# Resolving Shape Ambiguities using Heat Conduction and Shading

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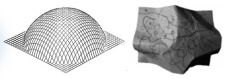
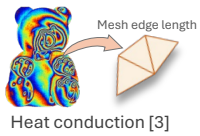
## Light, Heat and Shape

**Shape from Shading** [Light]




**Laplacian from Conduction** [Heat]




**Shape from Light and Heat**

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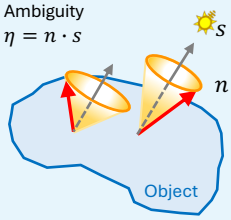
## Intersecting Shading and Laplacian Constraints

### Ambiguity in Shading

Normal lie along the cone

Per pixel Cone-shape Ambiguity

$\eta = n \cdot s$

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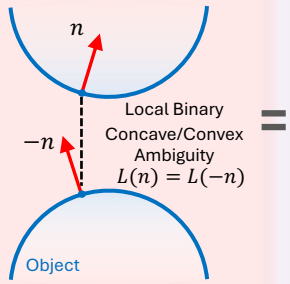
### Ambiguity in Laplacian

Laplacian preserves local curvature

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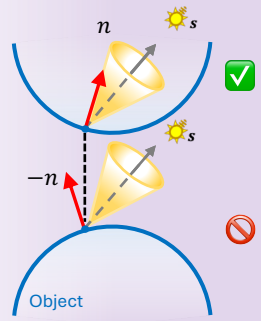
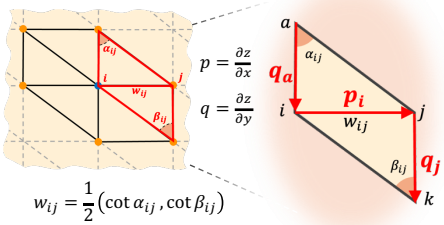
### Laplacian and Shading




## From Laplacian to Normal

**Overview:** We derived cotangent weight $w_{ij}$ in $p, q$ form to compute Laplacian.

Regular grid six-neighboring edges




$$w_{ij} = \frac{1}{2}(\cot \alpha_{ij}, \cot \beta_{ij})$$



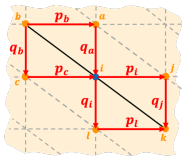

$$w_{ij} = \frac{1}{2}\left(\frac{1 + p_i q_a + q_a^2}{\sqrt{{p_i}^2 + {q_a}^2 + 1_j}} + \frac{1 + p_i q_j + q_j^2}{\sqrt{{p_i}^2 + {q_j}^2 + 1_j}}\right)$$

## Estimating Lighting and Shape

**Our approach:** We use locally planar-patches to estimate lighting, and then estimate full shape.

### Lighting from Laplacian and Shading

#### Near-Planar Initialization




$p_b \approx p_c \approx p_i \approx p_l$

$q_a \approx q_b \approx q_i \approx q_j$

⇨Locally planar patch

$$\frac{p(p+q)+1}{\sqrt{p^2+q^2+1}} = w_1$$

$$\frac{q(p+q)+1}{\sqrt{p^2+q^2+1}} = w_2$$

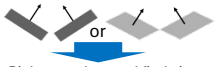
$$\frac{-pq}{\sqrt{p^2+q^2+1}} = w_3$$

$$|p| = \sqrt{(w_1 + w_3)x - 1}$$

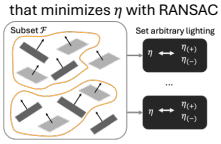
$$|q| = \sqrt{(w_2 + w_3)x - 1}$$

#### Sign Correction

Looking at the sign $w_3$, then




Pick n patches and find sign that minimizes $\eta$ with RANSAC



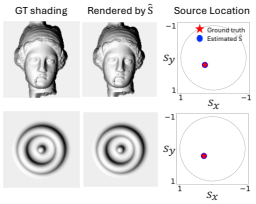

#### Reverse Photometric Stereo

$$\begin{bmatrix} \tilde{p}_1 & \tilde{q}_1 & \alpha_1 \\ \vdots & \vdots & \vdots \\ \tilde{p}_N & \tilde{q}_N & \alpha_N \end{bmatrix} \begin{bmatrix} \tilde{\beta}_x \\ \tilde{\beta}_y \\ \beta_z \end{bmatrix} = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}$$

