_i, \omega_o)} \overset{\text{Incident radiance}}{L_i(\omega_i)} \overset{\text{Shading wrt normal } n}{(n \cdot \omega_i)} d\sigma(\omega_i)$$

Unit hemisphere

Monte Carlo rendering:

- Sample random directions  $\omega_i^s$  from PDF  $p(\omega_i)$

# Direct illumination integral



Radiance from  $x$ :

$$I = \int_{\mathbb{H}^2} \overset{\text{Reflectance (BRDF)}}{f_r(\omega_i, \omega_o)} \overset{\text{Incident radiance}}{L_i(\omega_i)} \overset{\text{Shading wrt normal } n}{(n \cdot \omega_i)} d\sigma(\omega_i)$$

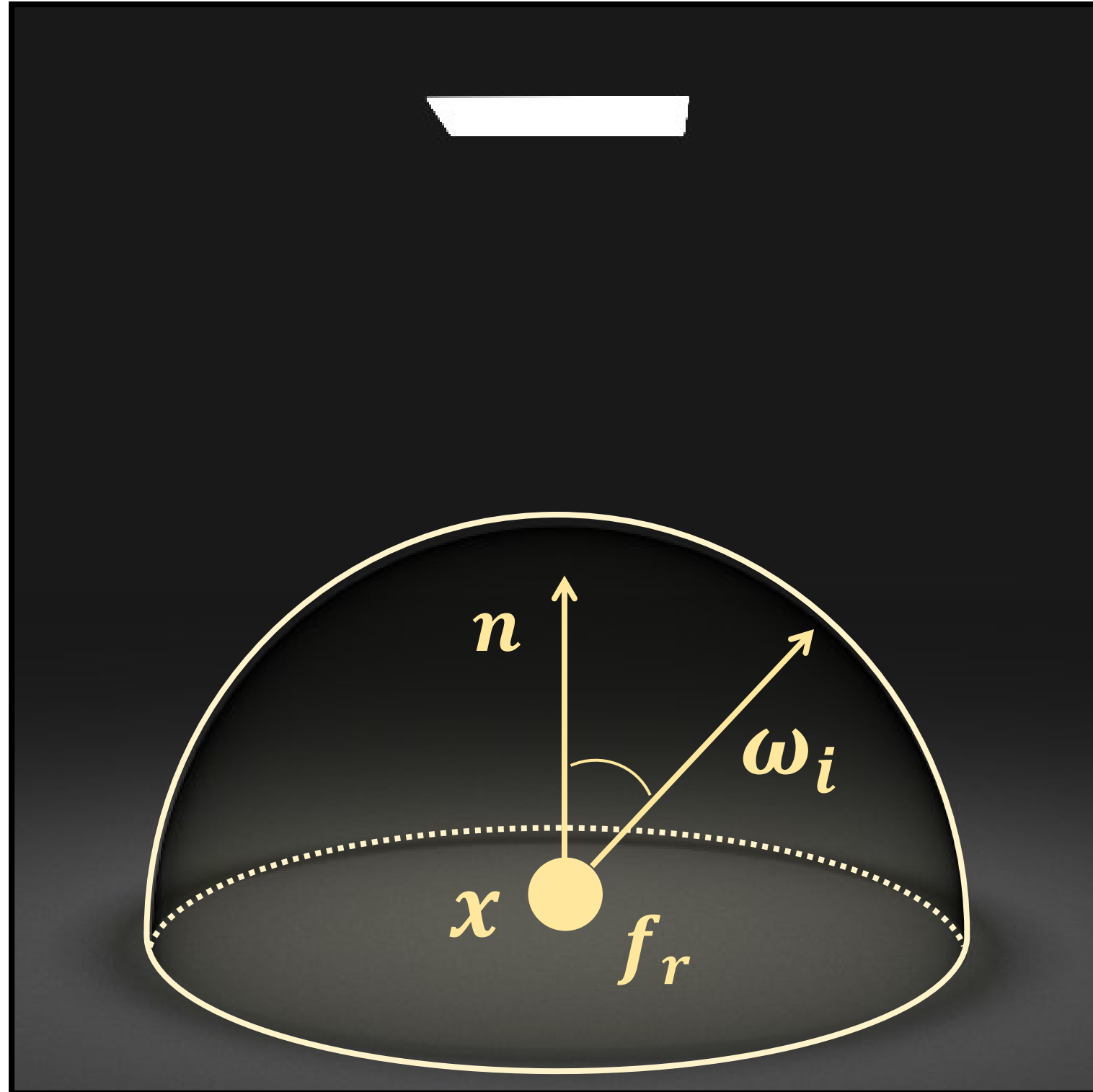
Unit hemisphere

Monte Carlo rendering:

- Sample random directions  $\omega_i^s$  from PDF  $p(\omega_i)$
- Form estimator

$$I \approx \sum_s \frac{\overset{\text{Reflectance (BRDF)}}{f_r(\omega_i^s, \omega_o)} \overset{\text{Incident radiance}}{L_i(\omega_i^s)} \overset{\text{Shading wrt normal } n}{(n \cdot \omega_i^s)}}{\underset{\text{PDF}}{p(\omega_i^s)}}$$

# Differential direct illumination

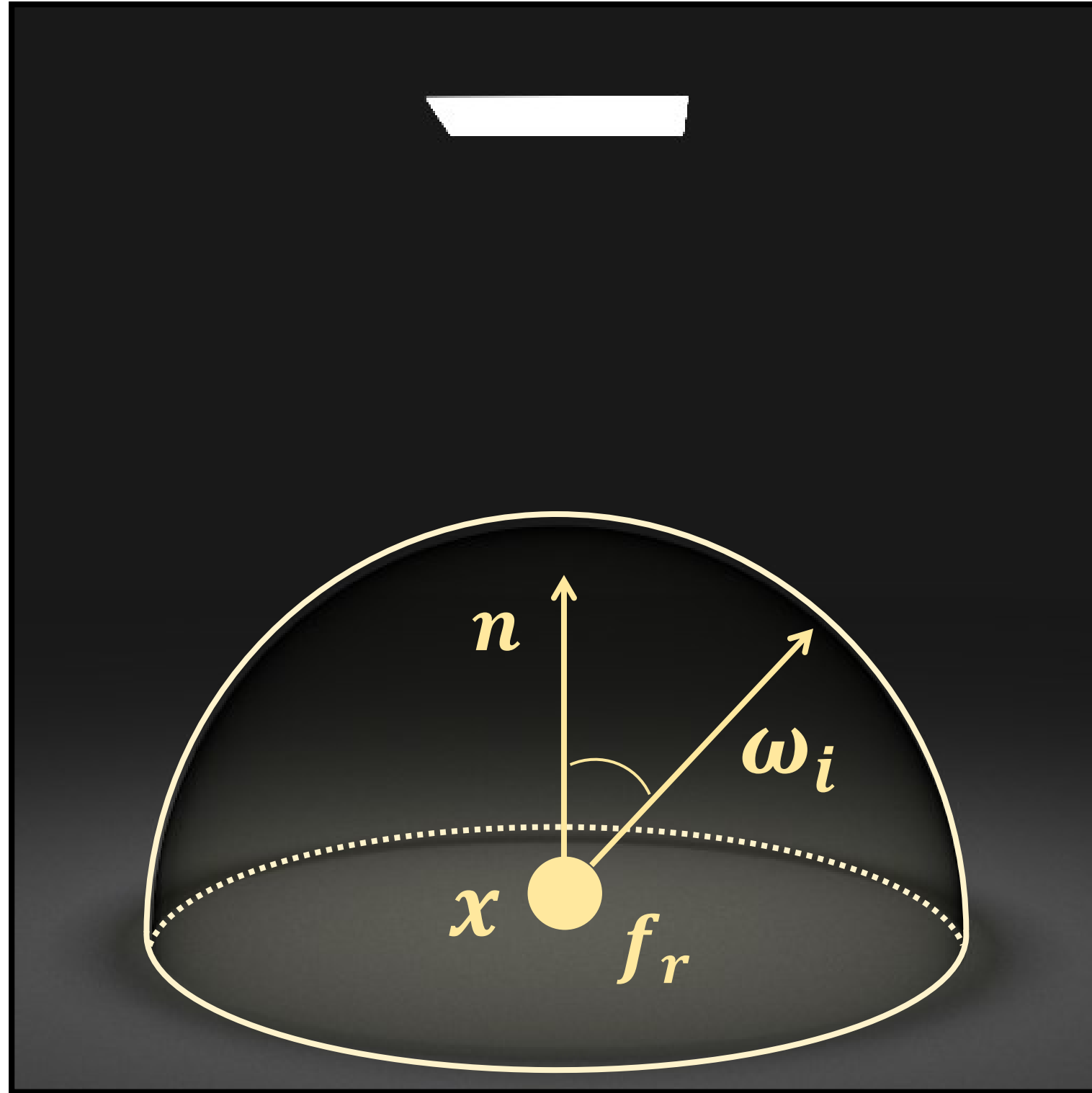


Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \frac{d}{d\pi} \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$



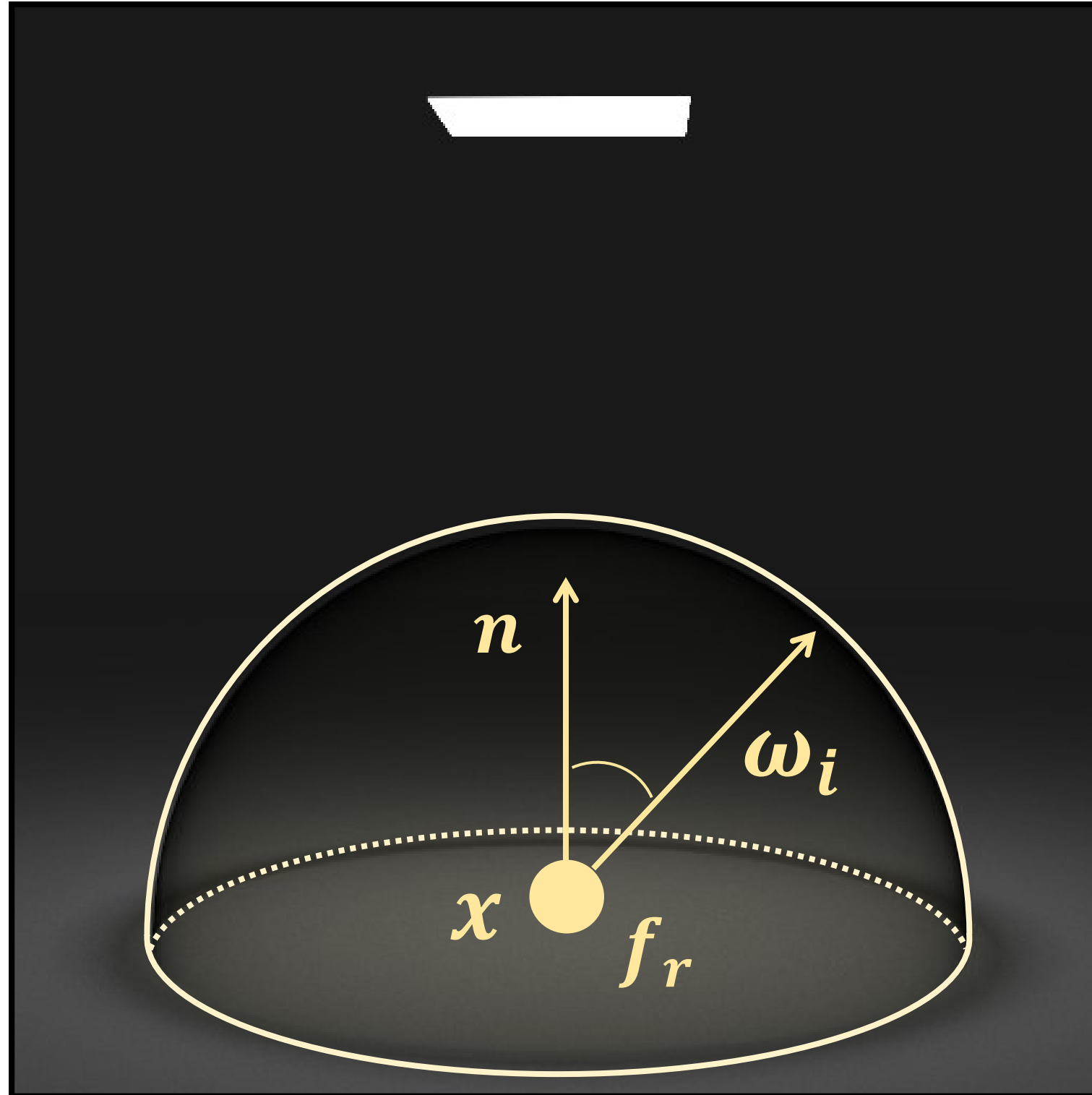
# Differential direct illumination: local parameters



Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \frac{d}{d\pi} \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$

# Differential direct illumination: local parameters



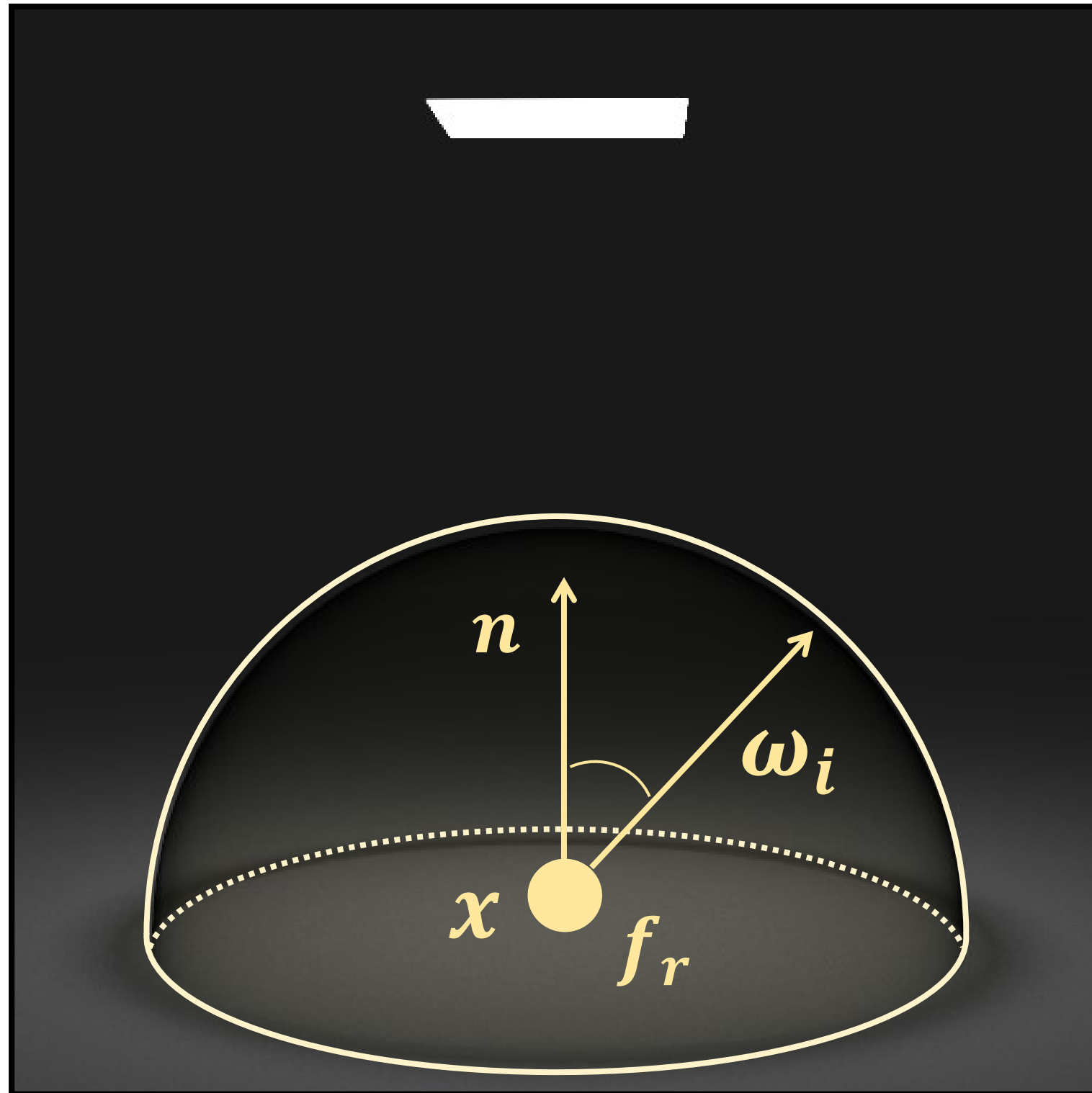
Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \frac{d}{d\pi} \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$

$\pi$ : local parameters

- BRDF parameters
- *shading* normal
- illumination brightness

# Differential direct illumination: local parameters



Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \int_{\mathbb{H}^2} \frac{d}{d\pi} \{f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i)\} d\sigma(\omega_i)$$

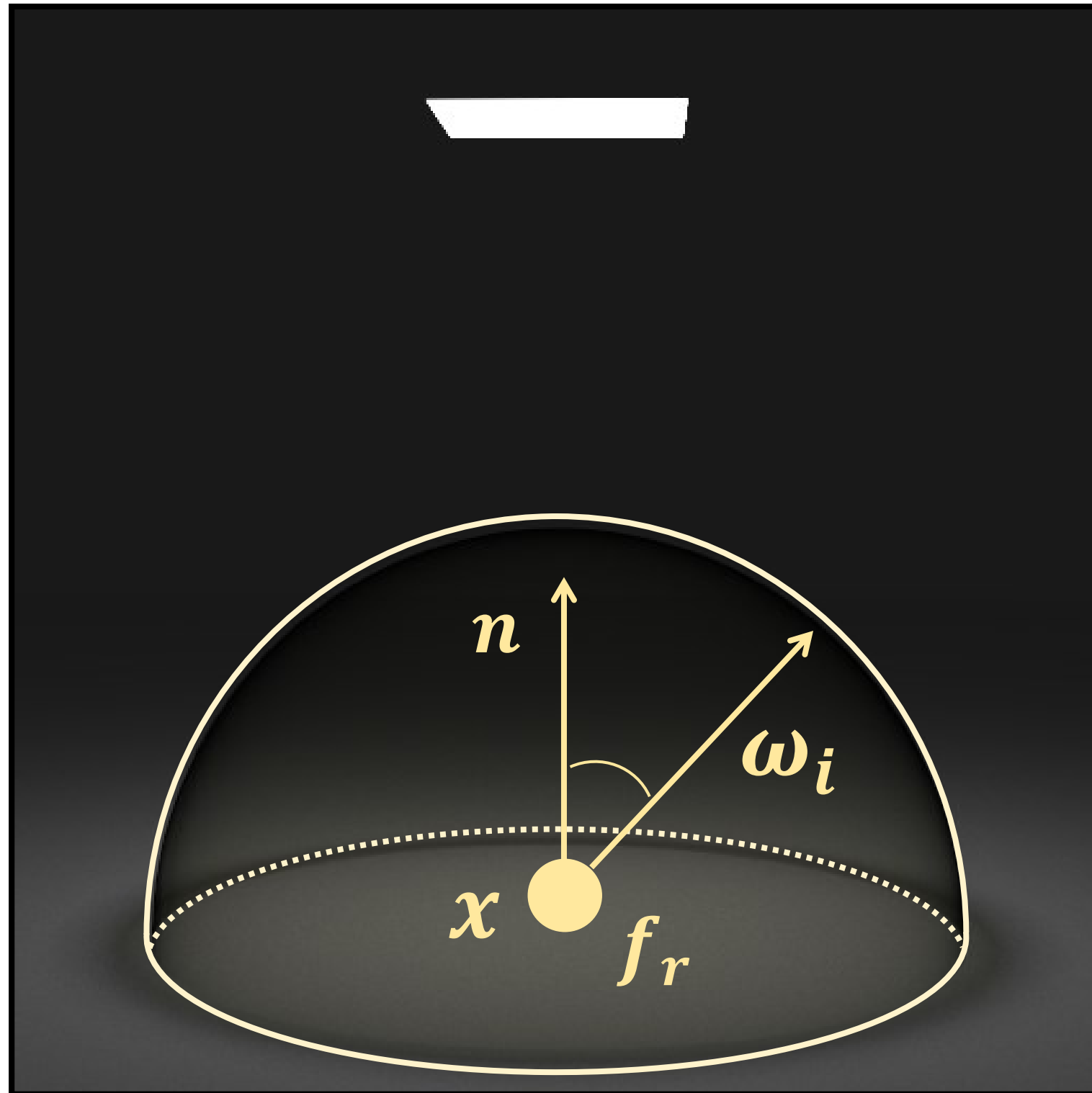
Just move derivative inside integral

$\pi$ : local parameters

- BRDF parameters
- *shading* normal
- illumination brightness



# Differential direct illumination: local parameters



$\pi$ : local parameters

- BRDF parameters
- *shading* normal
- illumination brightness

Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \int_{\mathbb{H}^2} \frac{d}{d\pi} \{f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i)\} d\sigma(\omega_i)$$

Just move derivative inside integral

Monte Carlo differentiable rendering:

- Sample random directions  $\omega_i^s$  from PDF  $p(\omega_i)$

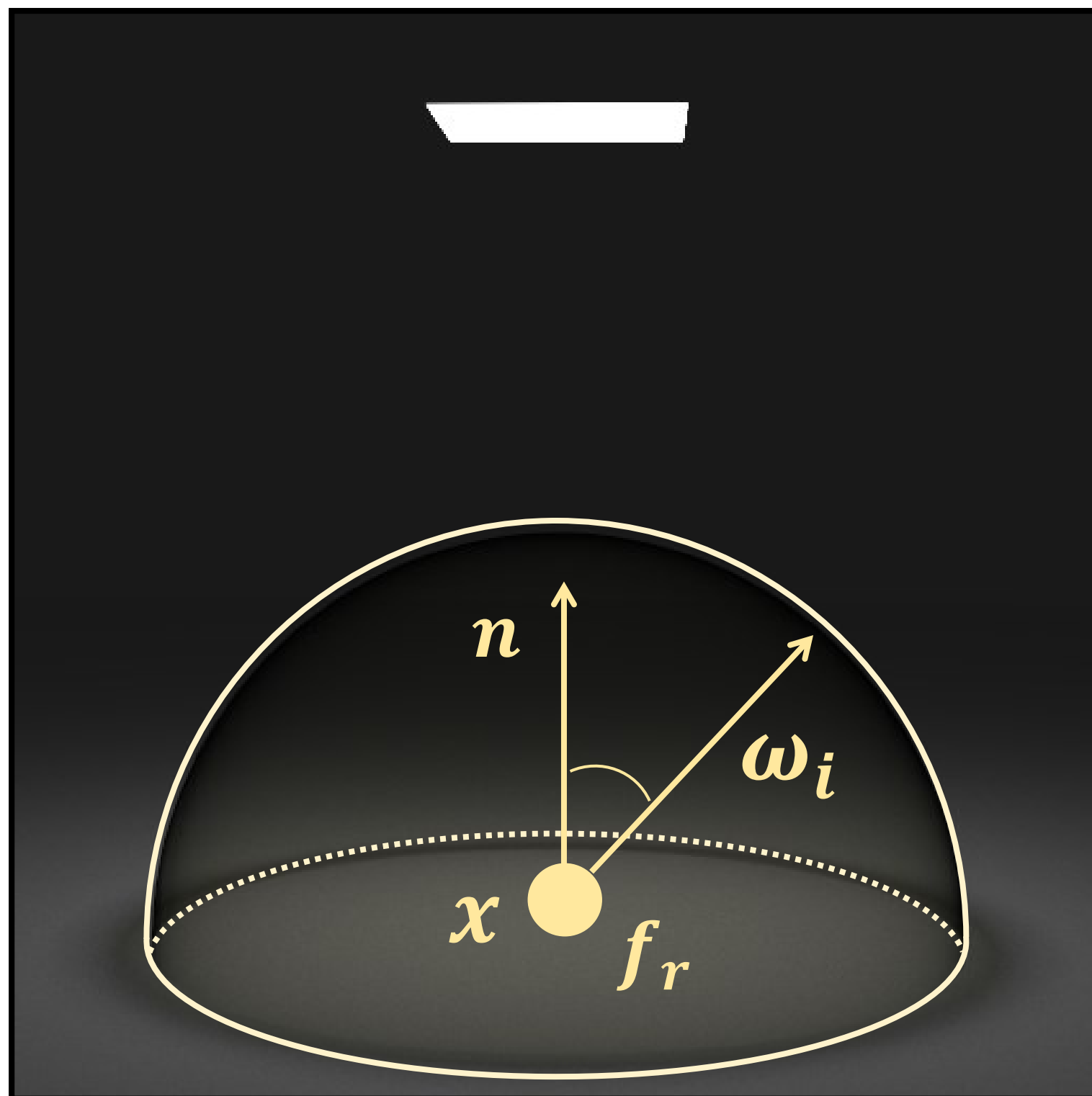
Just differentiate numerator

- Form estimator

[Khungurn et al. 2015, Gkioulekas et al. 2015]

$$\frac{dI}{d\pi} \approx \sum_s \frac{\frac{d}{d\pi} \{f_r(\omega_i^s, \omega_o) L_i(\omega_i^s) (n \cdot \omega_i^s)\}}{p(\omega_i^s)}$$

# Alternative estimator



$\pi$ : local parameters

- BRDF parameters

Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \int_{\mathbb{H}^2} \frac{d}{d\pi} \{f_r(\omega_i, \omega_o, \pi) L_i(\omega_i) (n \cdot \omega_i)\} d\sigma(\omega_i)$$

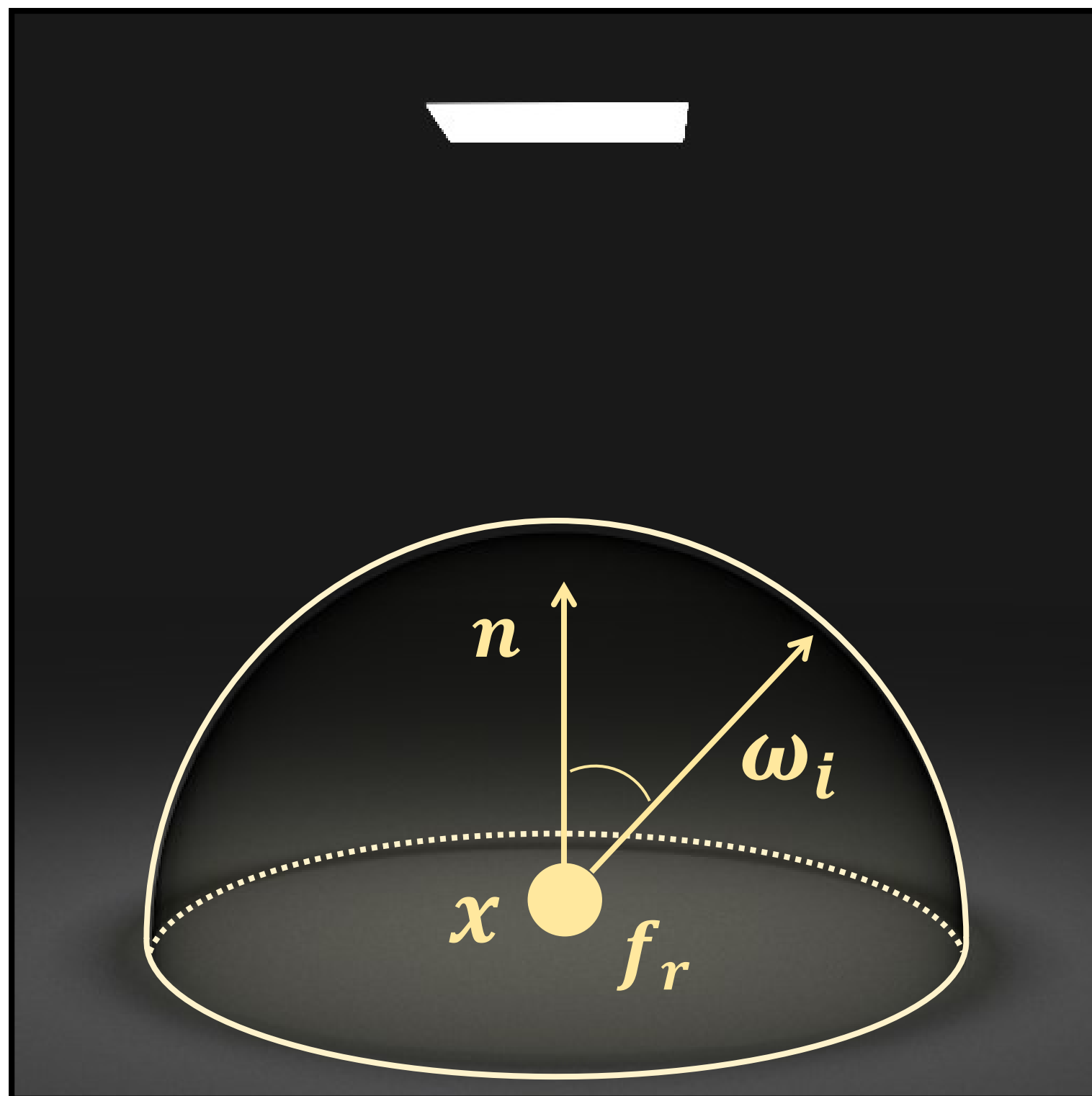
Just move derivative inside integral

Monte Carlo estimation:

- Sample random directions  $\omega_i^s$  from PDF  $p(\omega_i, \pi)$
- Form estimator

$$\frac{dI}{d\pi} \approx \sum_s \frac{\frac{d}{d\pi} \{f_r(\omega_i^s, \omega_o, \pi) L_i(\omega_i^s) (n \cdot \omega_i^s)\}}{p(\omega_i^s, \pi)}$$

# Alternative estimator



$\pi$ : local parameters

- BRDF parameters

Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \int_{\mathbb{H}^2} \frac{d}{d\pi} \{f_r(\omega_i, \omega_o, \pi) L_i(\omega_i) (n \cdot \omega_i)\} d\sigma(\omega_i)$$

Just move derivative inside integral

Monte Carlo estimation:

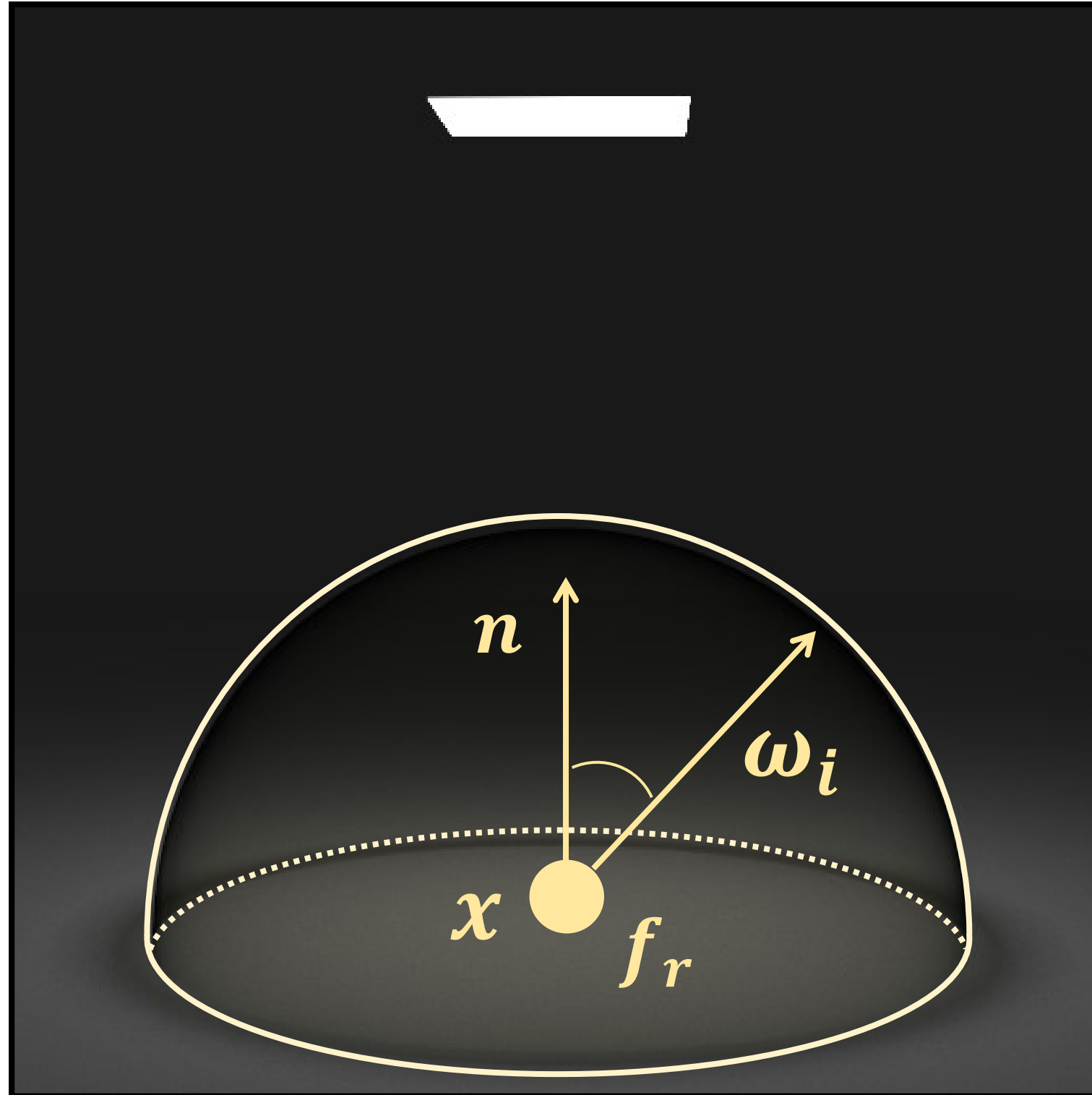
- Sample random directions  $\omega_i^s$  from PDF  $p(\omega_i, \pi)$
- Form estimator

Differentiate entire contribution  
[Zeltner et al. 2021]

$$\frac{dI}{d\pi} \approx \sum_s \frac{d}{d\pi} \left\{ \frac{f_r(\omega_i^s, \omega_o, \pi) L_i(\omega_i^s) (n \cdot \omega_i^s)}{p(\omega_i^s, \pi)} \right\}$$



# Differential direct illumination: global parameters



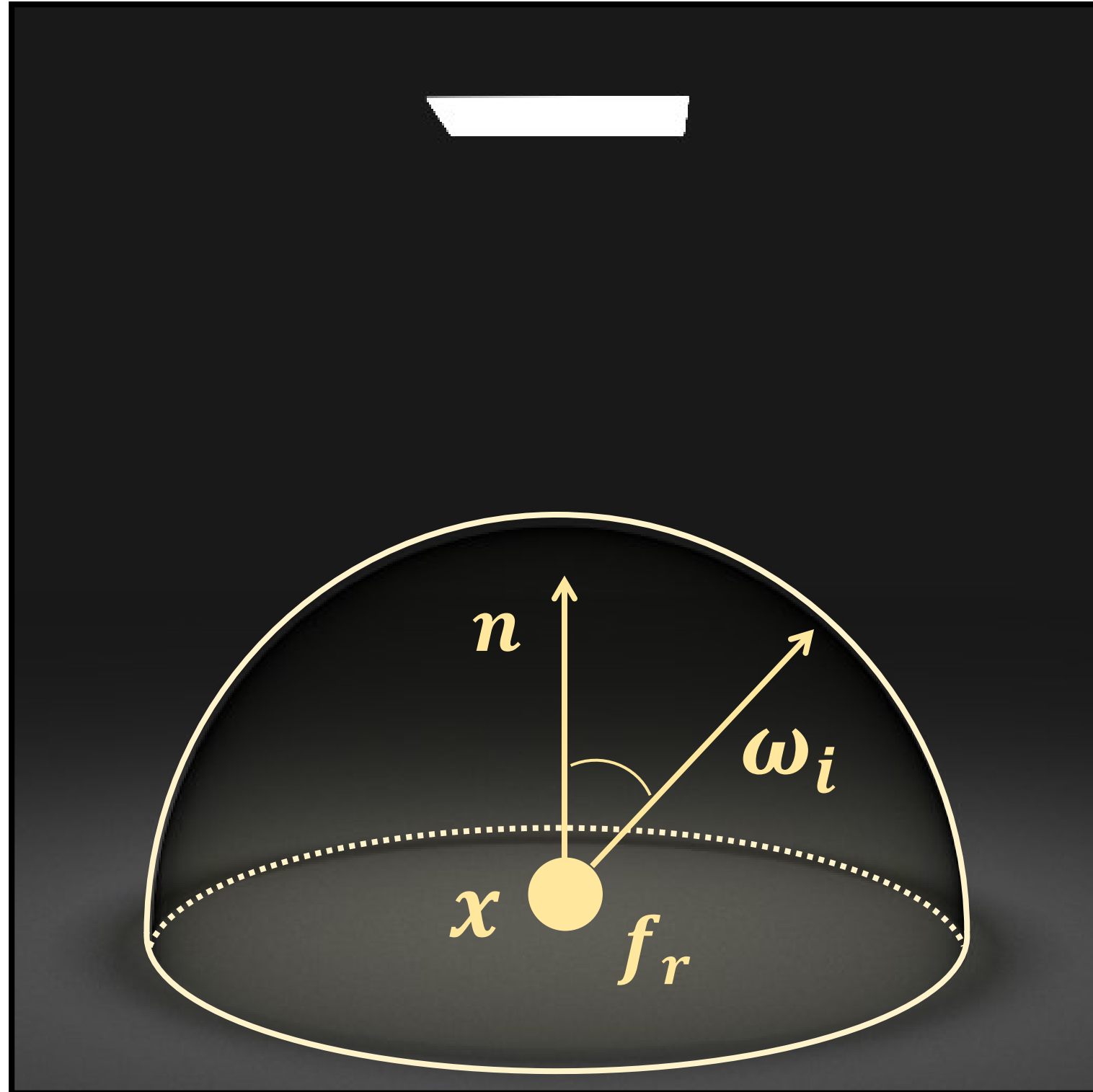
Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \frac{d}{d\pi} \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$

$\pi$ : global parameters

- shape and pose of different scene elements (camera, sources, objects)

# Differential direct illumination: global parameters



Differential radiance from  $x$ :

$$\frac{dI}{d\pi} = \frac{d}{d\pi} \int_{\mathbb{H}^2} f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i) d\sigma(\omega_i)$$

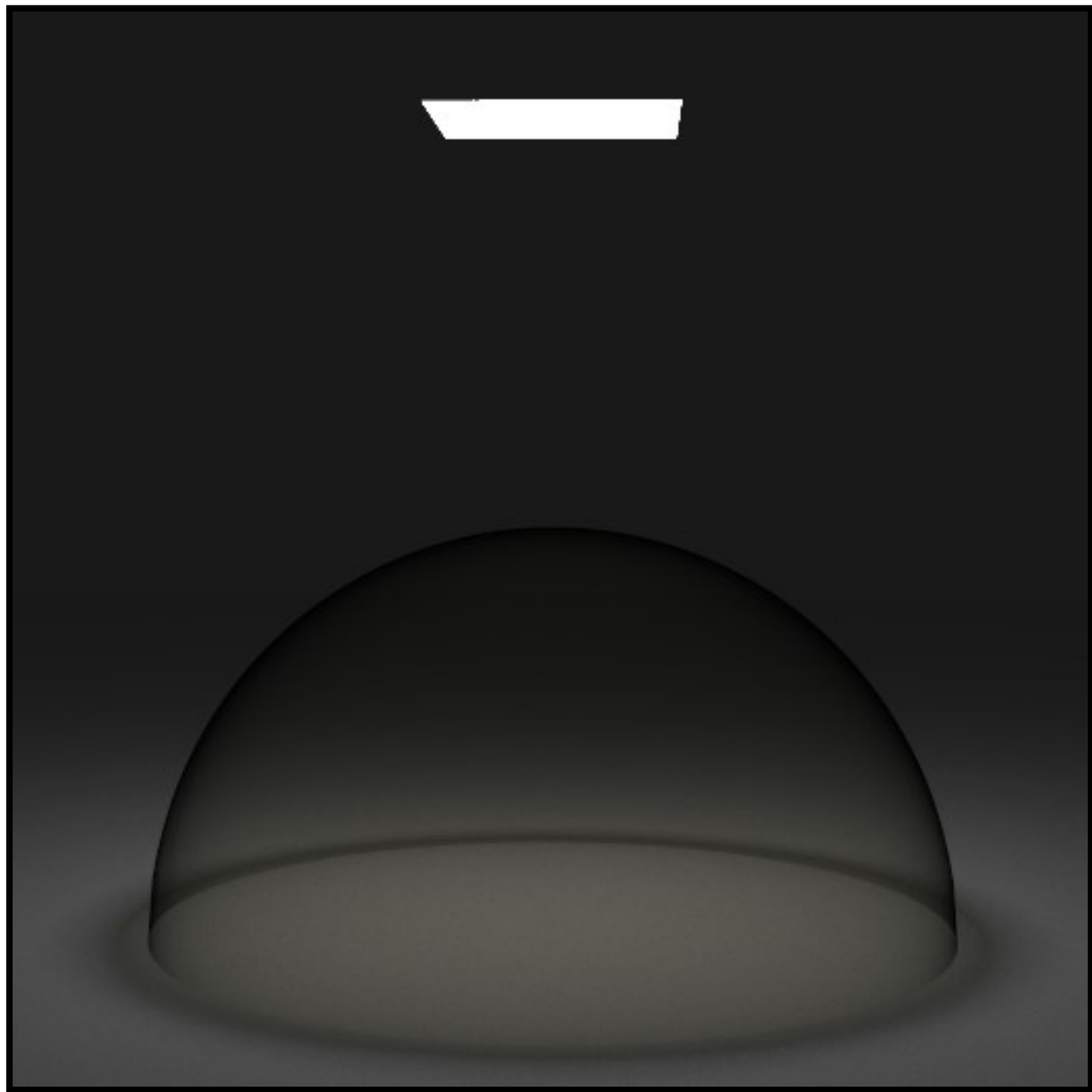
~~$$= \int_{\mathbb{H}^2} \frac{d}{d\pi} \{f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i)\} d\sigma(\omega_i)$$~~

Need to use full Reynolds transport theorem

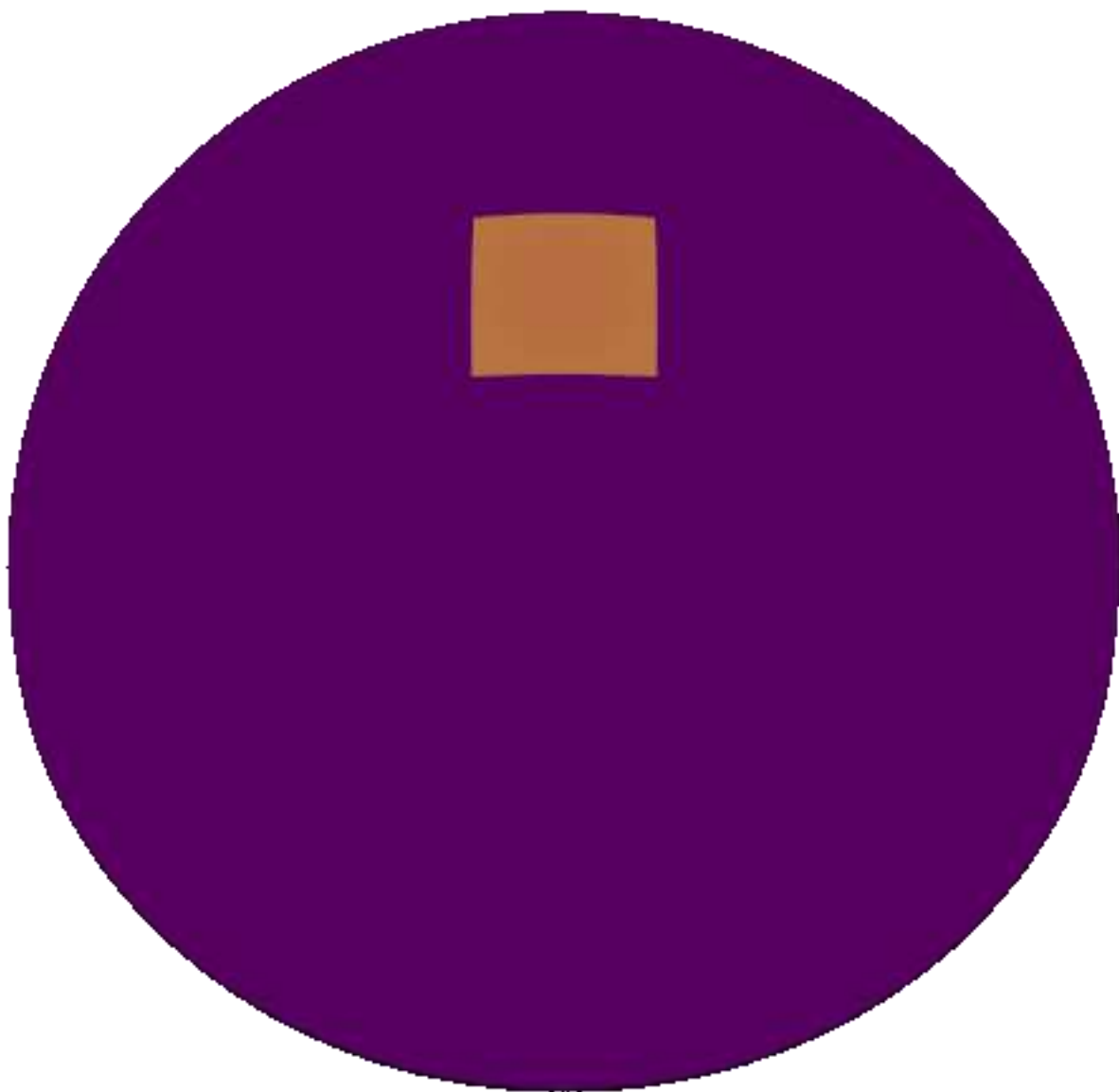
$\pi$ : global parameters

- shape and pose of different scene elements (camera, sources, objects)

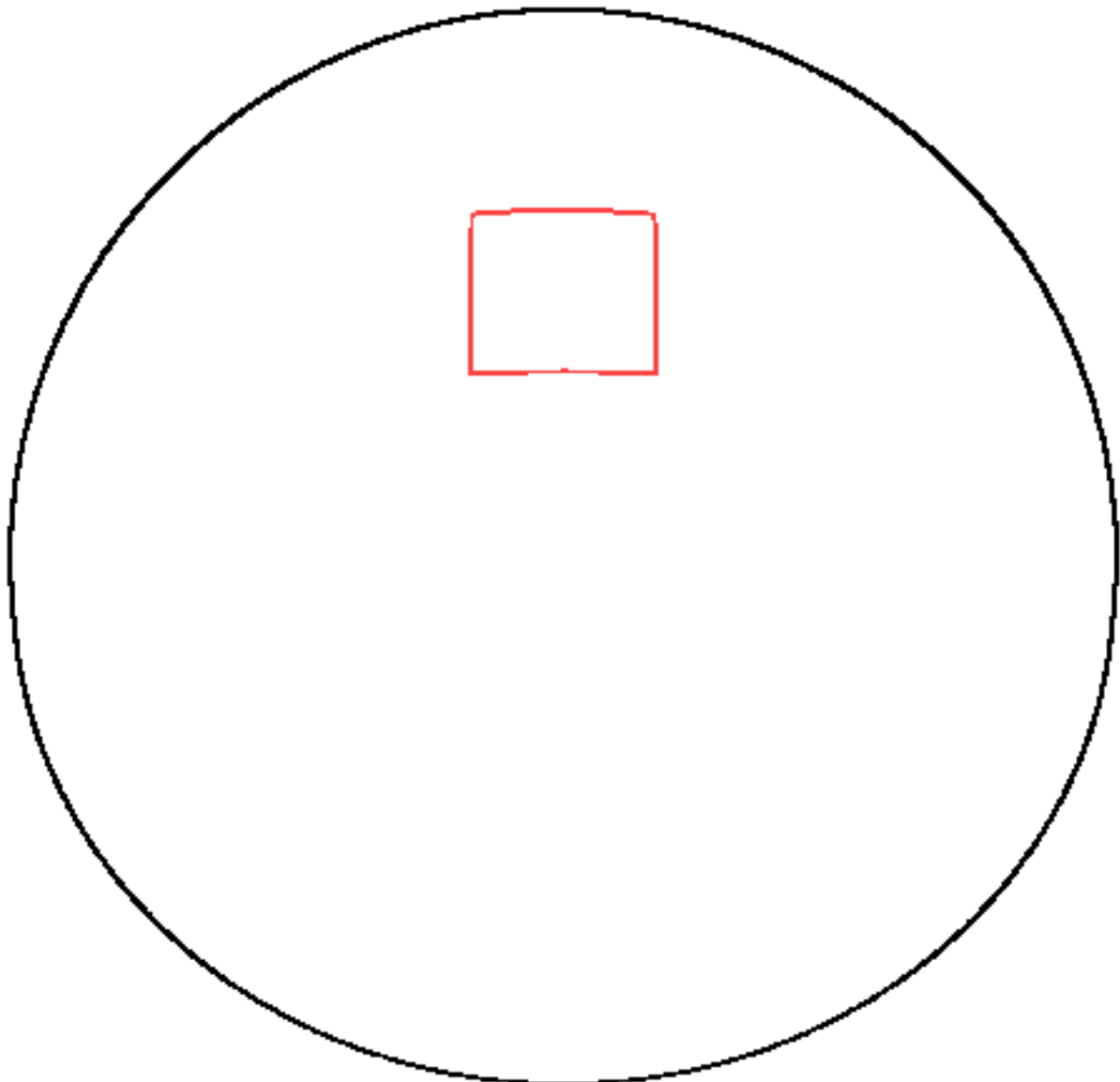
# Discontinuities in the integrand



Low  High



Integrand  
 $f(\omega_i)$



Discontinuous points  
( $\pi$ -dependent)

$\pi$ : size of the emitter

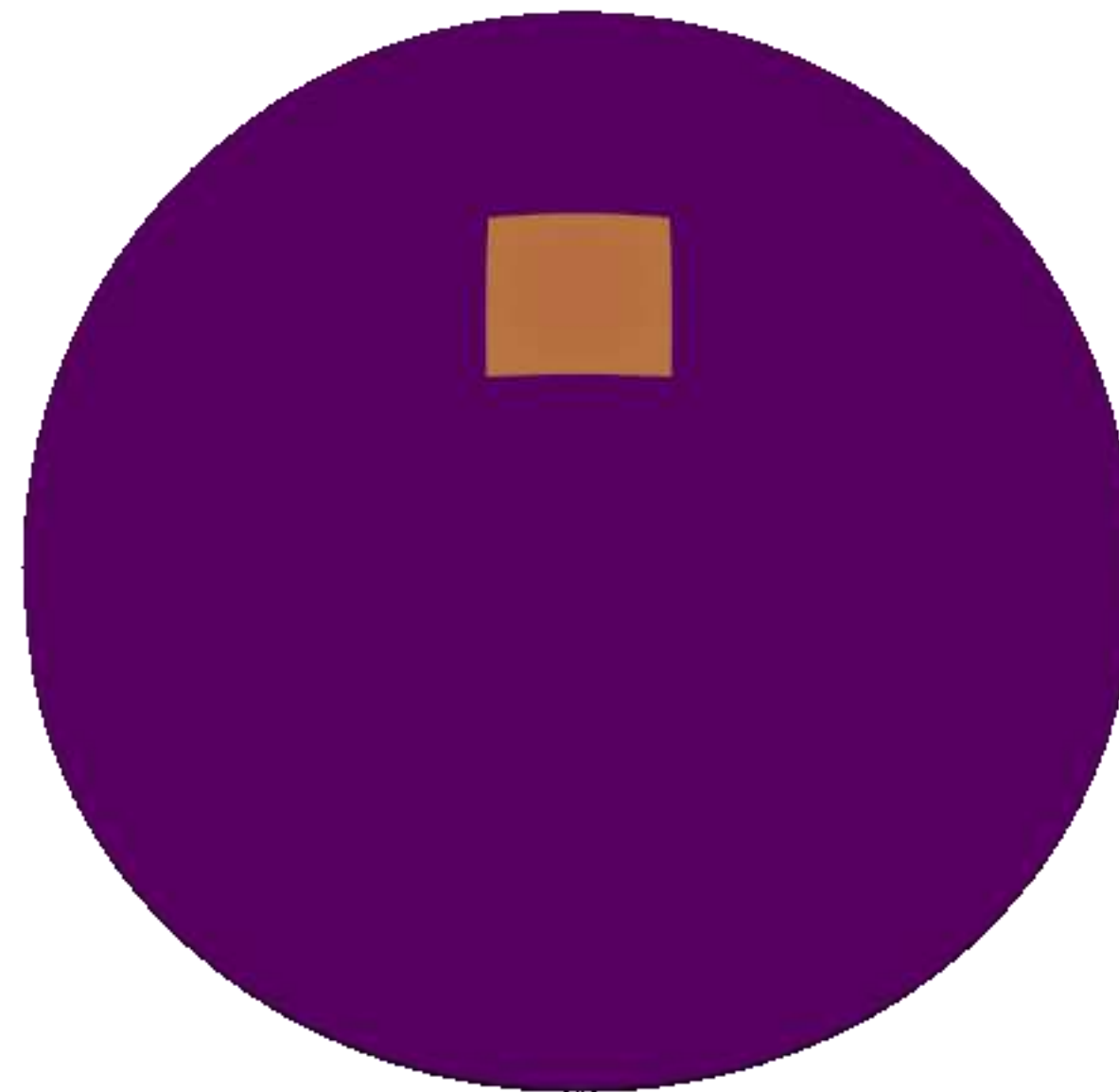
$$I = \int_{\mathbb{H}^2} \underbrace{f_r(\omega_i, \omega_o) L_i(\omega_i) (n \cdot \omega_i)}_{f(\omega_i)} d\sigma(\omega_i)$$



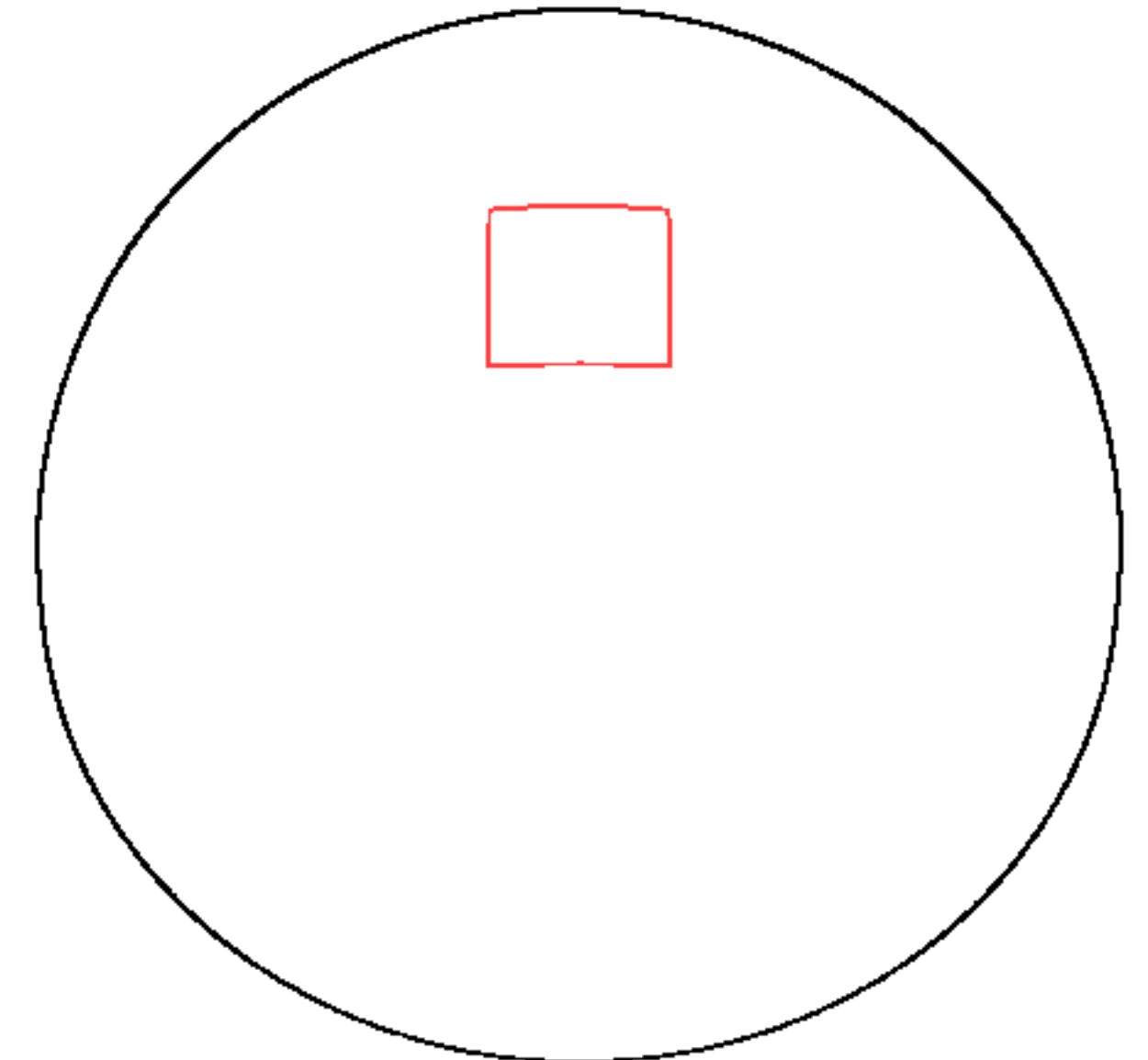
# Applying the Reynolds transport theorem

$$I = \int_{\mathbb{H}^2} f(\omega_i, \omega_o) d\sigma(\omega_i)$$

Low  High



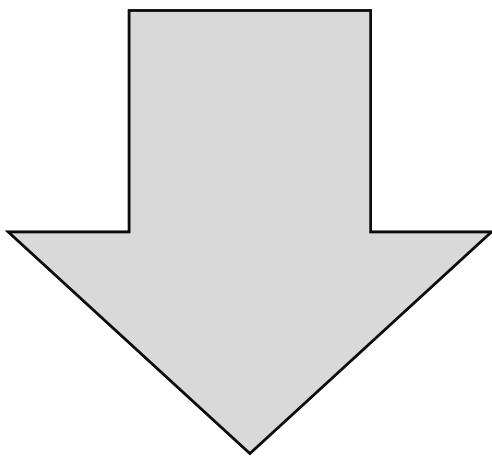
Integrand  
 $f(\omega_i)$



Discontinuous points  
( $\pi$ -dependent)

# Applying the Reynolds transport theorem

$$I = \int_{\mathbb{H}^2} f(\omega_i, \omega_o) d\sigma(\omega_i)$$



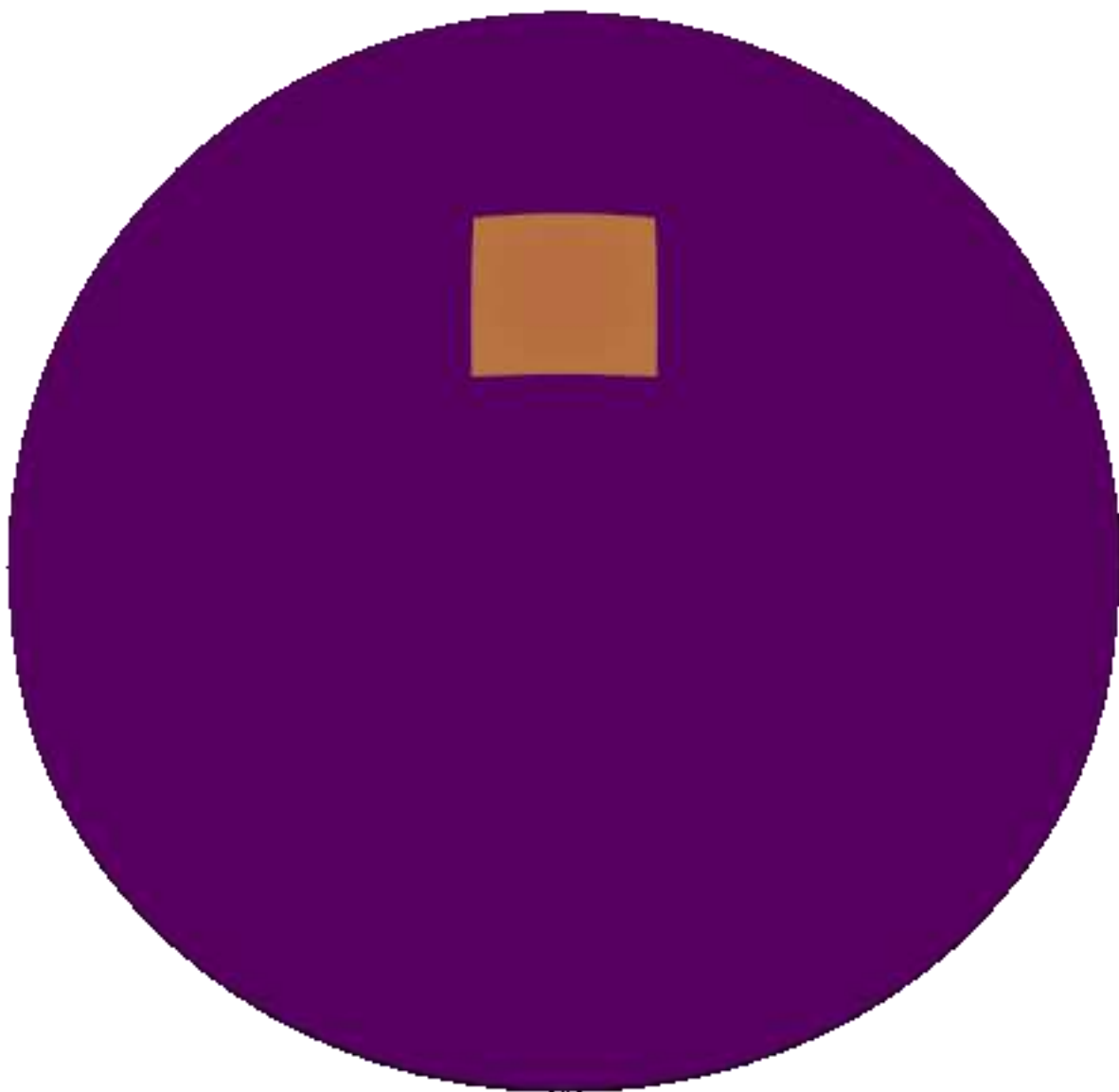
$$\frac{dI}{d\pi} = \int_{\mathbb{H}^2} \frac{df}{d\pi} d\sigma + \int_{\partial \mathbb{H}^2} g dl$$

Interior integral  
(same as for local  
parameters)

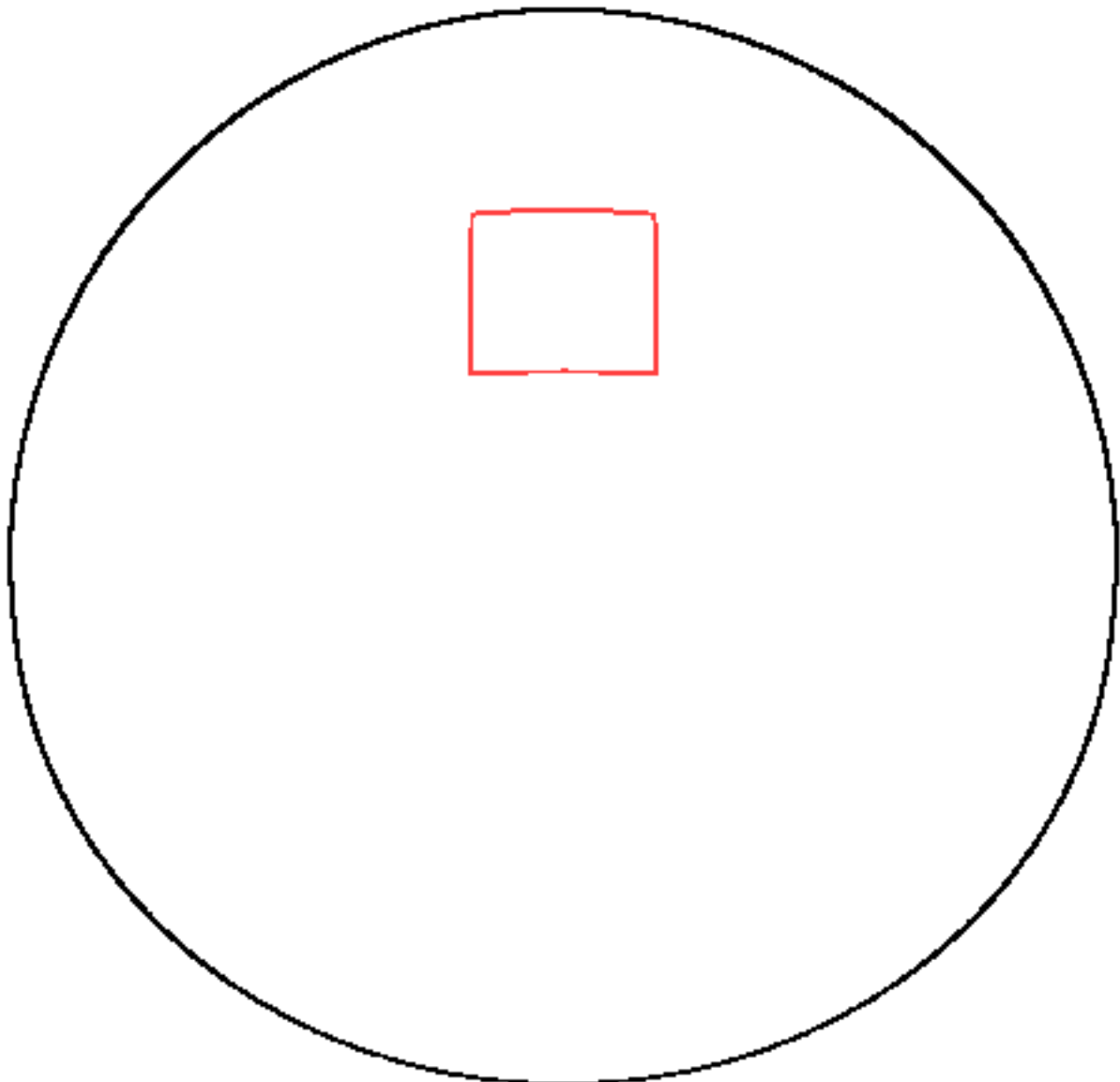
Boundary  
integral

[Ramamoorthi et al. 2007, Li et al. 2019]

Low  High



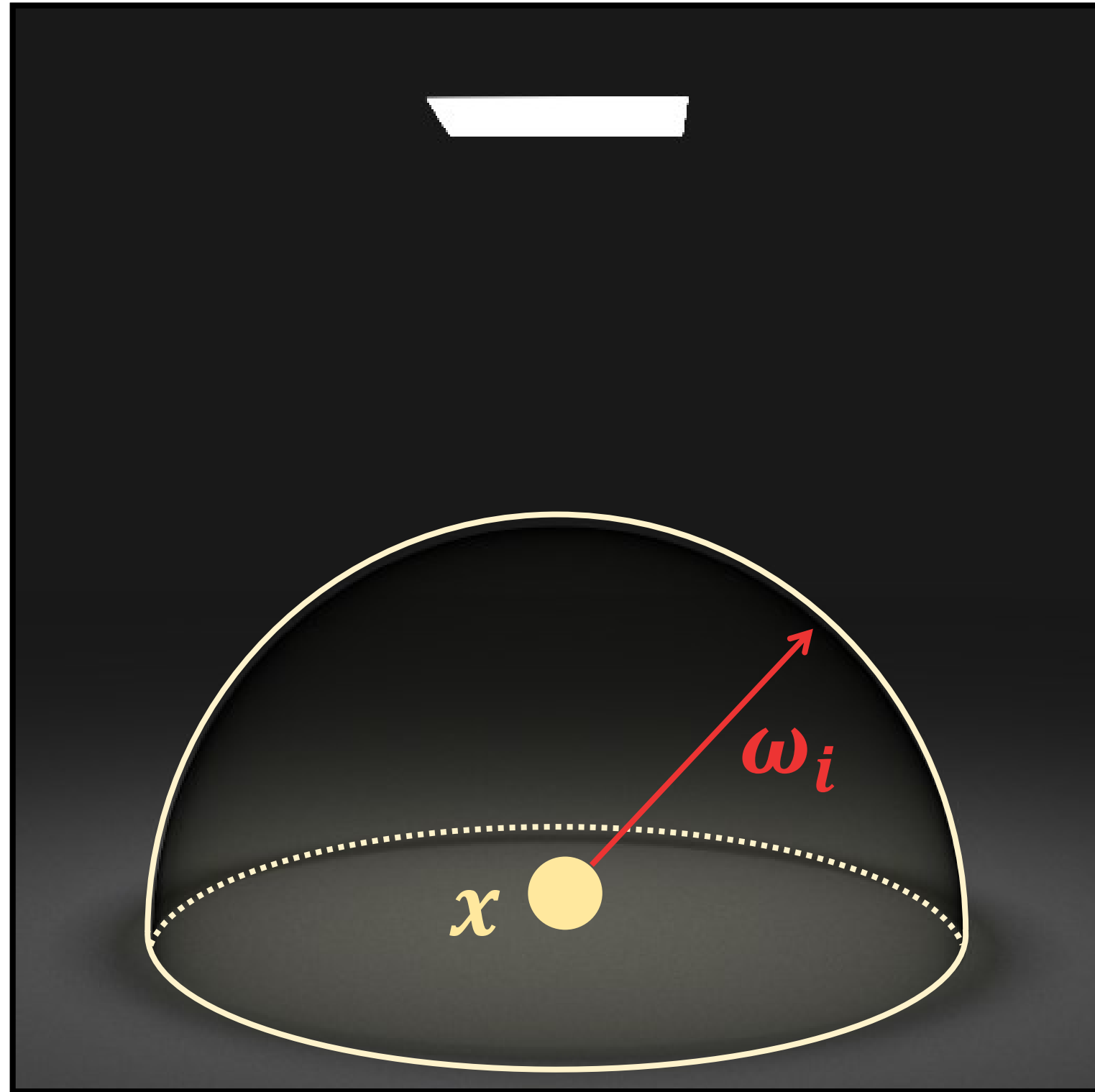
Integrand  
 $f(\omega_i)$



Discontinuous points  
( $\pi$ -dependent)

# Reparameterizing the direct illumination integral

Hemispherical integral

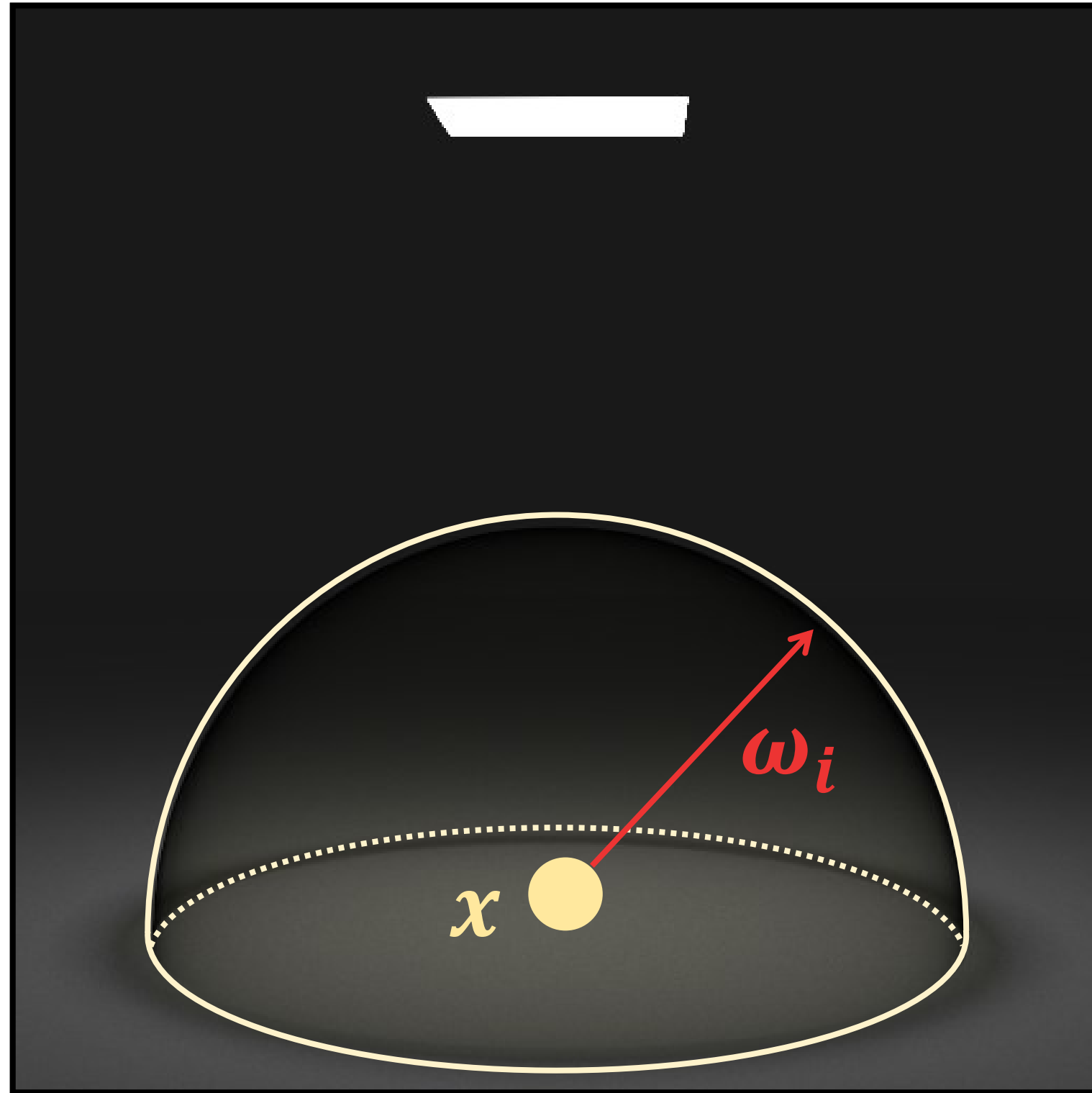


$$I = \int_{\mathbb{H}^2} f(\omega_i) \, d\sigma(\omega_i)$$



# Reparameterizing the direct illumination integral

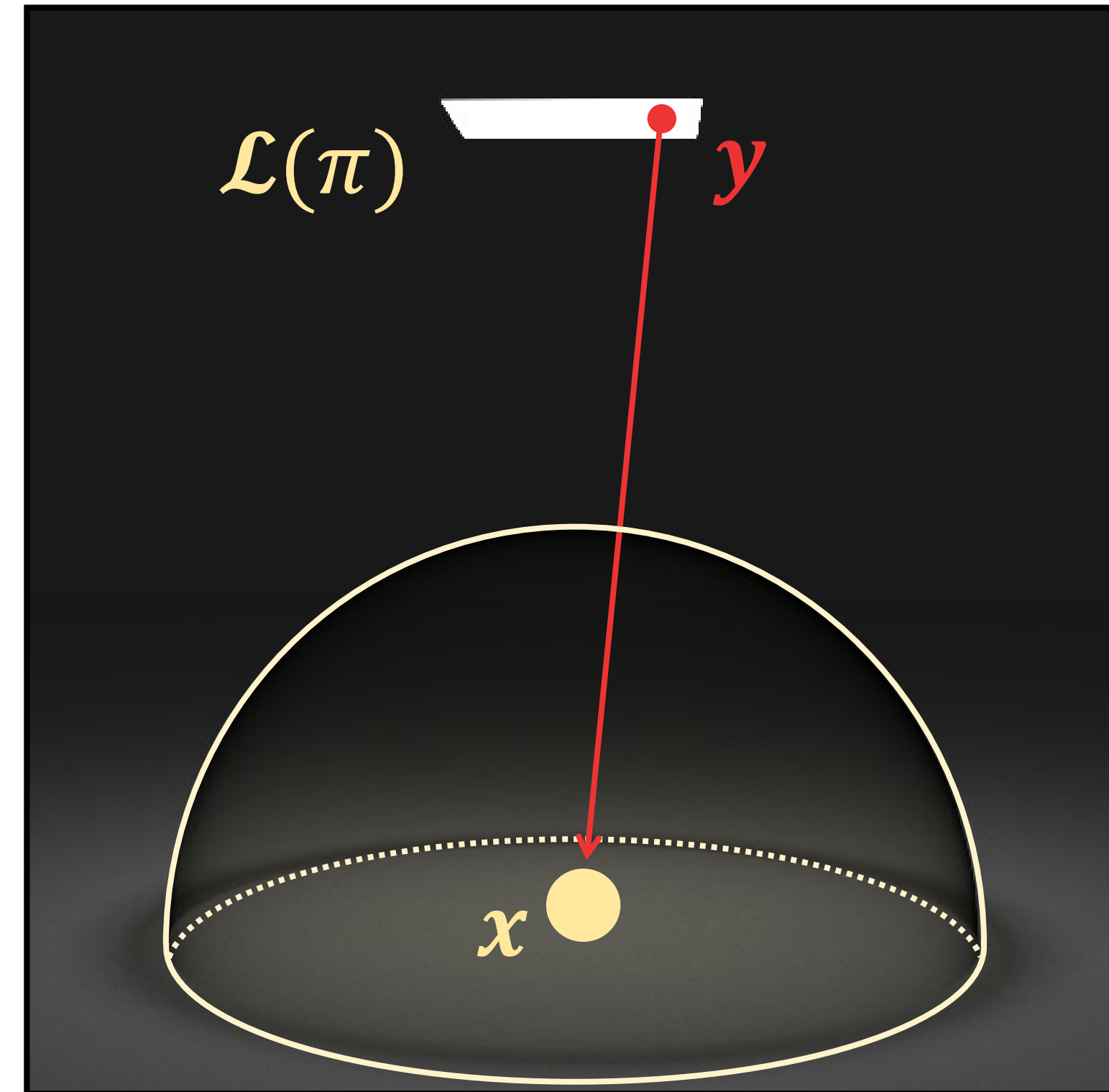
Hemispherical integral



$$I = \int_{\mathbb{H}^2} f(\omega_i) d\sigma(\omega_i)$$

Change of  
variables

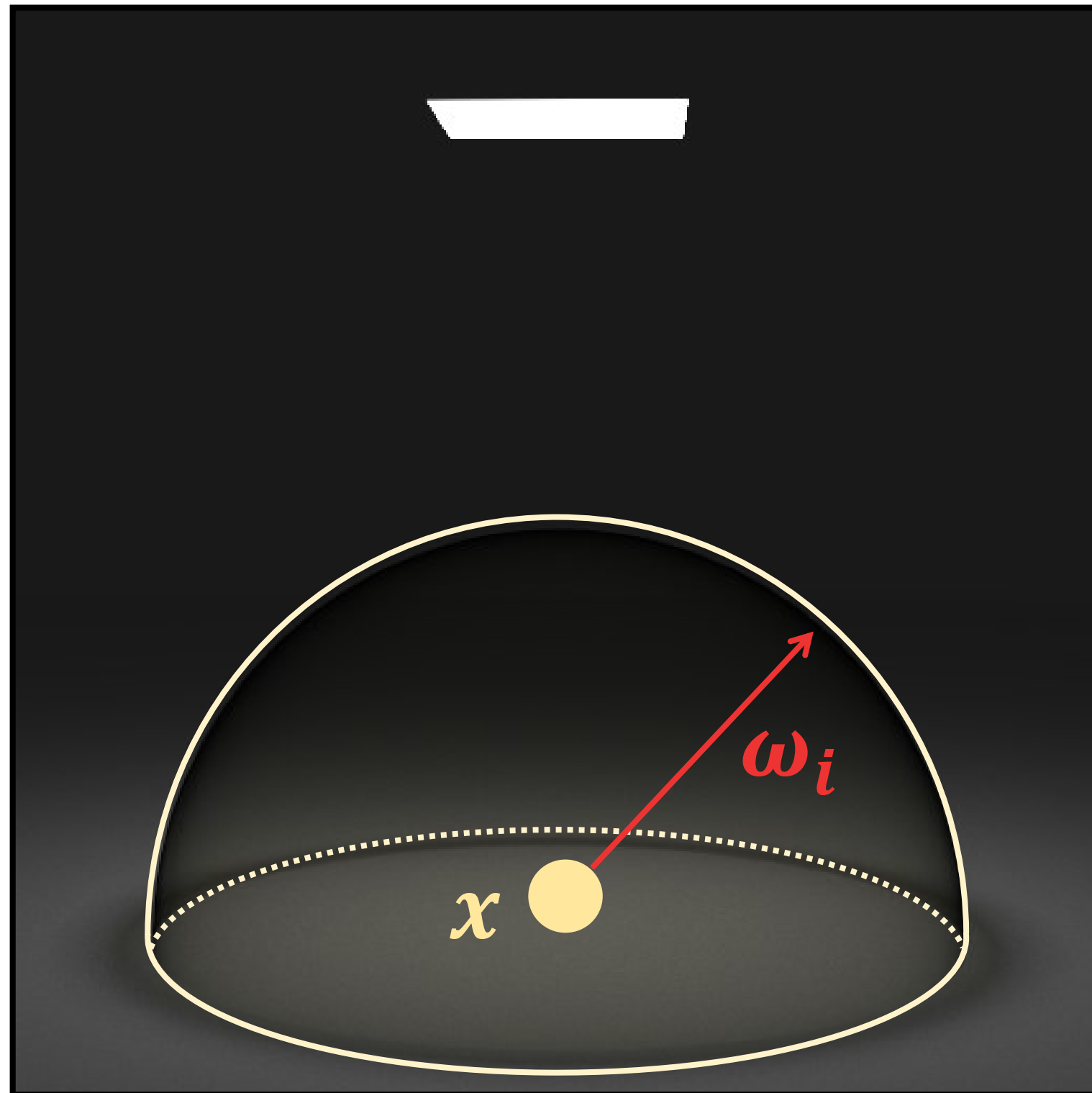
Surface integral



$$I = \int_{\mathcal{L}(\pi)} f(y \rightarrow x) G(x, y) dA(y)$$

# Reparameterizing the direct illumination integral

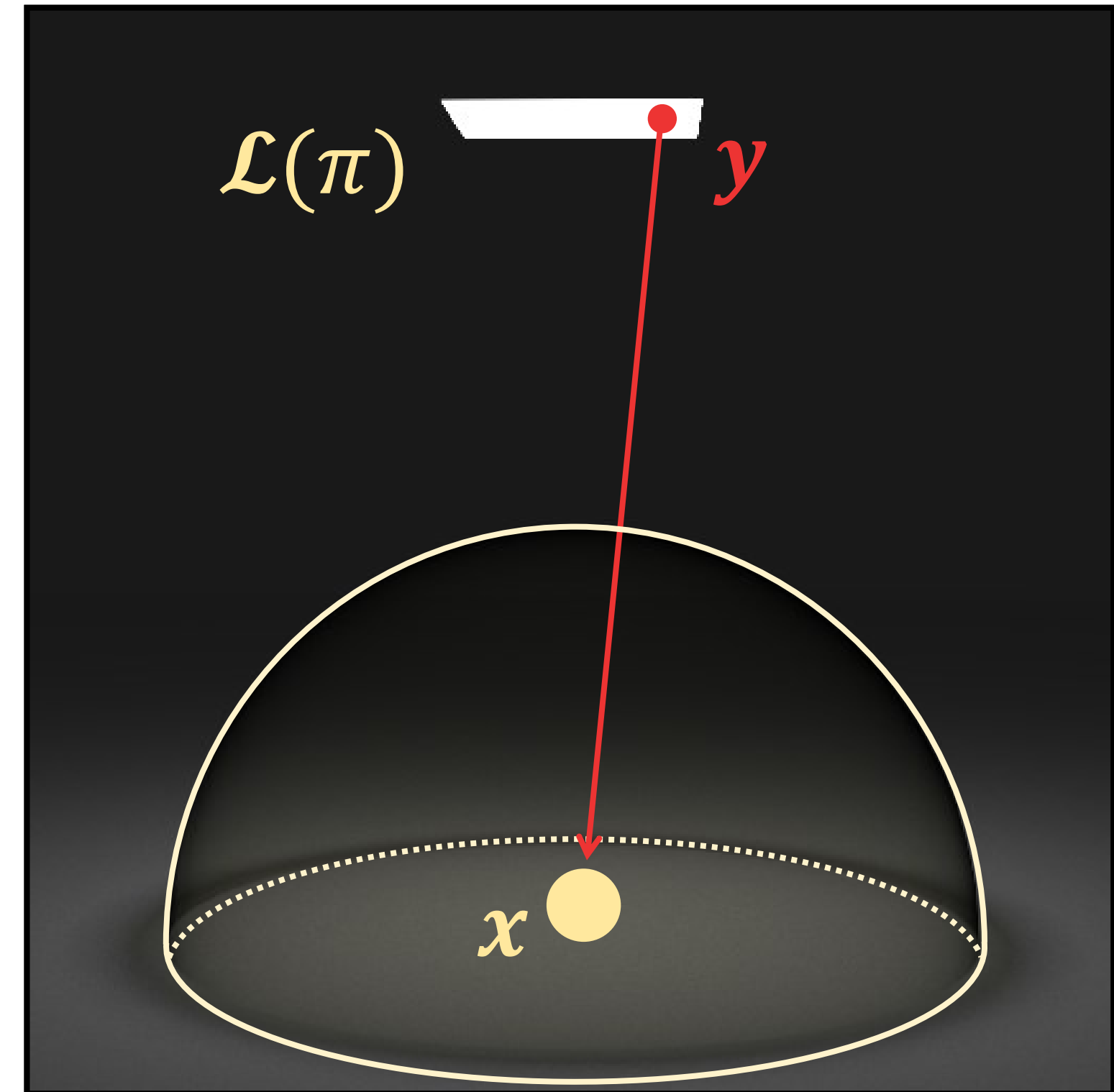
Hemispherical integral



$$I = \int_{\mathbb{H}^2} f(\omega_i) d\sigma(\omega_i)$$

Change of  
variables

Surface integral

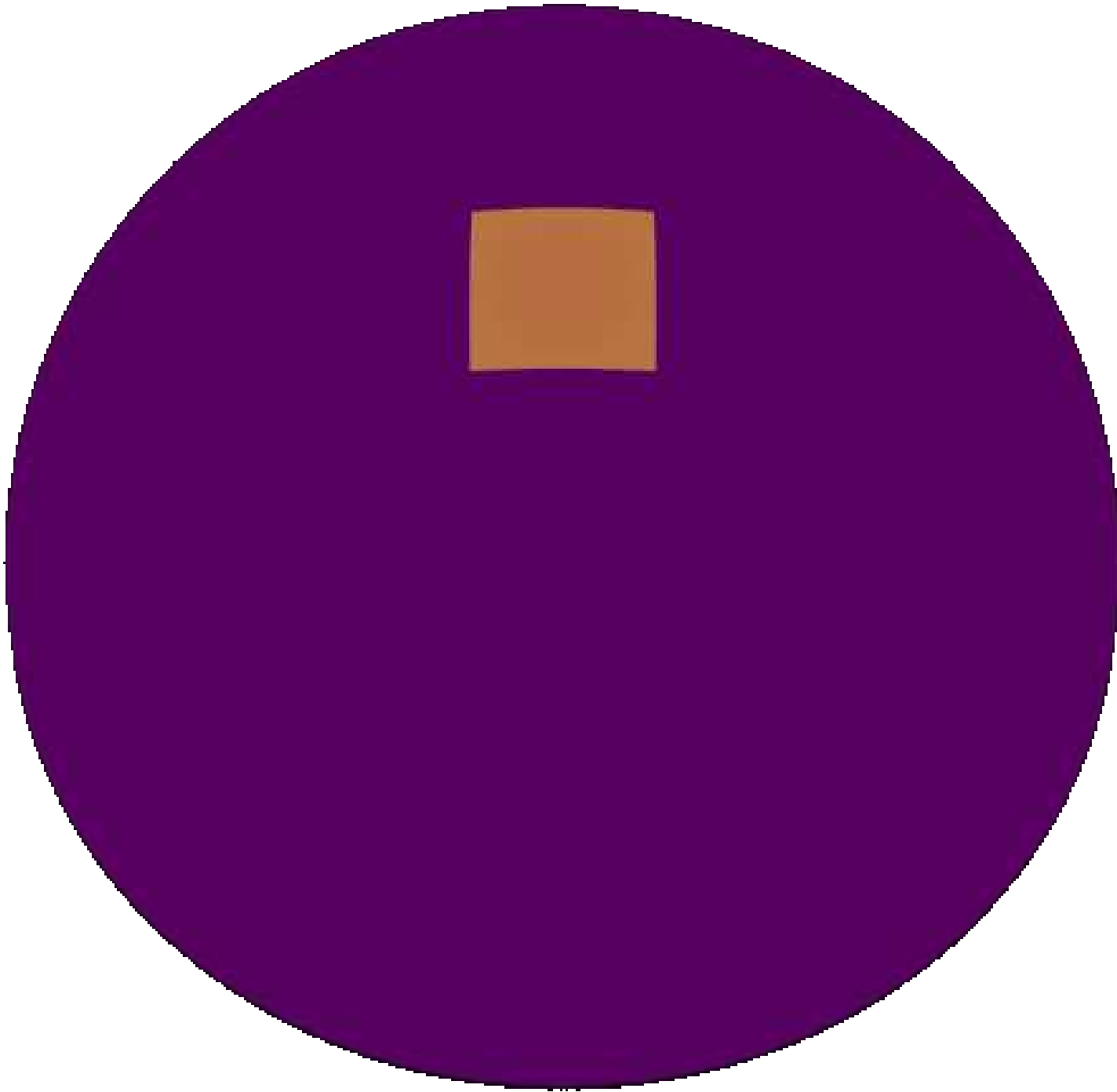


$$I = \int_{\mathcal{L}(\pi)} f(y \rightarrow x) G(x, y) dA(y)$$

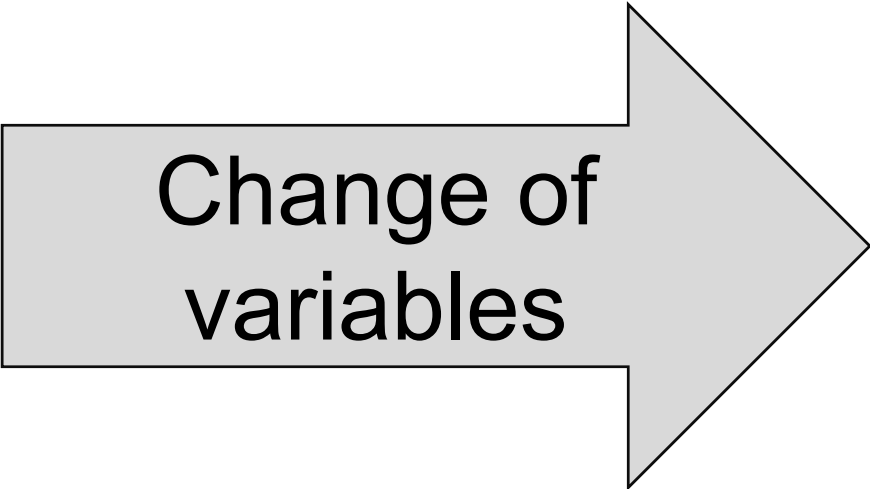
Includes visibility, fall-off,  
and foreshortening terms

# Reparameterizing the direct illumination integral

**Hemispherical integral**



Low  High



**Surface integral**



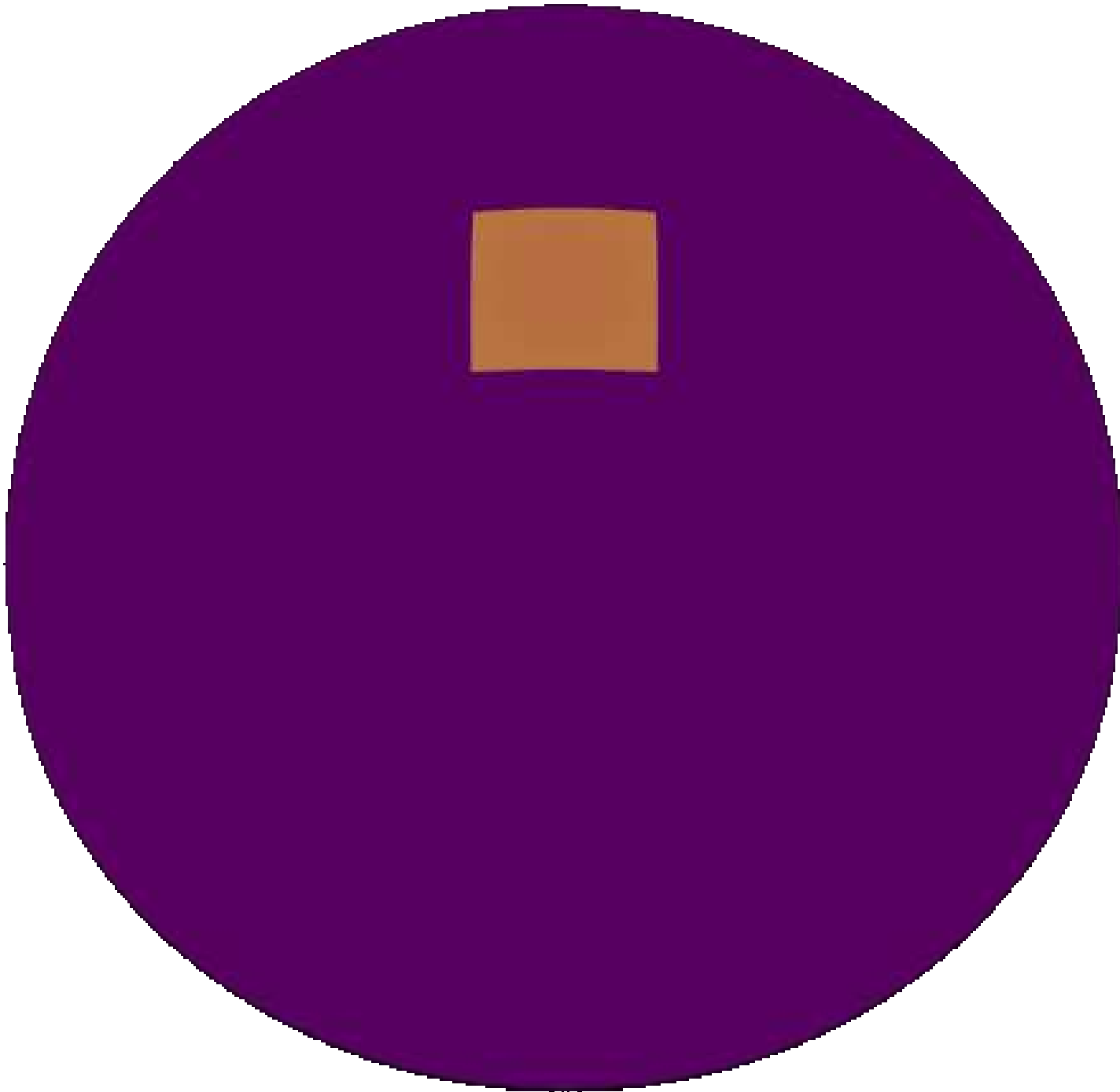
$$I = \int_{\mathbb{H}^2} f(\omega_i) \, d\sigma(\omega_i)$$

$$I = \int_{\mathcal{L}(\pi)} f(y \rightarrow x) \, G(x, y) \, dA(y)$$

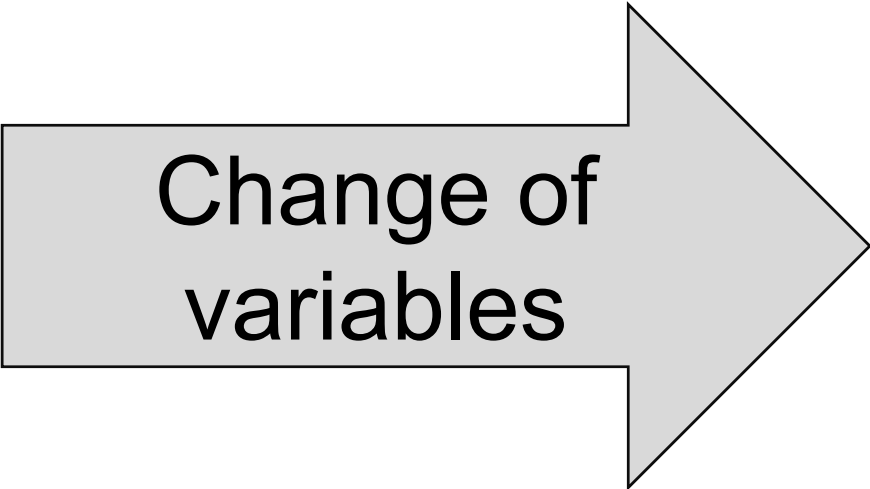


# Reparameterizing the direct illumination integral

Hemispherical integral



Low  High



Surface integral



discontinuous

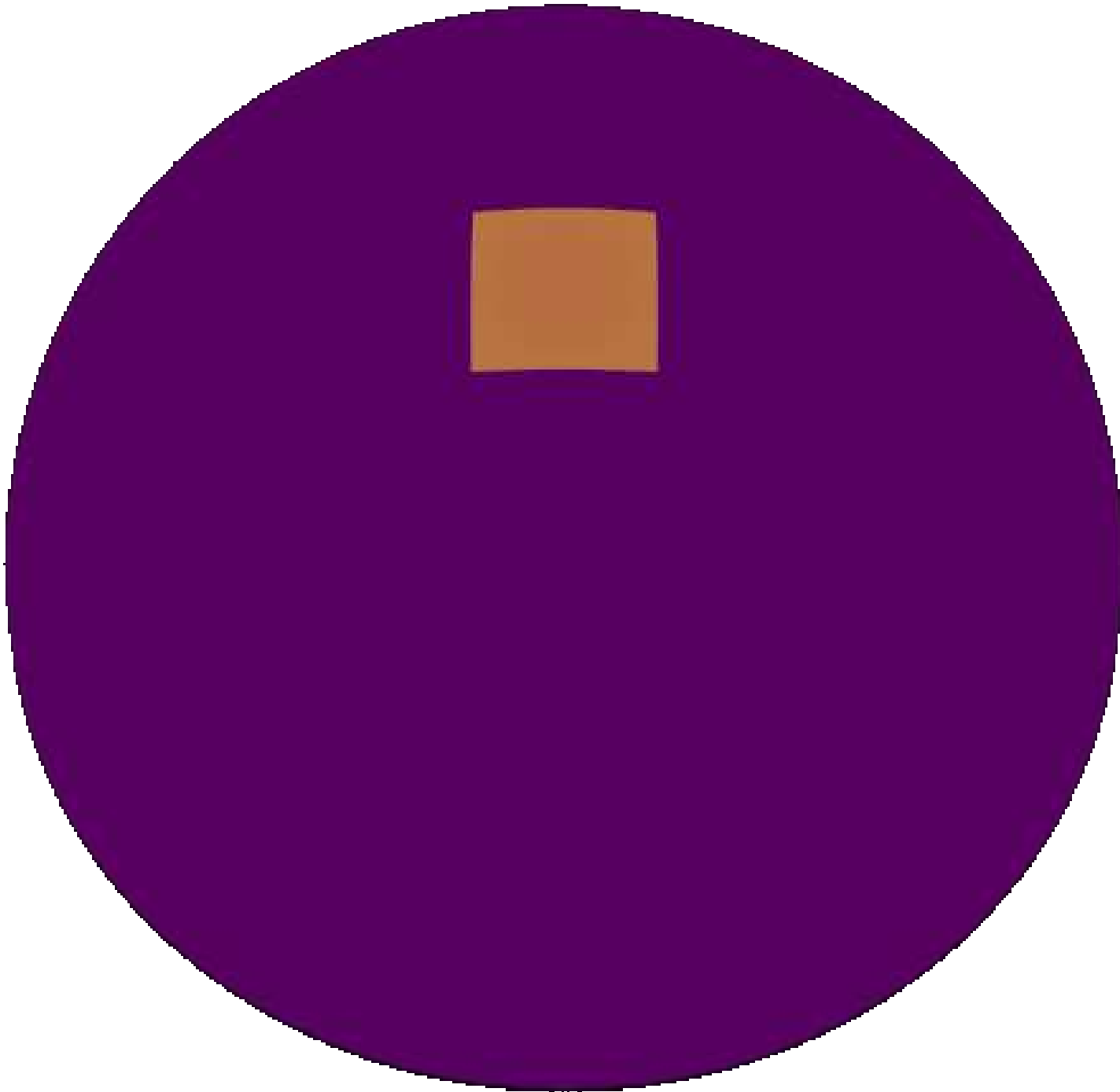
$$I = \int_{\mathbb{H}^2} f(\omega_i) \, d\sigma(\omega_i)$$

continuous

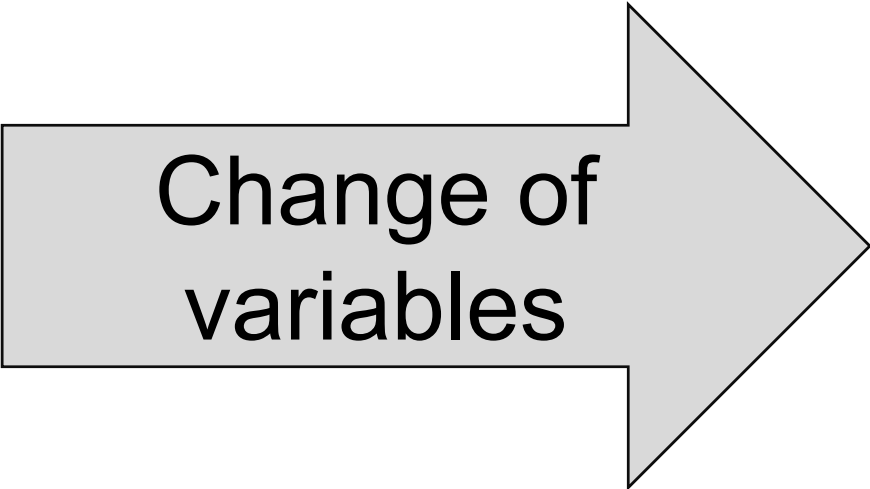
$$I = \int_{\mathcal{L}(\pi)} f(y \rightarrow x) G(x, y) \, dA(y)$$

# Reparameterizing the direct illumination integral

**Hemispherical integral**



Low  High



**Surface integral**



discontinuous

$$I = \int_{\mathbb{H}^2} f(\omega_i) \, d\sigma(\omega_i)$$

constant domain

continuous

$$I = \int_{\mathcal{L}(\pi)} f(y \rightarrow x) G(x, y) \, dA(y)$$

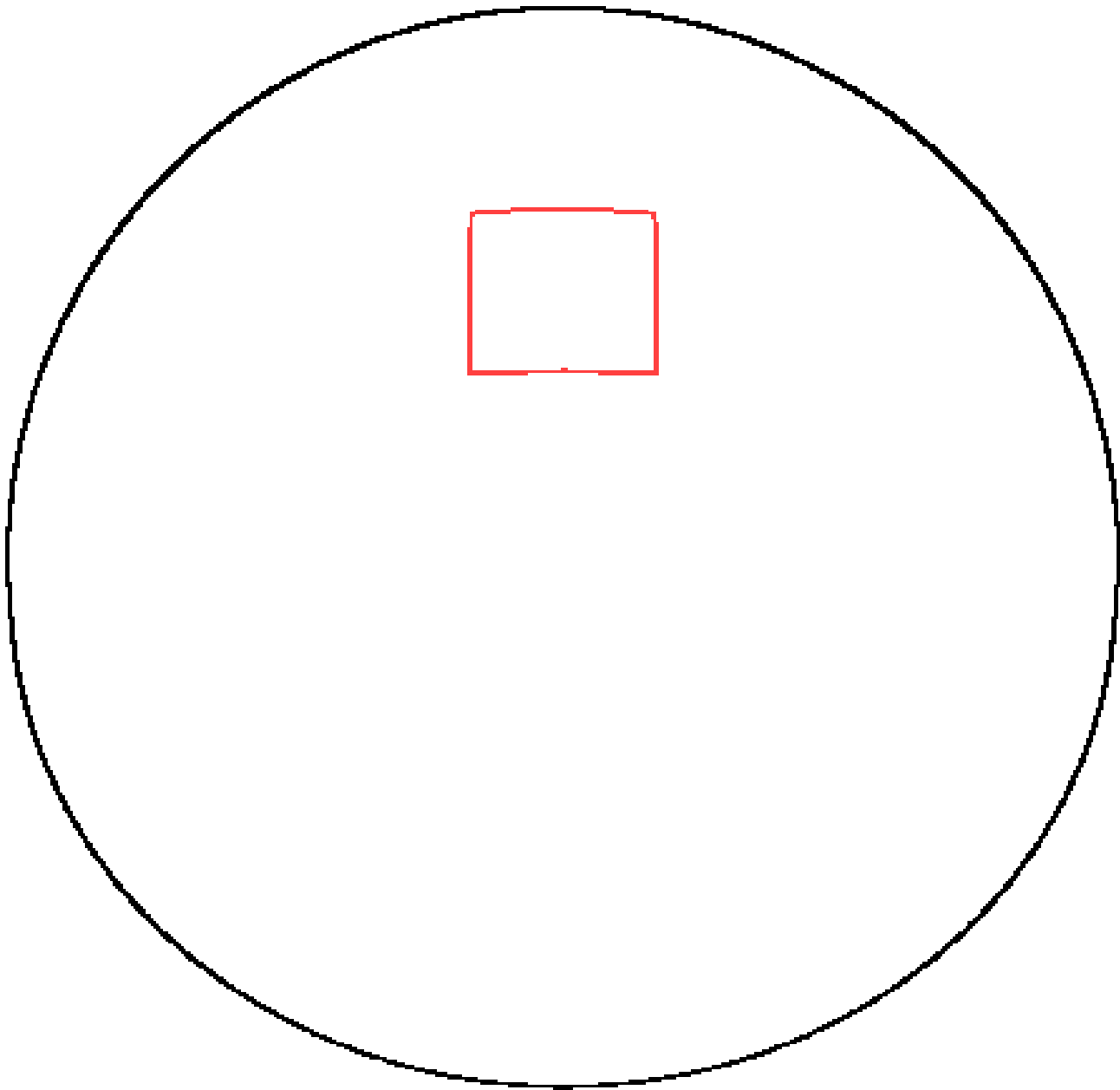
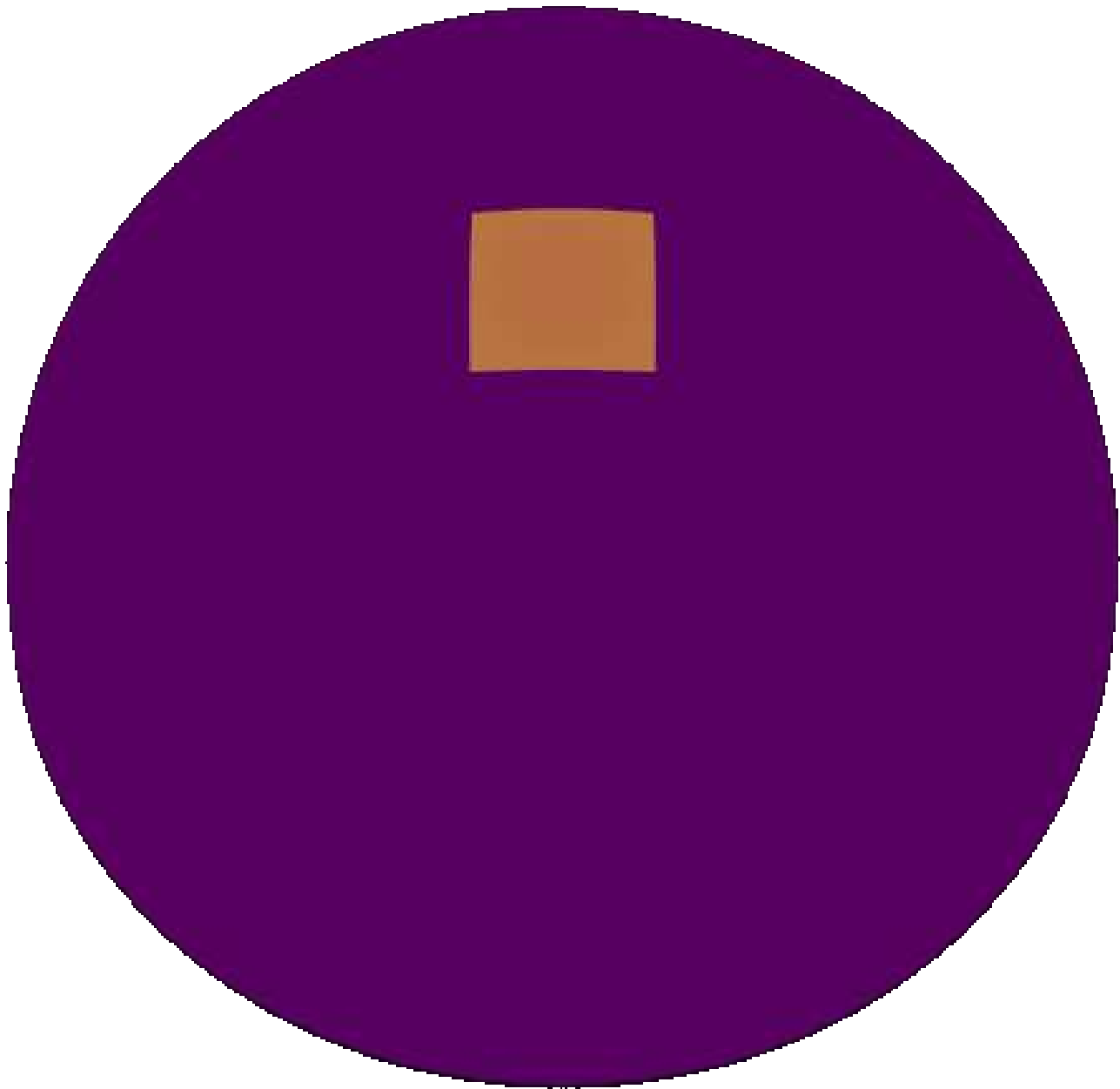
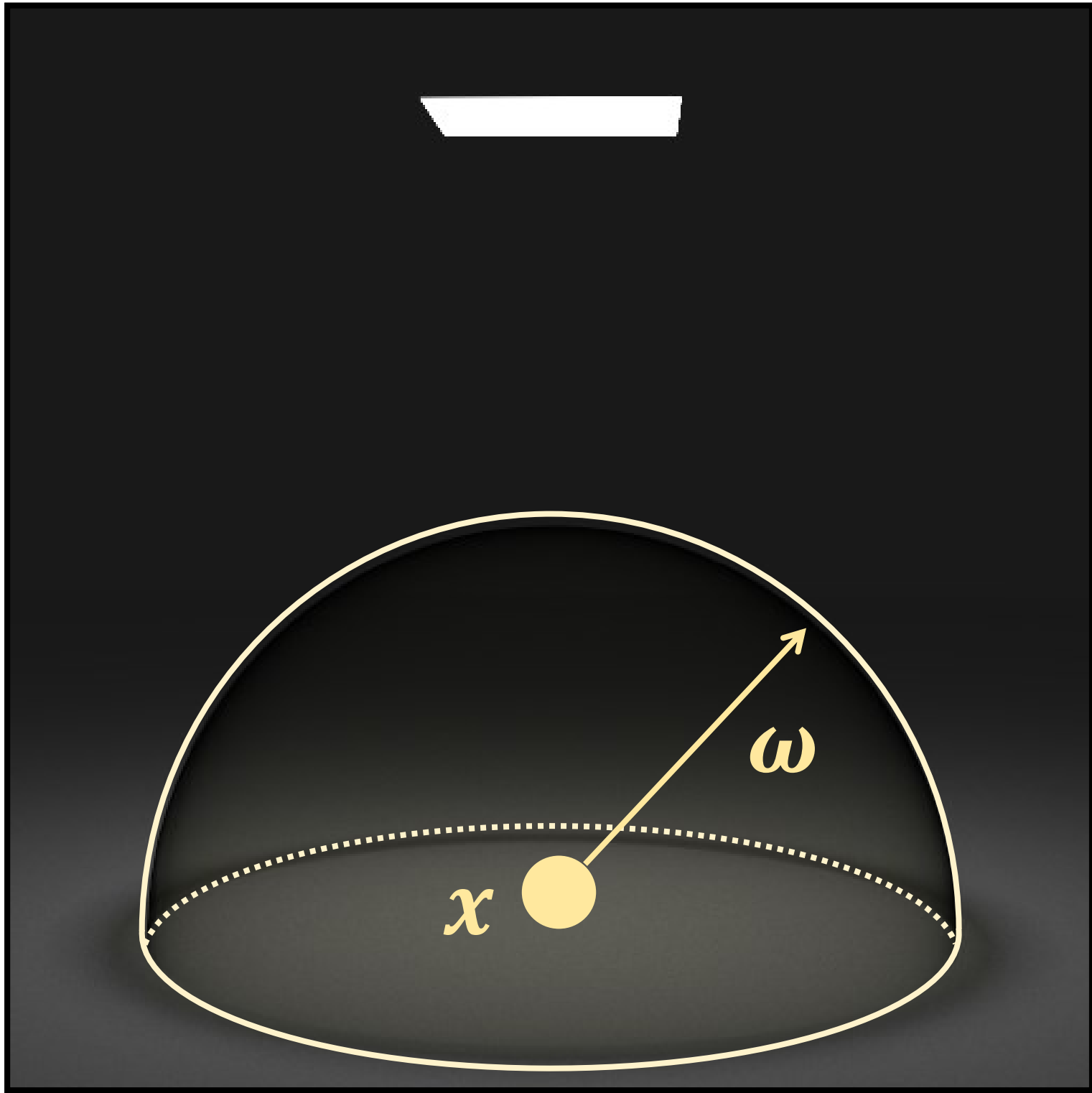
evolving domain

# Differentiating the hemispherical integral

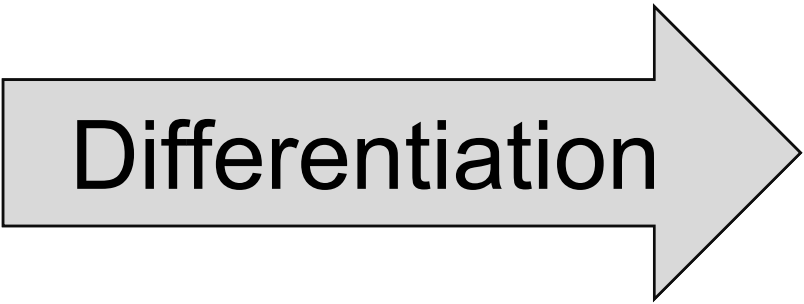
$\pi$ : size of the emitter

Low  High

Discontinuities of  $f$



$$I = \int_{\mathbb{H}^2} f(\omega_i) d\sigma(\omega_i)$$



Reynolds transport theorem

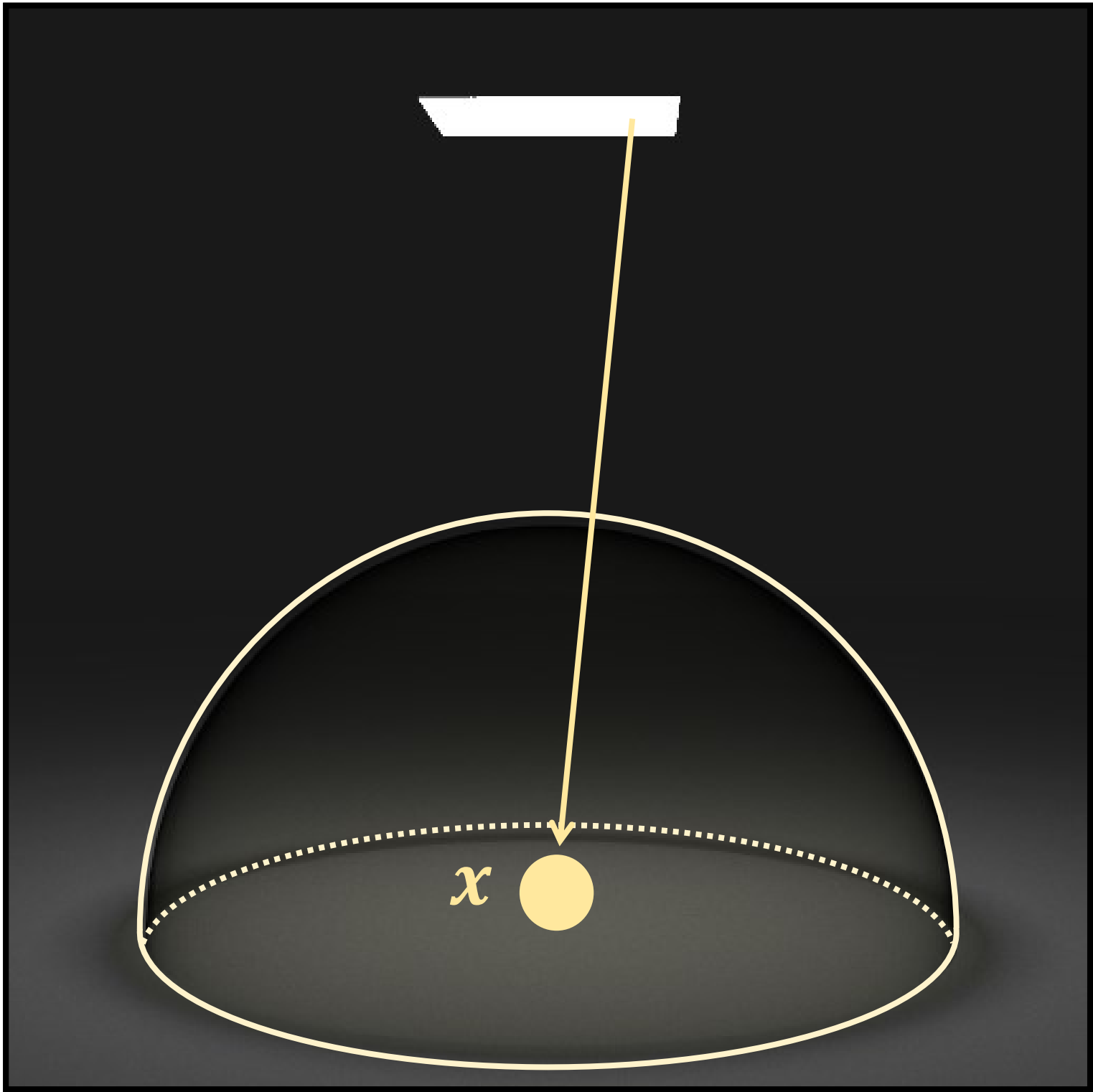
$$\frac{dI}{d\pi} = \underbrace{\int_{\mathbb{H}^2} \frac{d(f)}{d\pi} d\sigma}_{\text{Interior}} + \underbrace{\int_{\partial \mathbb{H}^2} g dl}_{\text{Boundary}}$$



# Differentiating the area integral

$\pi$ : size of the emitter

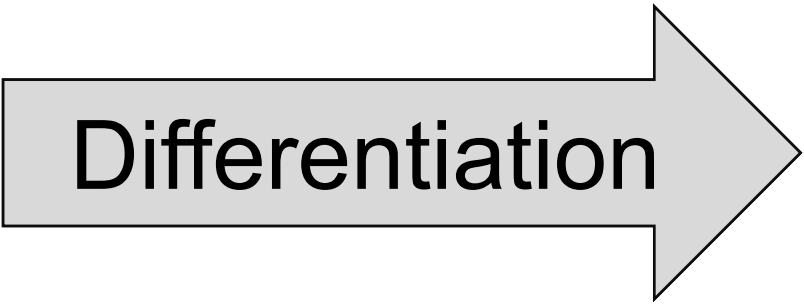
Low  High



Boundary of  $\mathcal{L}(\pi)$



$$I = \int_{\mathcal{L}(\pi)} f(y \rightarrow x) G(x, y) dA(y)$$



Reynolds transport theorem

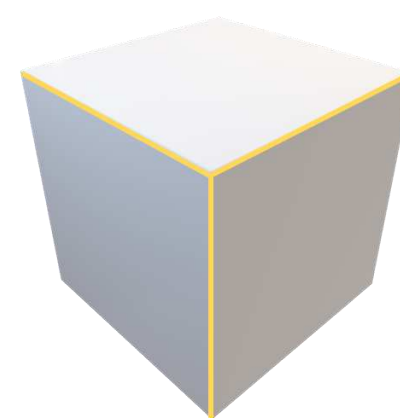
$$\frac{dI}{d\pi} = \underbrace{\int_{\mathcal{L}(\pi)} \frac{d(fG)}{d\pi} dA}_{\text{Interior}} + \underbrace{\int_{\partial \mathcal{L}(\pi)} g dl}_{\text{Boundary}}$$

# Sources of discontinuities

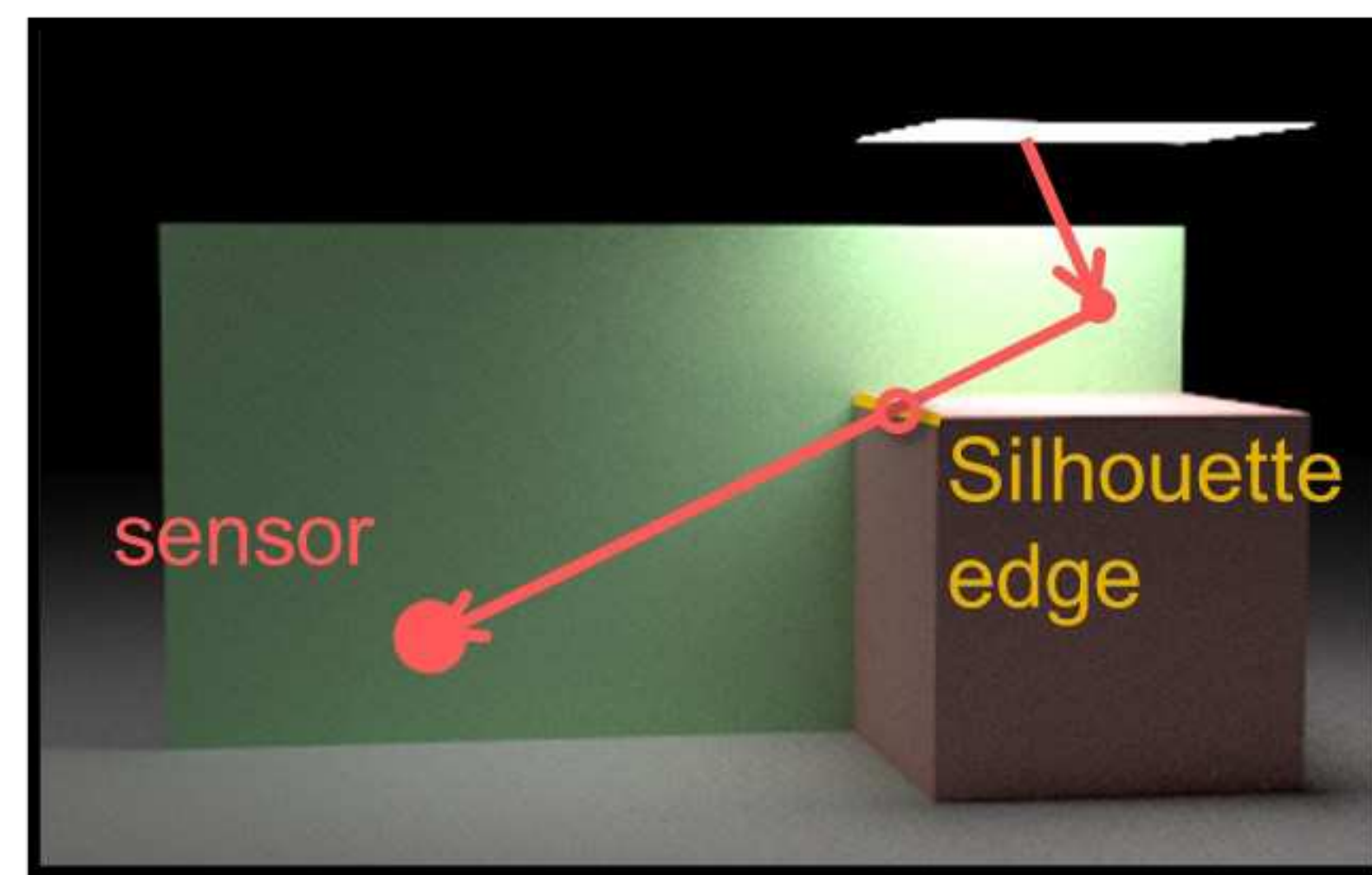
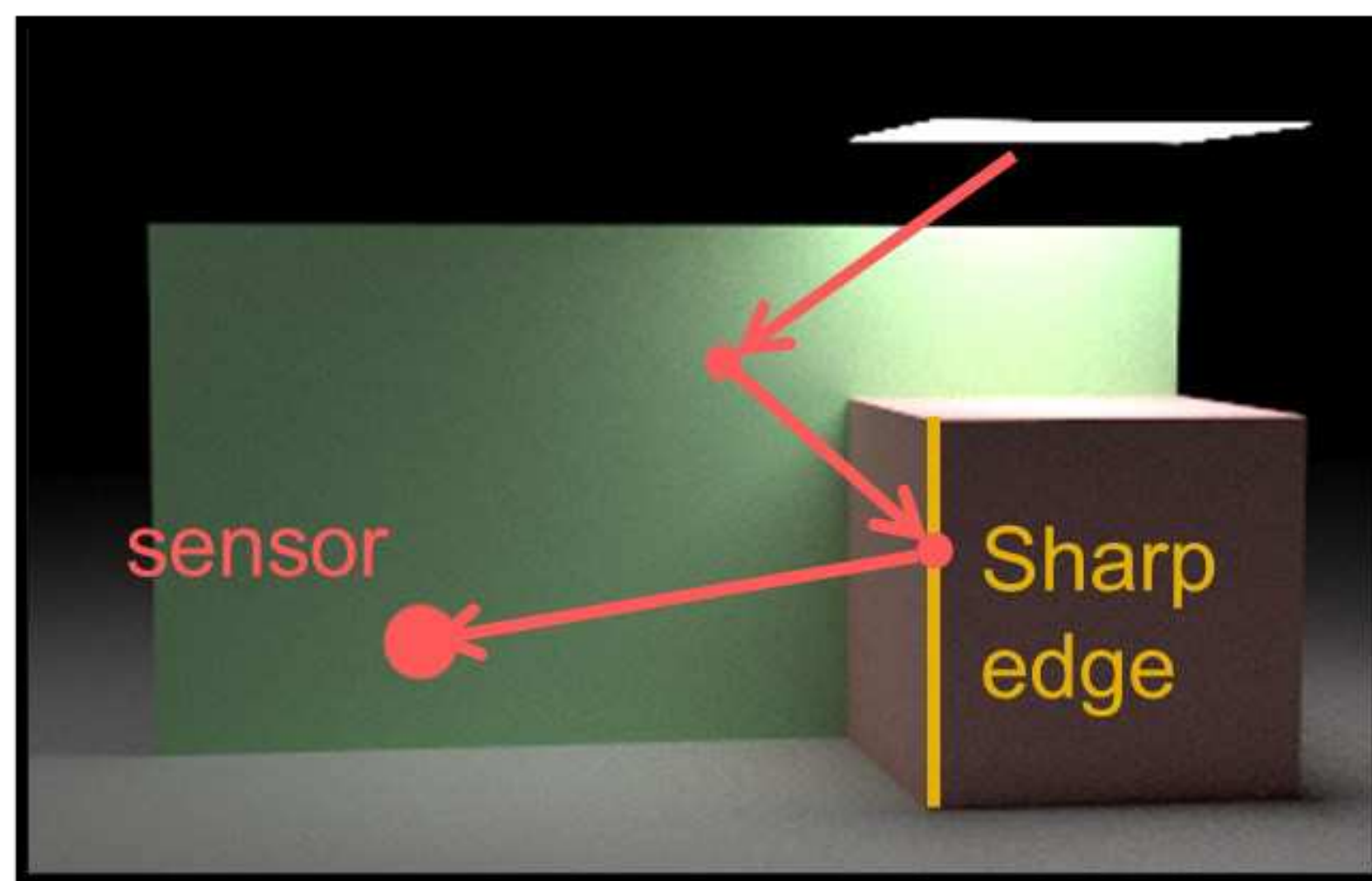
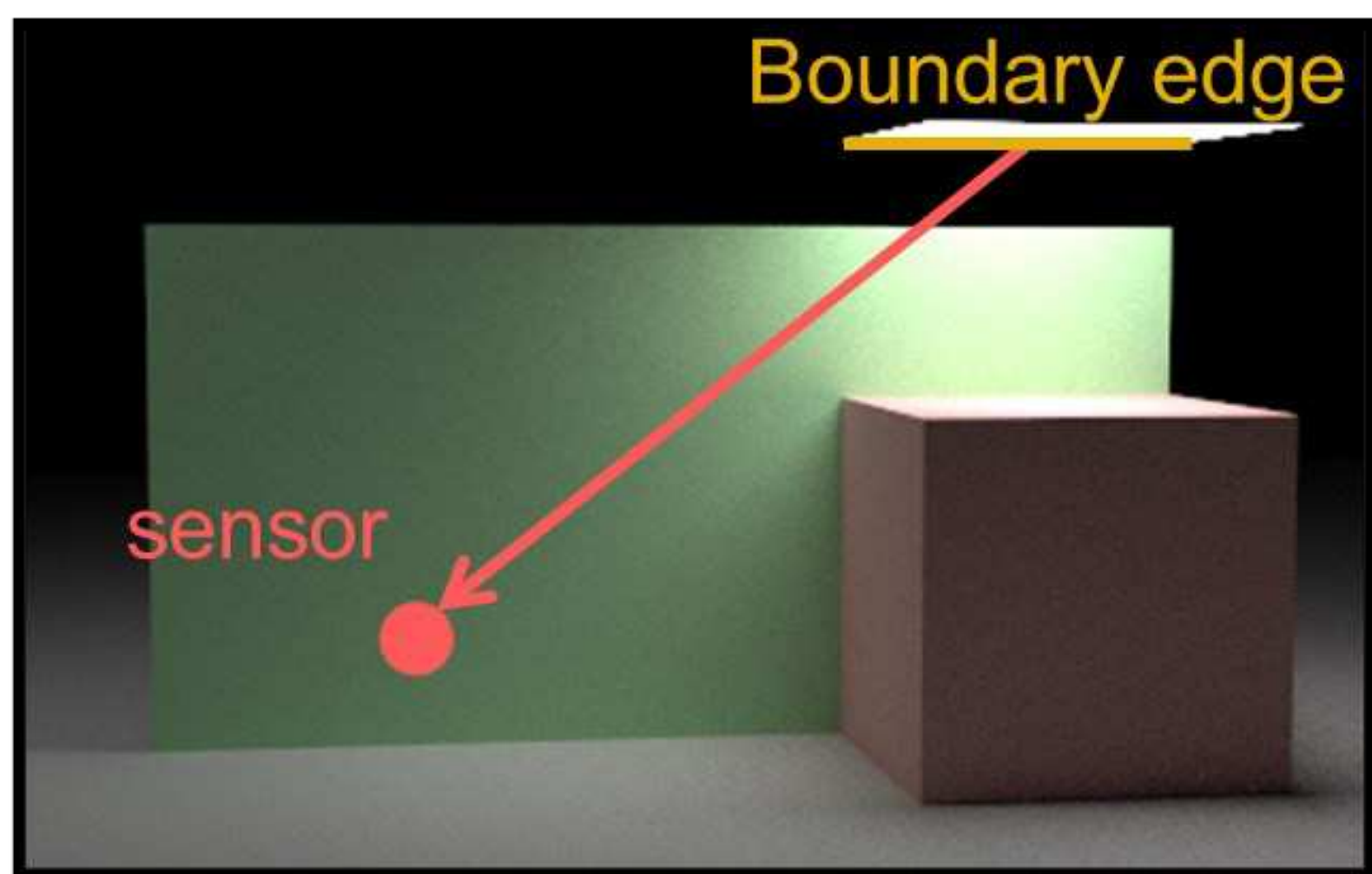
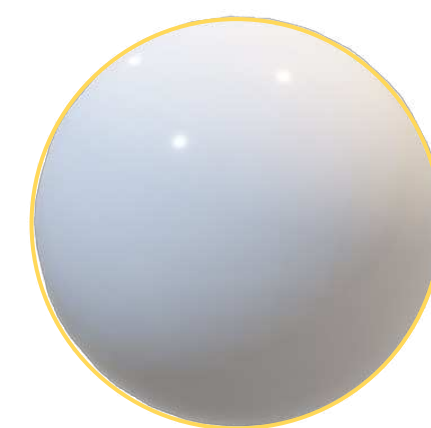
Boundary edge



Sharp edge



Silhouette edge

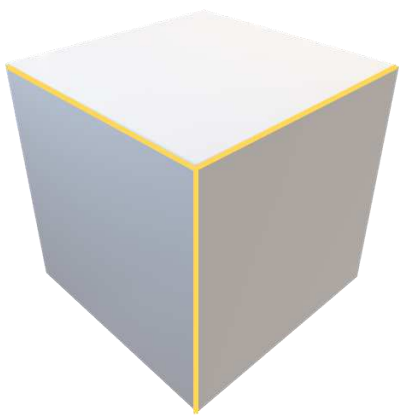


# Sources of discontinuities

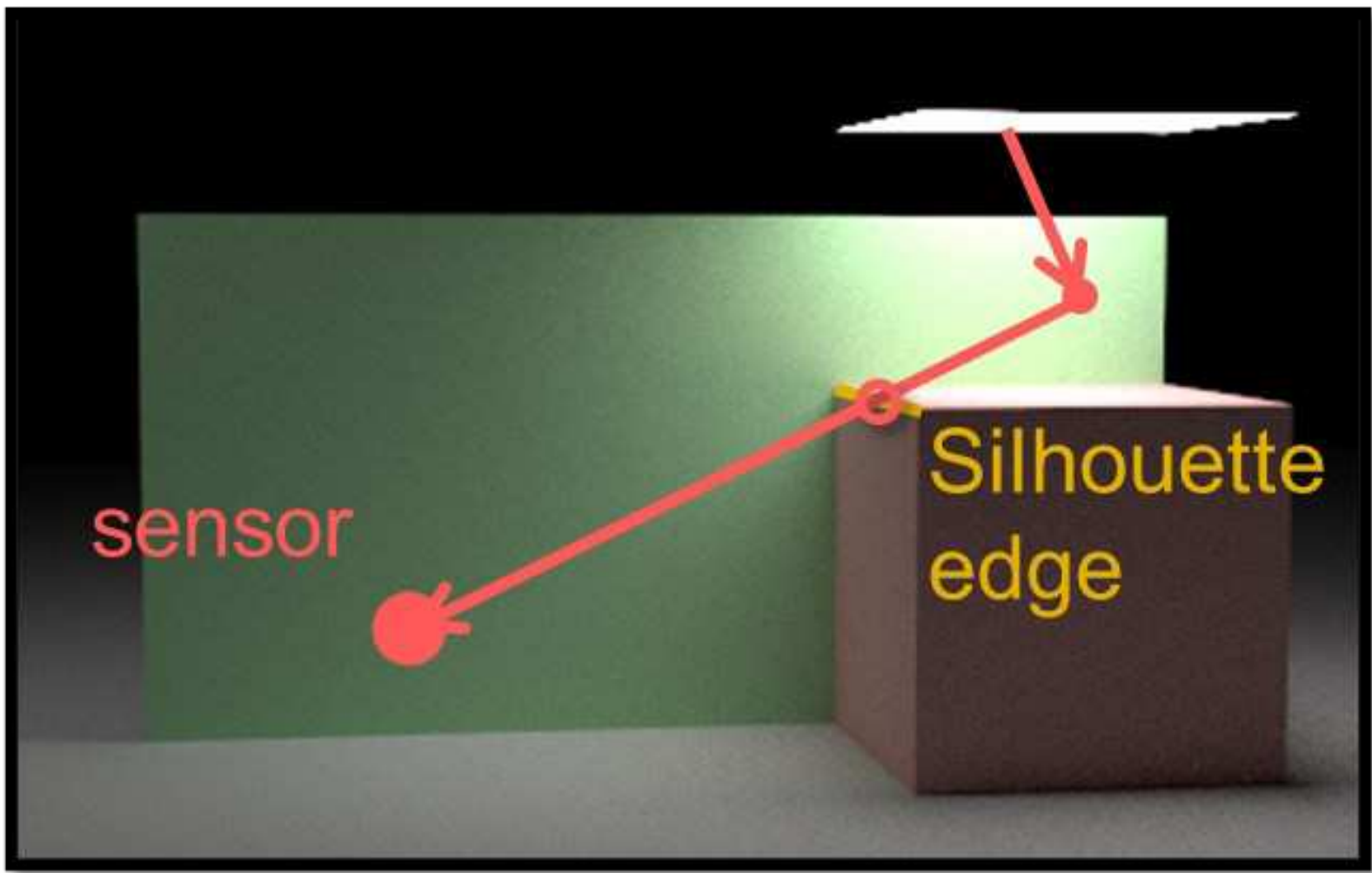
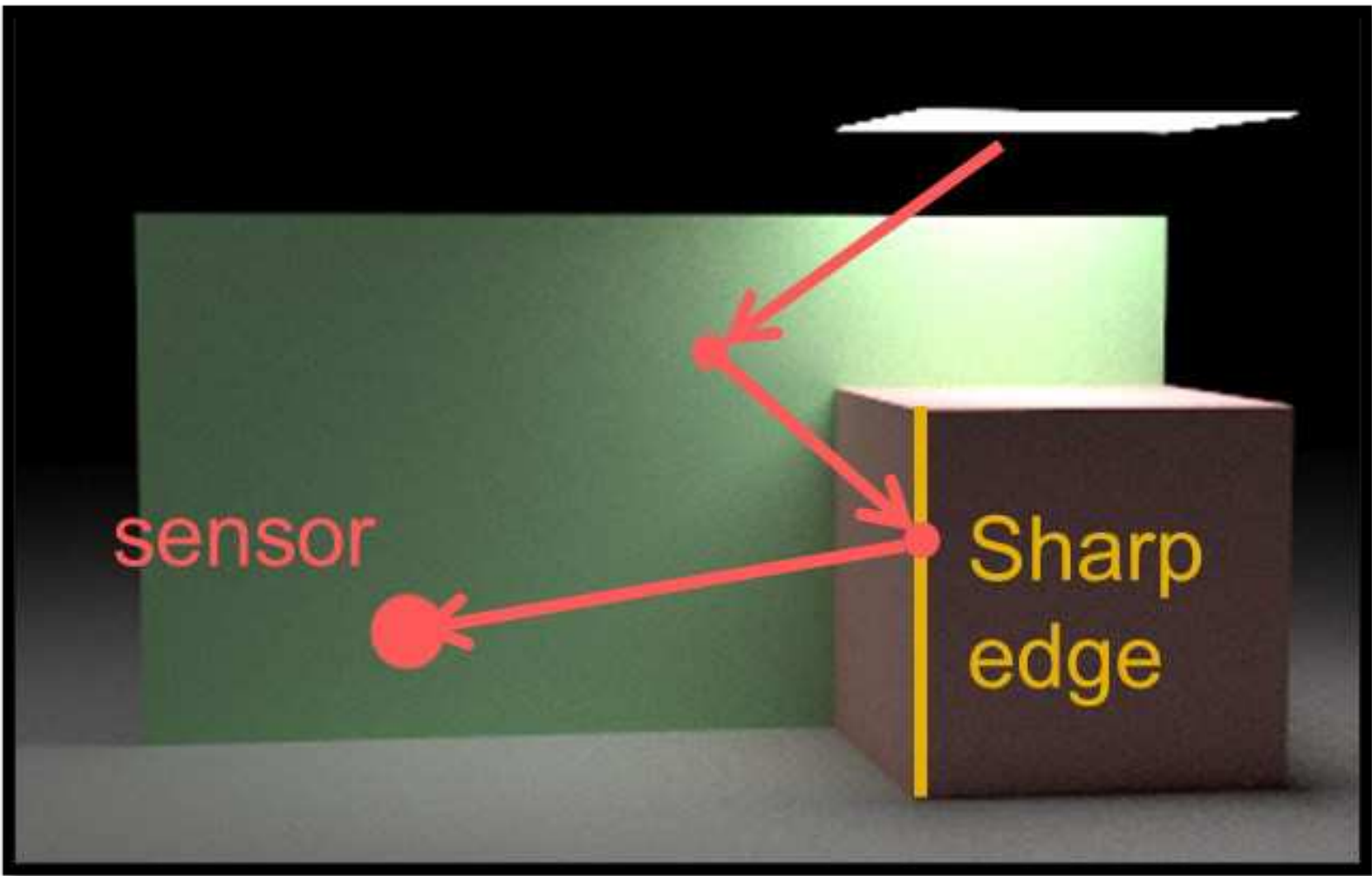
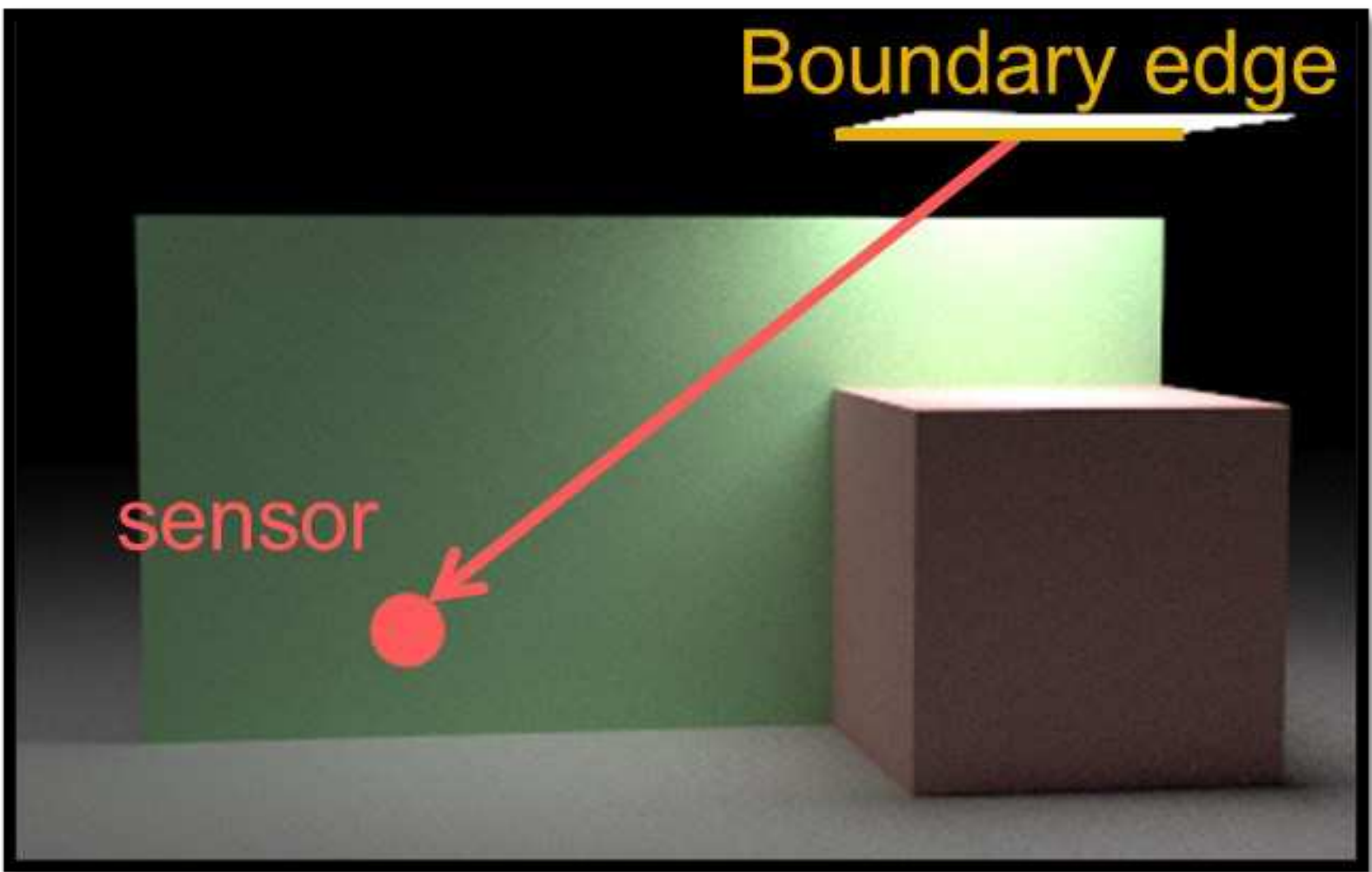
Boundary edge



Sharp edge



Silhouette edge



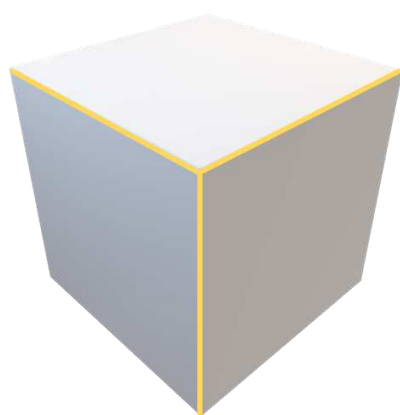


# Sources of discontinuities

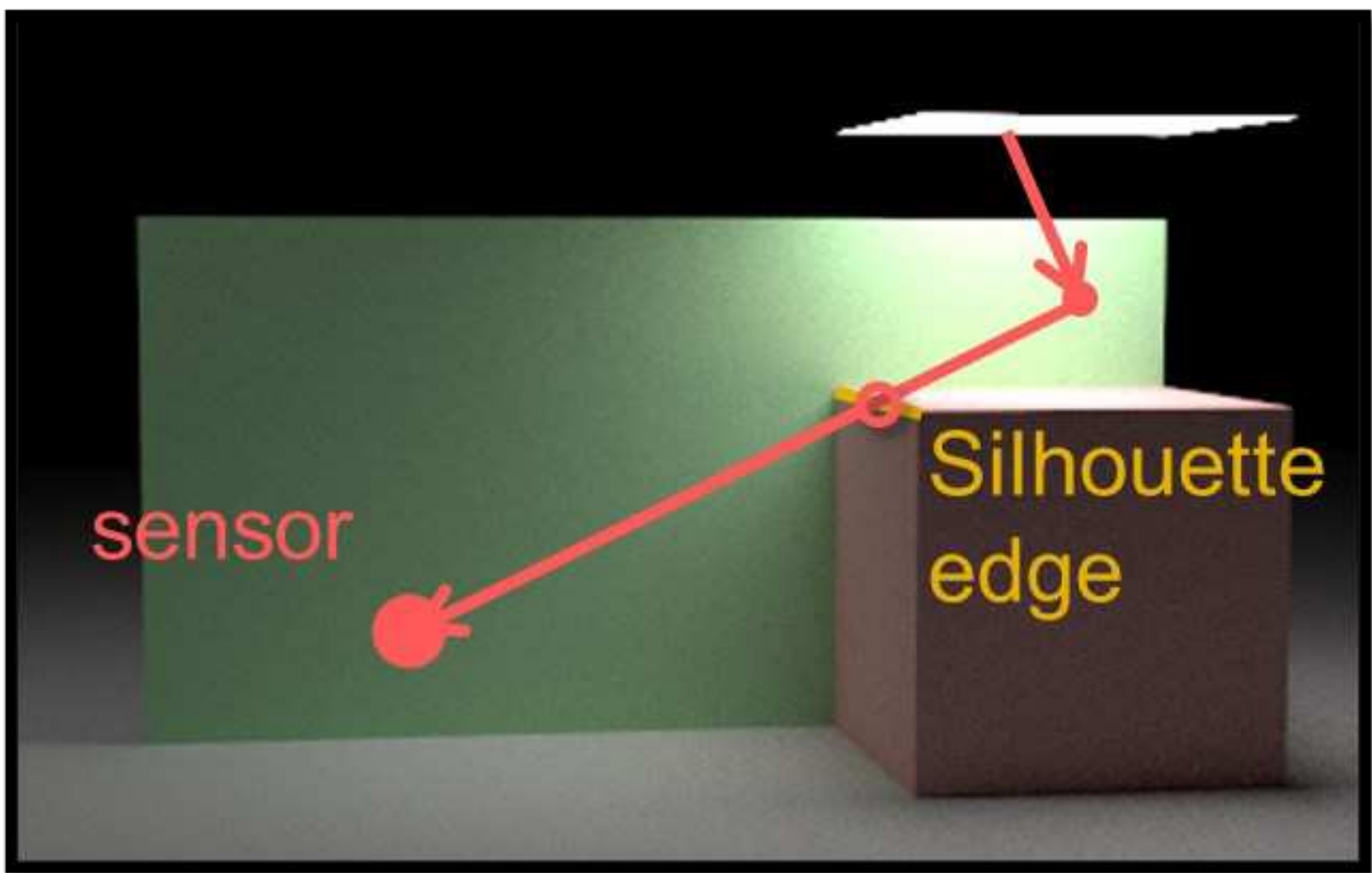
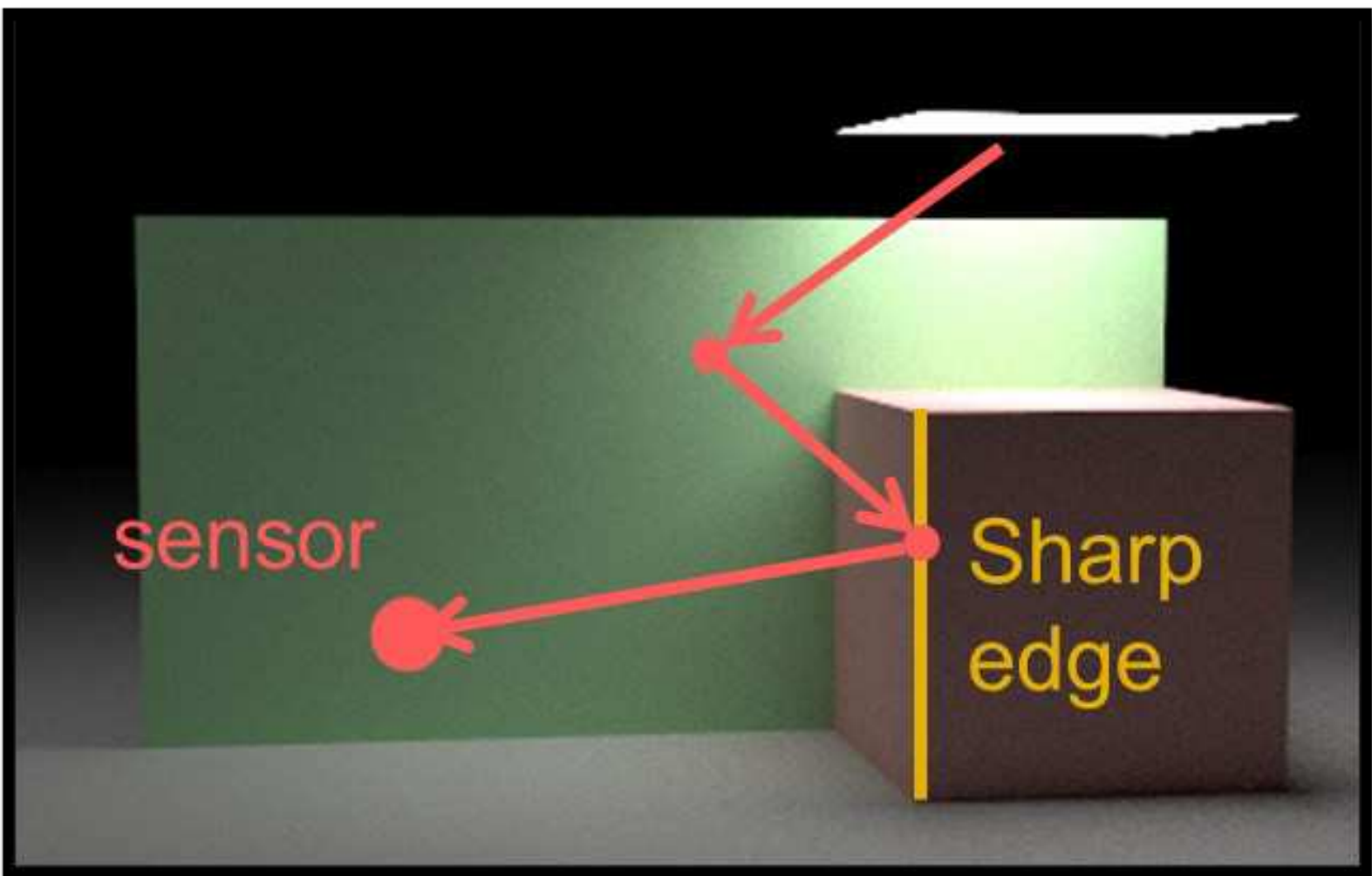
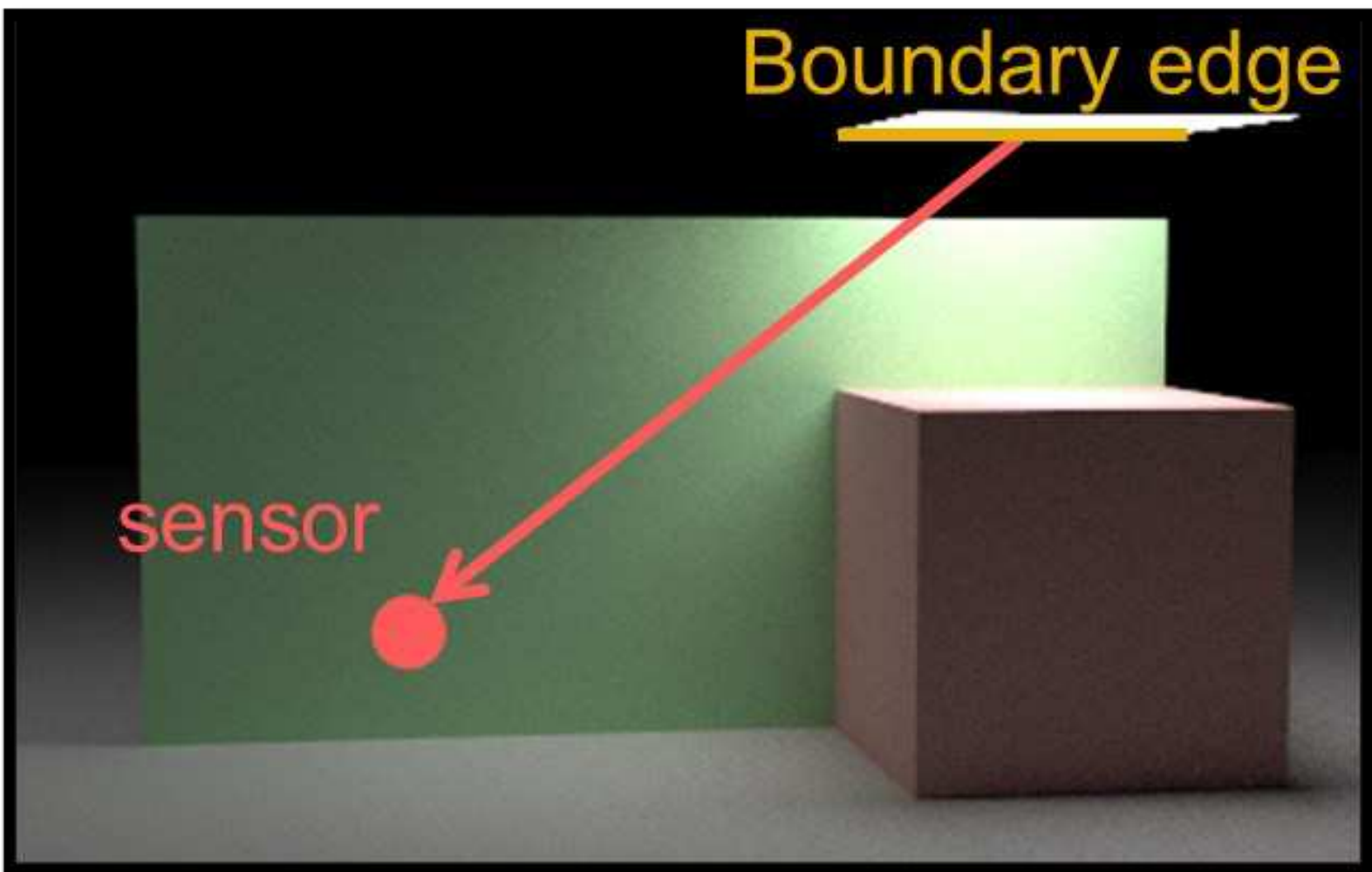
Boundary edge



Sharp edge

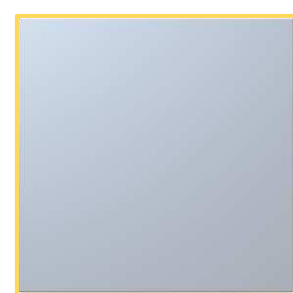


Silhouette edge

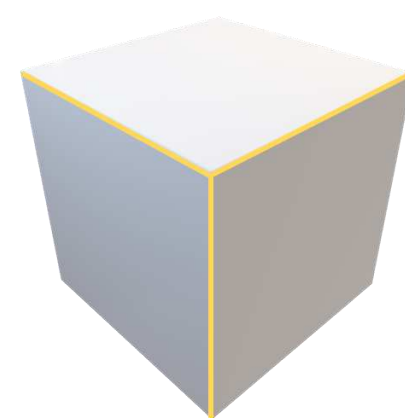


# Sources of discontinuities

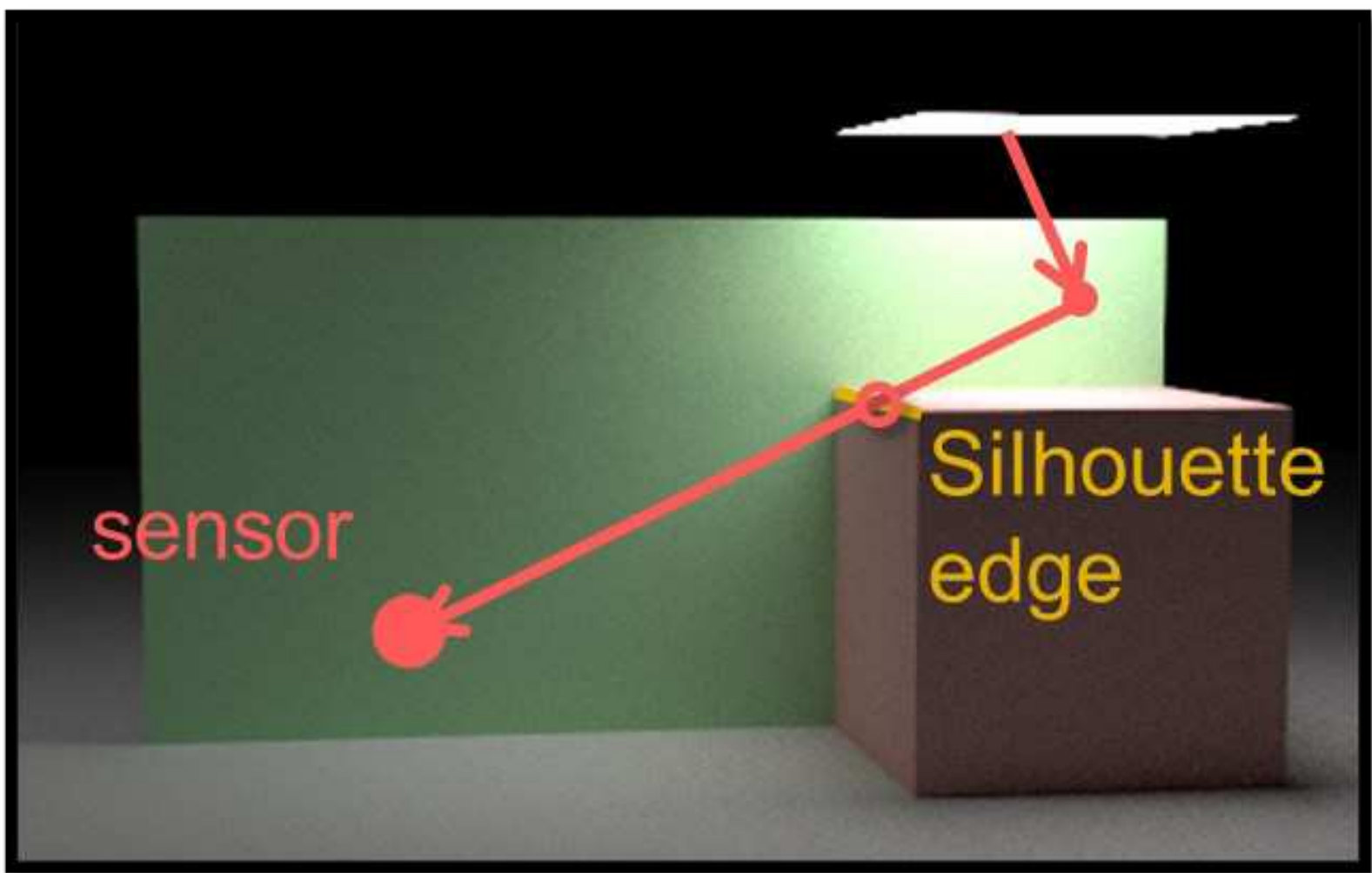
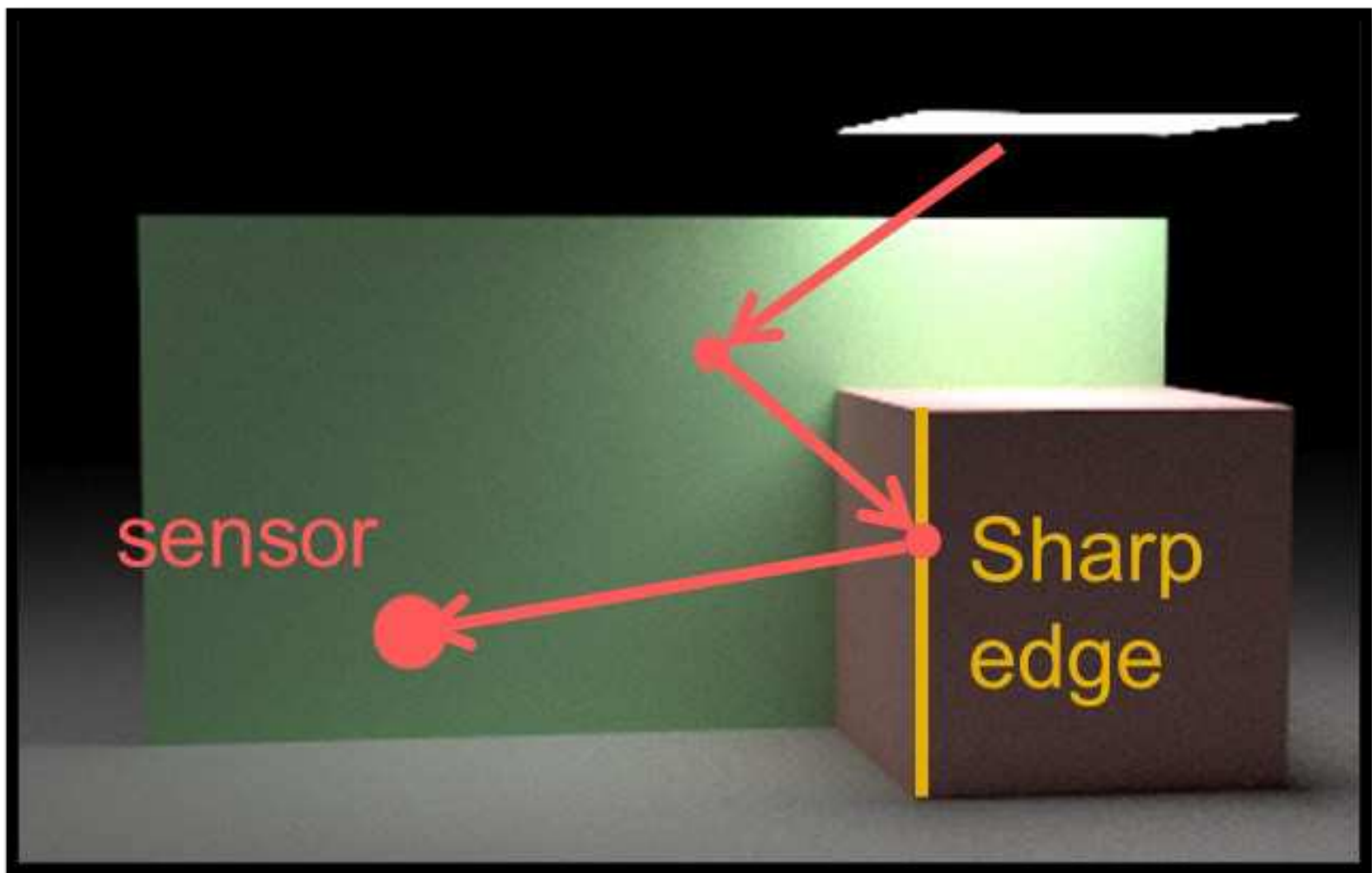
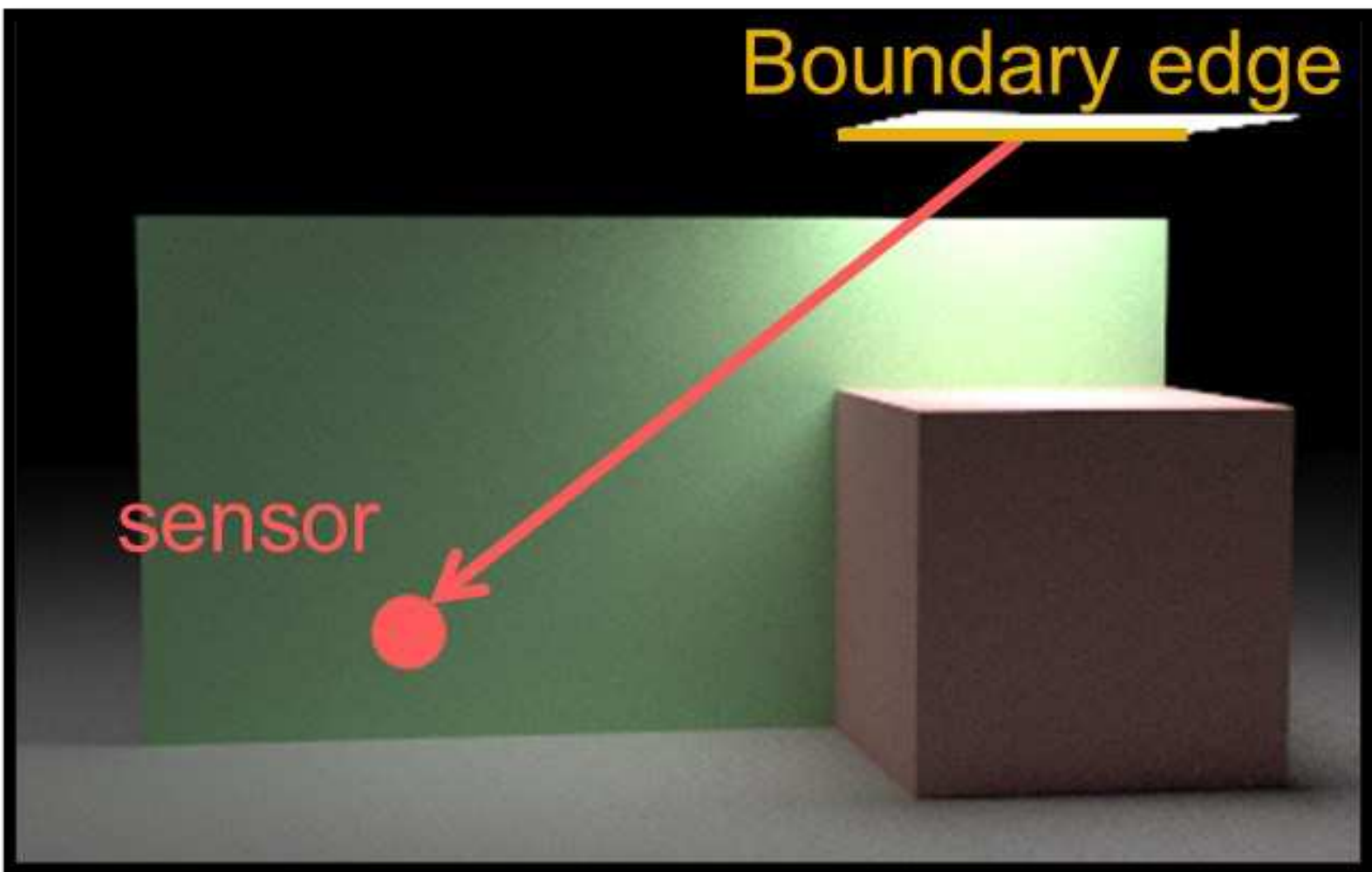
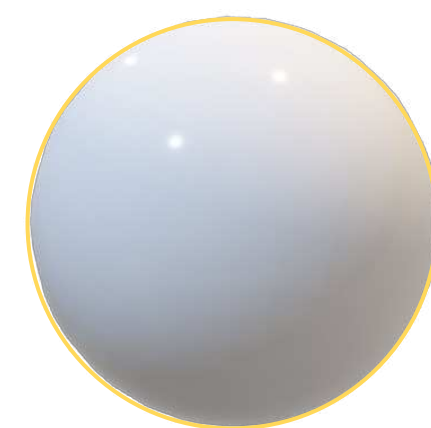
Boundary edge



Sharp edge



Silhouette edge



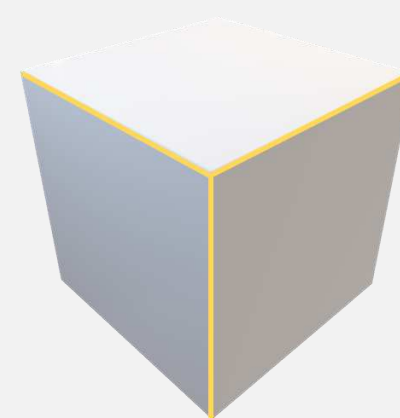


# Sources of discontinuities

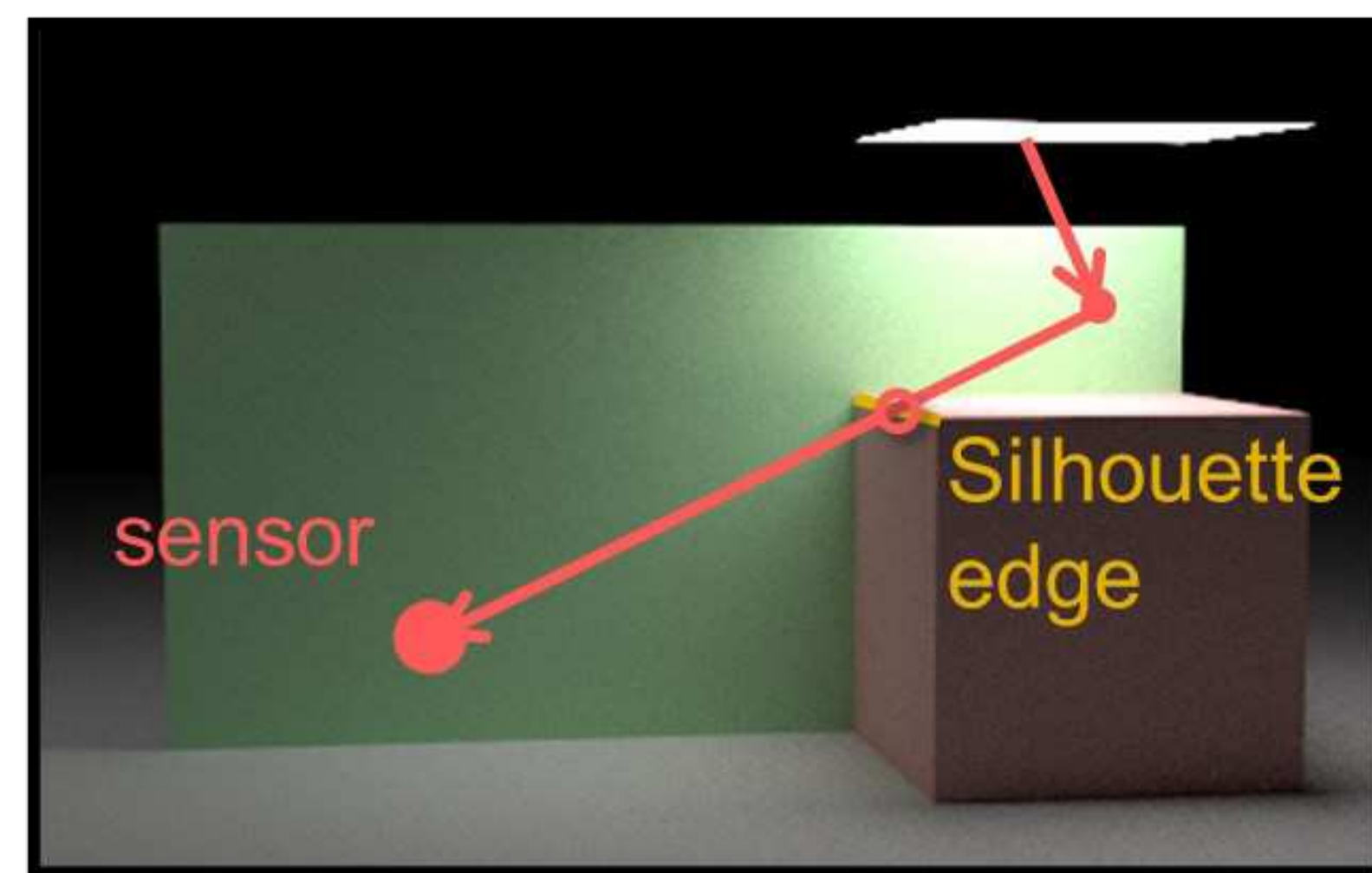
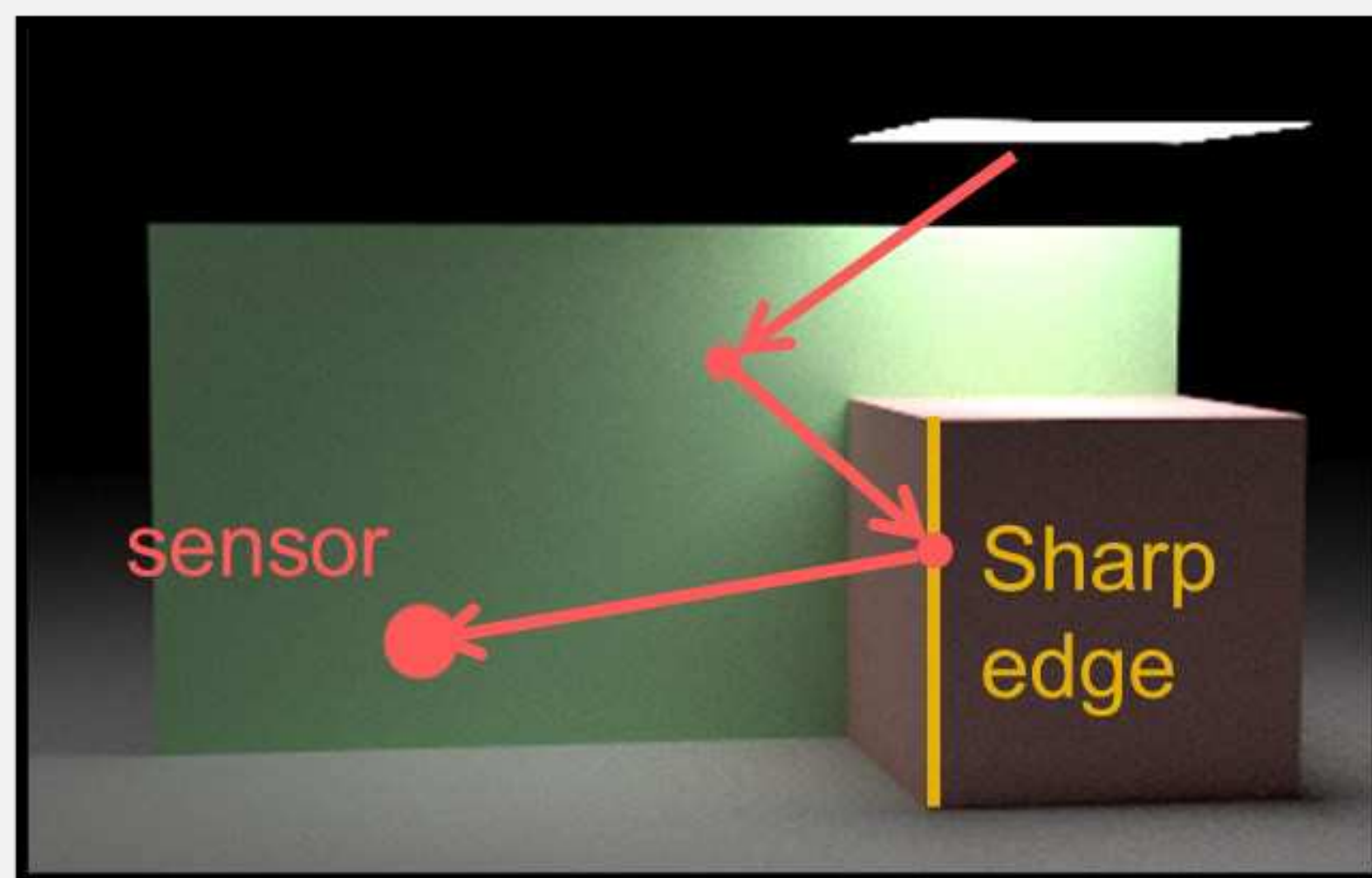
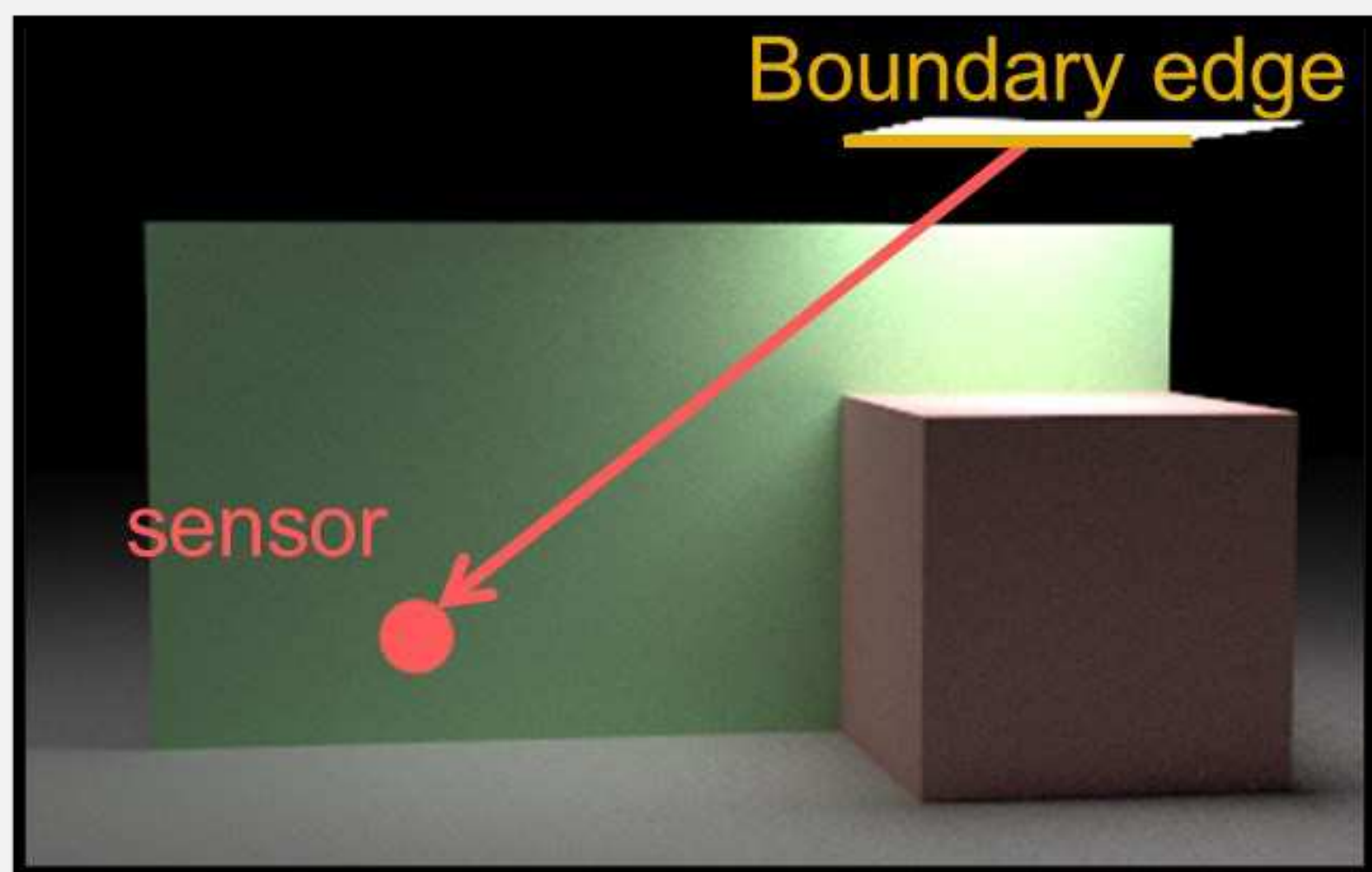
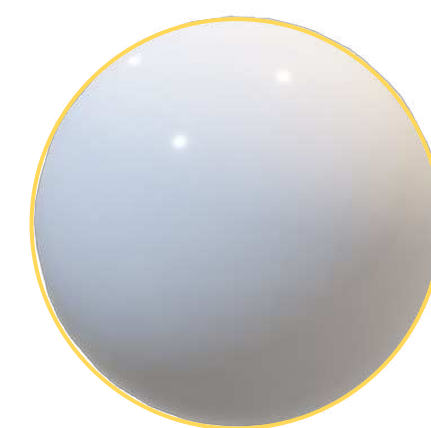
Boundary edge



Sharp edge



Silhouette edge



Topology-driven

Visibility-driven

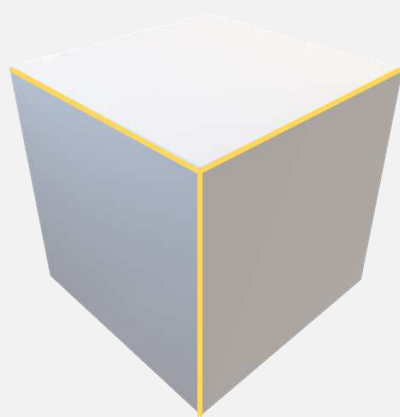


# Sources of discontinuities

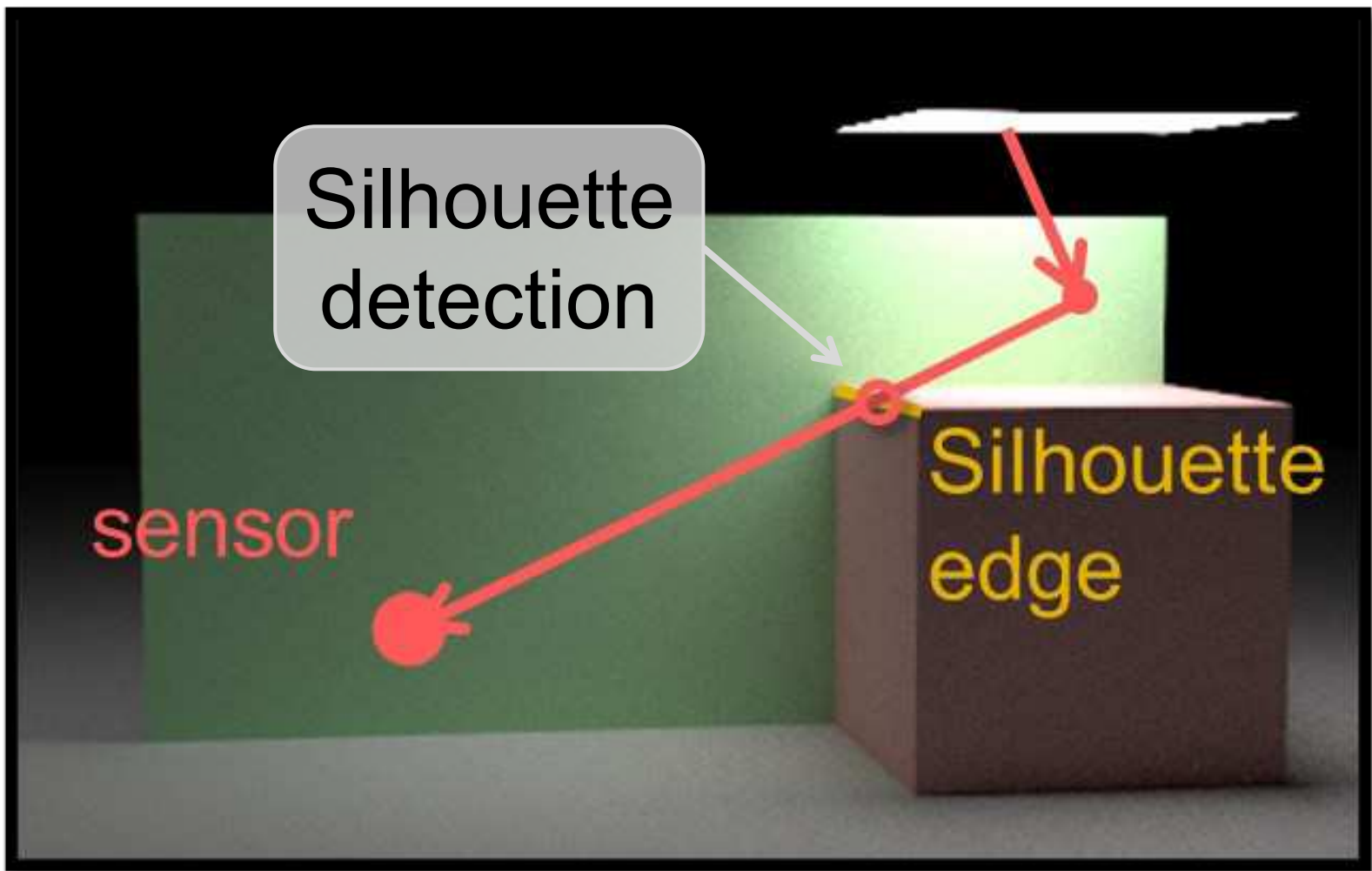
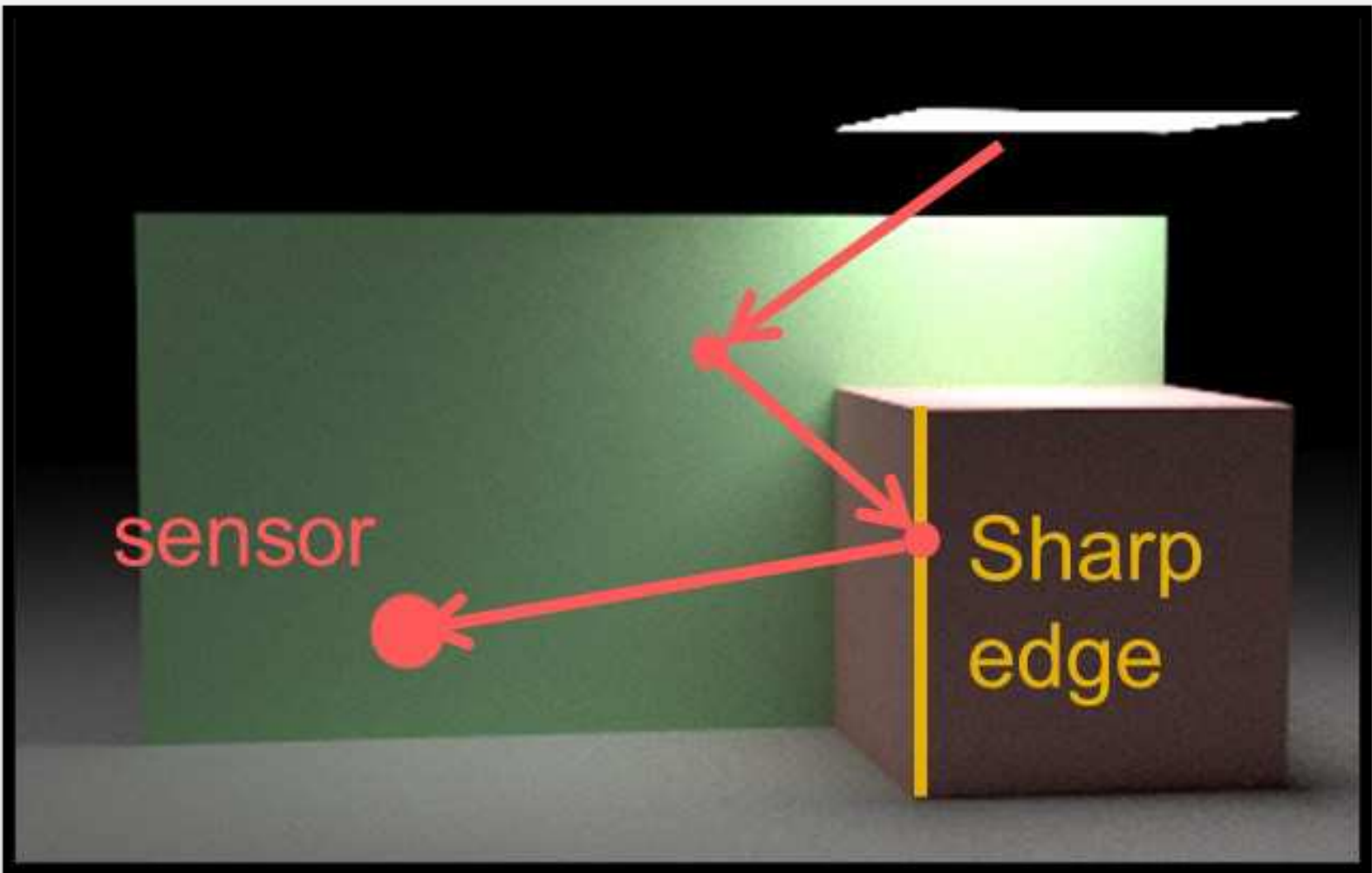
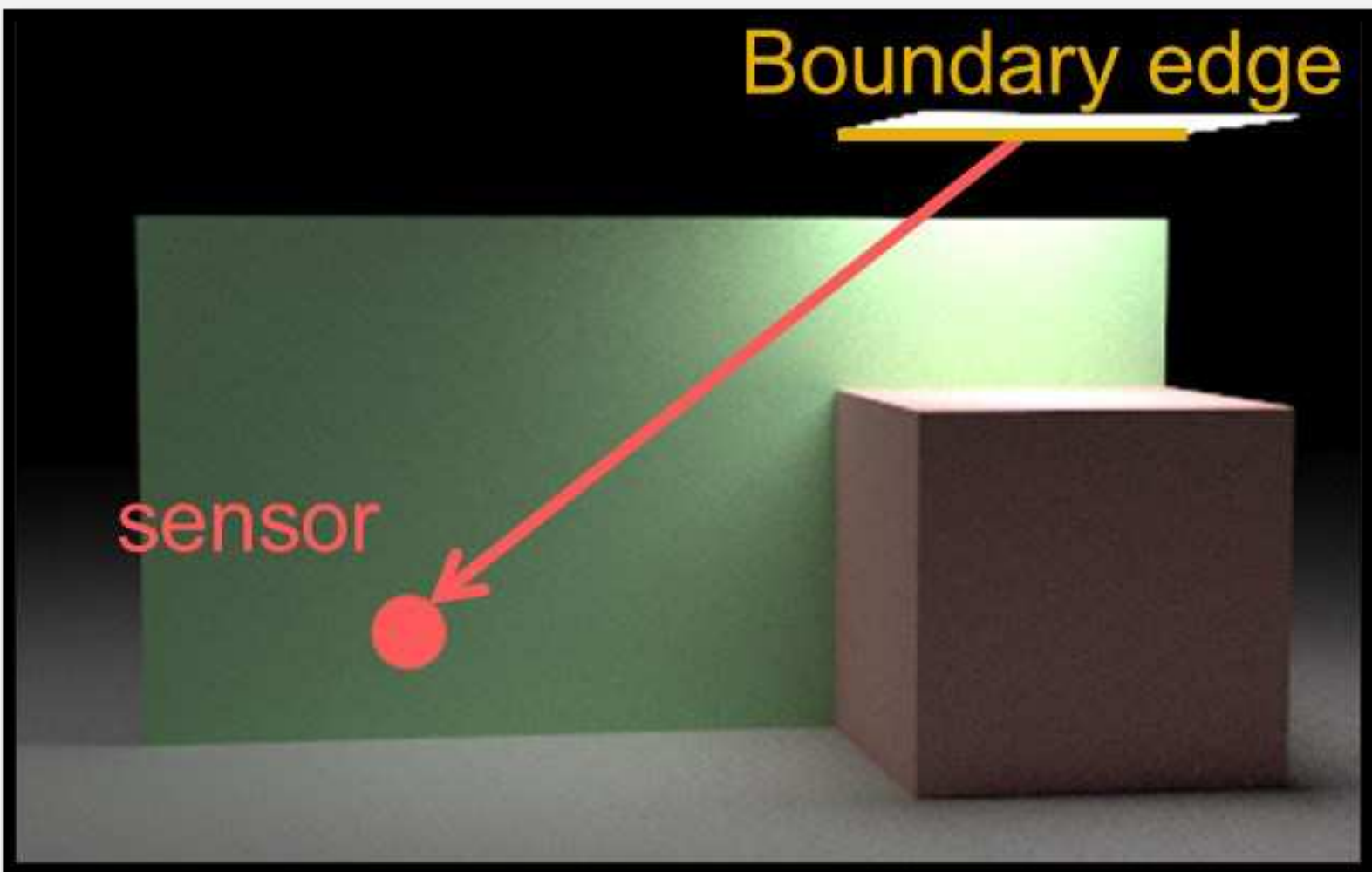
Boundary edge



Sharp edge



Silhouette edge



Topology-driven

Visibility-driven

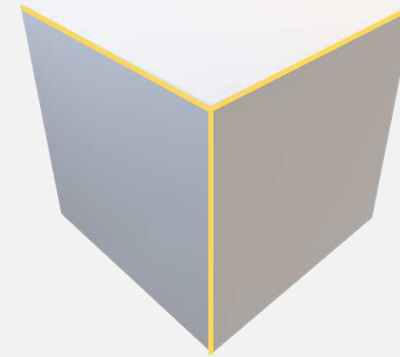
# Sources of discontinuities

- We still need to account for discontinuities when using smooth closed surfaces (e.g., neural SDFs)

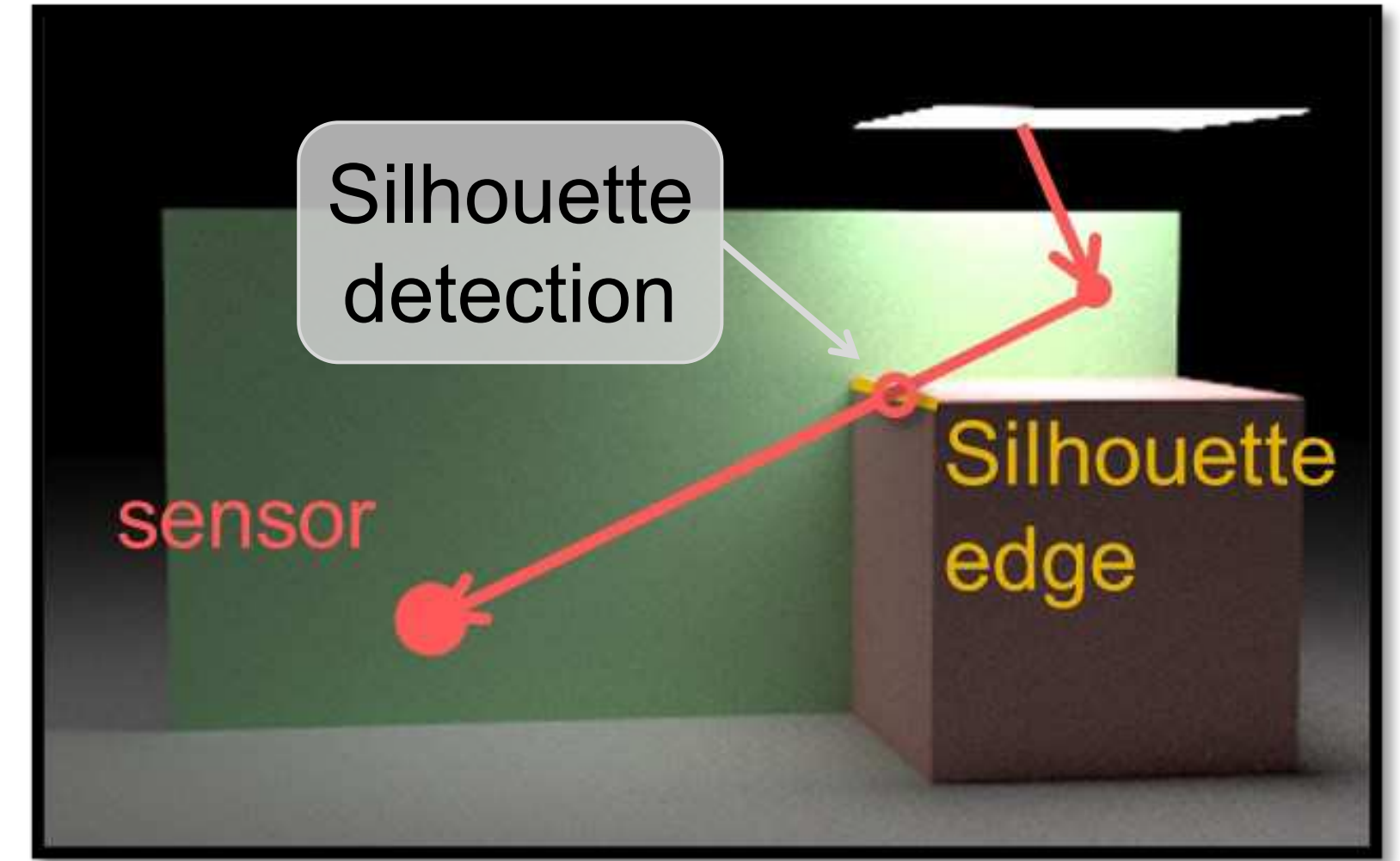
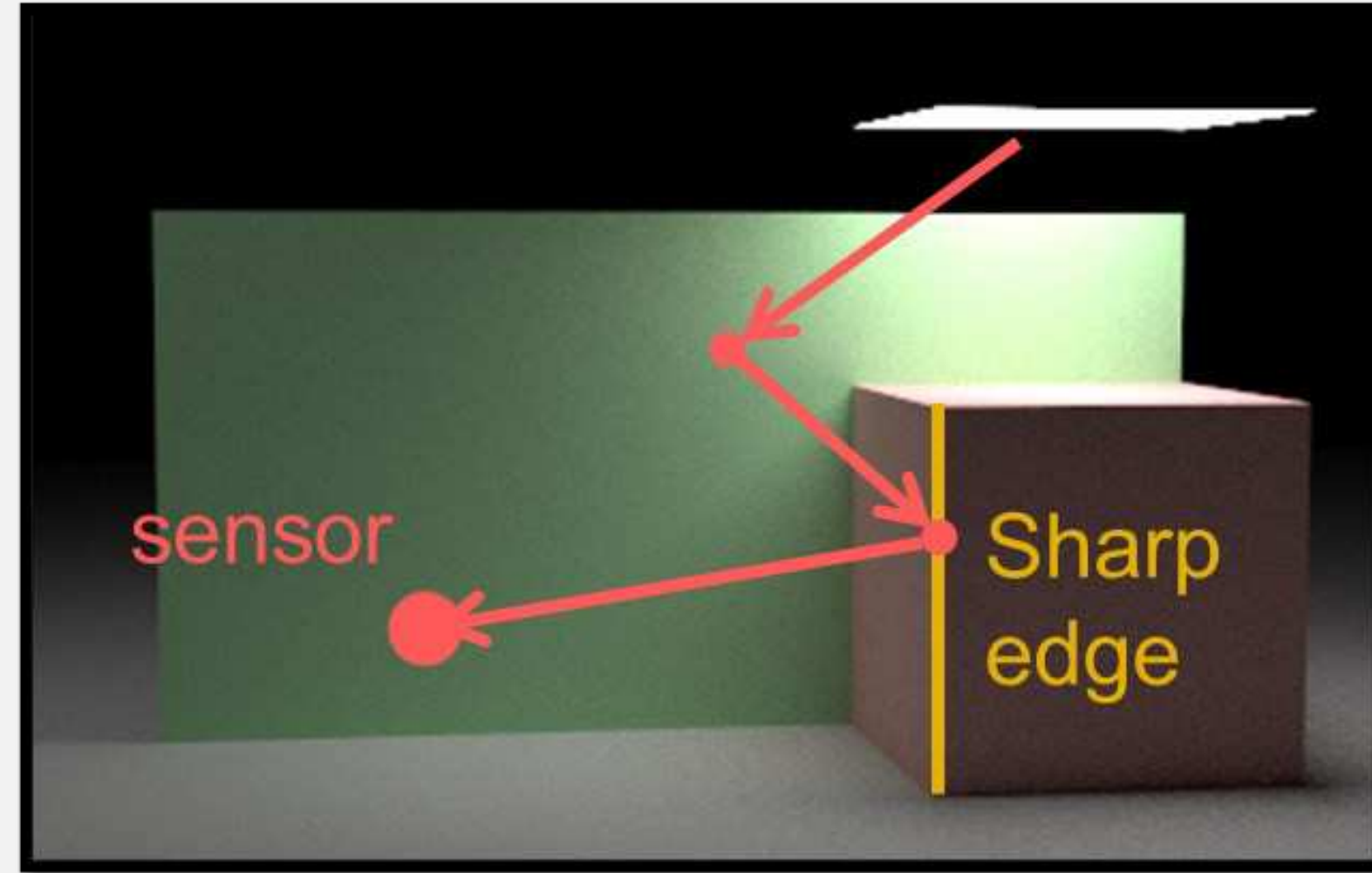
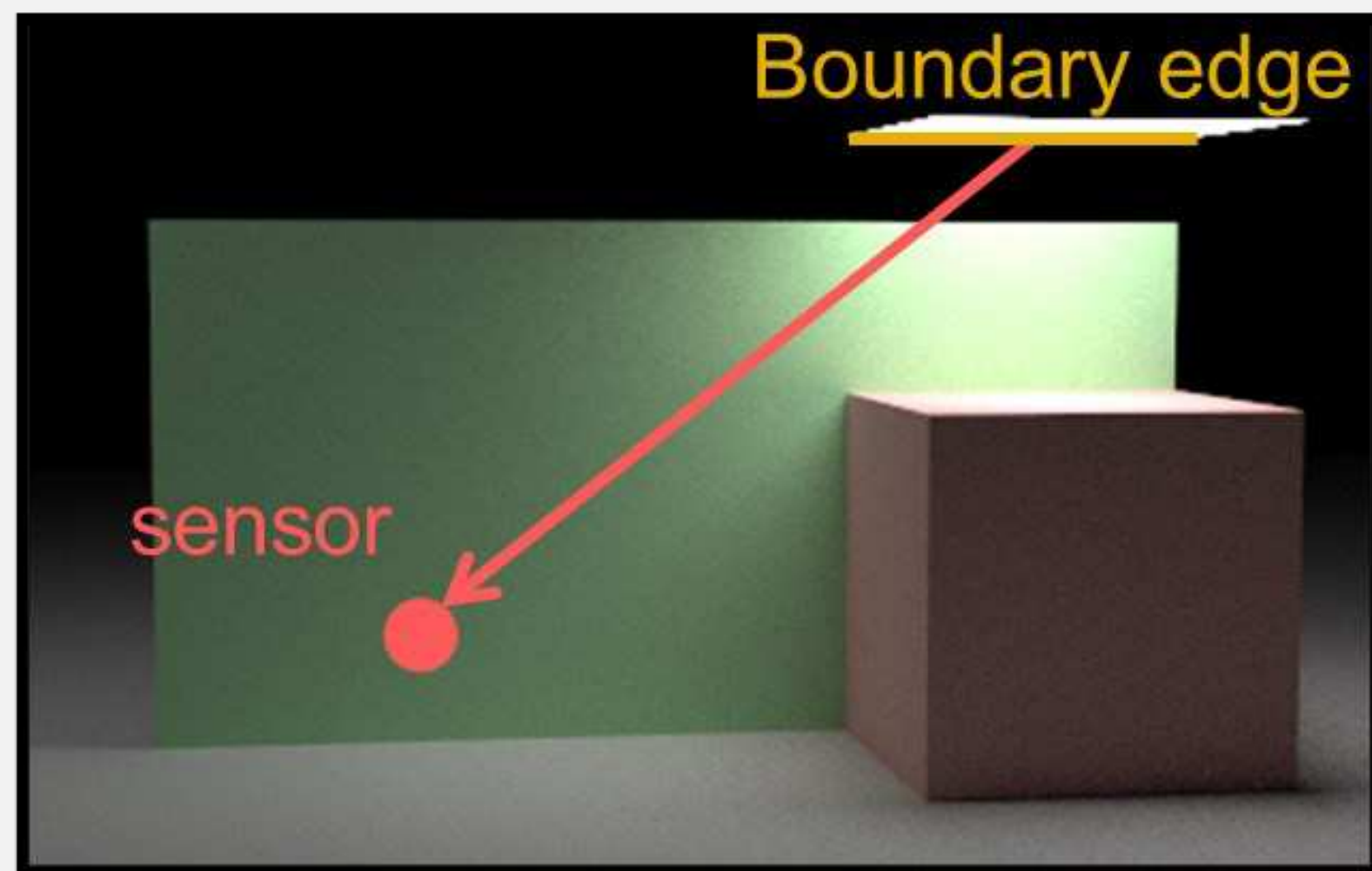
Boundary edge



Sharp edge



Silhouette edge



Topology-driven

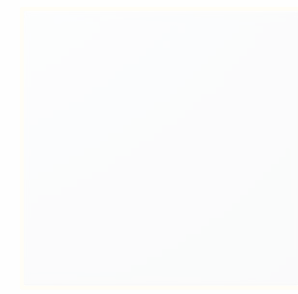
Visibility-driven



# Sources of discontinuities

- We still need to account for discontinuities when using smooth closed surfaces (e.g., neural SDFs)

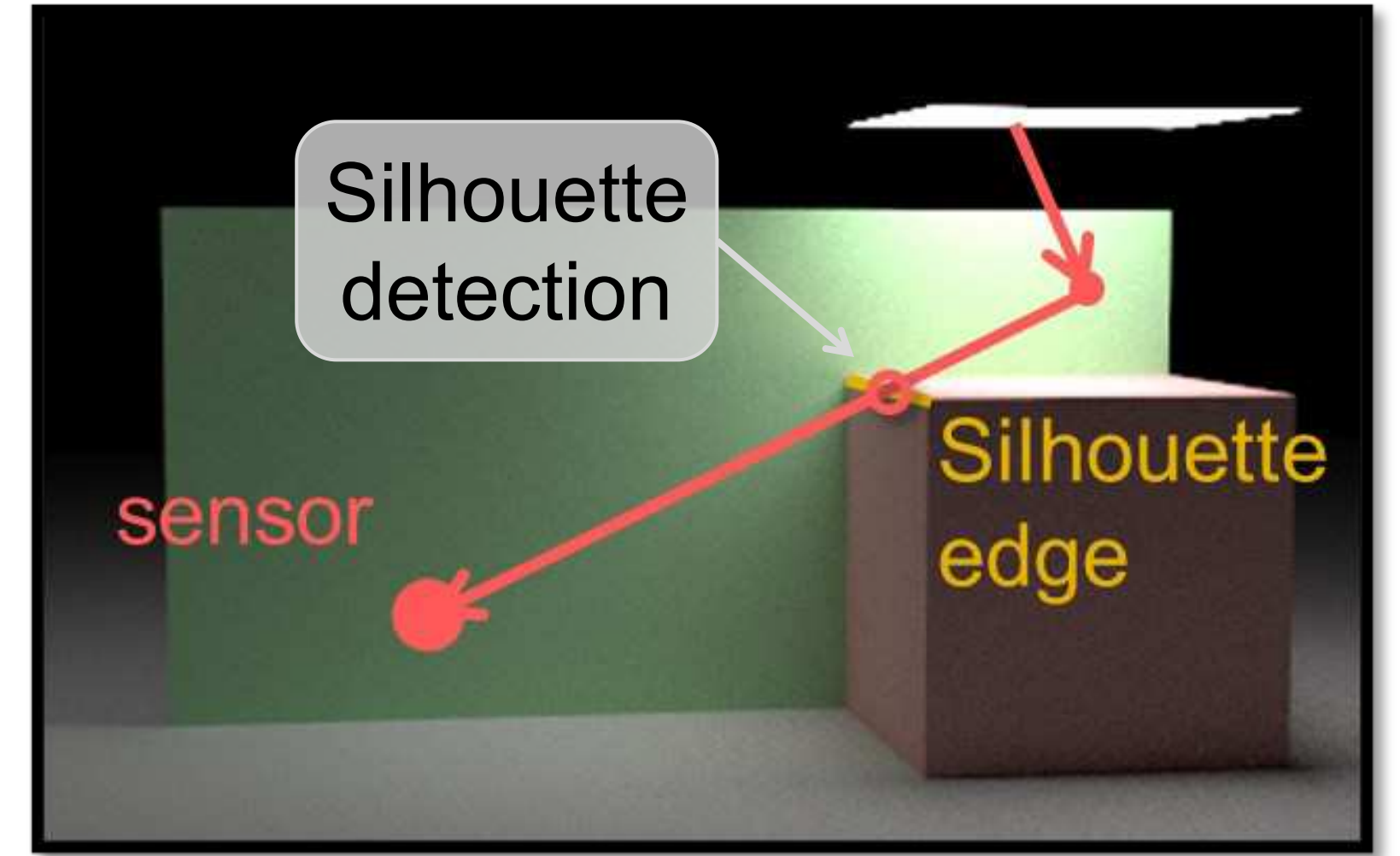
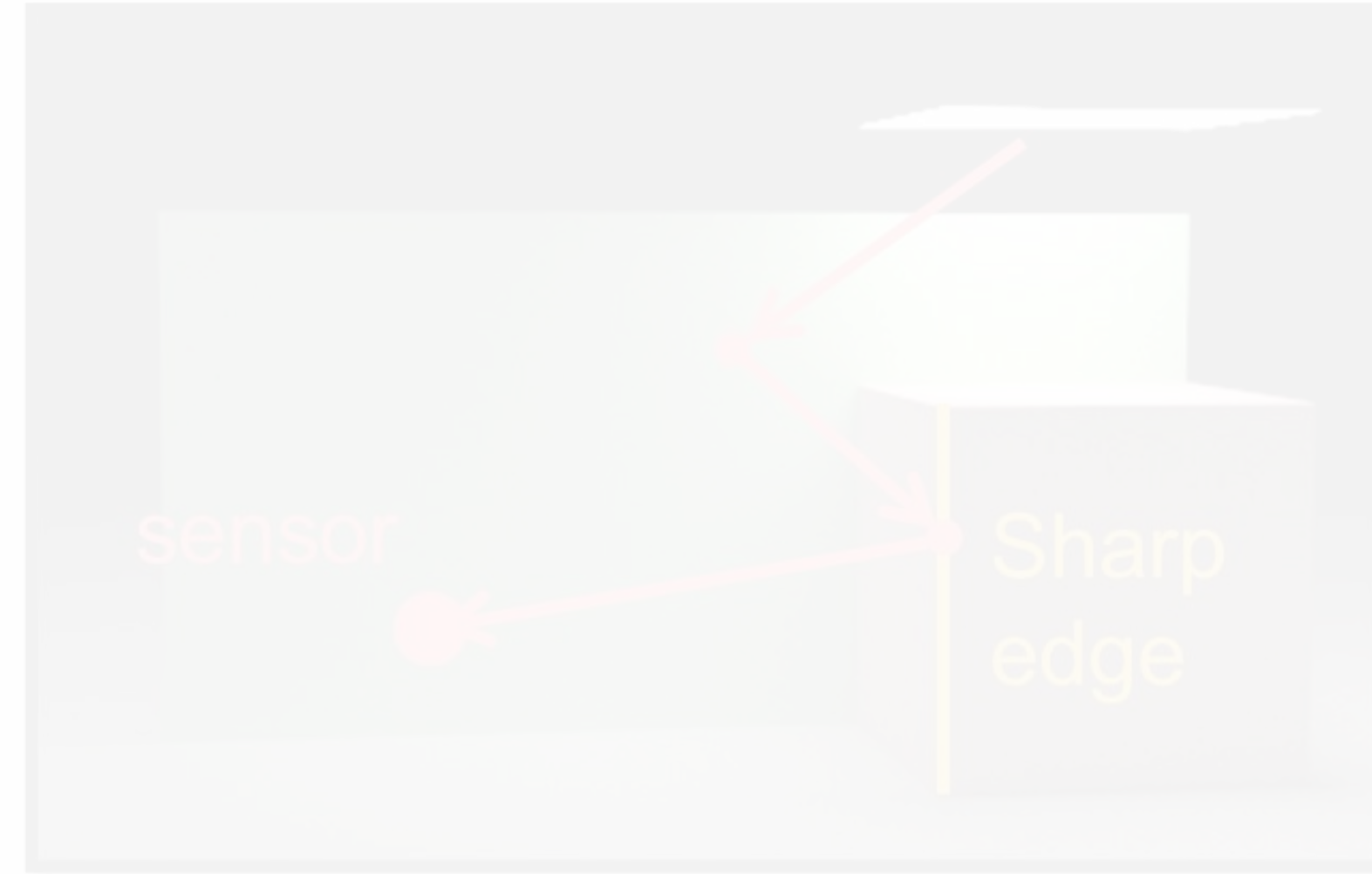
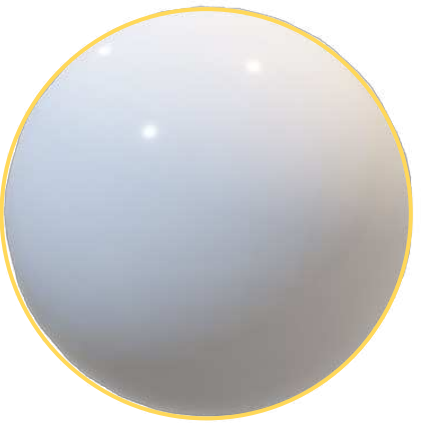
Boundary edge



Sharp edge



Silhouette edge



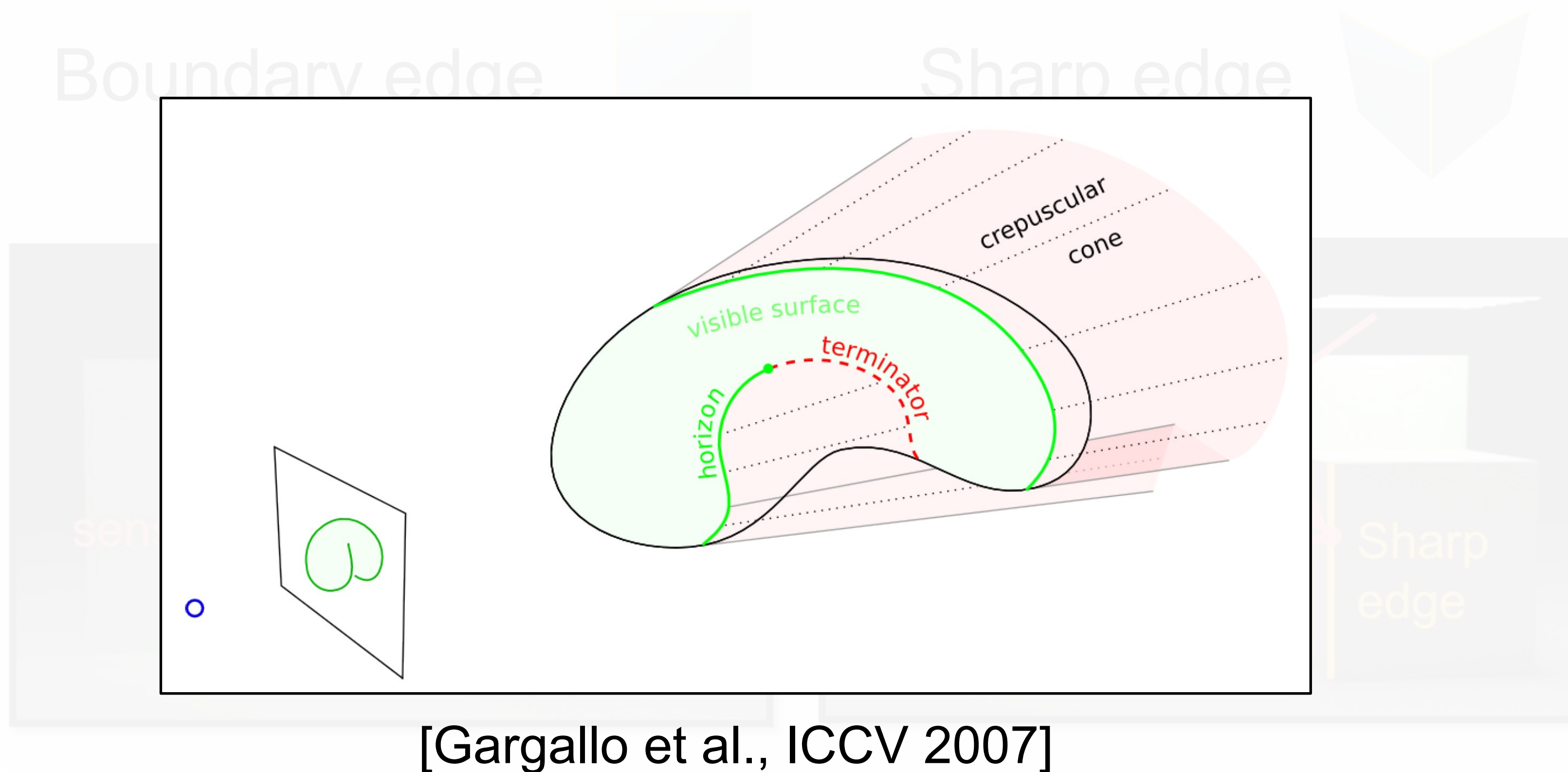
Topology-driven

Visibility-driven

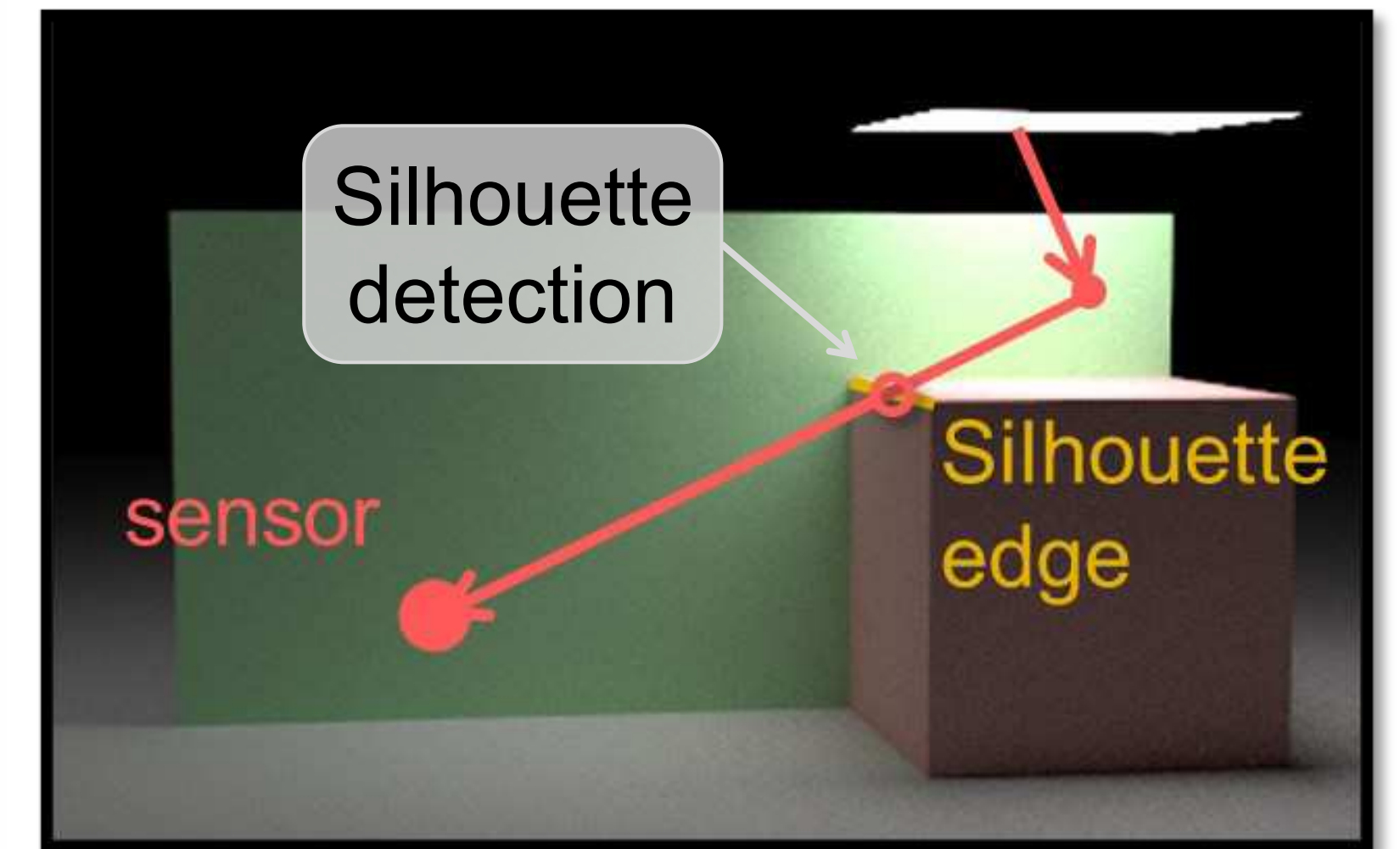


# Sources of discontinuities

- We still need to account for discontinuities when using smooth closed surfaces (e.g., neural SDFs)



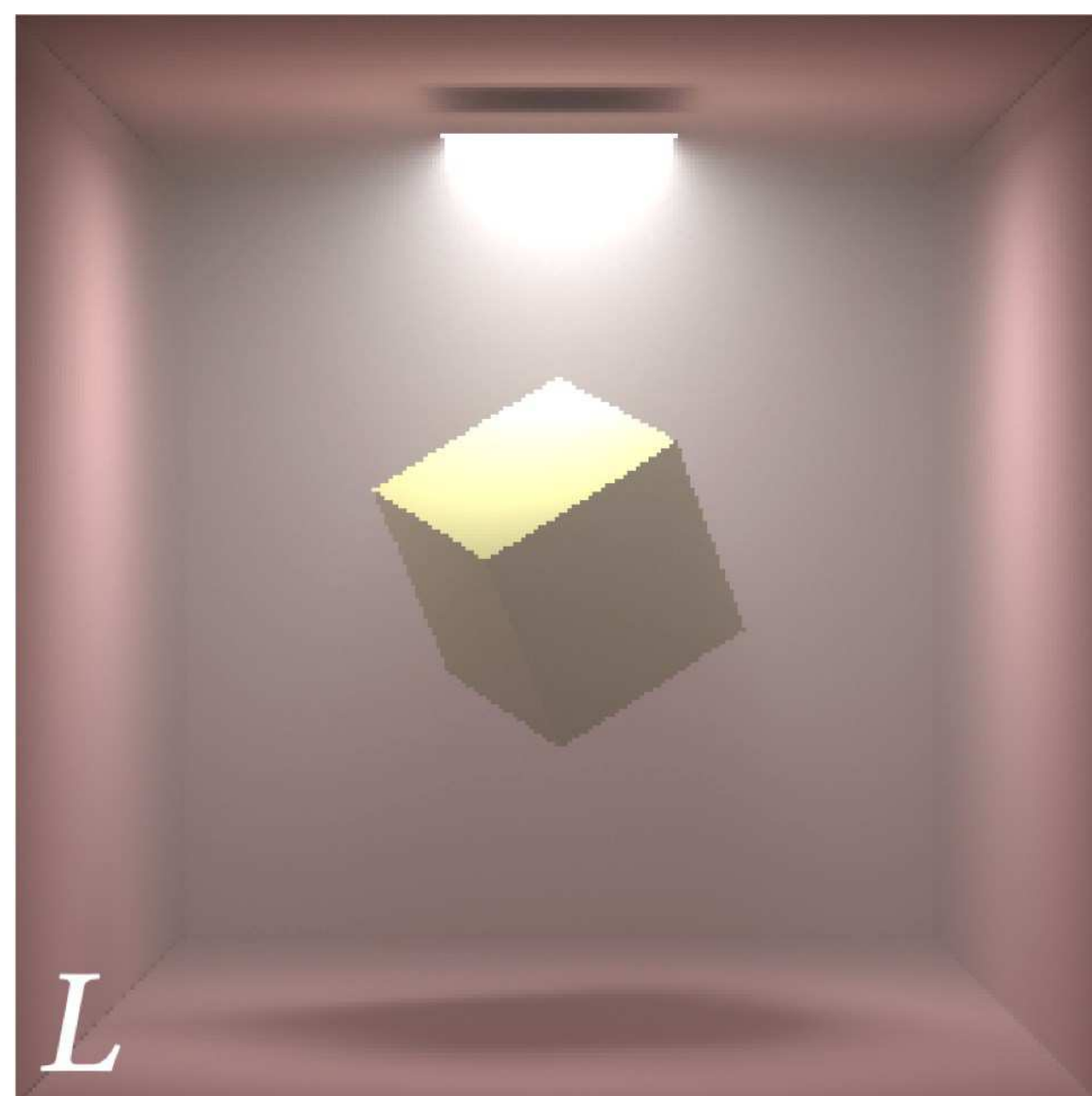
Silhouette edge



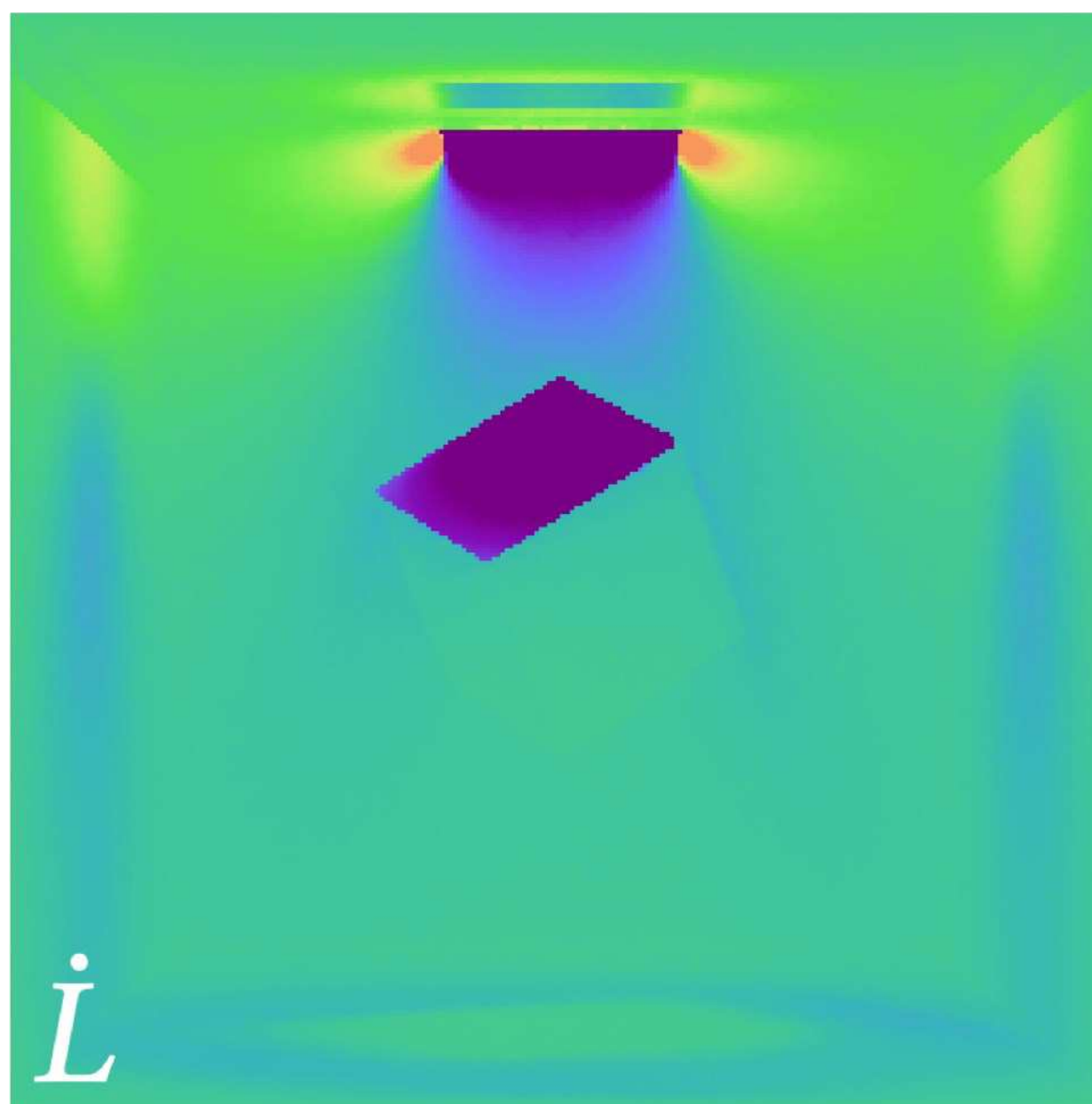
Visibility-driven

# Significance of the boundary integral

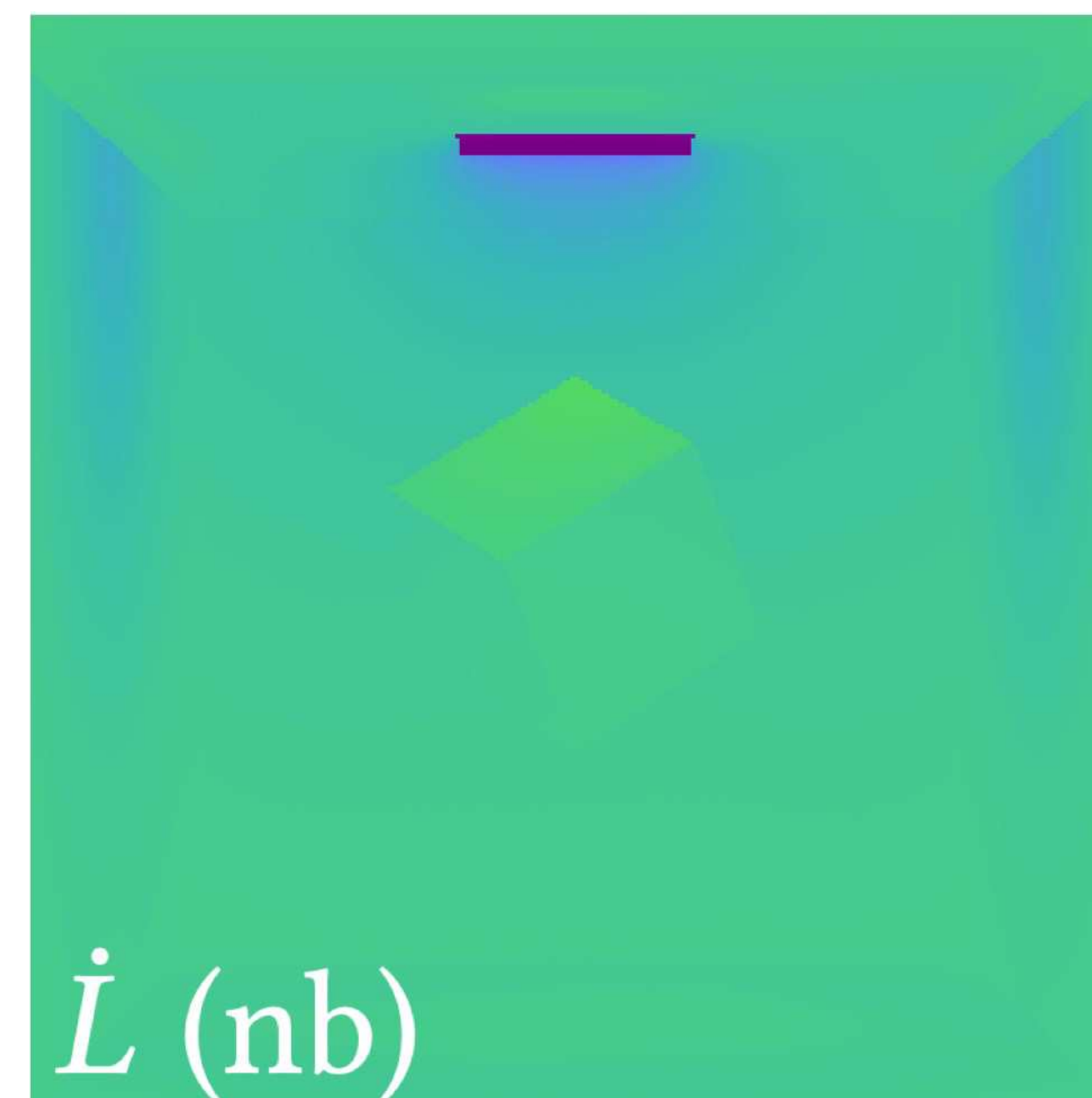
Negative  Zero Positive



**Original image**

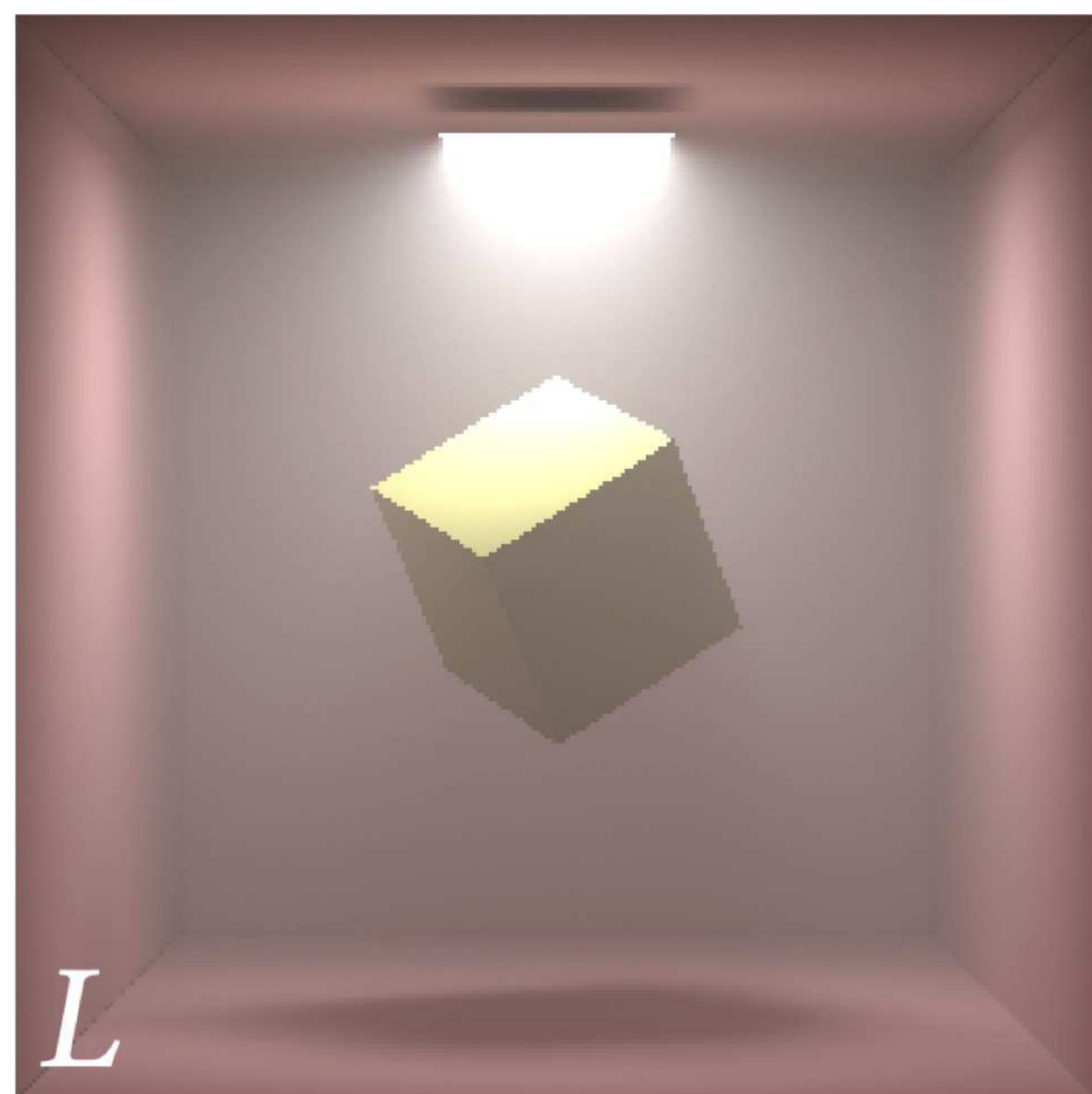


**Derivative image**  
w.r.t. vertical offset of  
the area light and the cube

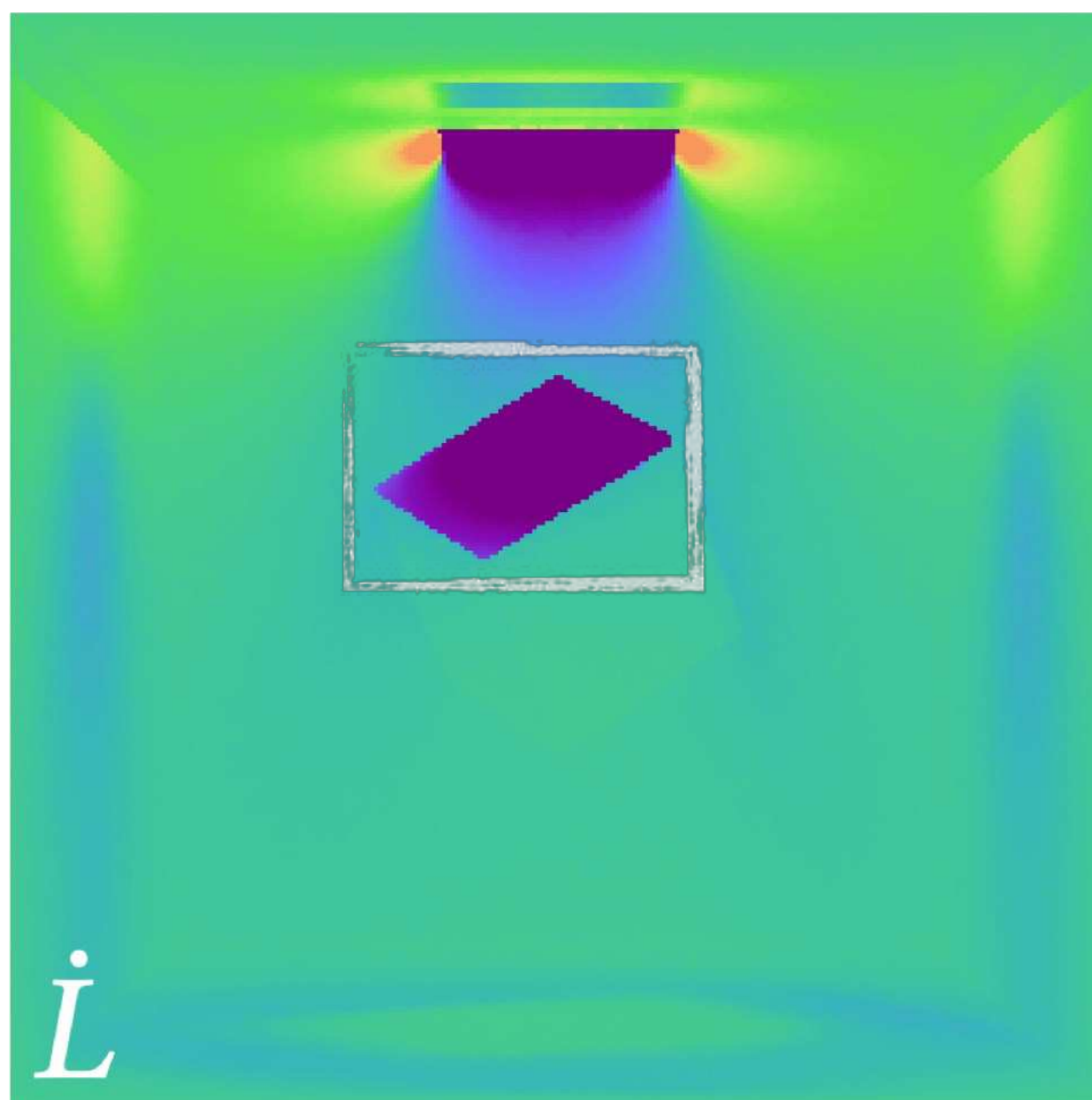


**Derivative image**  
w/o boundary integral

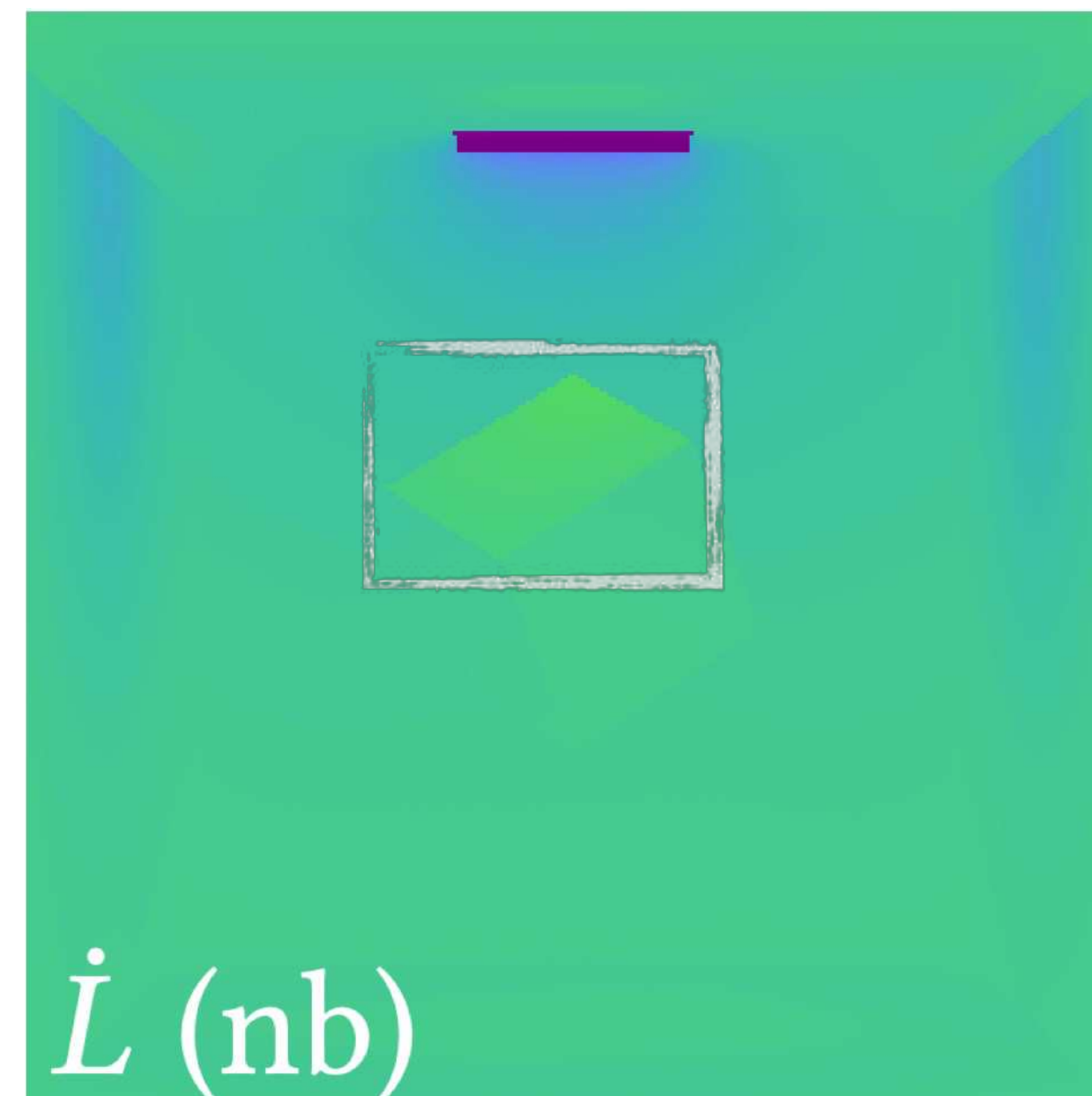
# Significance of the boundary integral



Original image



Derivative image  
w.r.t. vertical offset of  
the area light and the cube

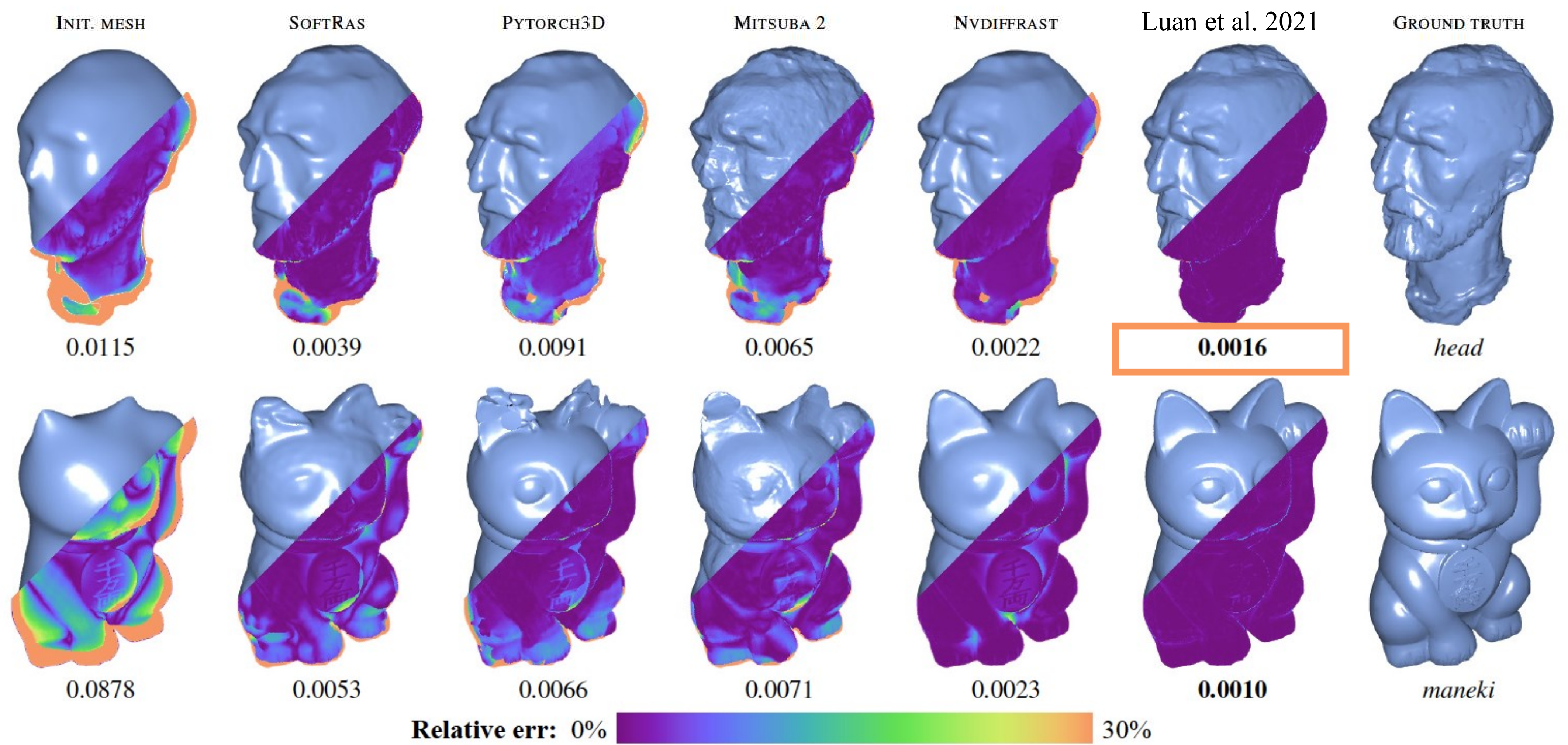


Derivative image  
w/o boundary integral



# Gradient Accuracy Matters

Inverse-rendering results with *identical* optimization settings





# Differential Global Illumination



# Very active area of research

## Path-Space Differentiable Rendering

CHENG ZHANG, University of California, Irvine  
BAILEY MILLER, Carnegie Mellon University  
KAI YAN, University of California, Irvine

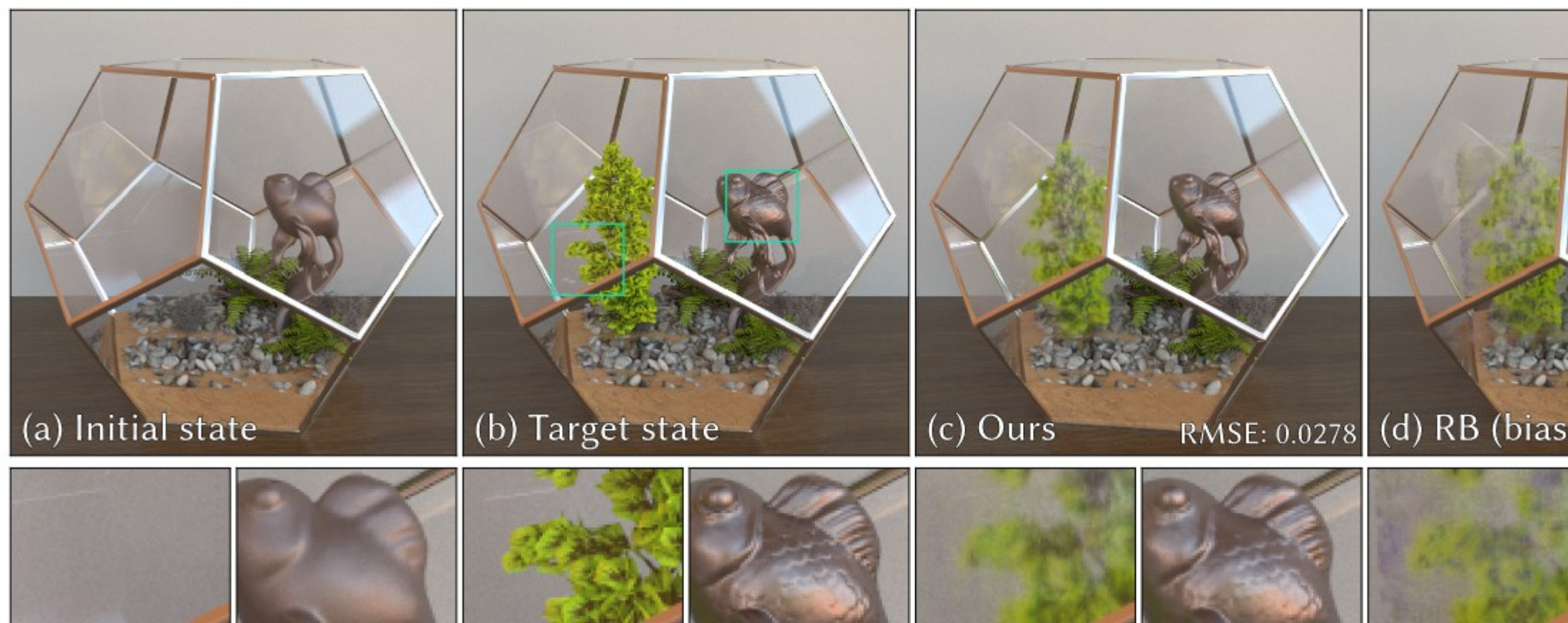
## Mitsuba 2: A Retargetable Forward and Inverse Renderer

MERLIN NIMIER-DAVID\*, École Polytechnique Fédérale de Lausanne  
DELIO VICINI\*, École Polytechnique Fédérale de Lausanne  
TIZIAN ZELTNER, École Polytechnique Fédérale de Lausanne  
WENZEL JAKOB, École Polytechnique Fédérale de Lausanne



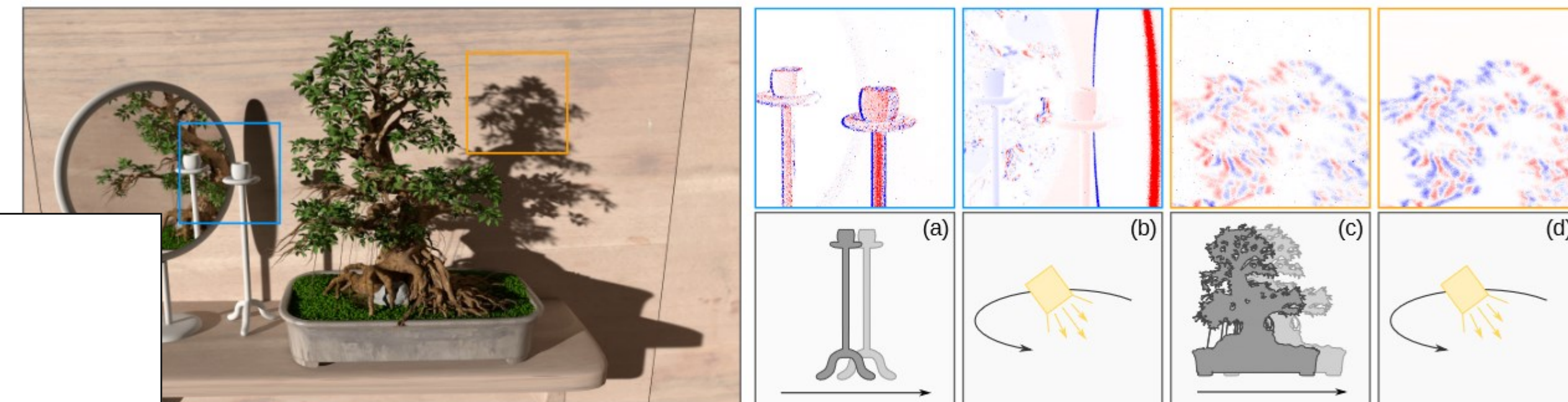
## Path Replay Backpropagation: Differentiating Light Paths using Constant Memory and Linear Time

DELIO VICINI, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland  
SÉBASTIEN SPEIERER, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland  
WENZEL JAKOB, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland



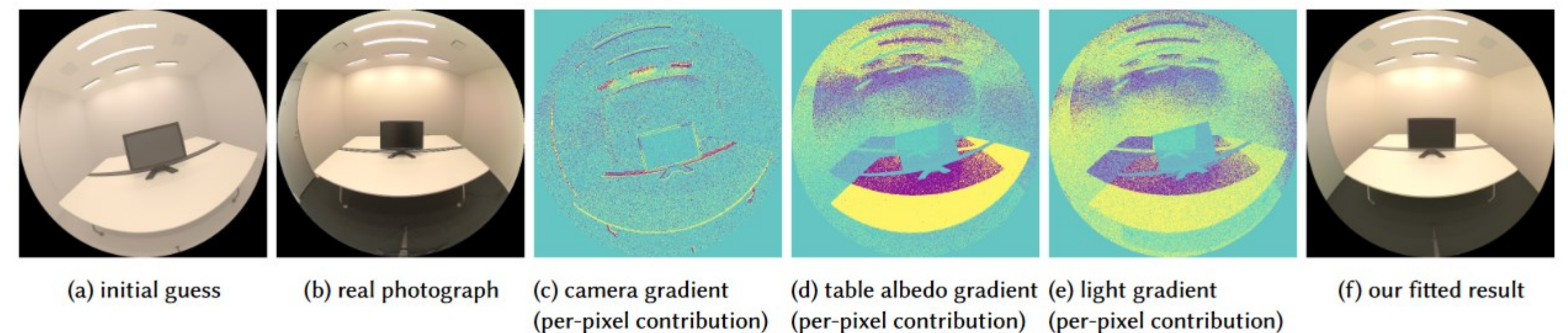
## Reparameterizing Discontinuous Integrands for Differentiable Rendering

GUILLAUME LOUBET, École Polytechnique Fédérale de Lausanne (EPFL)  
NICOLAS HOLZSCHUCH, Inria, Univ. Grenoble-Alpes, CNRS, LJK  
WENZEL JAKOB, École Polytechnique Fédérale de Lausanne (EPFL)



## Differentiable Monte Carlo Ray Tracing through Edge Sampling

TZU-MAO LI, MIT CSAIL  
MIIKA AITTALA, MIT CSAIL  
FRÉDO DURAND, MIT CSAIL  
JAAKKO LEHTINEN, Aalto University & NVIDIA





# Remember: Path Integral for Global Illumination

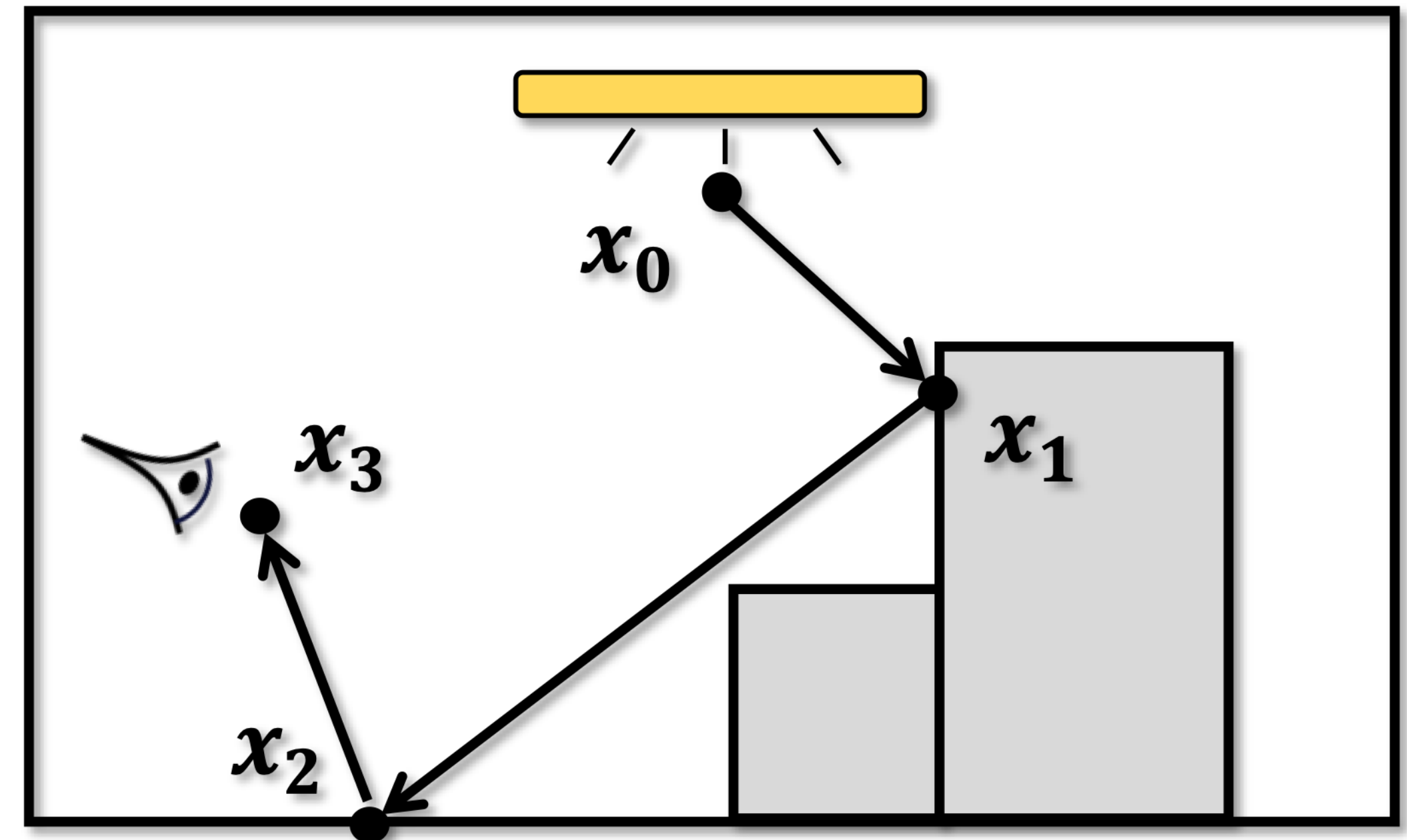
Pixel value

$$I = \int_{\Omega} f(\bar{x}) \, d\mu(\bar{x})$$

Measurement contribution

Path space

Area-product measure



Light path  $\bar{x} = (x_0, x_1, x_2, x_3)$

# Differential Path Integral

Path-space differentiable rendering

$$\frac{d}{d\theta} \left( \int_{\Omega} f(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}) \right) = \underbrace{\int_{\Omega} \dot{f}(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}})}_{\text{Interior integral}} + \underbrace{\int_{\partial\Omega} g(\bar{\mathbf{x}}) d\mu'(\bar{\mathbf{x}})}_{\text{Boundary integral}}$$

# Differential Path Integral

Path-space differentiable rendering

$$\frac{d}{d\theta} \left( \int_{\Omega} f(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}) \right) = \underbrace{\int_{\Omega} \dot{f}(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}})}_{\text{Interior integral}} + \underbrace{\int_{\partial\Omega} g(\bar{\mathbf{x}}) d\mu'(\bar{\mathbf{x}})}_{\text{Boundary integral}}$$

We now derive  $\partial I_N / \partial \pi$  in Eq. (25) using the recursive relations provided by Eqs. (21) and (24). Let

$$h_n^{(0)} := \left[ \prod_{n'=n+1}^N g(\mathbf{x}_{n'}; \mathbf{x}_{n'-2}, \mathbf{x}_{n'-1}) \right] W_e(\mathbf{x}_N \rightarrow \mathbf{x}_{N-1}), \quad (52)$$

$$h_n^{(1)} := \sum_{n'=n+1}^N \kappa(\mathbf{x}_{n'}) V(\mathbf{x}_{n'}), \quad (53)$$

$$\Delta h_{n,n'}^{(0)} := h_n^{(0)} \Delta g(\mathbf{x}_{n'}; \mathbf{x}_{n'-2}, \mathbf{x}_{n'-1}) / g(\mathbf{x}_{n'}; \mathbf{x}_{n'-2}, \mathbf{x}_{n'-1}), \quad (54)$$

for  $0 \leq n < n' \leq N$ . We omit the dependencies of  $h_n^{(0)}$ ,  $h_n^{(1)}$ , and  $\Delta h_{n,n'}^{(0)}$  on  $\mathbf{x}_{n+1}, \dots, \mathbf{x}_N$  for notational convenience.

We now show that, for all  $0 \leq n < N$ , it holds that

$$h_n(\mathbf{x}_n; \mathbf{x}_{n-1}) = \int_{\mathcal{M}^{N-n}} h_n^{(0)} \prod_{n'=n+1}^N dA(\mathbf{x}_{n'}), \quad (55)$$

and

$$\begin{aligned} \dot{h}_n(\mathbf{x}_n; \mathbf{x}_{n-1}) &= \int_{\mathcal{M}^{N-n}} \left[ \left( h_n^{(0)} \right)' - h_n^{(0)} h_n^{(1)} \right] \prod_{n'=n+1}^N dA(\mathbf{x}_{n'}) \\ &\quad + \sum_{n'=n+1}^N \int \Delta h_{n,n'}^{(0)} V_{\partial \mathcal{M}_{n'}}(\mathbf{x}_{n'}) d\ell(\mathbf{x}_{n'}) \prod_{\substack{n < i \leq N \\ i \neq n'}} dA(\mathbf{x}_i), \end{aligned} \quad (56)$$

where the integral domain of the second term on the right-hand side, which is omitted for notational clarity, is  $\mathcal{M}(\pi)$  for each  $\mathbf{x}_i$  with  $i \neq n'$  and  $\partial \mathcal{M}_{n'}(\pi)$ , which depends on  $\mathbf{x}_{n'-1}$ , for  $\mathbf{x}_{n'}$ .

It is easy to verify that Eqs. (55) and (56) hold for  $n = N - 1$ . We now show that, if they hold for some  $0 < n < N$ , then it is also the case for  $n - 1$ . Let  $g_{n-1} := g(\mathbf{x}_n; \mathbf{x}_{n-2}, \mathbf{x}_{n-1})$  for all  $0 < n \leq N$ . Then,

$$\begin{aligned} h_{n-1}(\mathbf{x}_{n-1}; \mathbf{x}_{n-2}) &= \int_{\mathcal{M}} g_{n-1} \int_{\mathcal{M}^{N-n}} h_n^{(0)} \prod_{n'=n+1}^N dA(\mathbf{x}_{n'}) dA(\mathbf{x}_n) \\ &= \int_{\mathcal{M}^{N-n+1}} h_{n-1}^{(0)} \prod_{n'=n}^N dA(\mathbf{x}_{n'}), \end{aligned} \quad (57)$$

and

$$\begin{aligned} \dot{h}_{n-1}(\mathbf{x}_{n-1}; \mathbf{x}_{n-2}) &= \int_{\mathcal{M}} \left[ \dot{g}_{n-1} h_n + g_{n-1} (\dot{h}_n - h_n \kappa(\mathbf{x}_n) V(\mathbf{x}_n)) \right] dA(\mathbf{x}_n) \\ &\quad + \int_{\partial \mathcal{M}_n} \Delta g_{n-1} h_n V_{\partial \mathcal{M}_n} d\ell(\mathbf{x}_n) \\ &= \int_{\mathcal{M}^{N-n+1}} \left\{ \dot{g}_{n-1} h_n^{(0)} + g_{n-1} \left[ \left( h_n^{(0)} \right)' - h_n^{(0)} h_n^{(1)} \right] \right\} \prod_{n'=k}^N dA(\mathbf{x}_{n'}) \\ &\quad + \sum_{n'=n+1}^N \int g_{n-1} \Delta h_{n,n'}^{(0)} V_{\partial \mathcal{M}_{n'}}(\mathbf{x}_{n'}) d\ell(\mathbf{x}_{n'}) \prod_{\substack{n \leq i \leq N \\ i \neq n'}} dA(\mathbf{x}_i) \\ &\quad + \int \Delta g_{n-1} h_n^{(0)} V_{\partial \mathcal{M}_n} d\ell(\mathbf{x}_n) \prod_{n'=n+1}^N dA(\mathbf{x}_{n'}) \\ &= \int_{\mathcal{M}^{N-n+1}} \left[ \left( h_{n-1}^{(0)} \right)' - h_{n-1}^{(0)} h_{n-1}^{(1)} \right] \prod_{n'=n}^N dA(\mathbf{x}_{n'}) \\ &\quad + \sum_{n'=n}^N \int \Delta h_{n-1,n'}^{(0)} V_{\partial \mathcal{M}_{n'}}(\mathbf{x}_{n'}) d\ell(\mathbf{x}_{n'}) \prod_{\substack{n \leq i \leq N \\ i \neq n'}} dA(\mathbf{x}_i). \end{aligned} \quad (58)$$

Thus, using mathematical induction, we know that Eqs. (55) and (56) hold for all  $0 \leq n < N$ .

Notice that  $h_0^{(0)} = f$  and  $\Delta h_{0,n'}^{(0)} = \Delta f_{n'}$ , where  $\Delta f_{n'}$  follows the definition in Eq. (28). Letting  $n = 0$  in Eq. (56) yields

$$\begin{aligned} \dot{h}_0(\mathbf{x}_0) &= \int_{\mathcal{M}^N} \left[ \dot{f}(\bar{\mathbf{x}}) - f(\bar{\mathbf{x}}) \sum_{n'=1}^N \kappa(\mathbf{x}_{n'}) V(\mathbf{x}_{n'}) \right] \prod_{n'=1}^N dA(\mathbf{x}_{n'}) \\ &\quad + \sum_{n'=1}^N \int \Delta f_{n'}(\bar{\mathbf{x}}) V_{\partial \mathcal{M}_{n'}} d\ell(\mathbf{x}_{n'}) \prod_{\substack{0 < i \leq N \\ i \neq n'}} dA(\mathbf{x}_i). \end{aligned} \quad (59)$$

Lastly, based on the assumption that  $h_0$  is continuous in  $\mathbf{x}_0$ , Eq. (25) can be obtained by differentiating Eq. (23):

$$\begin{aligned} \frac{\partial I_N}{\partial \pi} &= \frac{\partial}{\partial \pi} \int_{\mathcal{M}} h_0(\mathbf{x}_0) dA(\mathbf{x}_0) \\ &= \int_{\mathcal{M}} \left[ \dot{h}_0(\mathbf{x}_0) - h_0(\mathbf{x}_0) \kappa(\mathbf{x}_0) V(\mathbf{x}_0) \right] dA(\mathbf{x}_0) \\ &\quad + \int_{\partial \mathcal{M}_0} h_0(\mathbf{x}_0) V_{\partial \mathcal{M}_0}(\mathbf{x}_0) d\ell(\mathbf{x}_0) \\ &= \int_{\Omega_N} \left[ \dot{f}(\bar{\mathbf{x}}) - f(\bar{\mathbf{x}}) \sum_{K=0}^N \kappa(\mathbf{x}_K) V(\mathbf{x}_K) \right] d\mu(\bar{\mathbf{x}}) \\ &\quad + \sum_{K=0}^N \int_{\Omega_{N,K}} \Delta f_K(\bar{\mathbf{x}}) V_{\partial \mathcal{M}_K} d\mu'_{N,K}(\bar{\mathbf{x}}). \end{aligned} \quad (60)$$

(The full derivation is quite involved...)

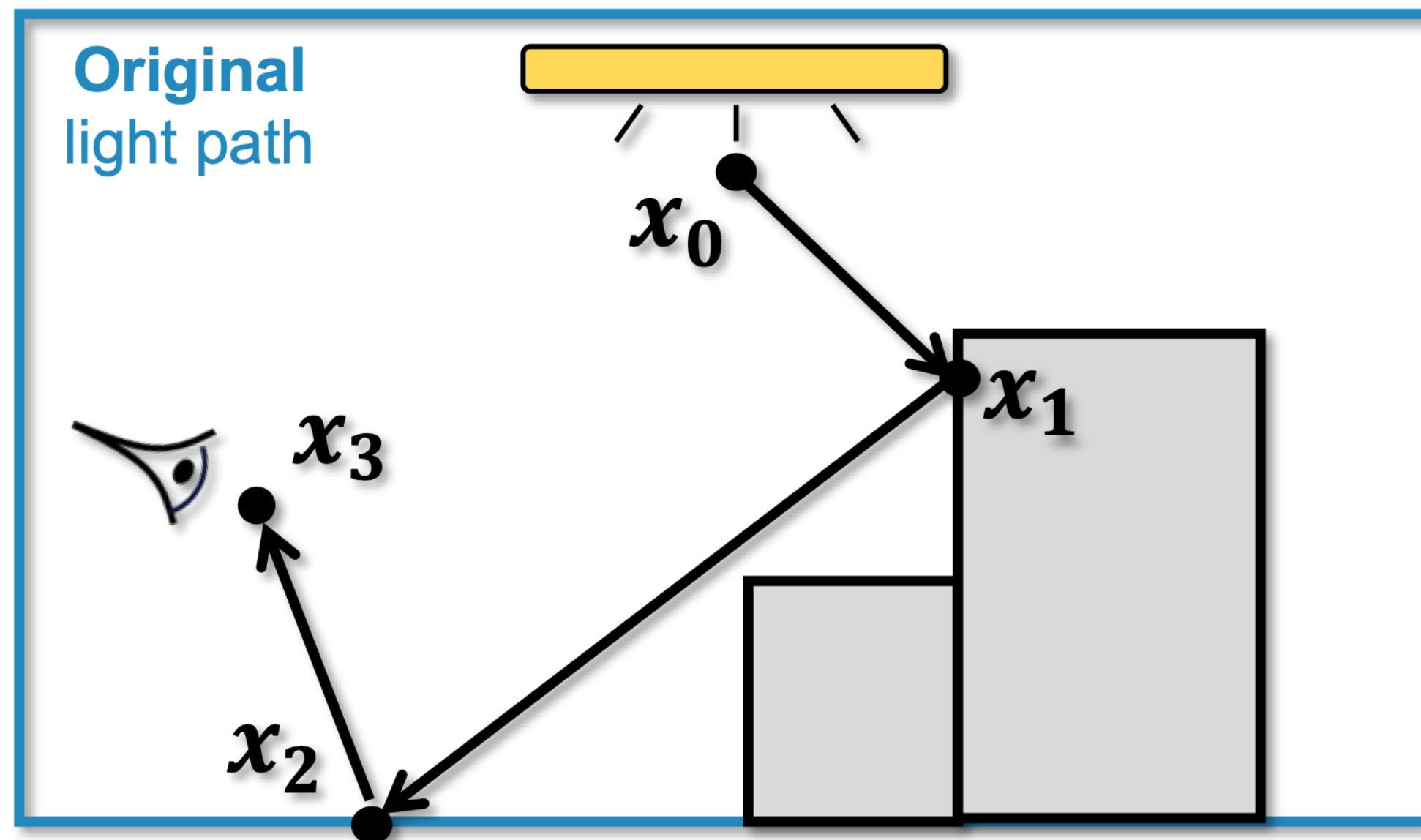


# Differential Path Integral

Path-space differentiable rendering

$$\frac{d}{d\theta} \left( \int_{\Omega} f(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}) \right) = \int_{\Omega} \dot{f}(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}) + \int_{\partial\Omega} g(\bar{\mathbf{x}}) d\mu'(\bar{\mathbf{x}})$$

Interior integral



Interior integral

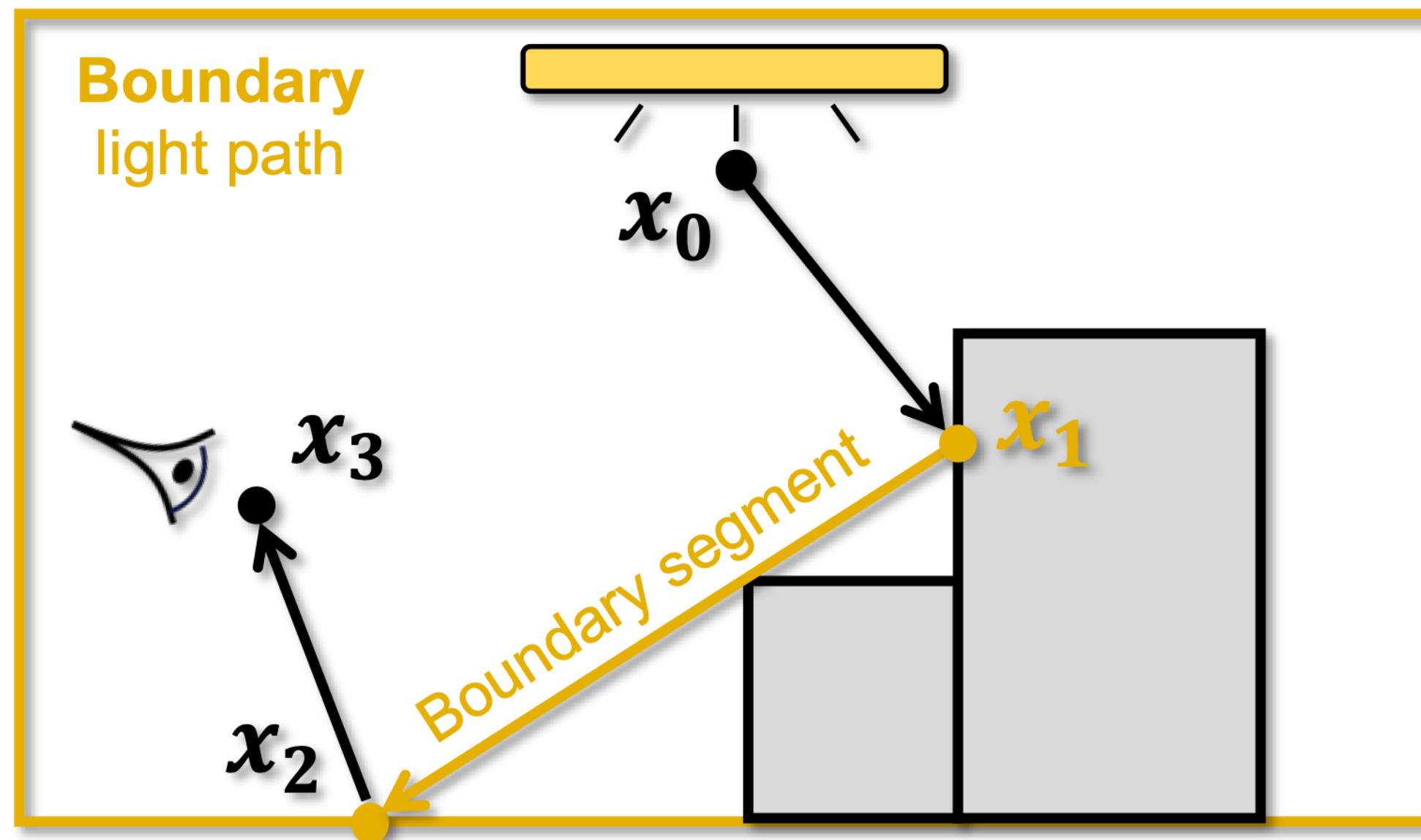
- Defined on the ordinary path space  $\Omega$
- The integrand  $\dot{f}$  can be obtained by differentiating the ordinary *measurement contribution function*  $f$

# Differential Path Integral

Path-space differentiable rendering

$$\frac{d}{d\theta} \left( \int_{\Omega} f(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}) \right) = \int_{\Omega} \dot{f}(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}) + \int_{\partial\Omega} g(\bar{\mathbf{x}}) d\mu'(\bar{\mathbf{x}})$$

**Boundary** integral



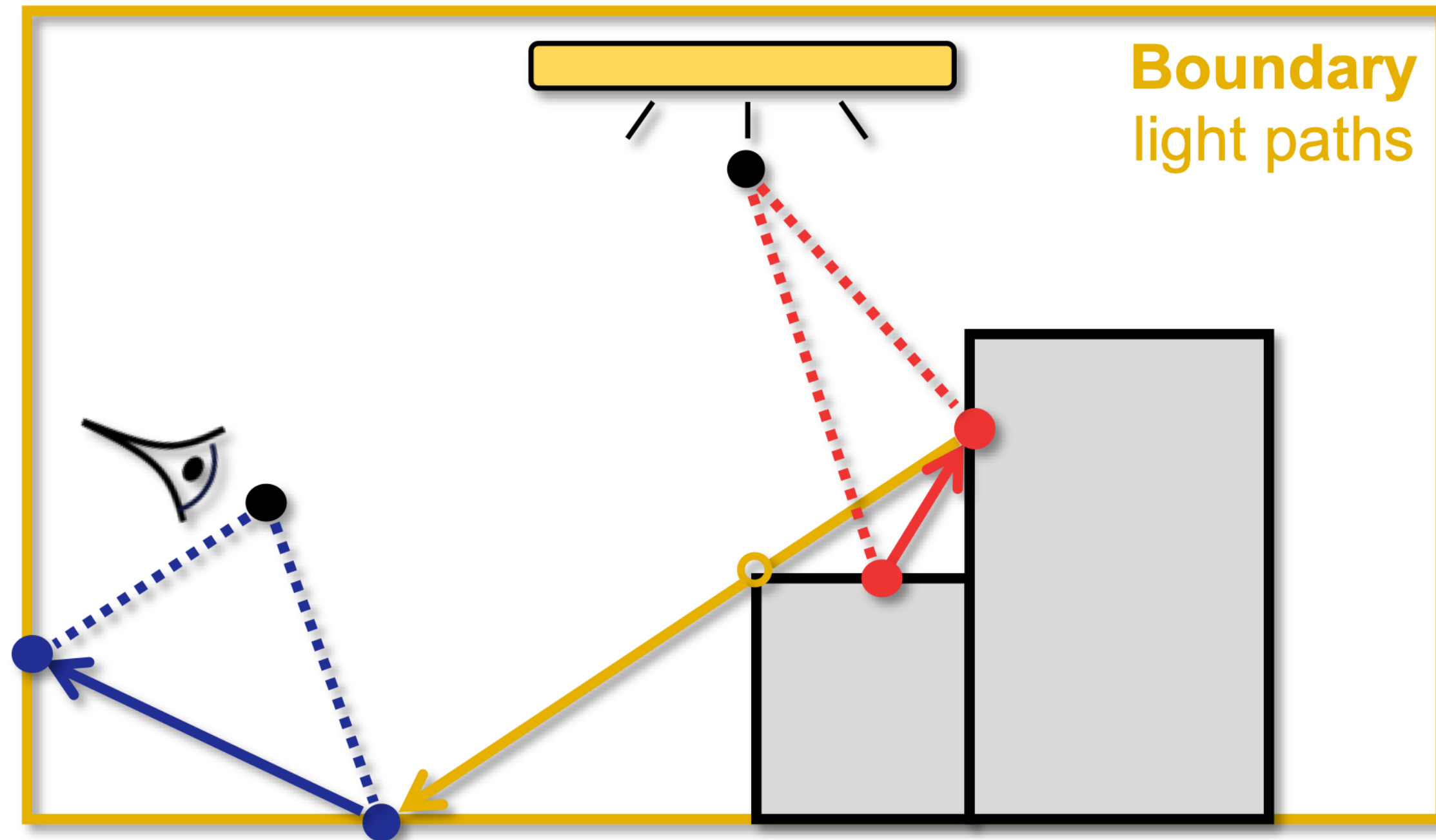
## **Boundary** integral

- Defined on the boundary path space  $\partial\Omega$
- A **boundary** light path is the same as an original one except having exactly one **boundary** segment

# Path-Space Differentiable Path Tracing

## **Unidirectional** estimator

- **Interior**: *unidirectional* path tracing
- **Boundary**: *unidirectional* sampling of subpaths



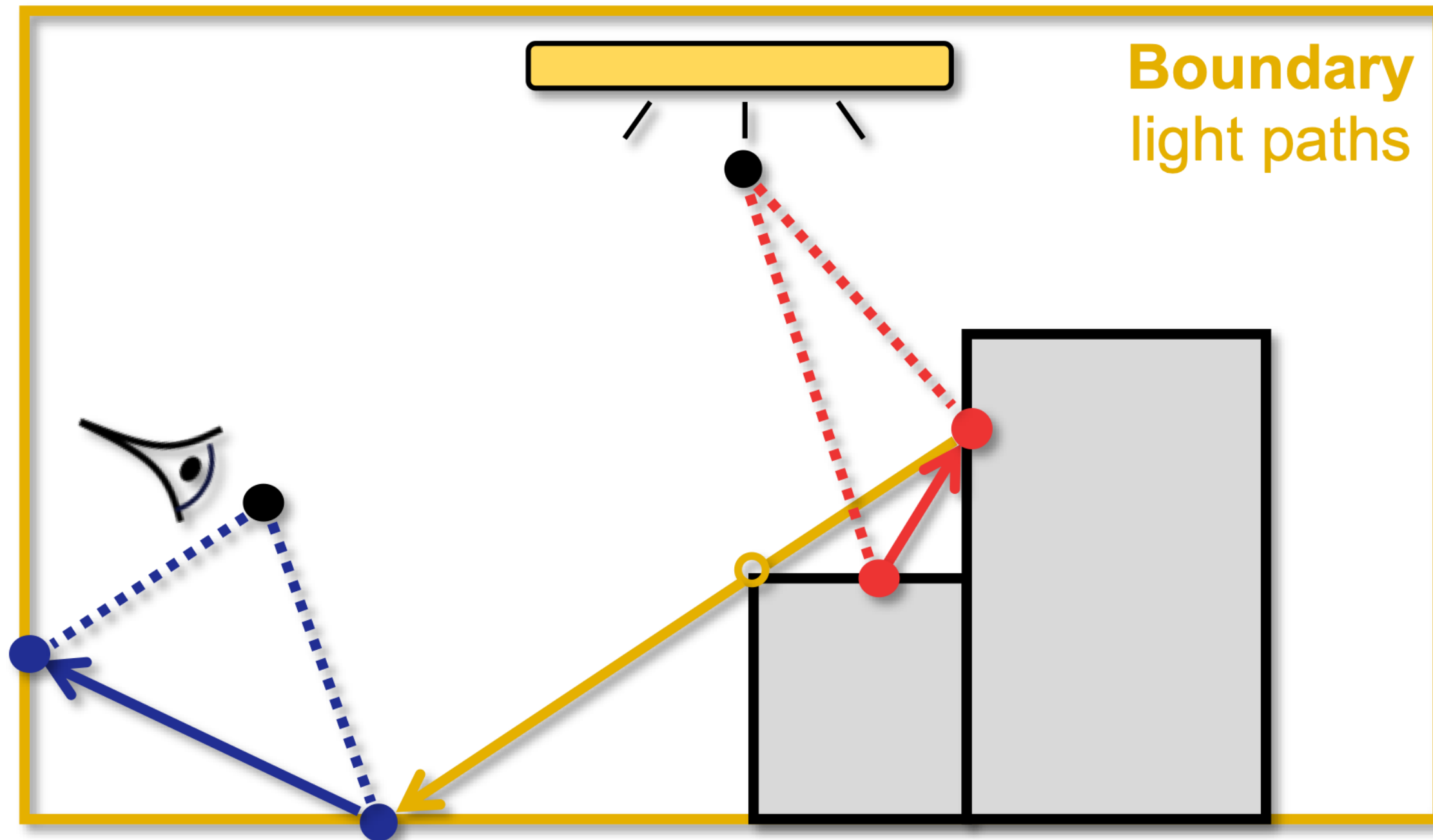
*Unidirectional* path tracing + NEE



# Path-Space Differentiable Path Tracing

## **Unidirectional** estimator

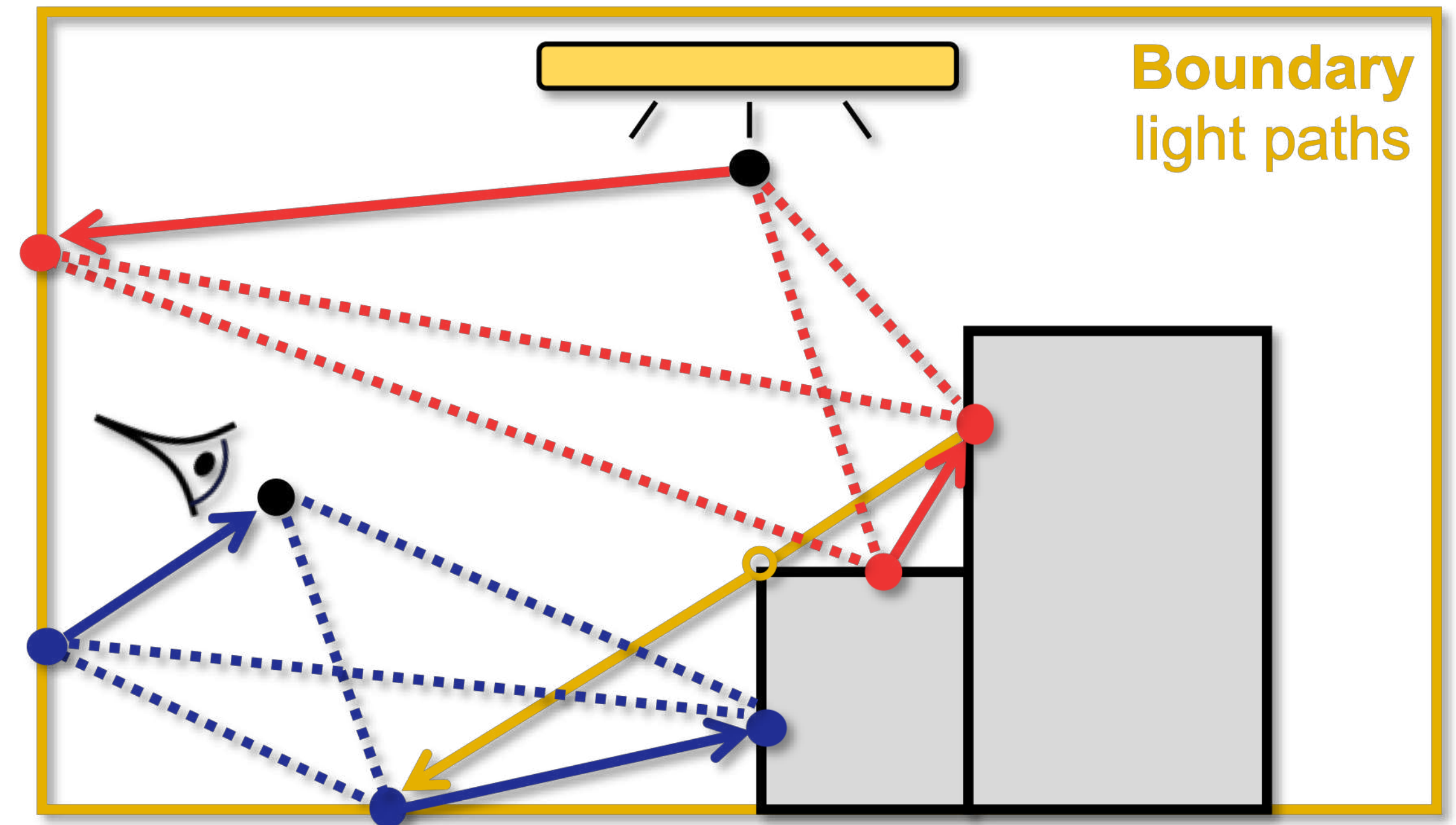
- **Interior**: *unidirectional* path tracing
- **Boundary**: *unidirectional* sampling of subpaths



*Unidirectional* path tracing + NEE

## **Bidirectional** estimator

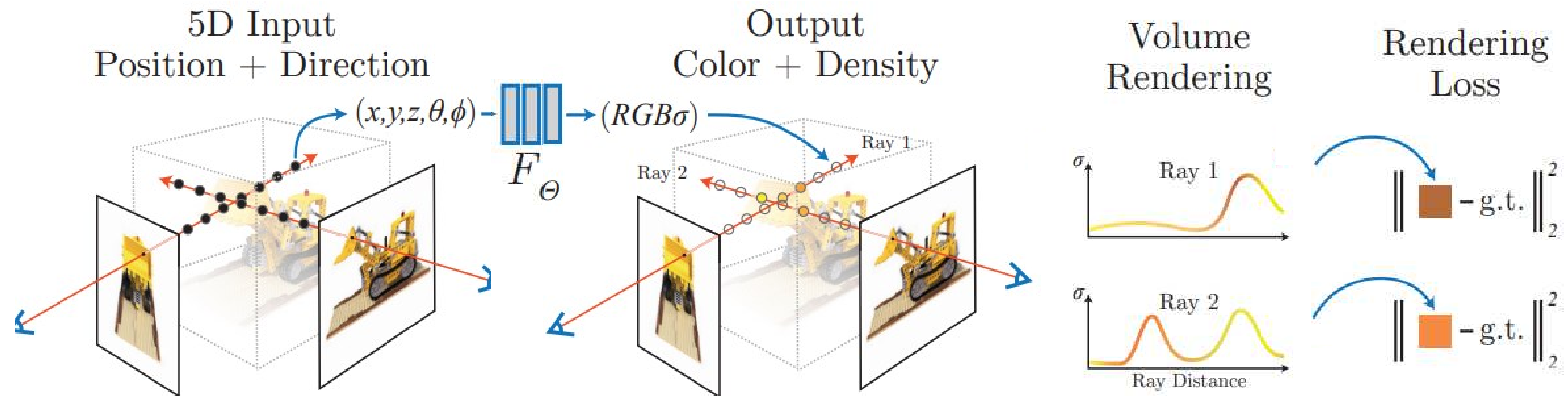
- **Interior**: *bidirectional* path tracing
- **Boundary**: *bidirectional* sampling of subpaths



*Bidirectional* path tracing



# Application: neural rendering



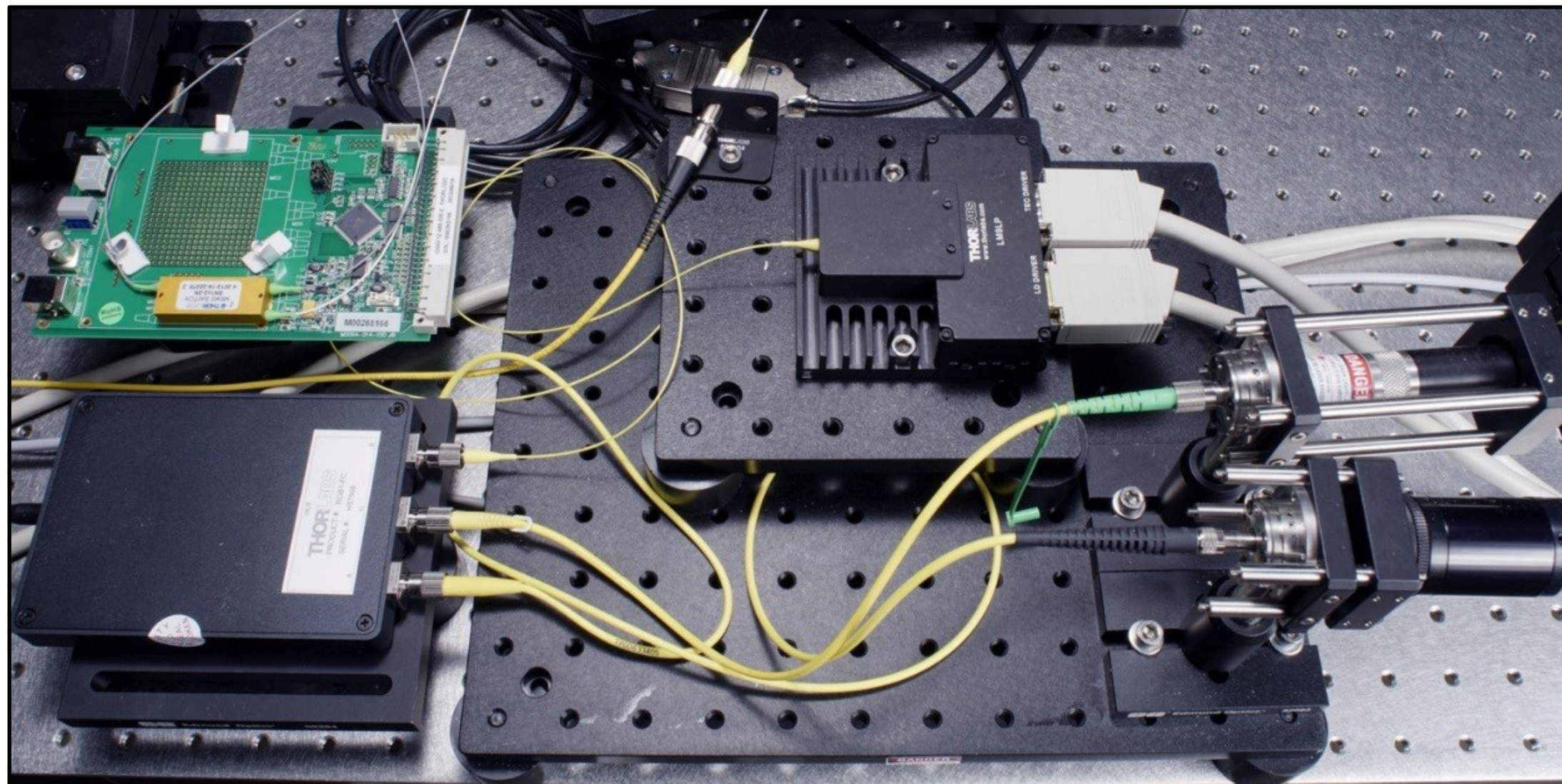
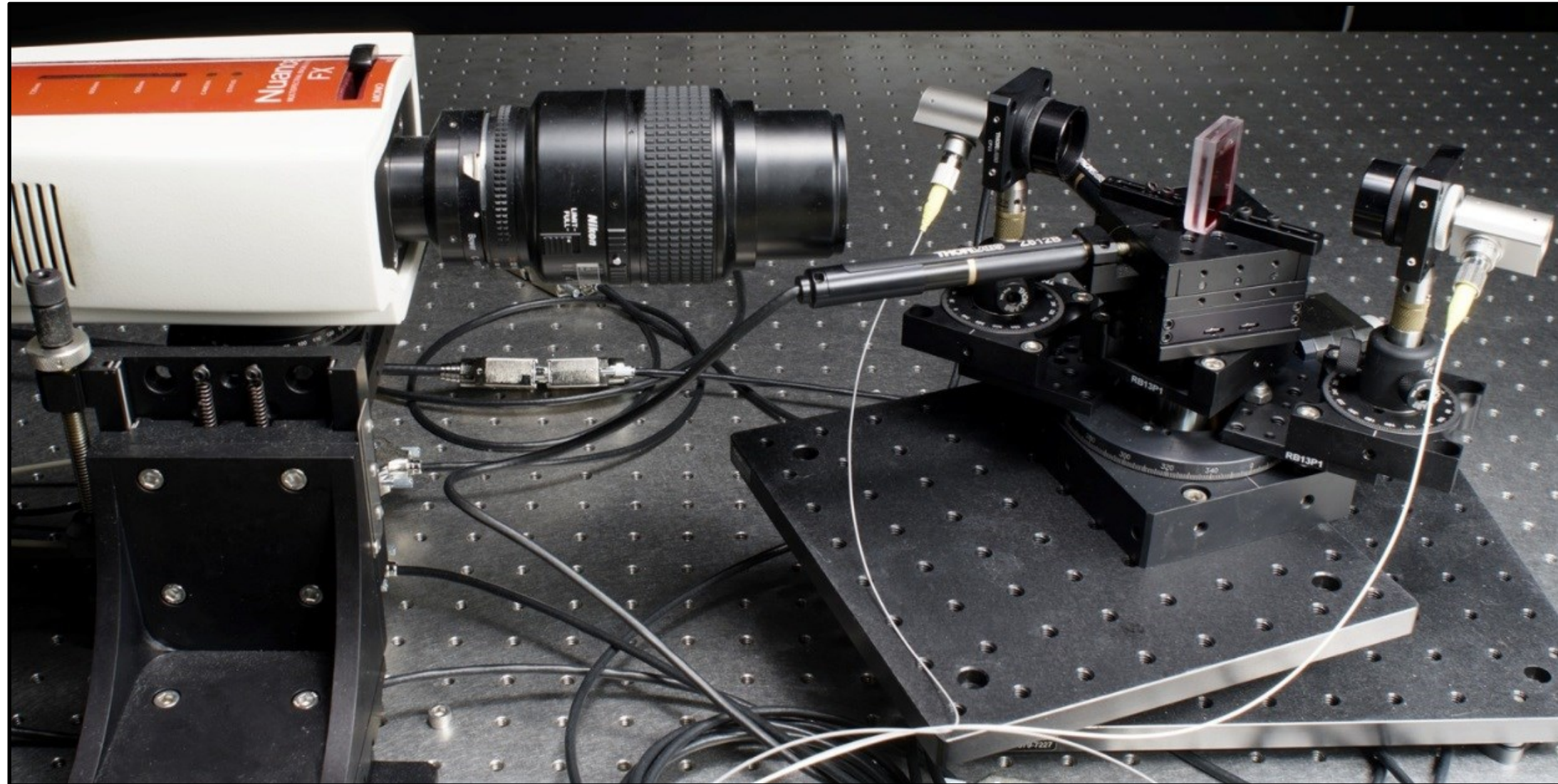


# Acquisition of scattering materials



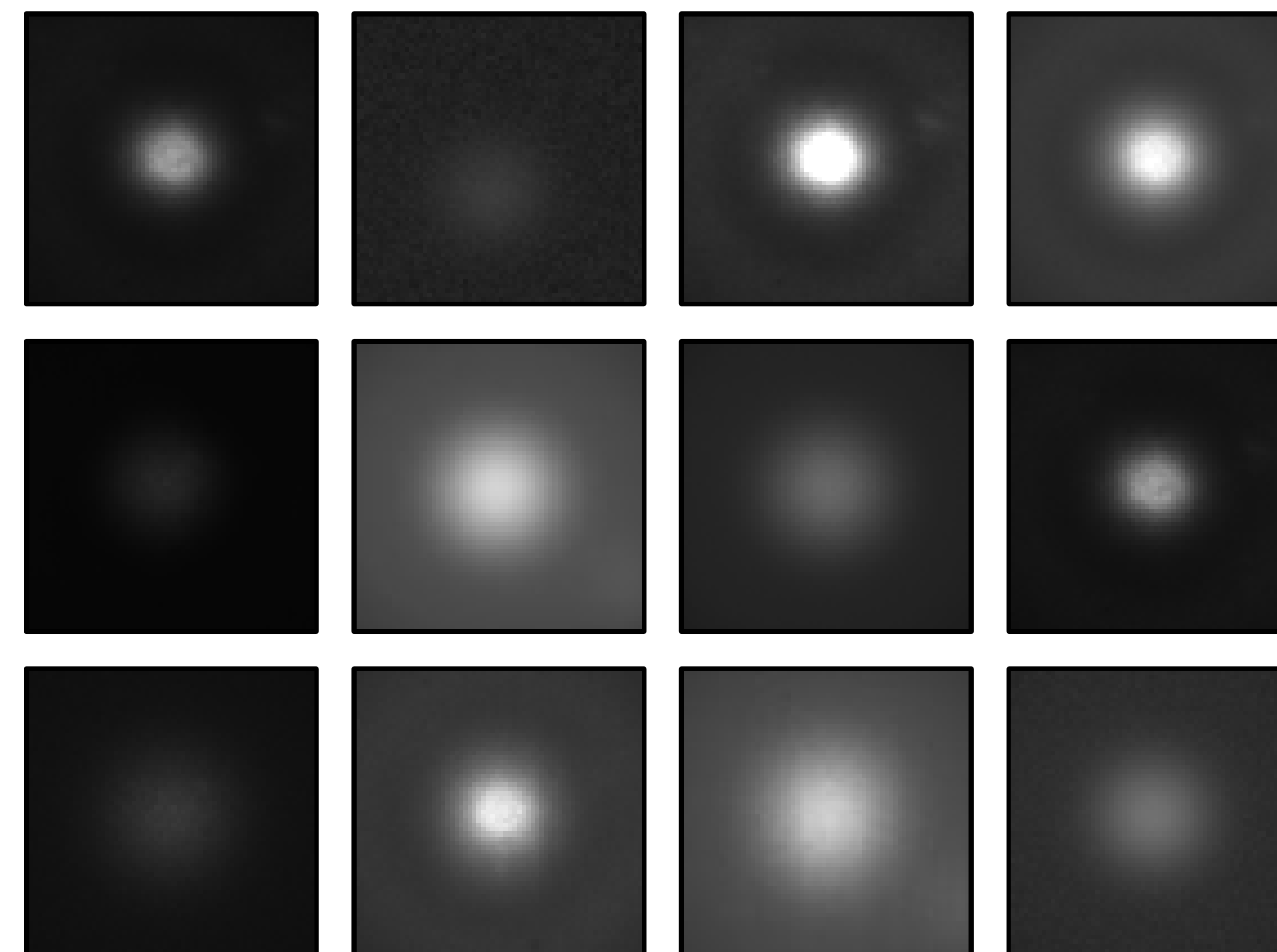
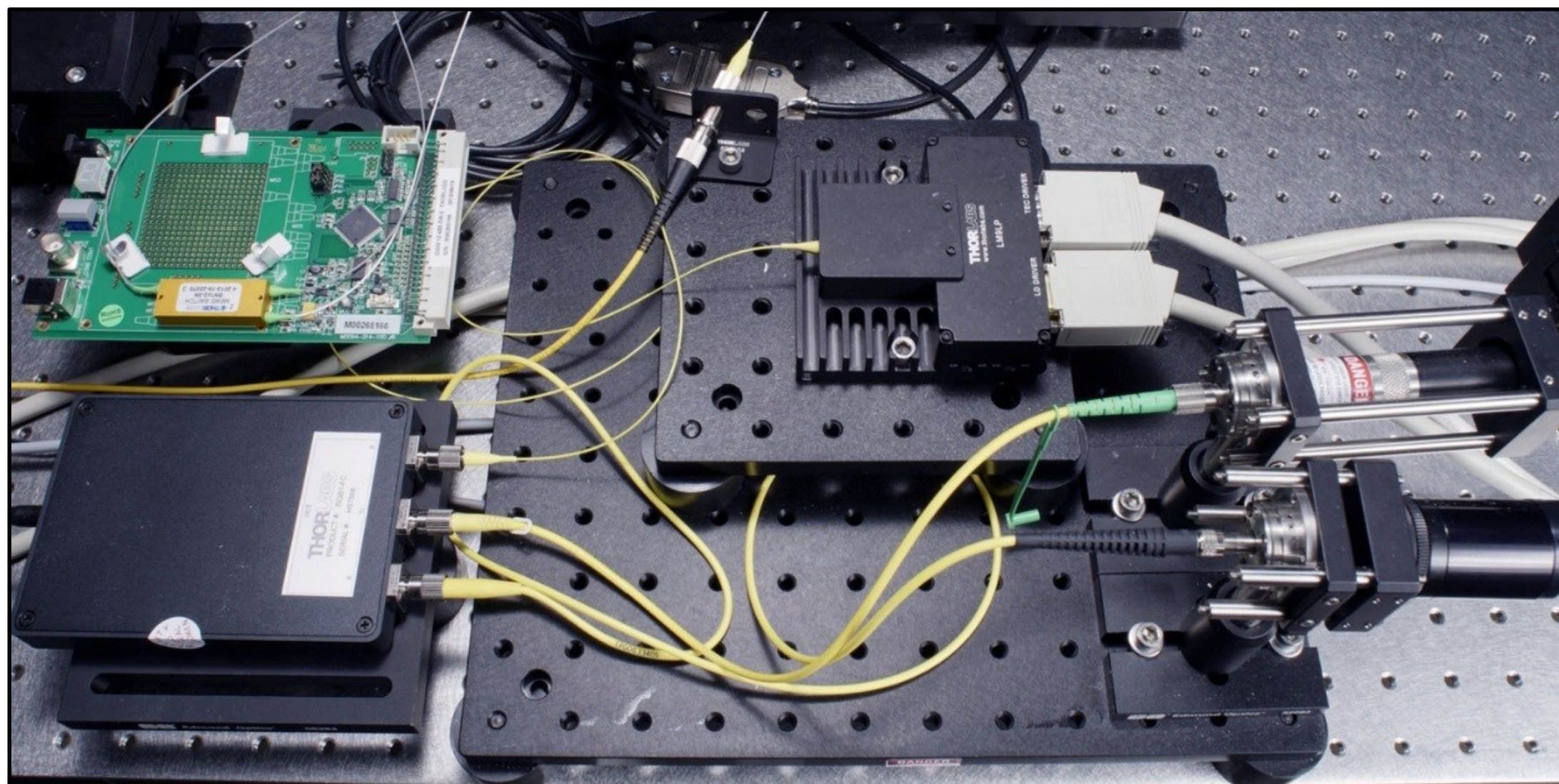
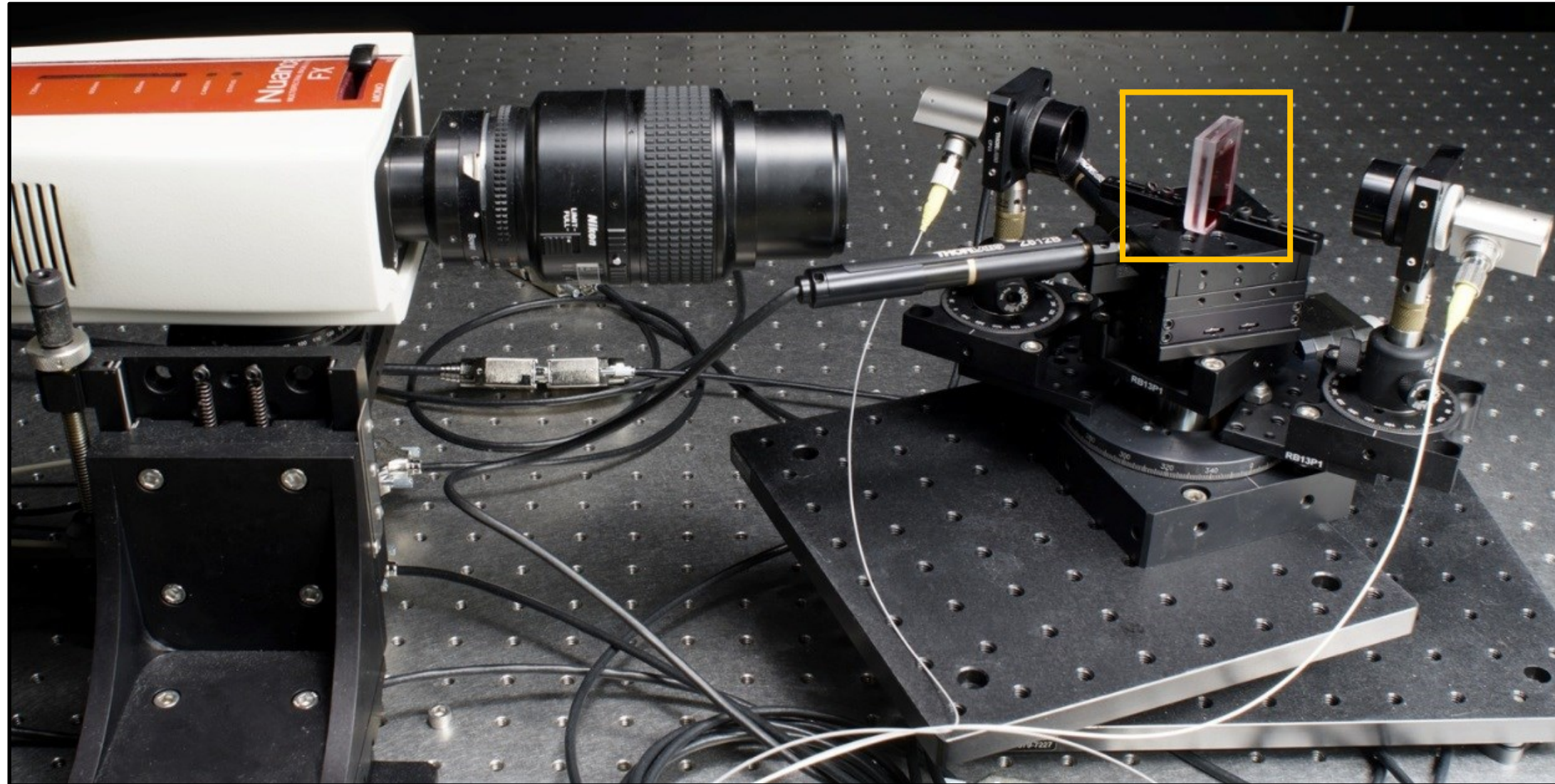


# Acquisition setup





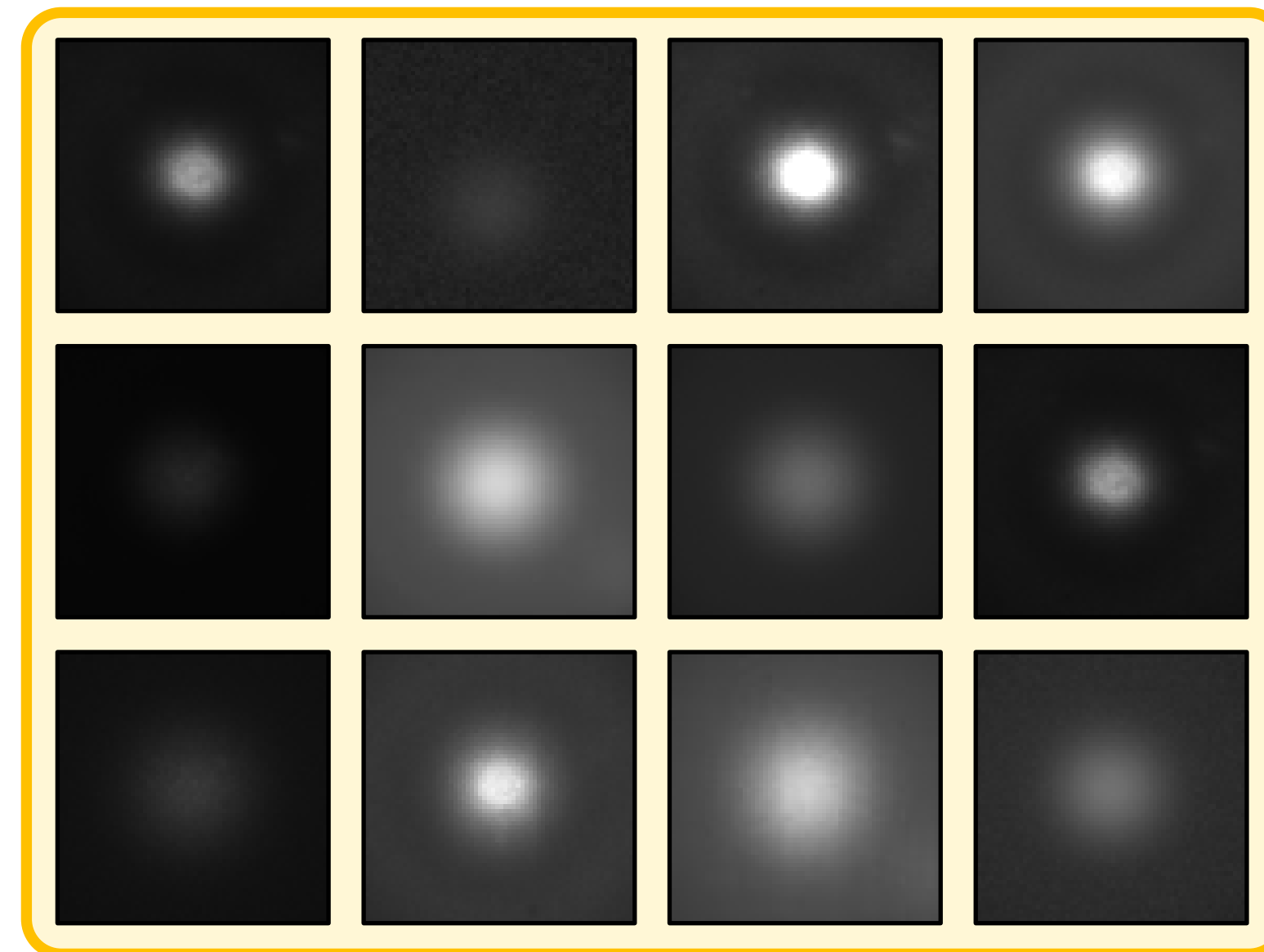
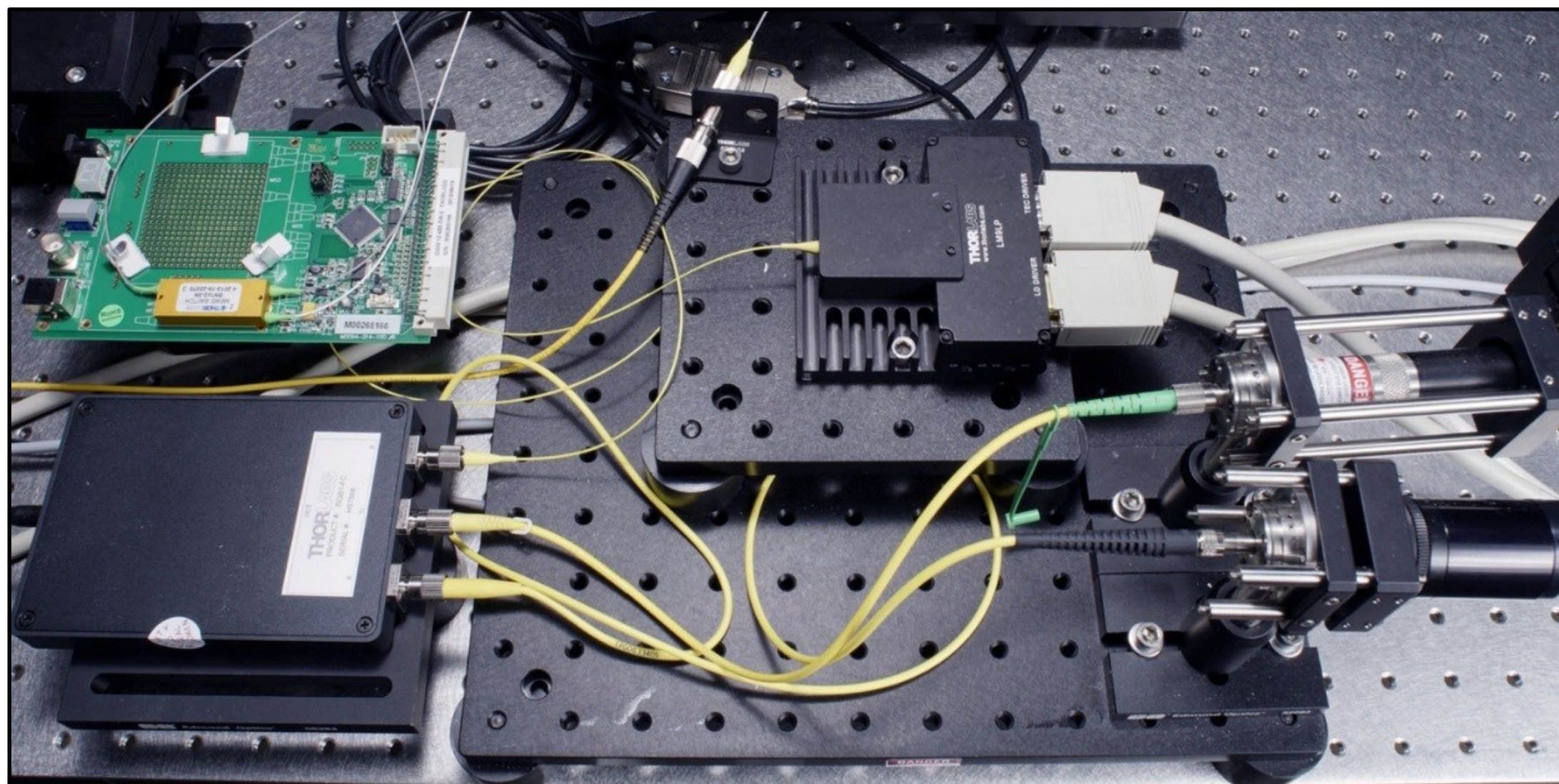
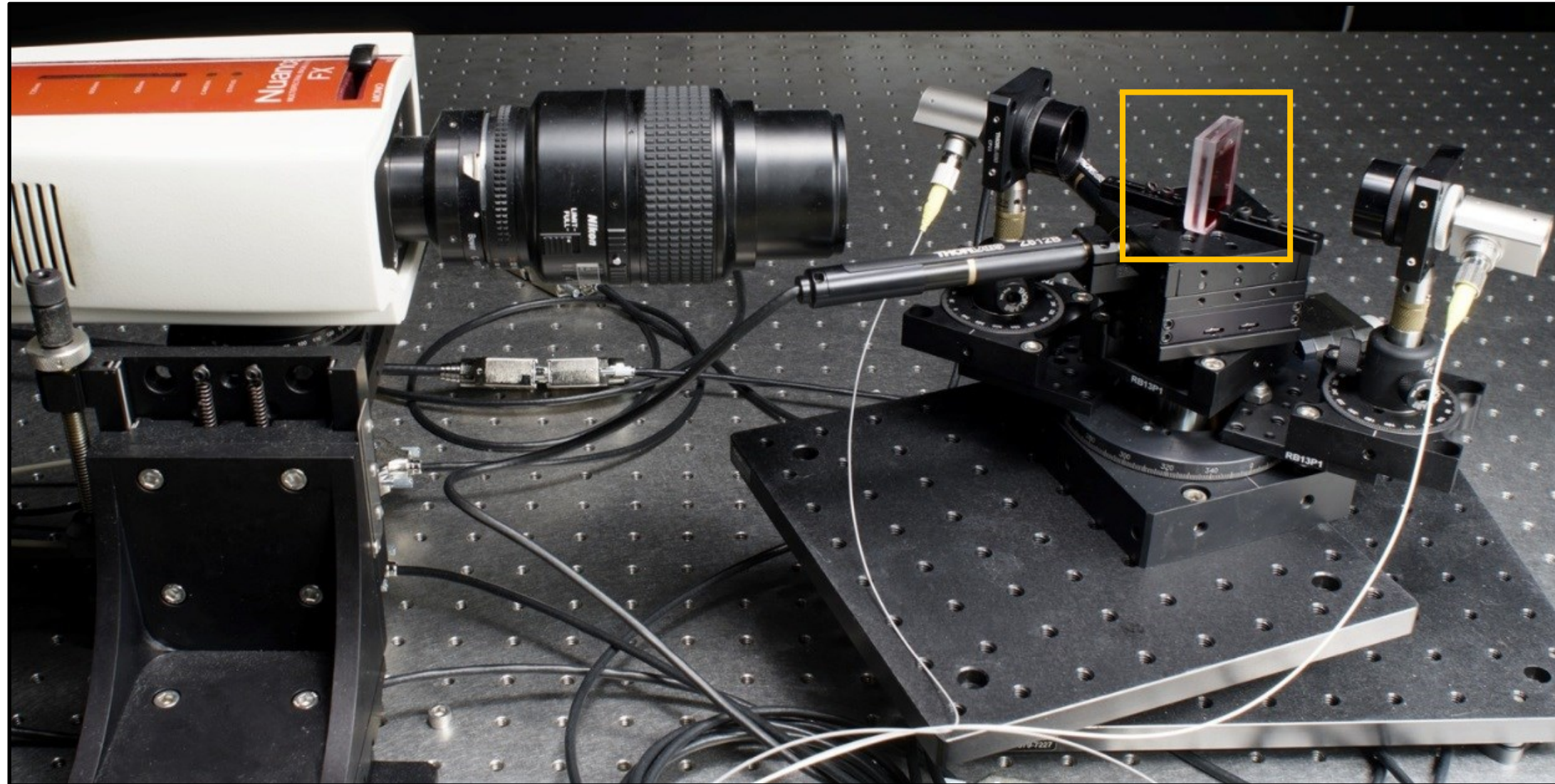
# Acquisition setup



[Gkioulekas et al., 2013]



# Acquisition setup

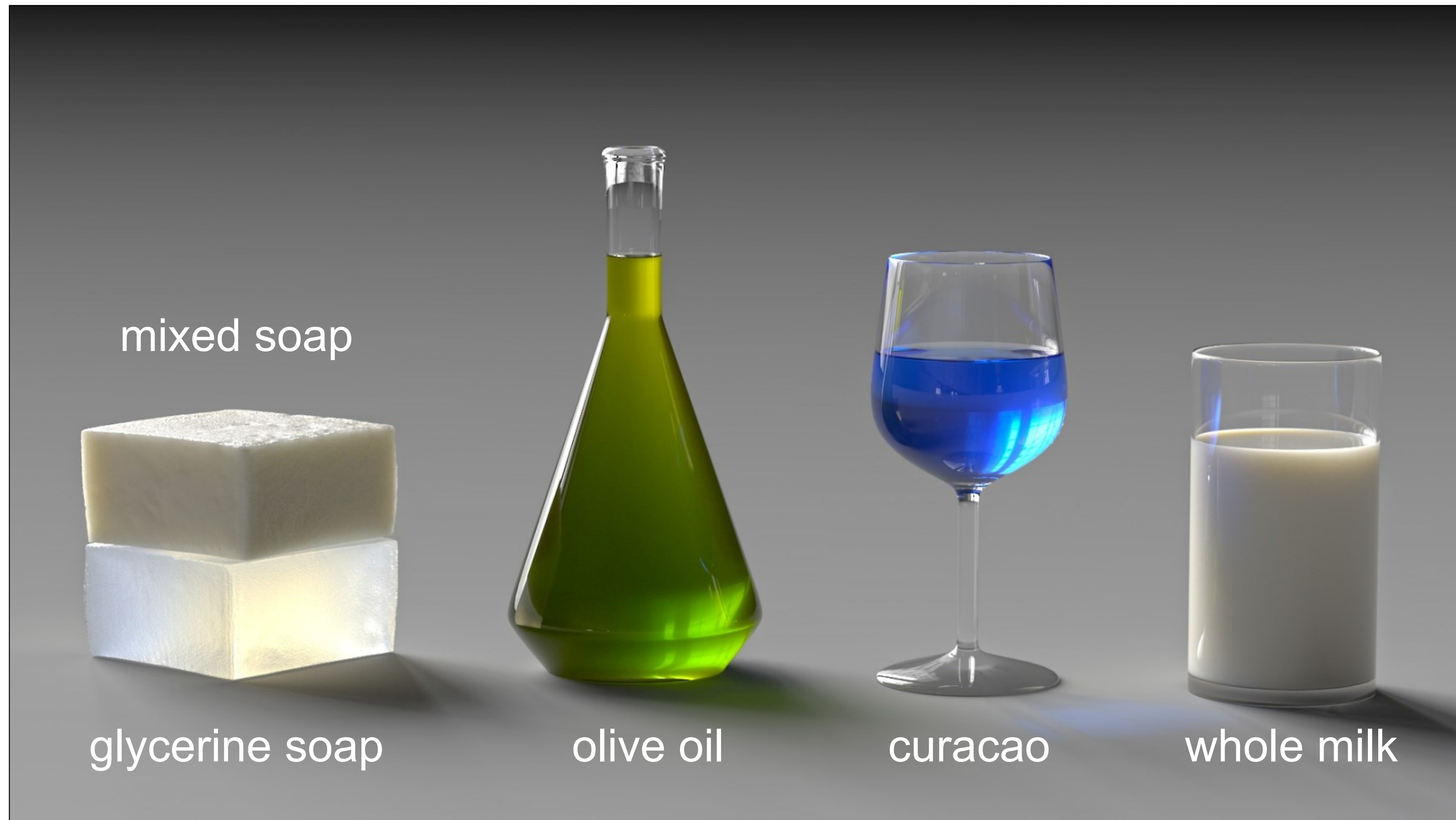


Invert using  
differentiable  
rendering

[Gkioulekas et al., 2013]



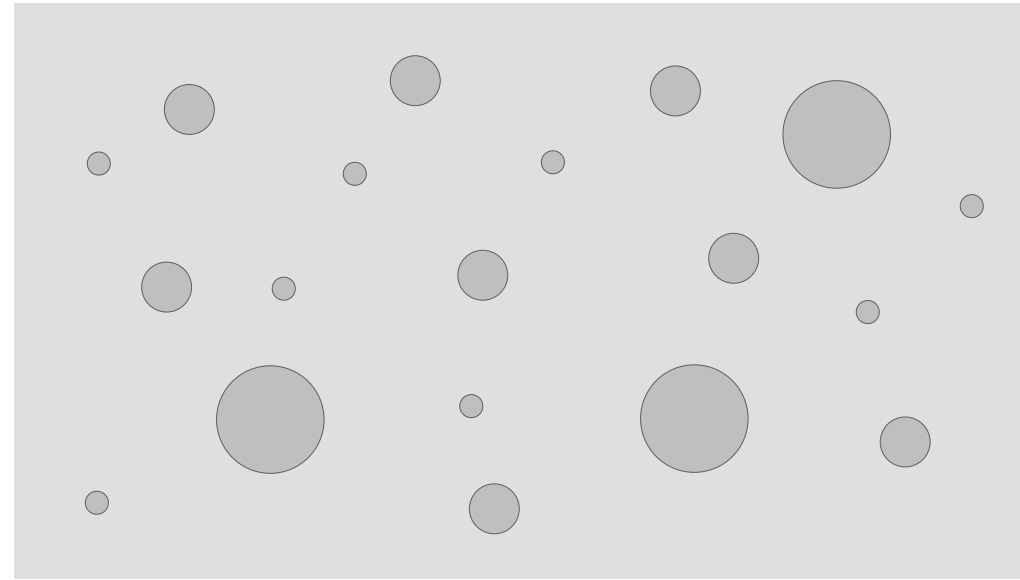
# Synthetic renderings


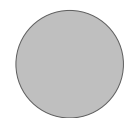


# Particle sizing of industrial nanodispersions

# Particle sizing of industrial nanodispersions

unknown nanodispersion

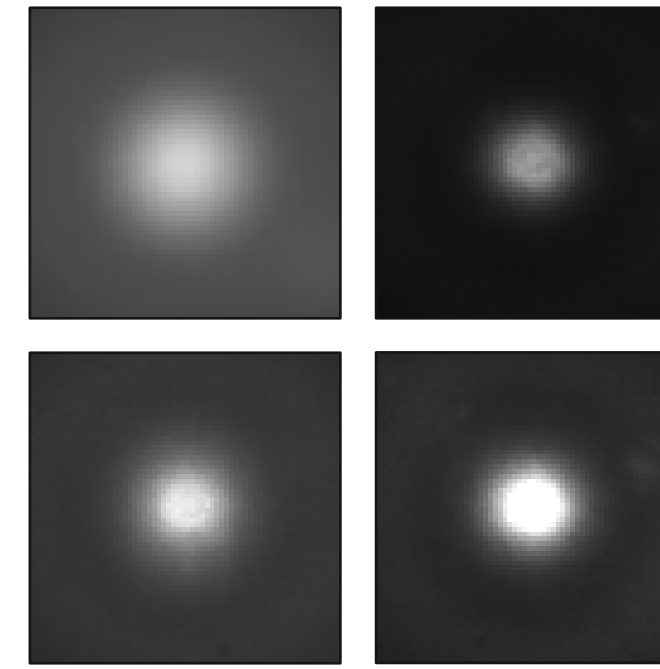
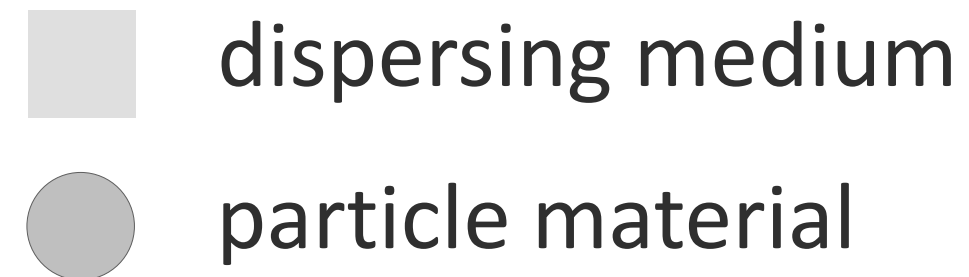
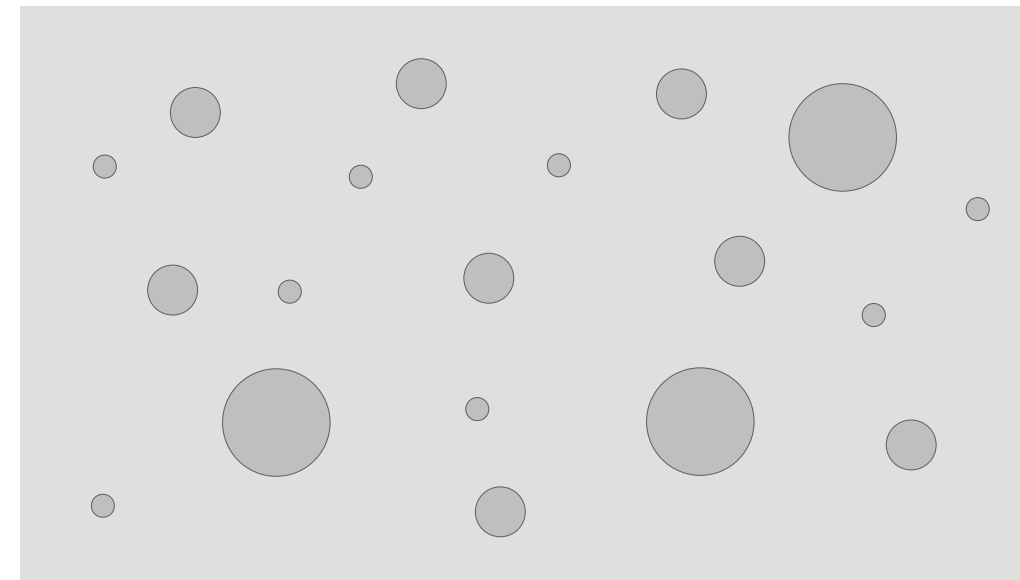


-  dispersing medium
-  particle material

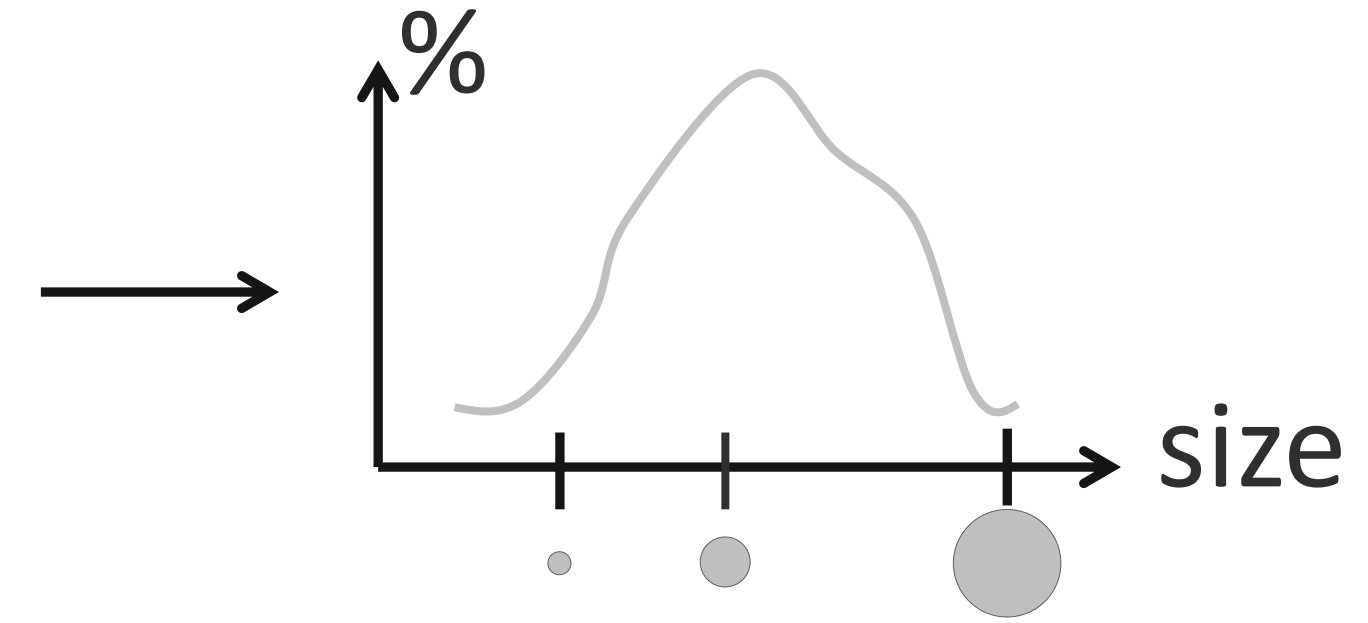


# Particle sizing of industrial nanodispersions

unknown nanodispersion

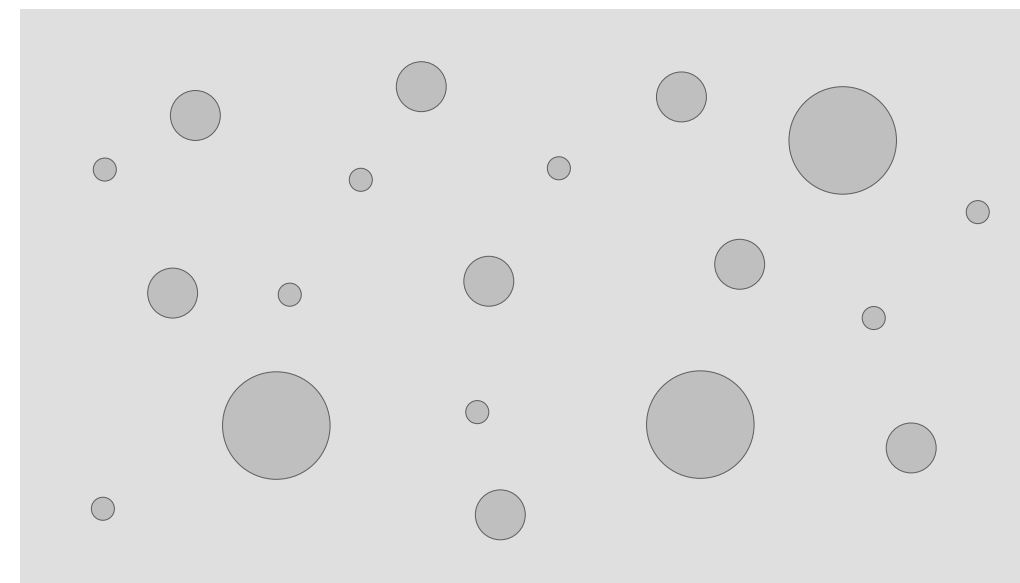


measurements

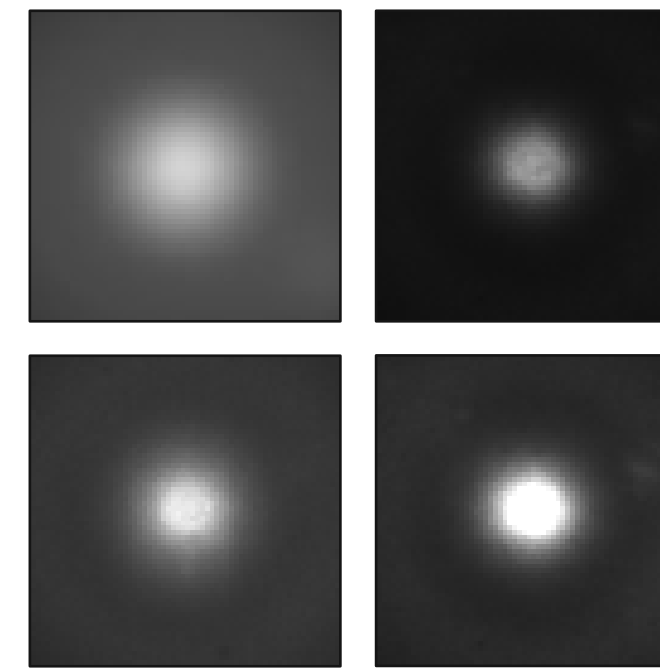


# Particle sizing of industrial nanodispersions

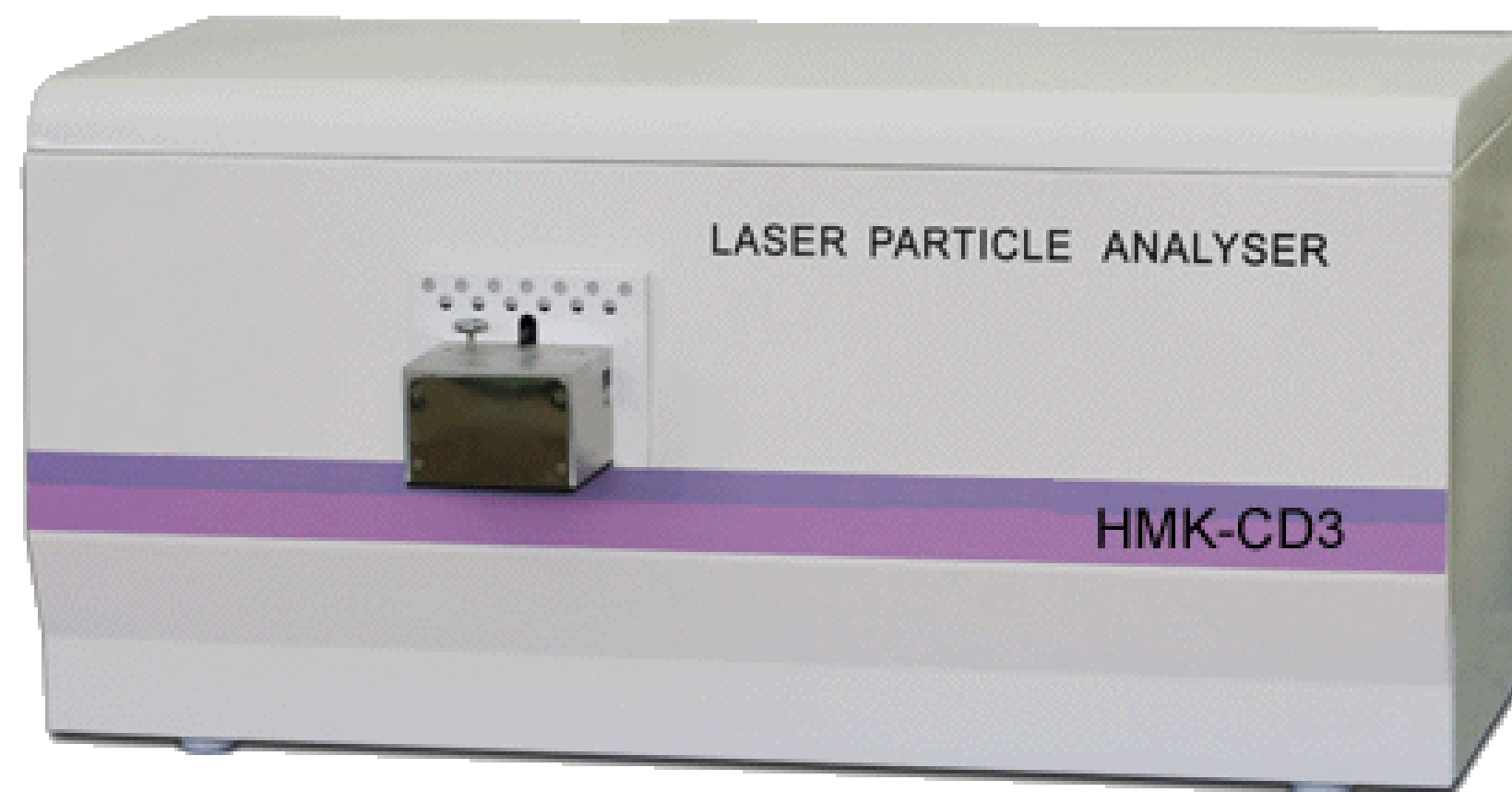
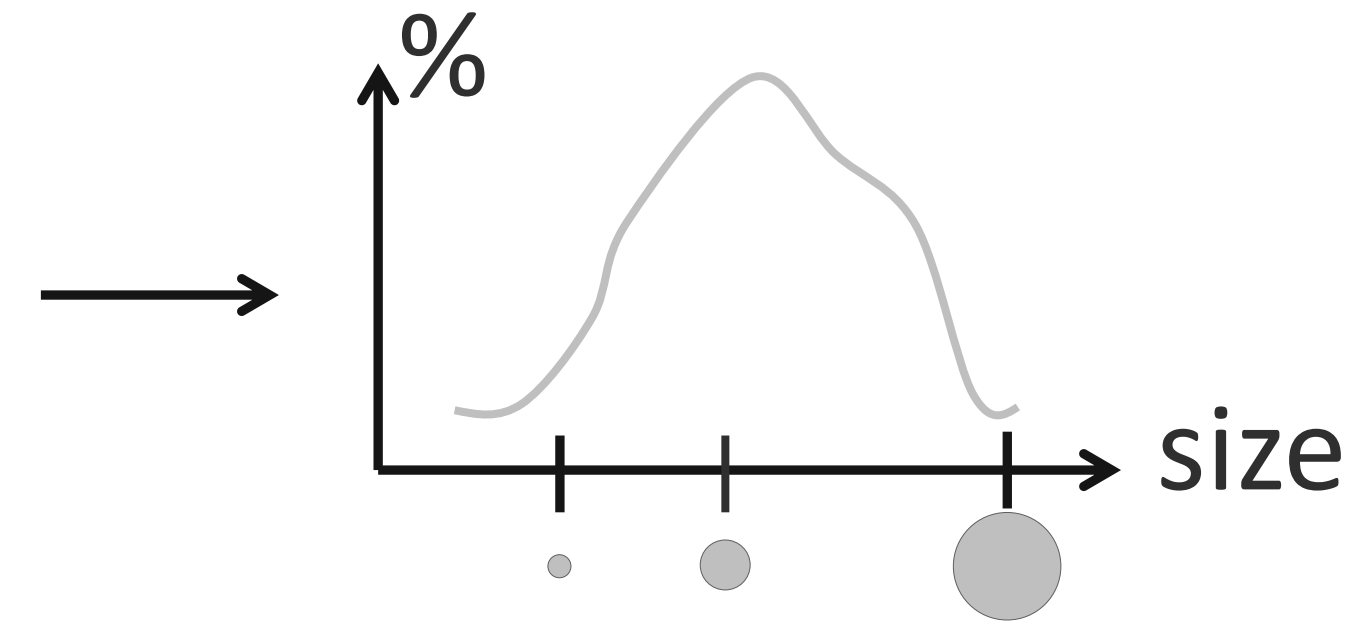
unknown nanodispersion



■ dispersing medium  
● particle material



measurements



# Particle sizing of industrial nanodispersions



polystyrene



aluminum oxide

very precise dispersions (NIST  
Traceable Standards)



# Particle sizing of industrial nanodispersions

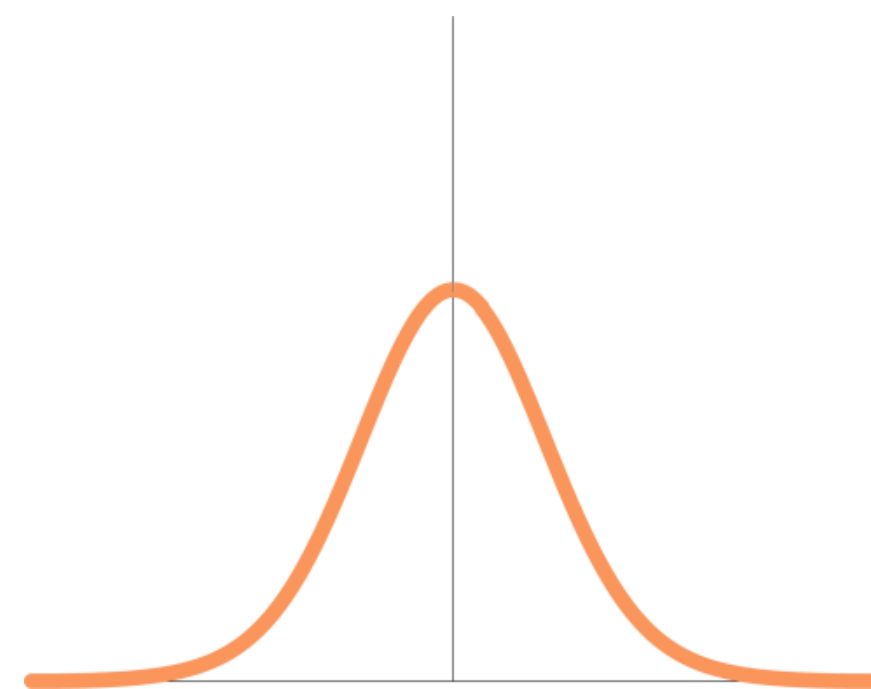


polystyrene

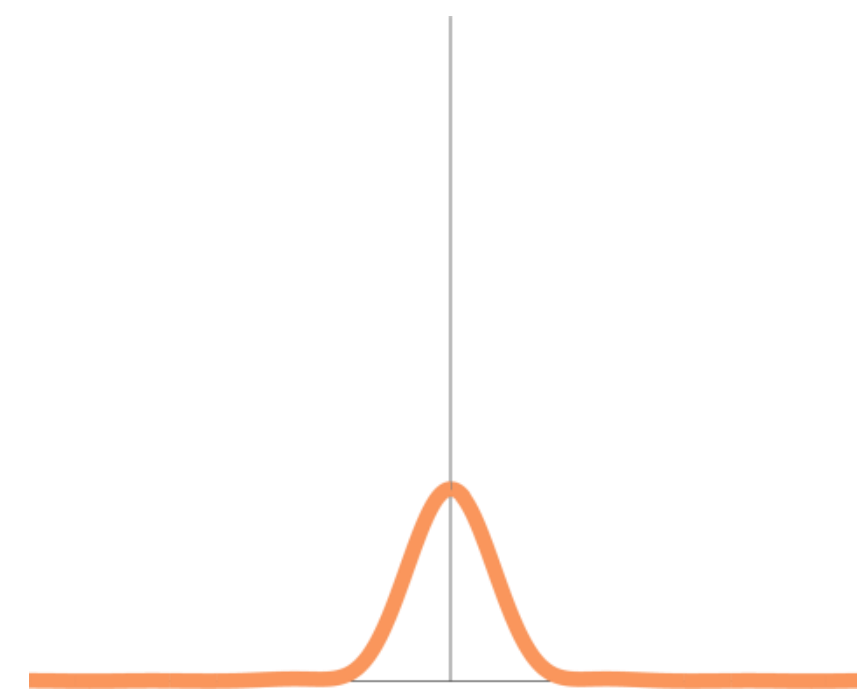


aluminum oxide

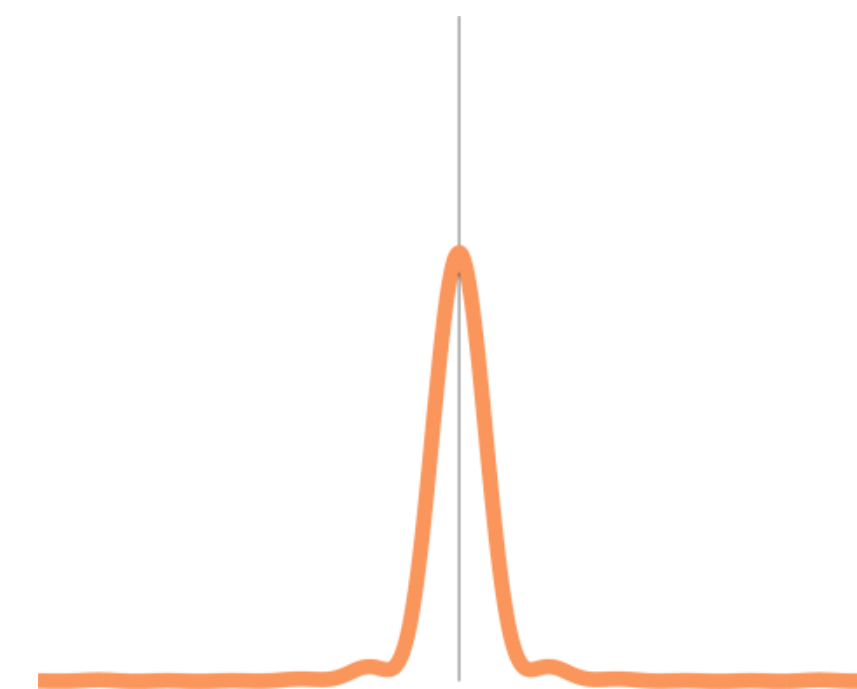
very precise dispersions (NIST  
Traceable Standards)



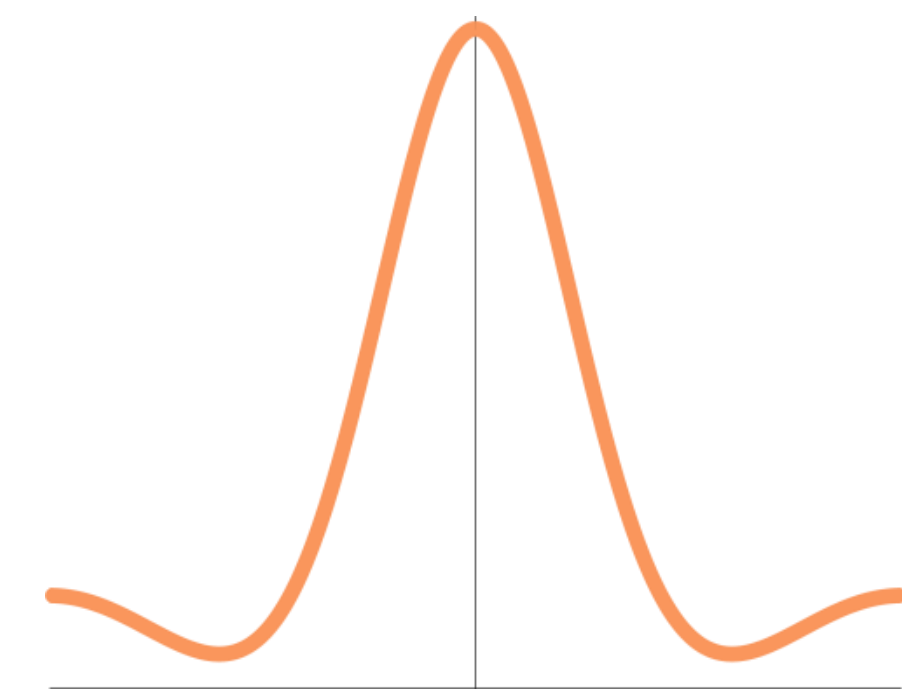
polystyrene 1



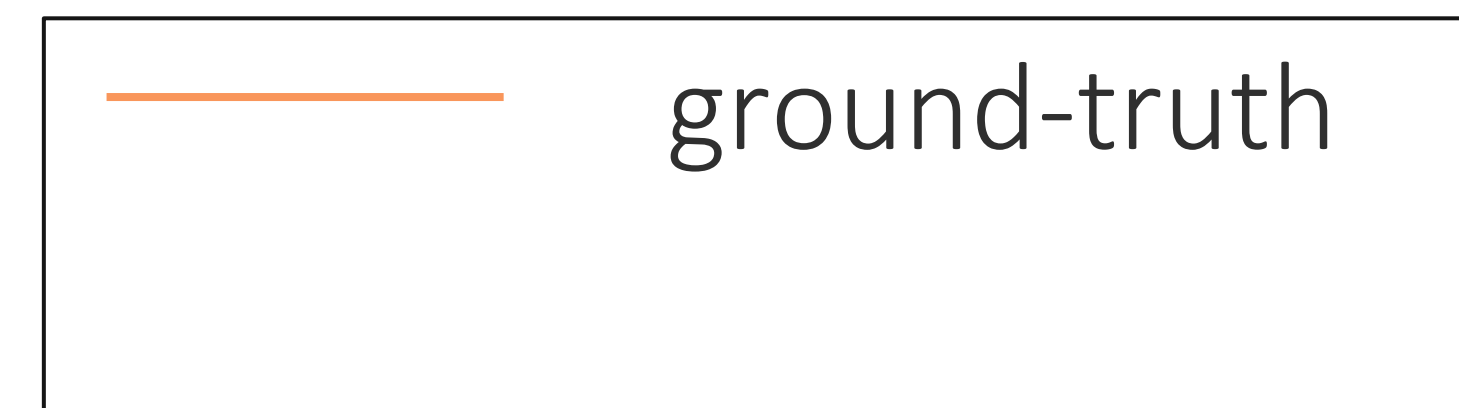
polystyrene 2



polystyrene 3



aluminum oxide



# Particle sizing of industrial nanodispersions

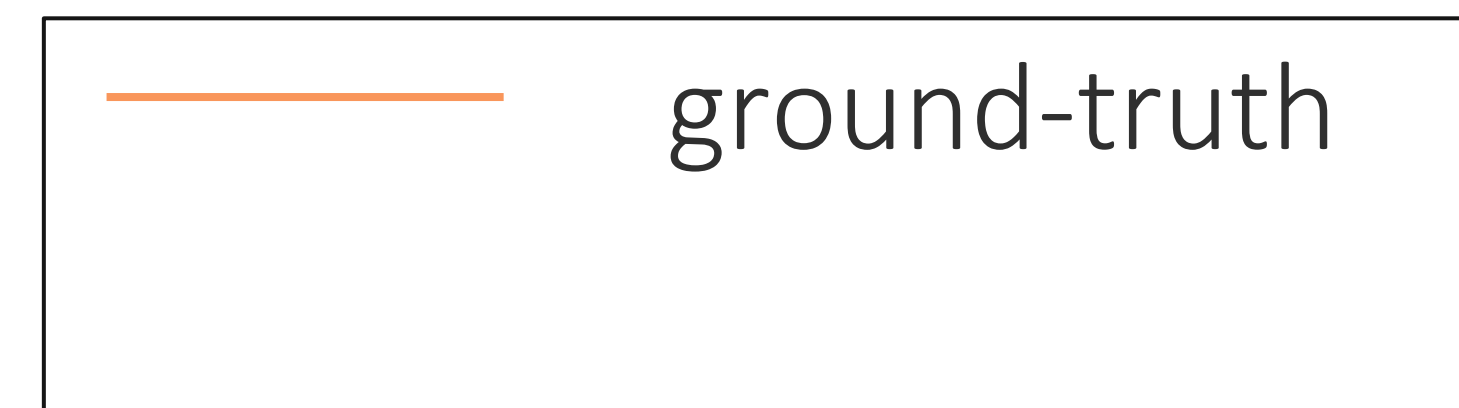
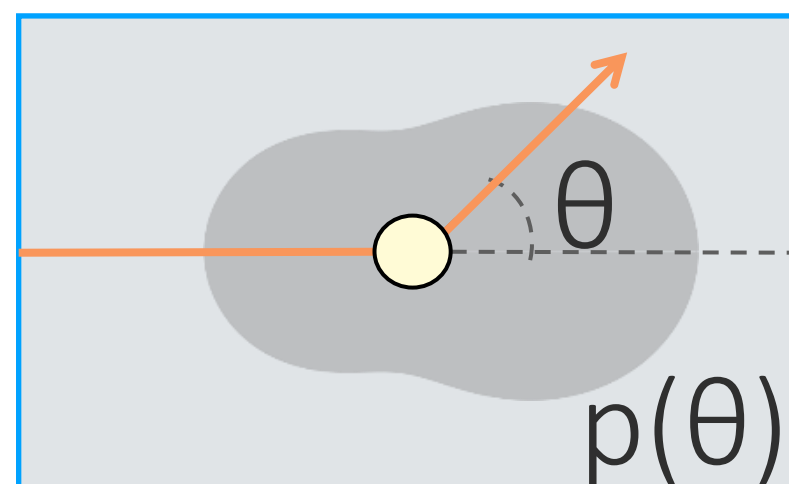
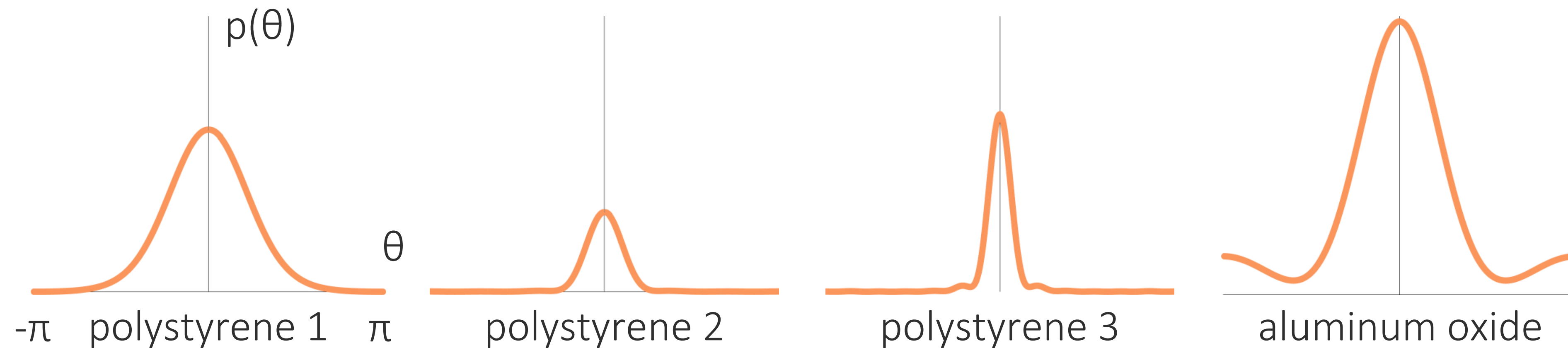


polystyrene



aluminum oxide

very precise dispersions (NIST  
Traceable Standards)



# Particle sizing of industrial nanodispersions

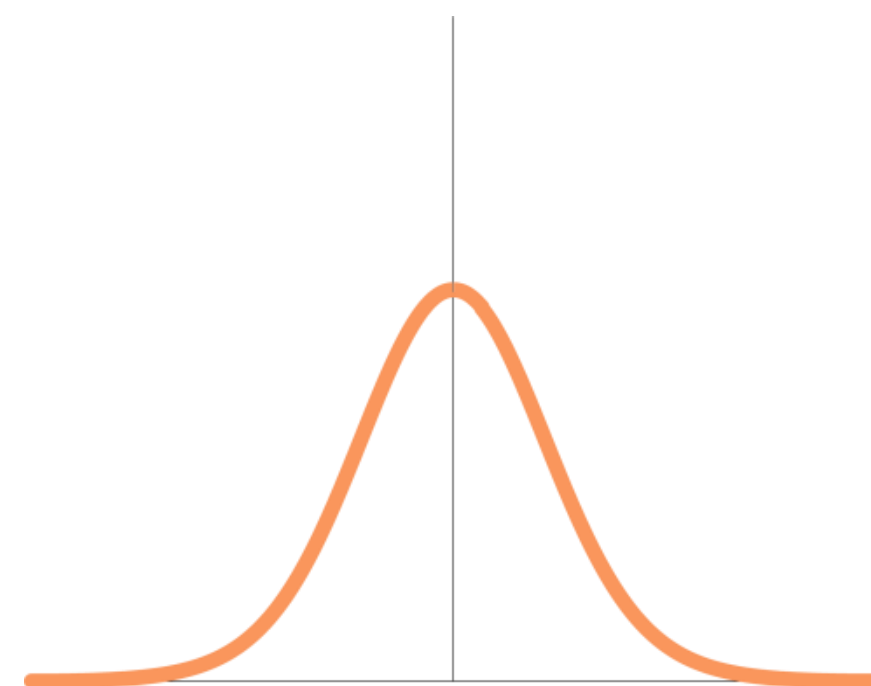


polystyrene

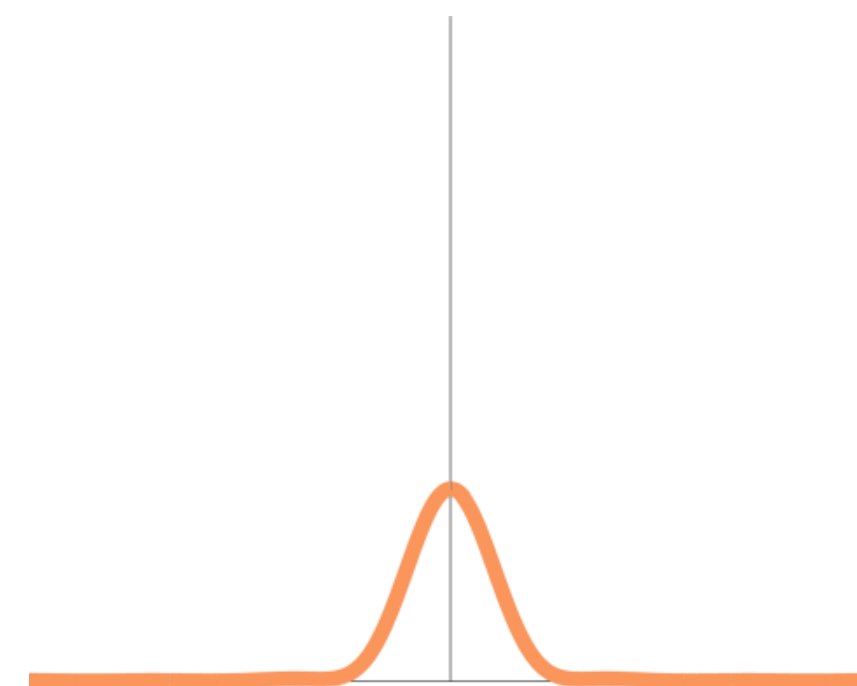


aluminum oxide

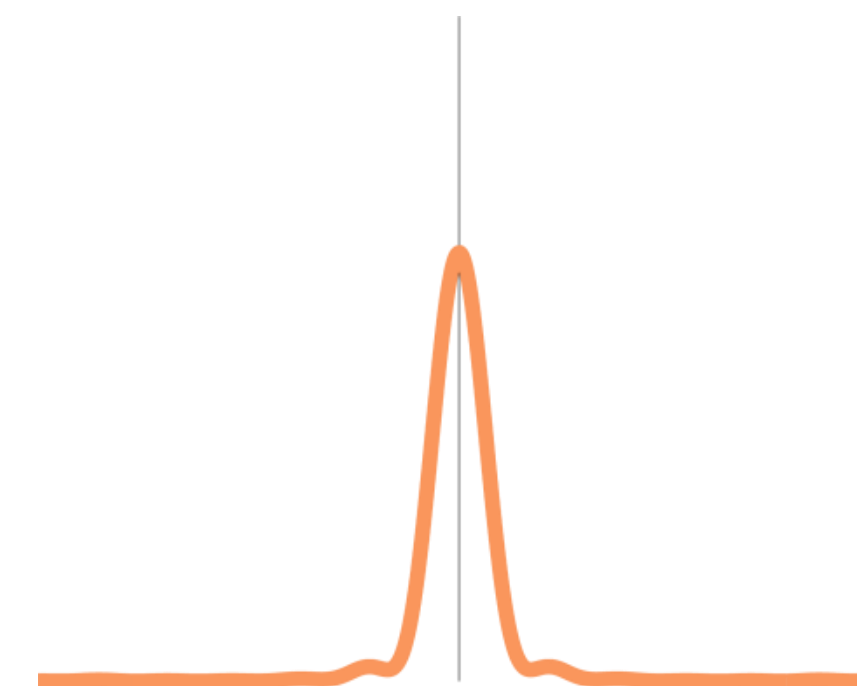
very precise dispersions (NIST  
Traceable Standards)



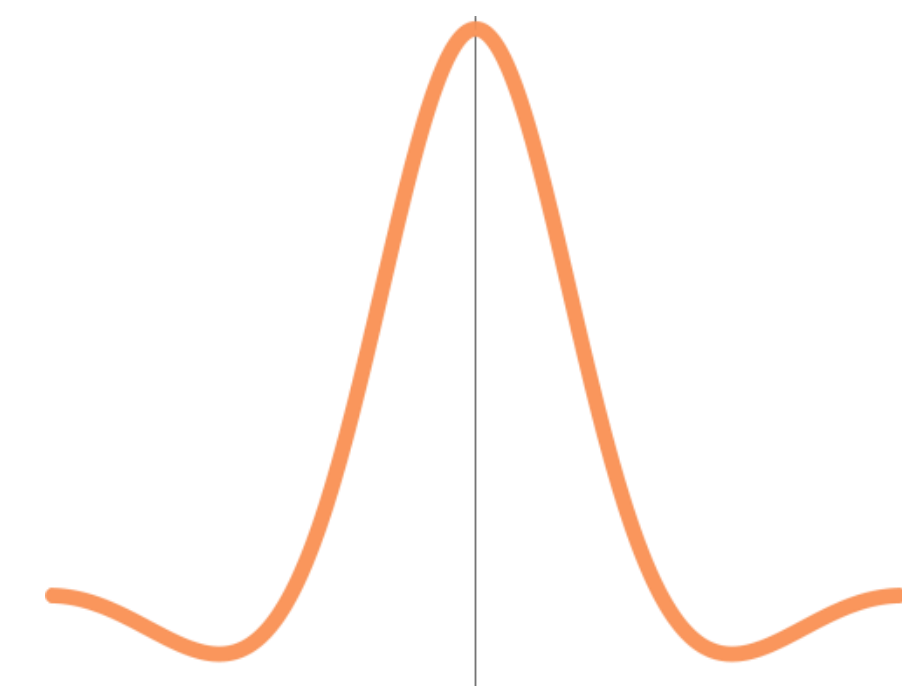
polystyrene 1



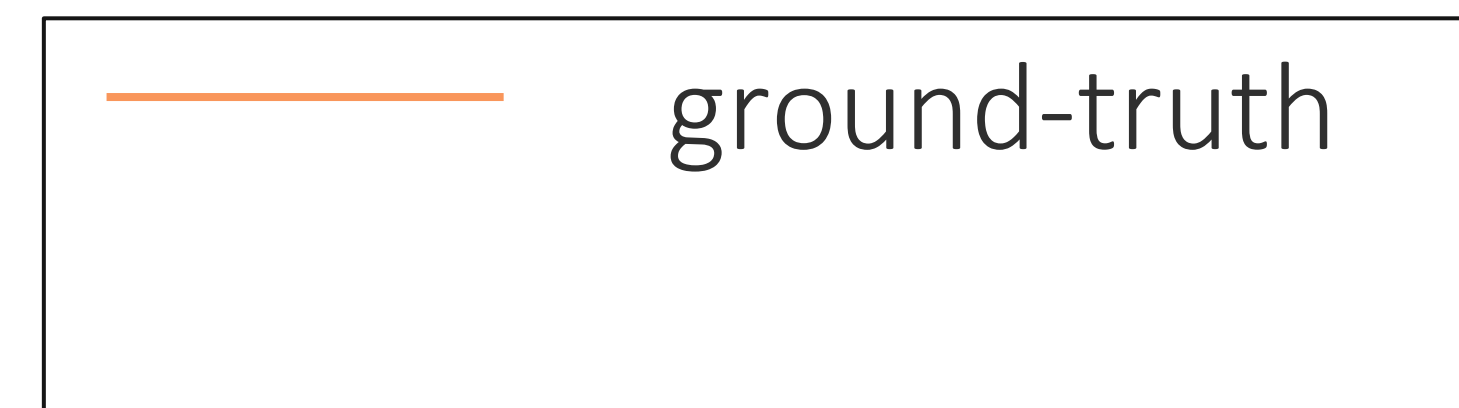
polystyrene 2



polystyrene 3



aluminum oxide





# Particle sizing of industrial nanodispersions

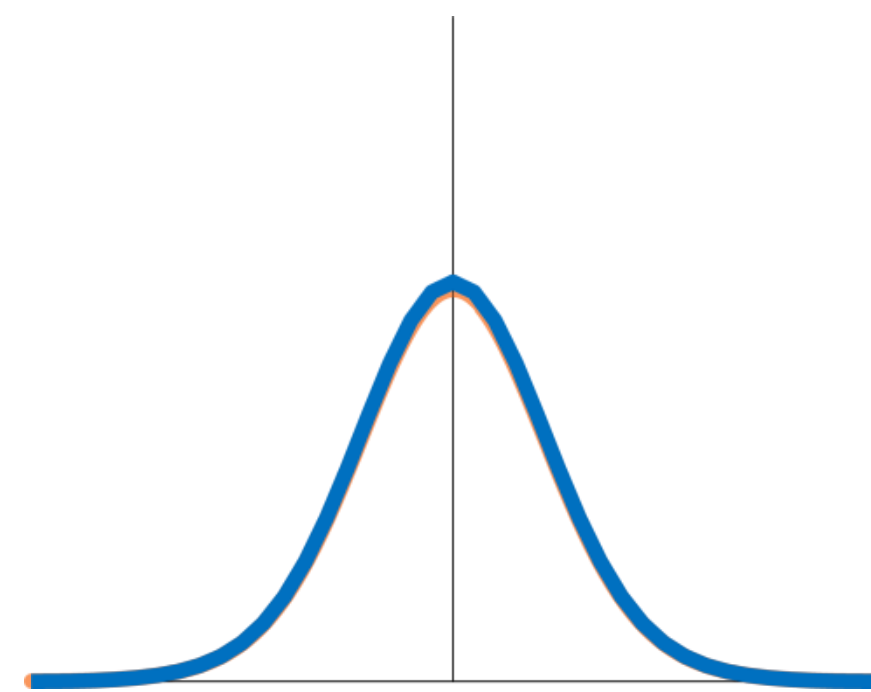


polystyrene

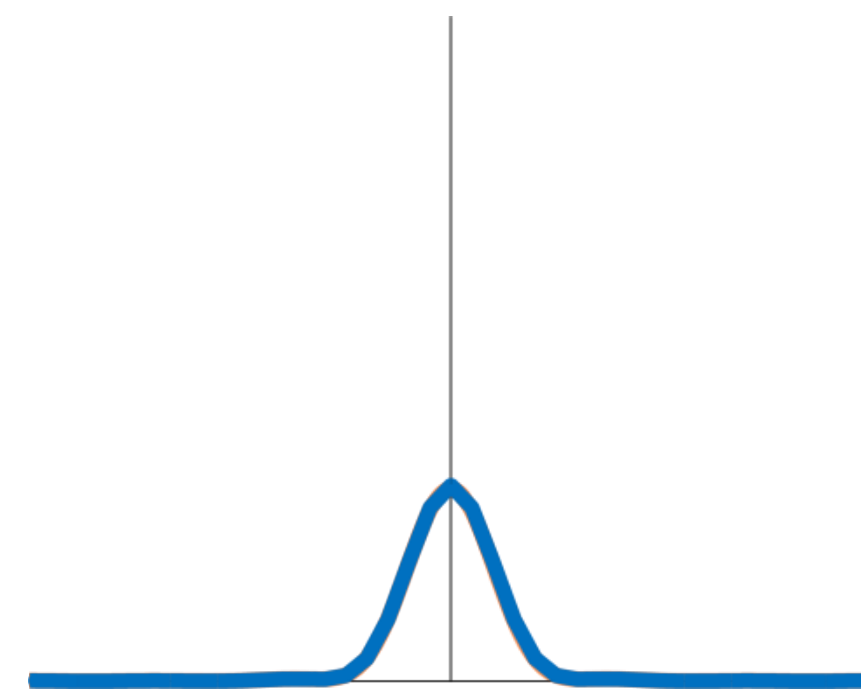


aluminum oxide

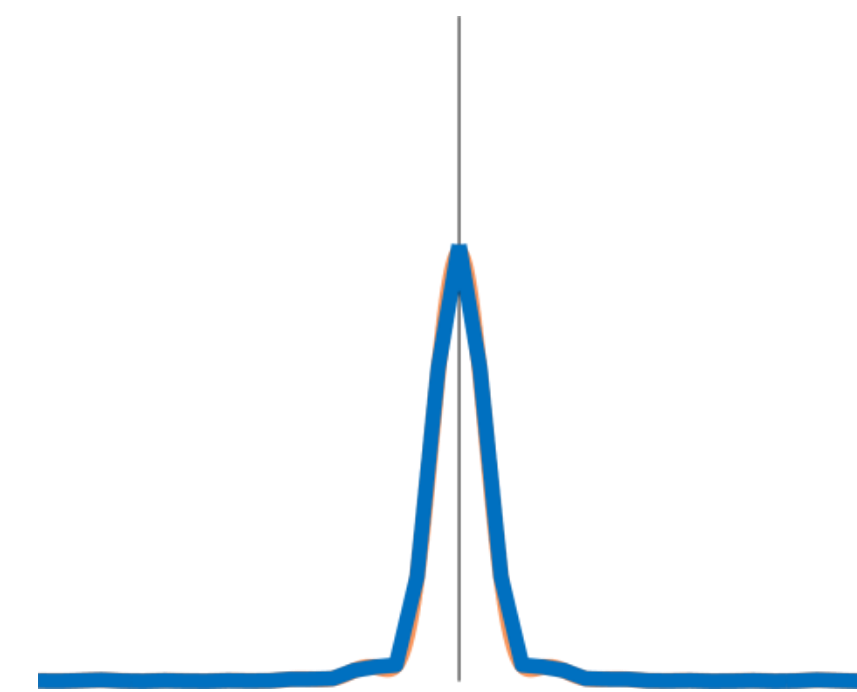
very precise dispersions (NIST  
Traceable Standards)



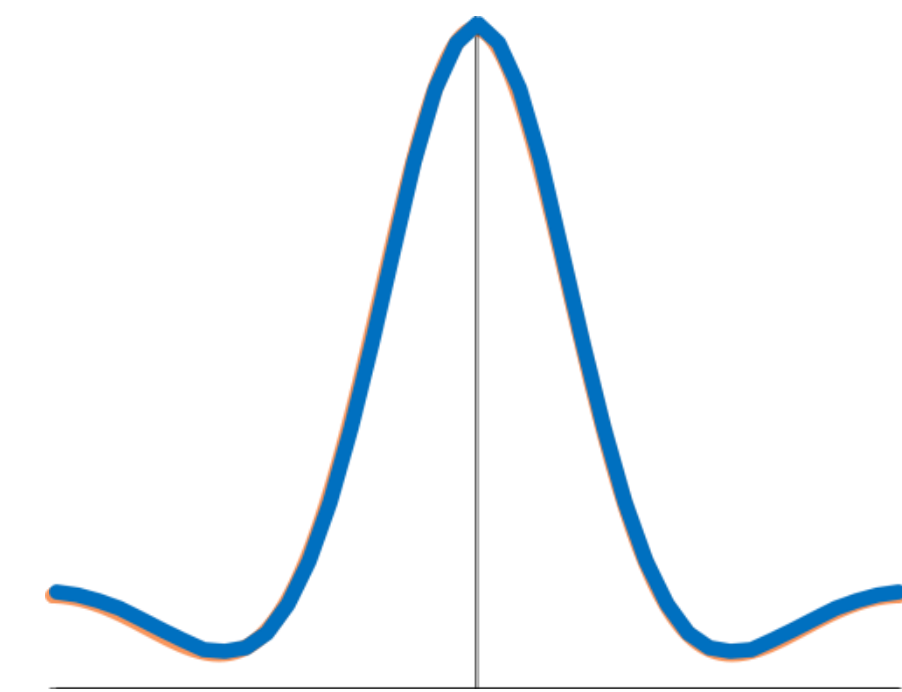
polystyrene 1



polystyrene 2



polystyrene 3



aluminum oxide



# Particle sizing of industrial nanodispersions

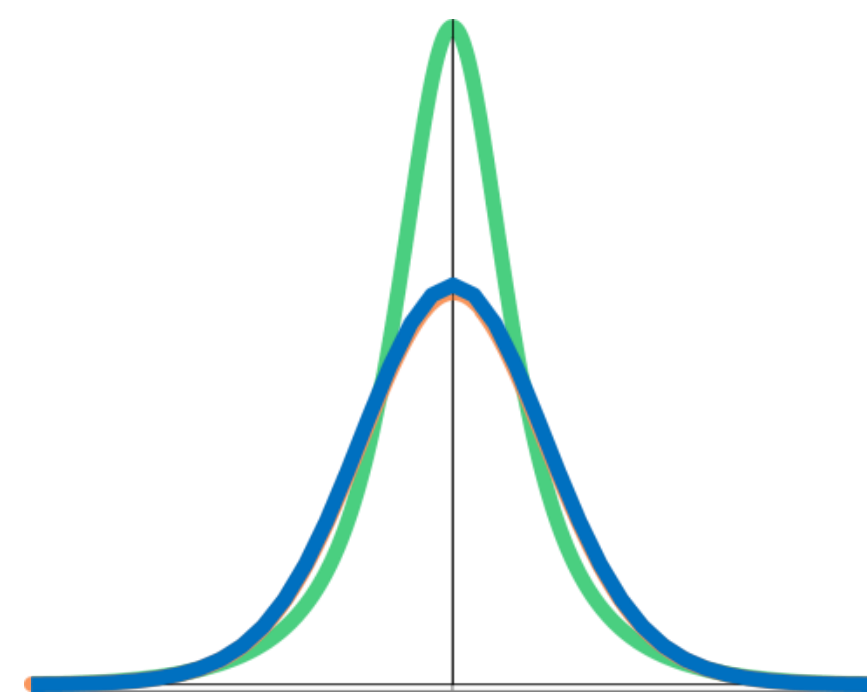


polystyrene

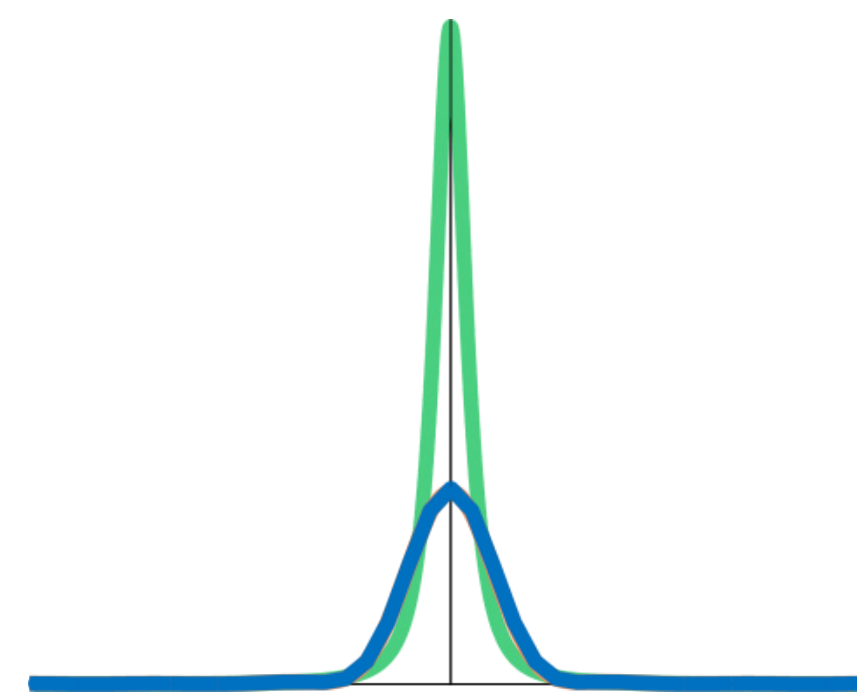


aluminum oxide

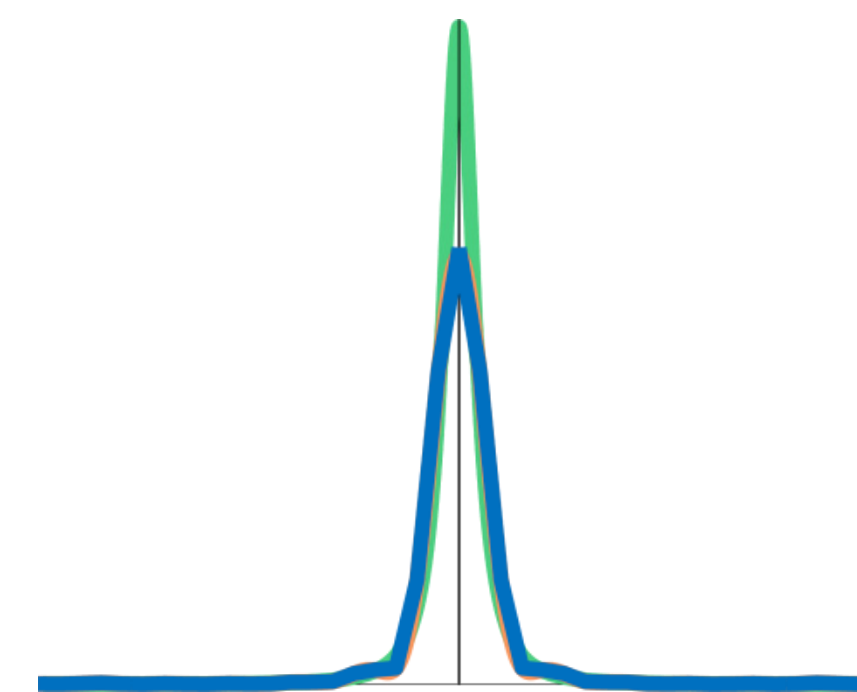
very precise dispersions (NIST  
Traceable Standards)



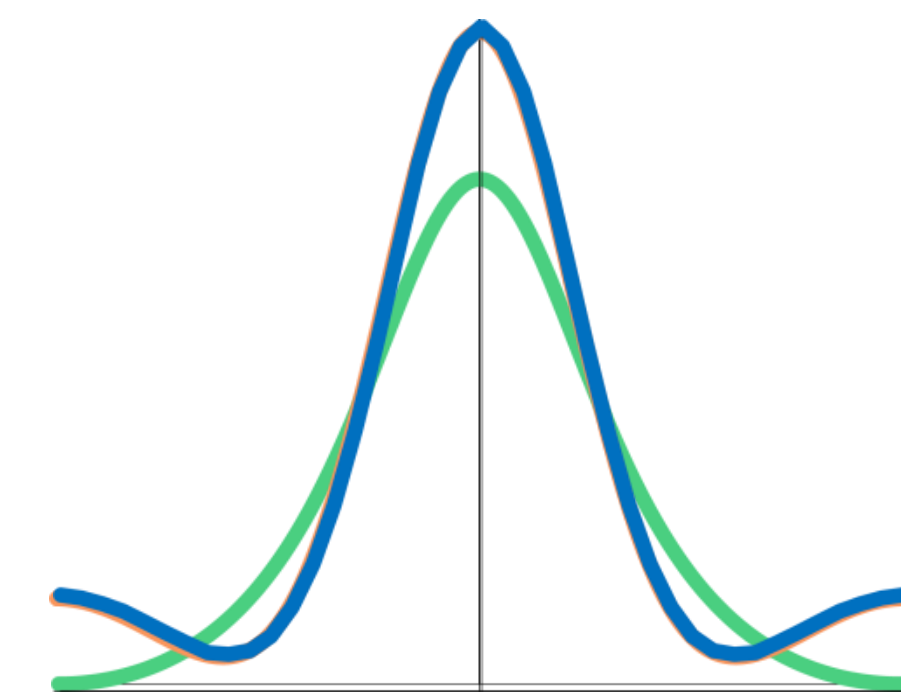
polystyrene 1



polystyrene 2



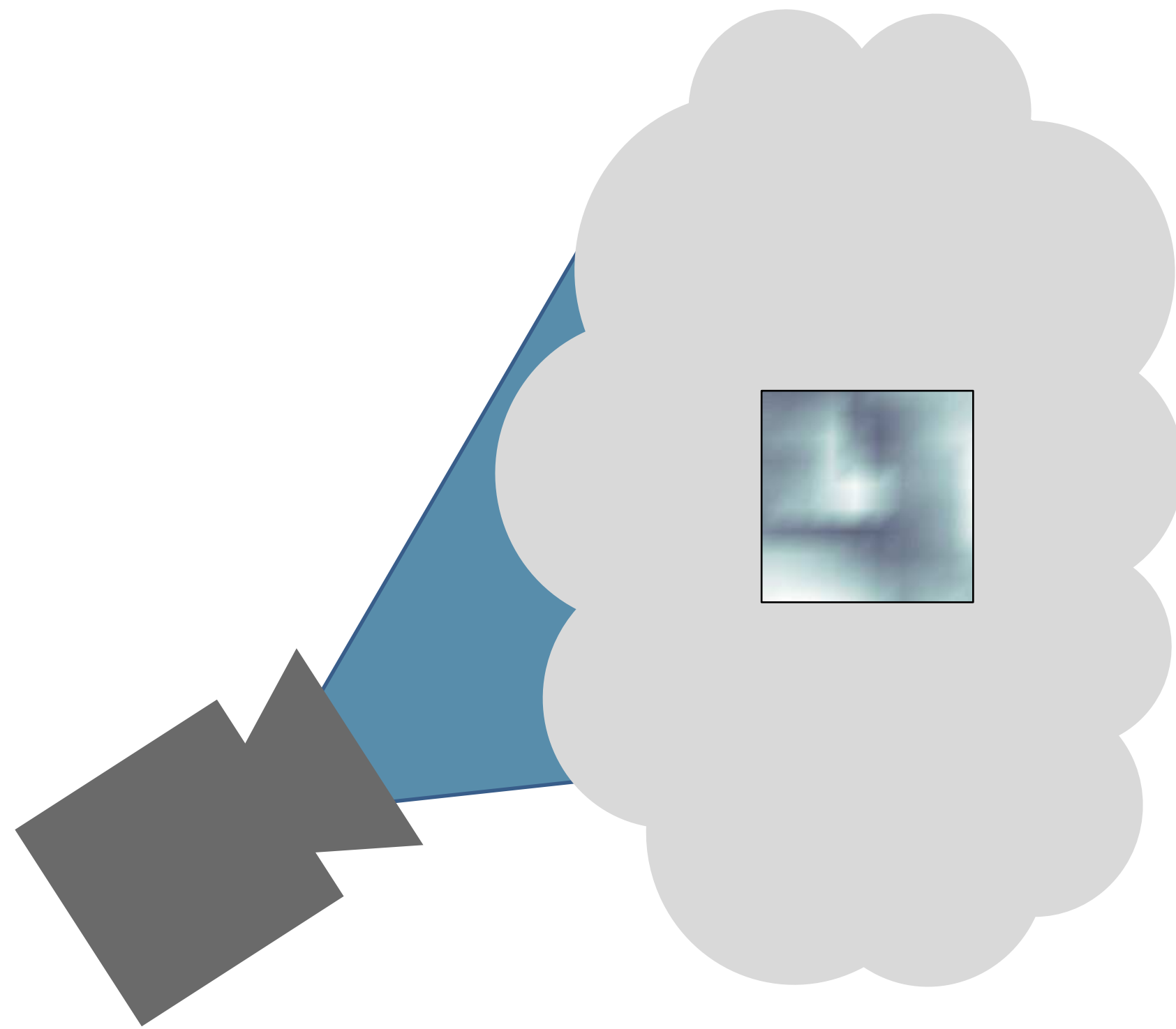
polystyrene 3



aluminum oxide



# Optical tomography



camera

thick smoke cloud



simulated camera  
measurements



reconstructed  
cloud volume



slice through  
the cloud



# Computational Fabrication

Determining the material configuration for individual voxels in full-color inkjet 3D printing



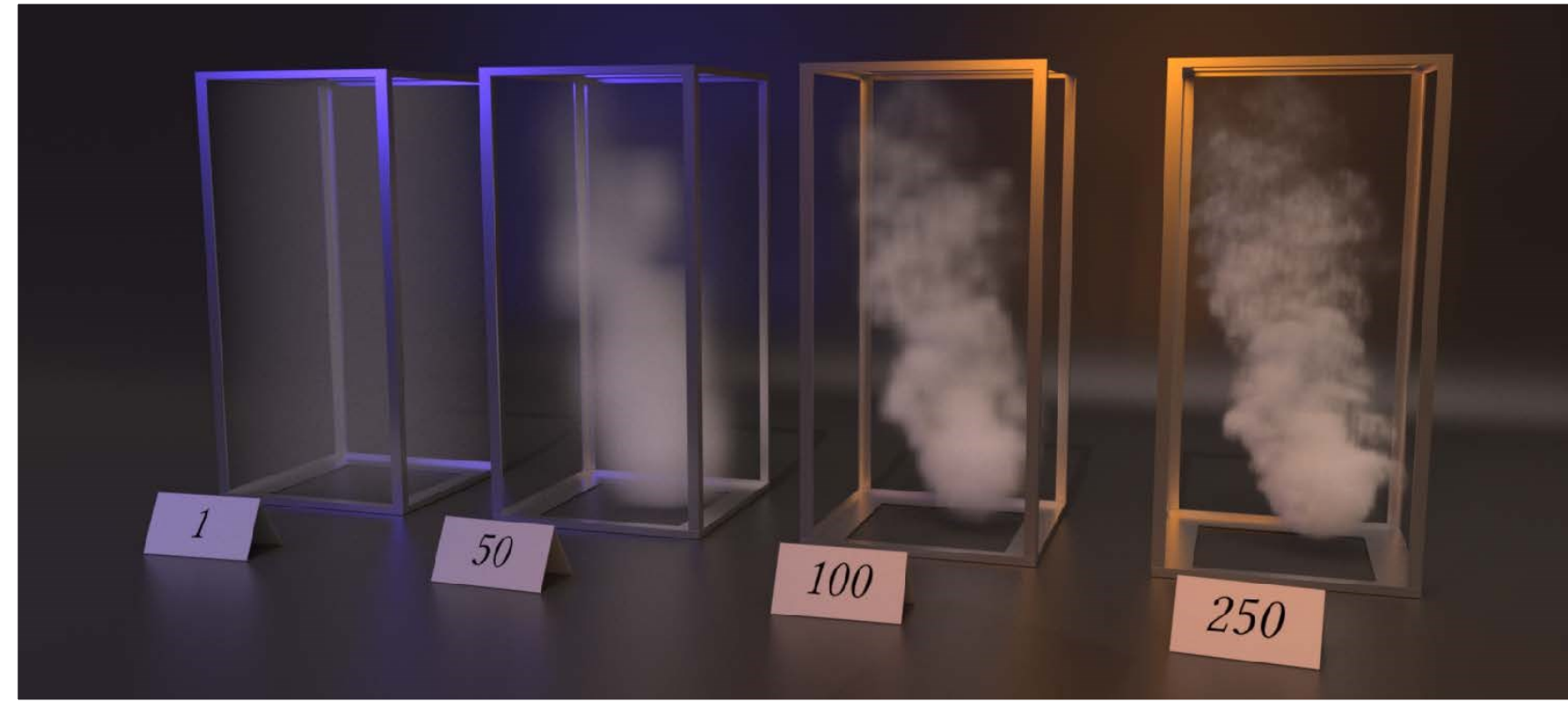
[Nindel et al. 2021]



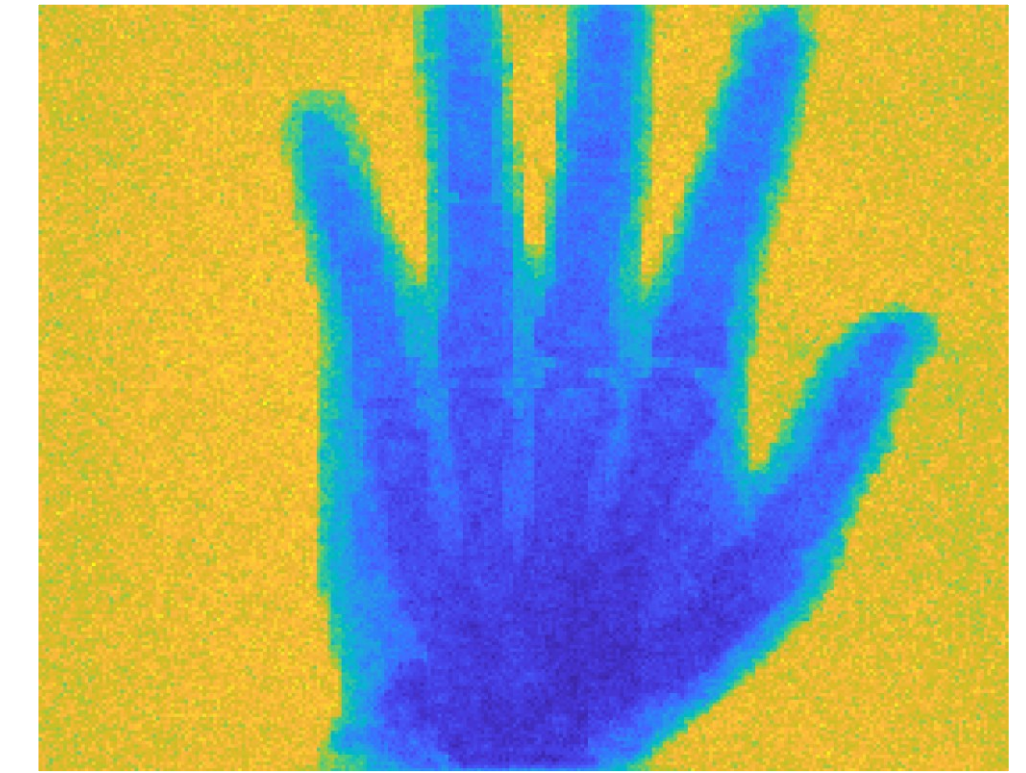
# Active area of research



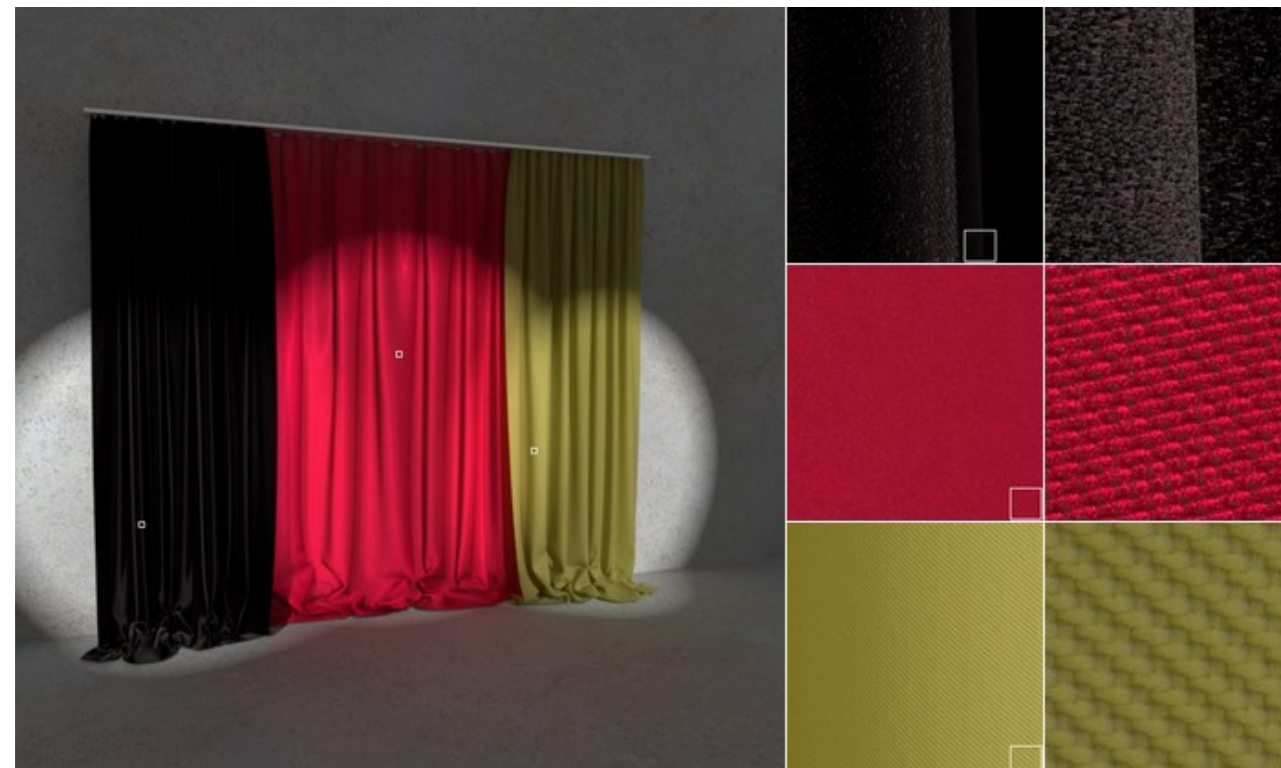
industrial dispersions  
[Gkioulekas et al. 2013]



efficient algorithms  
[Nimier-David et al. 2019, 2020]



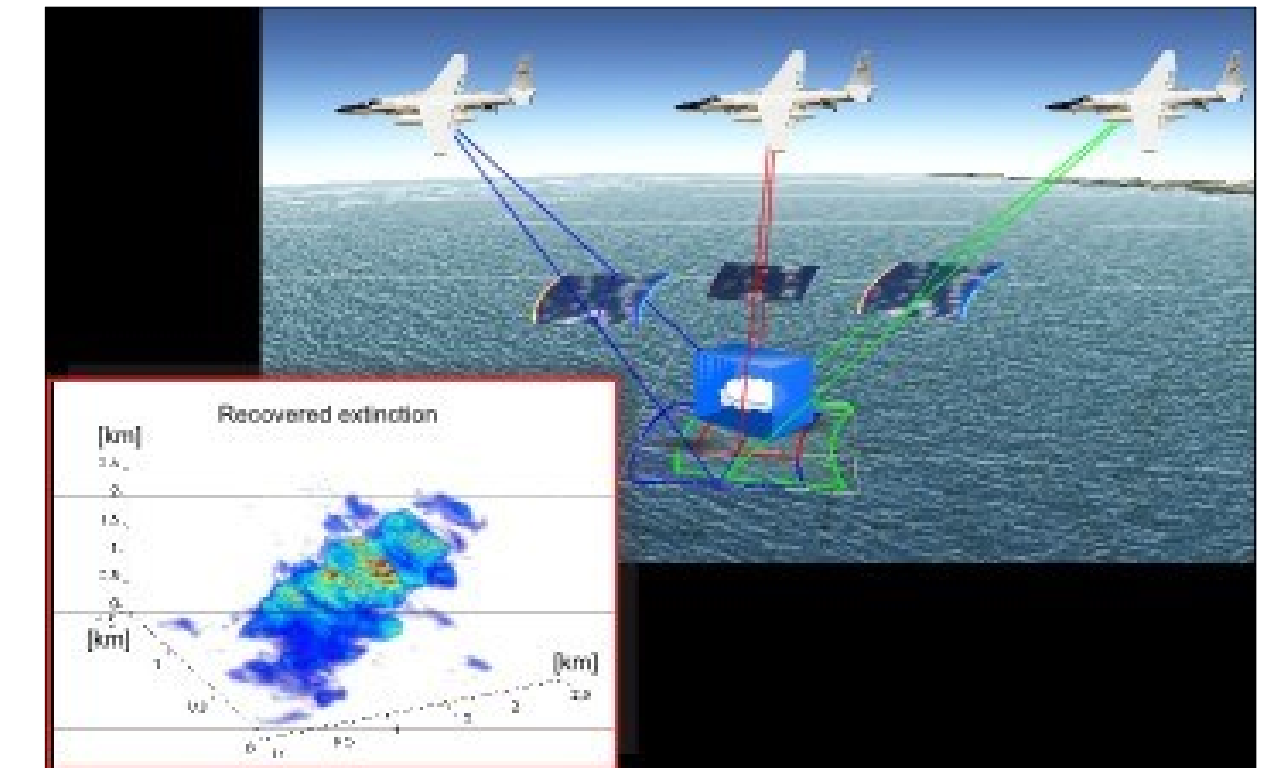
computed tomography  
[Geva et al. 2018]



woven fabrics  
[Khungurn et al. 2015,  
Zhao et al. 2016]

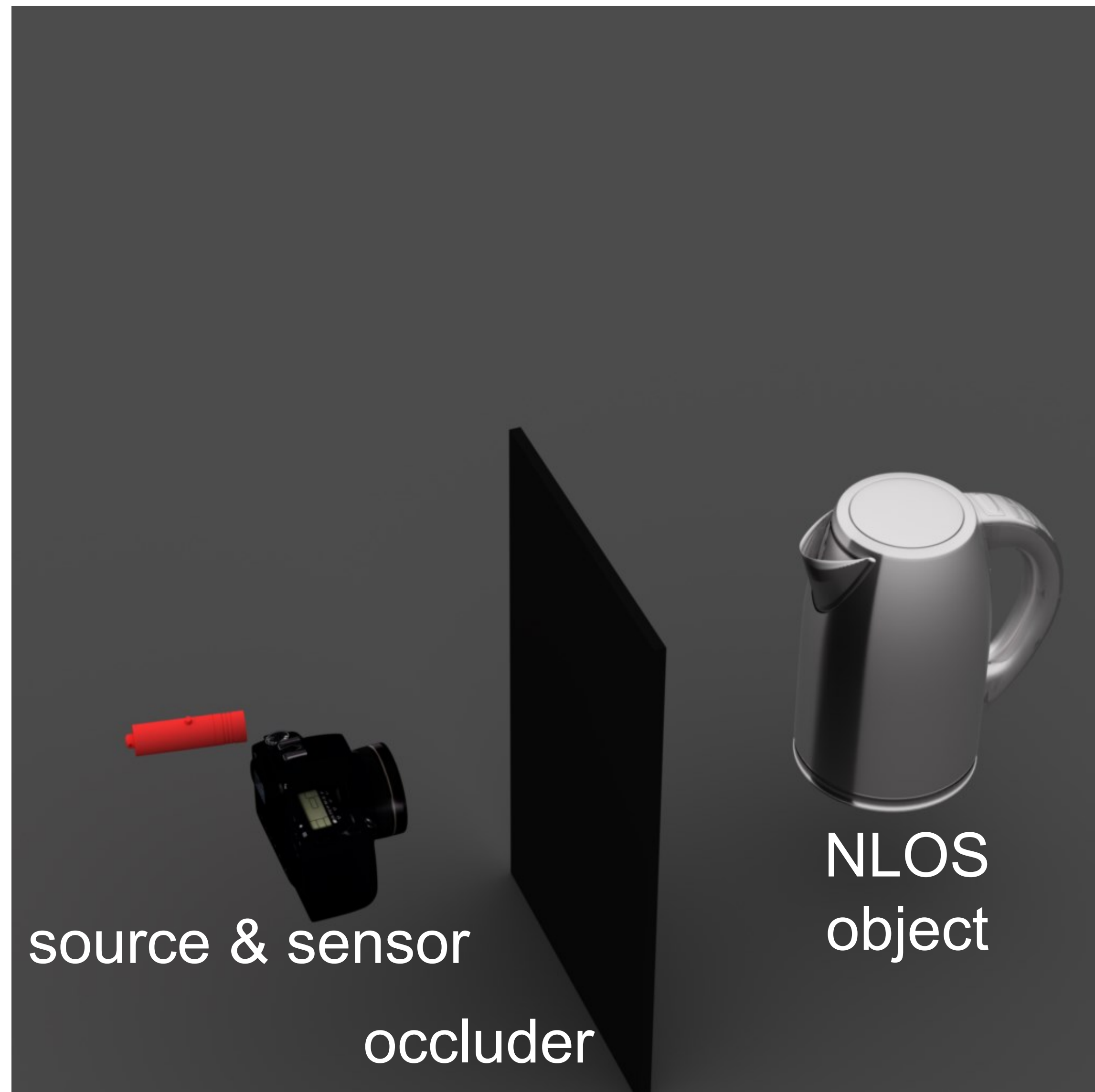


3D printing  
[Elek et al. 2019,  
Nindel et al. 2021]



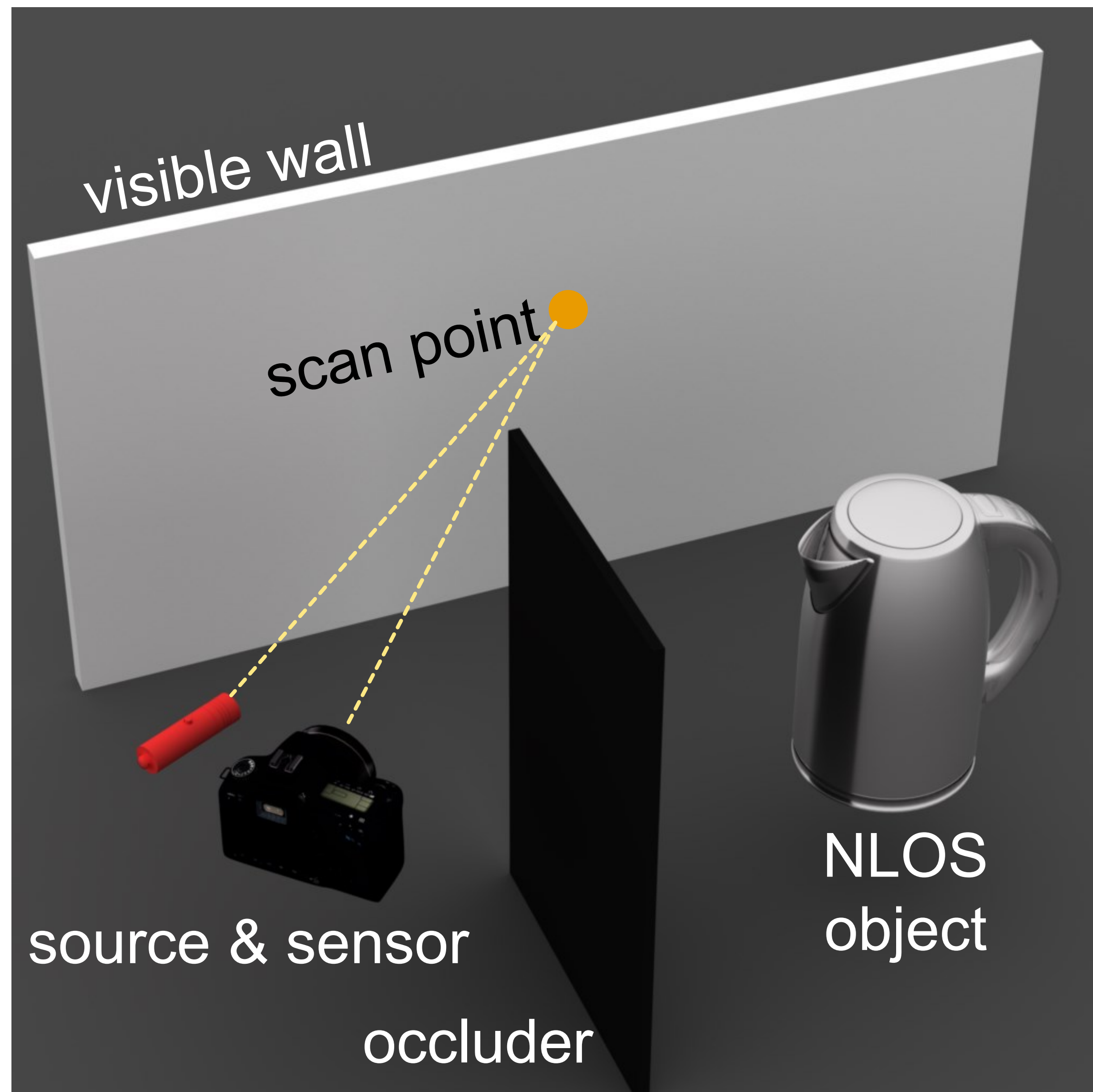
cloud tomography  
[Levis et al. 2015,  
2017, 2020]

# Non-line-of-sight (NLOS) imaging

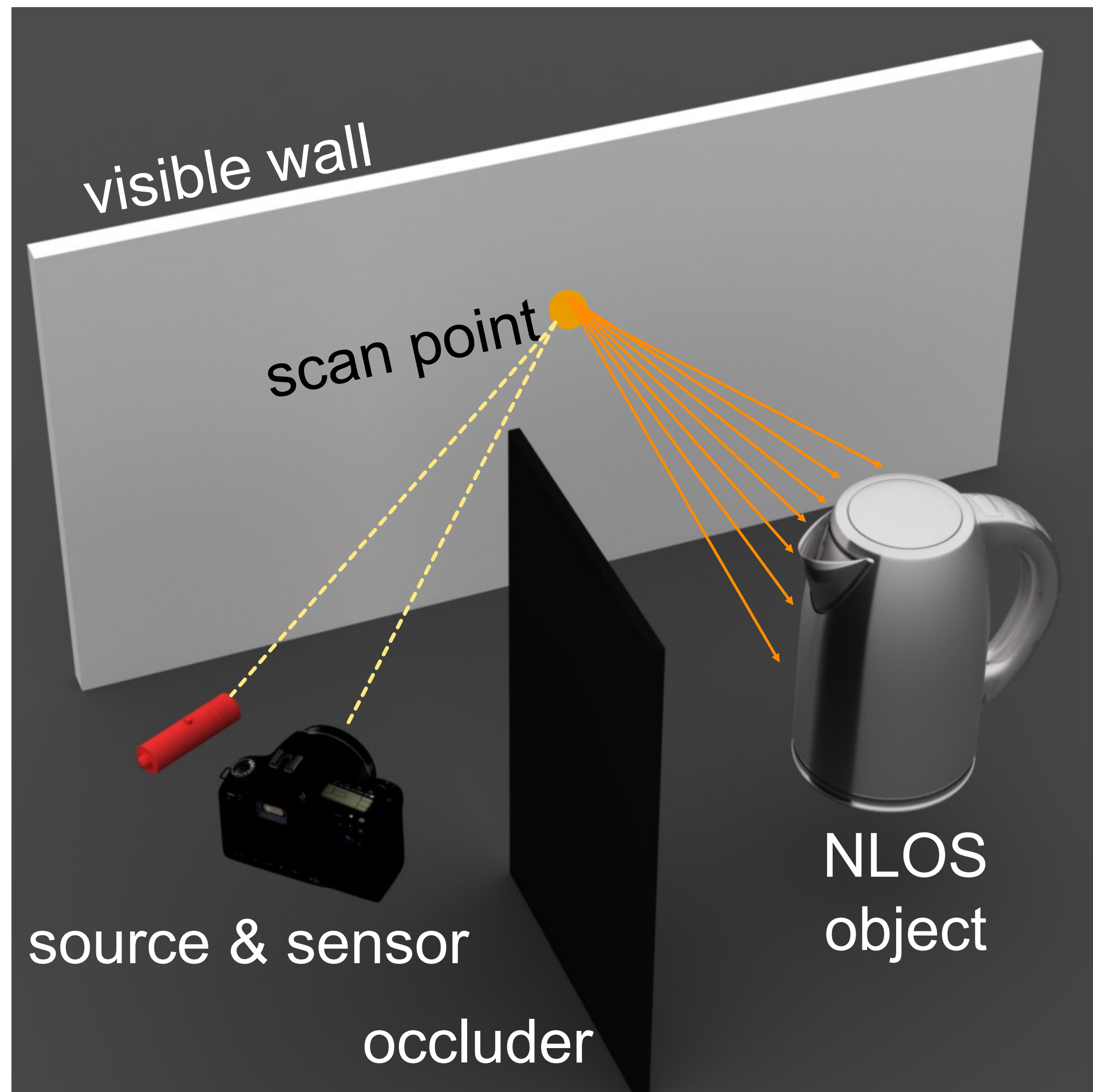




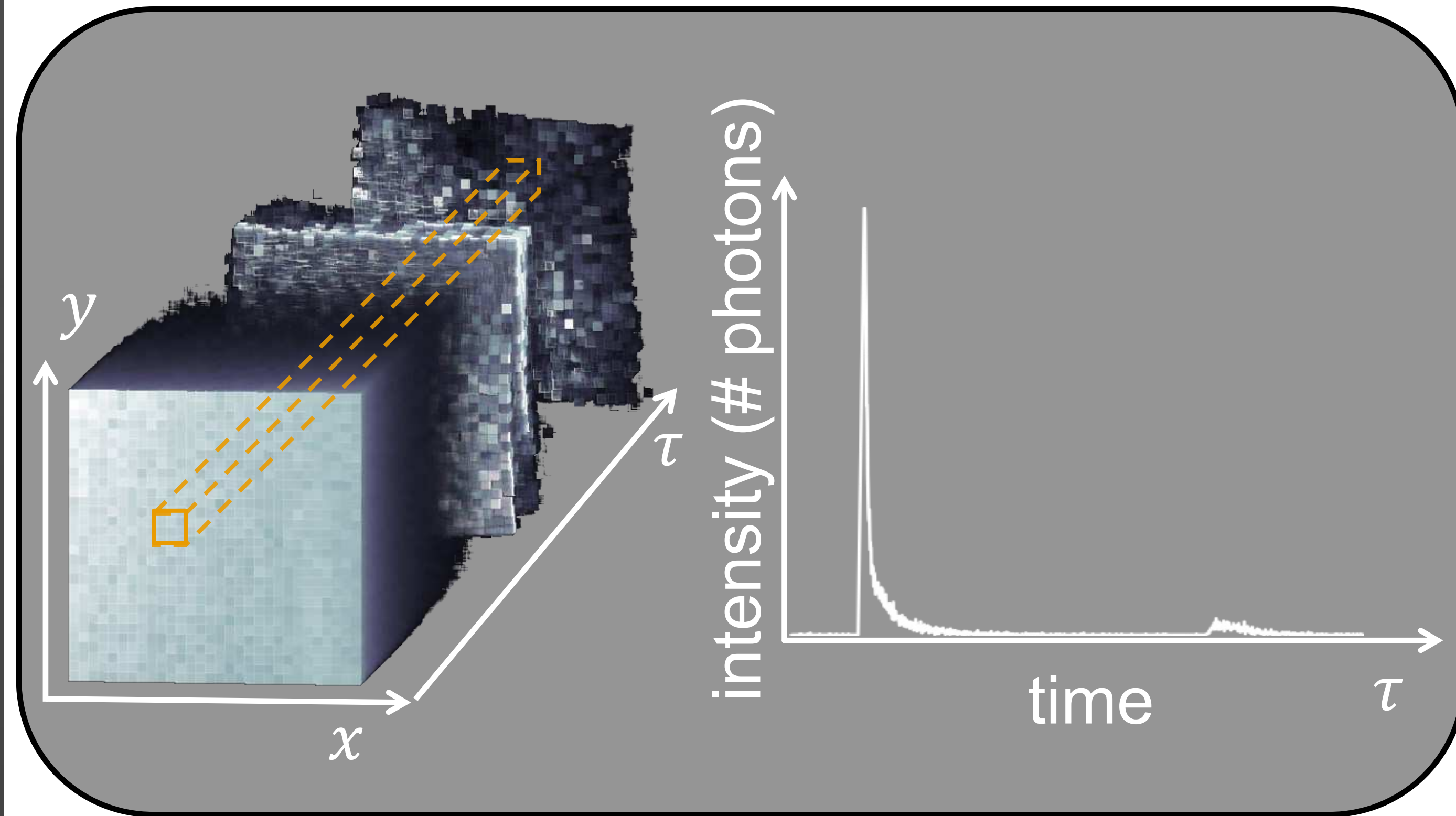
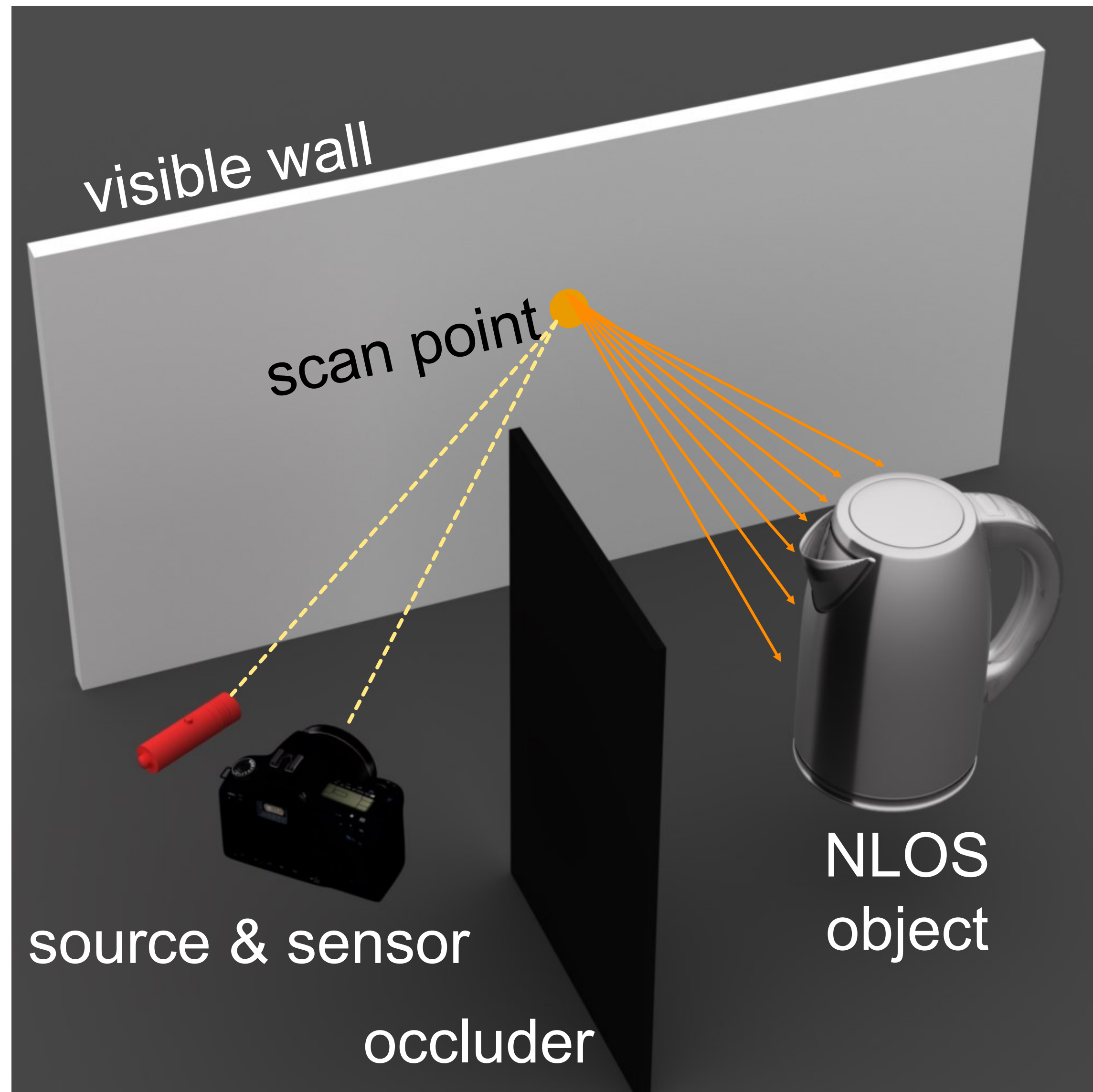
# Non-line-of-sight (NLOS) imaging



# Non-line-of-sight (NLOS) imaging



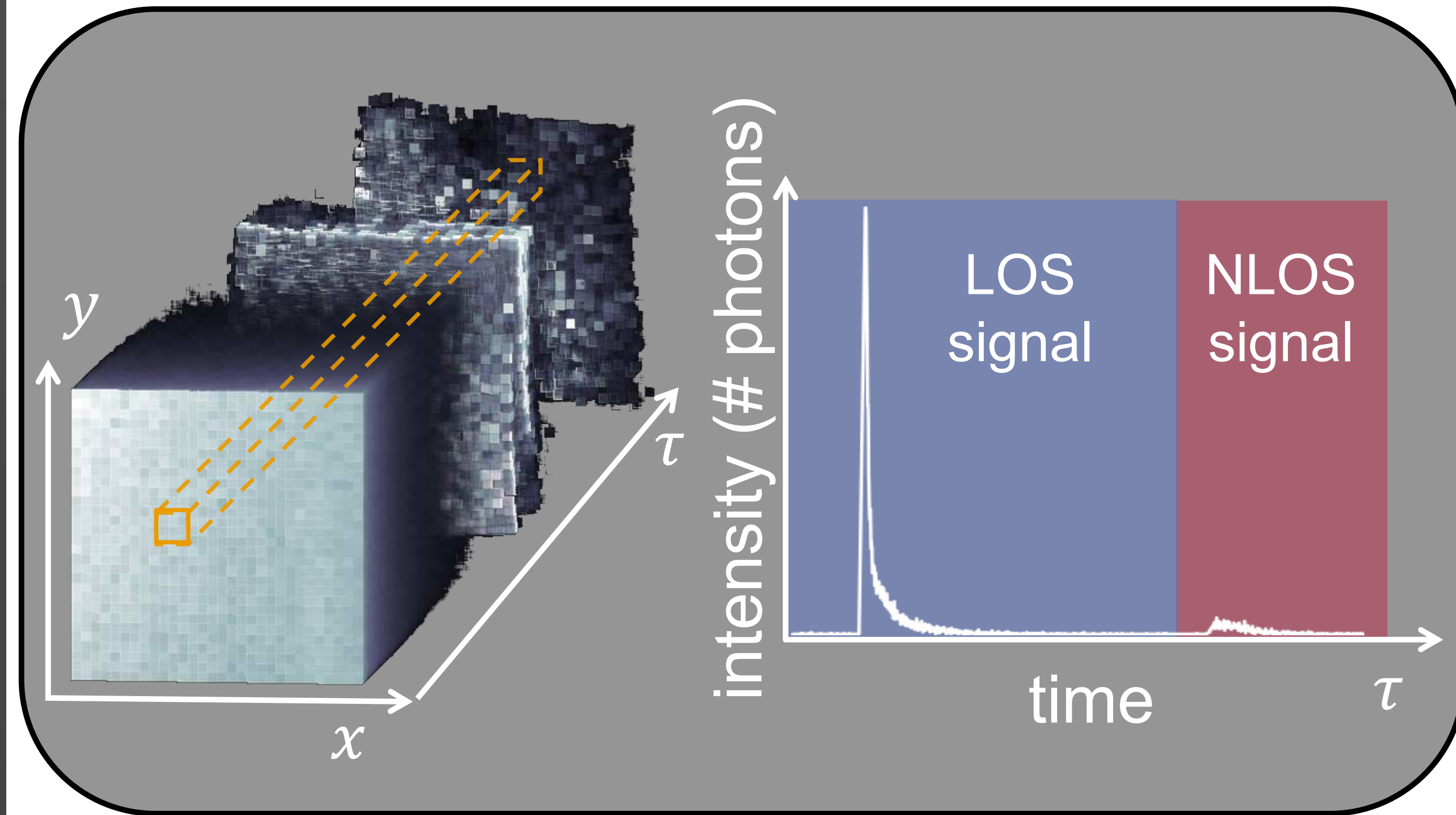
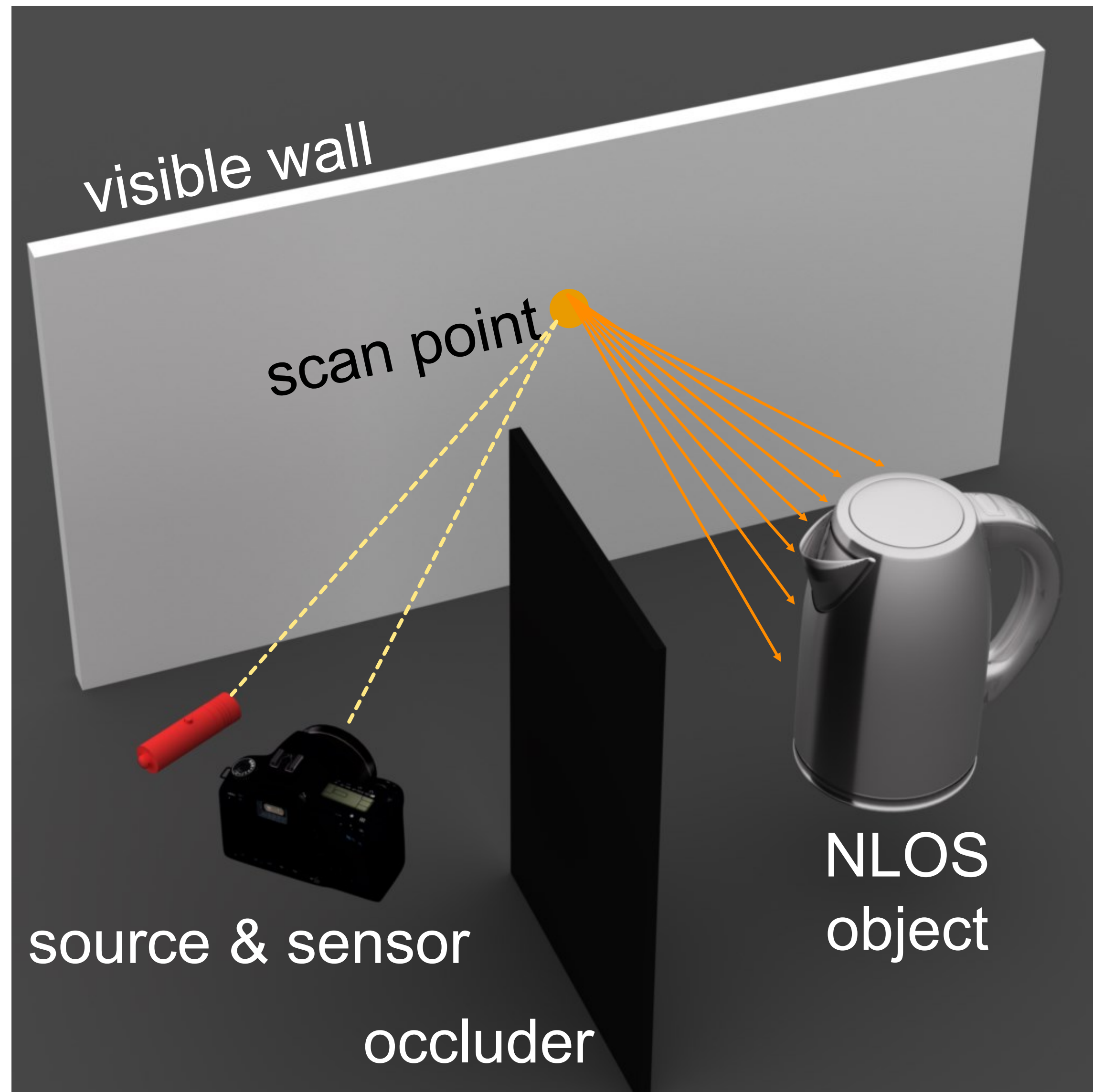
# Non-line-of-sight (NLOS) imaging



Time-of-flight measurements



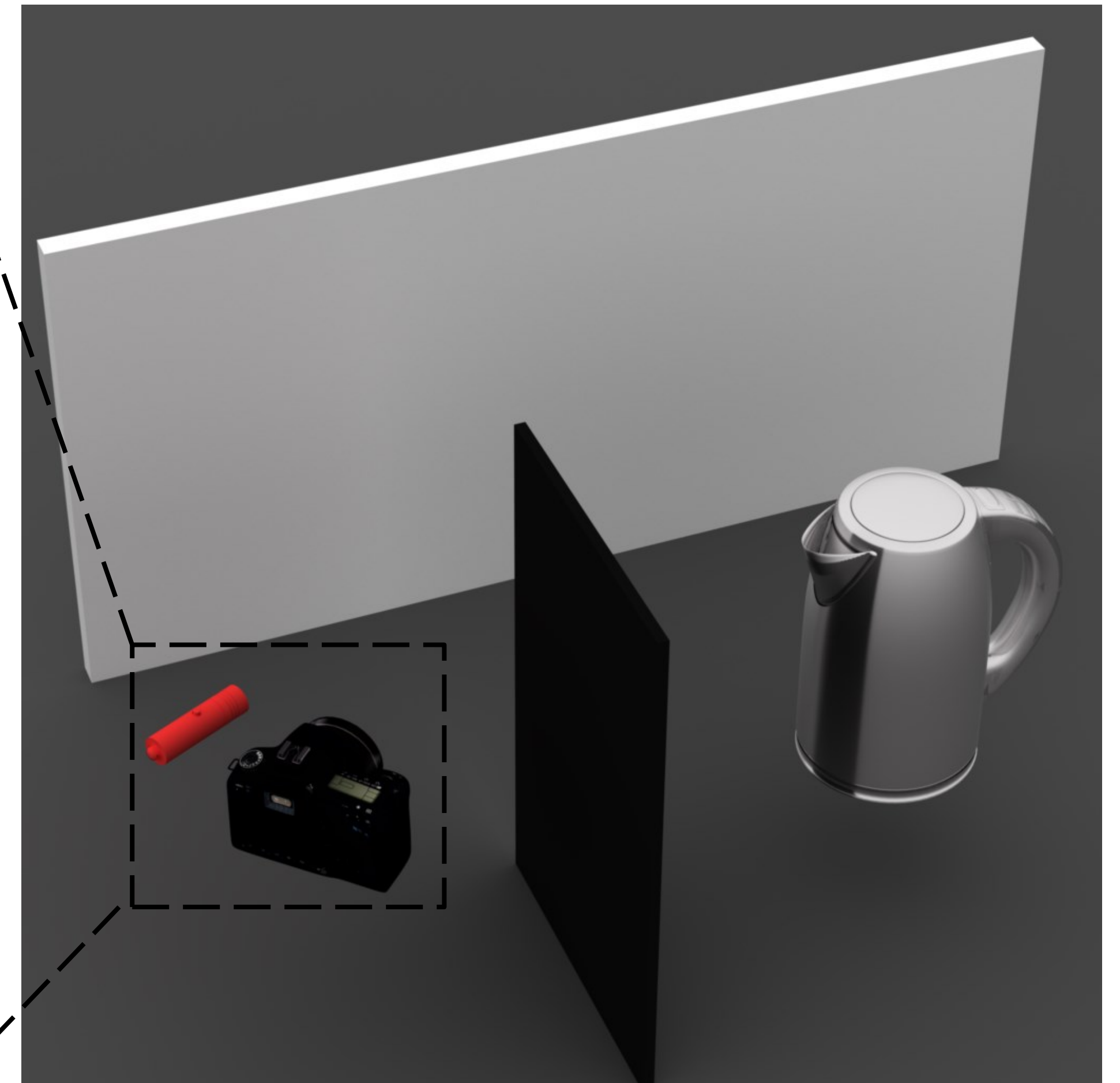
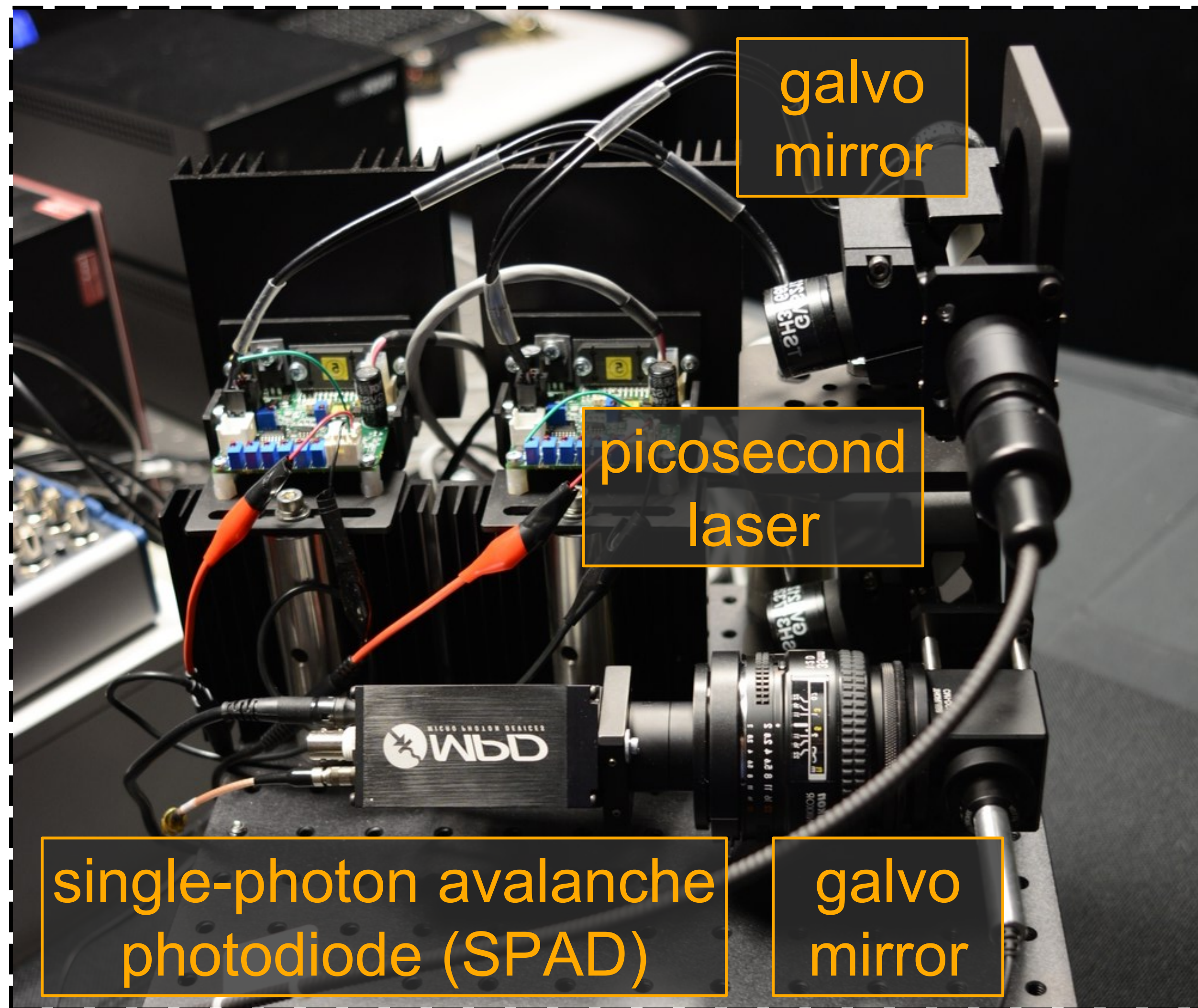
# Non-line-of-sight (NLOS) imaging



Time-of-flight measurements

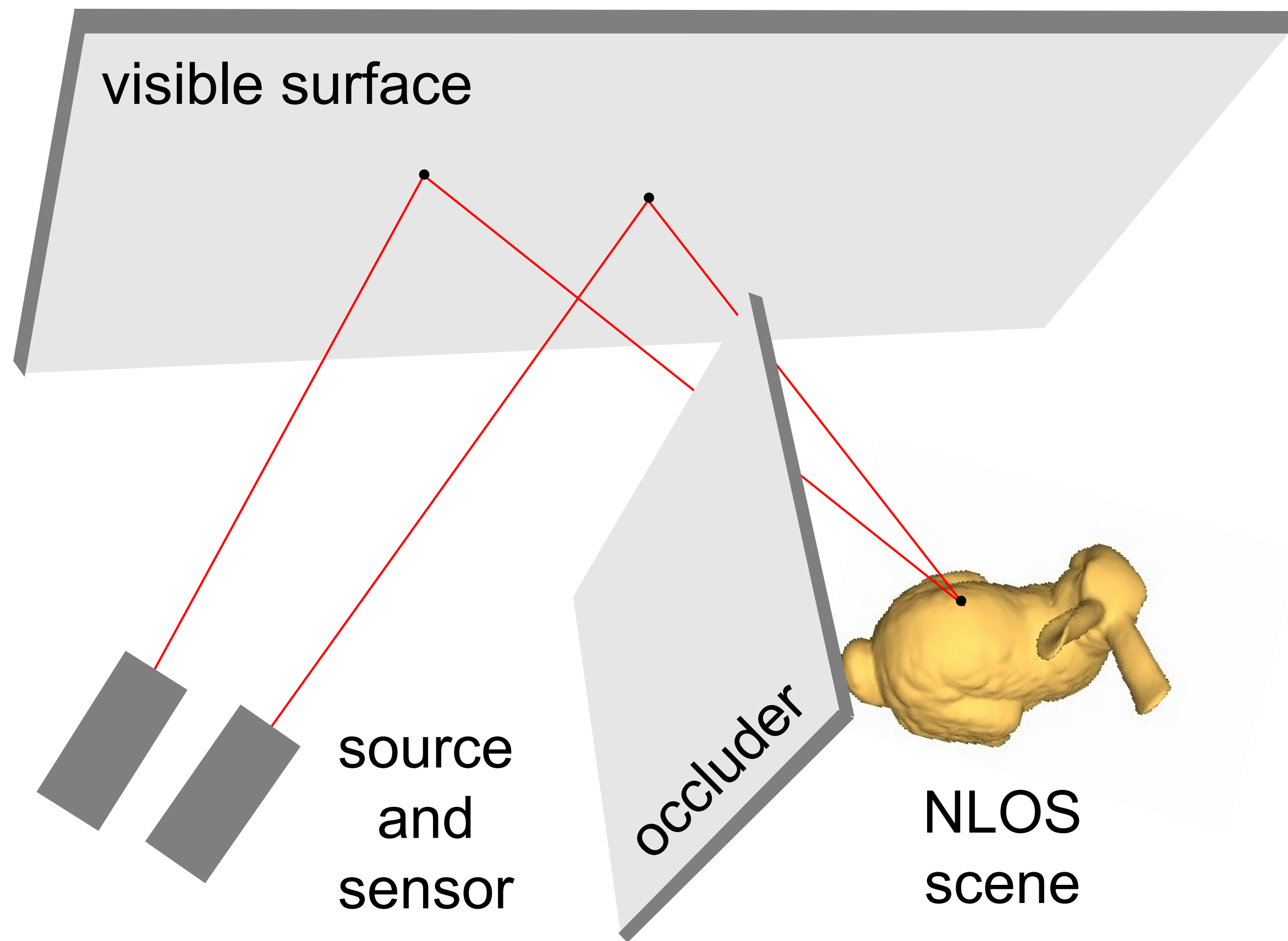


# SPAD-based lidar

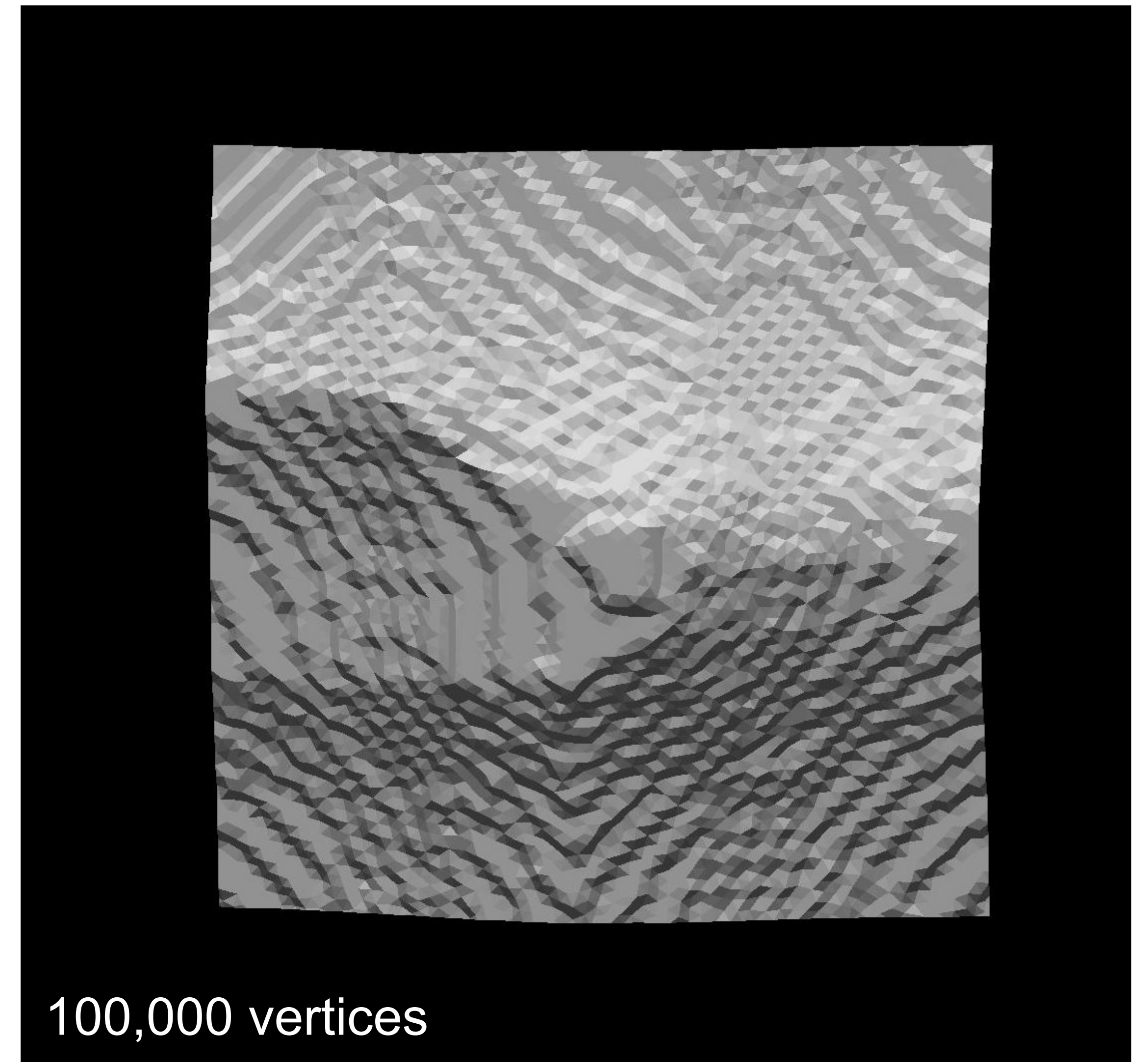




# NLOS shape optimization

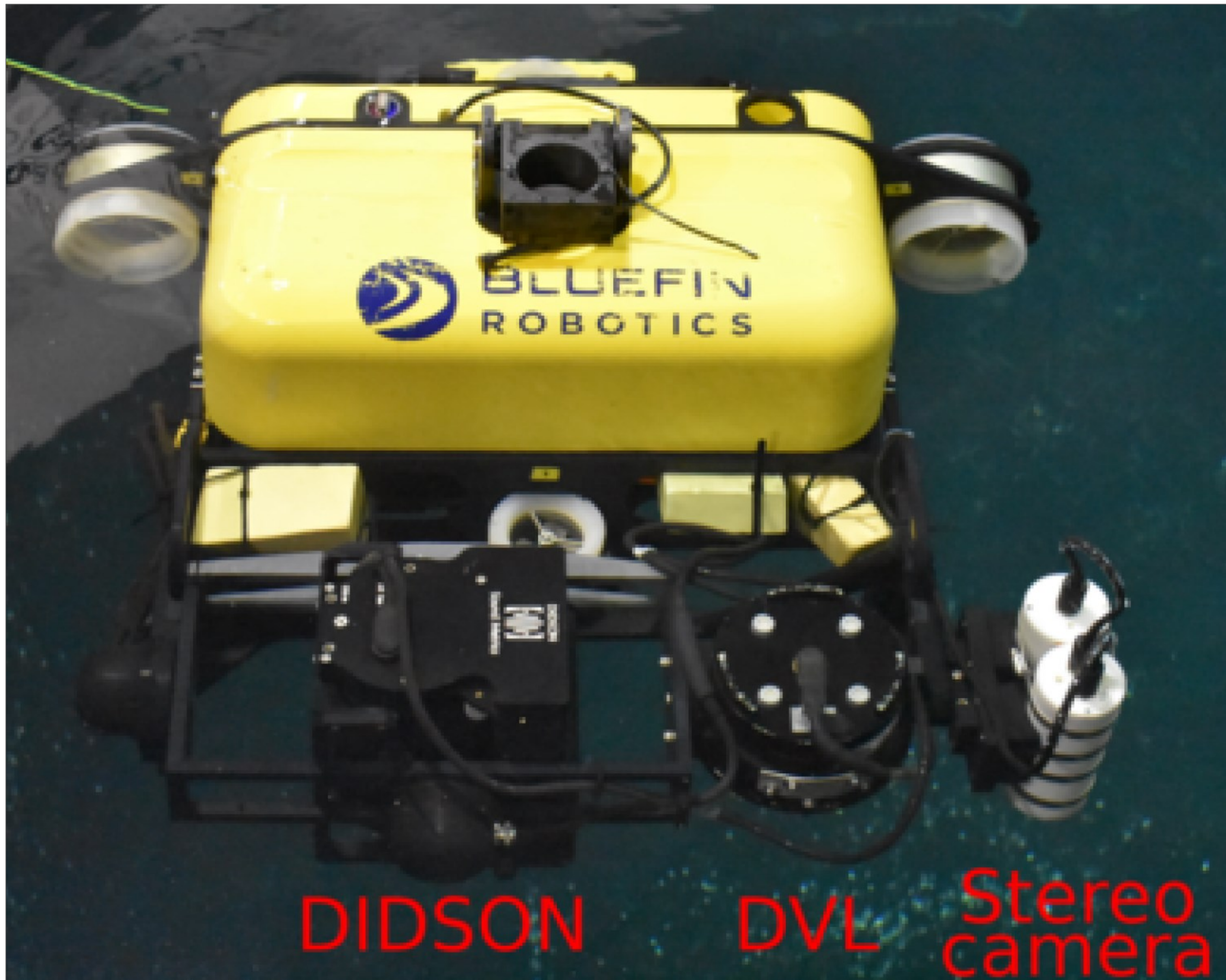


Simulated time-of-flight data

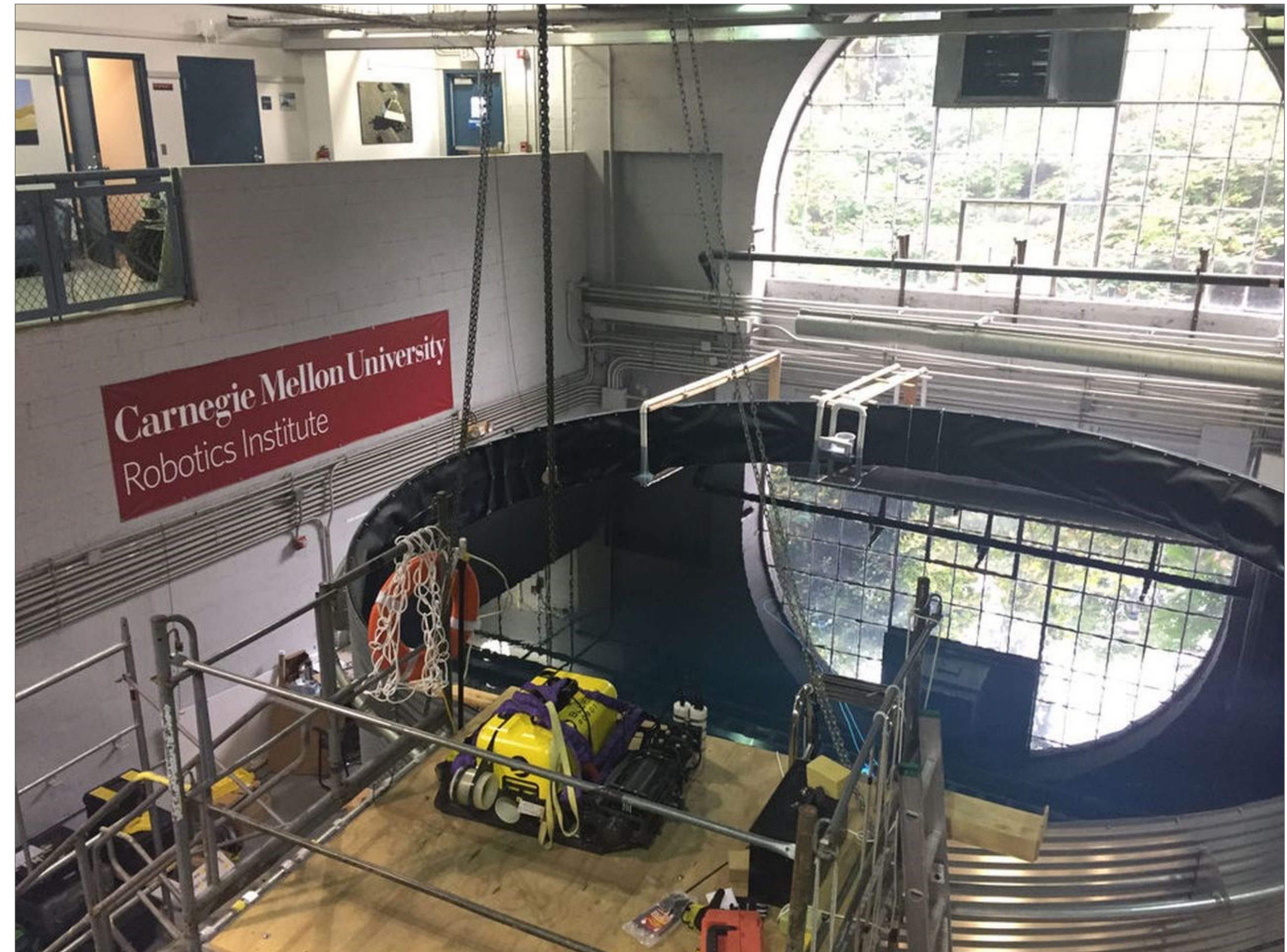




# Underwater 3D using imaging SONAR



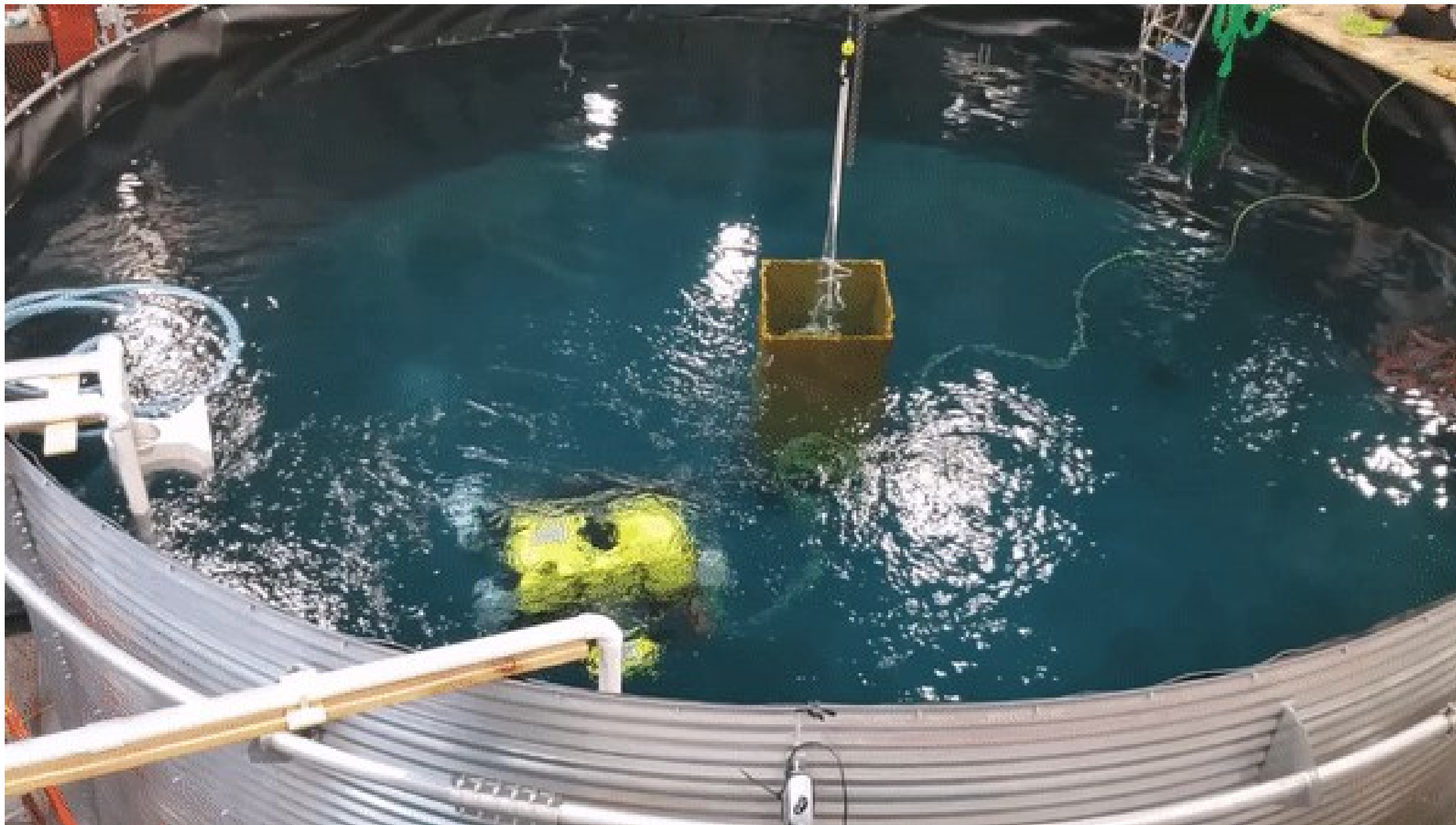
Underwater robot with sonar



Robotics Institute High Bay



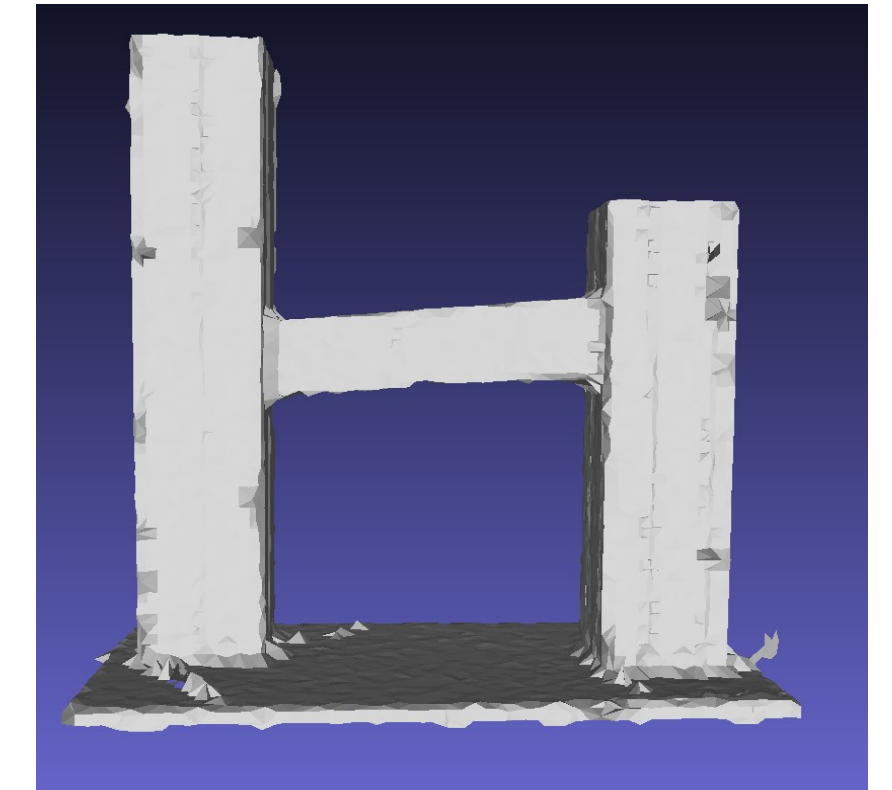
# Underwater 3D using imaging SONAR



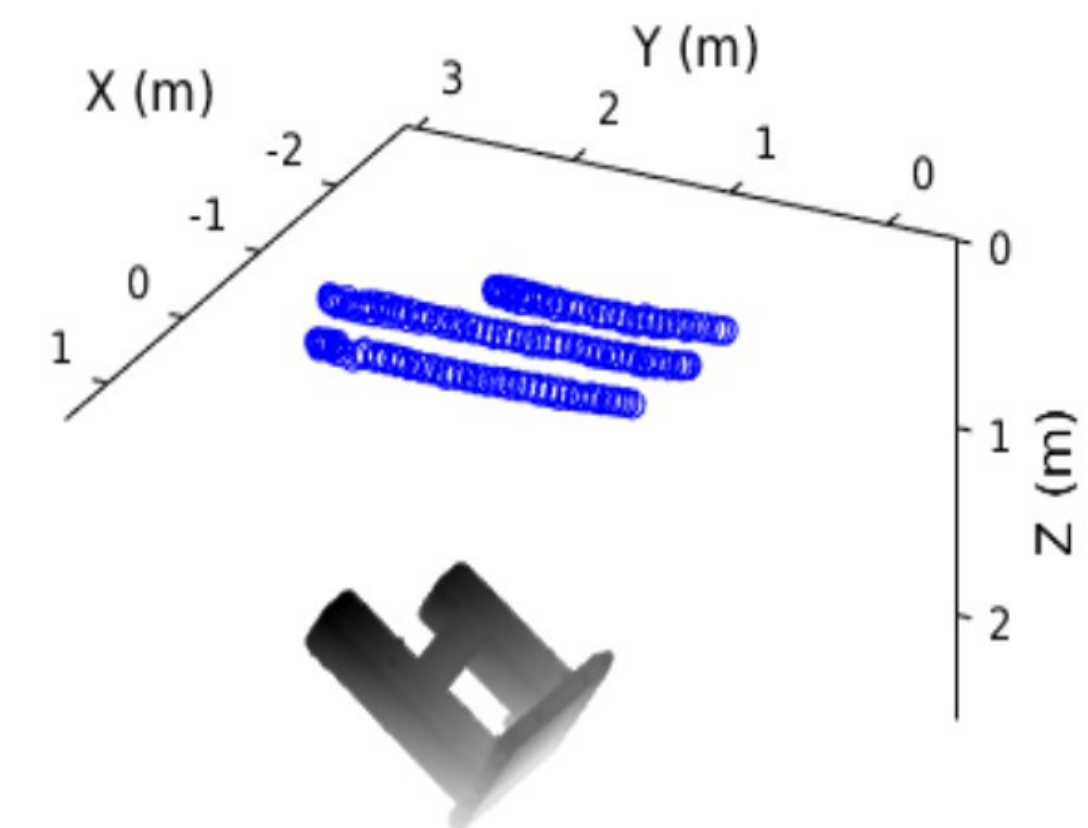
Test structure



Ground truth mesh  
obtained using a laser scan



Sonar image collection points



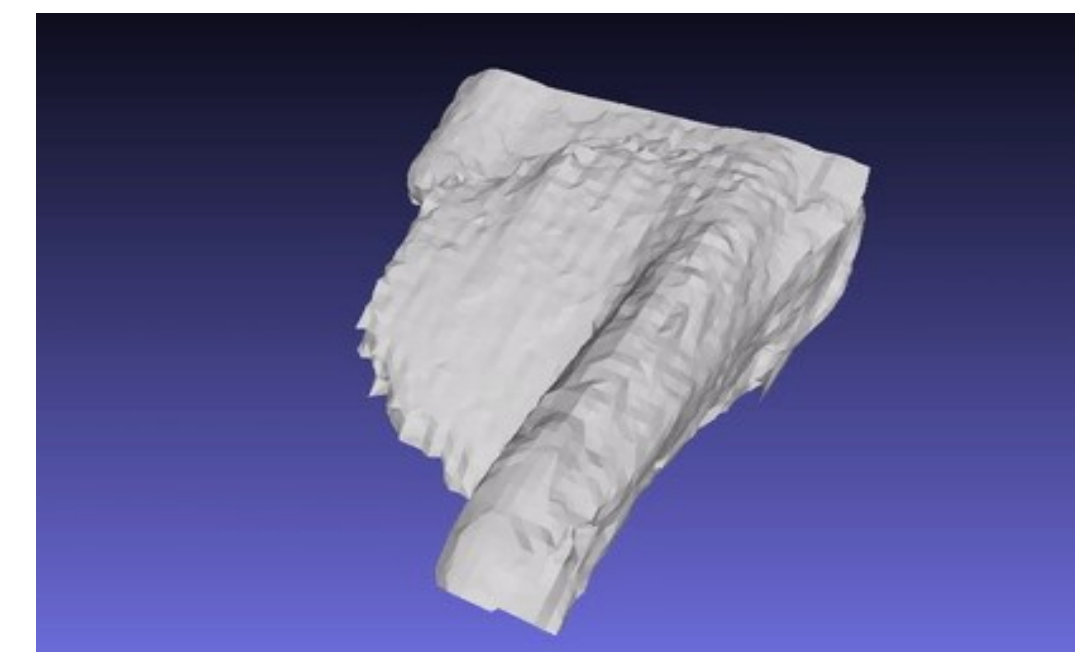
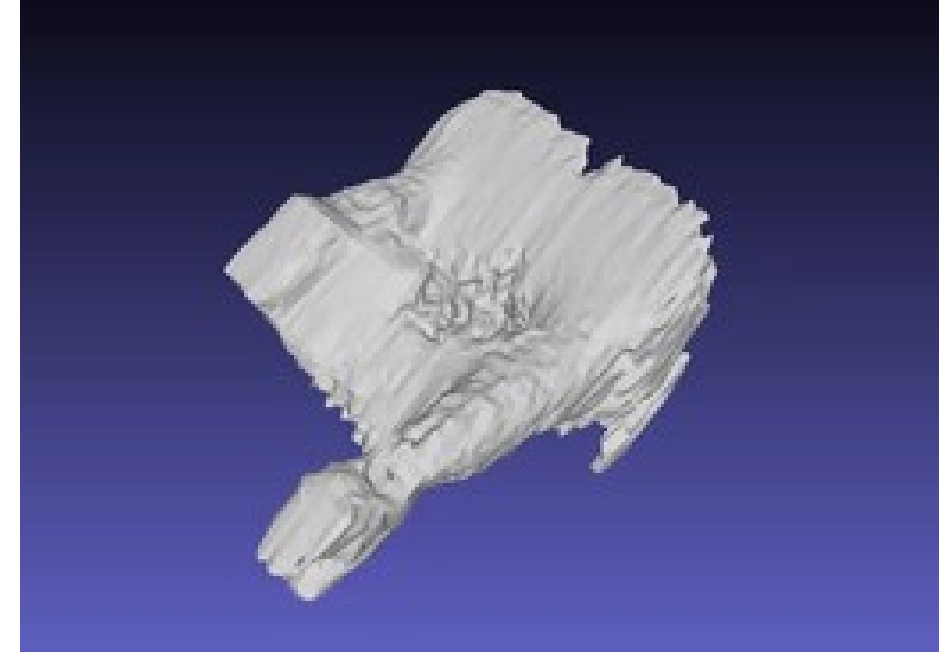
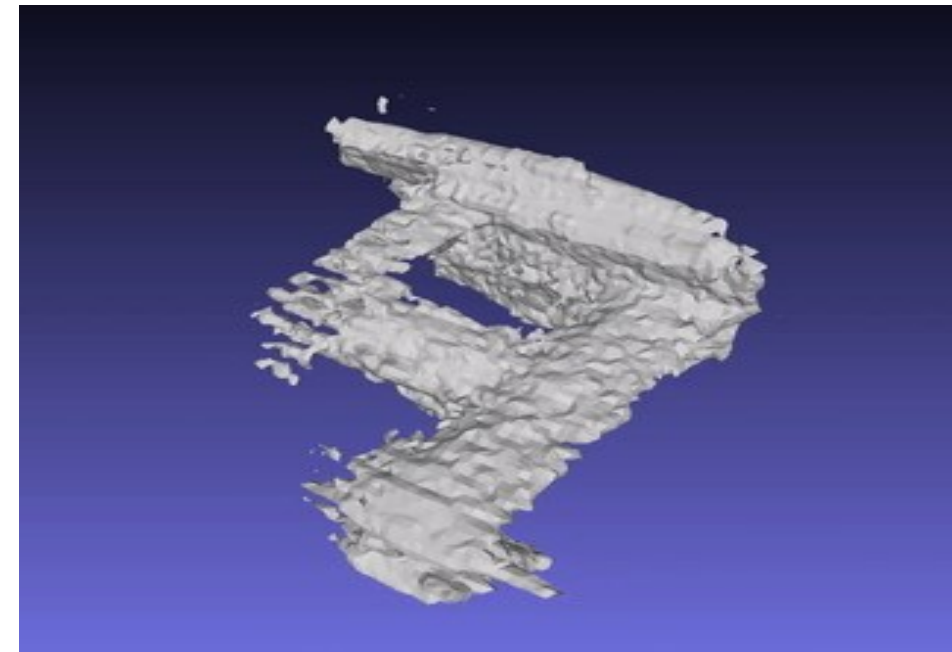
# Underwater 3D using imaging SONAR

1 degree

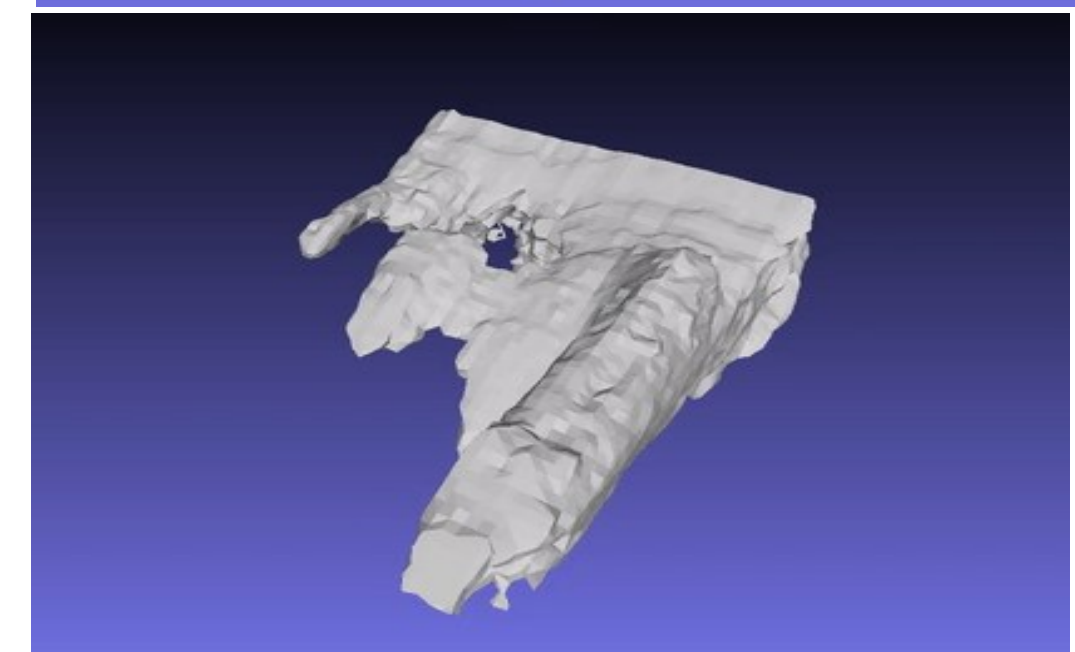
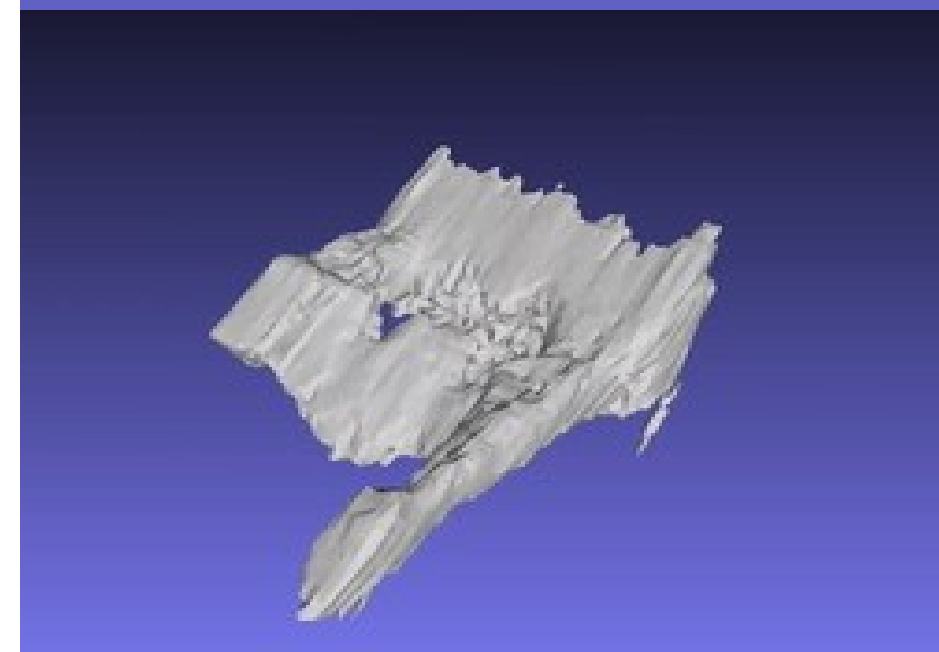
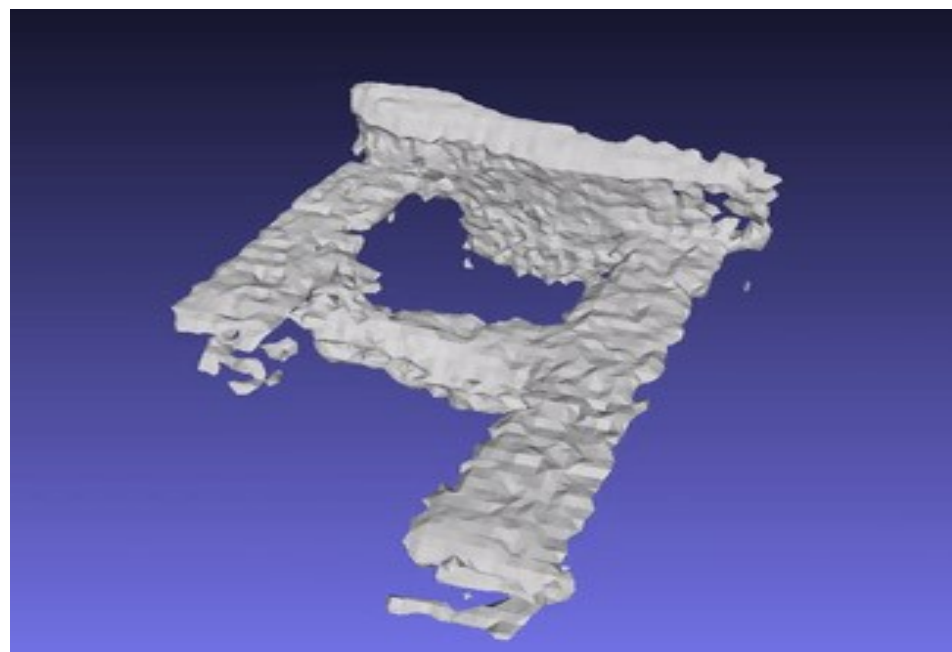
14 degrees

28 degrees

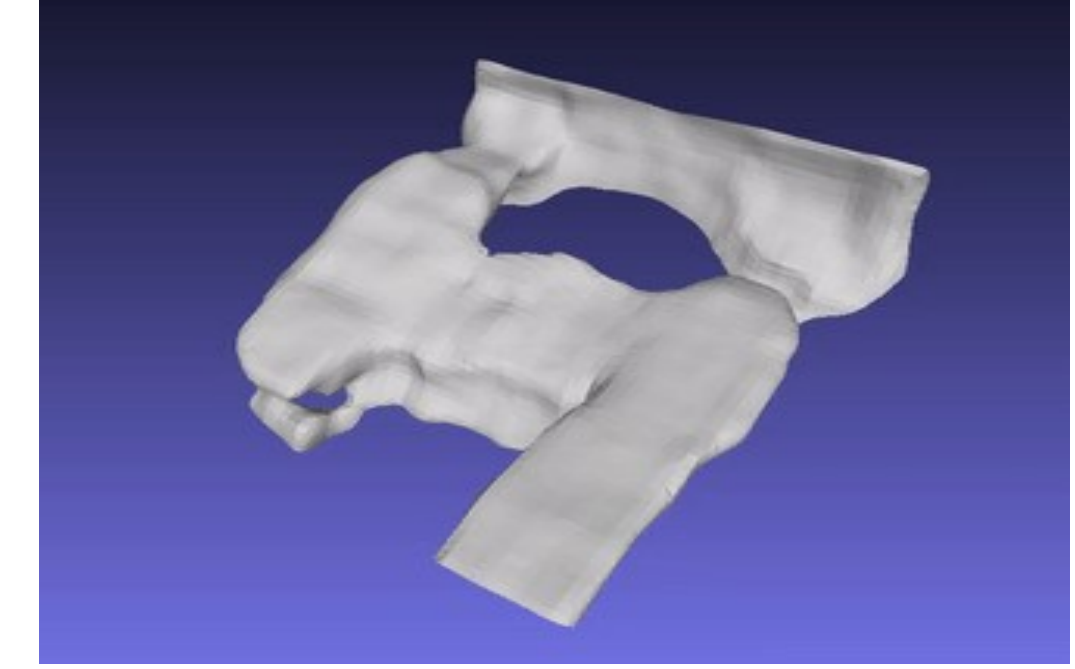
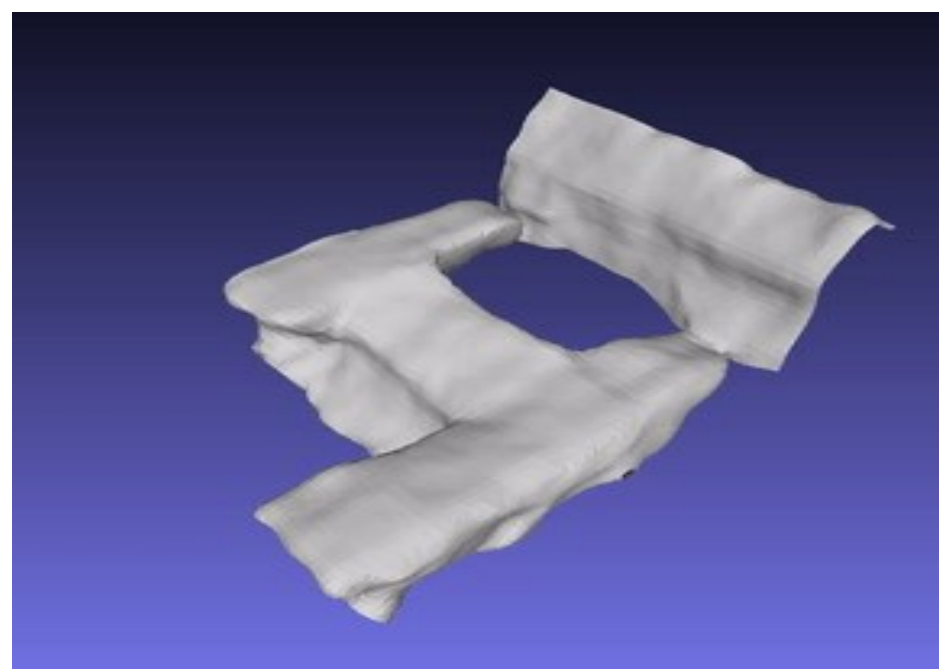
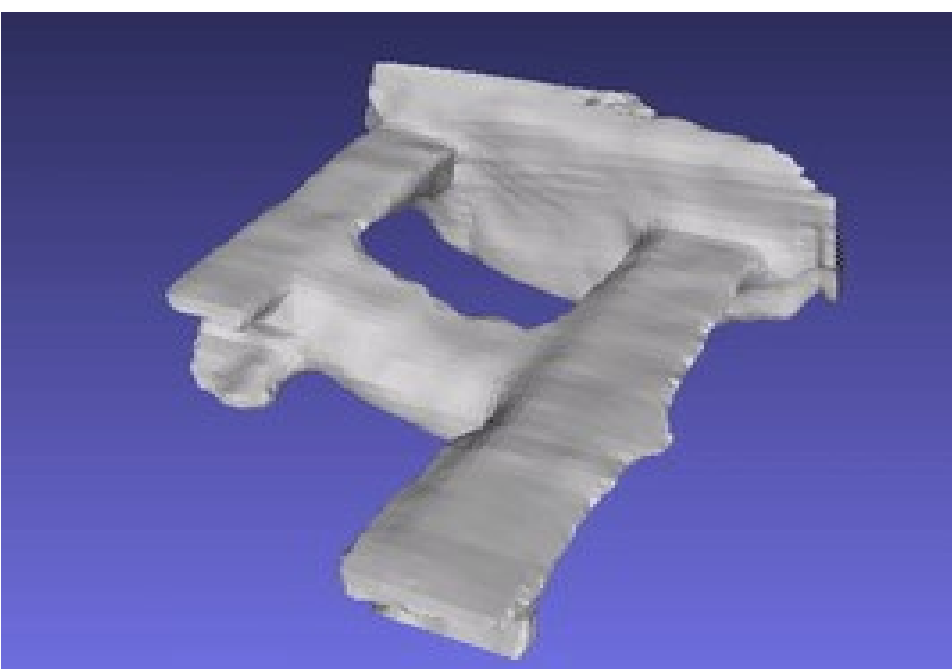
back-  
projection



virtual  
aperture

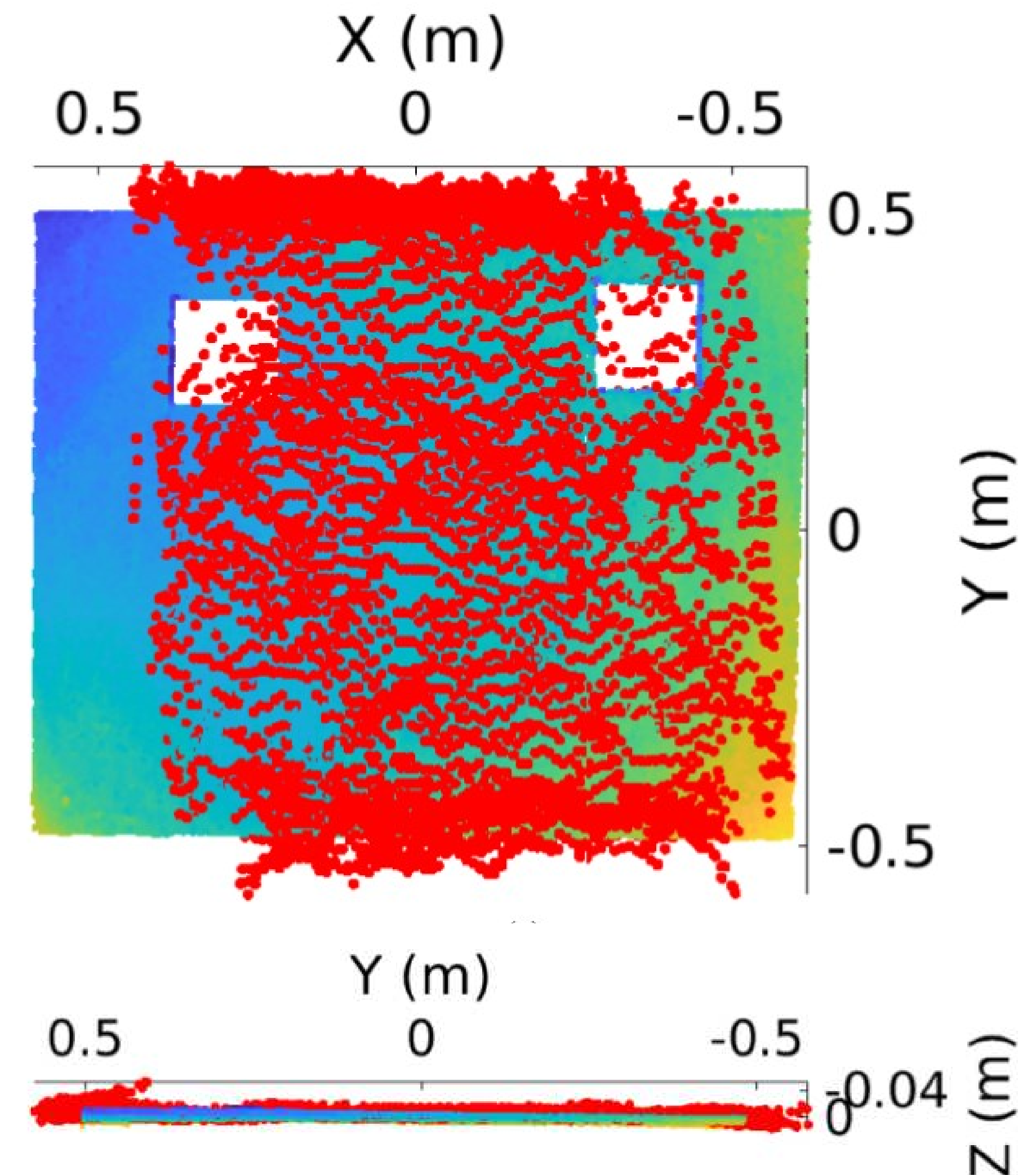
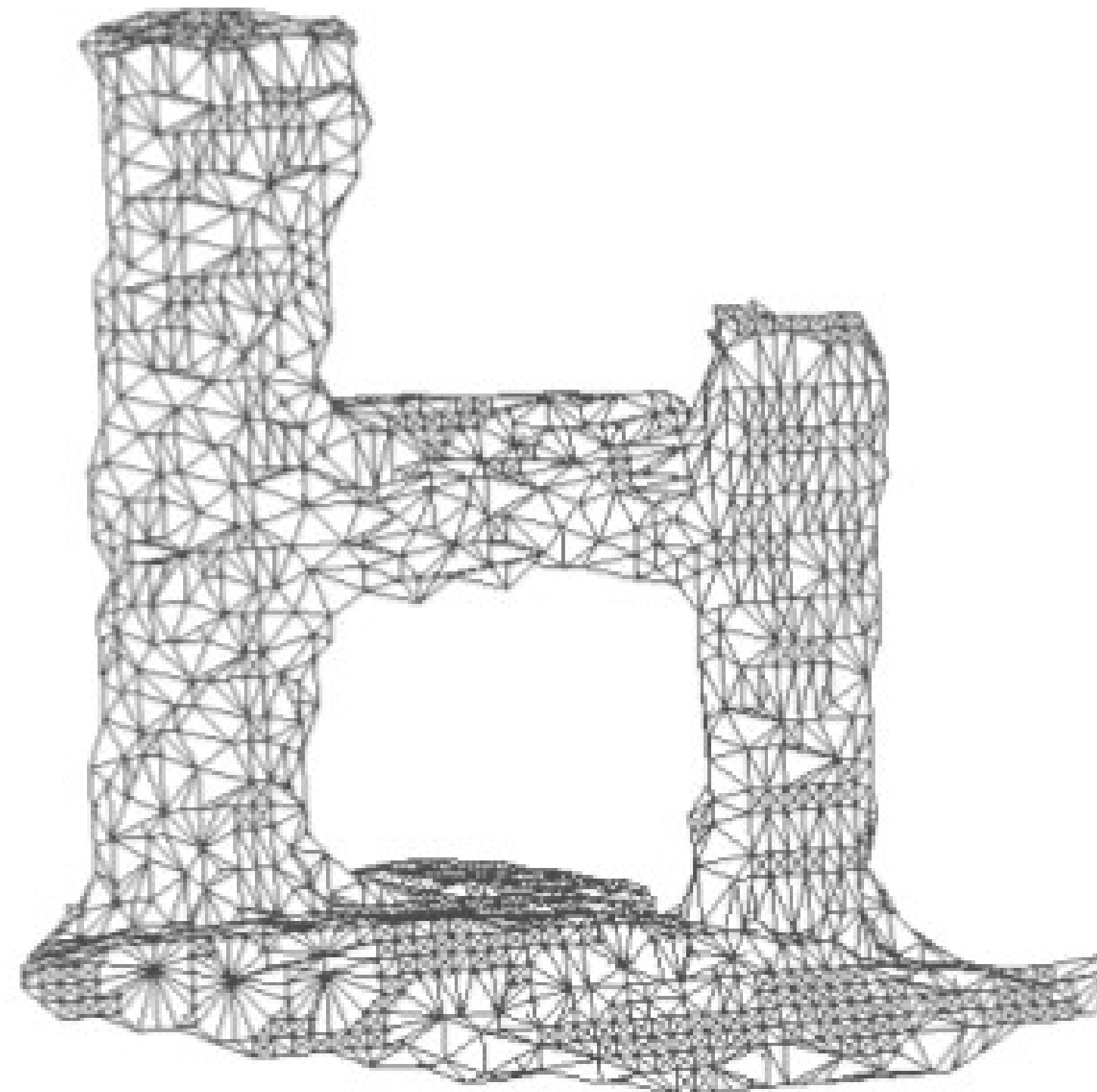


differentiable  
rendering





# Underwater 3D using imaging SONAR



Millimeter-accuracy underwater 3D reconstructions using data captured with an acoustic sonar mounted on robot.

# Kaleidoscopic 3D scanning



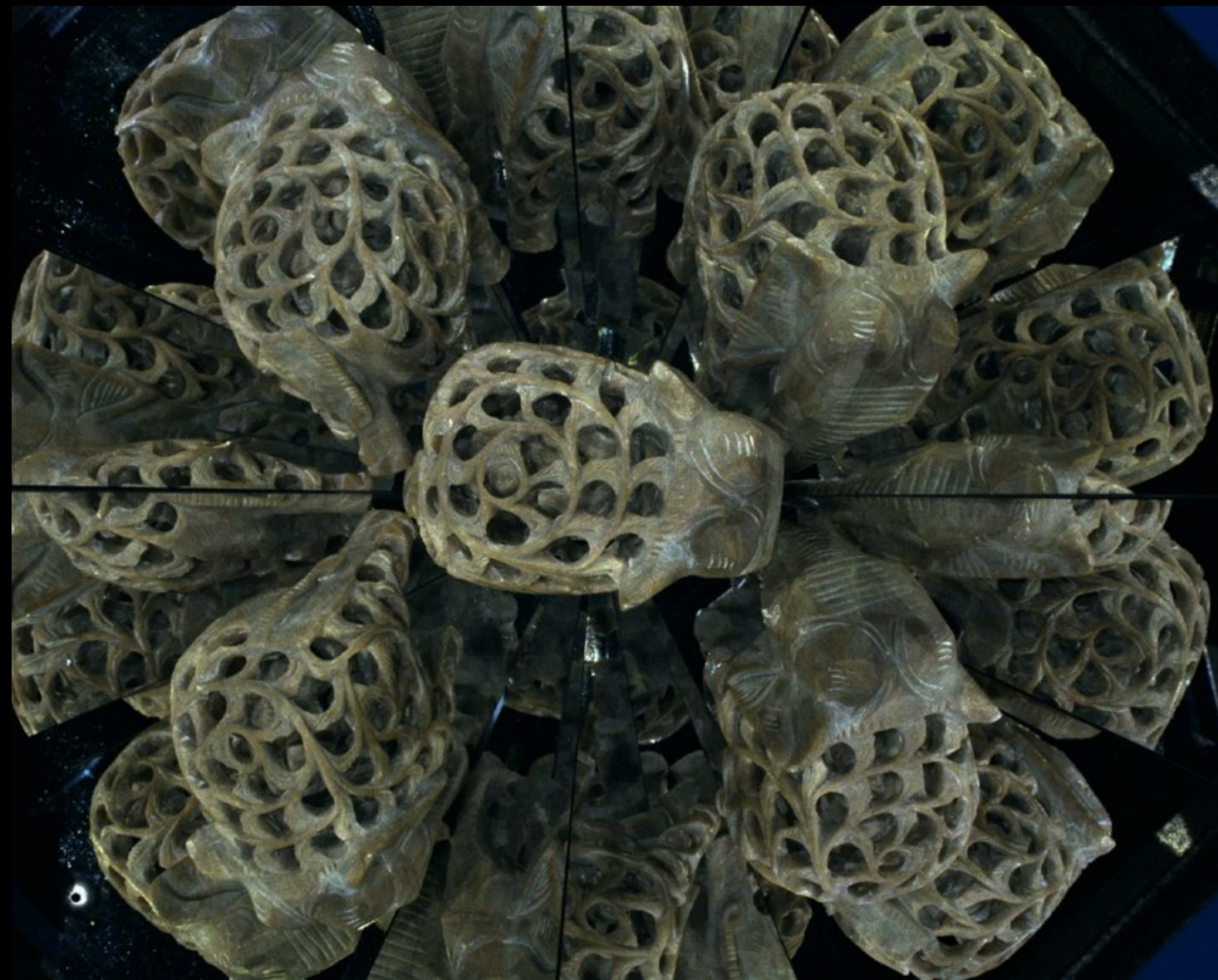
kaleidoscopic system



# Kaleidoscopic 3D scanning



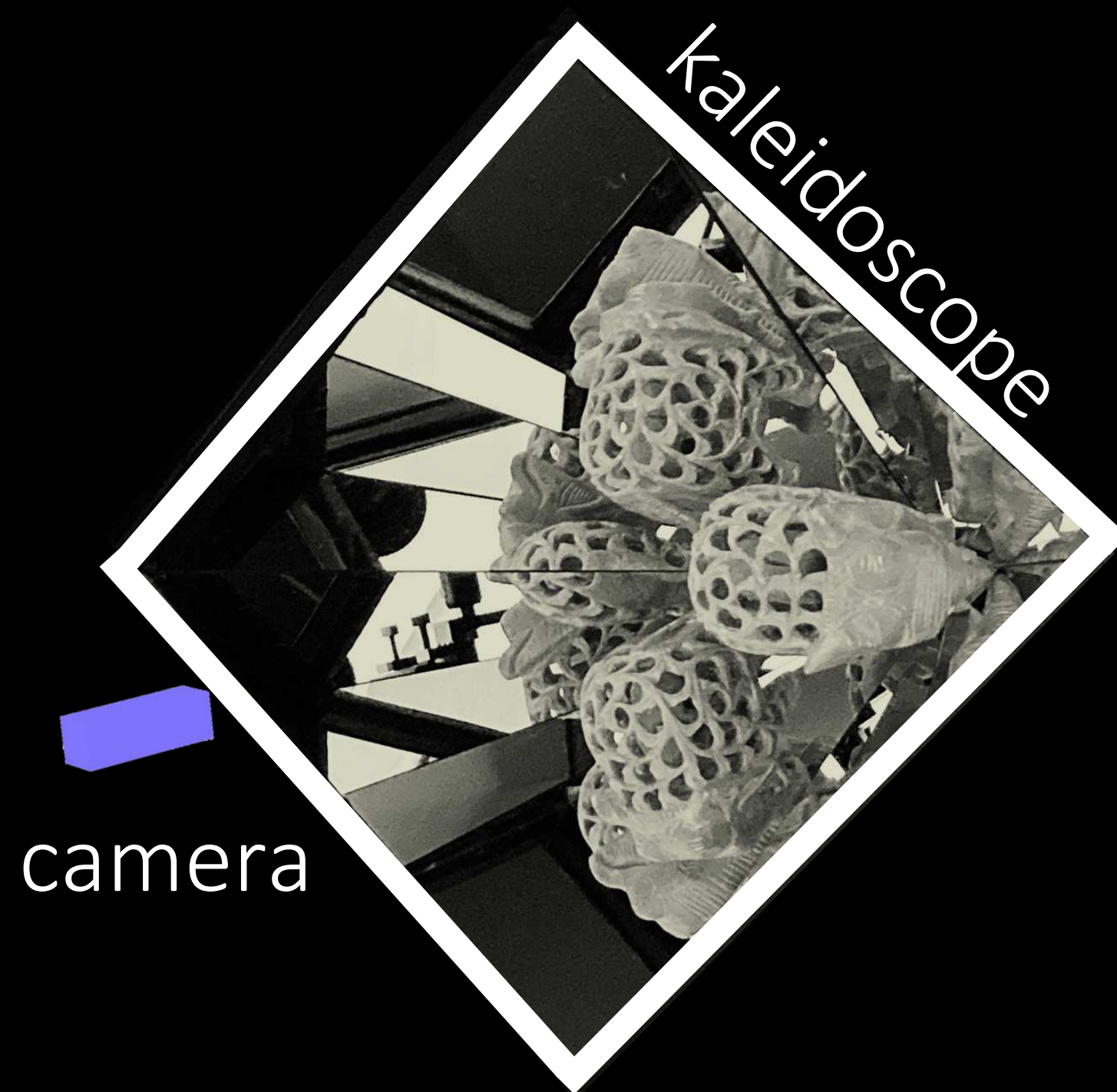
kaleidoscopic system



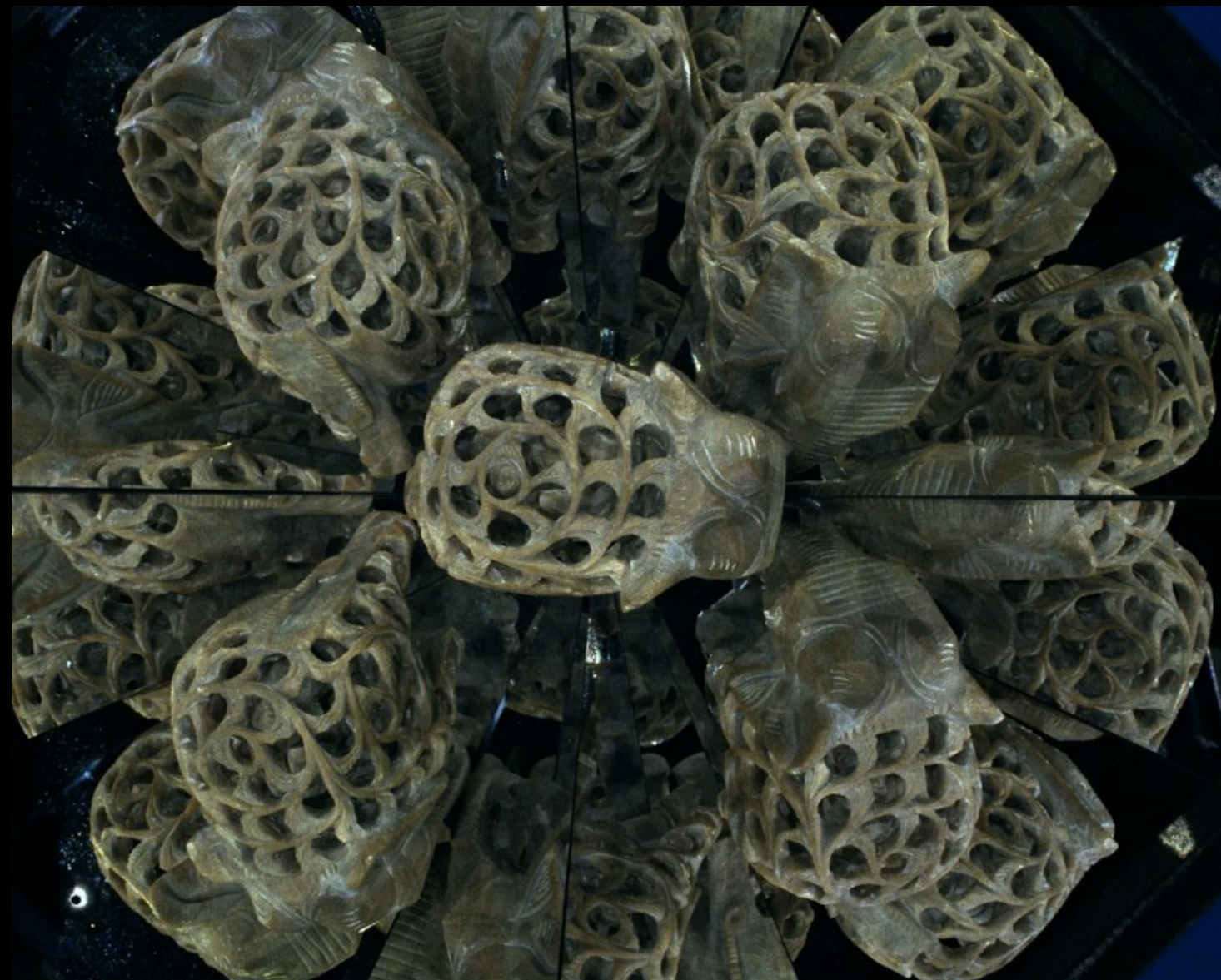
camera view



# Kaleidoscopic 3D scanning



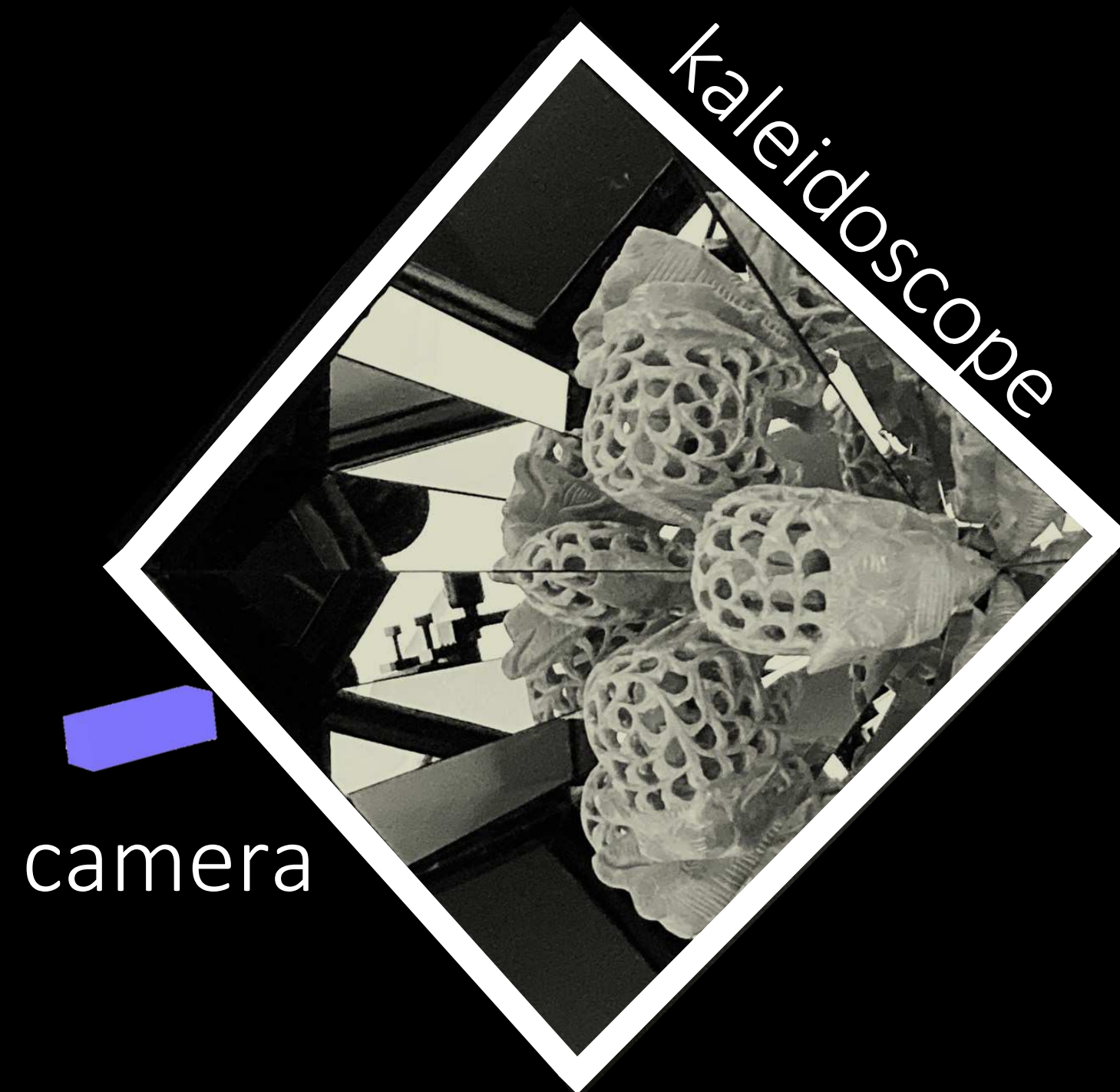
kaleidoscopic system



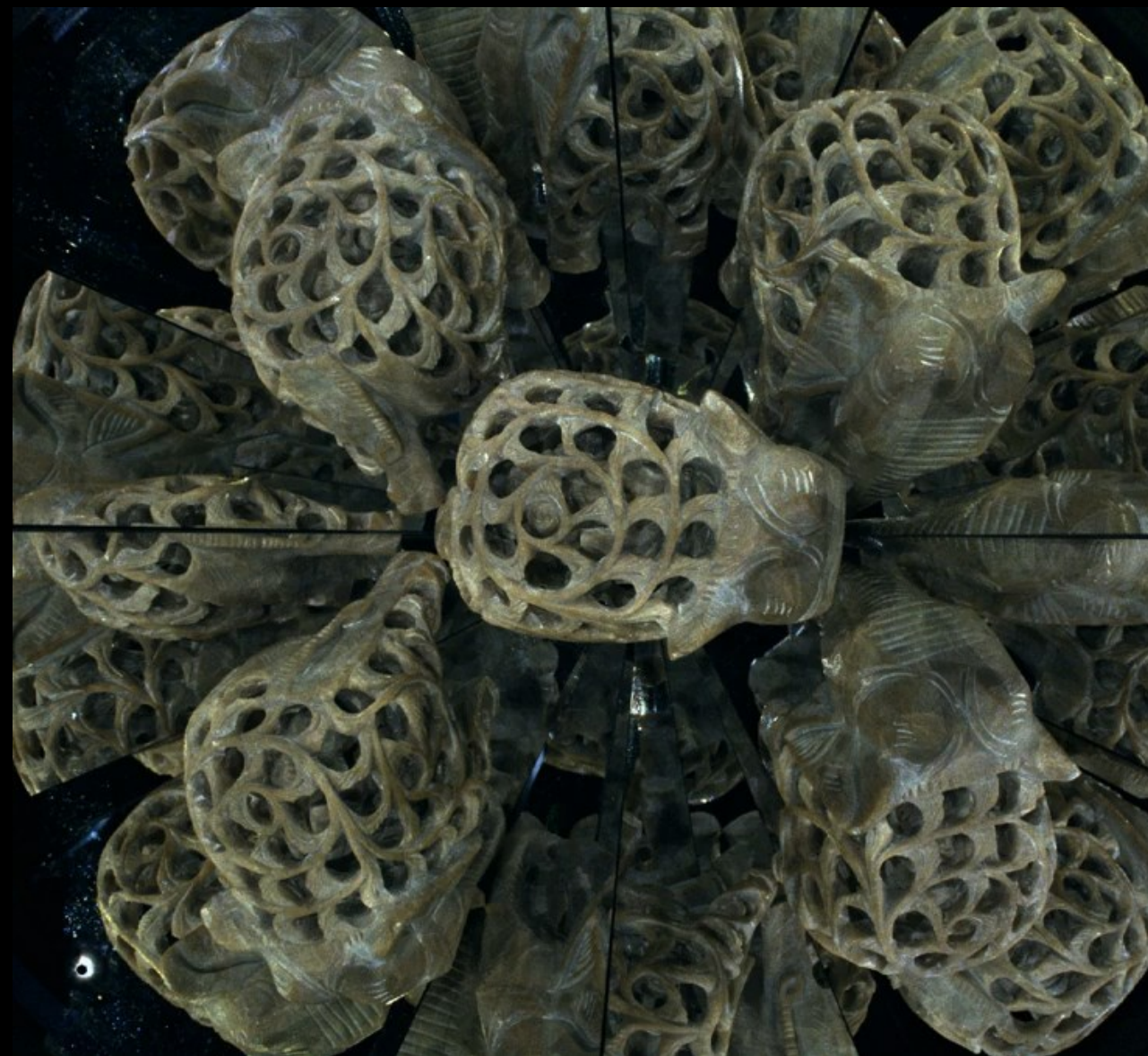
camera view



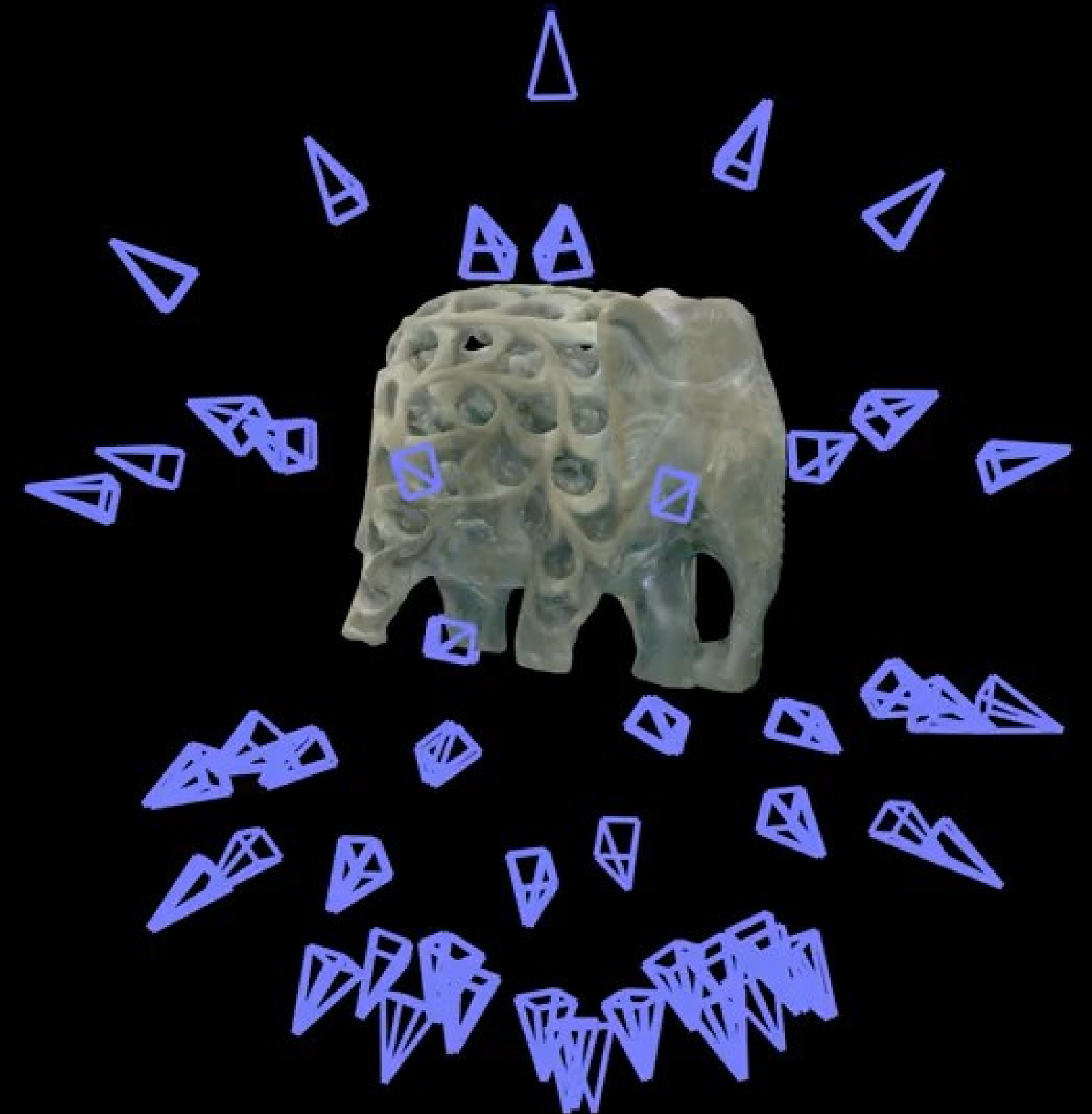
# Kaleidoscopic 3D scanning



kaleidoscopic system



camera view

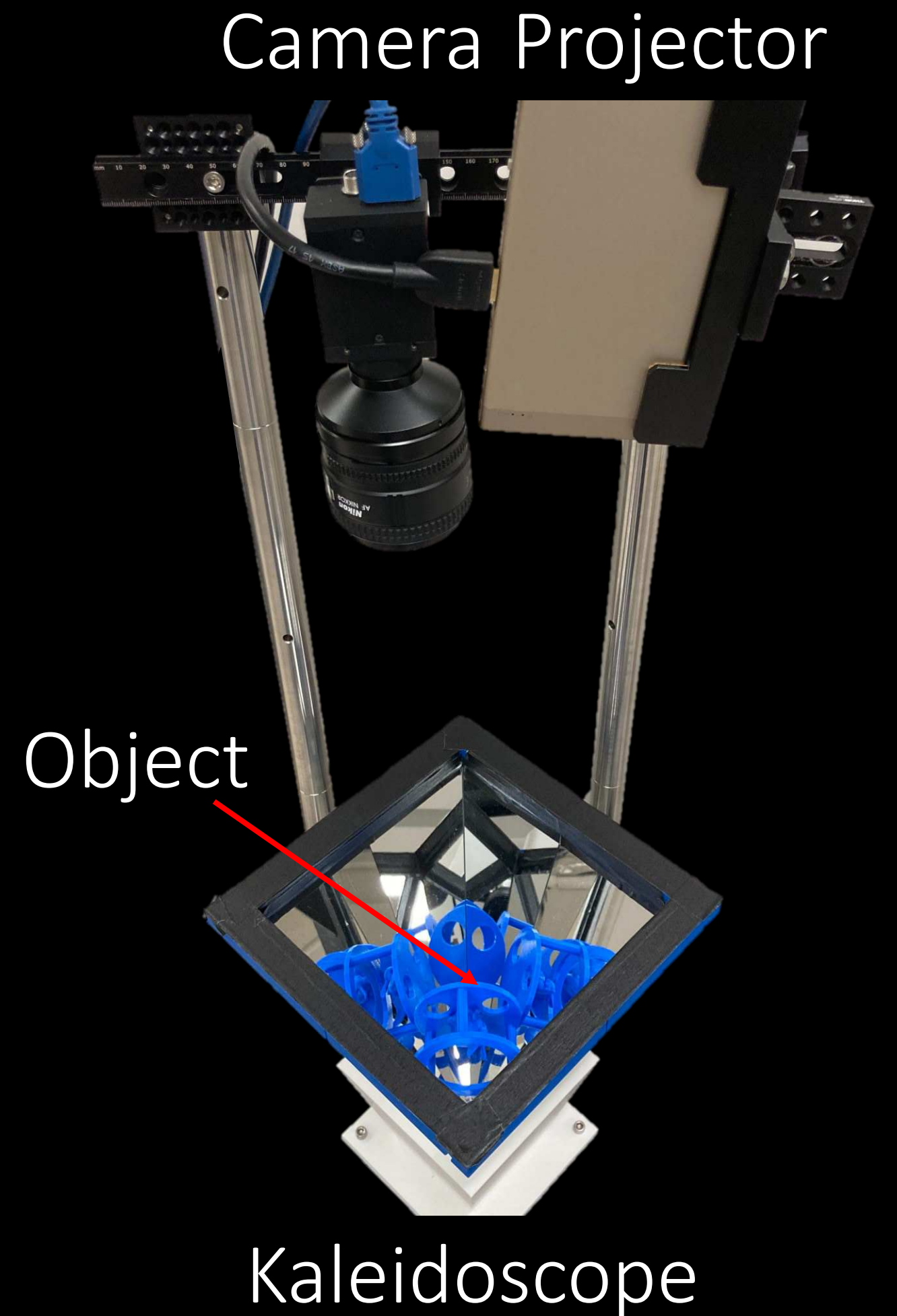


virtual cameras

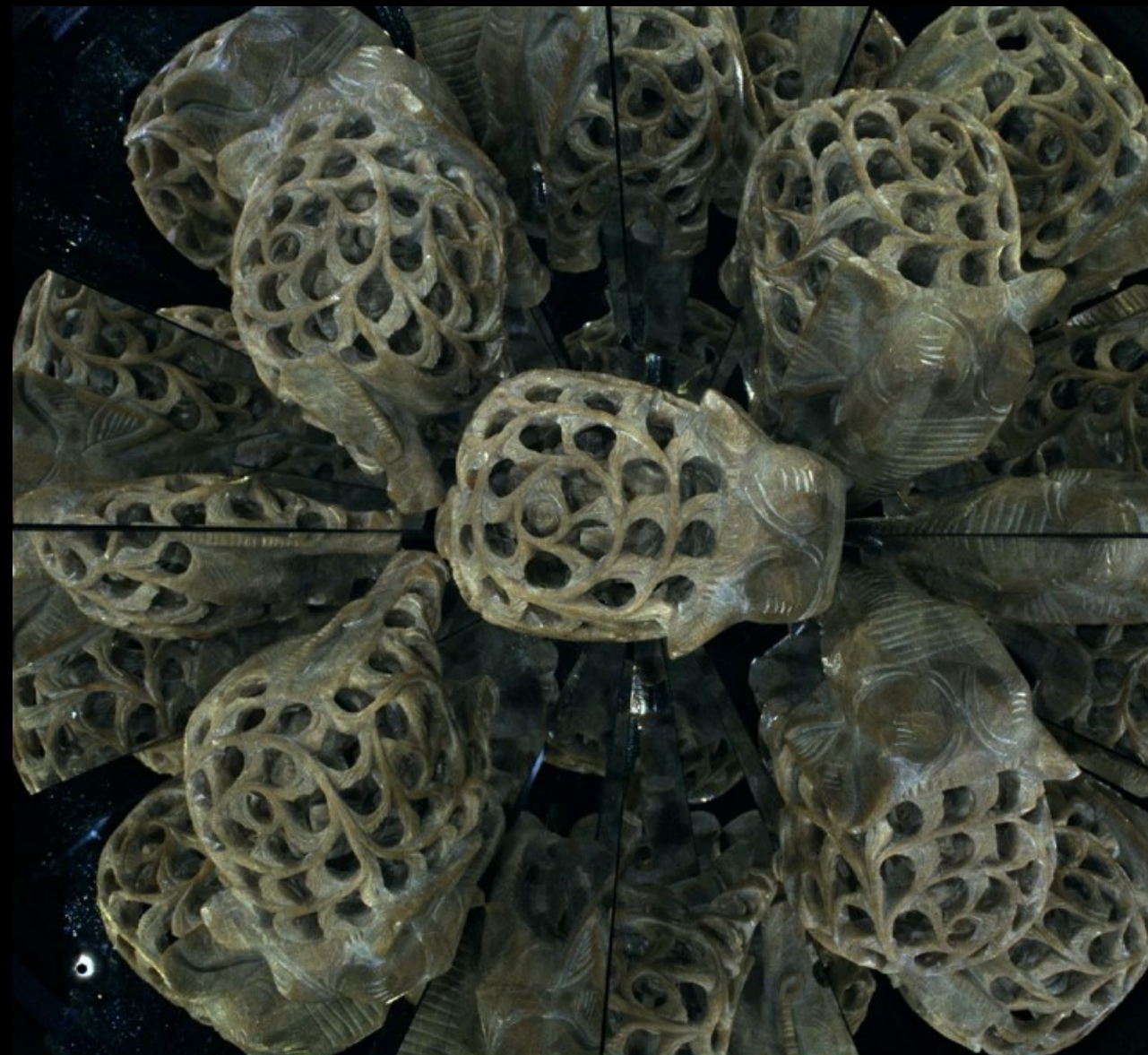
[Ahn et al., CVPR 2023, Tuesday PM]



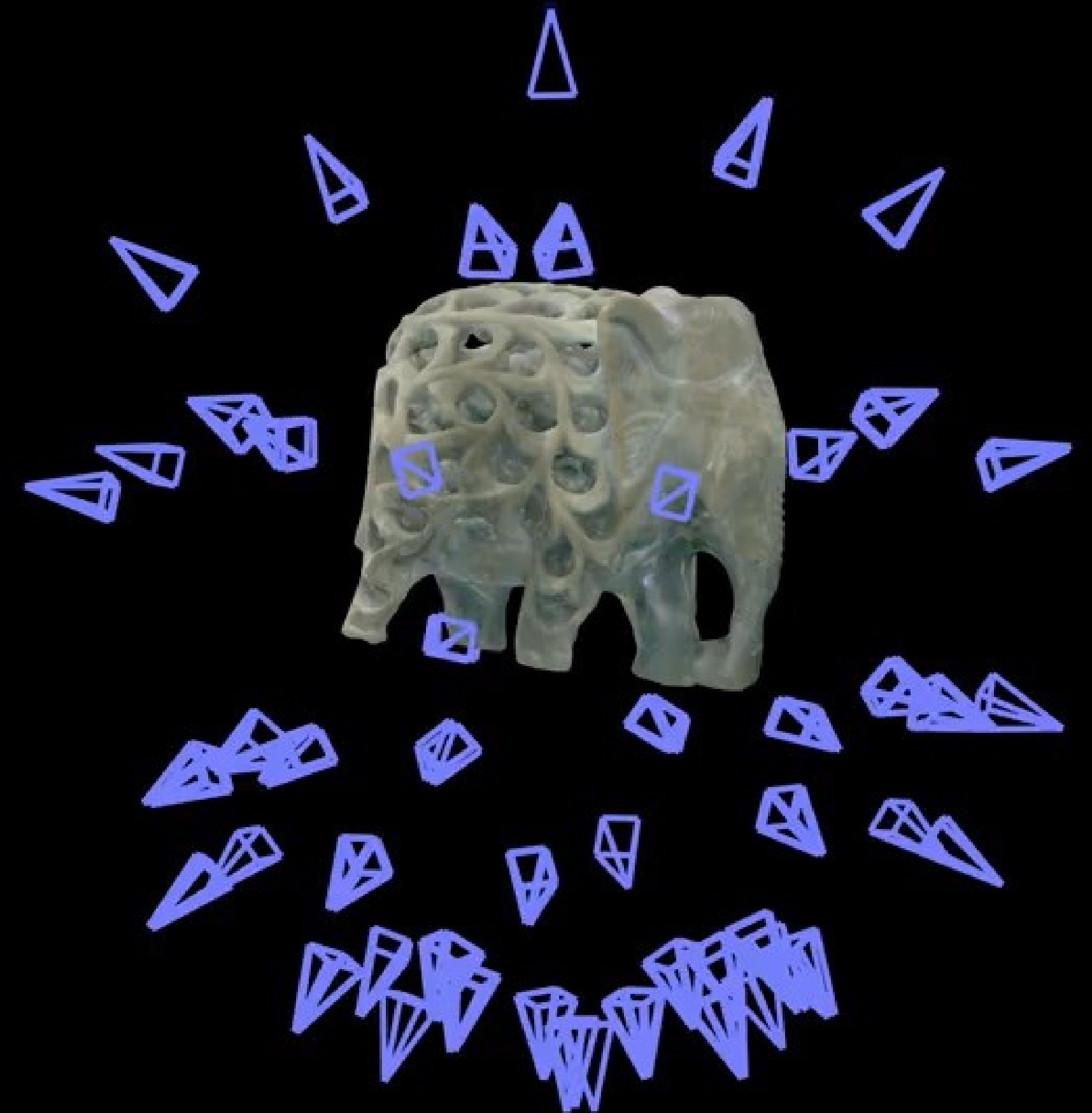
# Kaleidoscopic 3D scanning



kaleidoscopic system



camera view



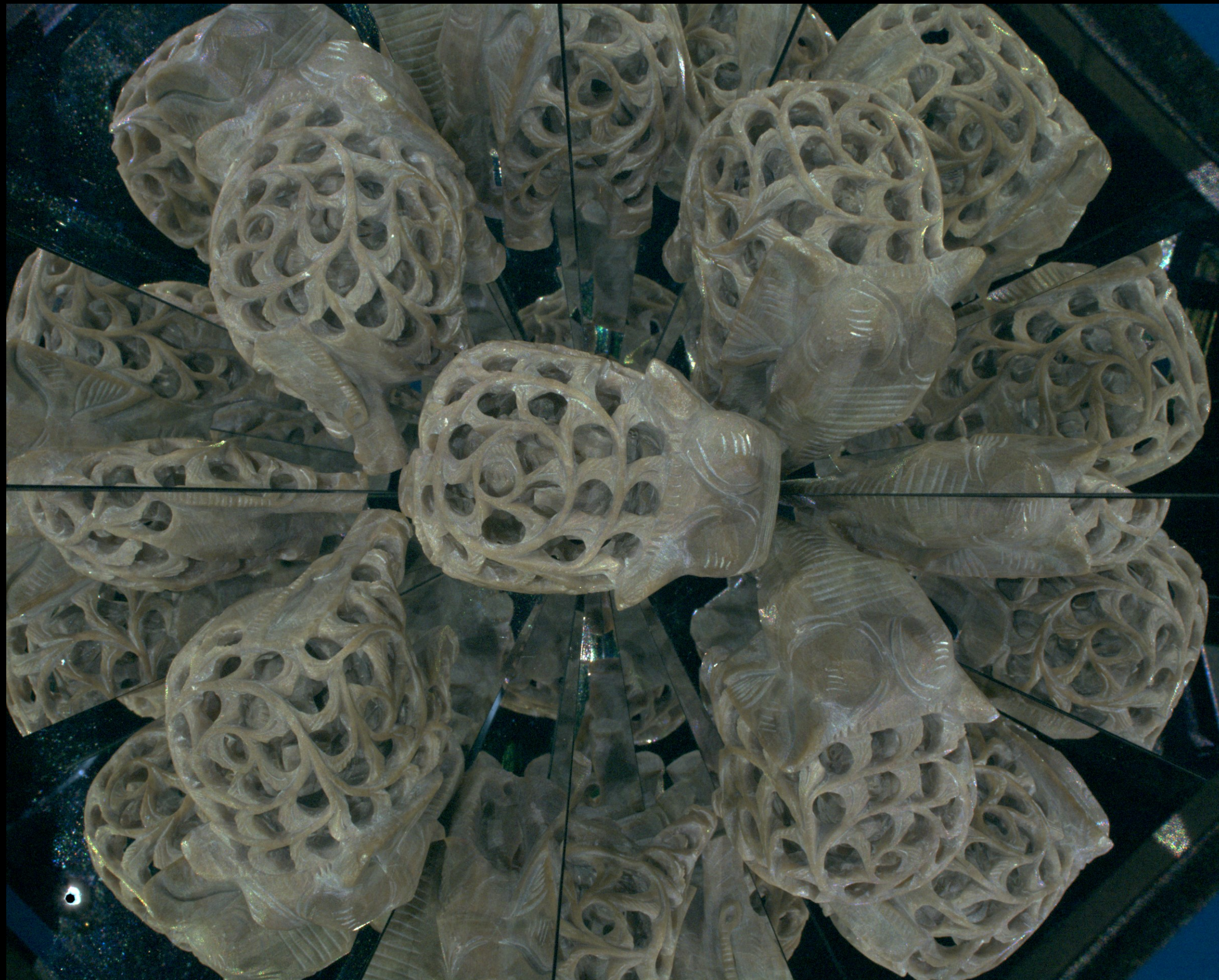
virtual cameras

[Ahn et al., CVPR 2023, Tuesday PM]



# Example 3D scans

photograph

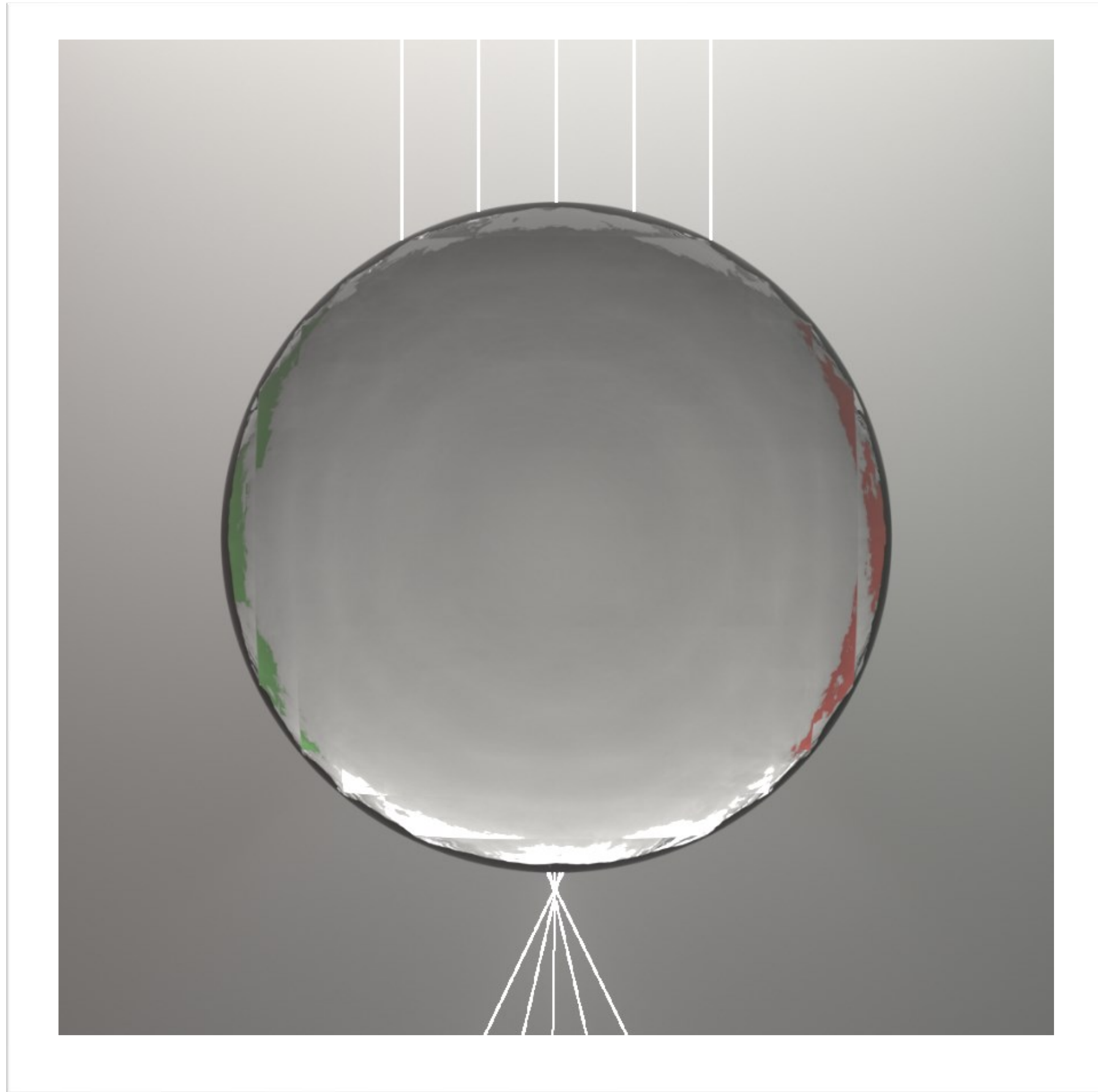


3D reconstruction

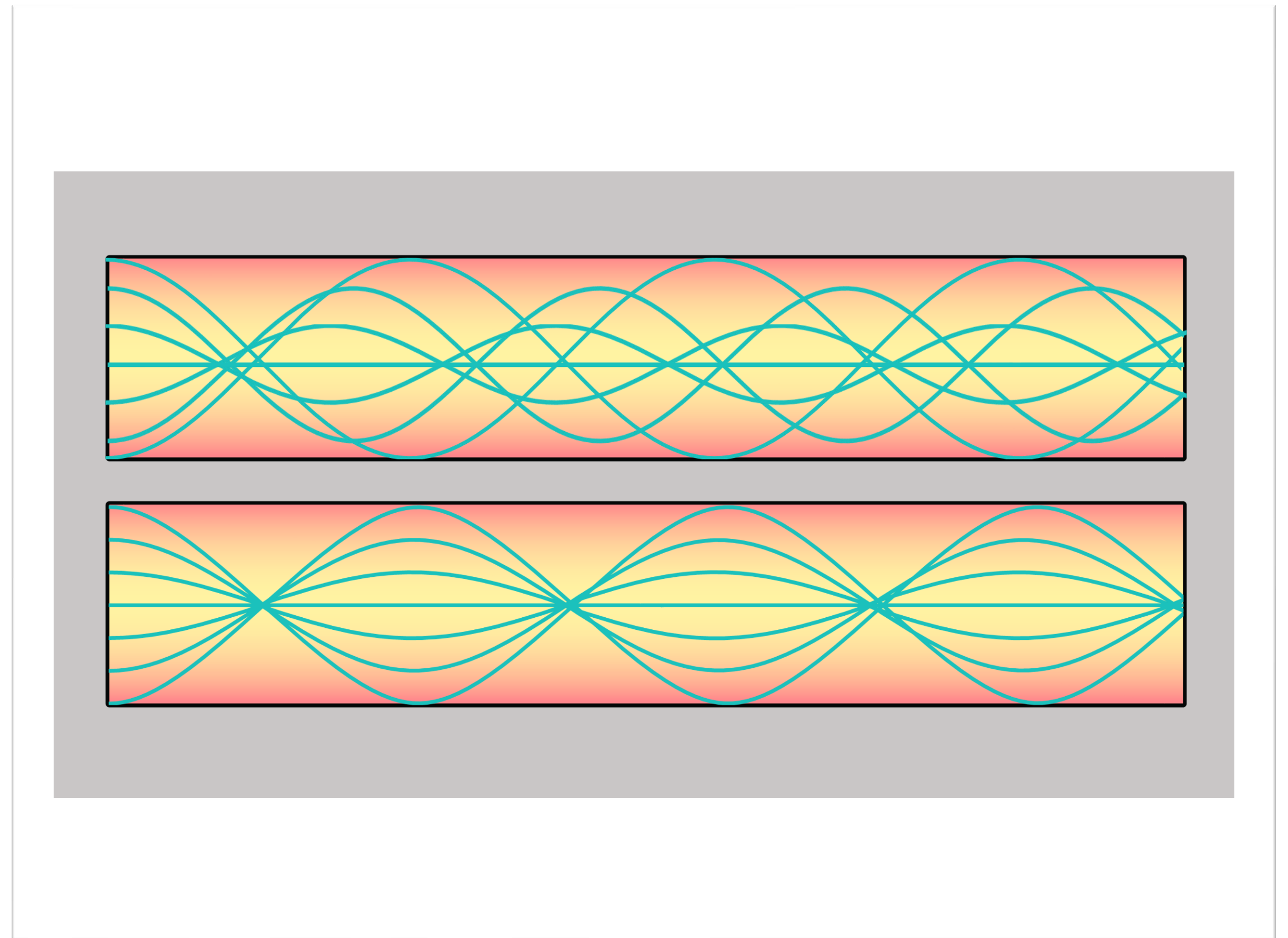




# Optimizing Gradient-Index (GRIN) Optics



Luneburg Lens



GRIN Fiber

# Nonlinear Ray Tracing



$\eta(\mathbf{x})$  : refractive index of the volume at location,  $\mathbf{x}$



# Nonlinear Ray Tracing



# Nonlinear Ray Tracing



# Nonlinear Ray Tracing





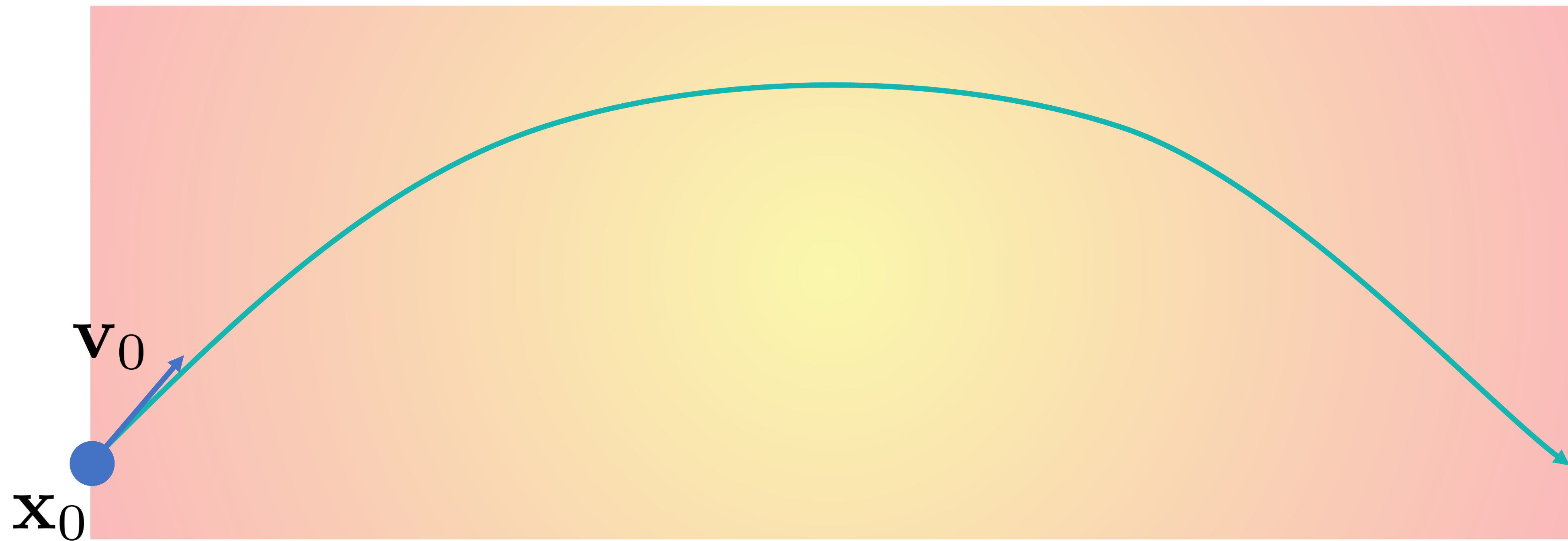
# Nonlinear Ray Tracing



$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \eta \nabla \eta$$

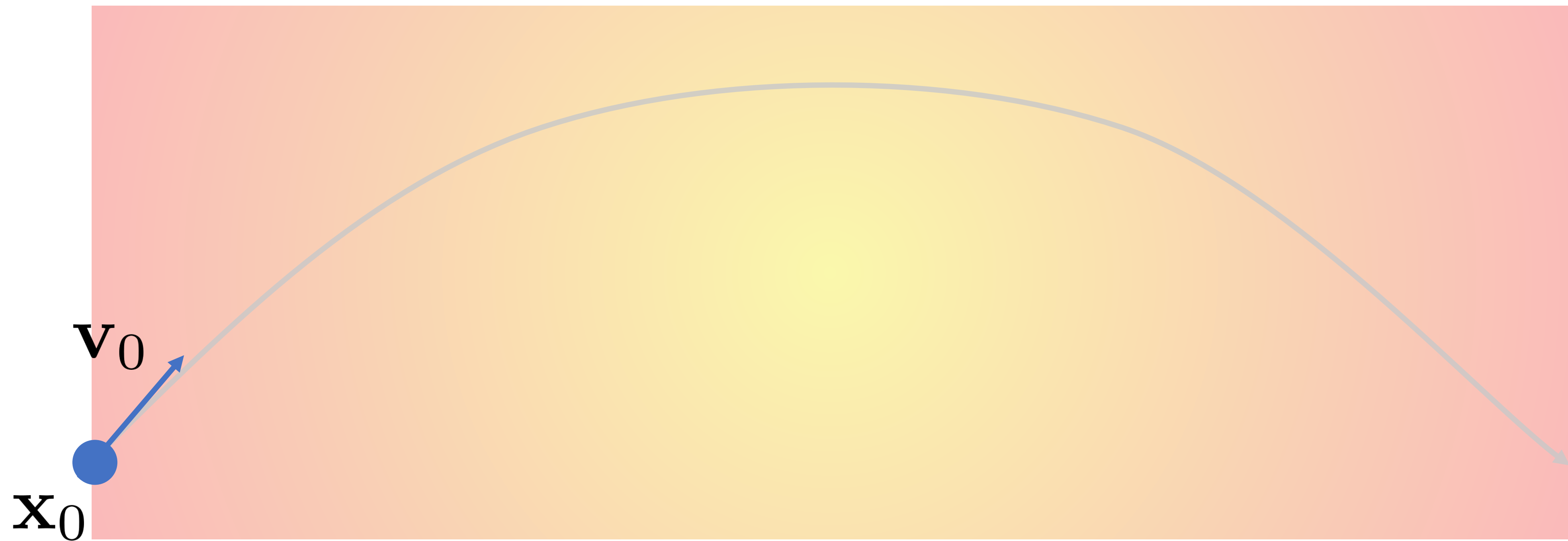
# Nonlinear Ray Tracing



$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \eta \nabla \eta$$

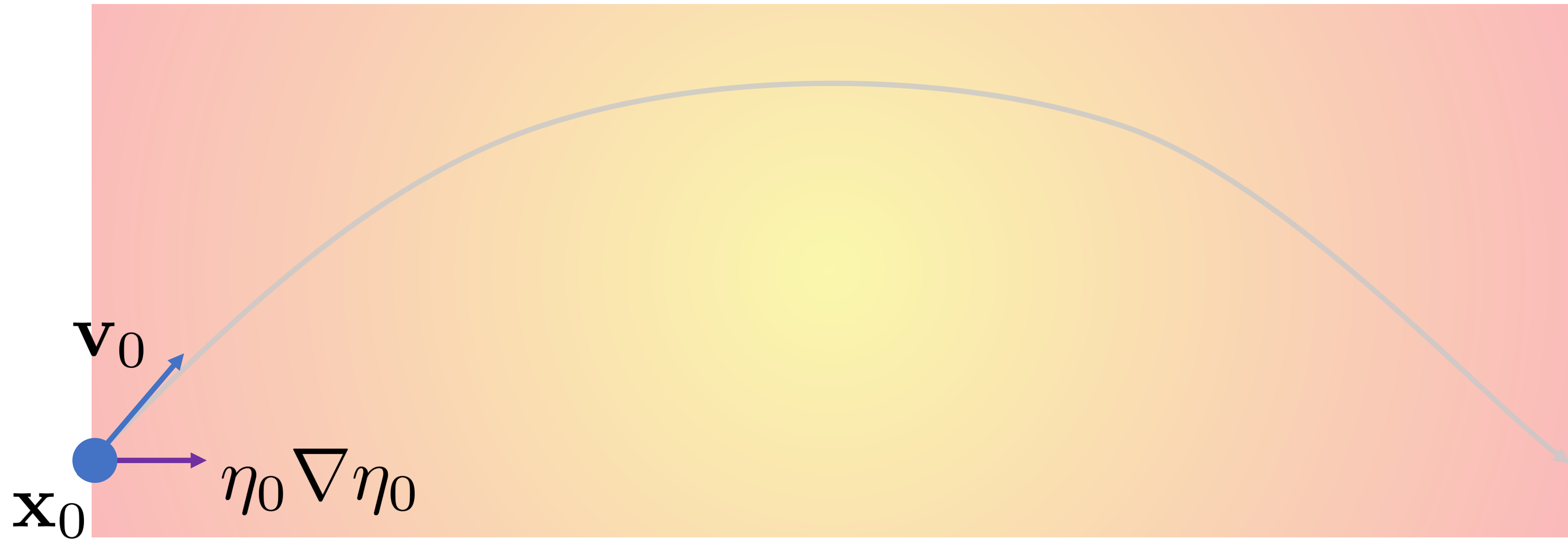
# Nonlinear Ray Tracing



$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t \quad \mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$

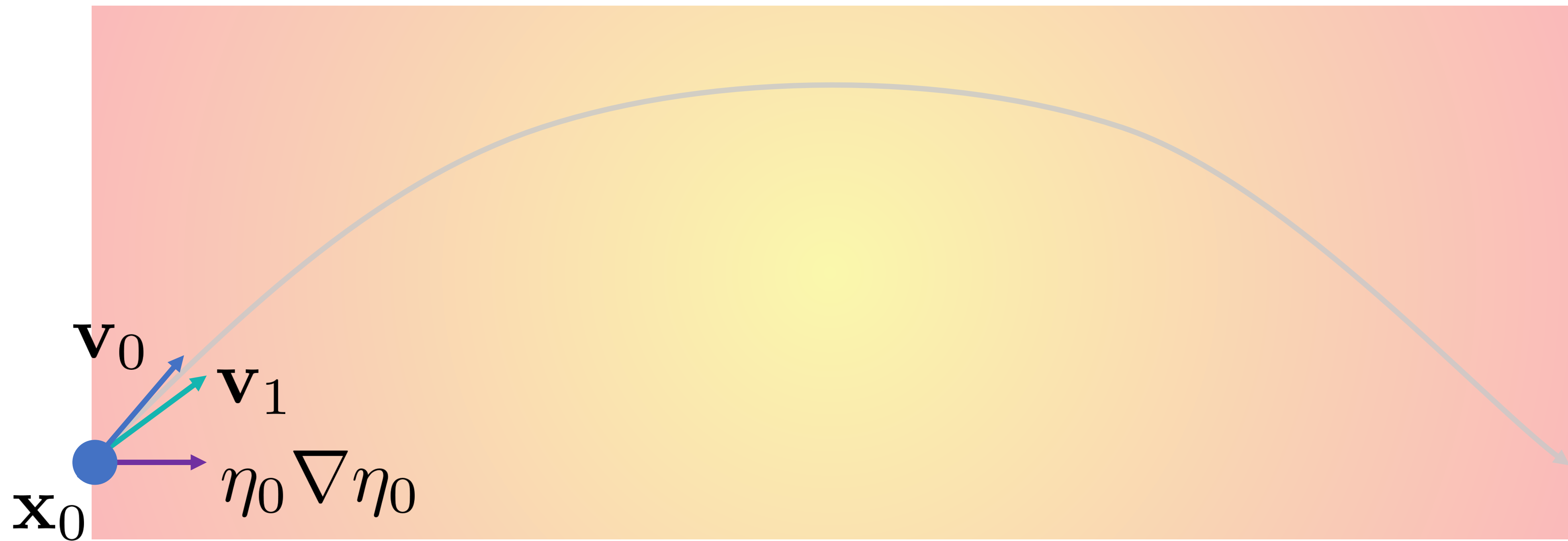


# Nonlinear Ray Tracing



$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t \quad \mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$

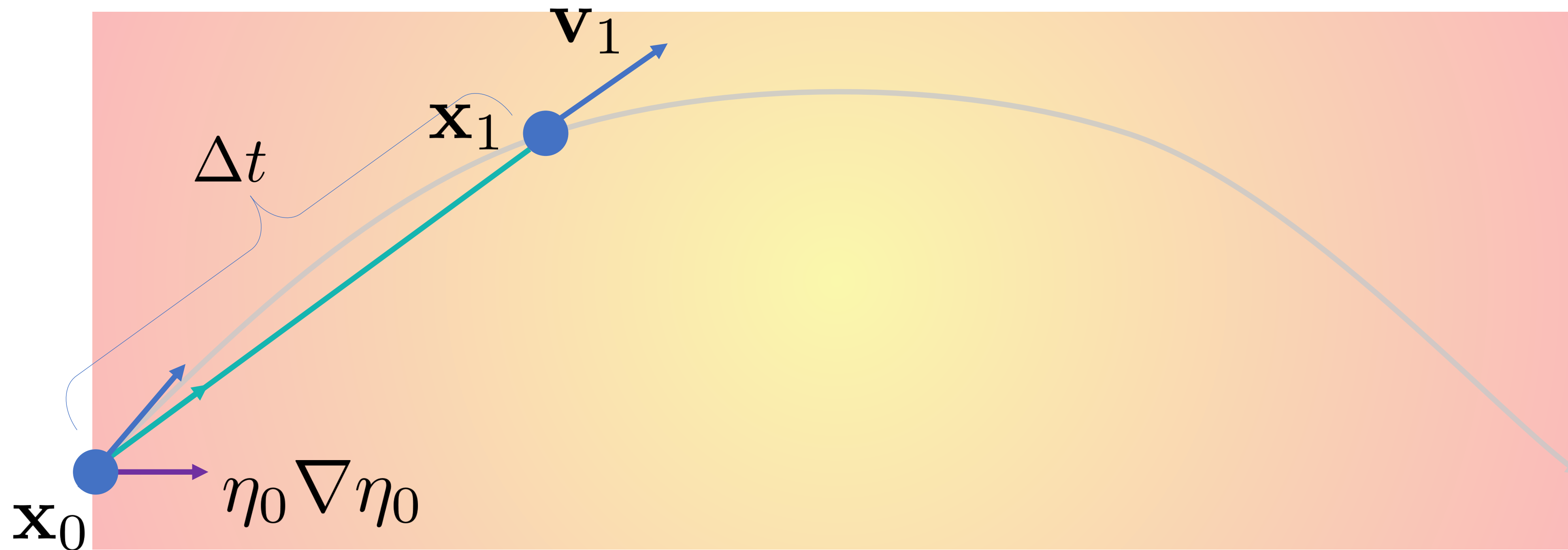
# Nonlinear Ray Tracing



$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t$$

$$\mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$

# Nonlinear Ray Tracing

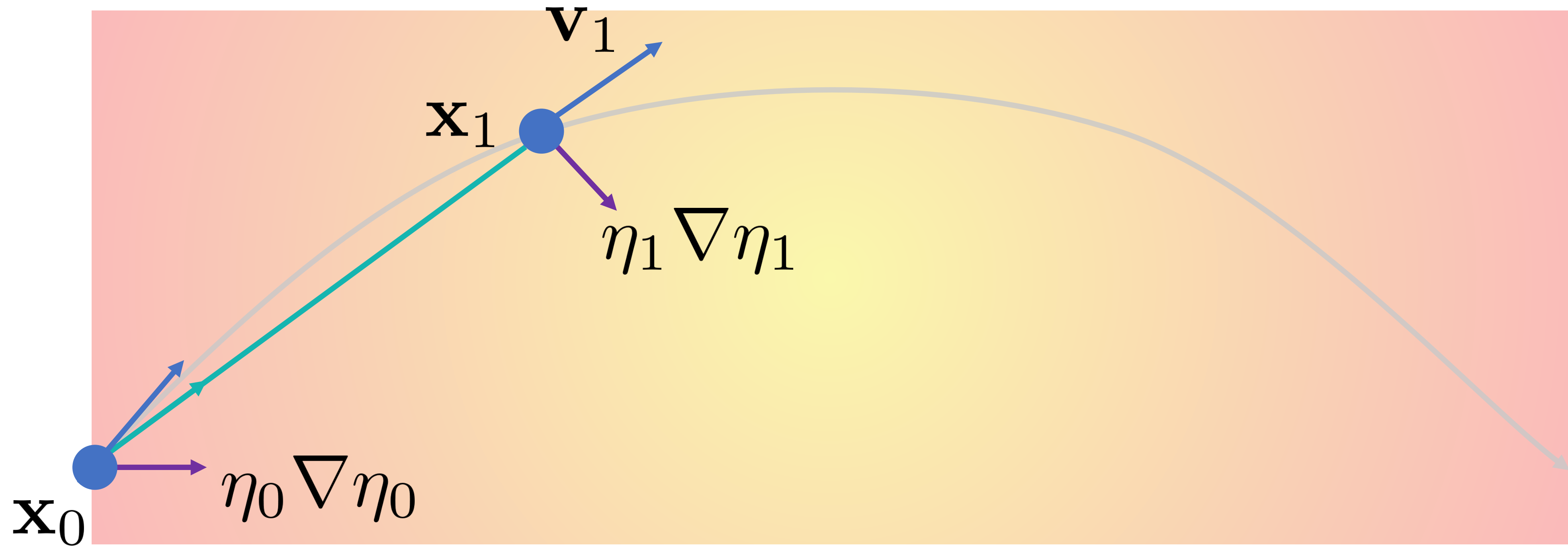


$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t$$

$$\mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$



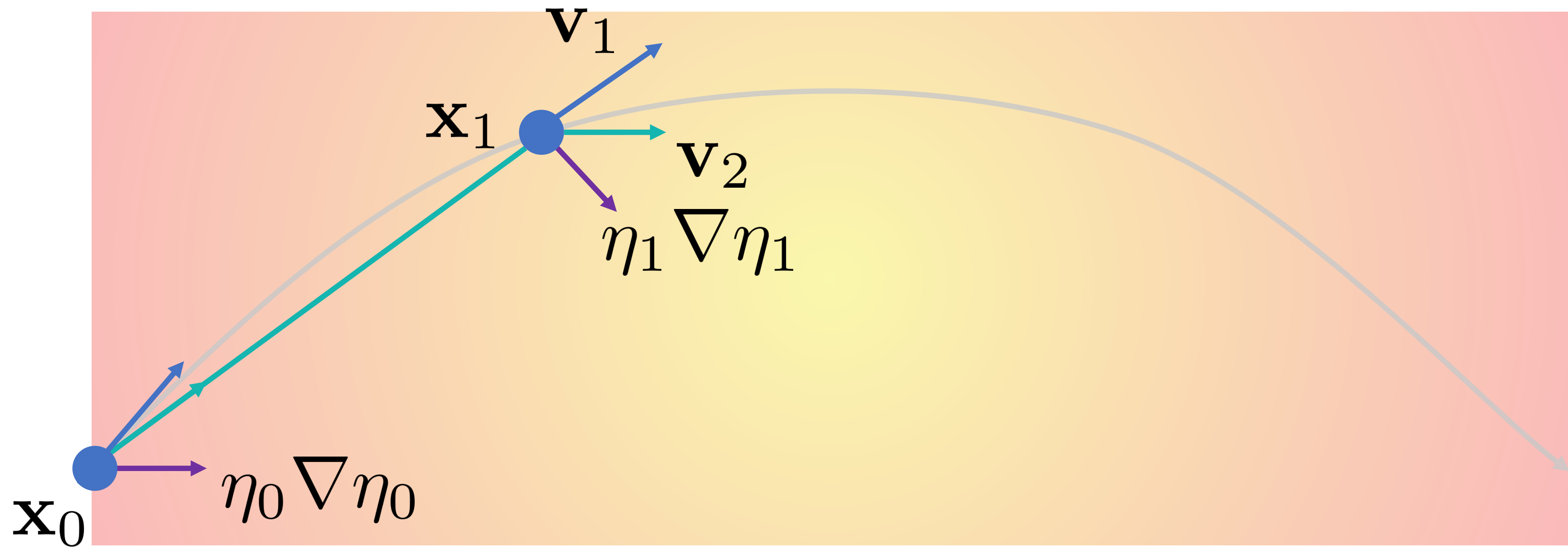
# Nonlinear Ray Tracing



$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t$$

$$\mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$

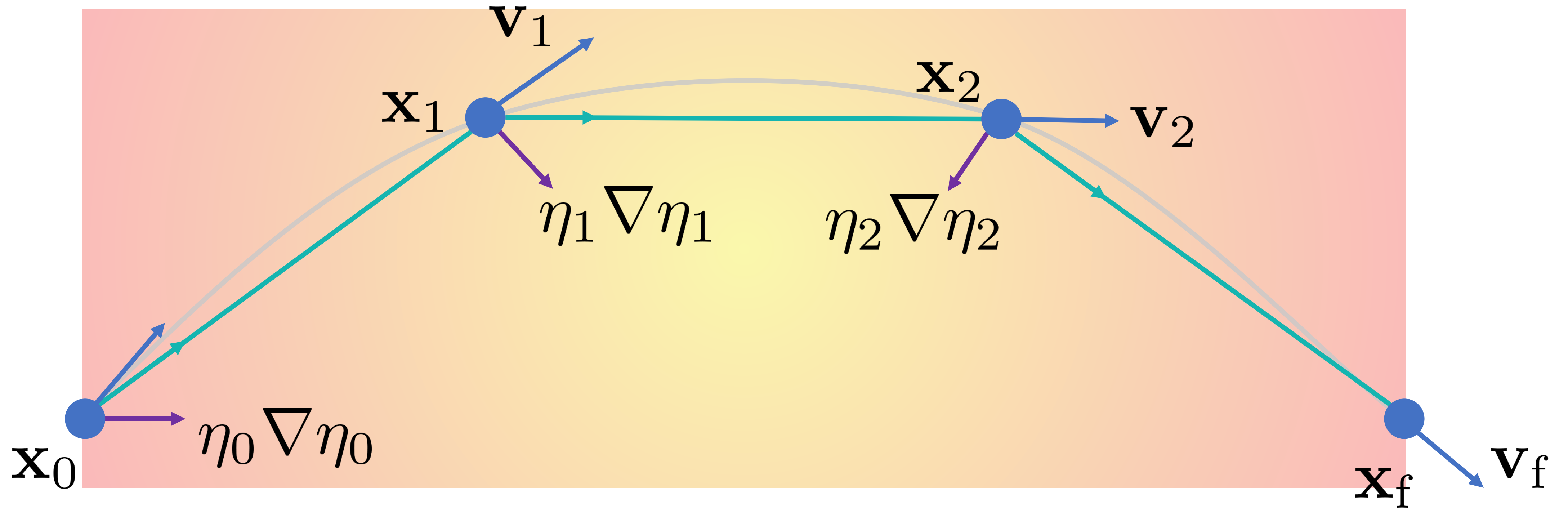
# Nonlinear Ray Tracing



$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t$$

$$\mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$

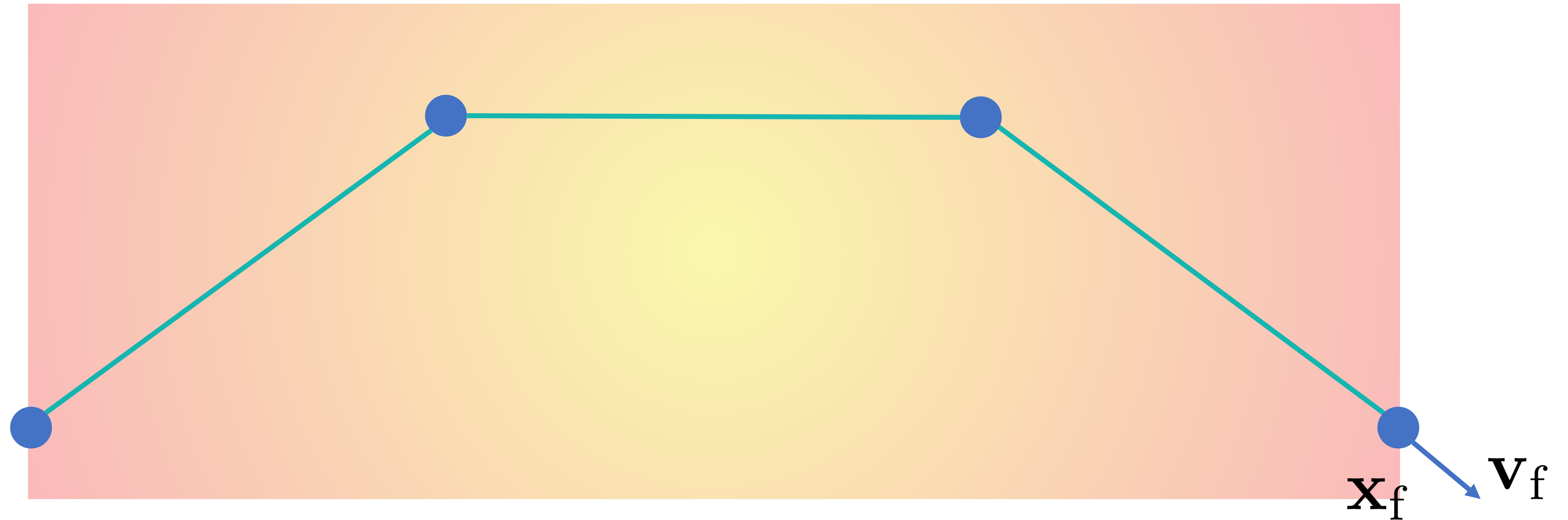
# Nonlinear Ray Tracing



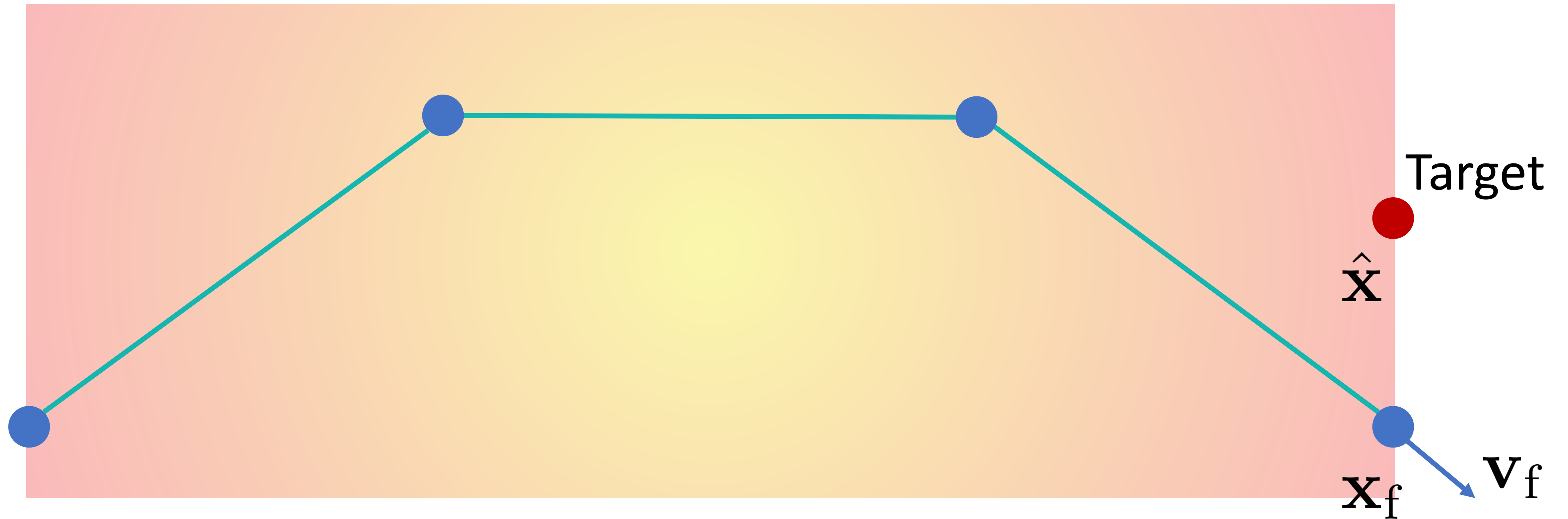
$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t \quad \mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$



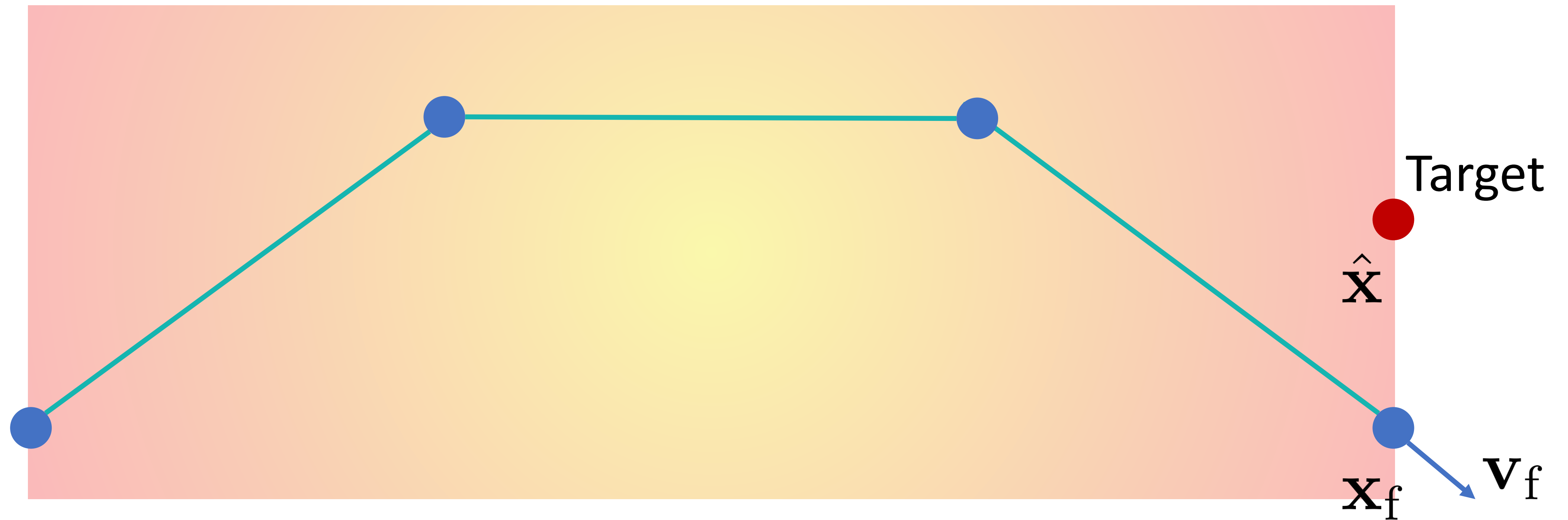
# Nonlinear Ray Tracing



# Nonlinear Ray Tracing



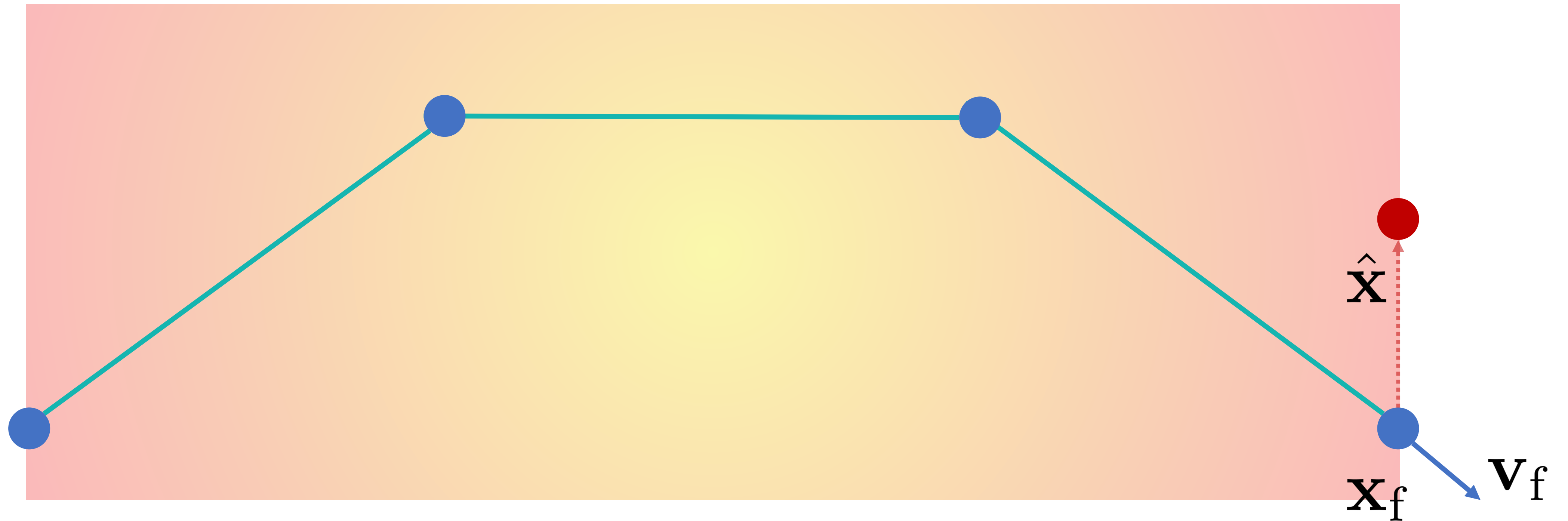
# Nonlinear Ray Tracing



$$\min_{\eta} \|\hat{\mathbf{x}} - \mathbf{x}_f\|^2$$

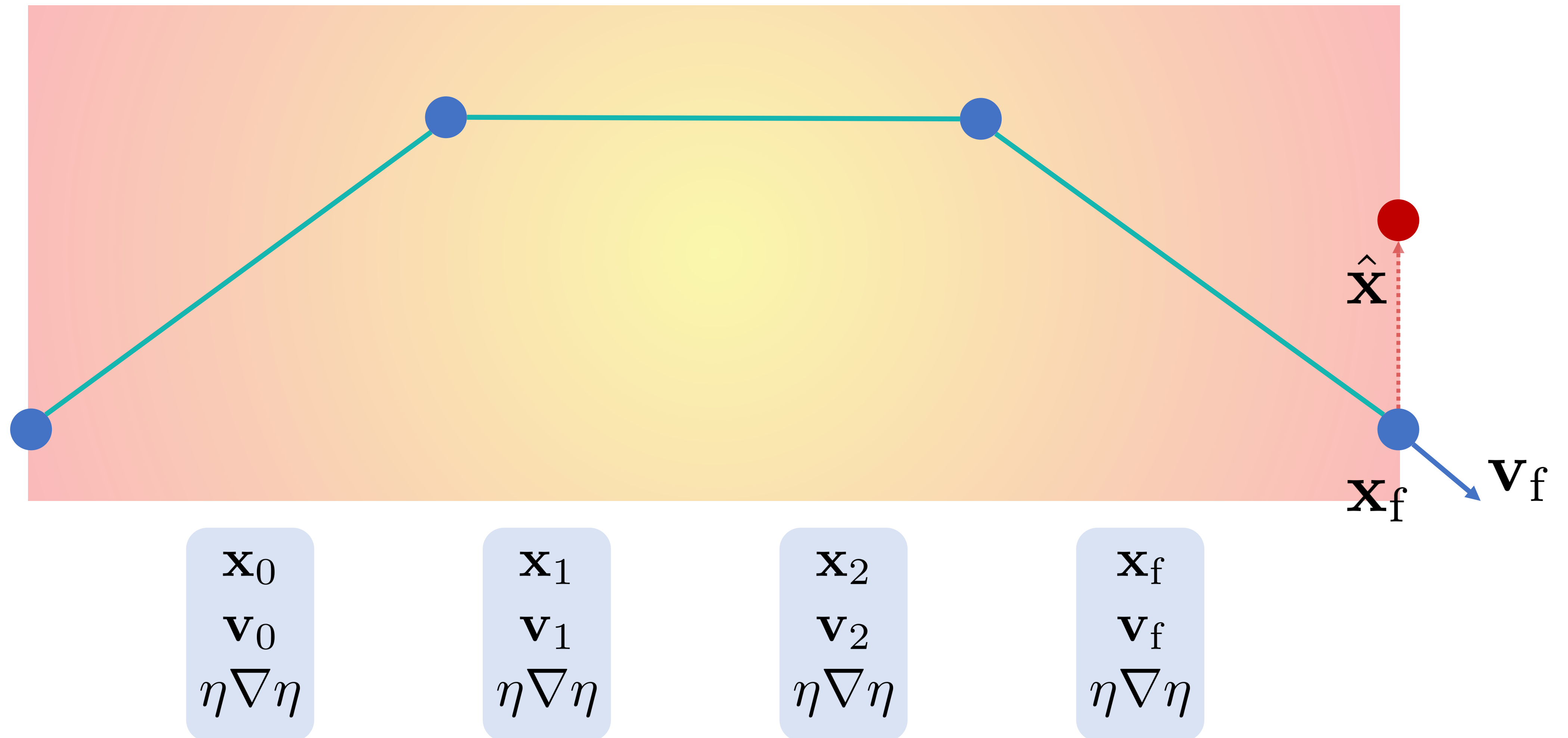


# Nonlinear Ray Tracing

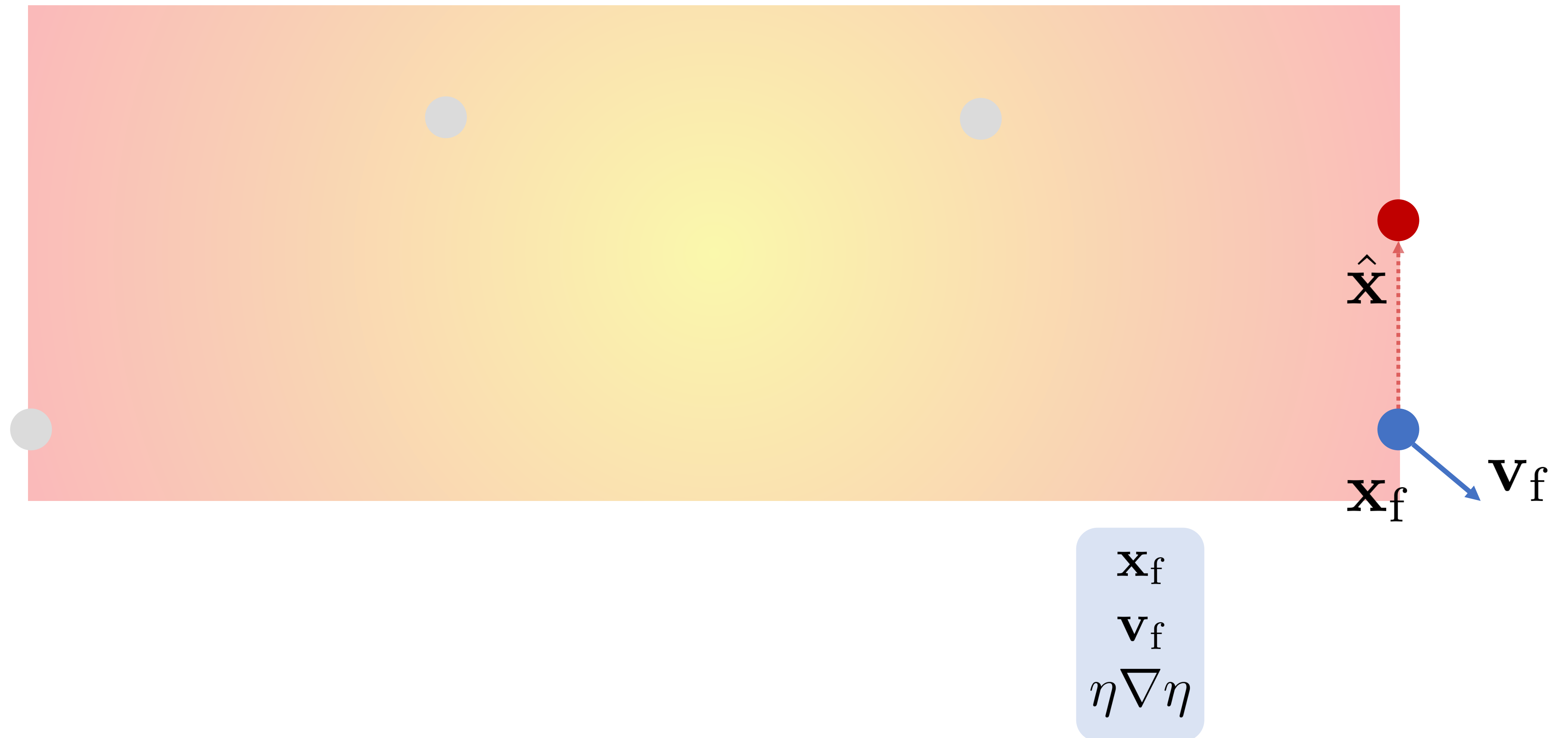


$$\frac{d}{d\eta} \|\hat{\mathbf{x}} - \mathbf{x}_f\|^2$$

# Nonlinear Ray Tracing in reverse

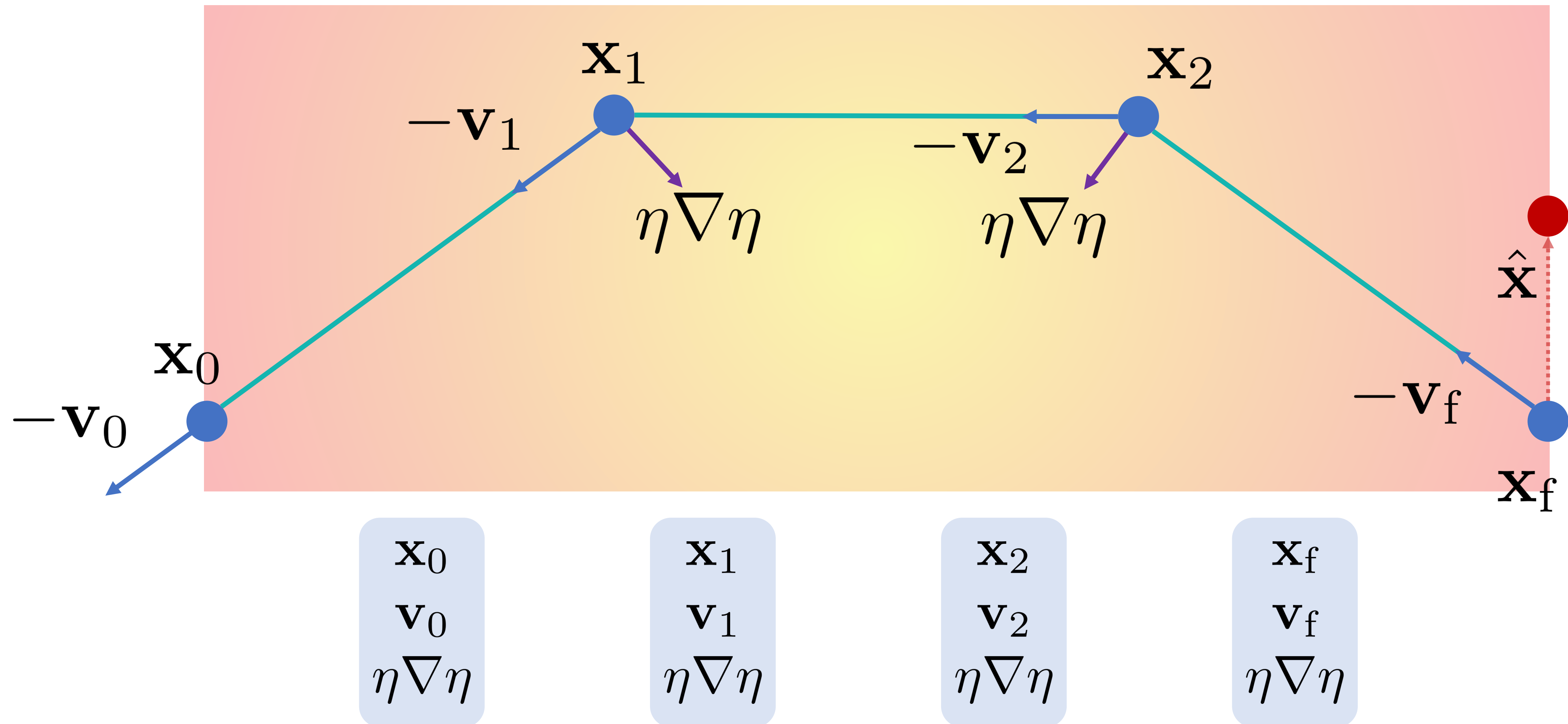


# Nonlinear Ray Tracing in reverse





# Nonlinear Ray Tracing in reverse



# Nonlinear Ray Tracing in reverse

$$\begin{aligned} \min_{\eta} \sum_{i=1}^N \mathcal{F}_i & \left[ \iint_{(\mathbf{x}_0, \mathbf{v}_0) \in \Omega} C_i \left( \mathbf{x} \left( \sigma_f; \eta, \mathbf{x}_0, \mathbf{v}_0 \right), \mathbf{v} \left( \sigma_f; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) \right) d\mathbf{x}_0 d\mathbf{v}_0 \right] \\ \text{s.t. } \dot{\mathbf{x}} \left( \sigma; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \mathbf{v}, \quad \forall \sigma \in [0, \sigma_f], \\ \dot{\mathbf{v}} \left( \sigma; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \eta \nabla \eta, \quad \forall \sigma \in [0, \sigma_f], \\ \mathbf{x} \left( 0; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \mathbf{x}_0, \\ \mathbf{v} \left( 0; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \mathbf{v}_0, \end{aligned} \quad (15)$$

$\mathbf{x}_0$

$\hat{\mathbf{x}}$

$$\dot{\lambda} = - \left( \nabla \eta (\nabla \eta)^\top + \eta \text{Hess} (\eta) \right) \mu, \quad \forall \sigma \in [0, \sigma_f] \quad (19)$$

$$\dot{\mu} = -\lambda, \quad \forall \sigma \in [0, \sigma_f] \quad (20)$$

$$\lambda \left( \sigma_f \right) = \frac{\partial \mathcal{C}}{\partial \mathbf{x}}, \quad (21)$$

$$\mu \left( \sigma_f \right) = \frac{\partial \mathcal{C}}{\partial \mathbf{v}}. \quad (22)$$

$\eta \nabla \eta$

$\eta \nabla \eta$

$$\mathbf{x}_{i-1} = \mathbf{x}_i - \mathbf{v}_i \Delta \sigma, \quad (28)$$

$$\mathbf{v}_{i-1} = \mathbf{v}_i - \eta \left( \mathbf{x}_{i-1} \right) \nabla \eta \left( \mathbf{x}_{i-1} \right) \Delta \sigma, \quad (29)$$

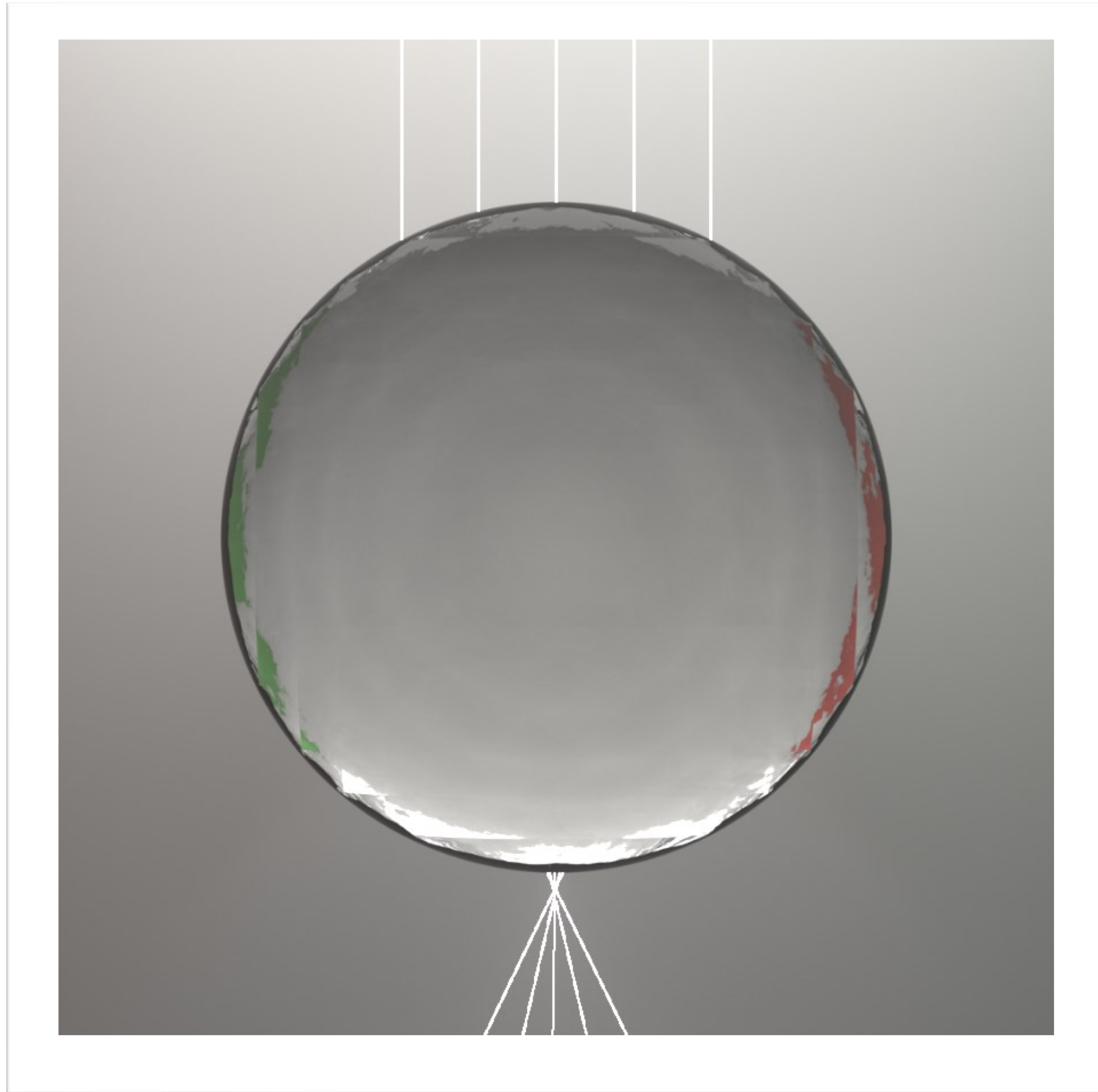
$$\begin{aligned} \lambda_{i-1} &= \lambda_i \\ &\quad + \left( \nabla \eta \left( \mathbf{x}_{i-1} \right) \left( \nabla \eta \left( \mathbf{x}_{i-1} \right) \right)^\top + \eta \left( \mathbf{x}_{i-1} \right) \text{Hess} \left( \eta \left( \mathbf{x}_{i-1} \right) \right) \right) \mu_i \Delta \sigma, \end{aligned} \quad (30)$$

$$\mu_{i-1} = \mu_i + \lambda_{i-1} \Delta \sigma. \quad (31)$$

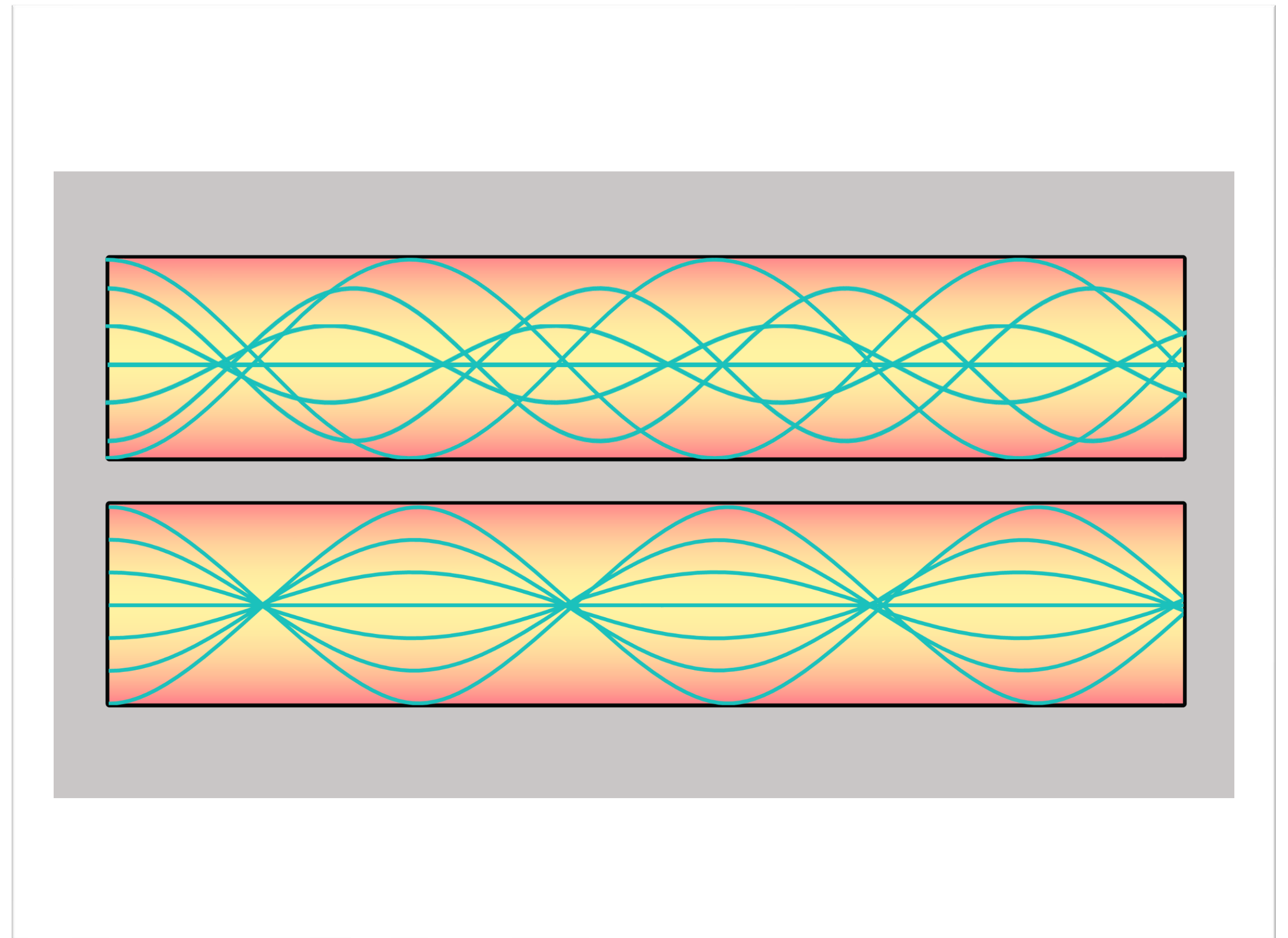
$\eta \nabla \eta$

$\eta \nabla \eta$

# Optimizing Gradient-Index (GRIN) Optics



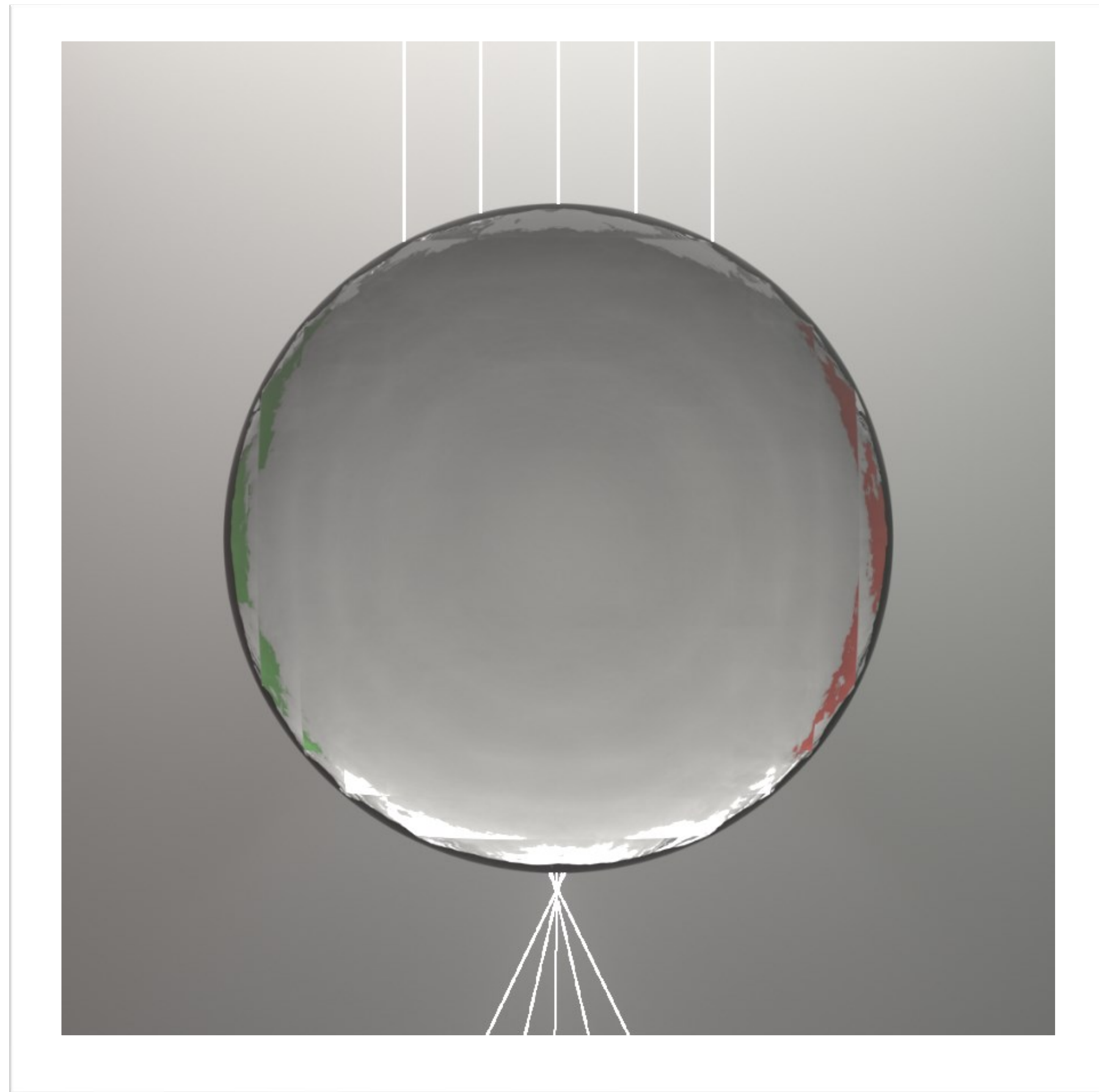
Luneburg Lens



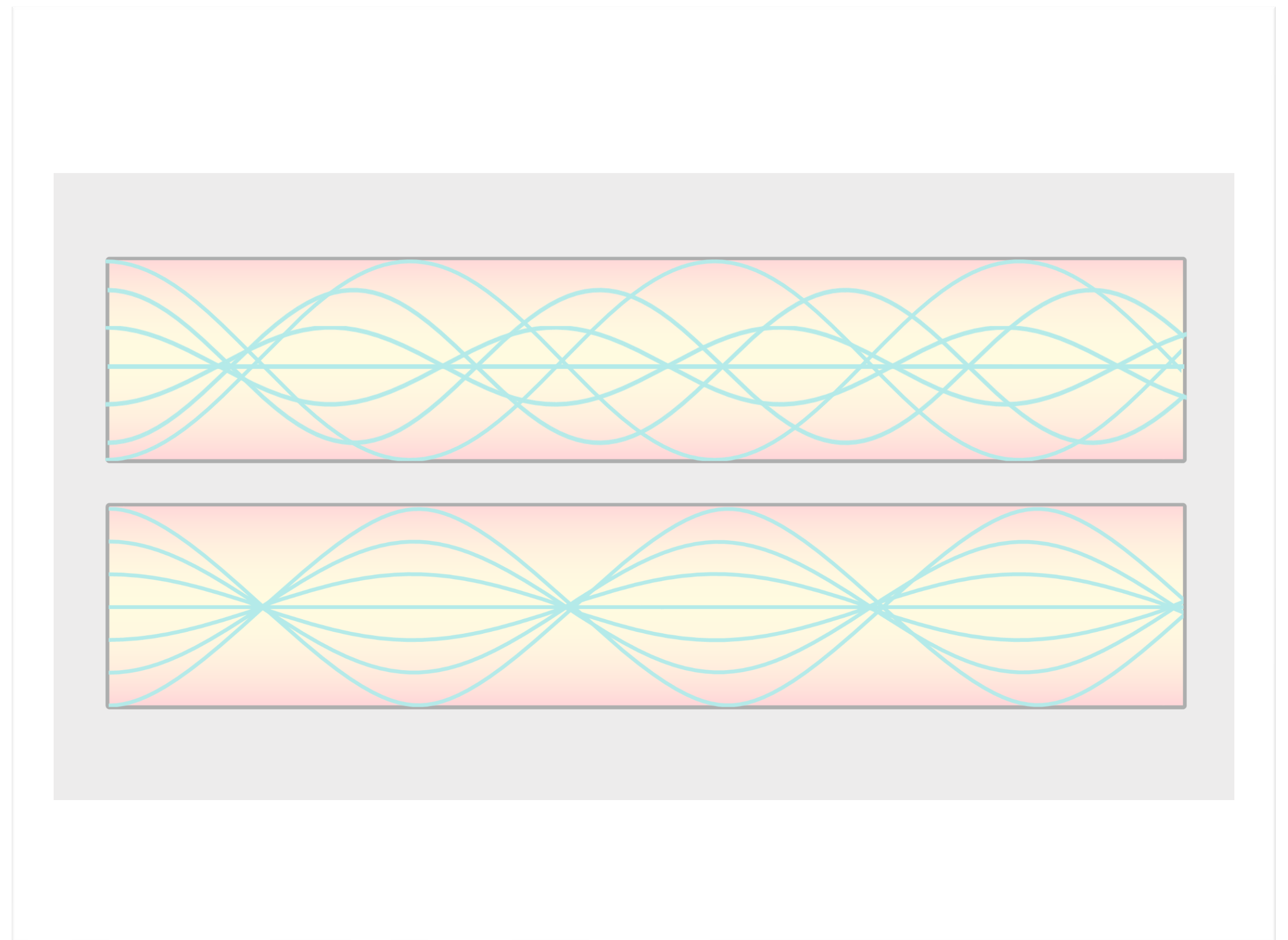
GRIN Fiber



# Optimizing Gradient-Index (GRIN) Optics

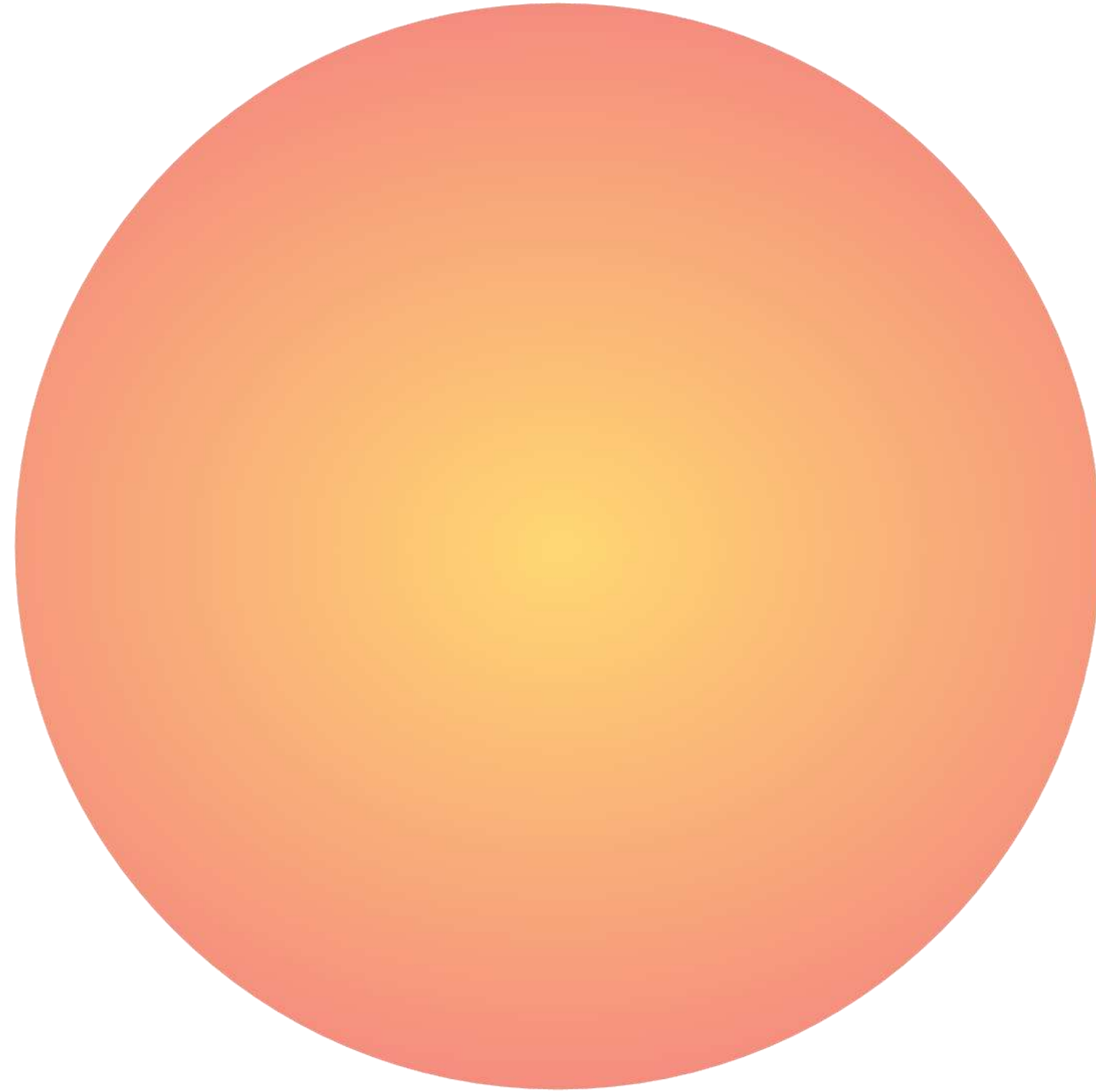


Luneburg Lens

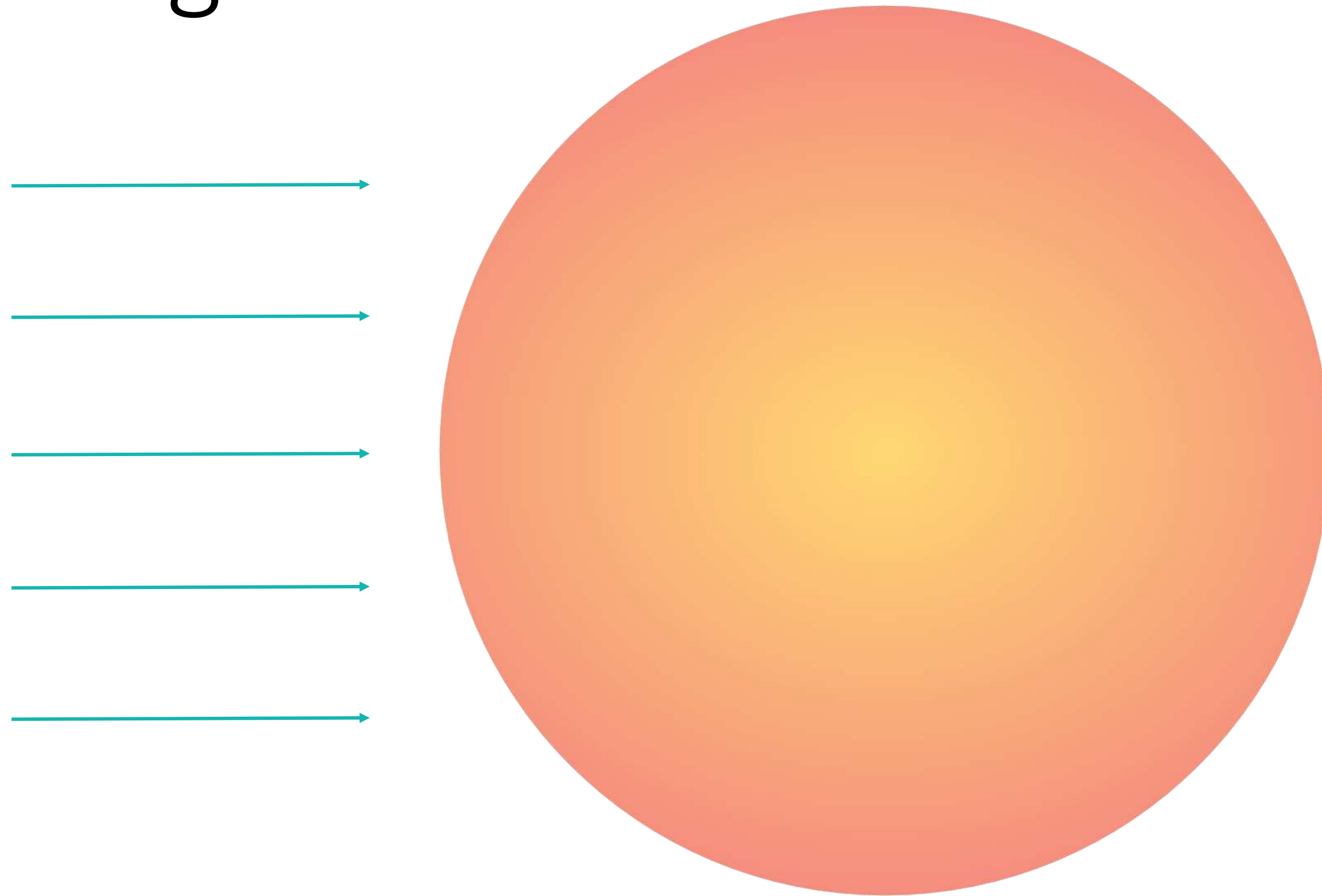


GRIN Fiber

# Luneburg Lens

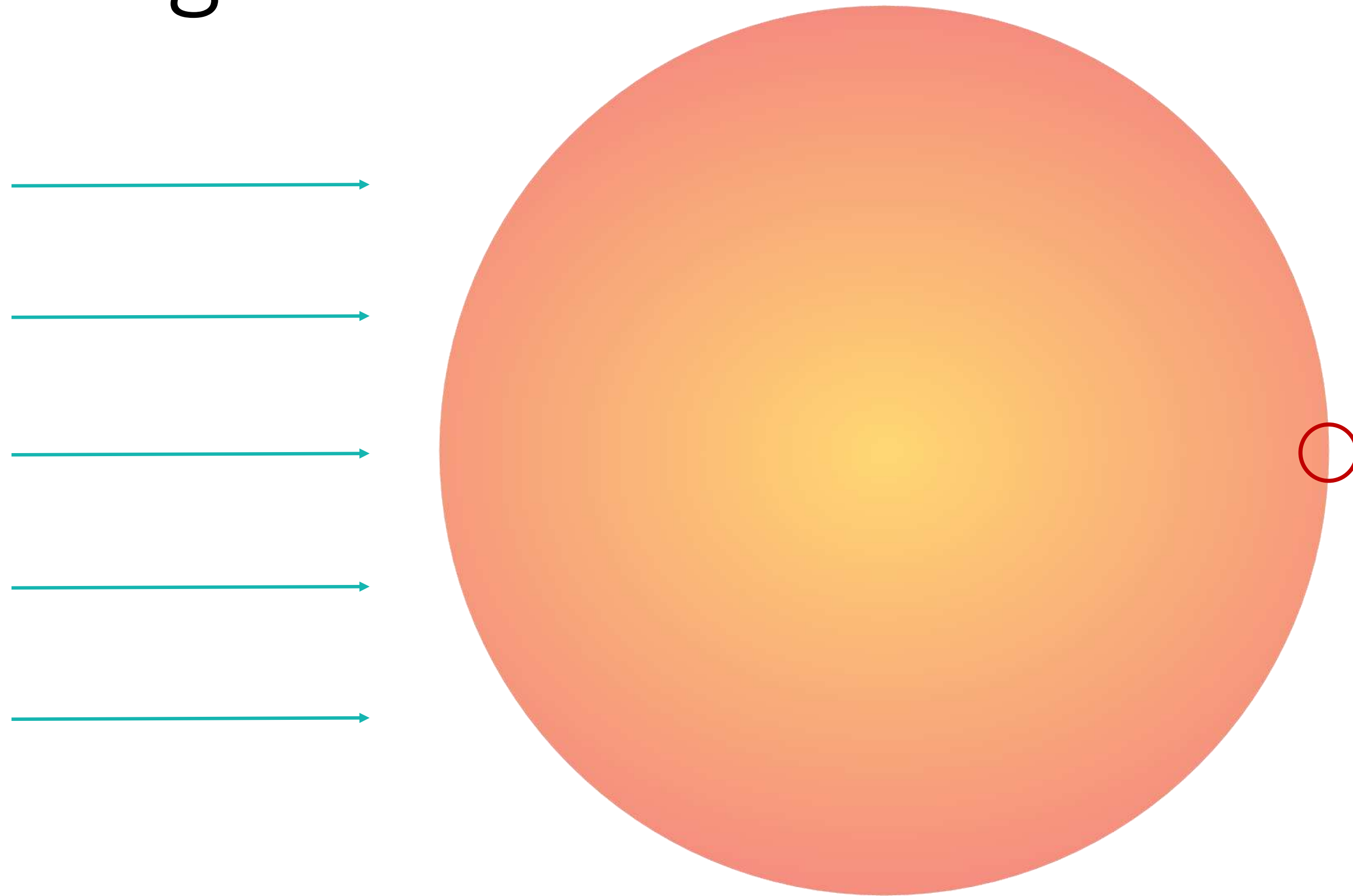


# Luneburg Lens

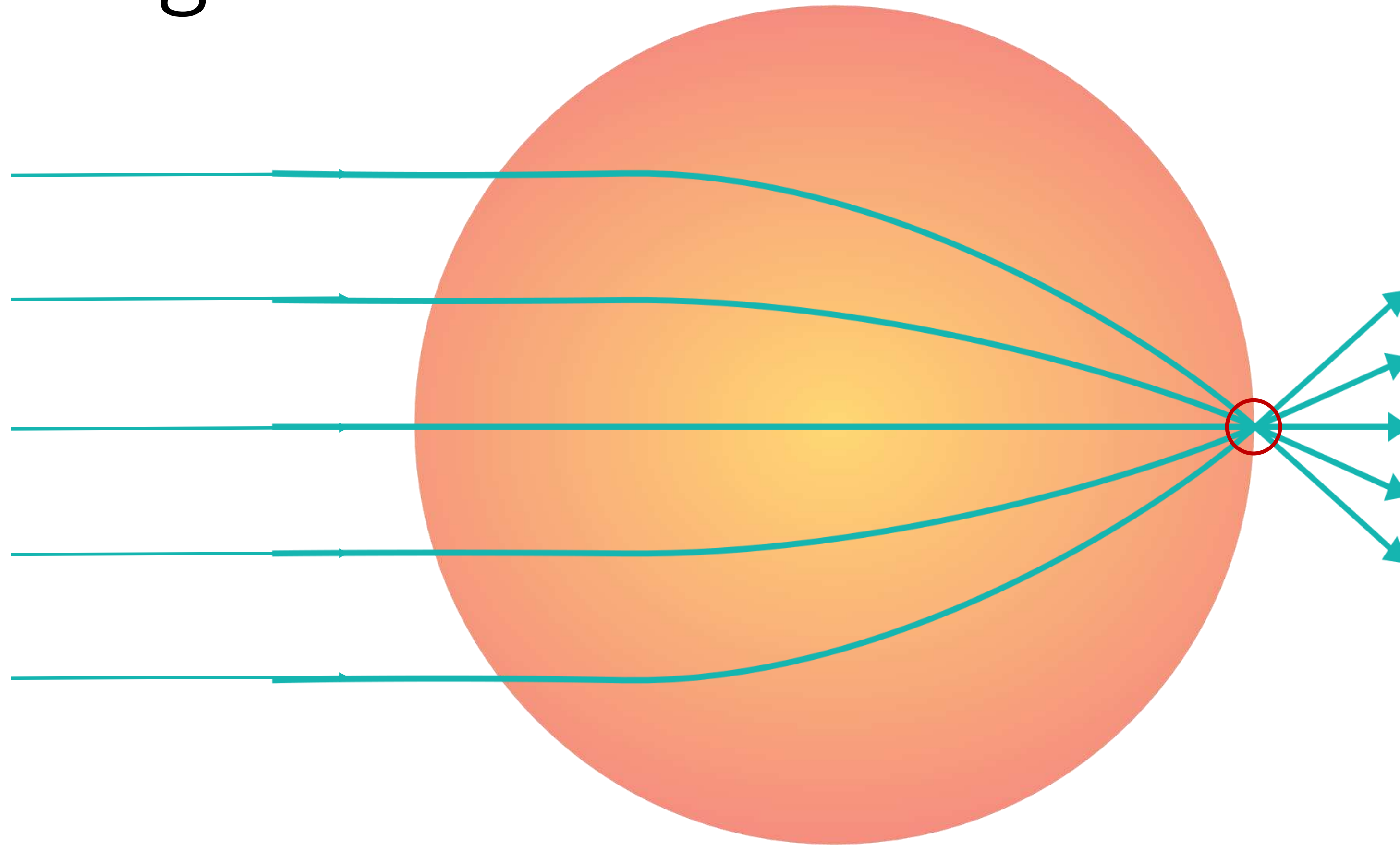




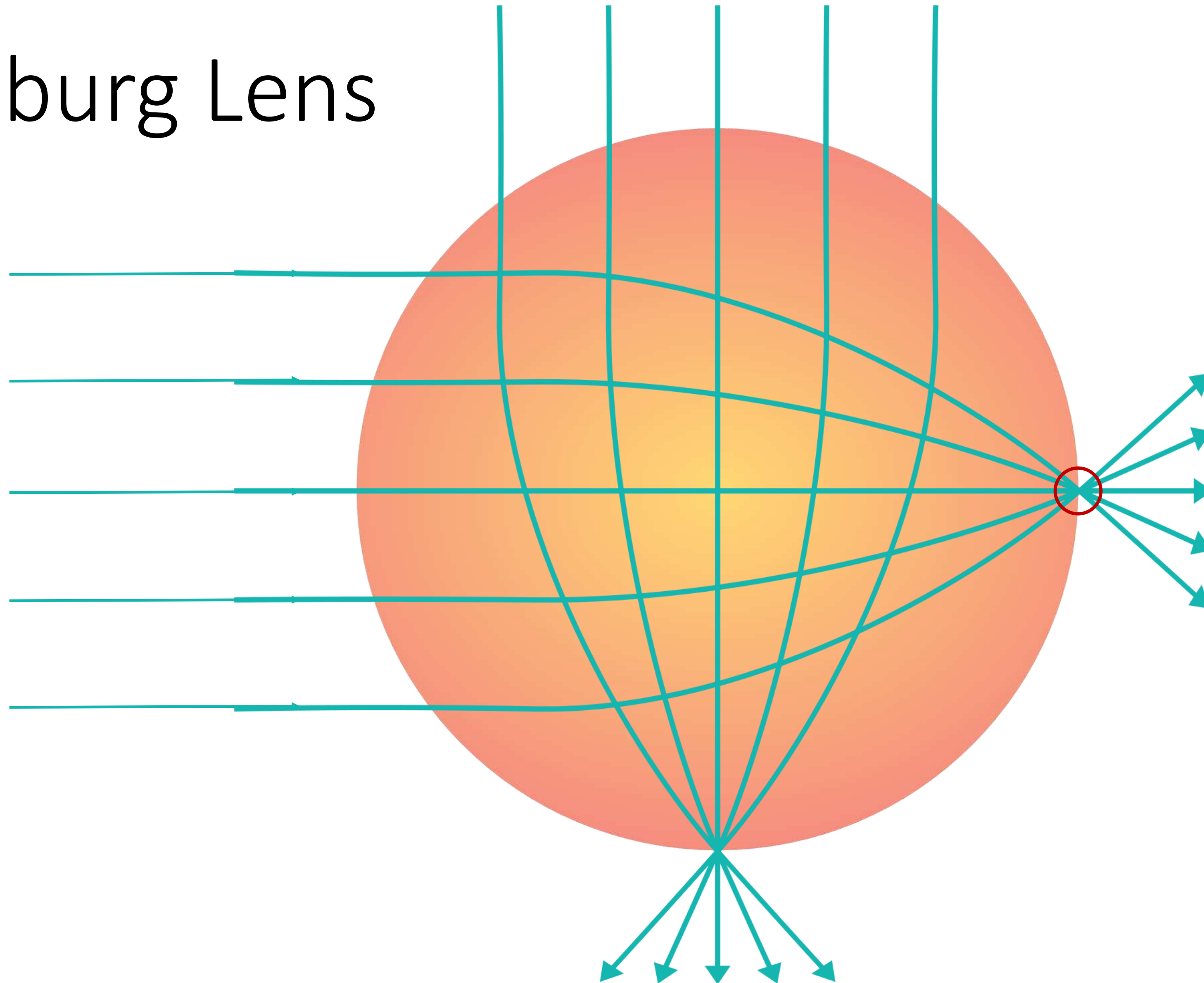
# Luneburg Lens



# Luneburg Lens

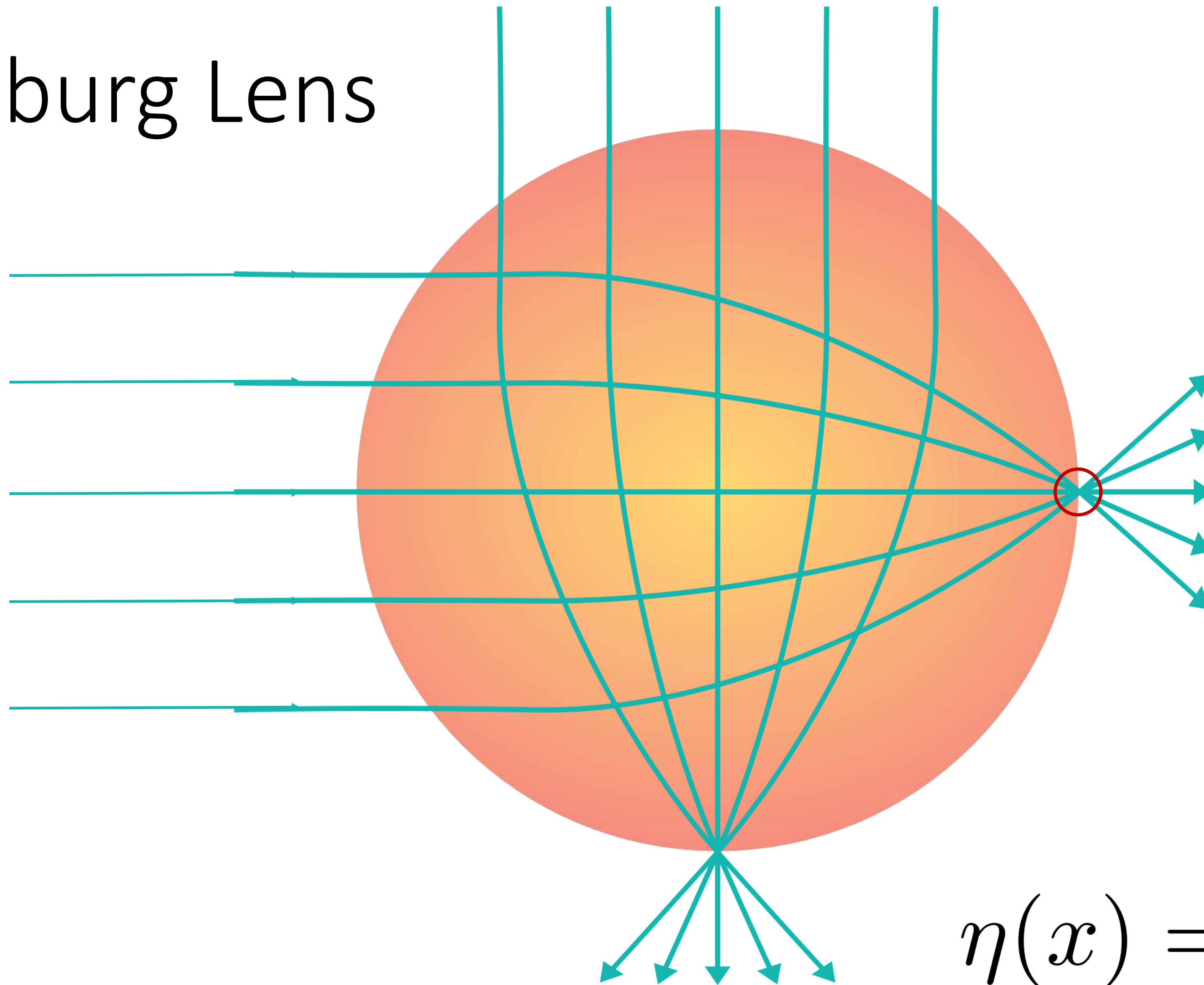


# Luneburg Lens





# Luneburg Lens



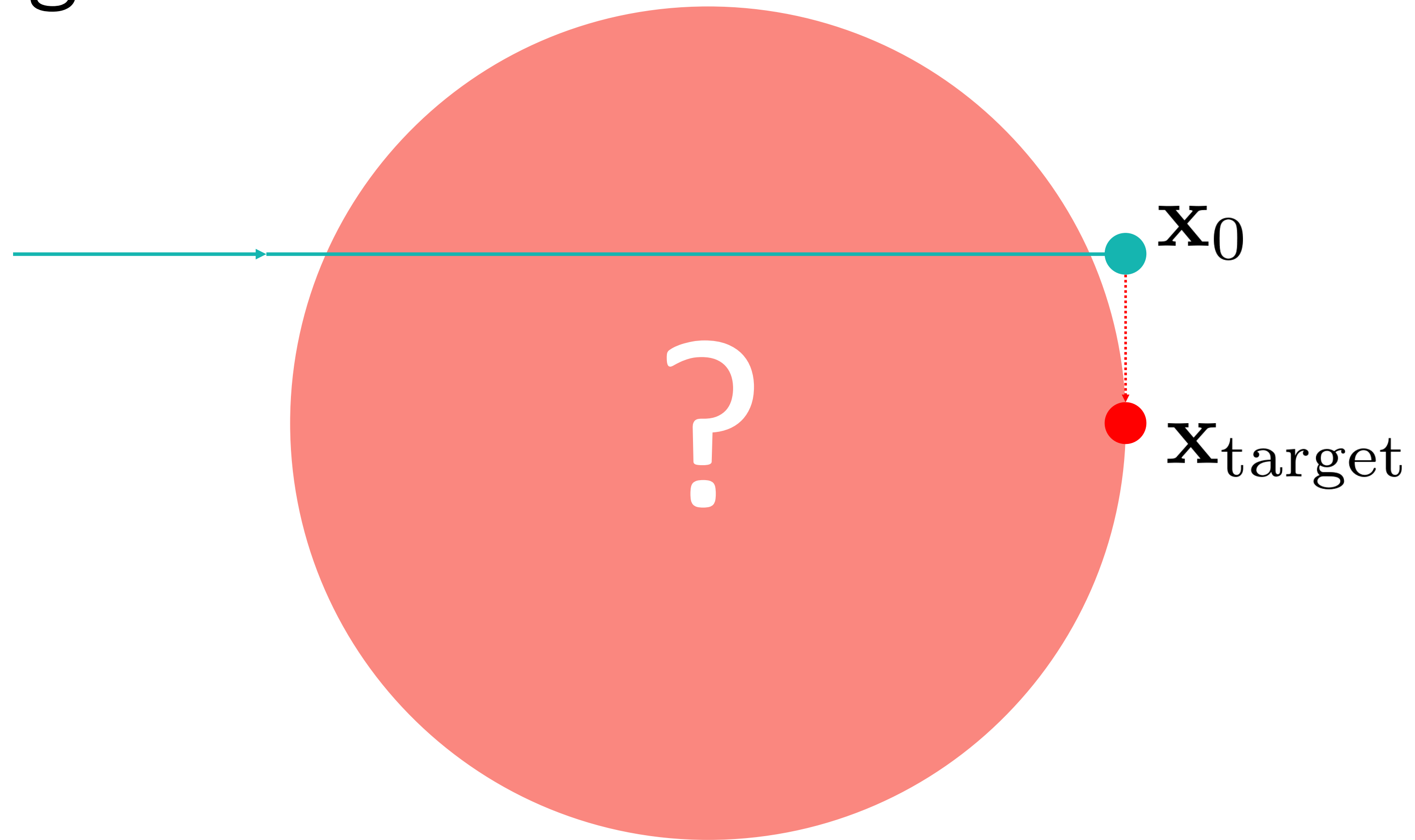
$$\eta(x) = \sqrt{2 - \|x\|^2}$$

[Luneburg, R. K. 1944]

# Luneburg Lens

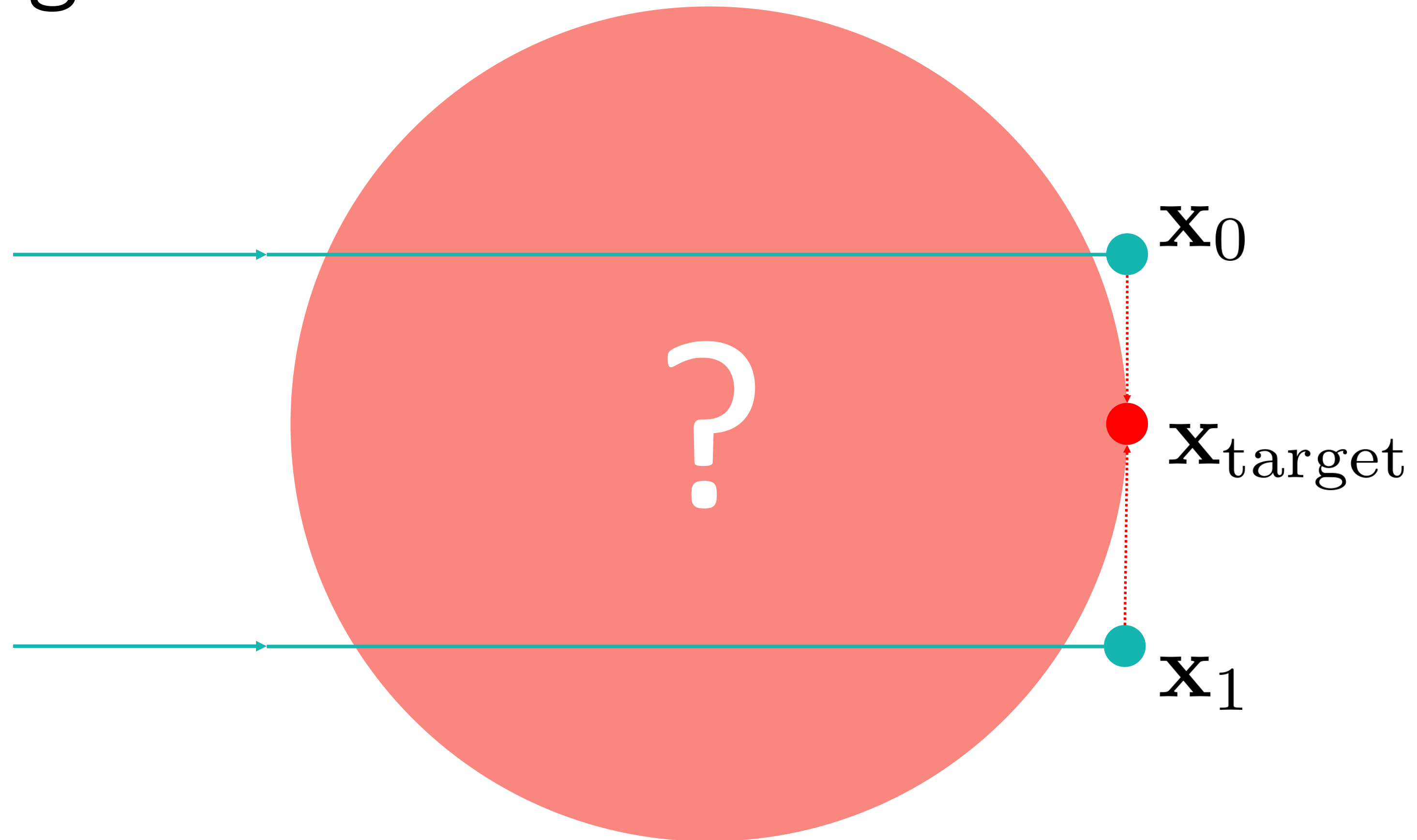


# Luneburg Lens



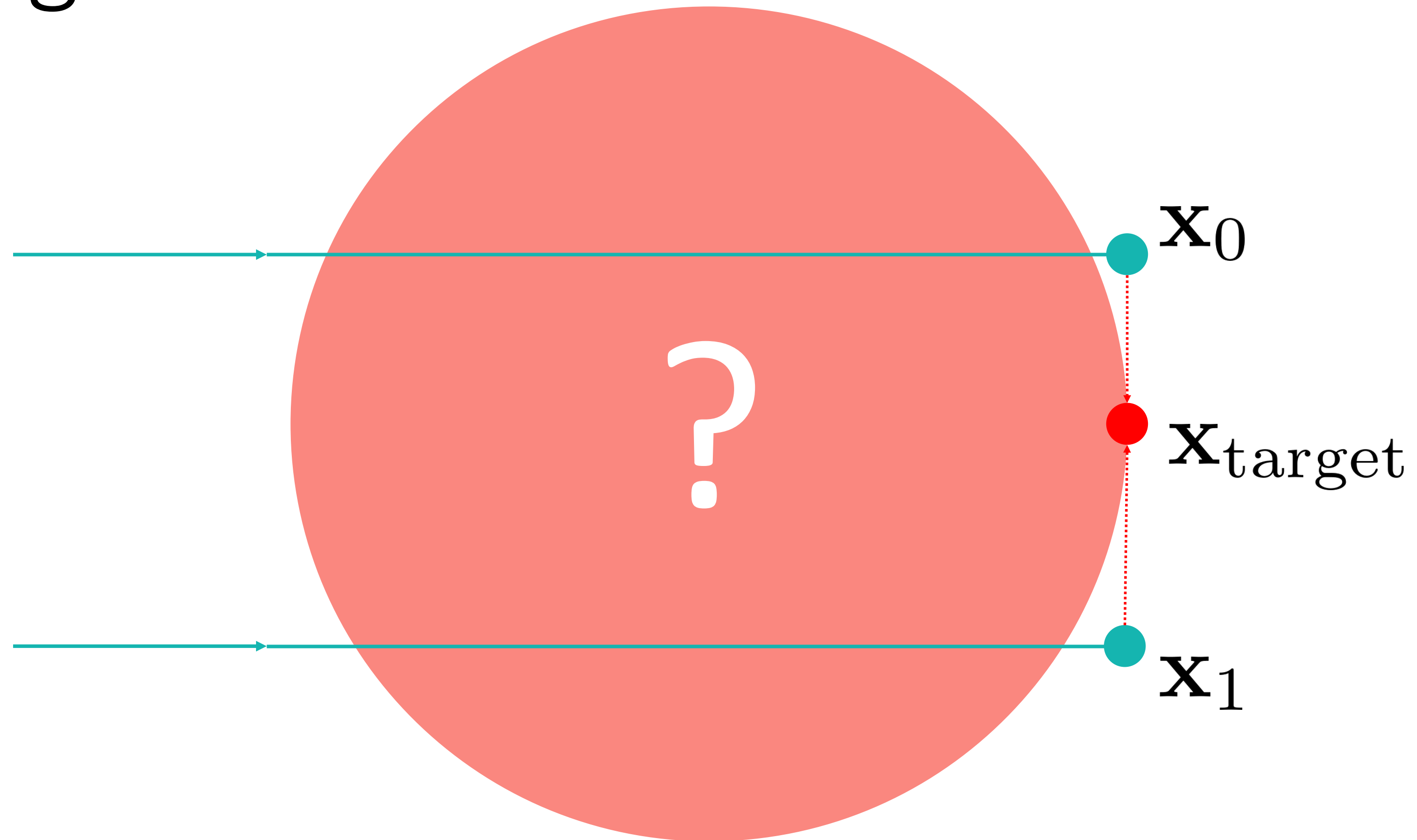


# Luneburg Lens



$$\min_{\eta} \sum_{i \in \text{rays}} \|\mathbf{x}_{\text{target}} - \mathbf{x}_i\|^2$$

# Luneburg Lens



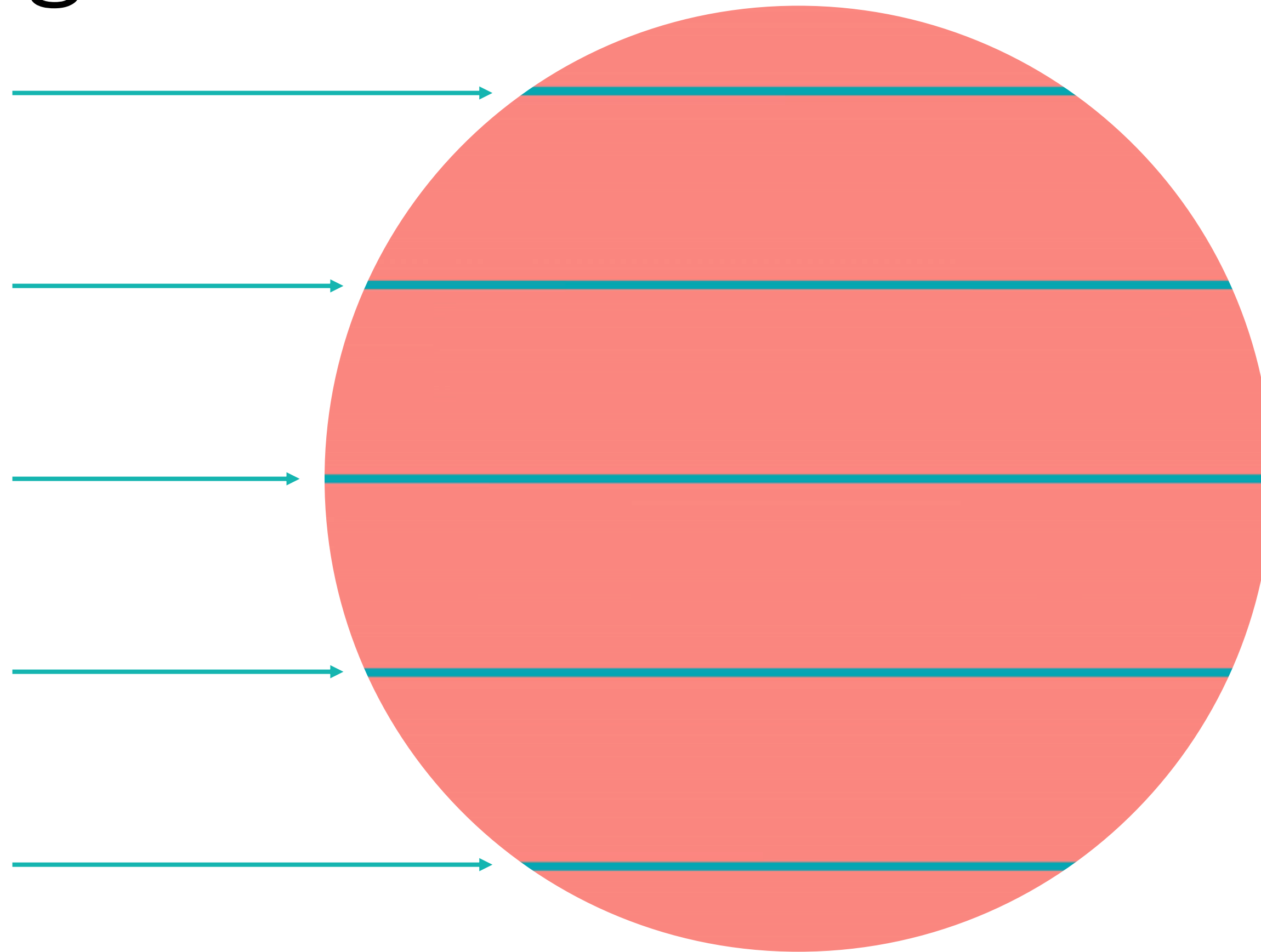
$$\min_{\eta} \sum_{i \in \text{rays}} \|\mathbf{x}_{\text{target}} - \mathbf{x}_i\|^2$$

# Luneburg Lens

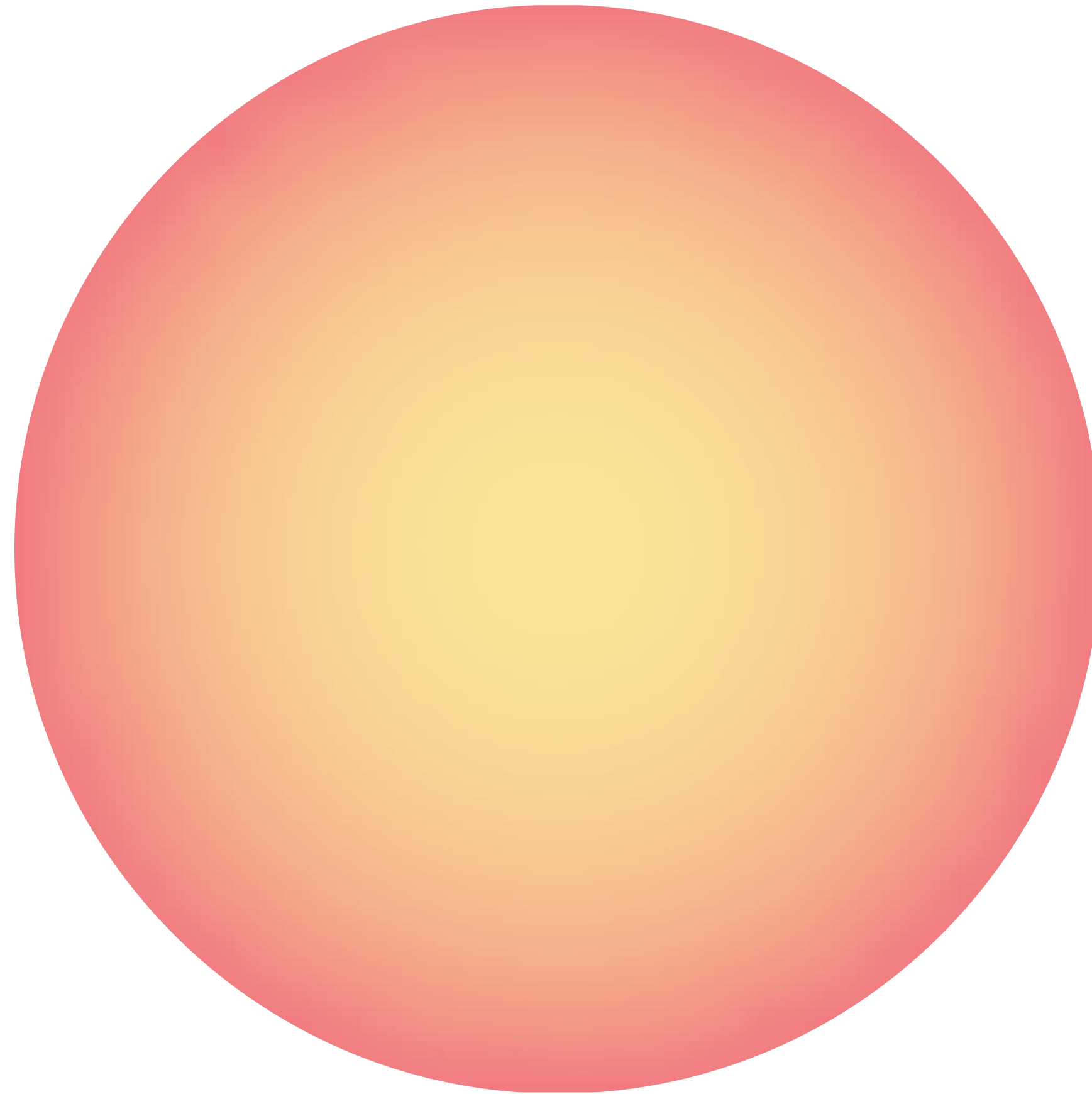




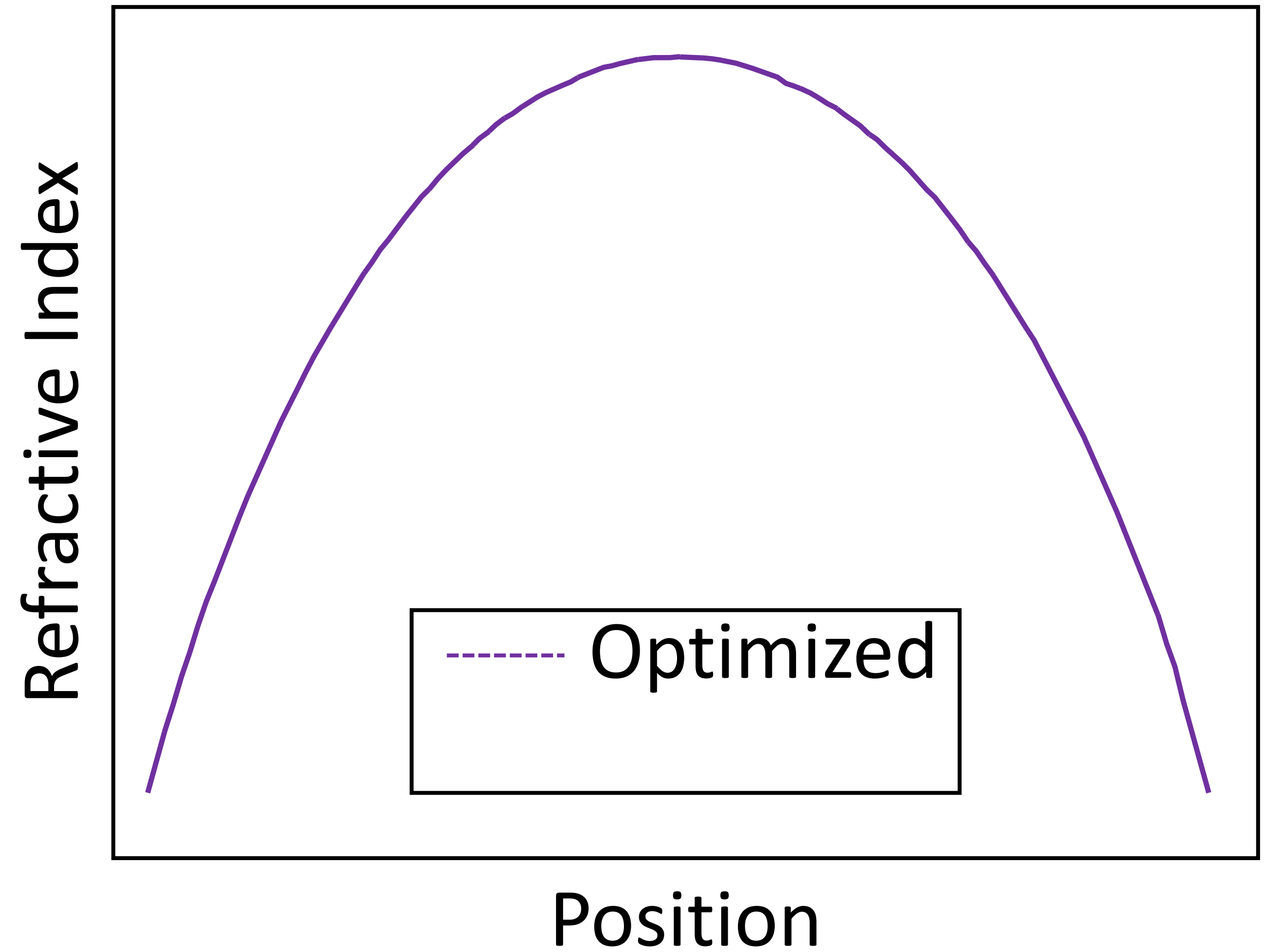
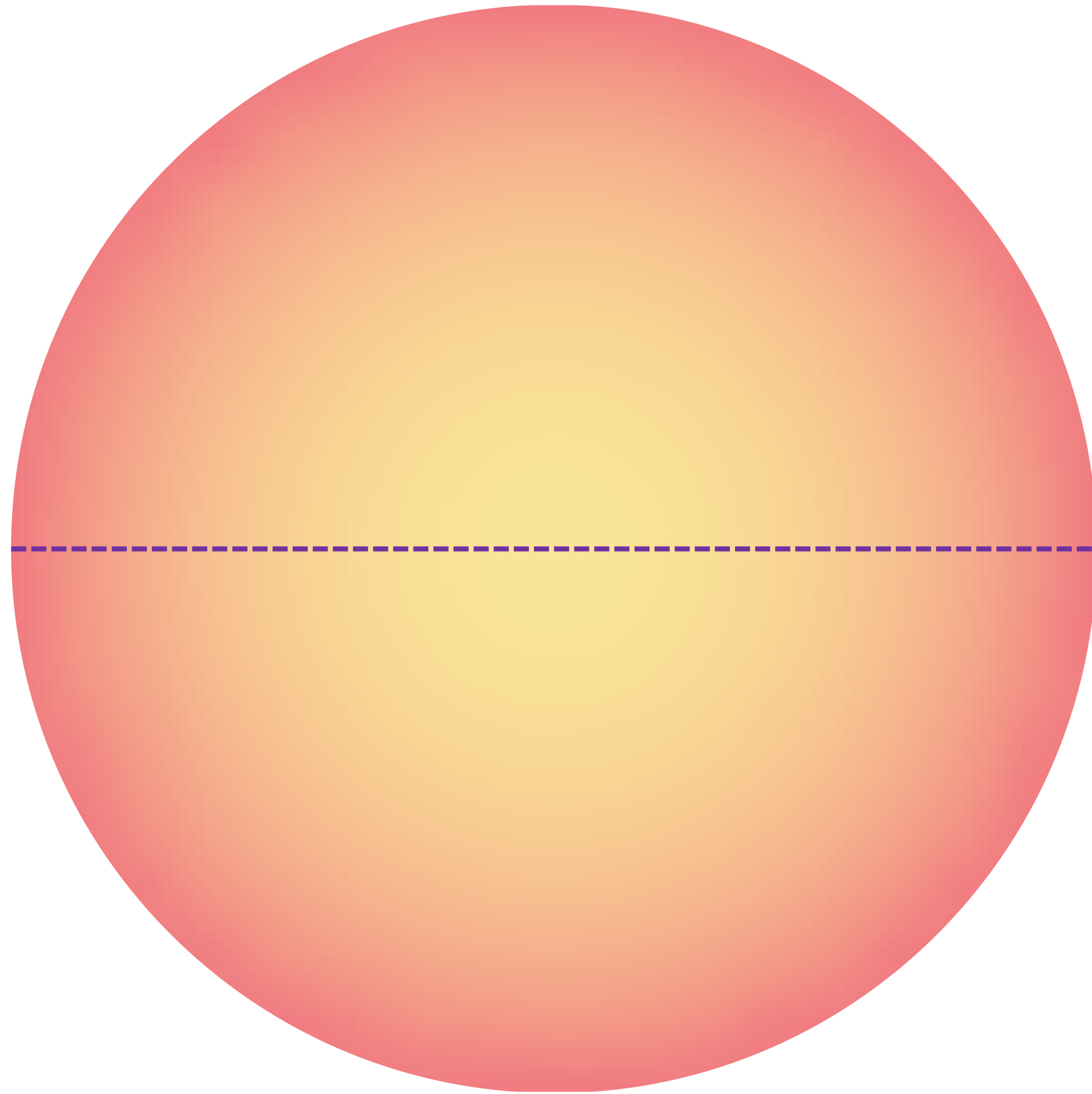
# Luneburg Lens



# Luneburg Lens

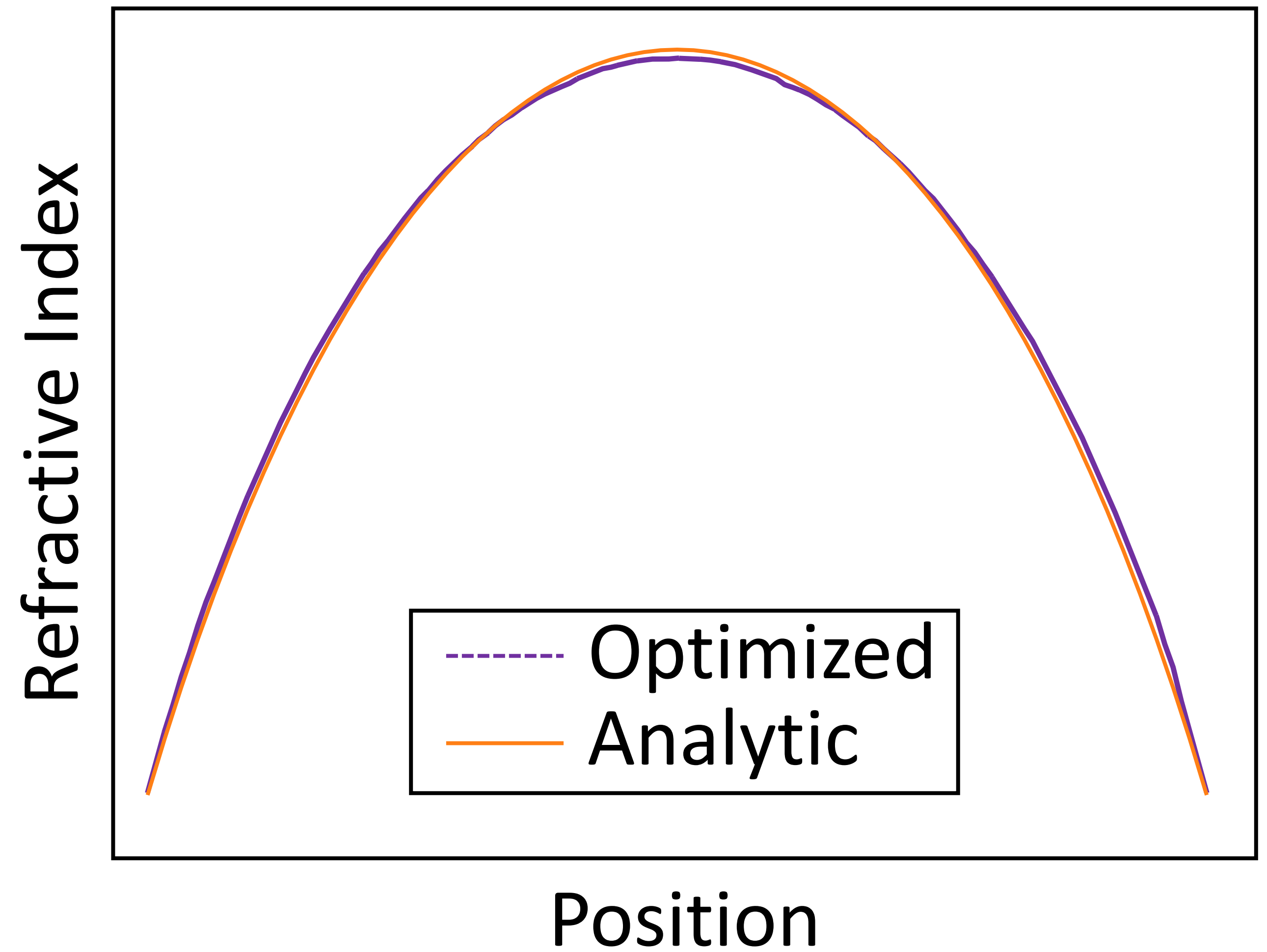
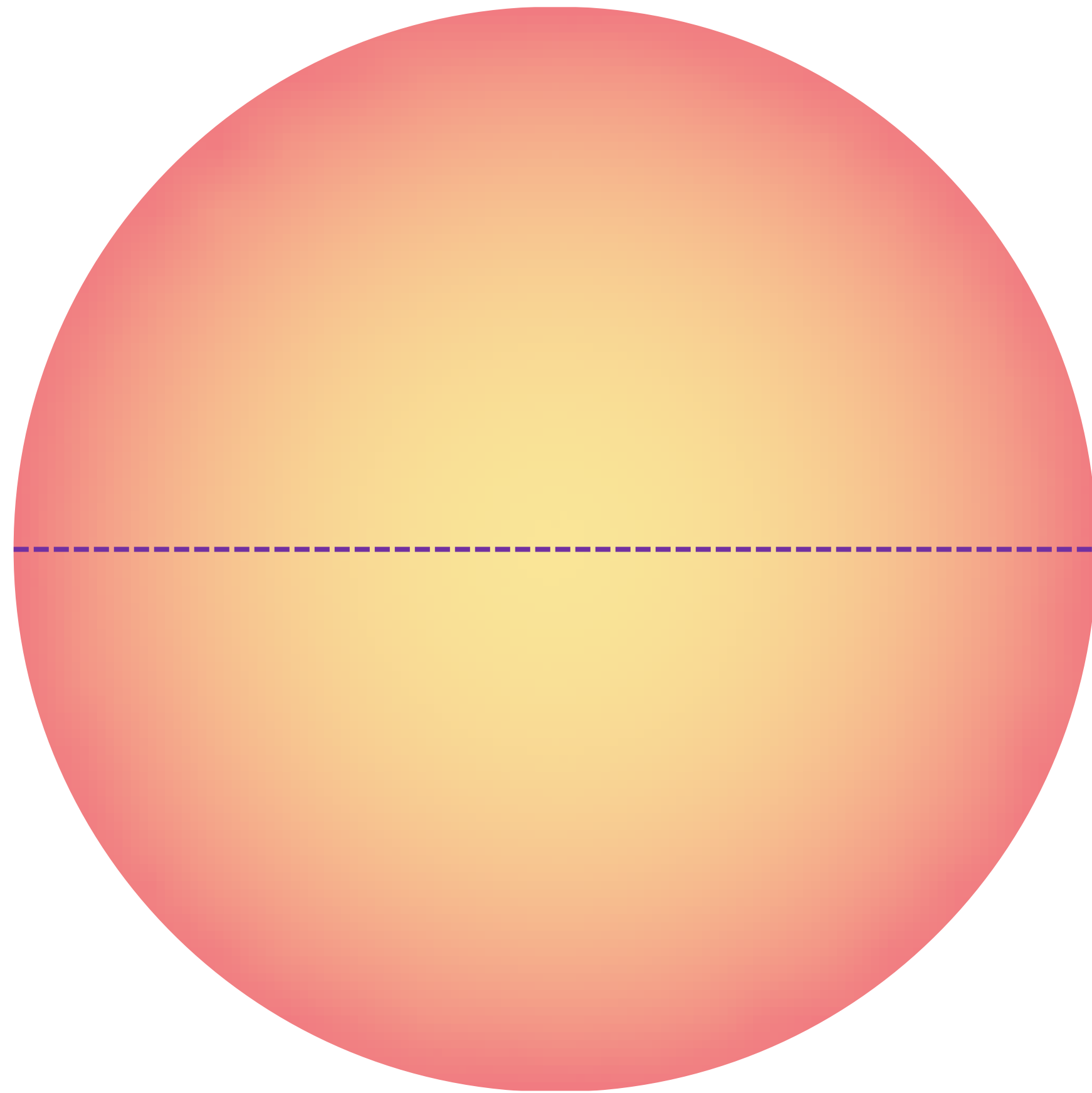


# Luneburg Lens

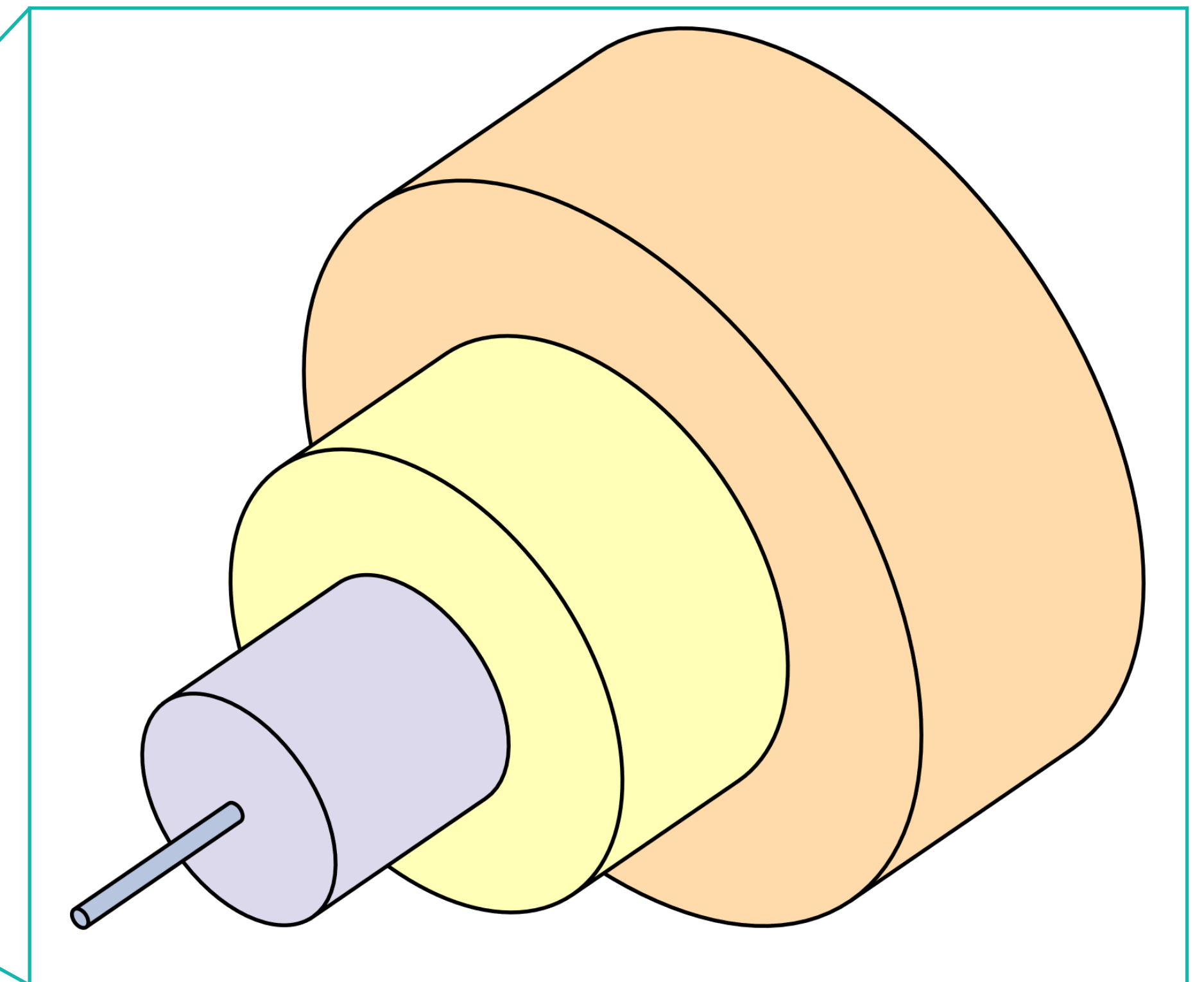
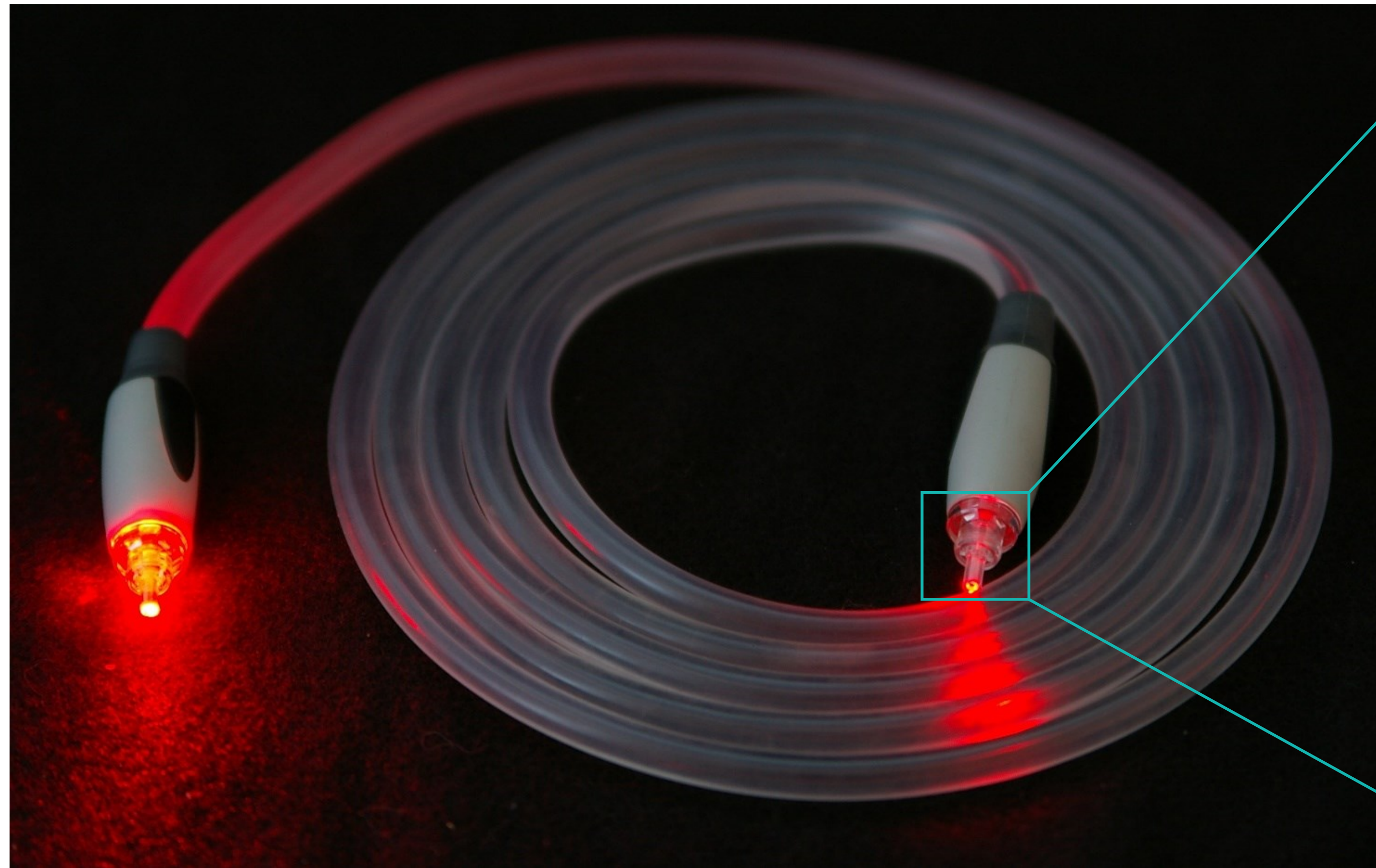




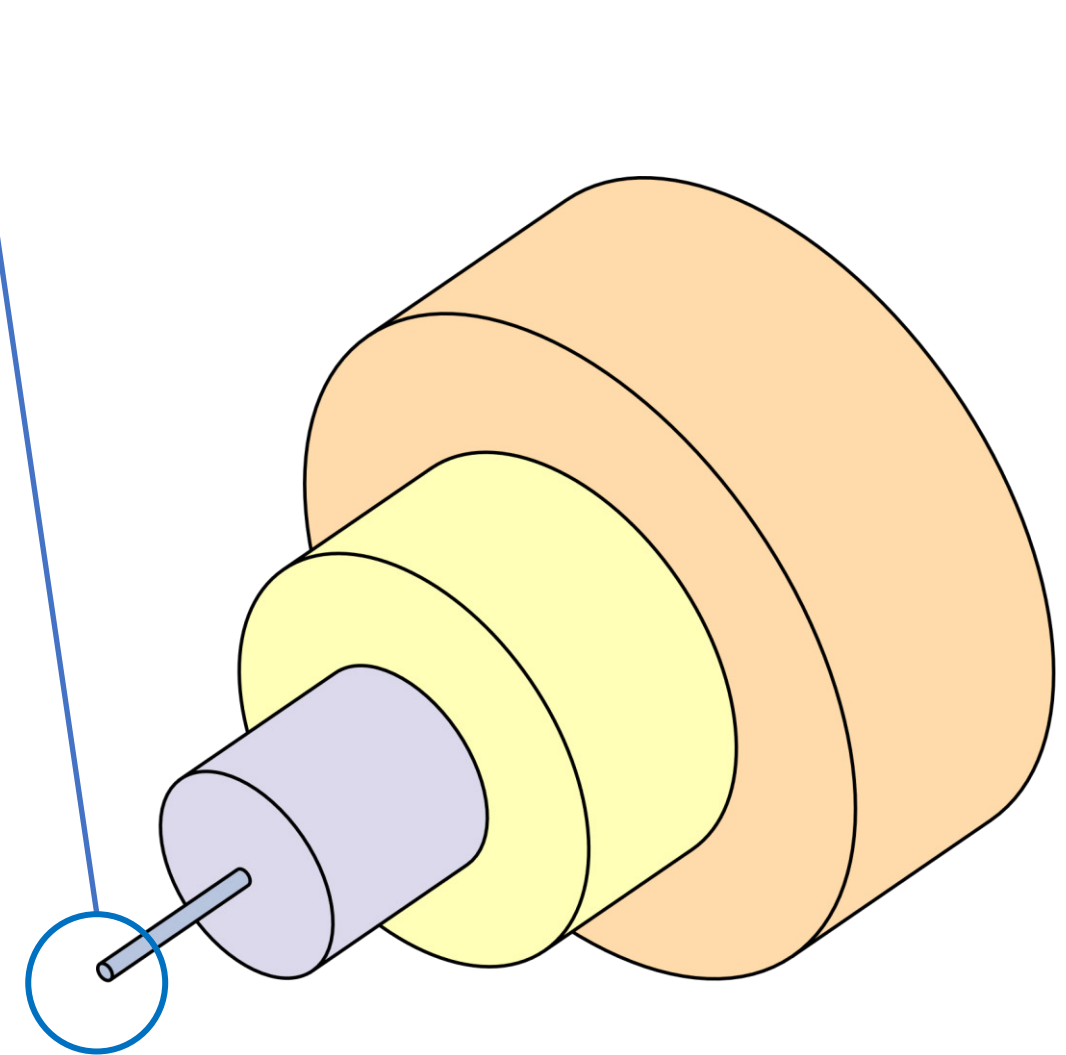
# Luneburg Lens



# GRIN Fiber

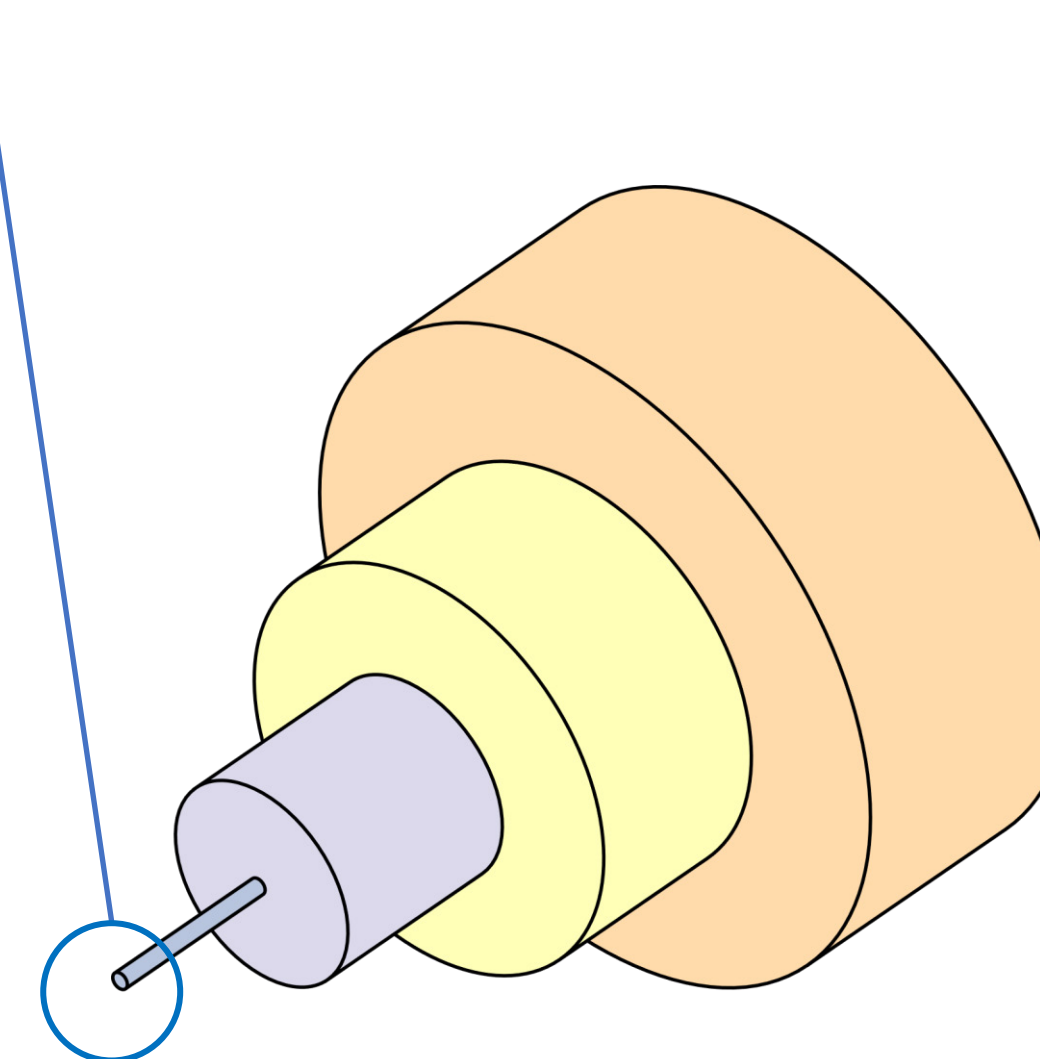
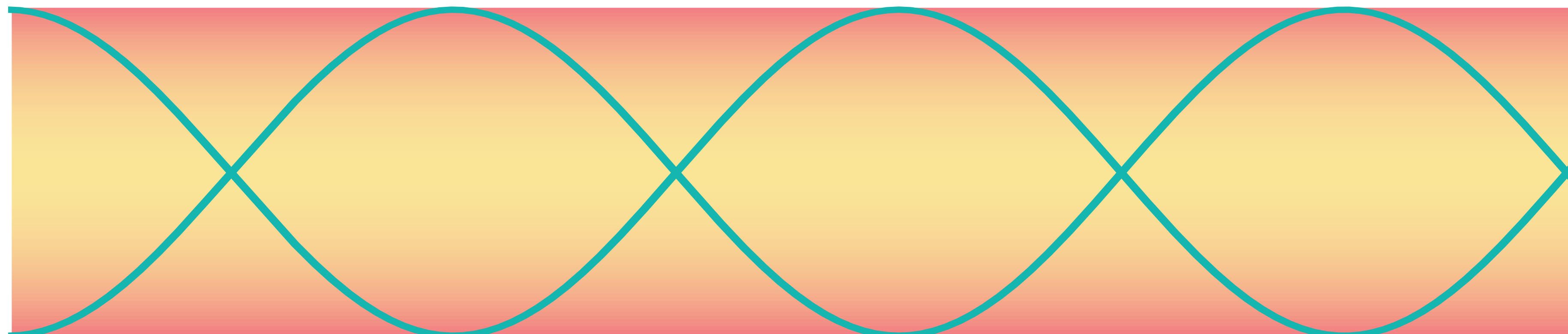


# GRIN Fiber

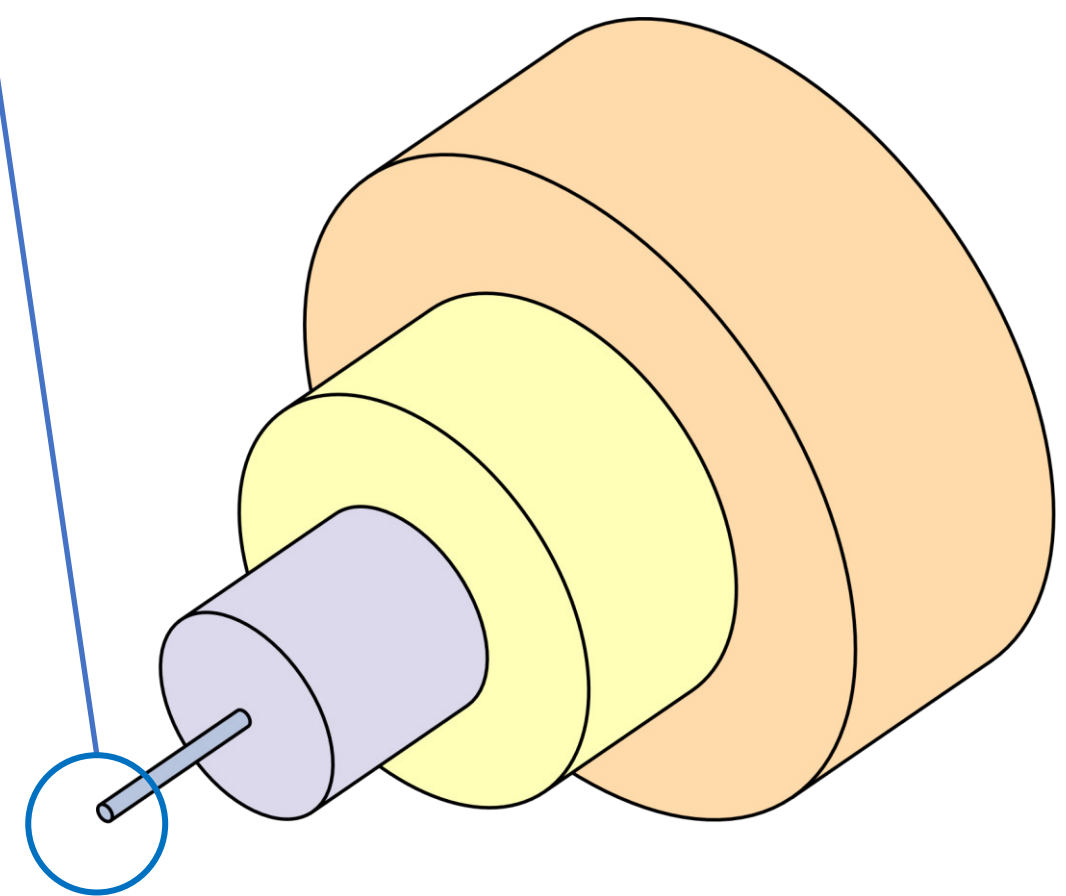
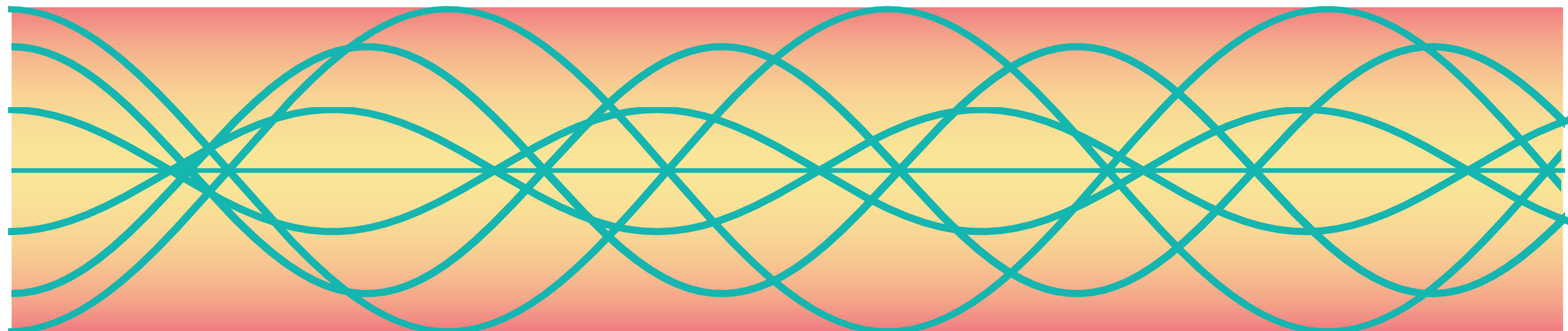




# GRIN Fiber

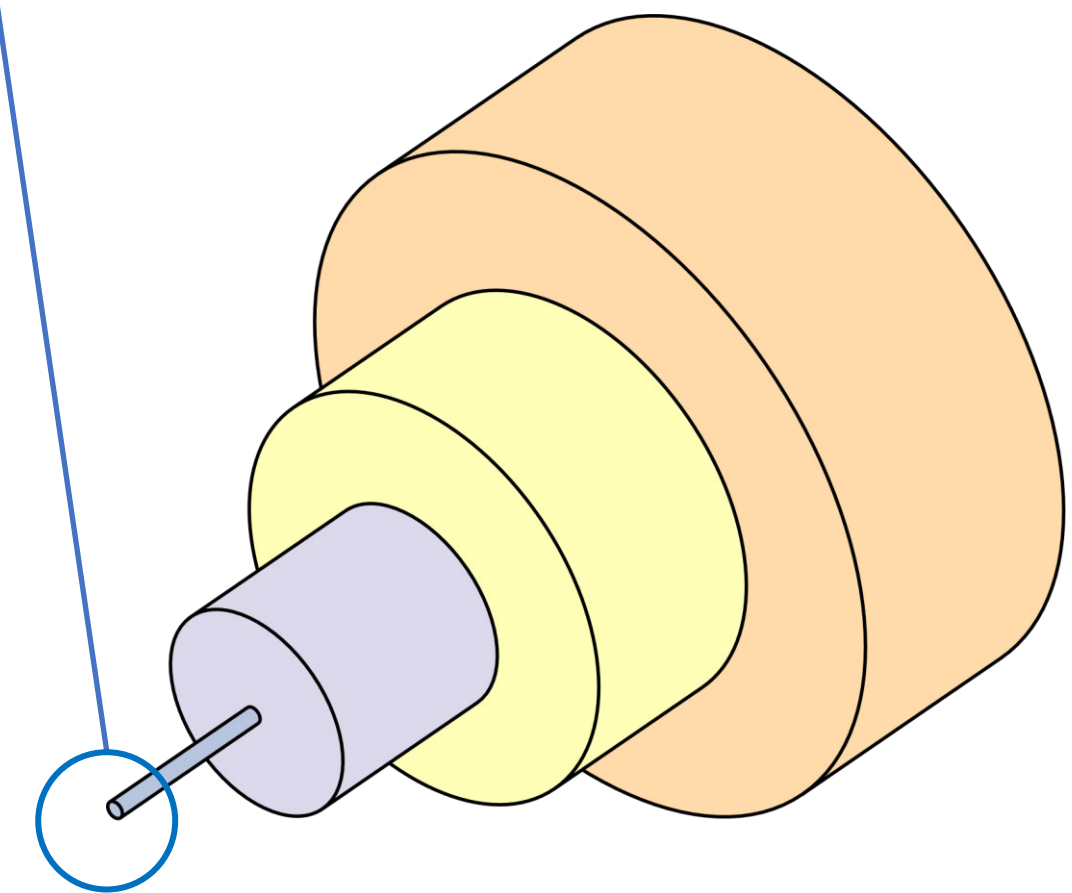
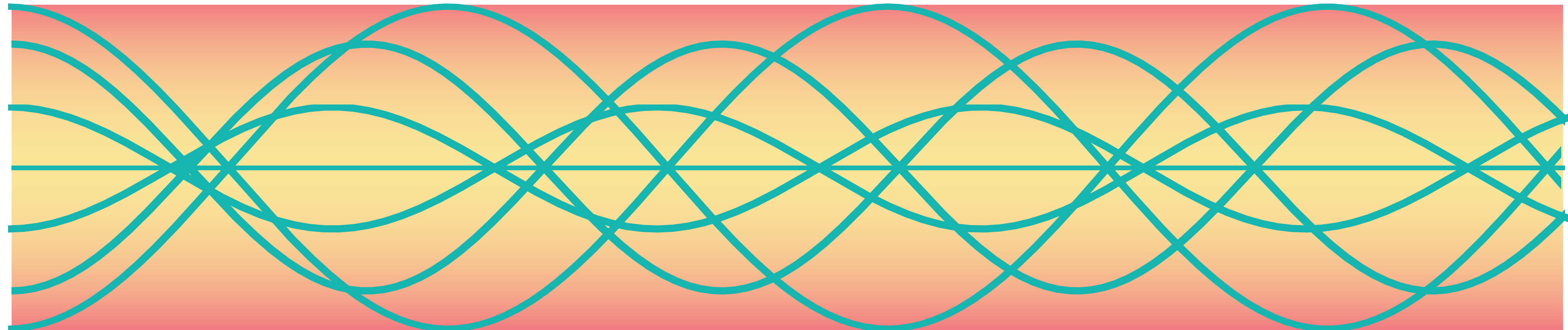


# GRIN Fiber



# GRIN Fiber

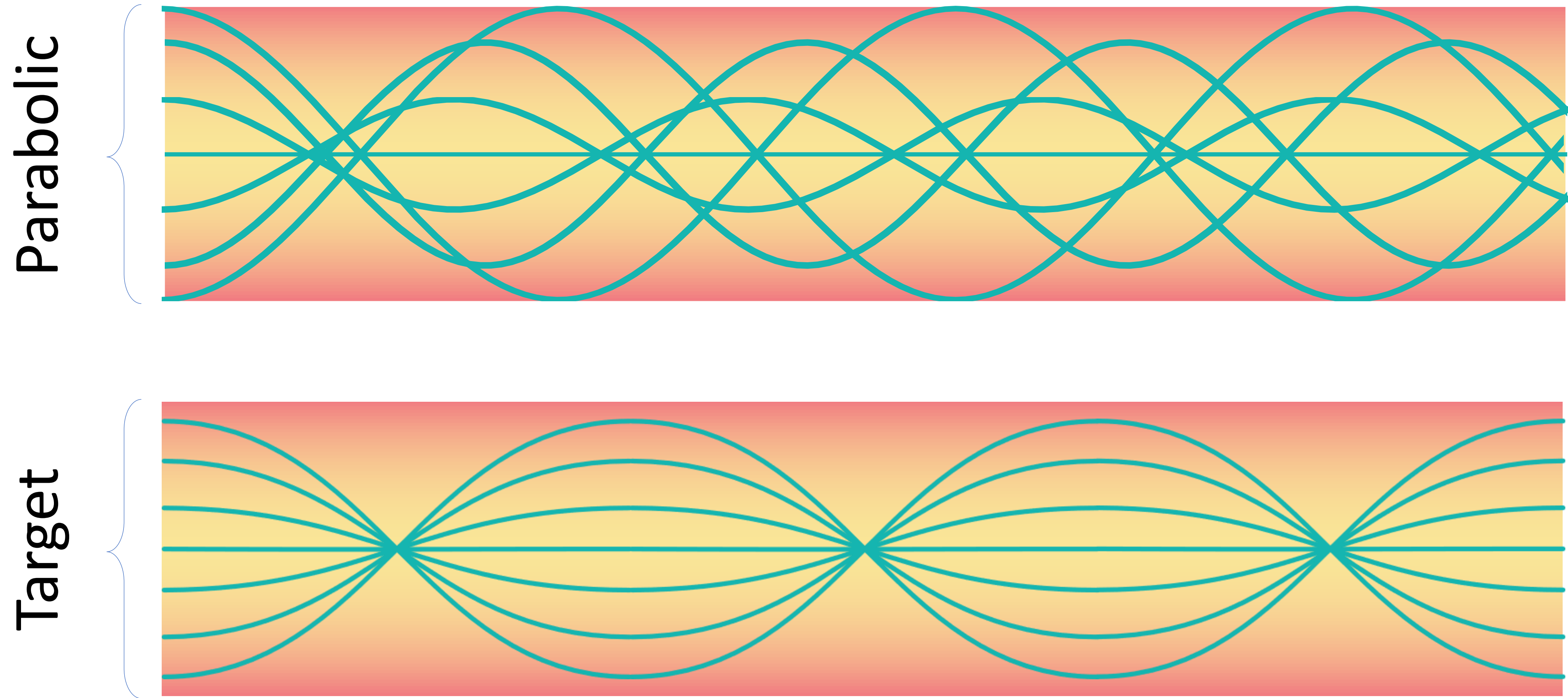
Modal dispersion



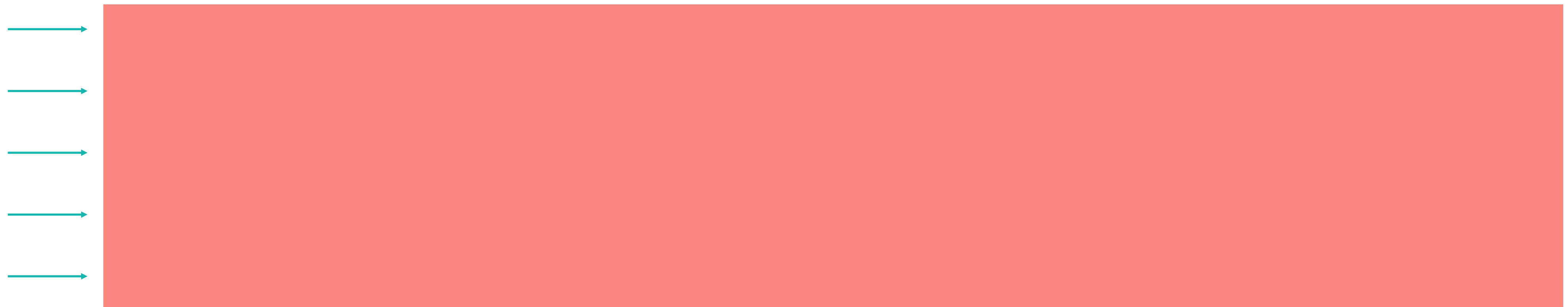
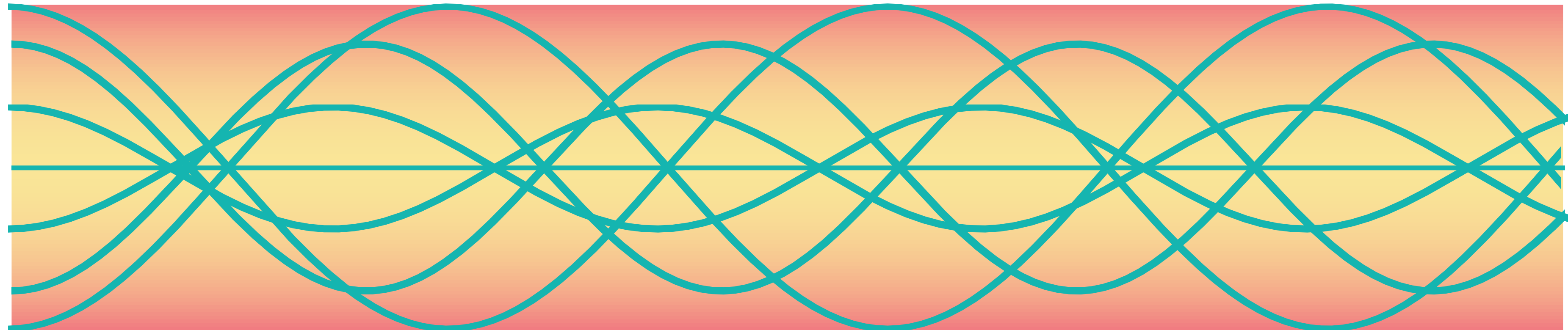


# GRIN Fiber

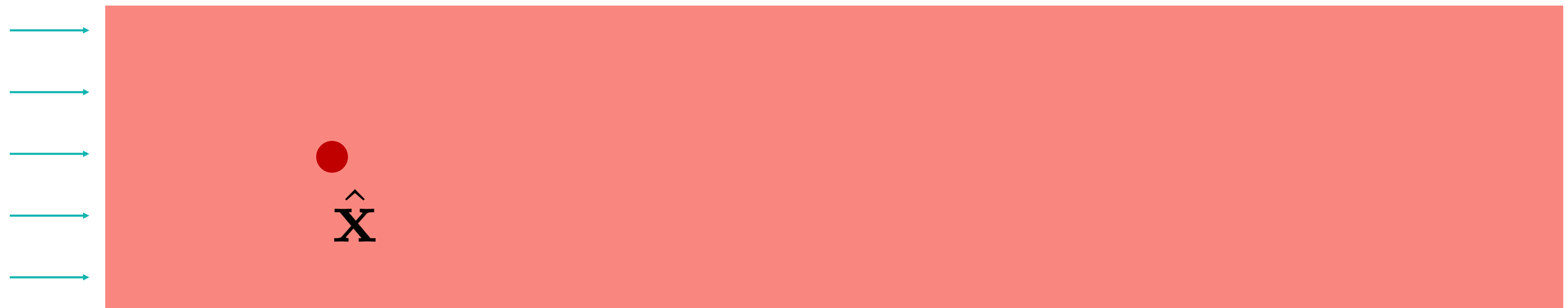
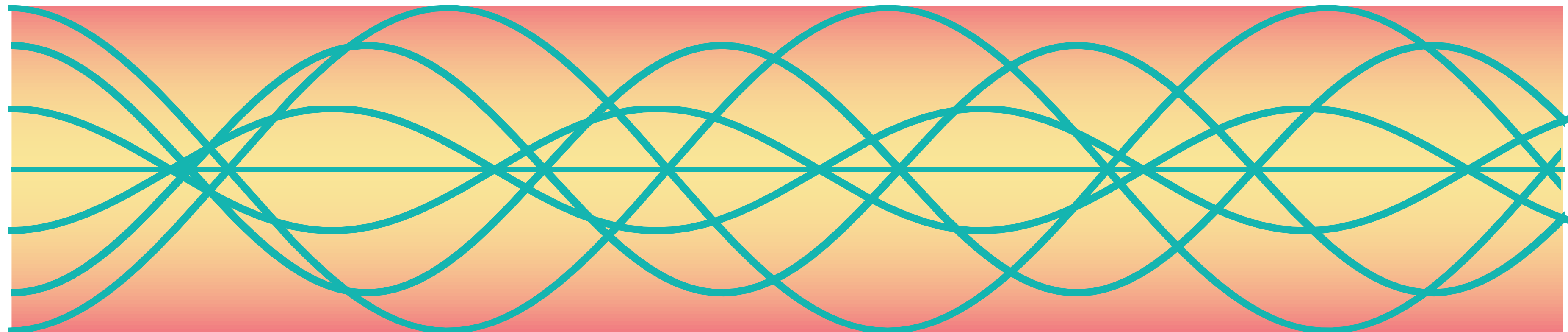
Modal dispersion



# GRIN Fiber

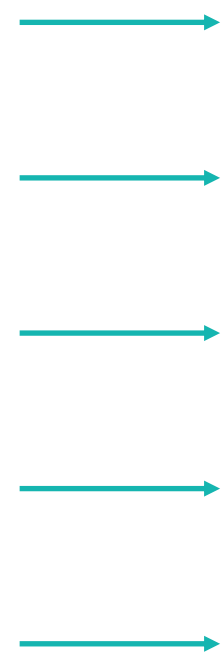
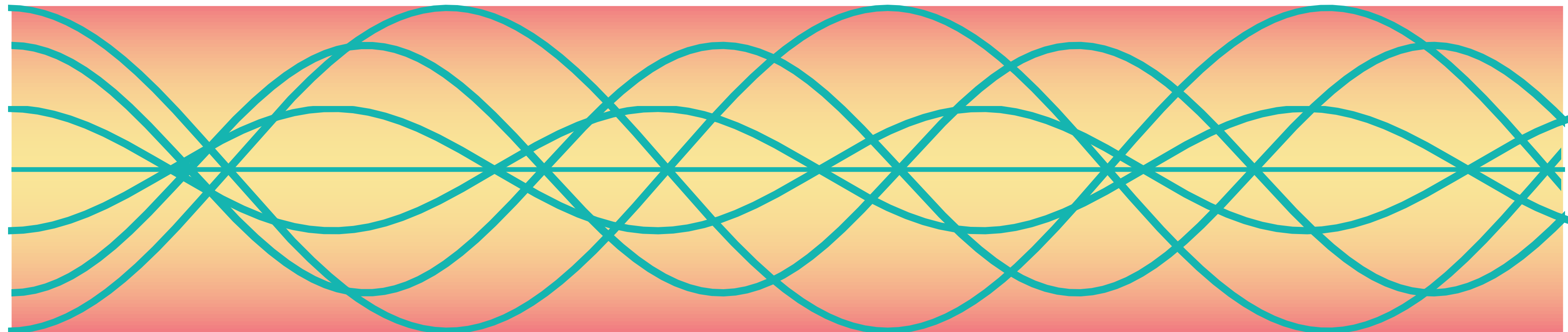


# GRIN Fiber



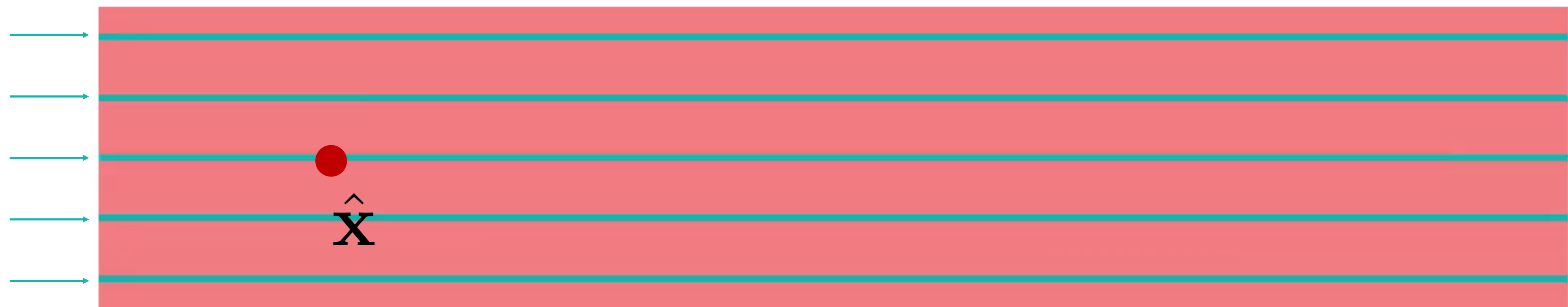
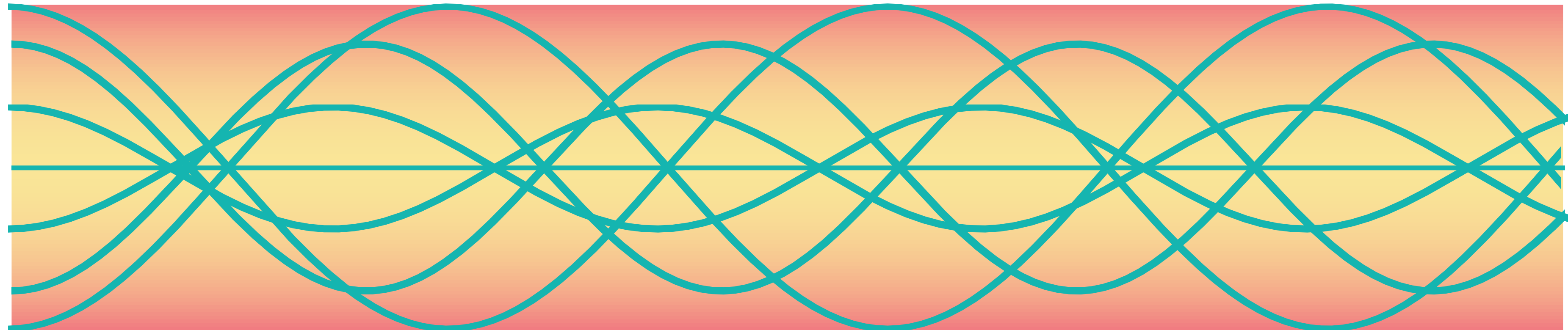


# GRIN Fiber

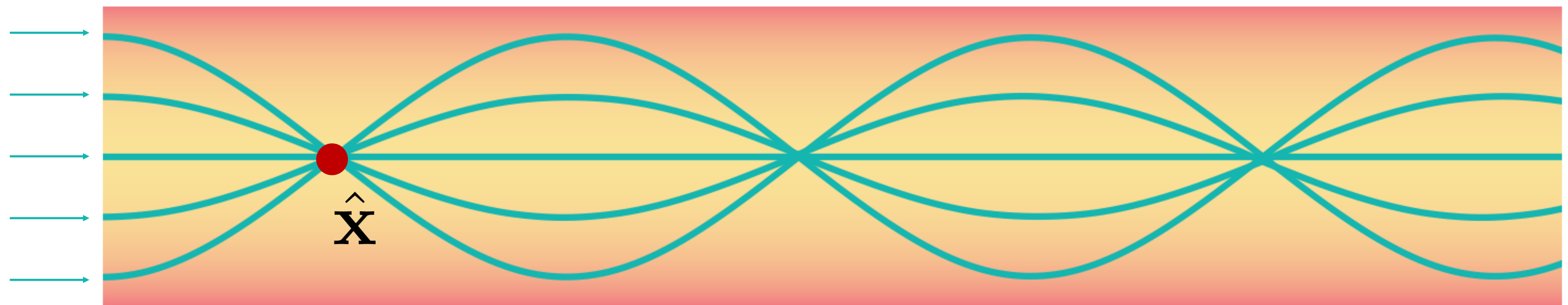
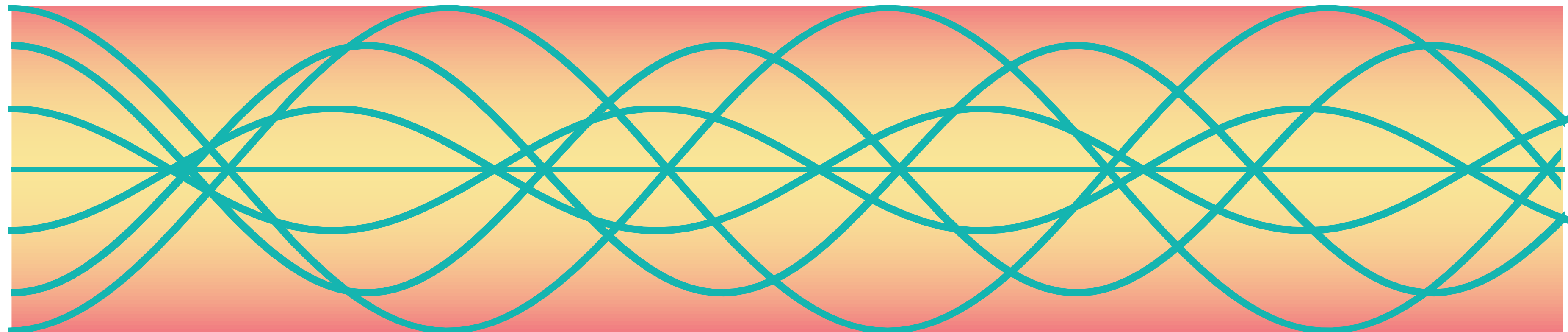


$\hat{\mathbf{x}}$

# GRIN Fiber

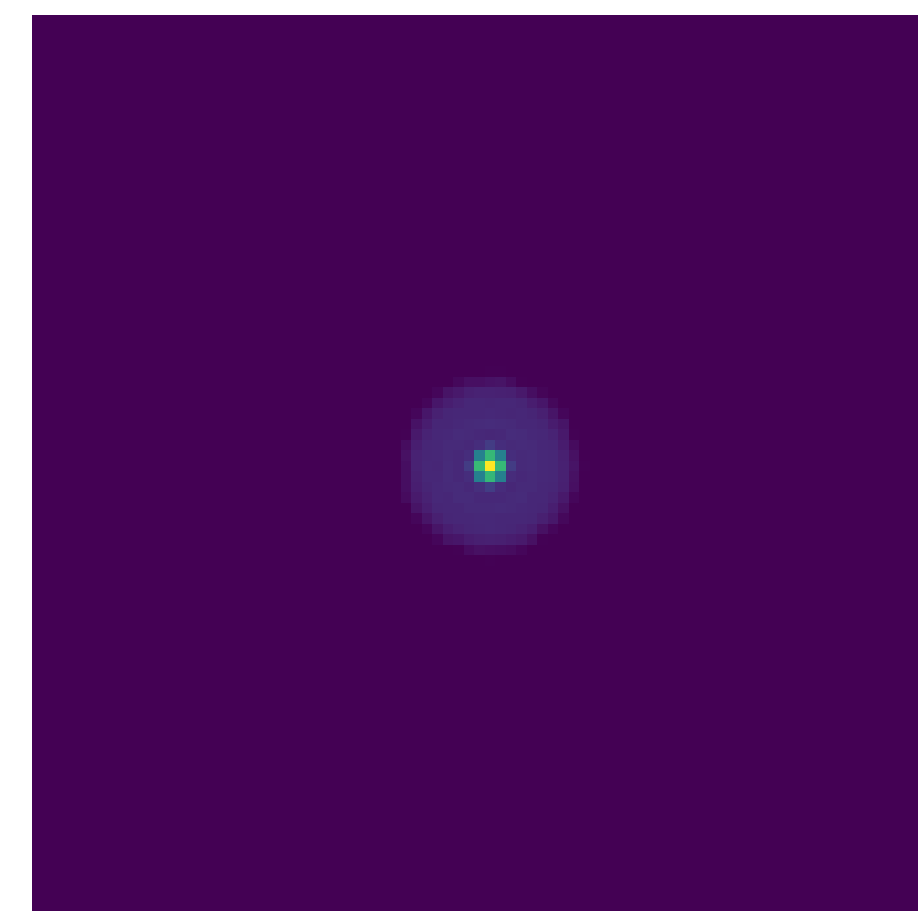
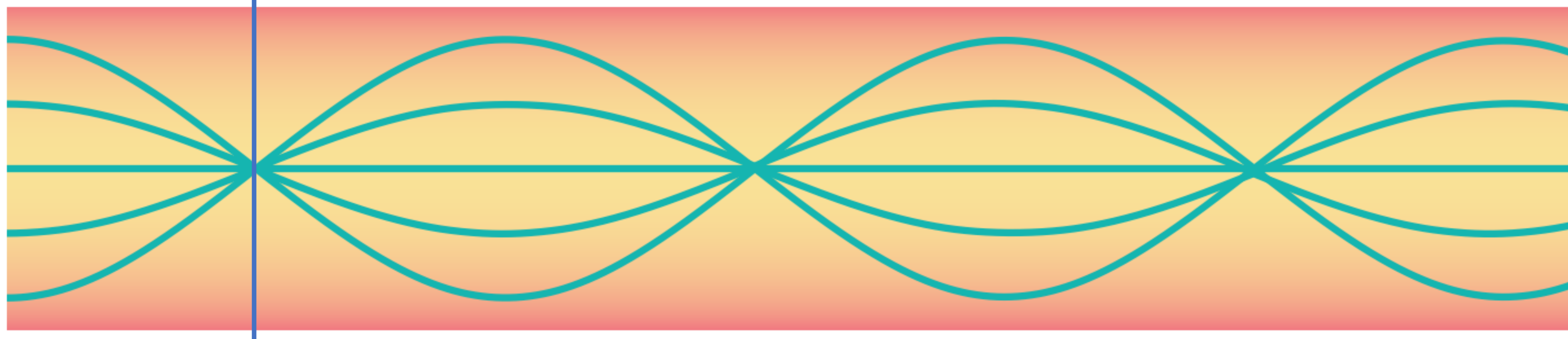
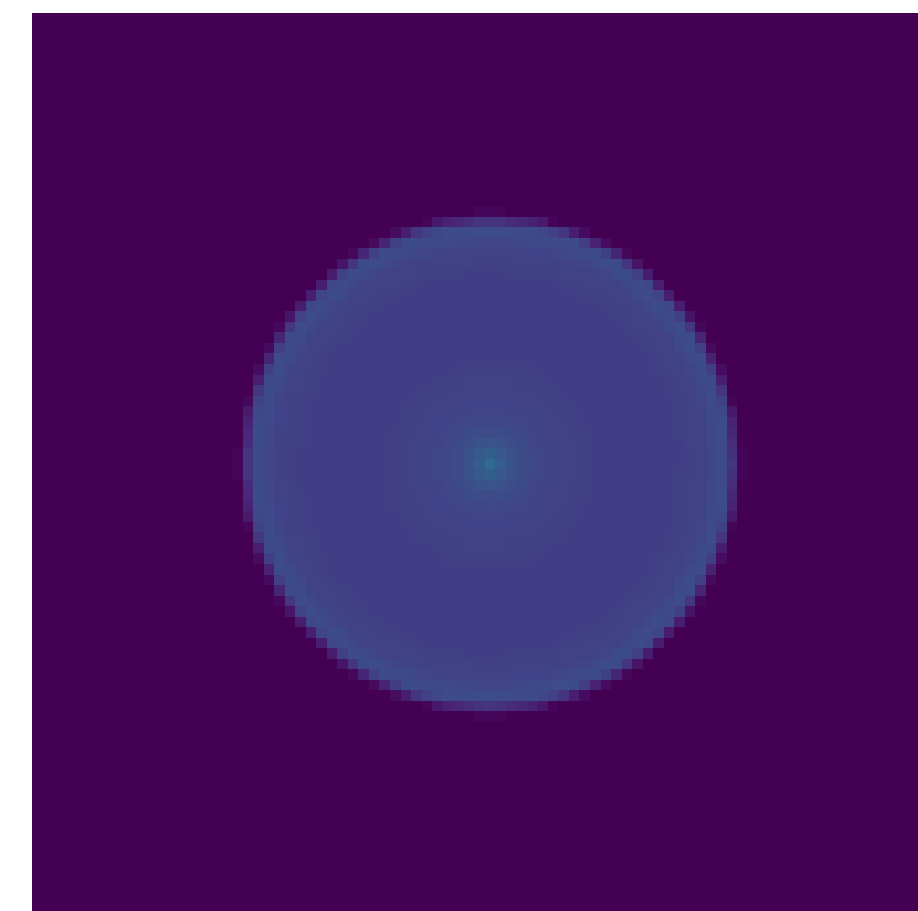
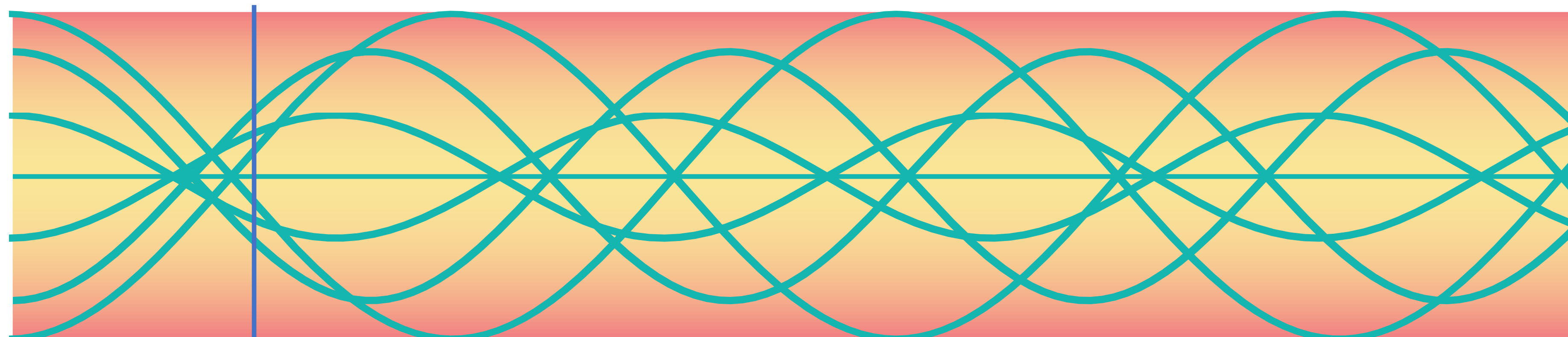


# GRIN Fiber

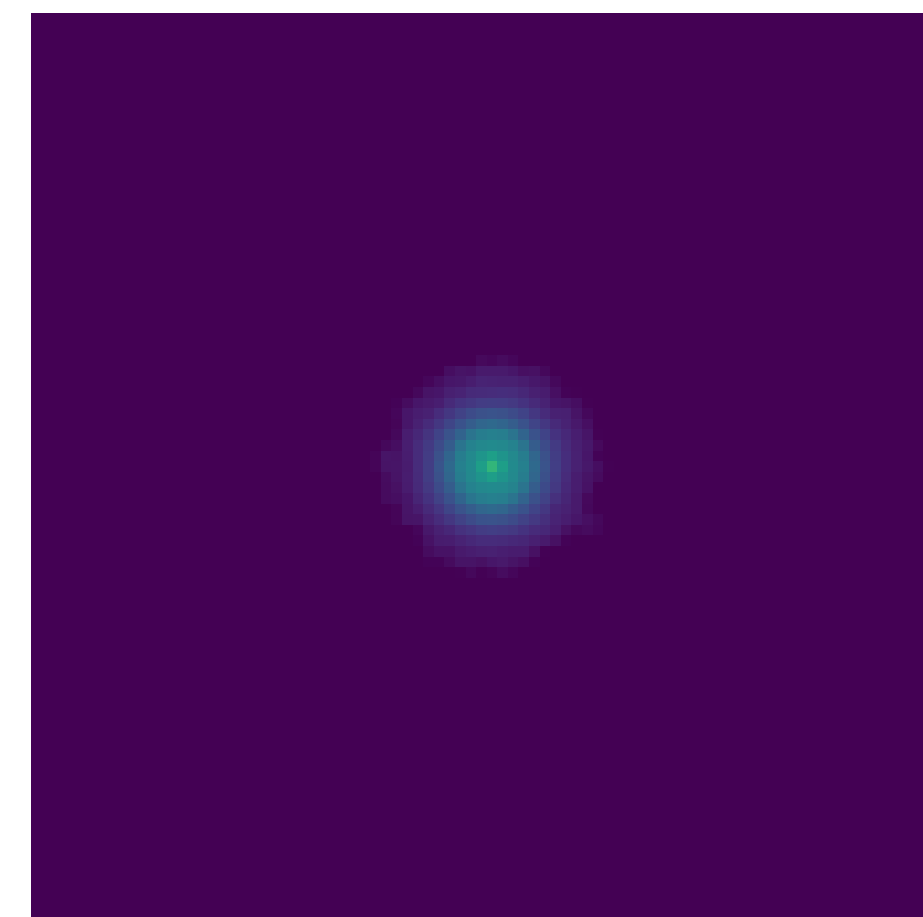
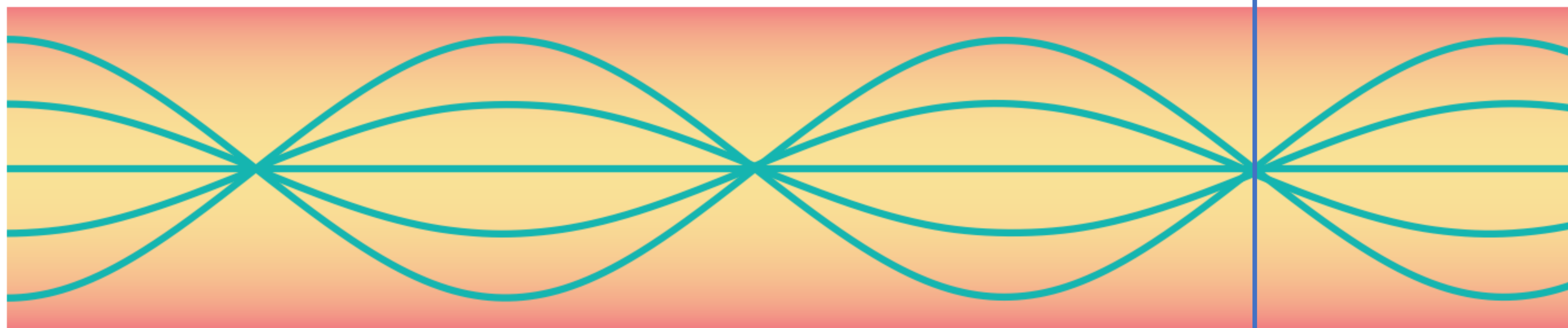
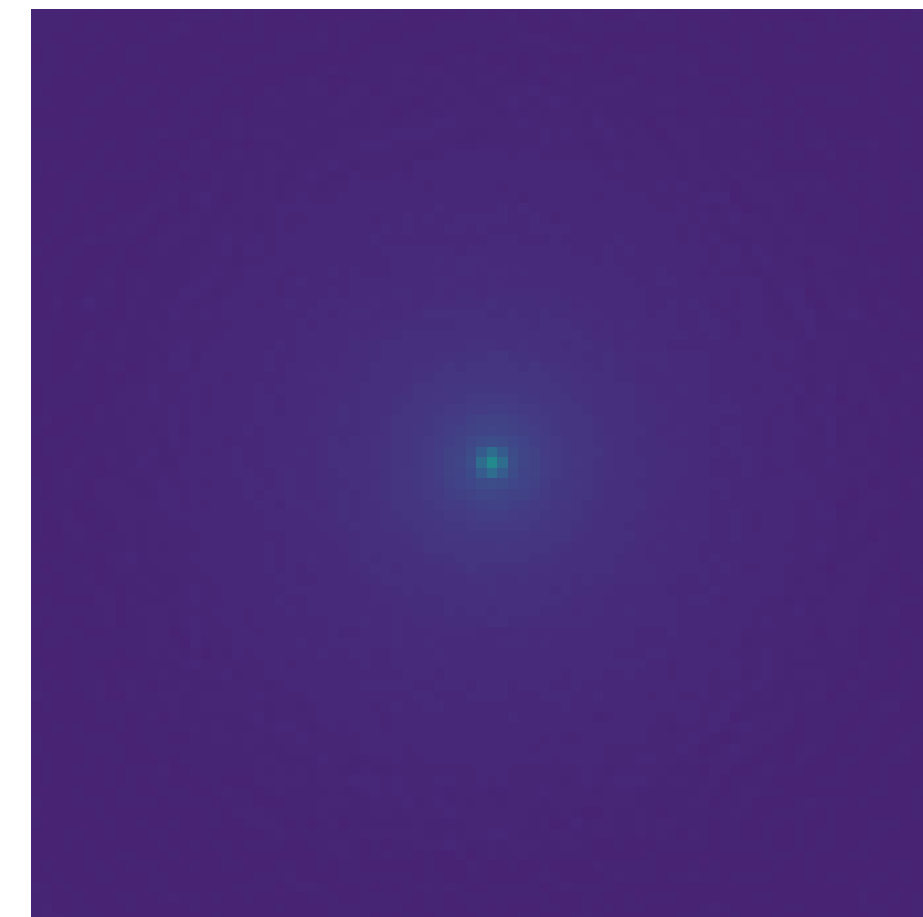
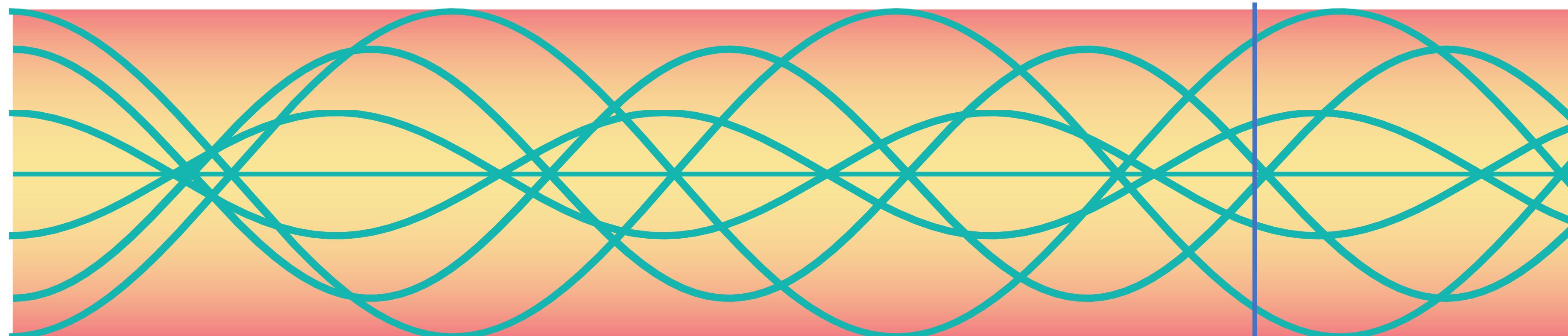




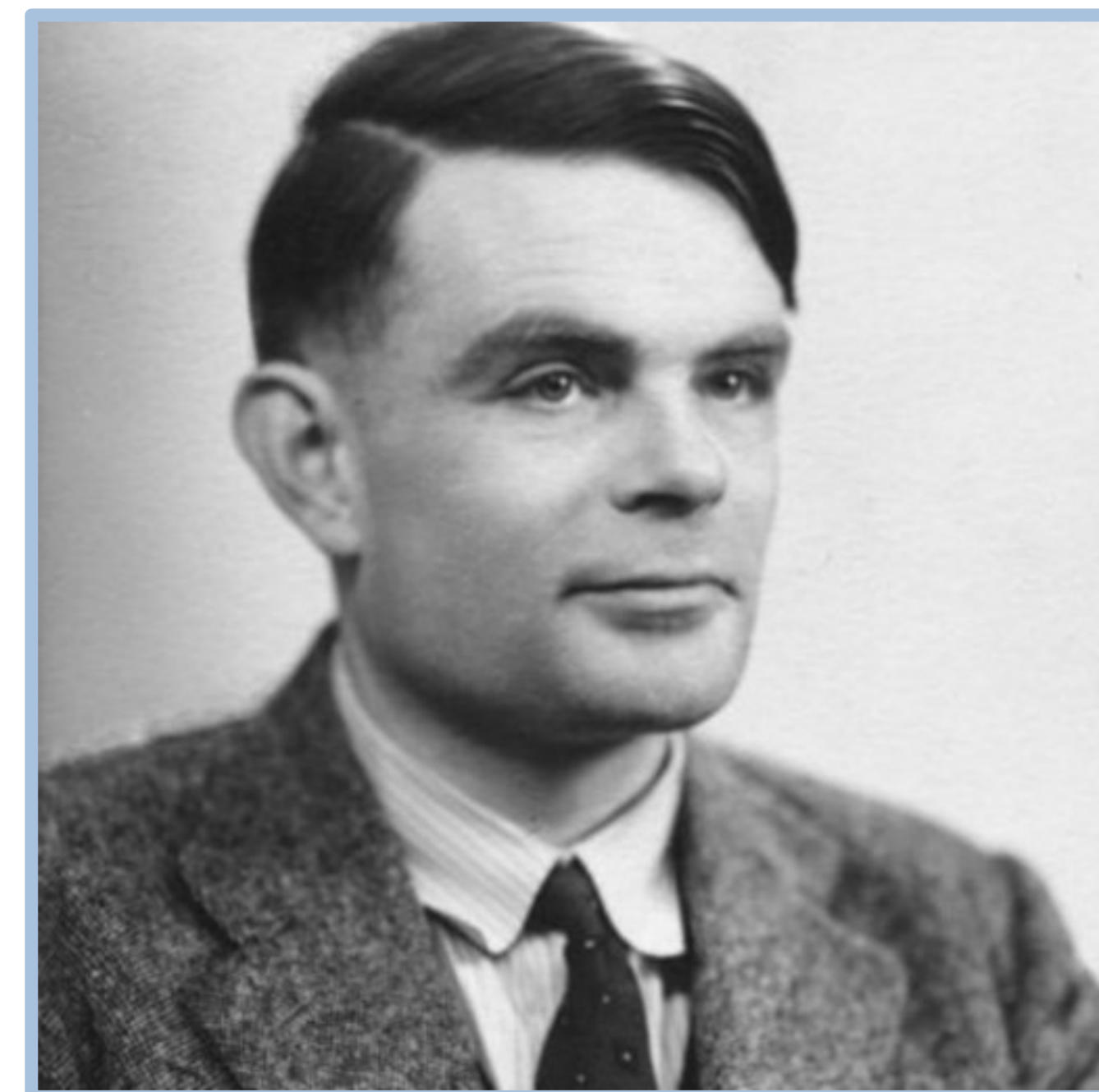
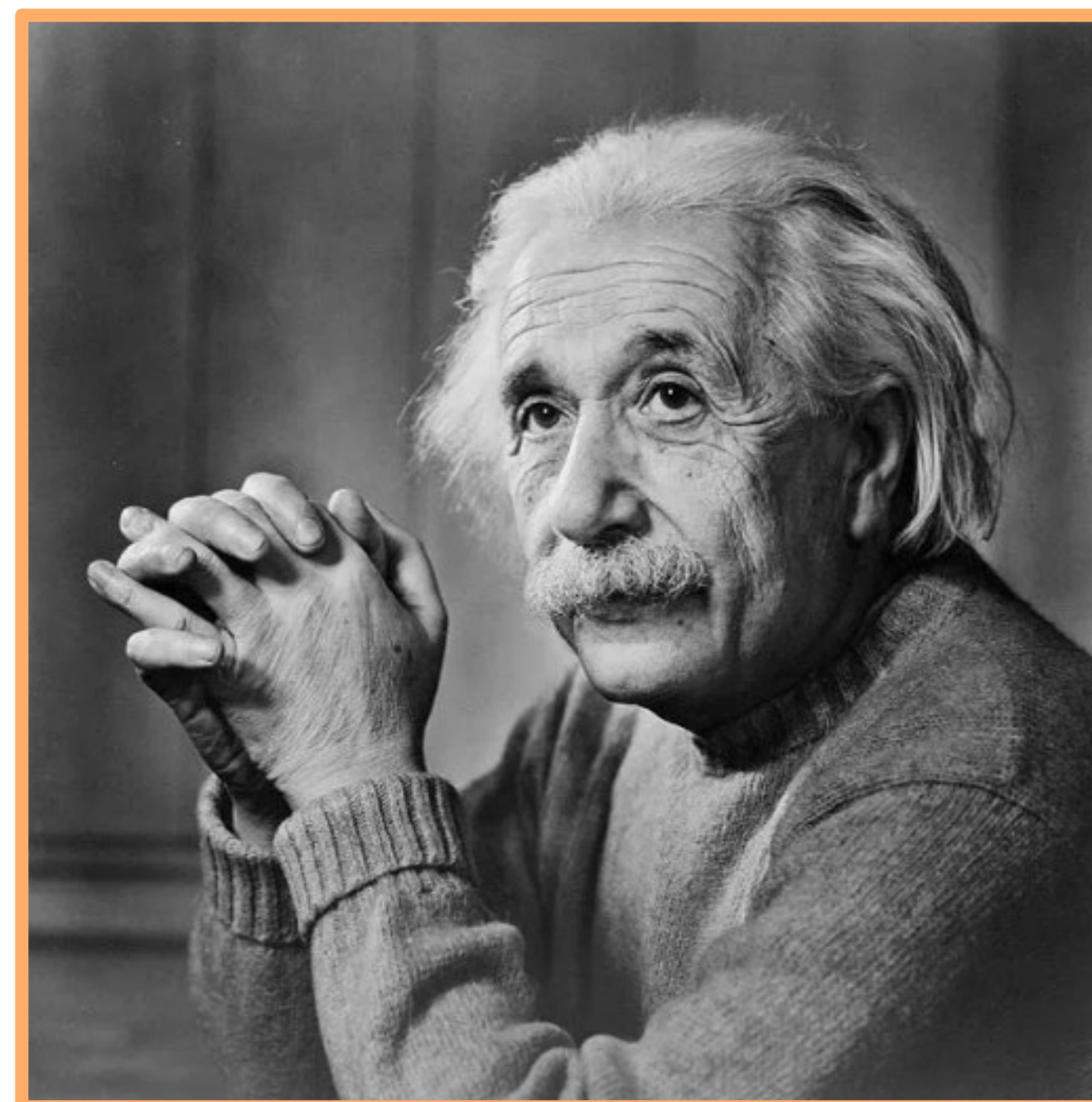
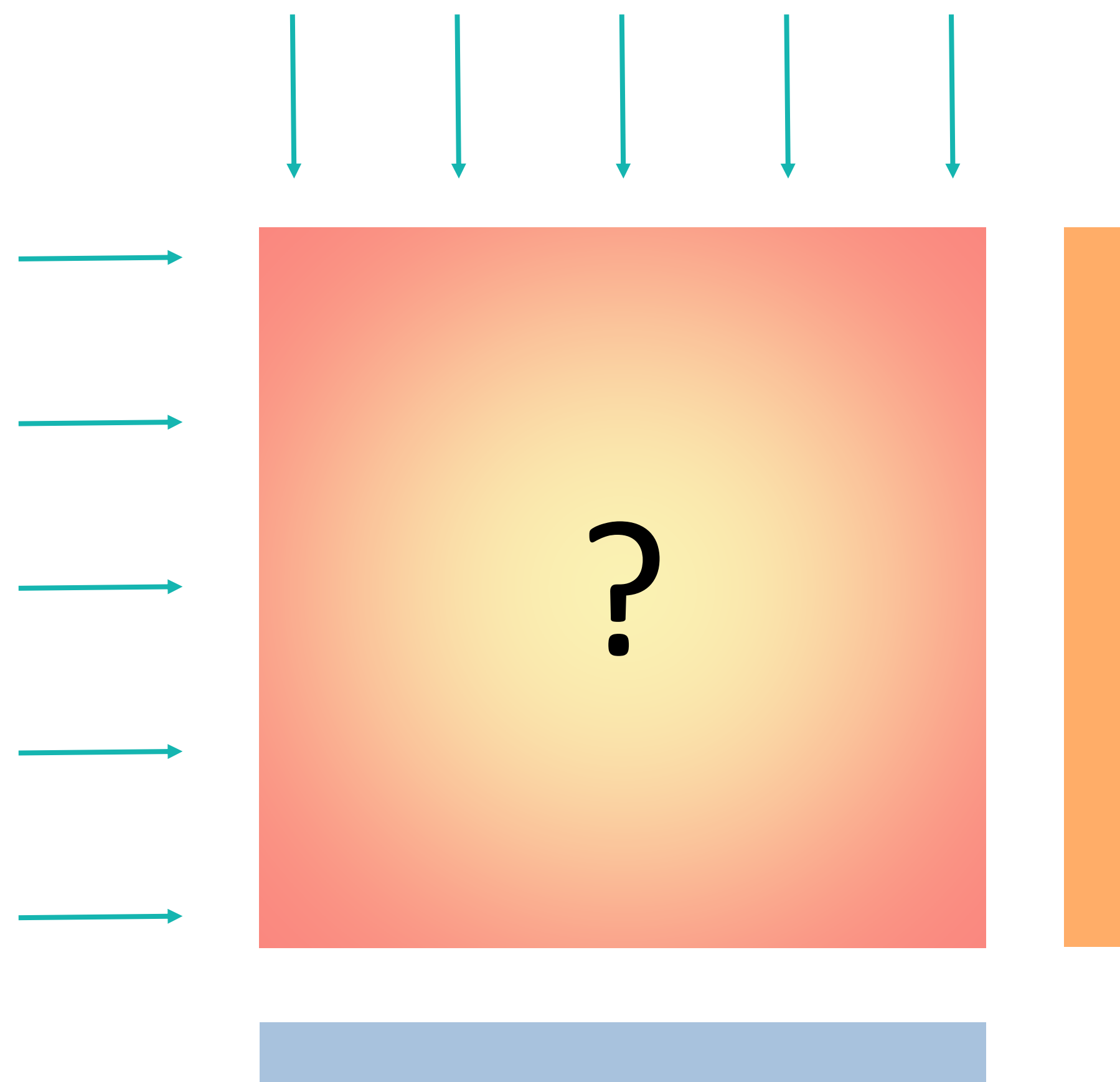
# GRIN Fiber



# GRIN Fiber

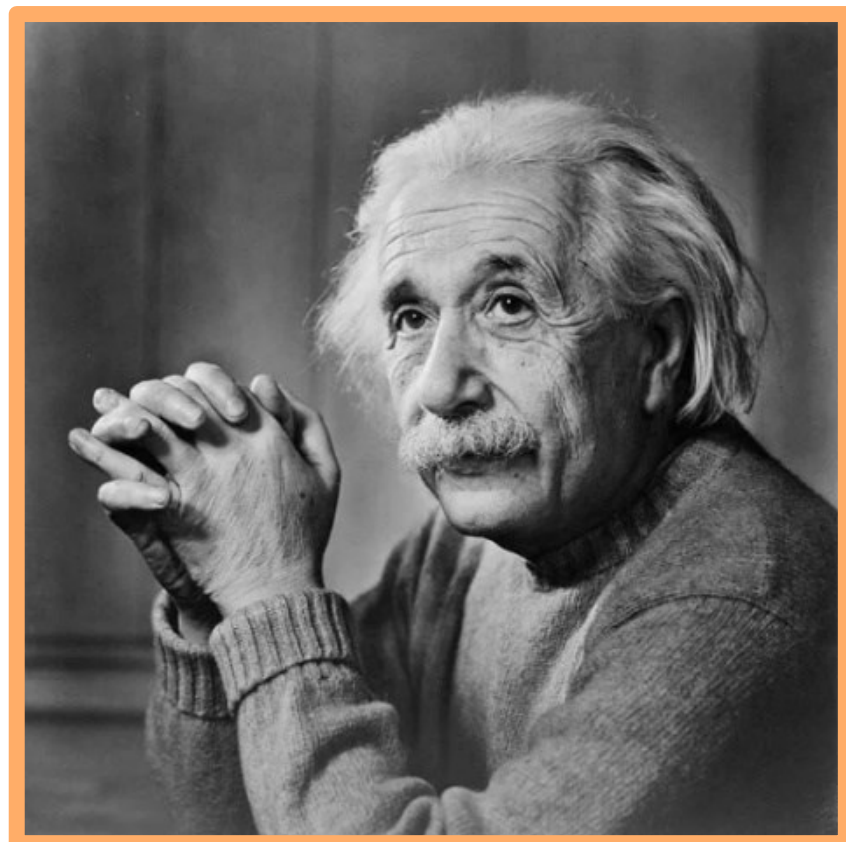


# Multiview Display

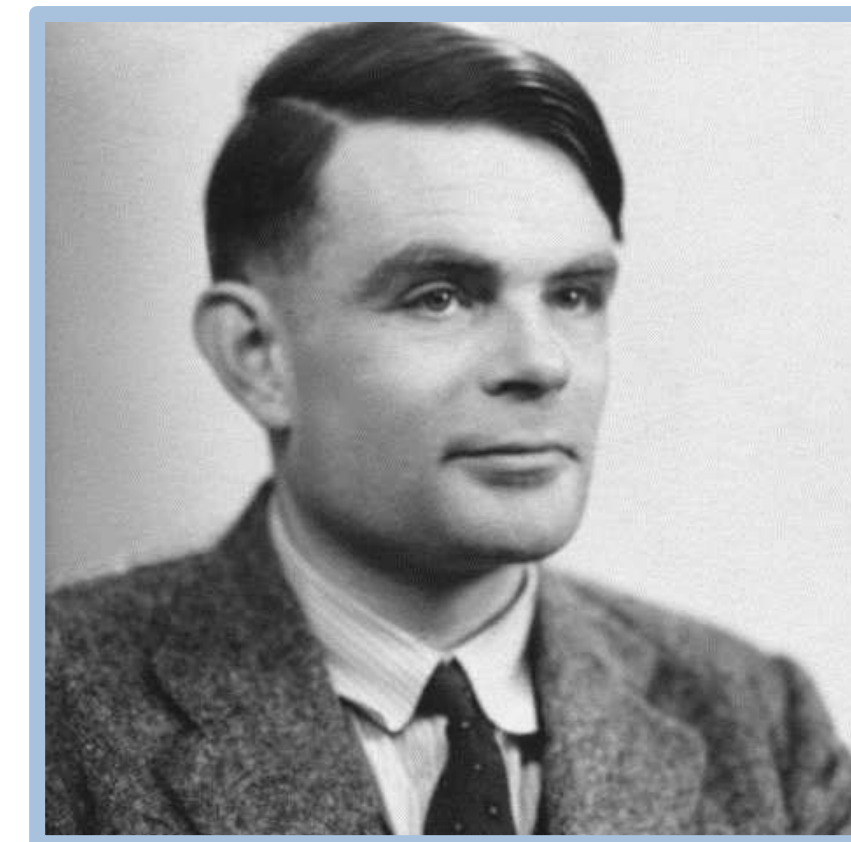
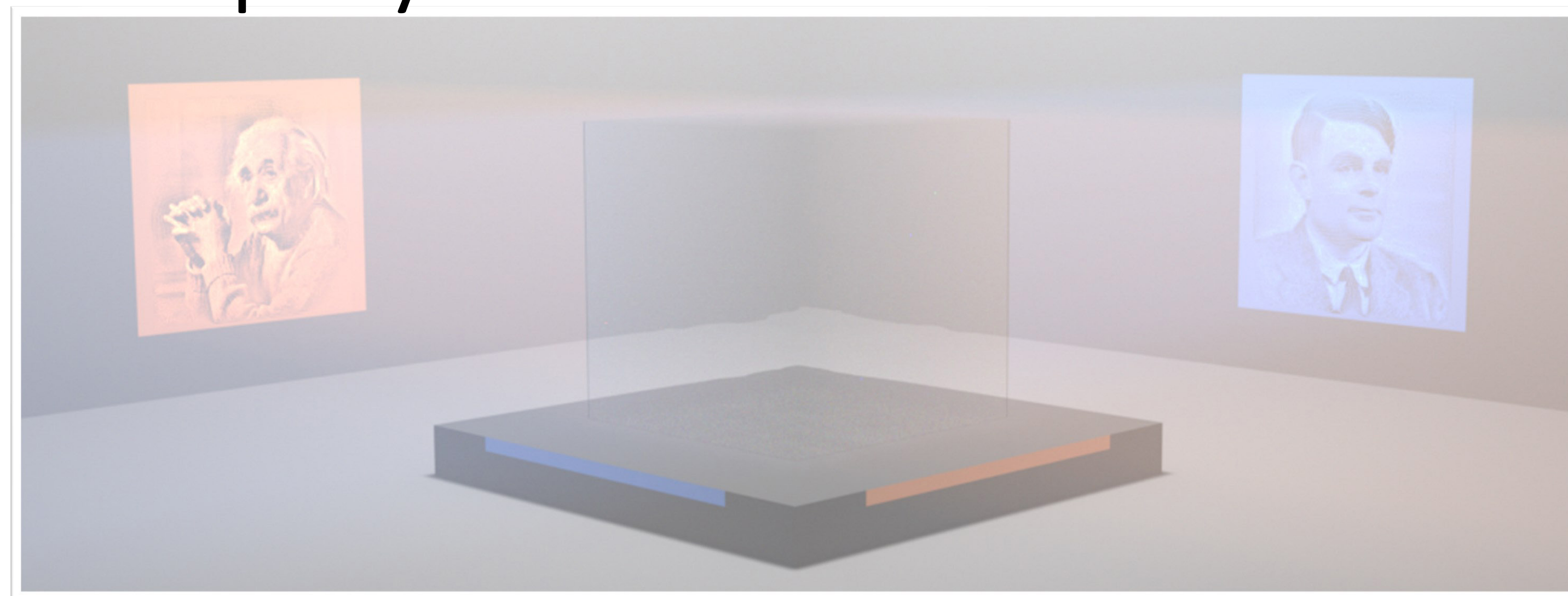




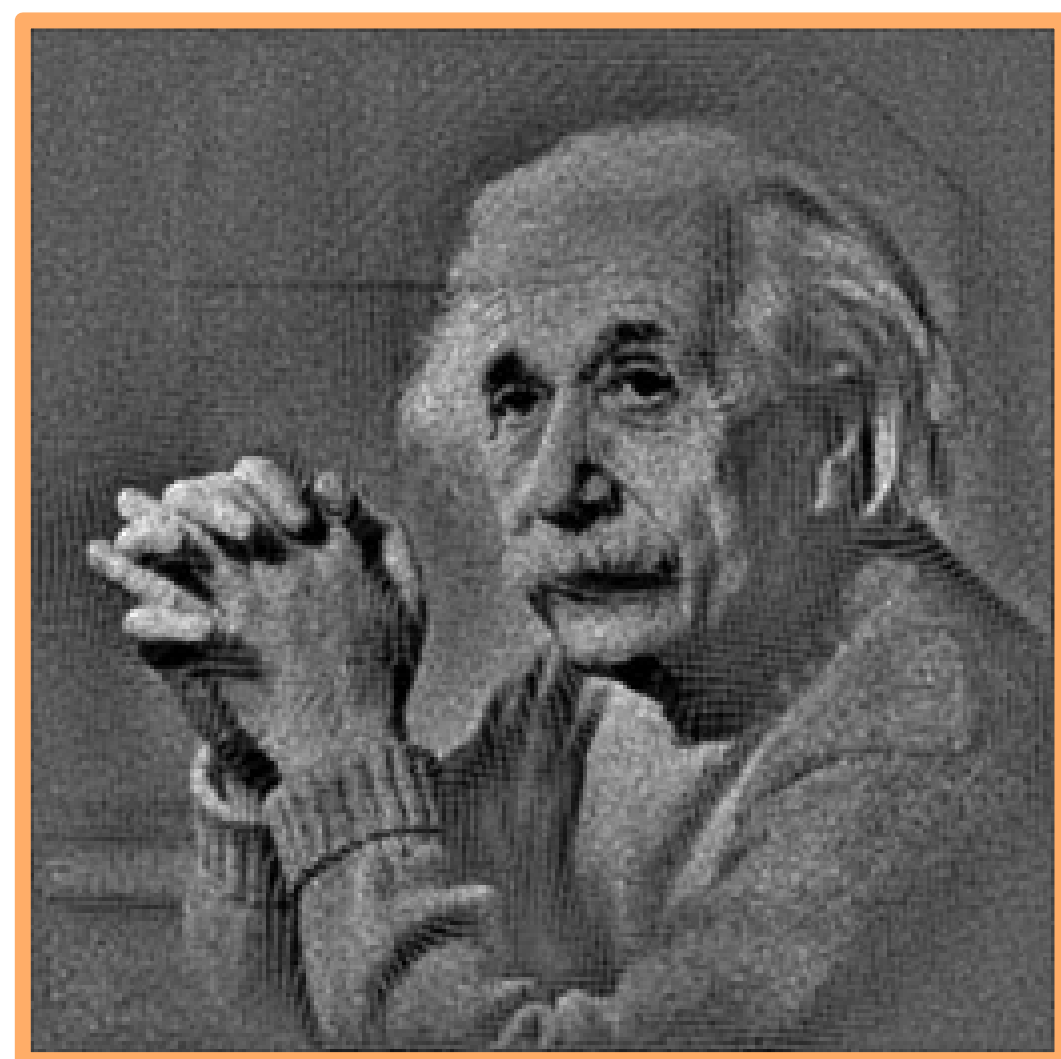
# Multiview Display



Target



Target



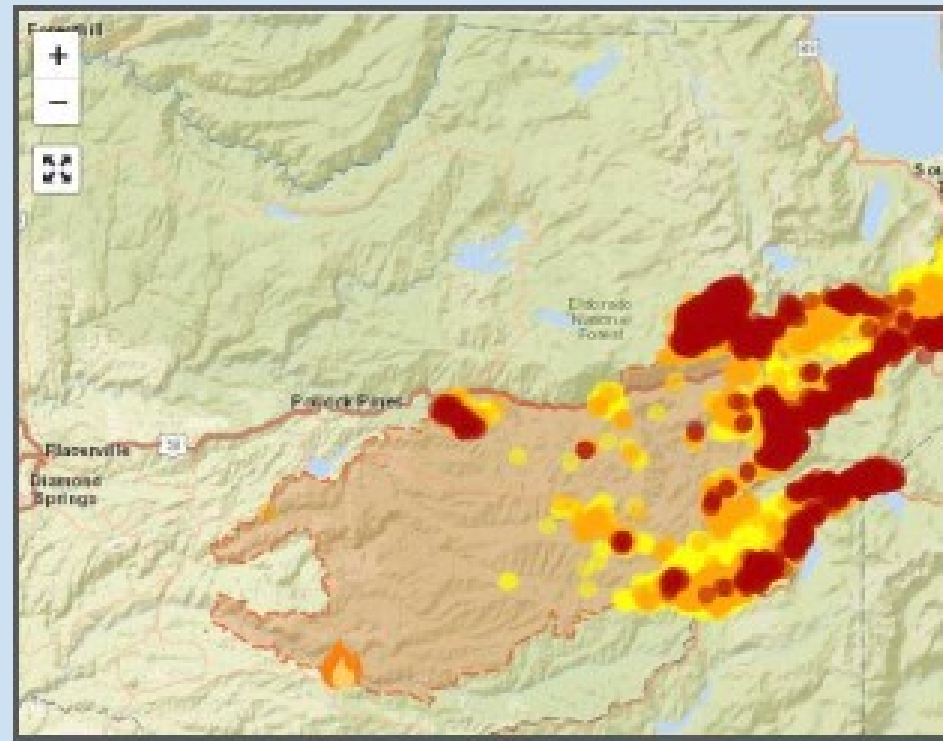
optimization  
results



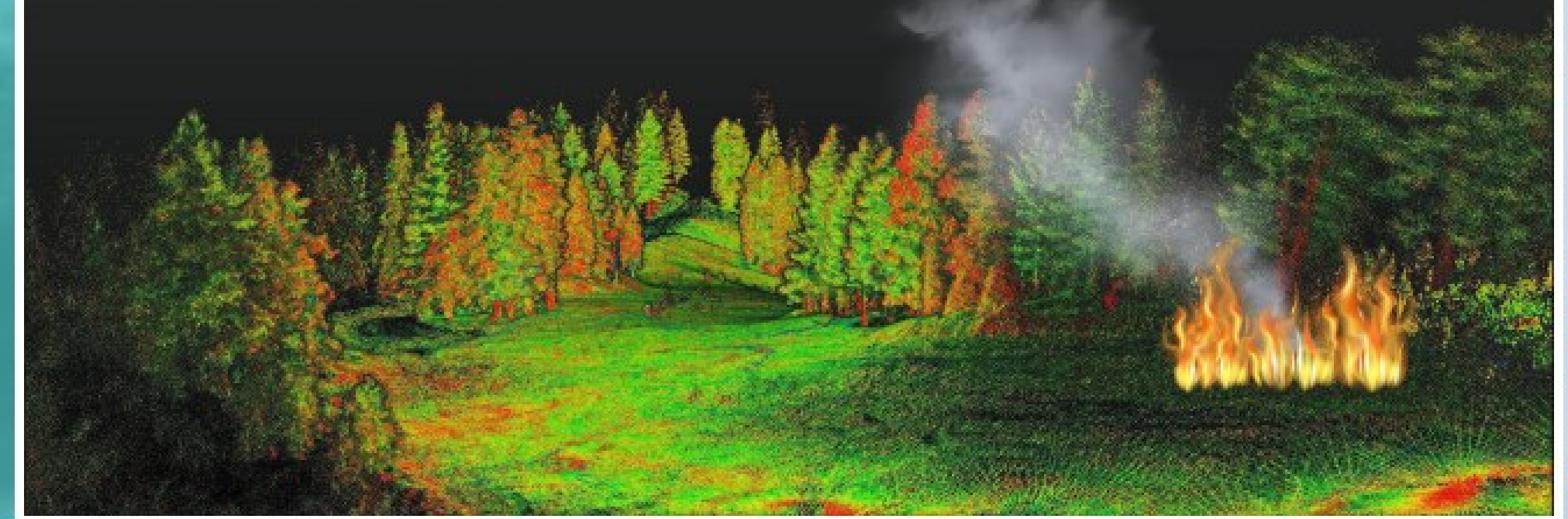


# Differentiable rendering for wildfire monitoring

Other's product: 2D fire map, low-res



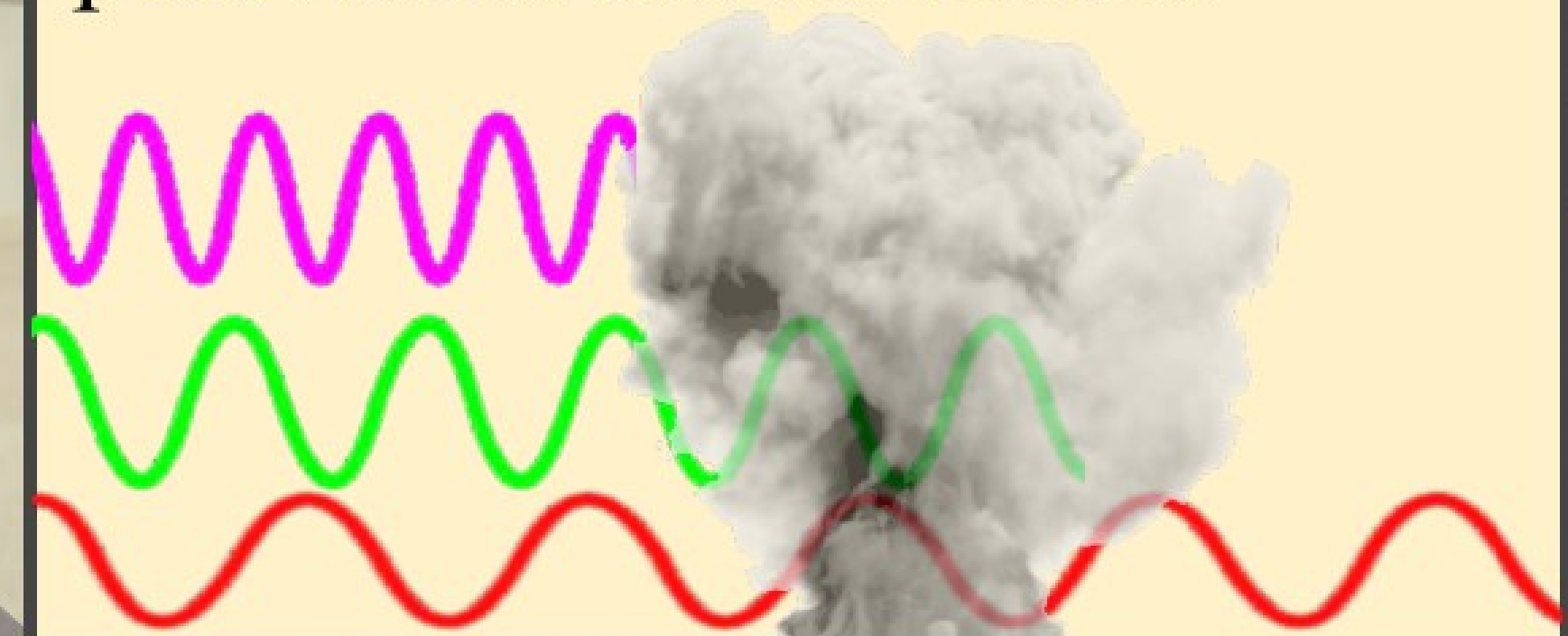
**Our Product:** granular 3D map of environment and fire plume



**Ours:** Low-flying, granular resolution; team scouts fire plume and environment up-close



Sensor wavelength trades smoke penetration and resolution



[USDA NIFA project jointly with Sebastian Scherer and Katia Sycara]

What differentiable rendering does  
not give us

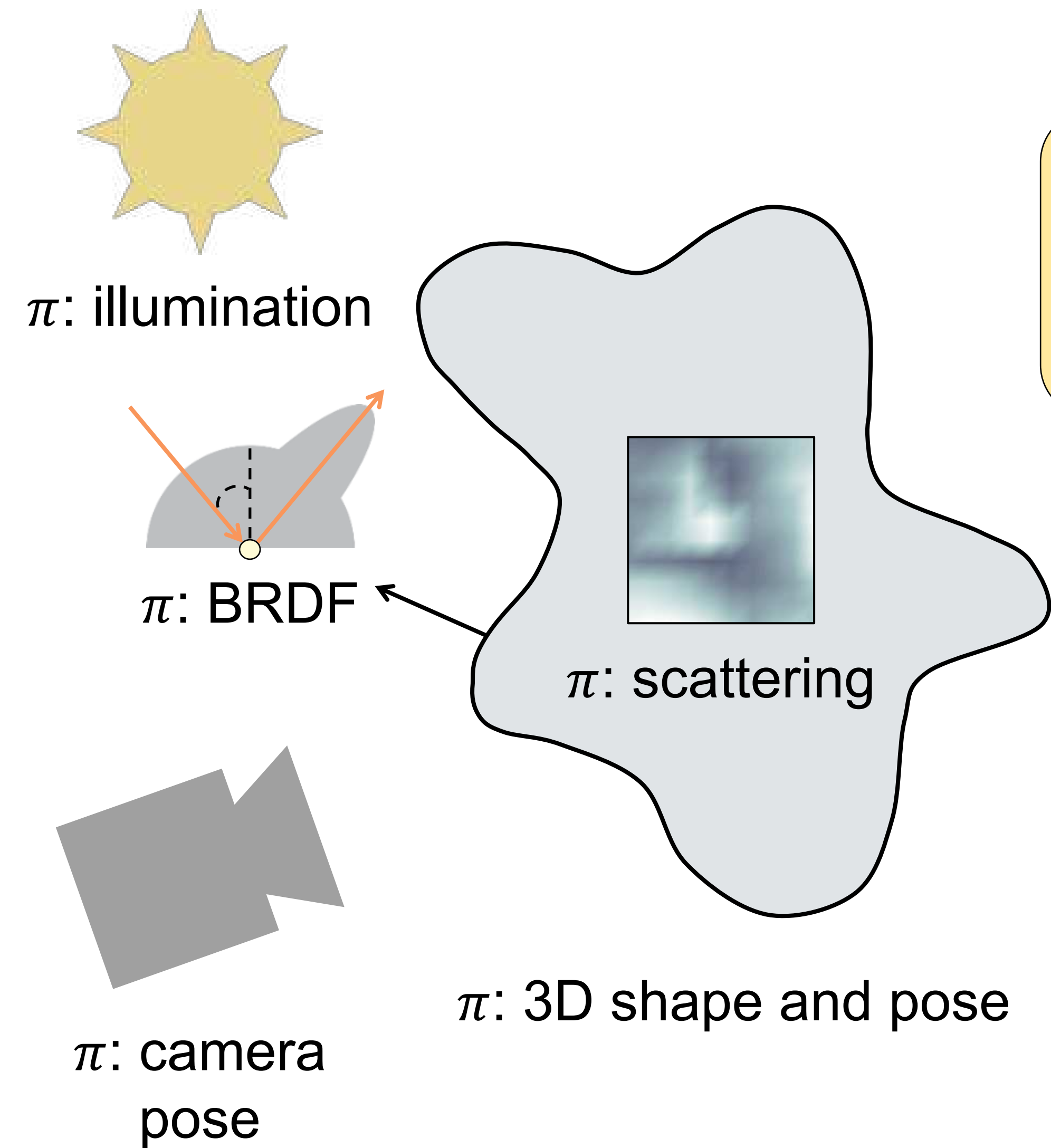


# Inverse rendering (a.k.a. analysis by synthesis)

Analysis-by-synthesis optimization:


$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{img}, \text{render} \left( \begin{matrix} \text{scene} \\ \text{unknowns } \pi \end{matrix} \right) \right]$$


# Inverse rendering (a.k.a. analysis by synthesis)

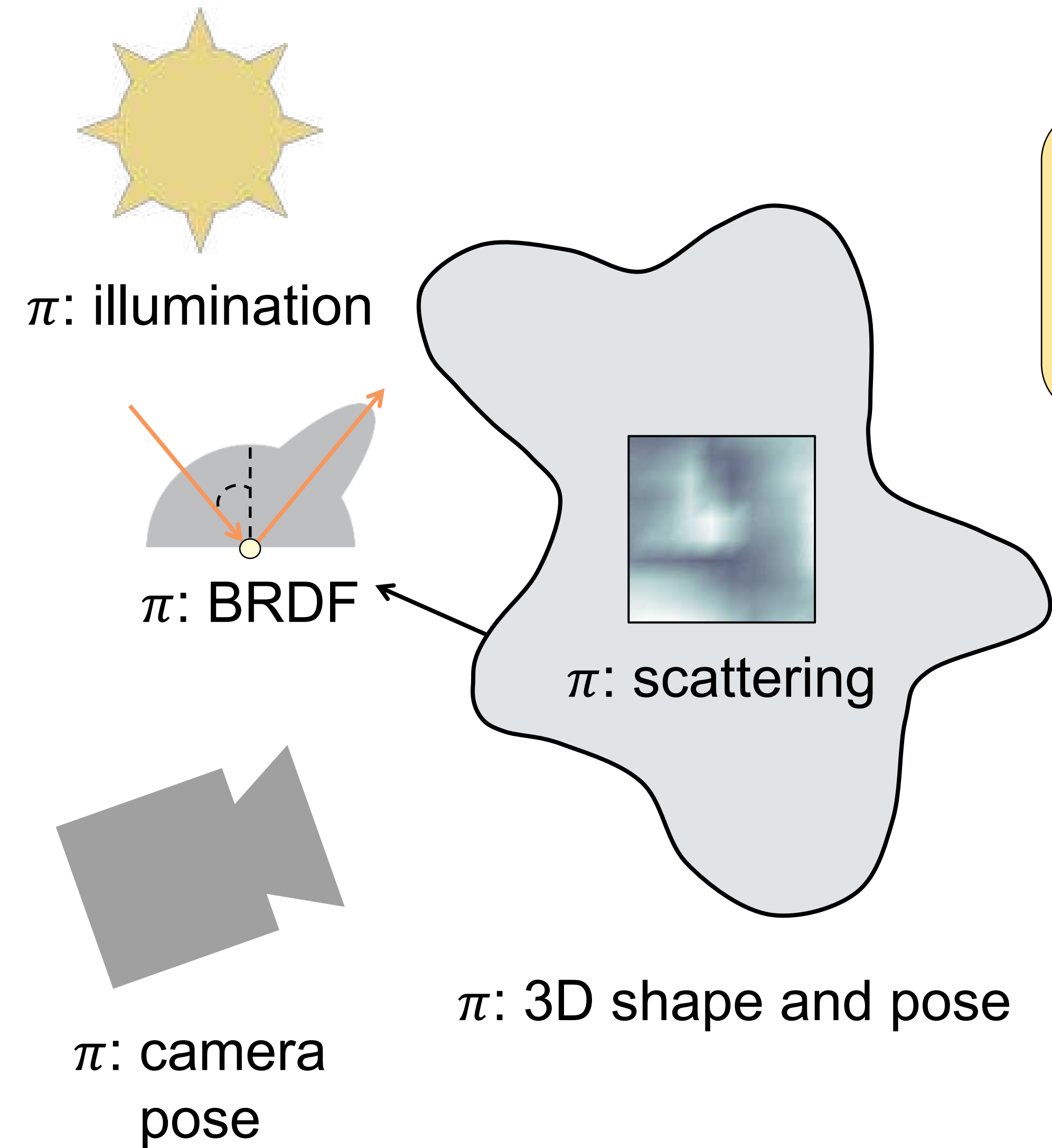


Analysis-by-synthesis optimization:

$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{img}, \text{render} \left( \begin{matrix} \text{scene} \\ \text{unknowns } \pi \end{matrix} \right) \right]$$

The equation is contained within a yellow rounded rectangle. The image  represents the ground truth image used in the loss function.

# Inverse rendering (a.k.a. analysis by synthesis)



Analysis-by-synthesis optimization:

$$\min_{\text{scene unknowns } \pi} \text{loss} \left[ \text{scene image}, \text{render} \left( \begin{matrix} \text{scene} \\ \text{unknowns } \pi \end{matrix} \right) \right]$$

Stochastic gradient descent (e.g., Adam):

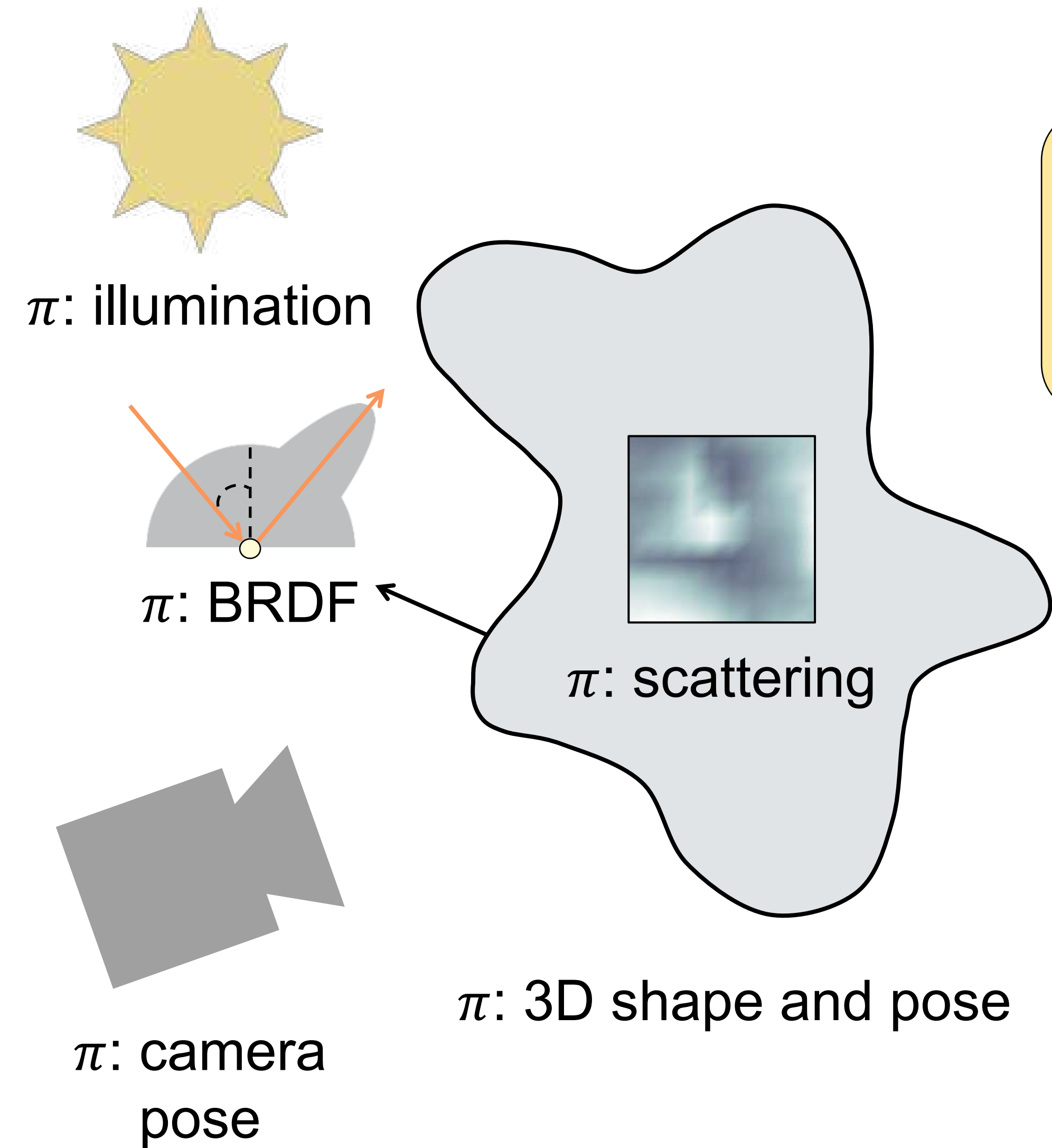
initialize  $\pi \leftarrow \pi_0$

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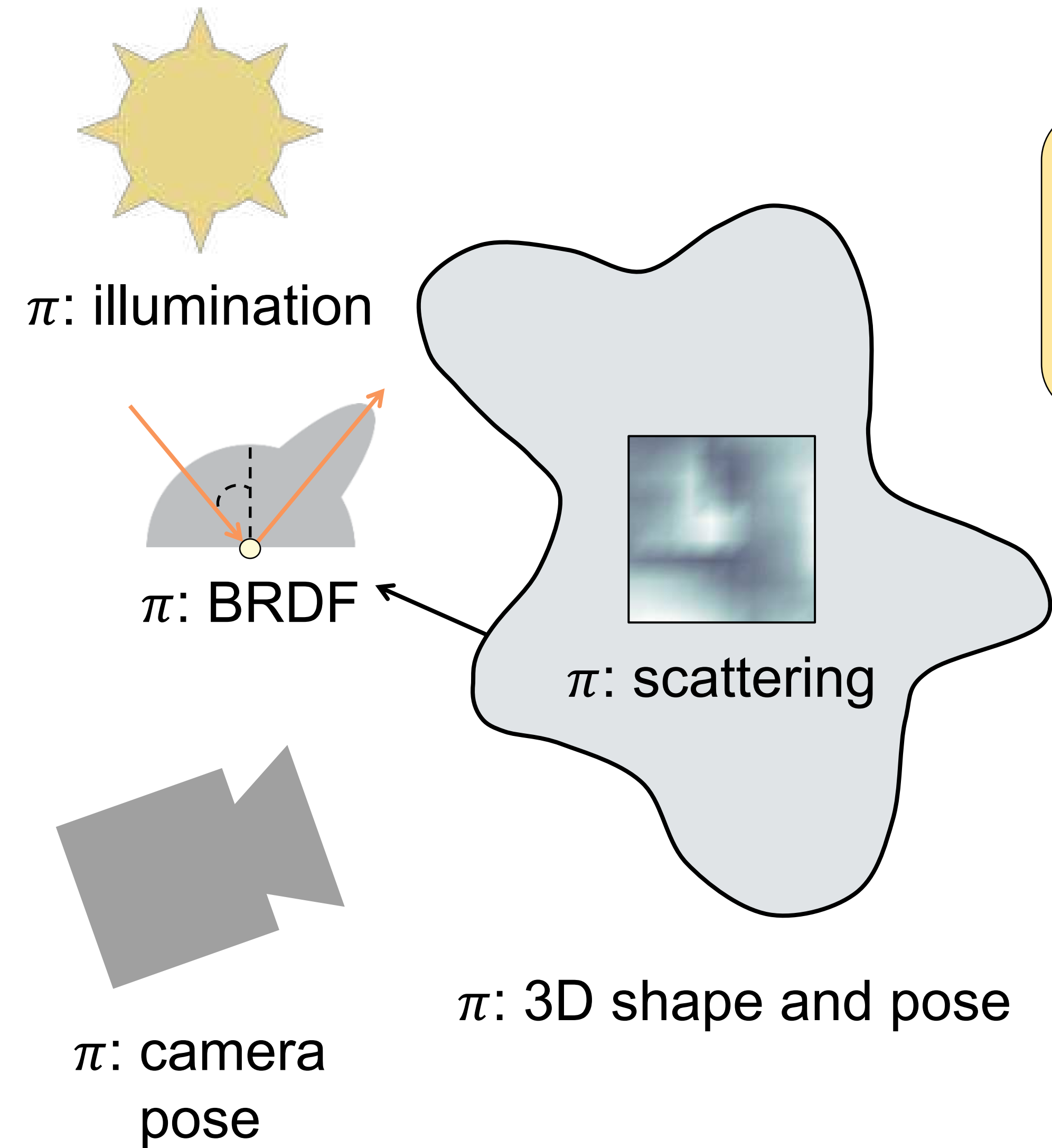
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Differentiable  
rendering

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# Why we need good initializations

- Analysis-by-synthesis objectives are highly non-convex, non-linear
  - Multiple *local* minima

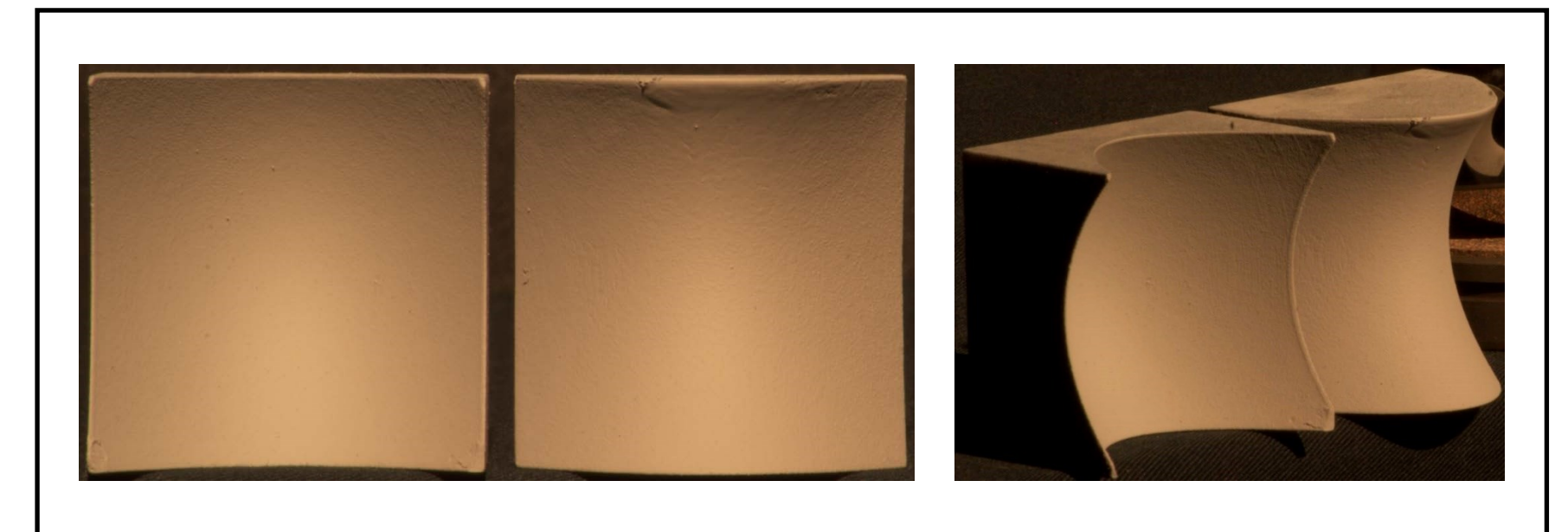


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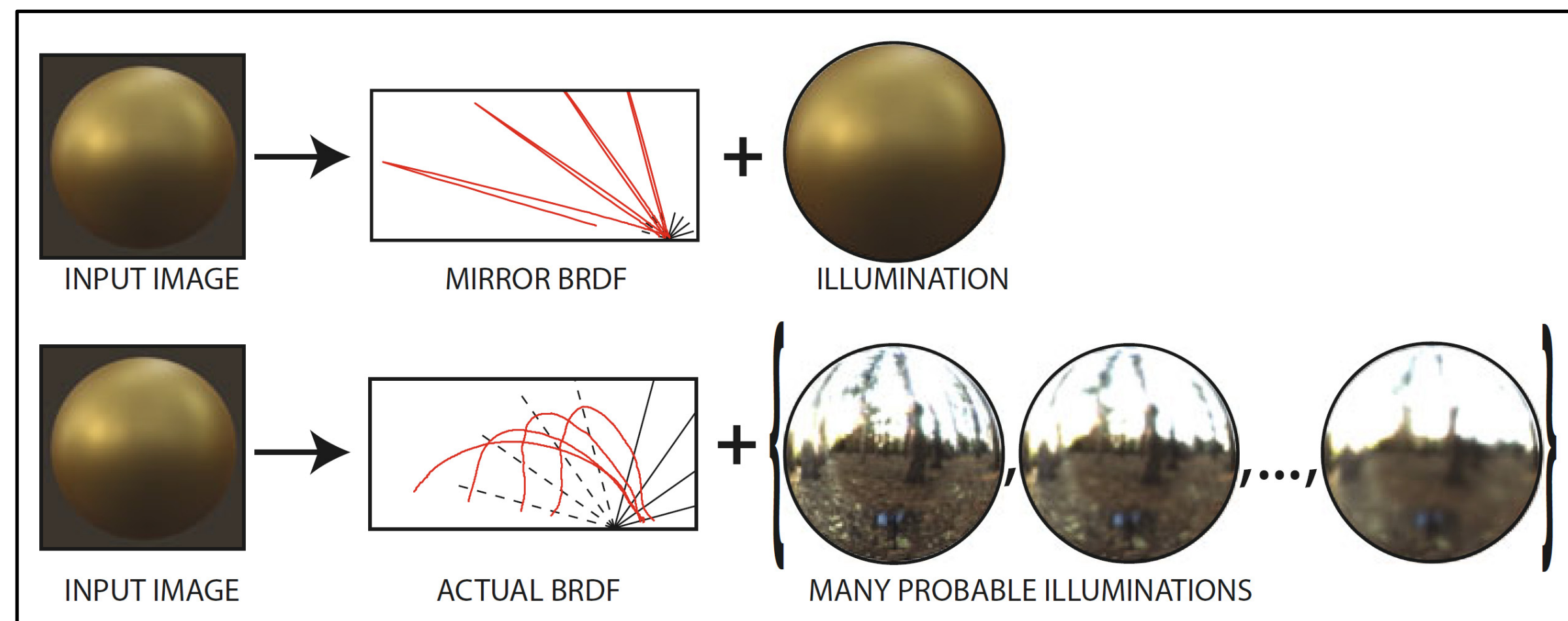
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Ambiguities between shape and lighting  
[Xiong et al. 2015]



Ambiguities between scattering  
parameters [Zhao et al. 2014]



Ambiguities between BRDF and lighting  
[Romeiro and Zickler 2010]

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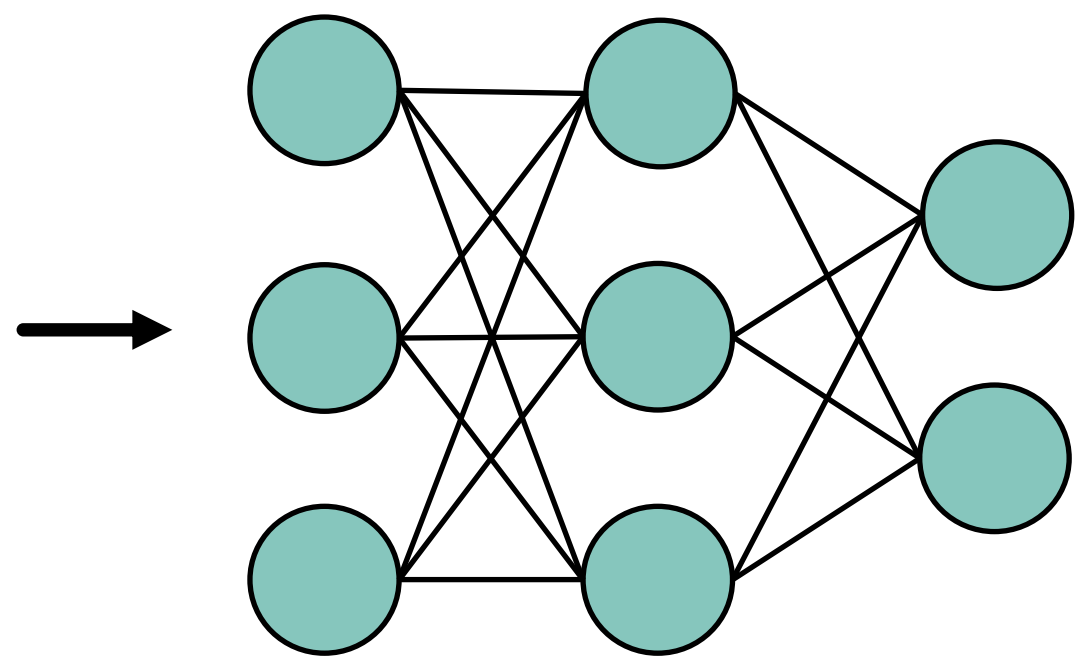
Differentiable  
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Neural network

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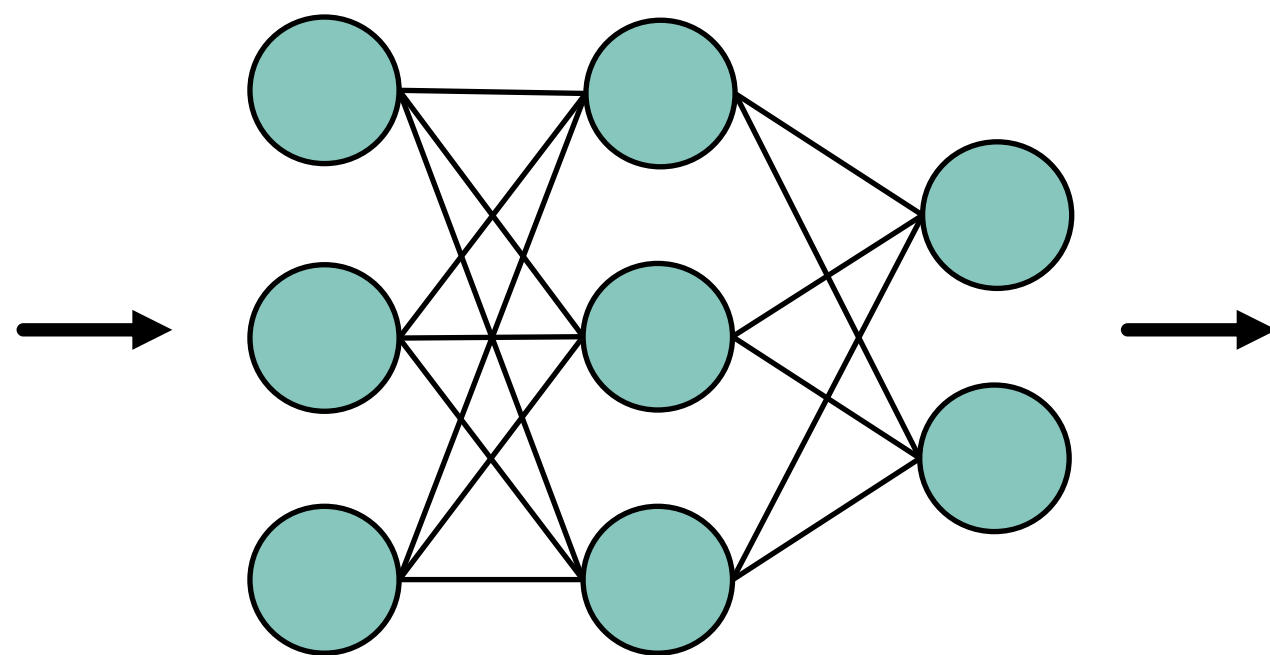
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Learned initializations help:

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- accelerate convergence



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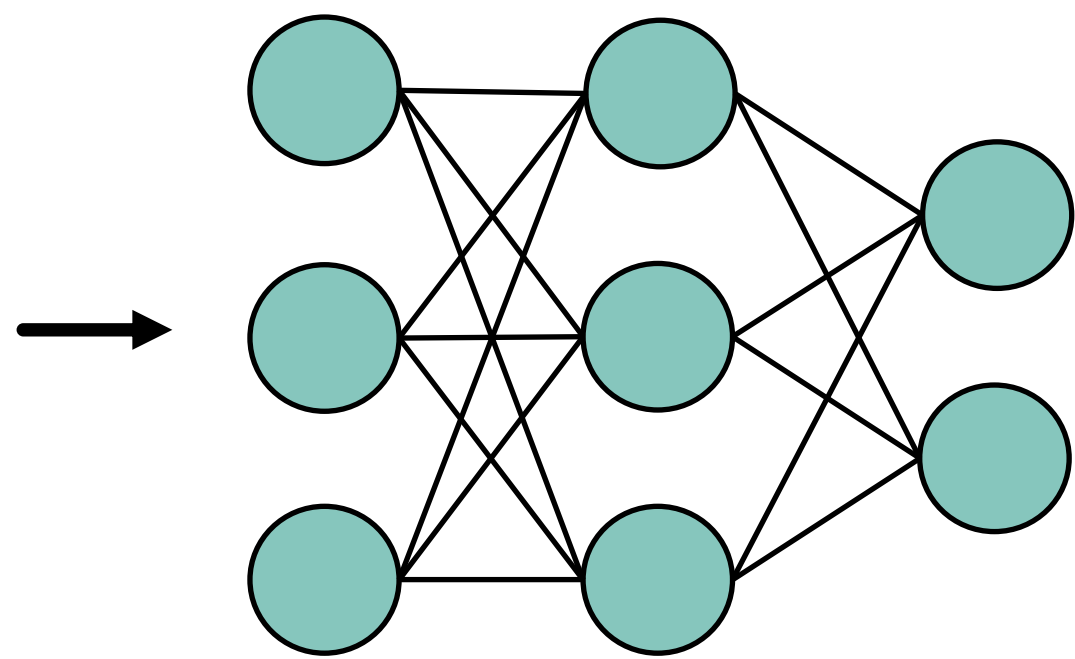
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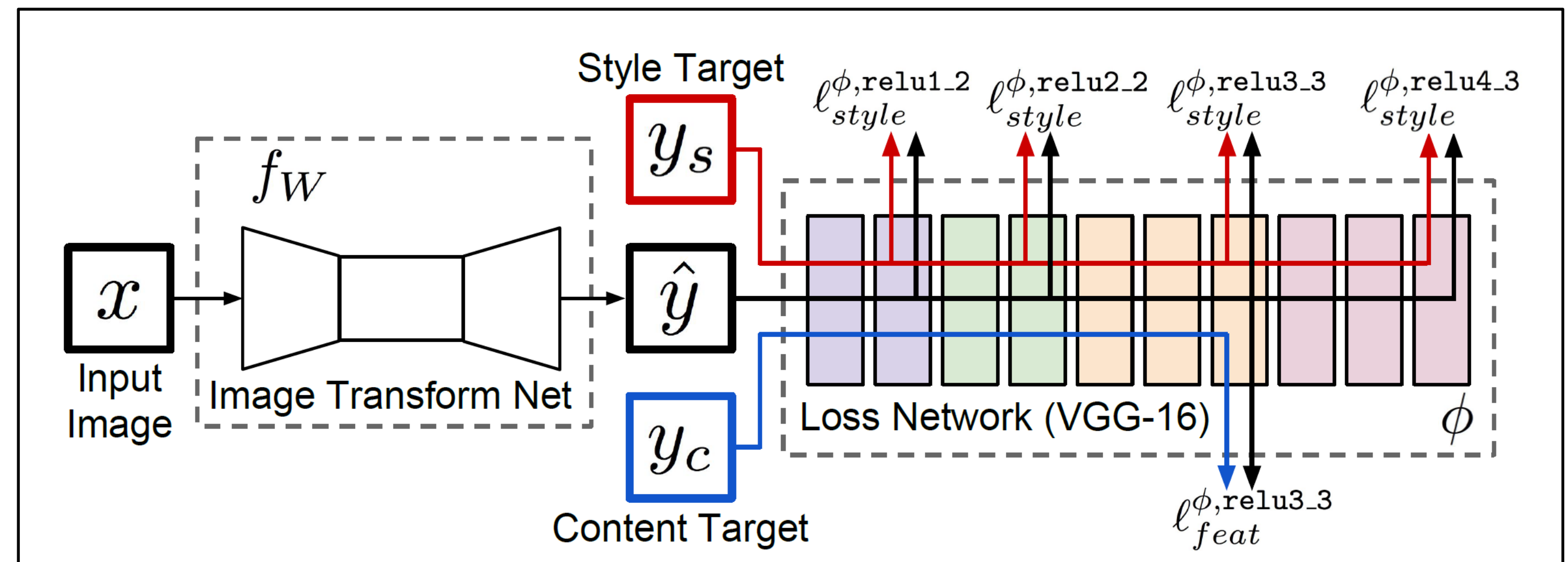
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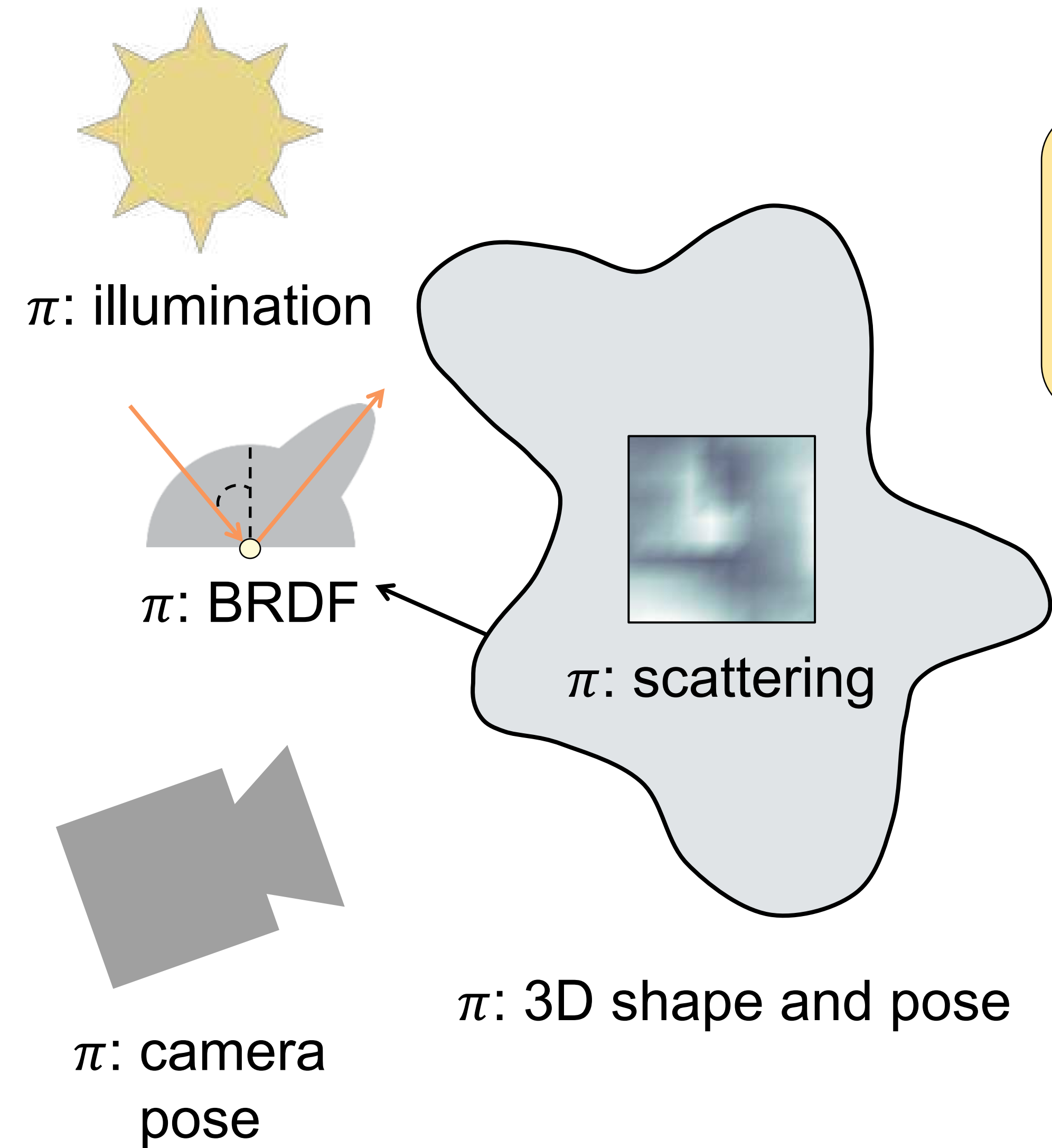
# Why we need discriminative loss functions

- Well-designed loss functions can help reduce ambiguities
- Perceptual losses can help emphasize design aspects that matter
- Differentiable rendering can be combined with any loss function that can be backpropagated through



VGG-based *perceptual loss* [Johnson et al. 2016]

# Inverse rendering (a.k.a. analysis by synthesis)



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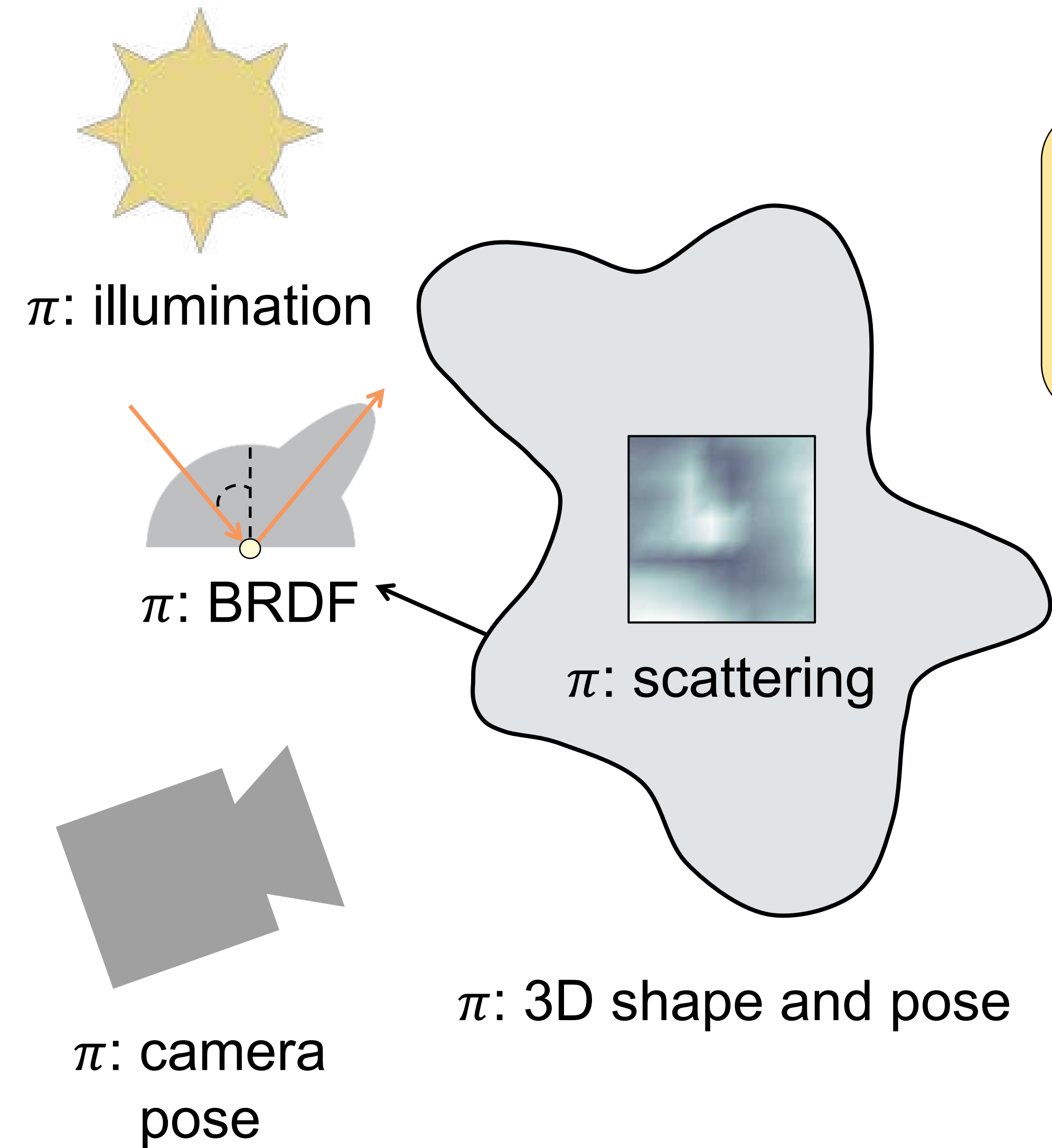
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Differentiable  
rendering

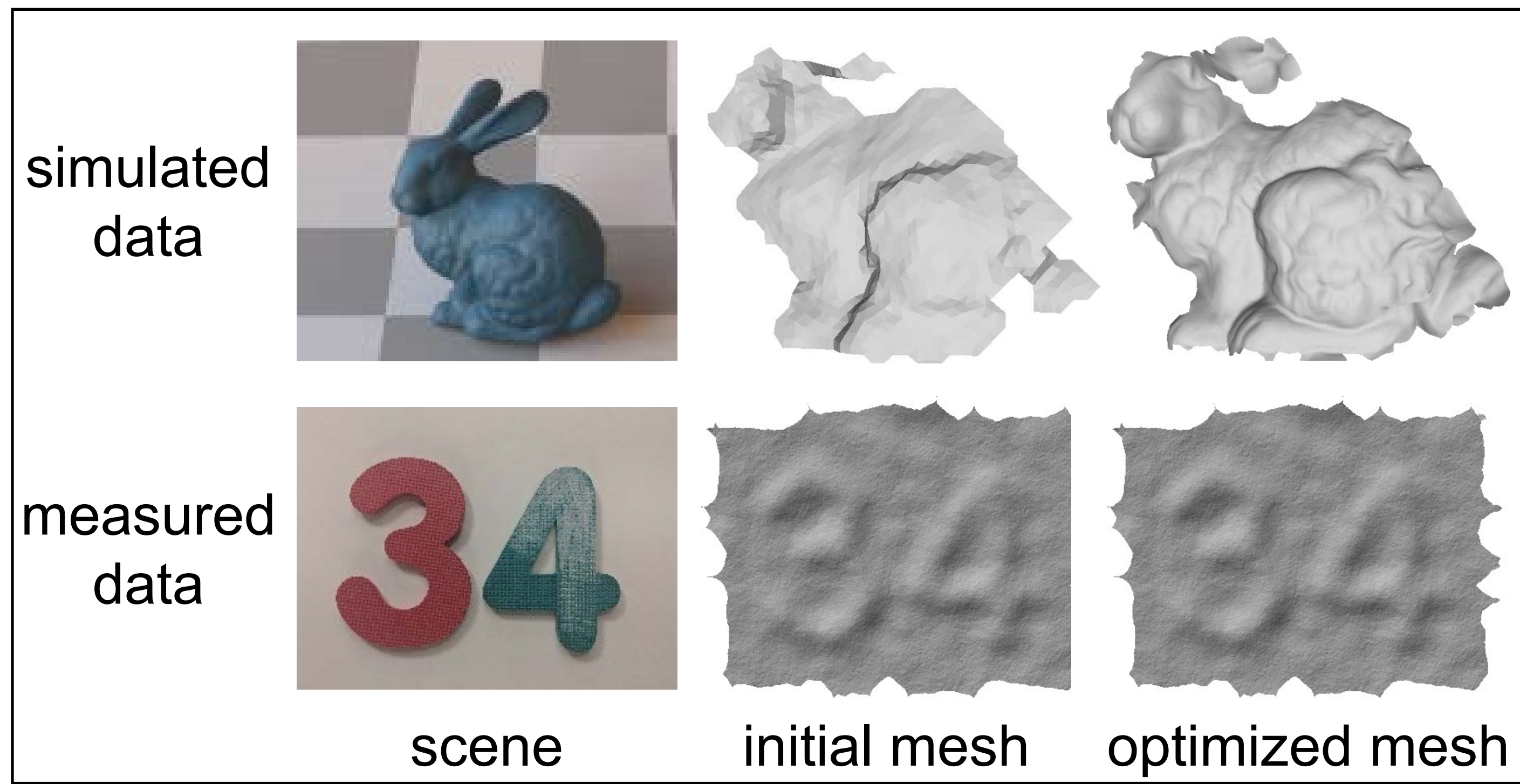


# High signal-to-noise ratio is critical

- The extent to which we can improve upon an initialization strongly depends on the signal-to-noise ratio of our measurements

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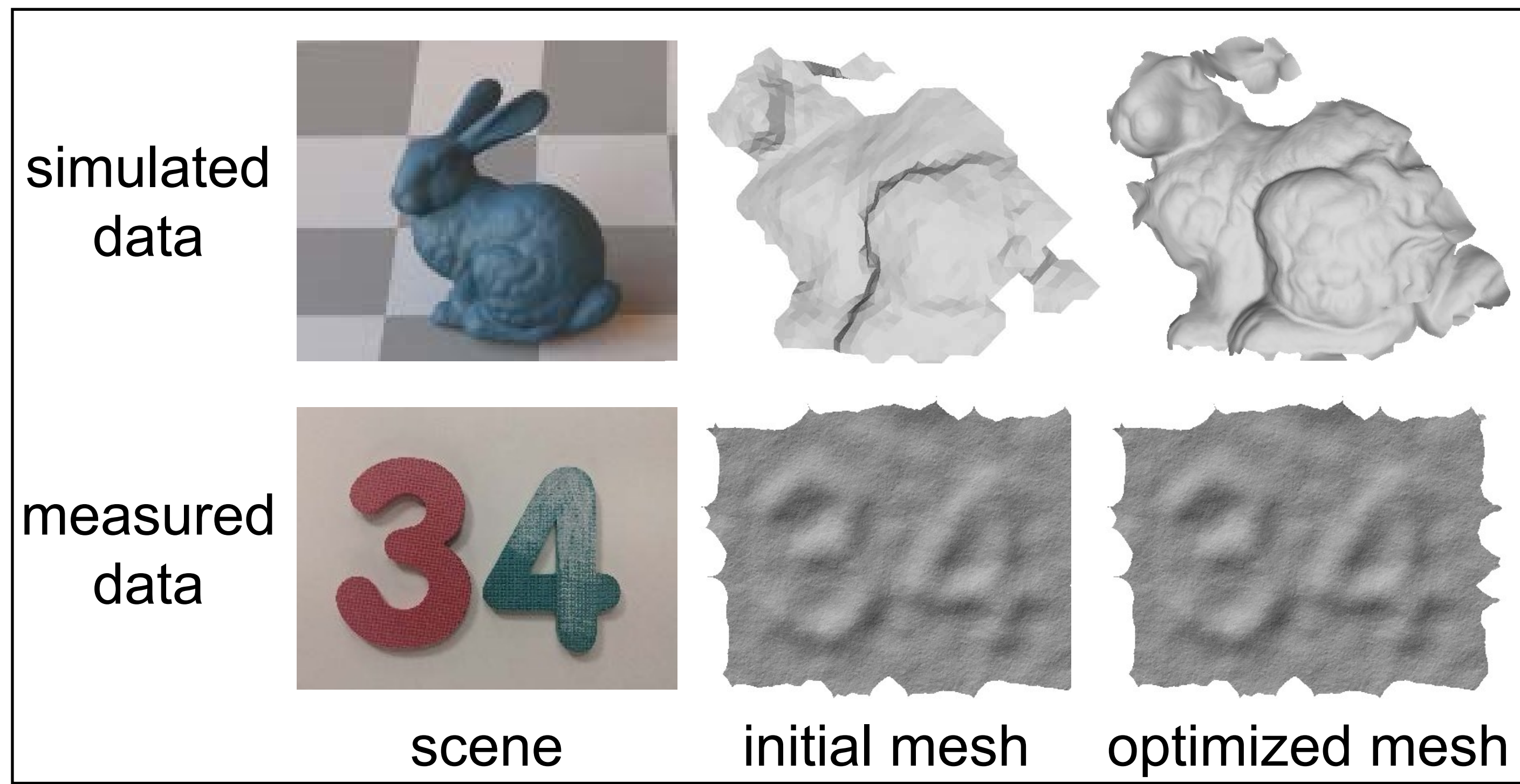
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Non-line-of-sight imaging [Tsai et al. 2019]

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- We need reliable camera models (noise, aberrations, other non-idealities)

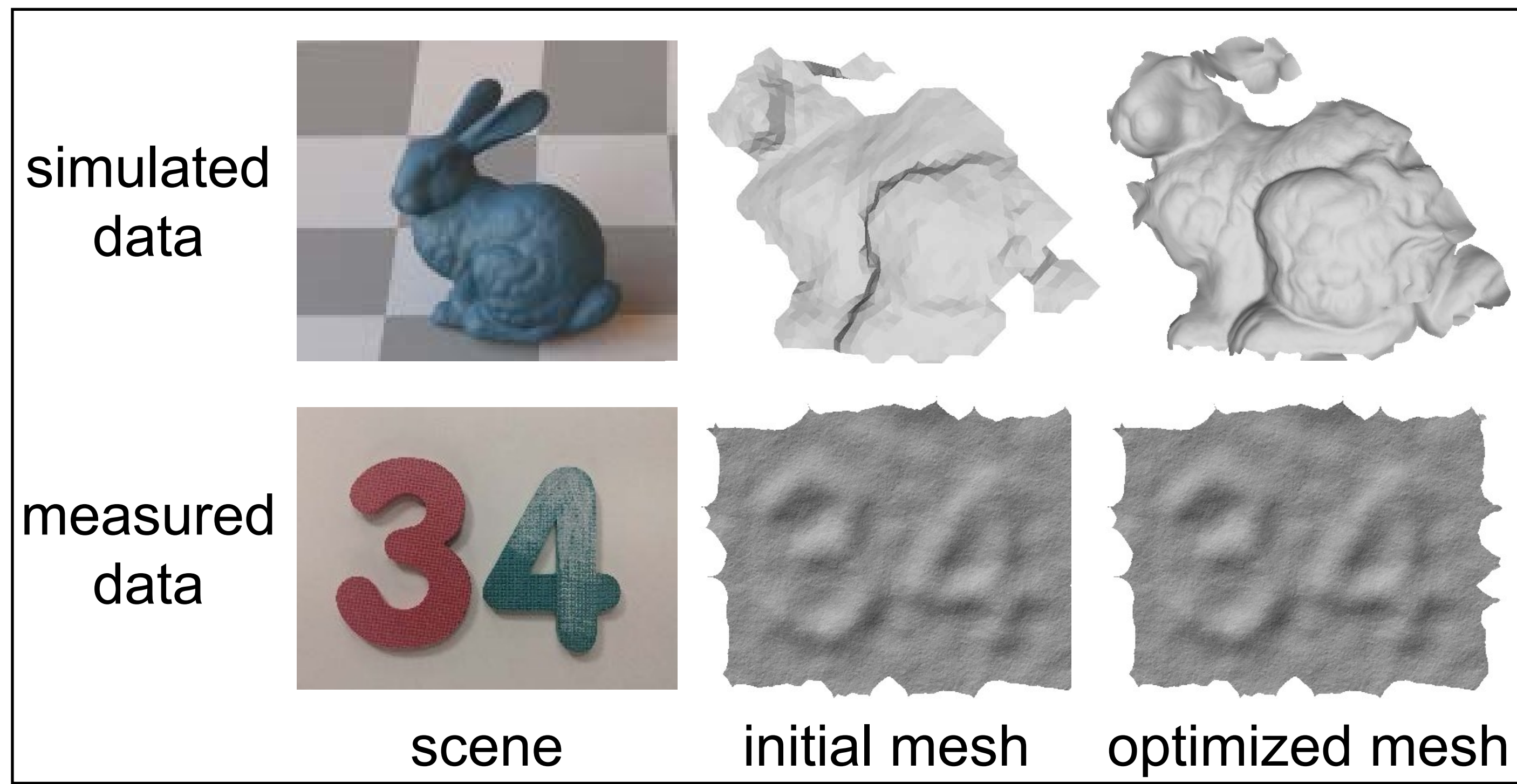


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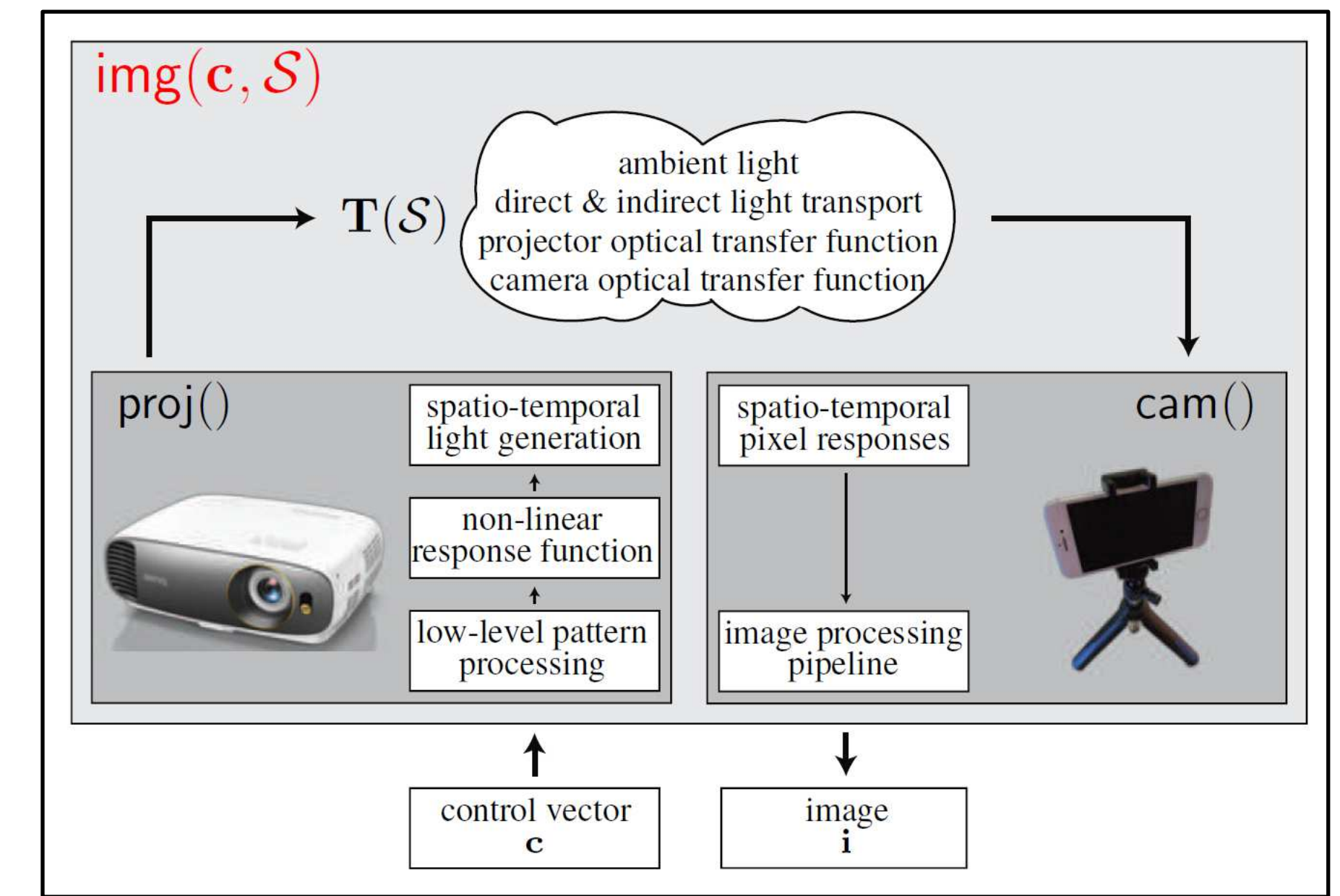


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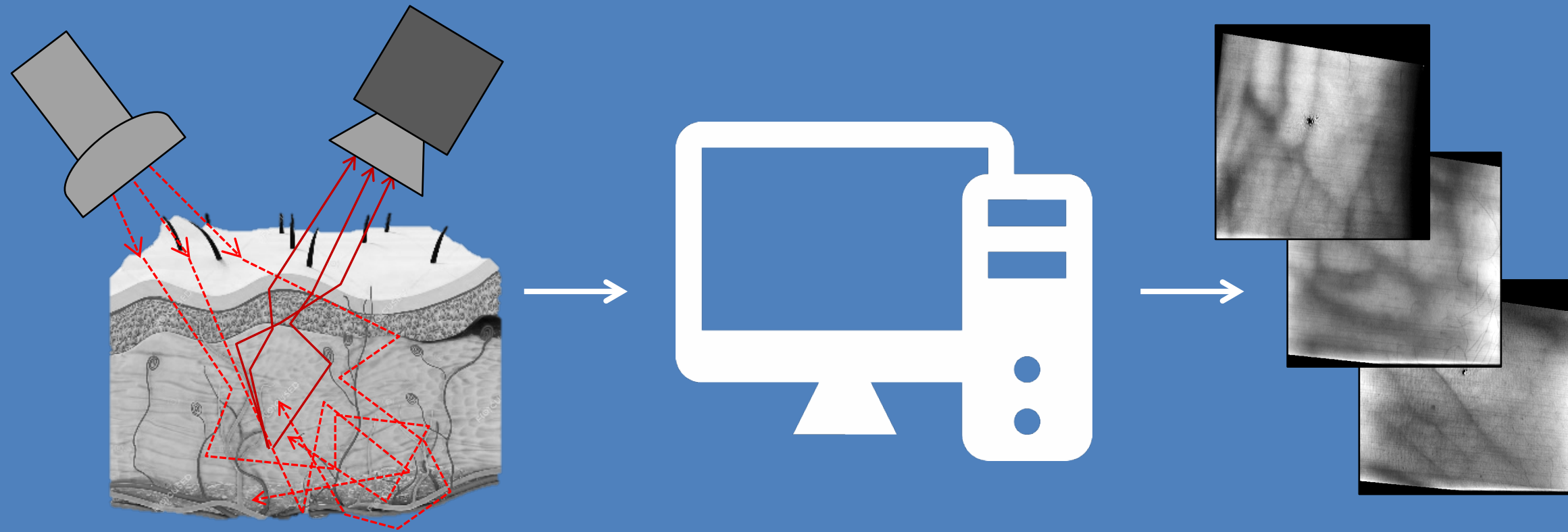


Optical gradient descent [Chen et al. 2020]



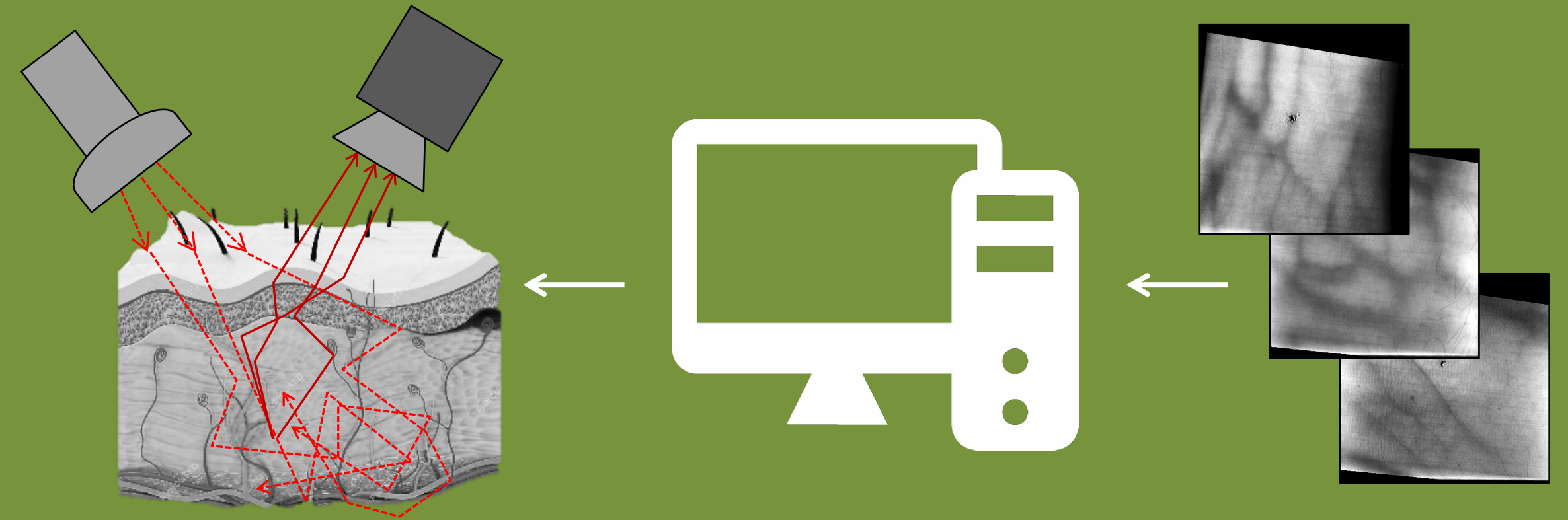
# Physics-based rendering and its applications to computational imaging

## forward rendering

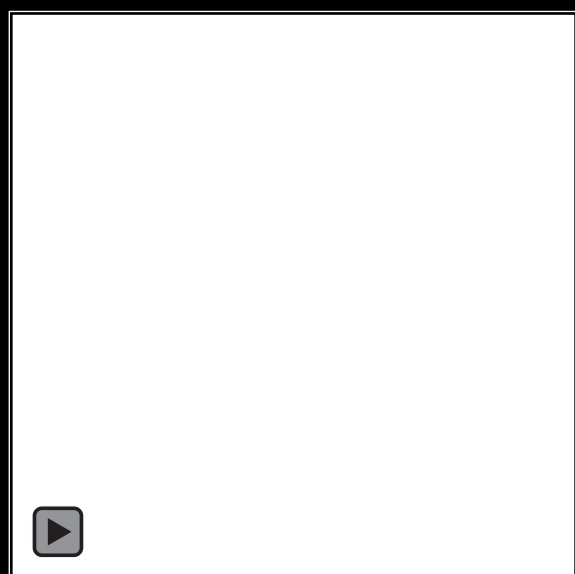


- accurate and efficient simulation
- virtually design sensors, optics, and algorithms

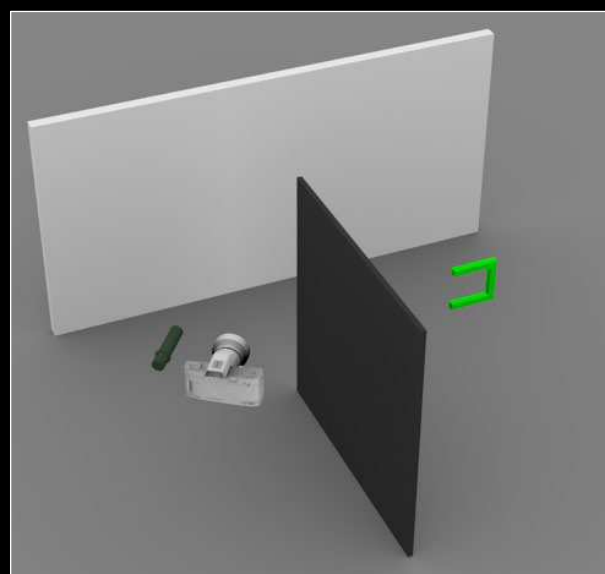
## inverse rendering



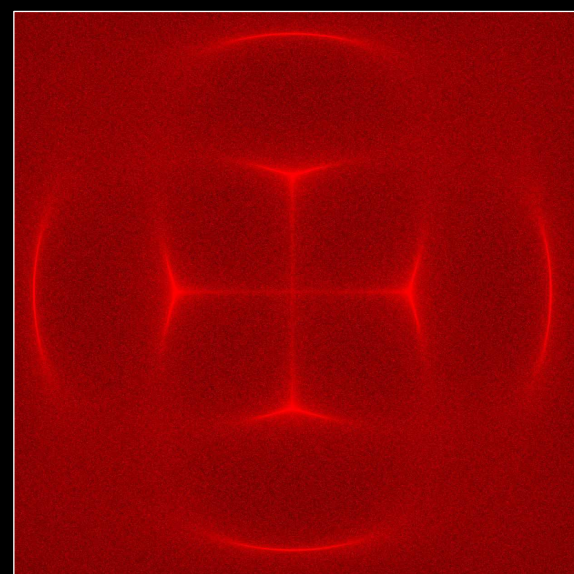
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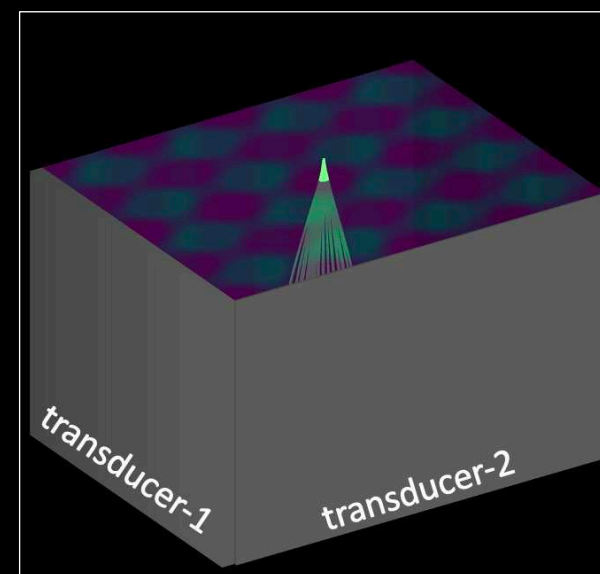
time-of-flight  
imaging



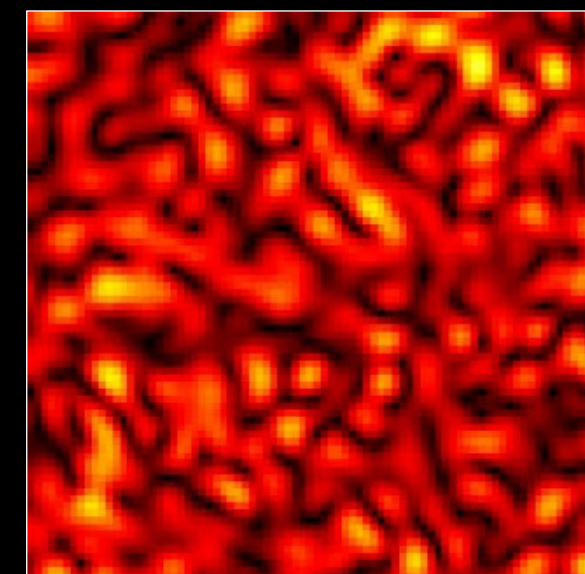
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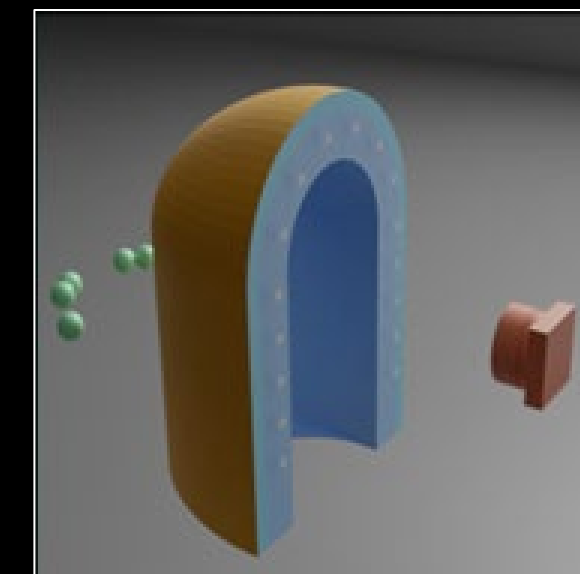
acousto-optic  
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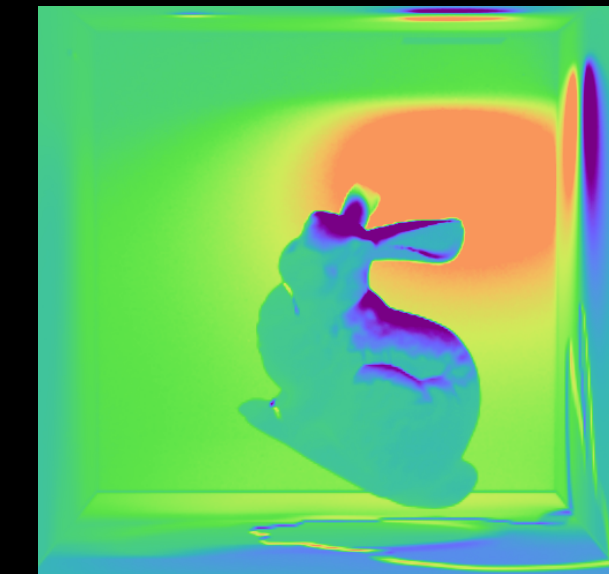
ultrafast light  
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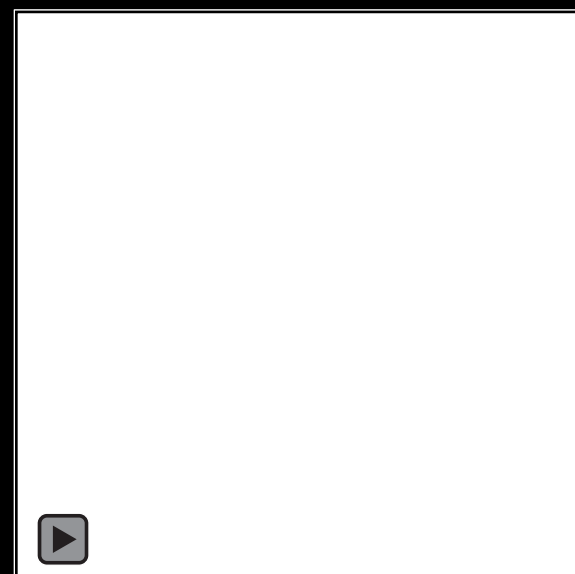
speckle  
imaging



tactile sensor  
design



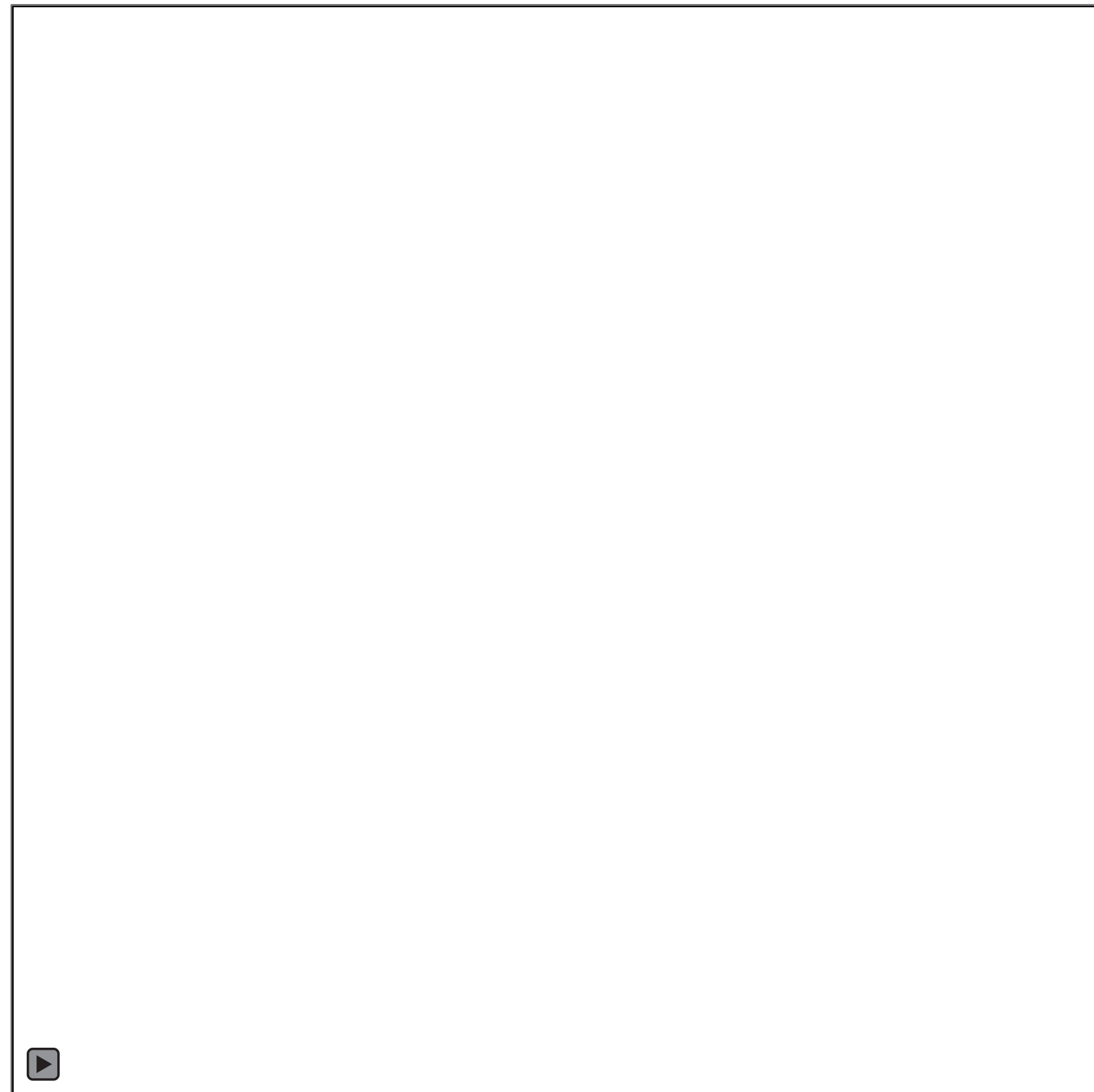
differentiable  
rendering



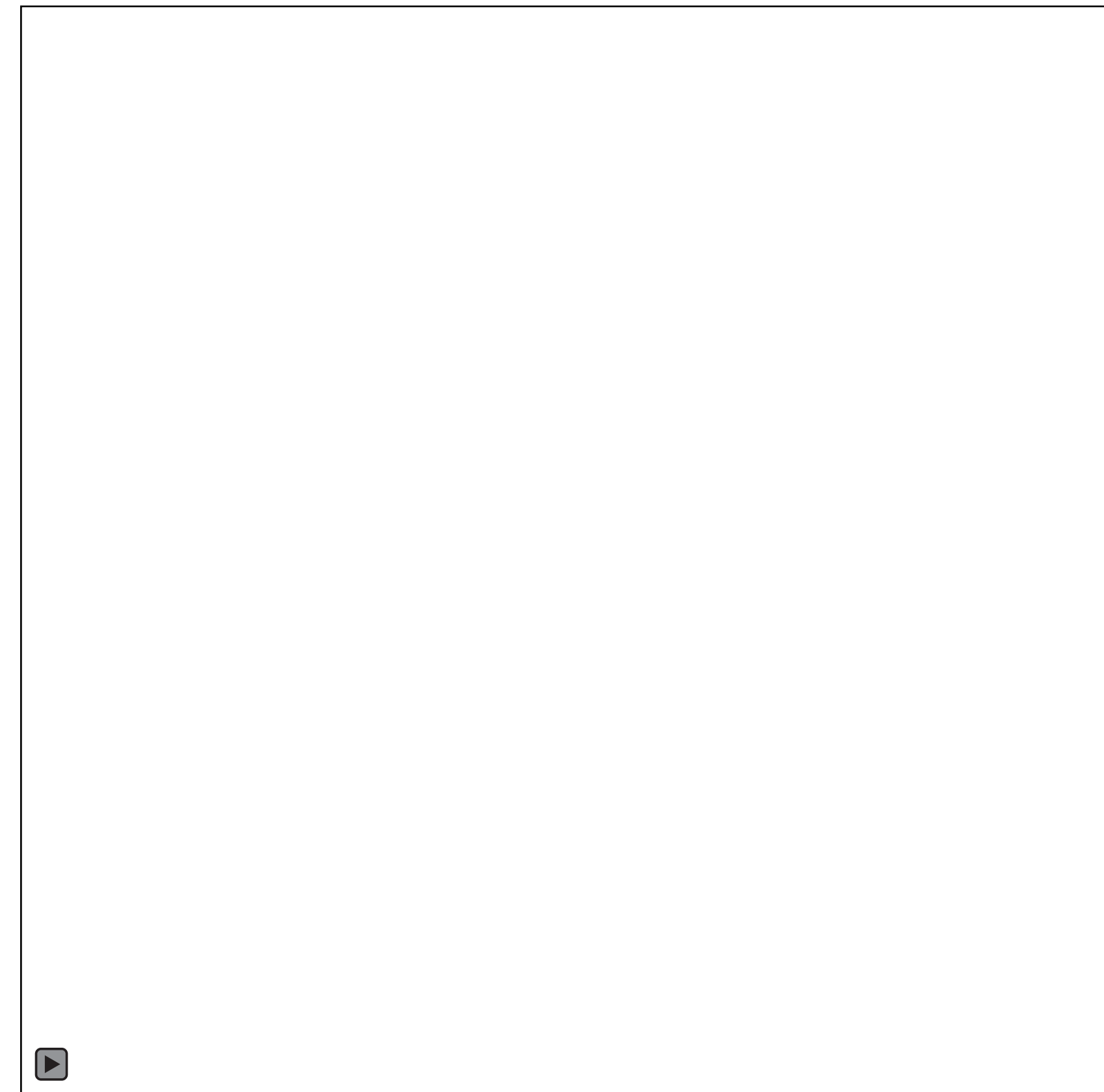
inverse  
problems

# Take-Home Messages

- Great progress has been made in physics-based rendering
  - Capable of handling **multiple types of imaging systems beyond RGB cameras (e.g., time-of-flight, sonar, tactile sensors)**.



transient rendering of static scene

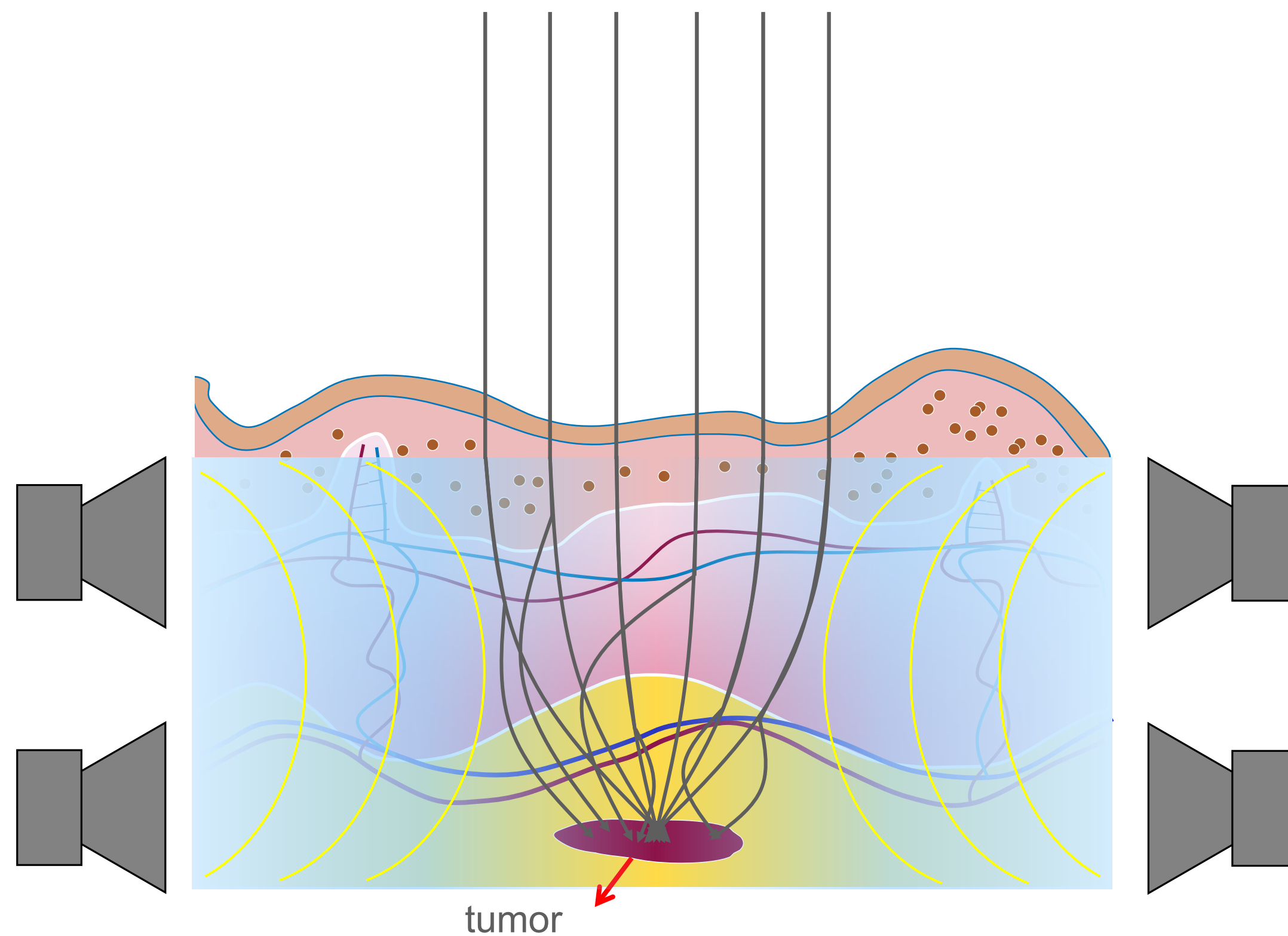


time-gated rendering of dynamic scene

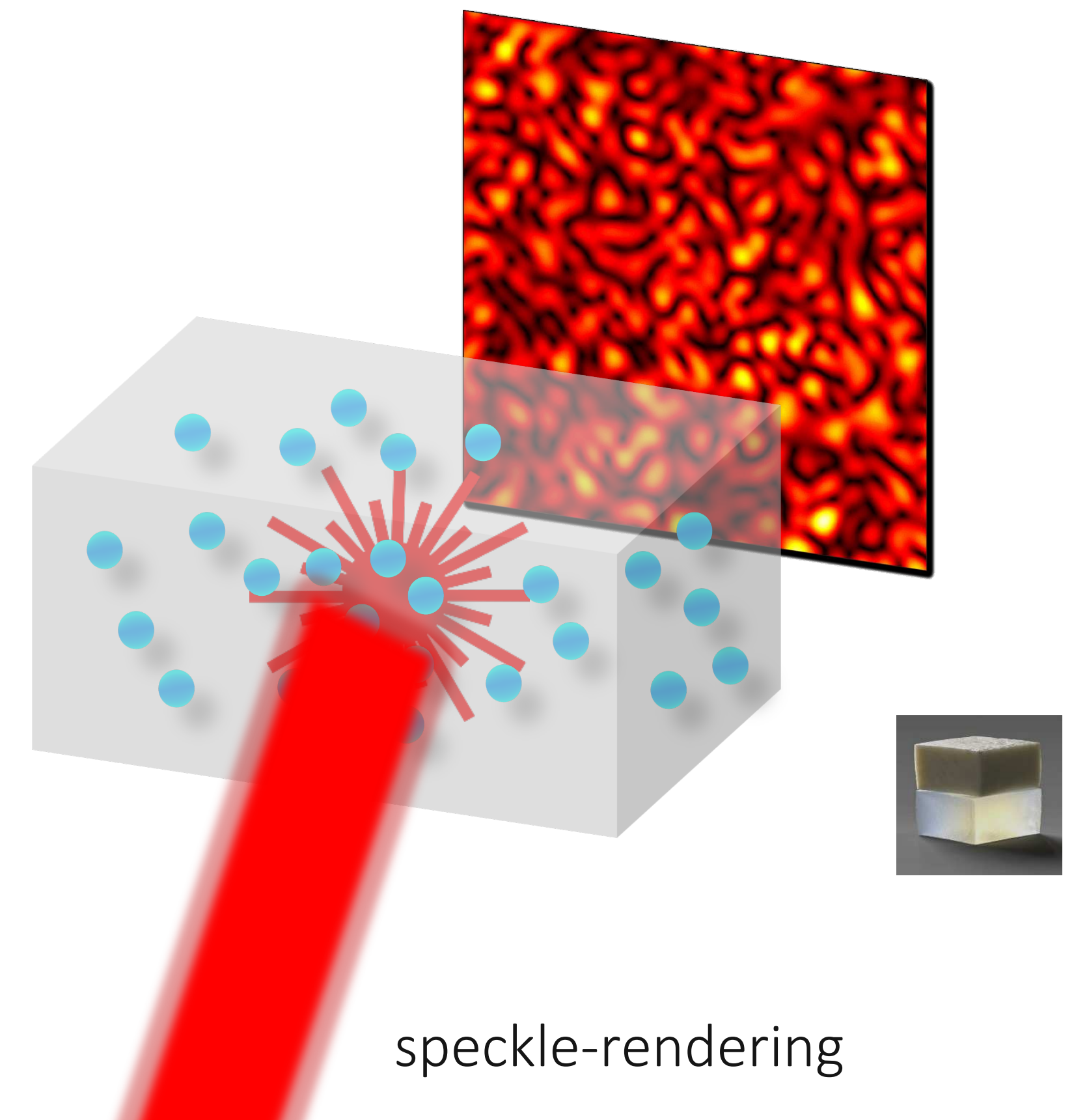


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continuous refraction

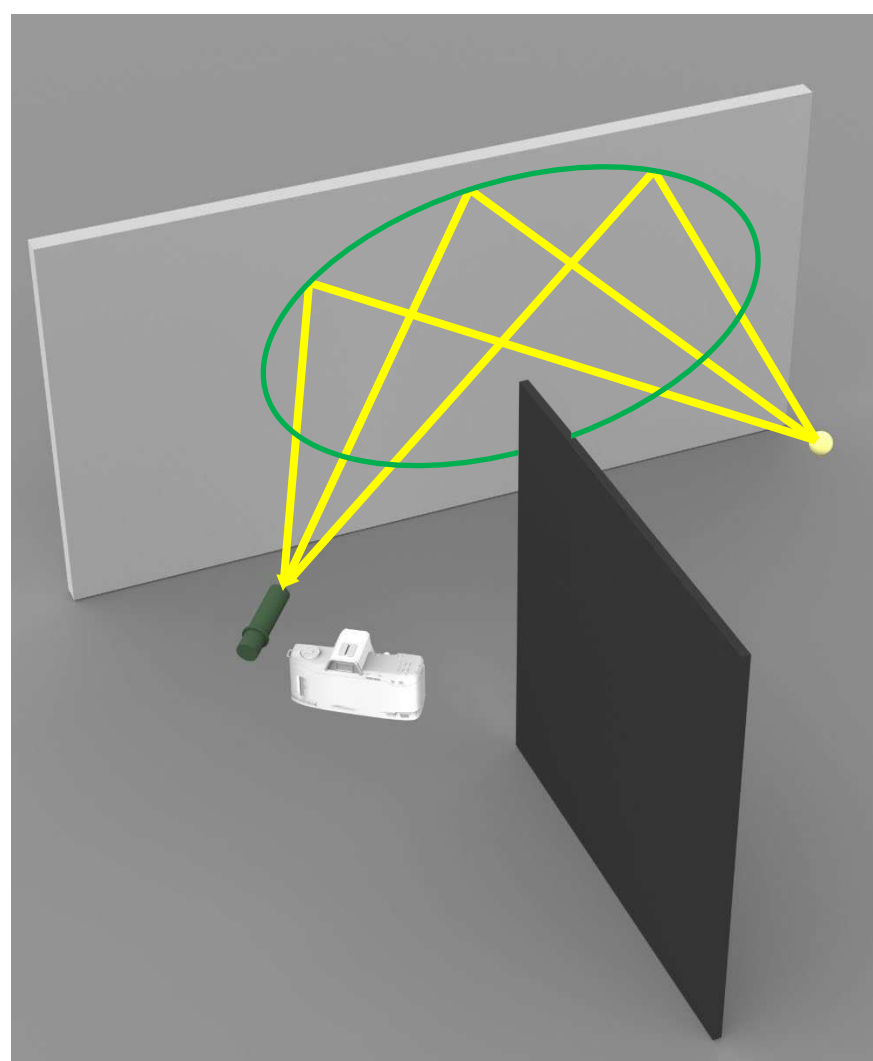


speckle-rendering

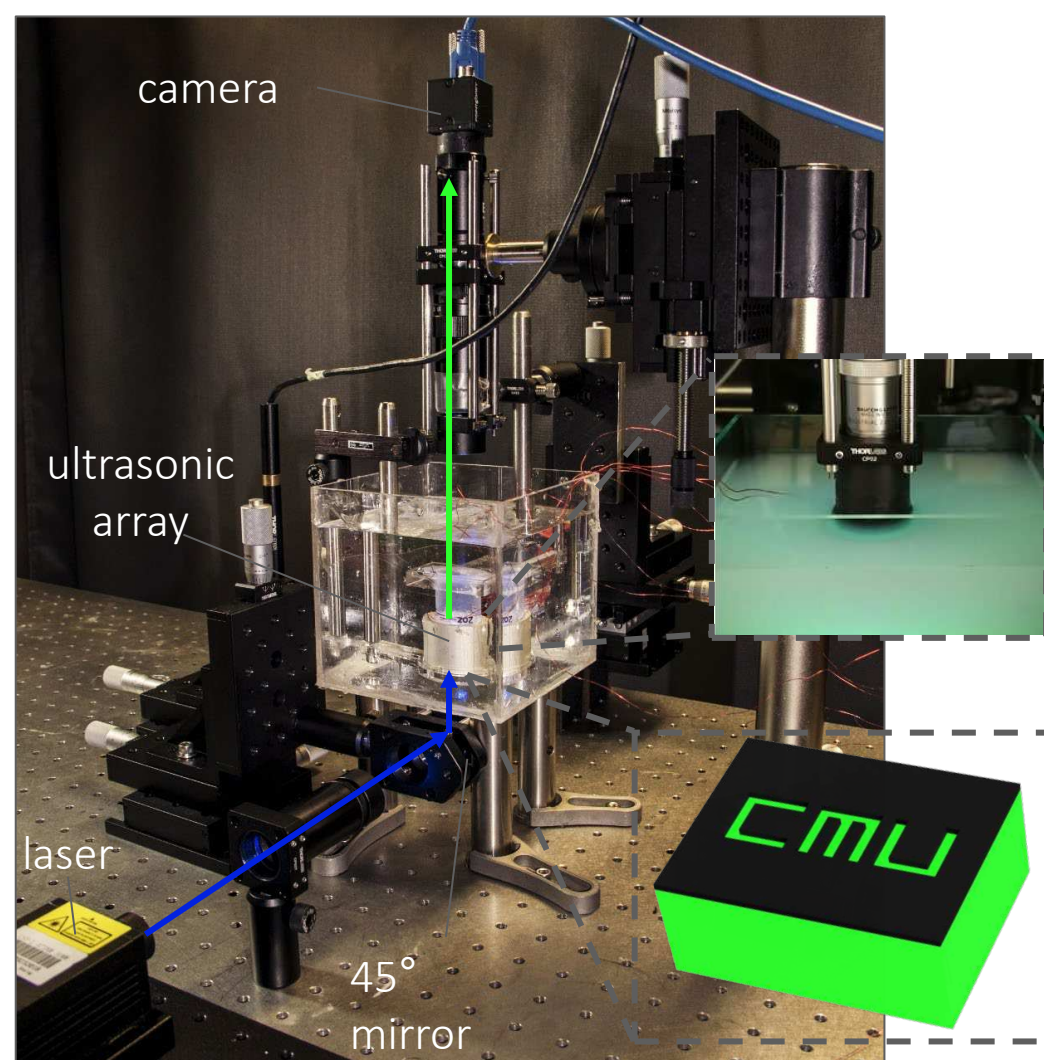


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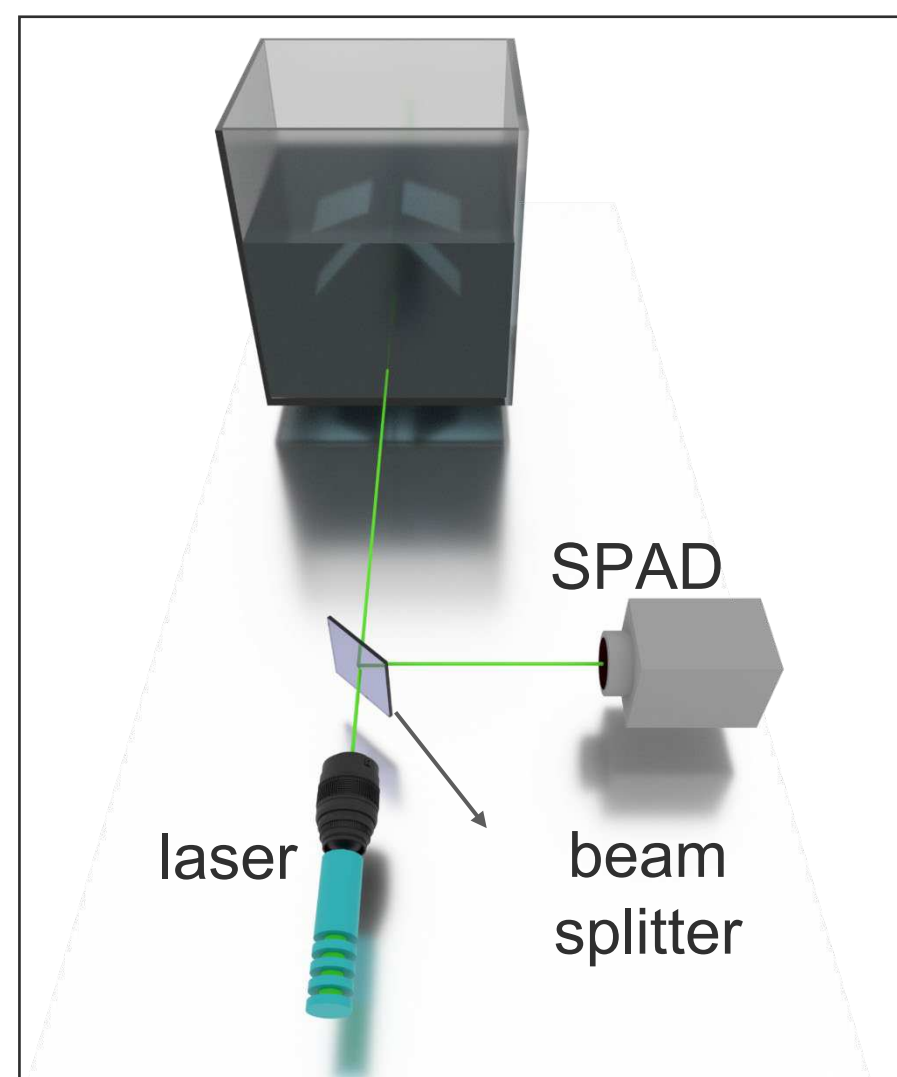
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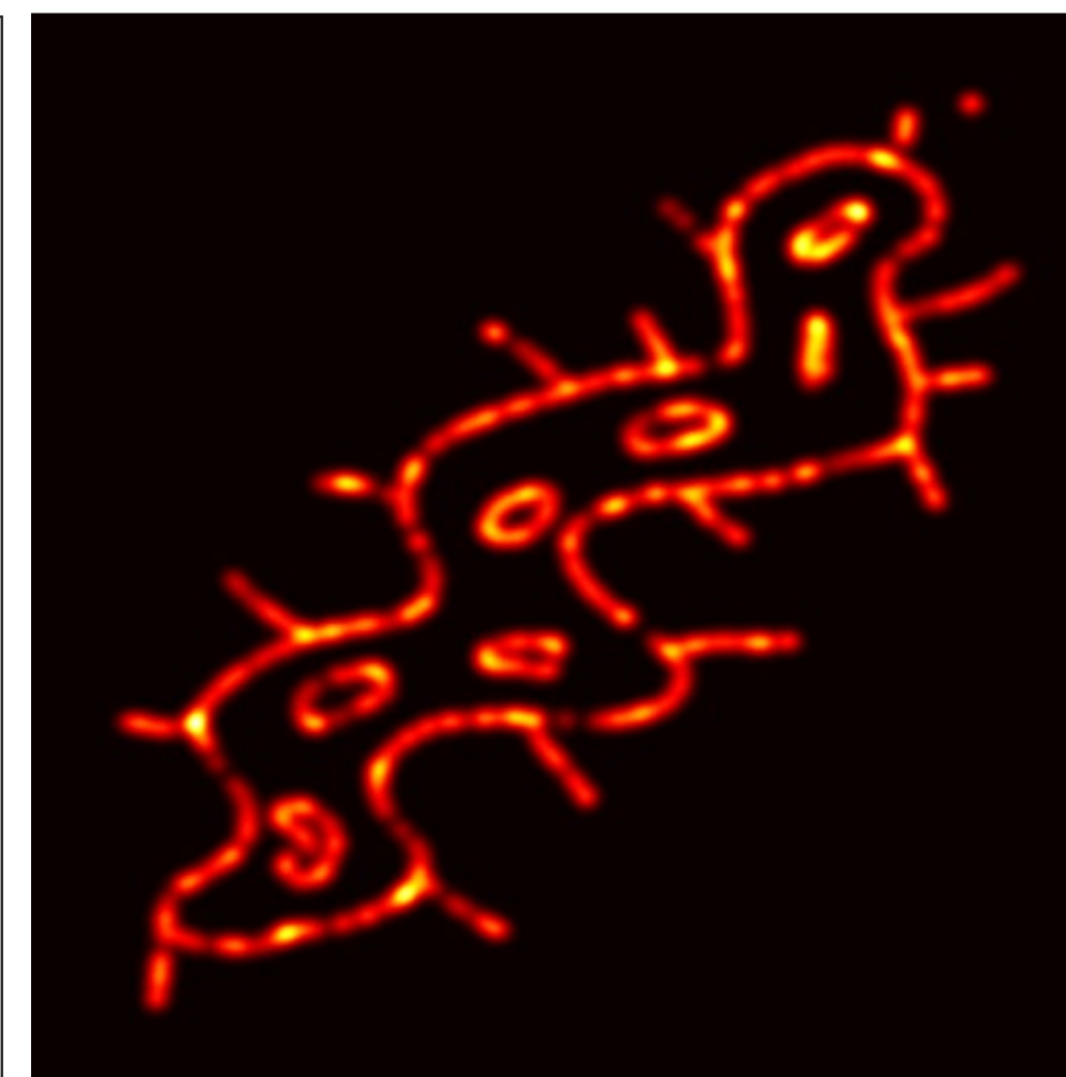
non-line-of-sight  
imaging



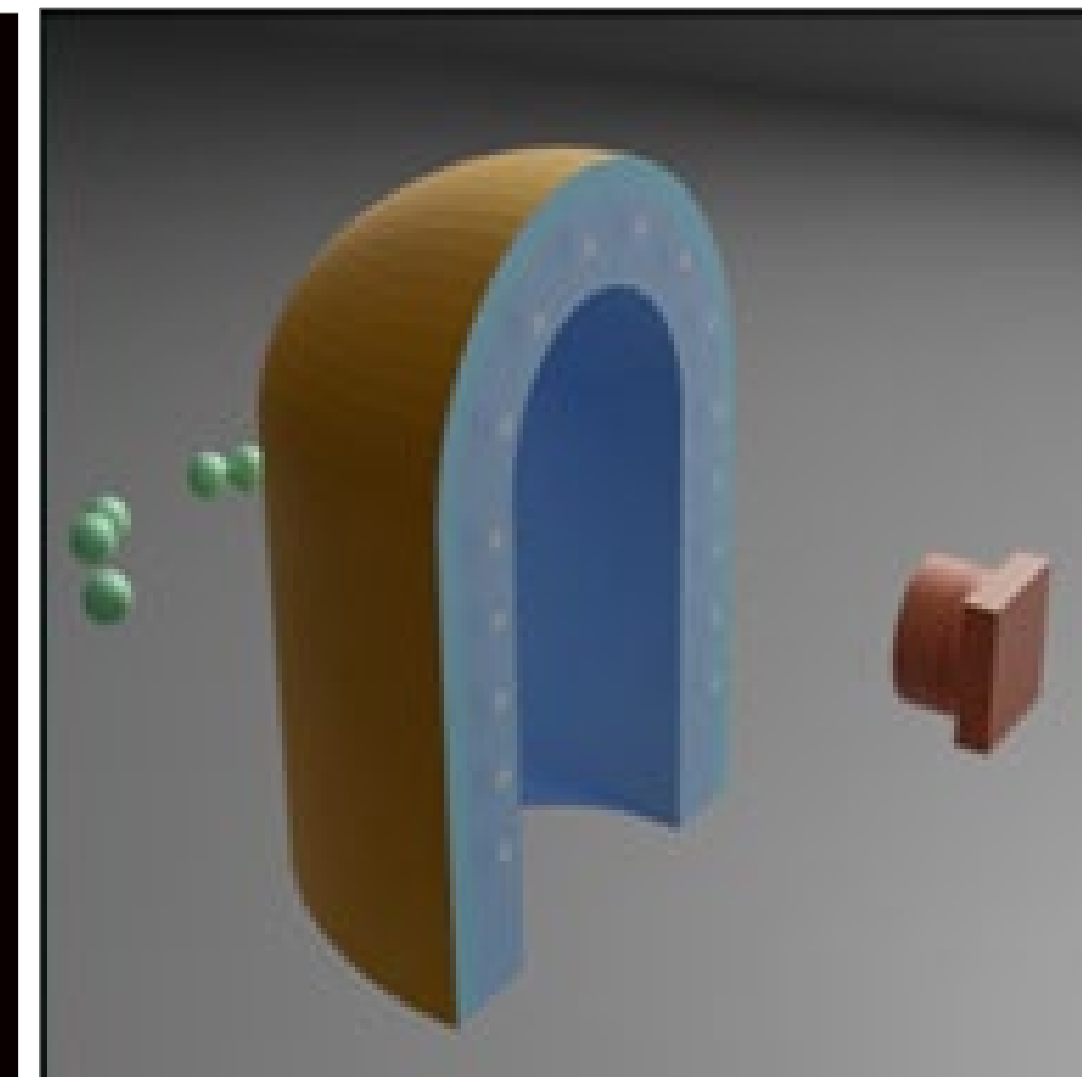
light throughput  
enhancement



ultrafast  
light scanners



imaging through  
scattering media

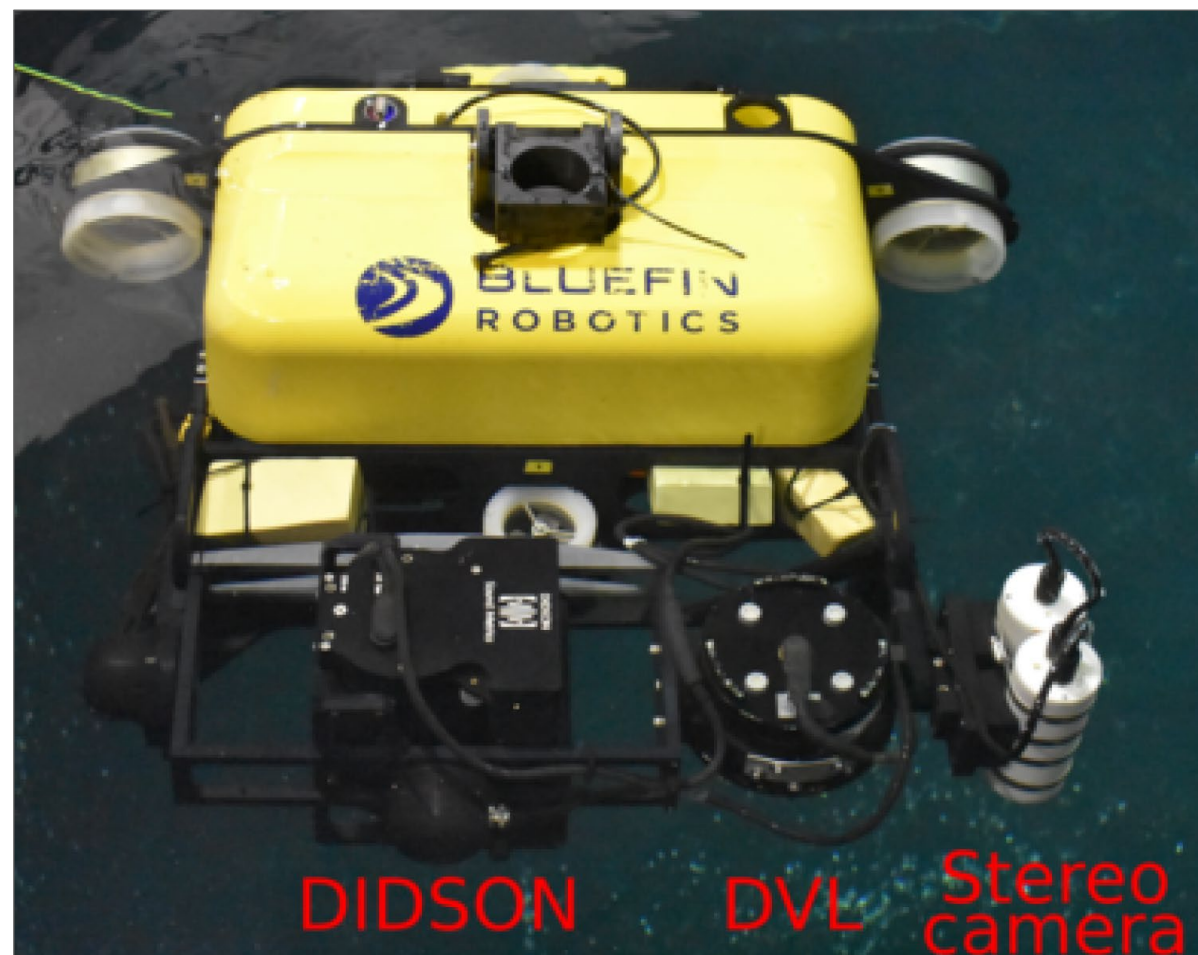


tactile  
sensor design

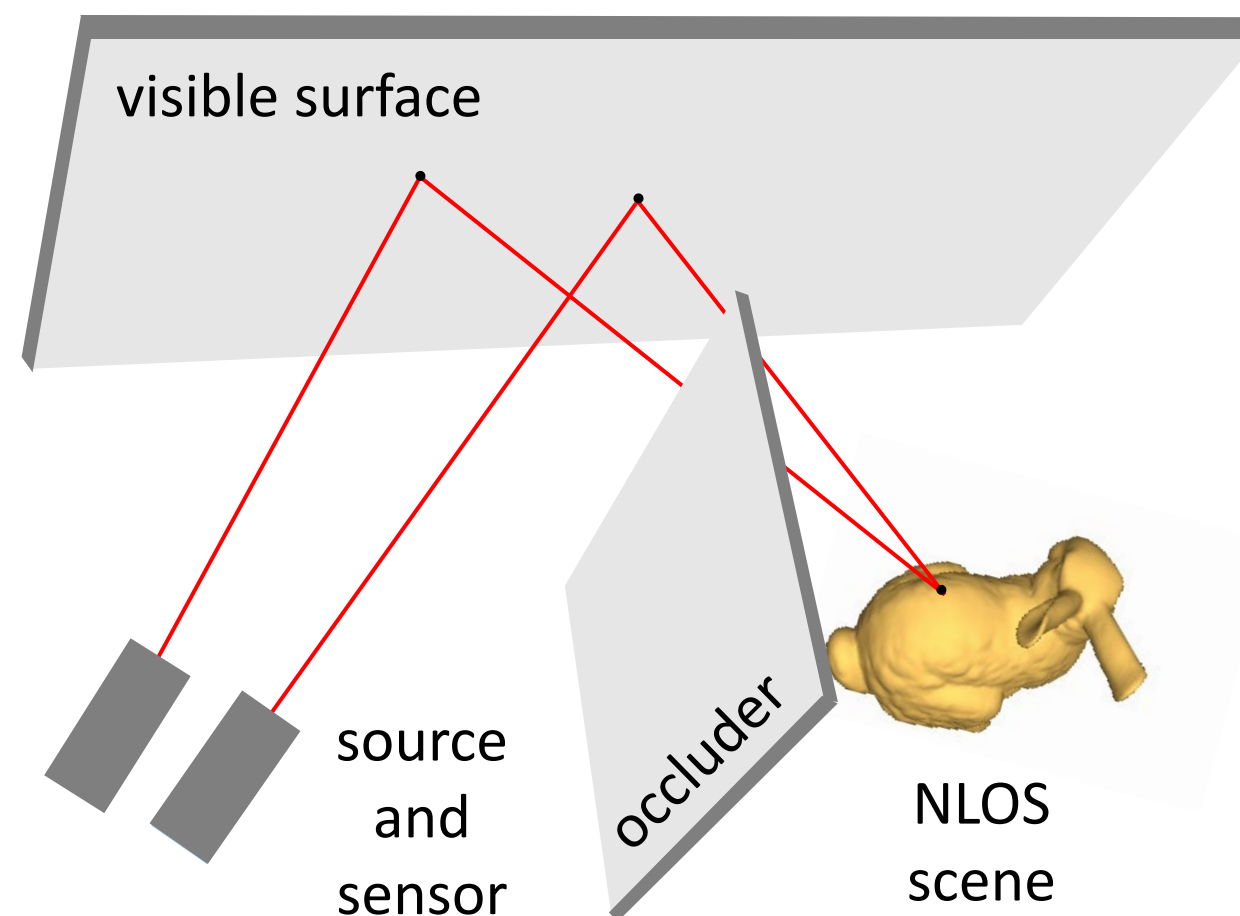


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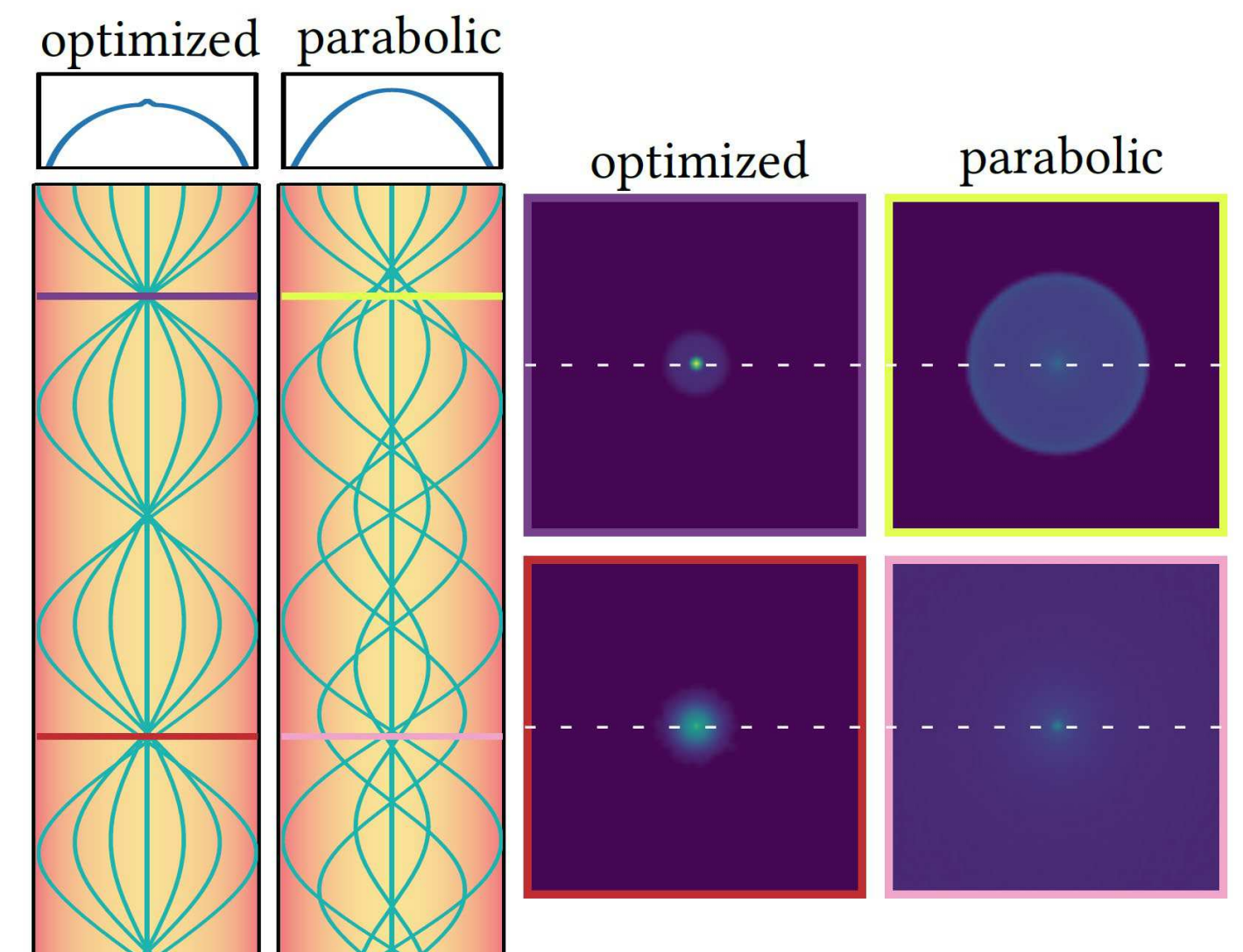
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underwater  
sonar



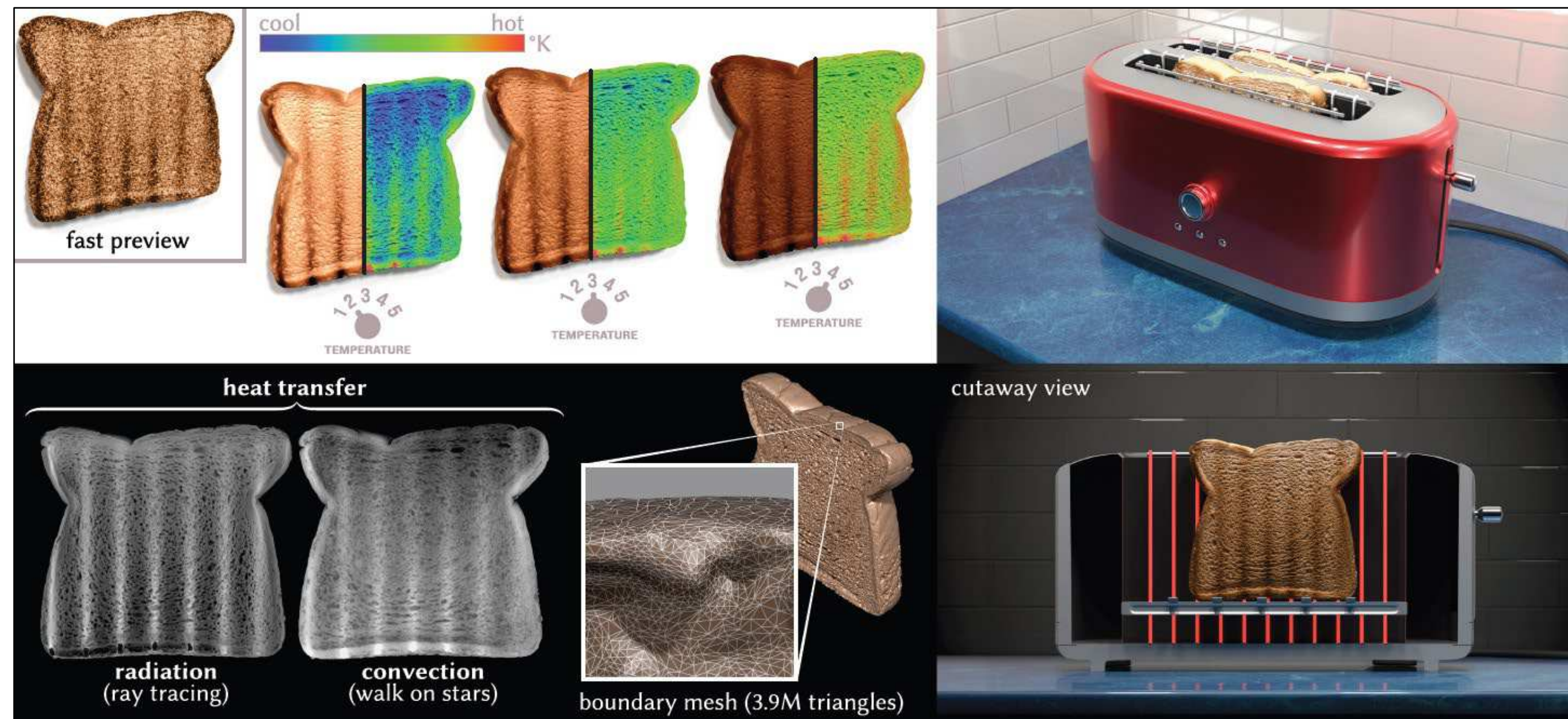
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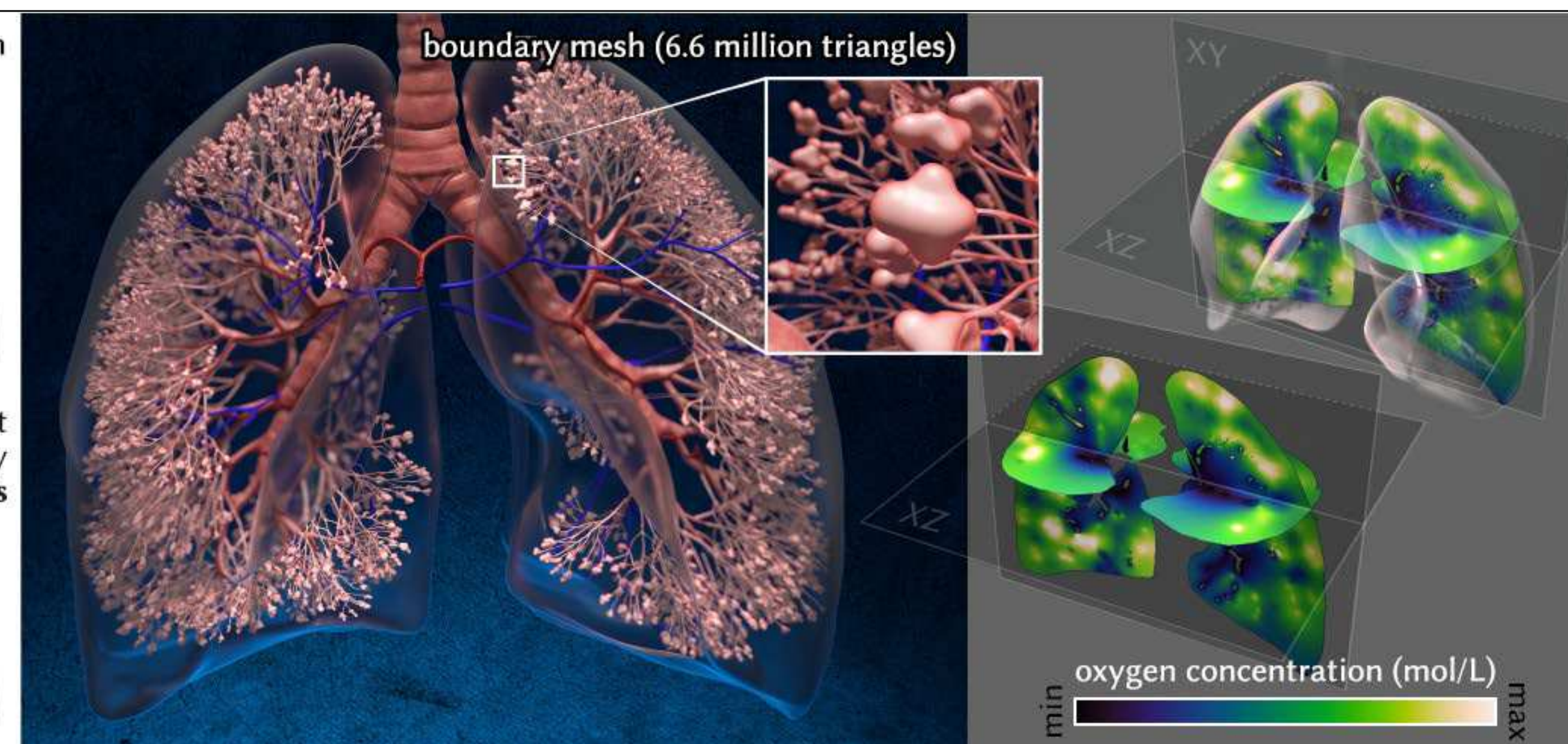
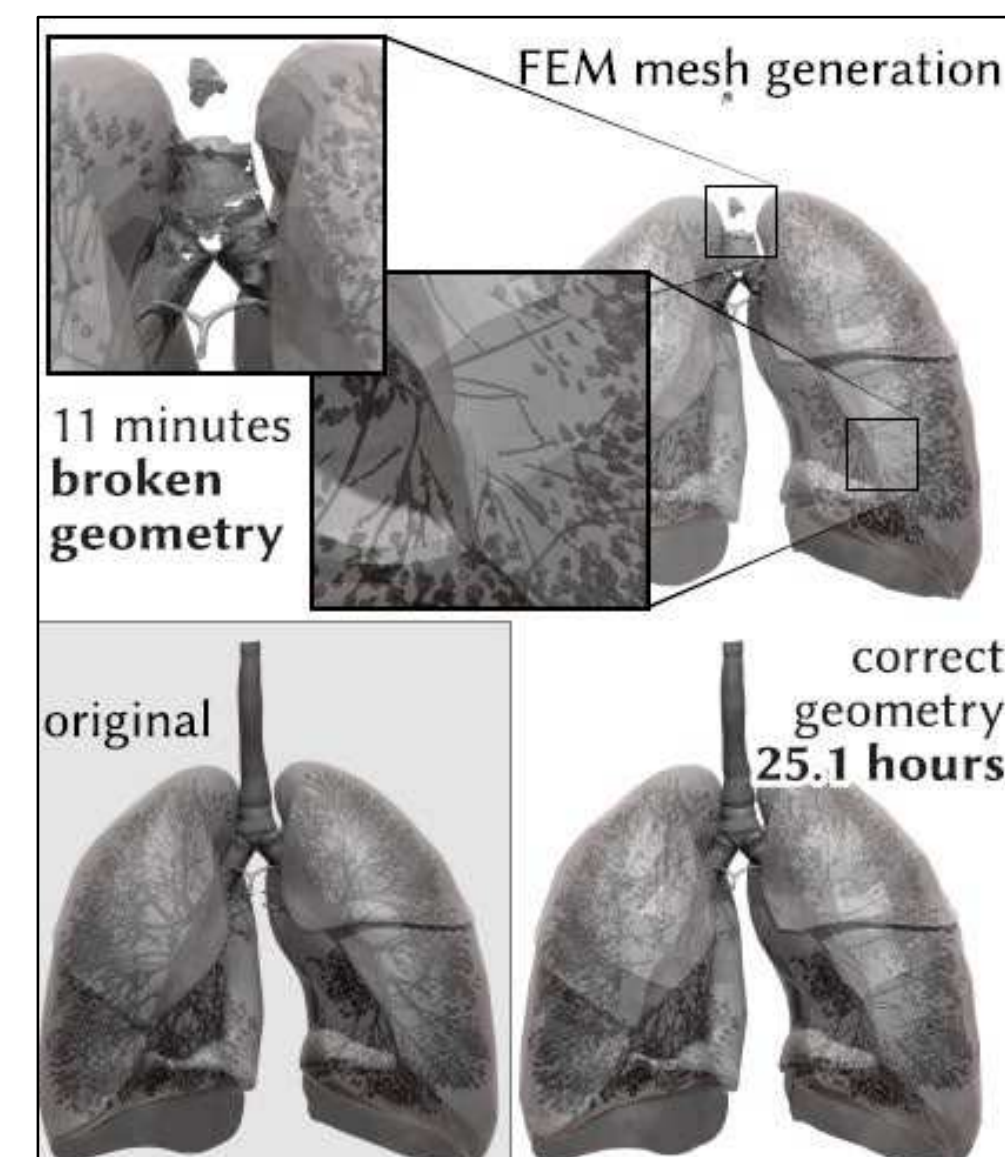
GRIN optic  
design



# Monte Carlo rendering for more general physics and sensing



Simulation of general diffusion processes like heat transfer and oxygen flow



Joint work with  
Rohan Sawhney, Bailey Miller,  
Keenan Crane  
SIGGRAPH 2023





Many thanks to our collaborators



Many thanks to our sponsors



ALFRED P. SLOAN  
FOUNDATION

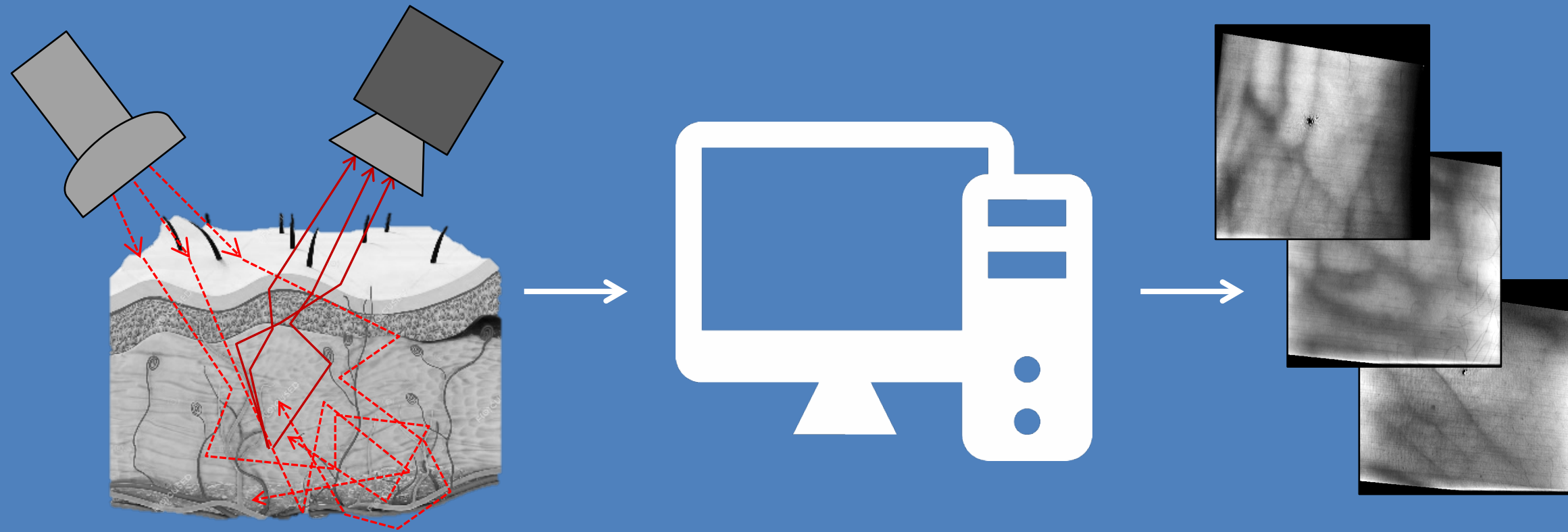


SEE BELOW THE SKIN



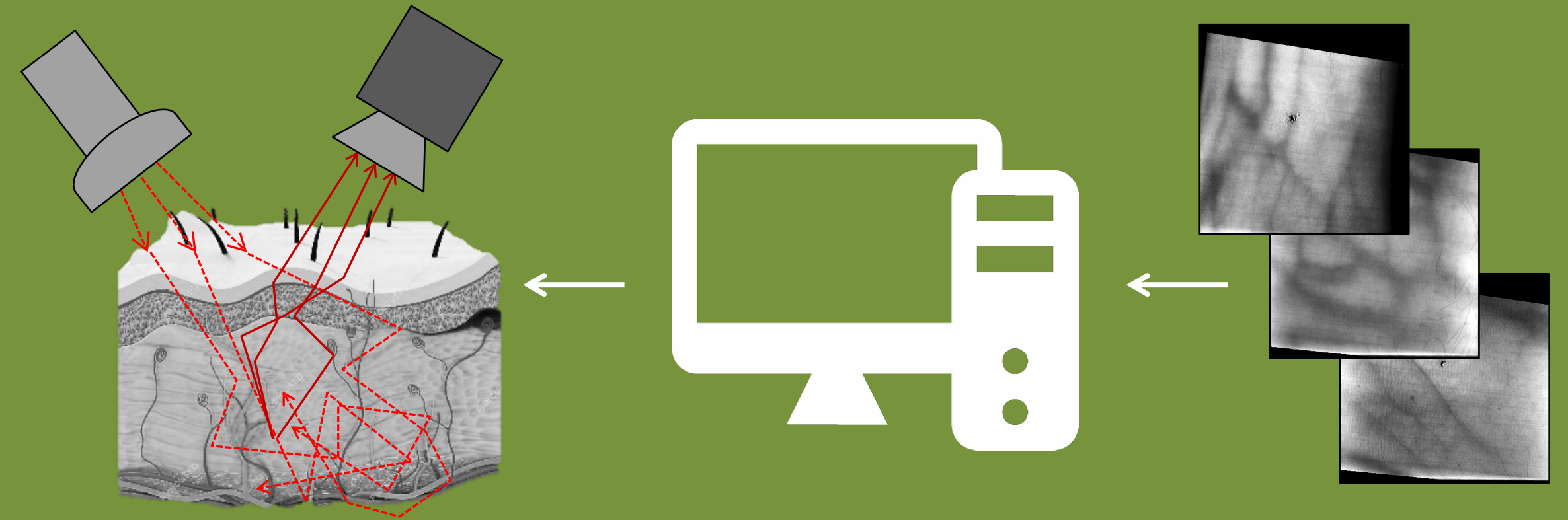
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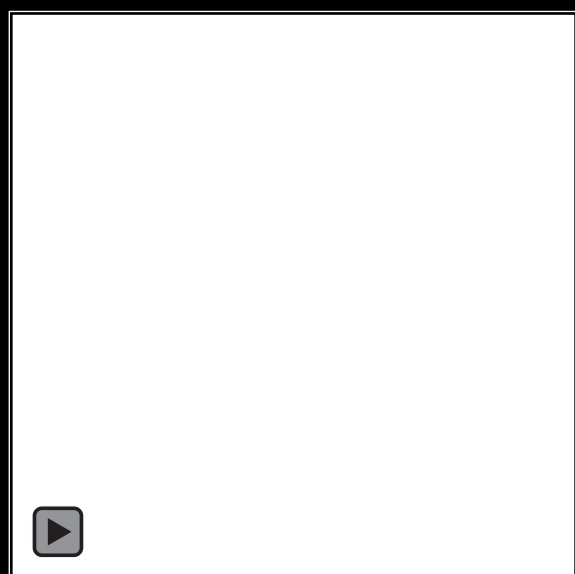


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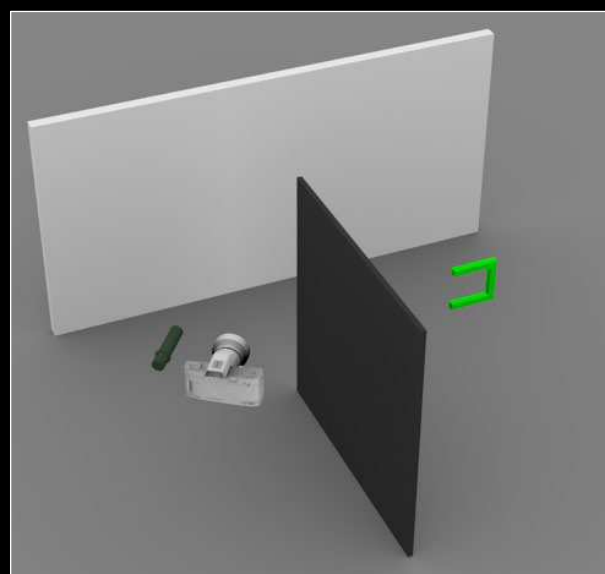
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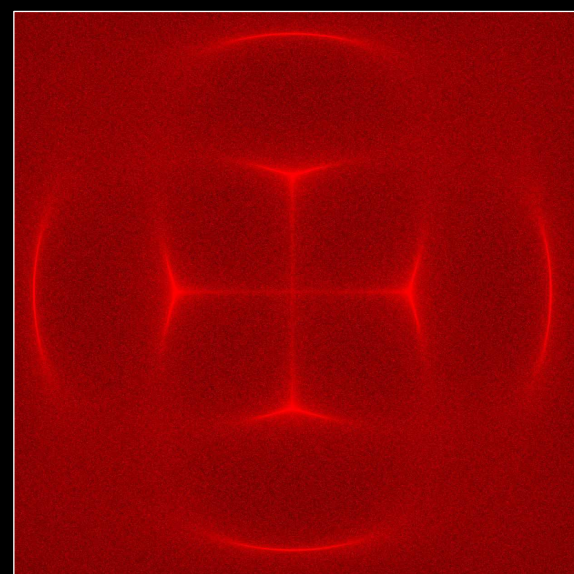
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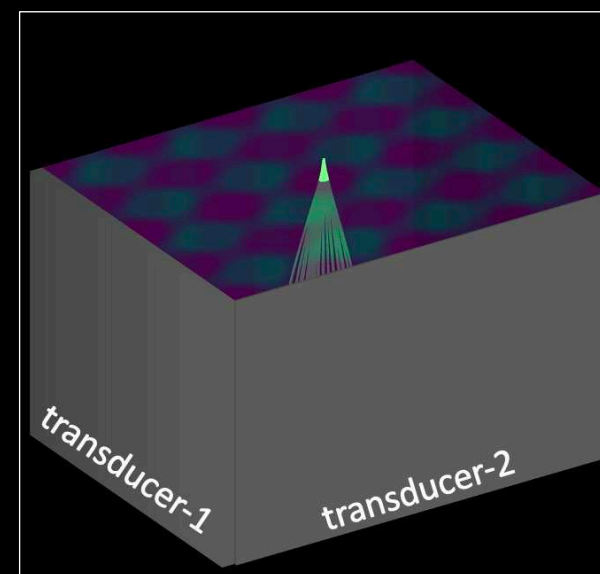
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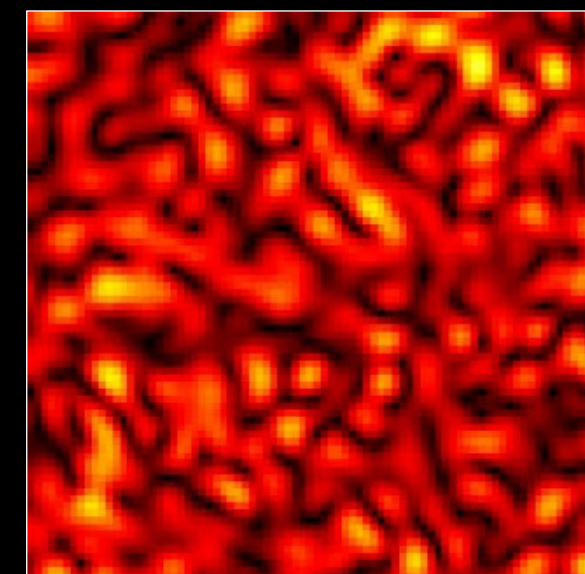
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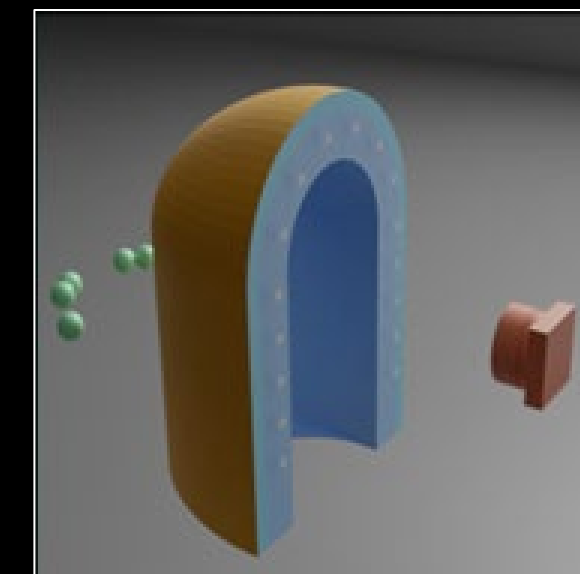
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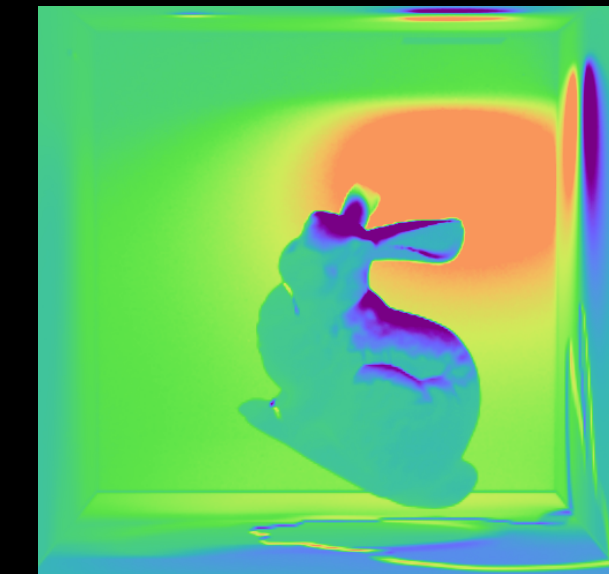
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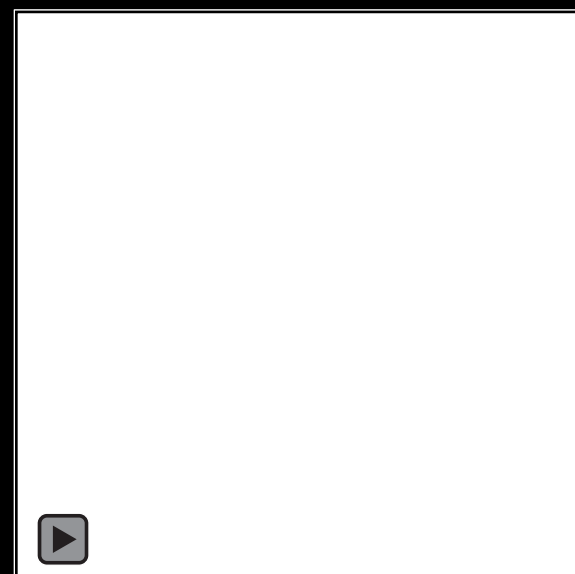
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