Estimated $\tilde{\mathbf{N}}$ | Unknown $\tilde{\mathbf{S}}$ | Known $\eta$

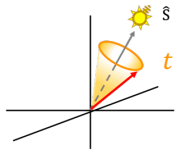
Solution: Least Square Estimation

$$\hat{\mathbf{S}} = (\tilde{\mathbf{N}}^T \tilde{\mathbf{N}})^{-1} \tilde{\mathbf{N}}^T \eta$$




### Shape Estimation

$t$ parametrization




Objective function:

$$\mathbf{t}^* = \arg\min_{\mathbf{t}} \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{N}(i)} \|w_{ik} - \hat{w}_{ik}(\mathbf{t})\|^2$$

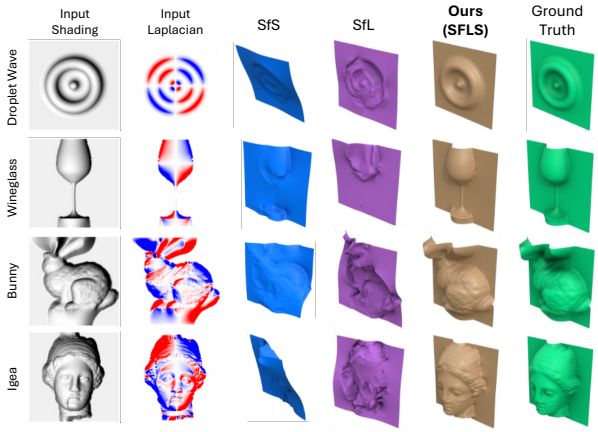
Sign Corrected Subset

Laplacian Error

**No data driven or smoothness prior**

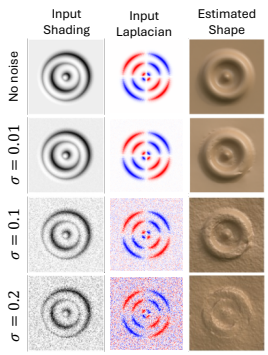
## Results

### Baseline comparison

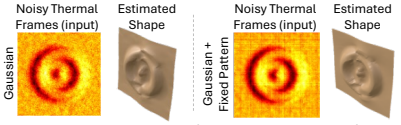



| | Droplet Wave | Wineglass | Bunny | Igea |
|---|---|---|---|---|
| Shape from Shading | 4.17 % | 2.75 % | 6.20 % | 8.60 % |
| Shape from Laplacian | 3.05 % | 3.16 % | 8.51 % | 6.60 % |
| Ours (SFLS) | 0.05 % | 0.23 % | 0.26 % | 1.58 % |

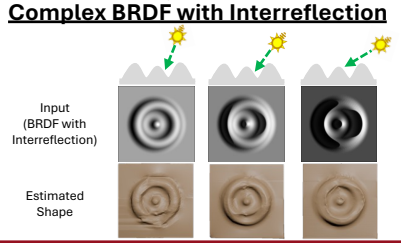
### Gaussian Noise




### Noisy Thermal Frames




### Complex BRDF with Interreflection




## Conclusion

- Both Laplacian and shading suffer from shape ambiguity.
- Combining Laplacian and shading resolves shape ambiguity and enables shape reconstruction without priors.
- Lighting can be estimated from Laplacian and shading.

[1] Ikeuchi and Horn, Numerical Shape from Shading and Occluding Boundaries, *Artificial Intelligence*, 1981.
[2] Barron and Malik. Shape illumination, and reflectance from shading, *IEEE PAMI*, 2014.
[3] Narayanan et al., Shape from heat conduction, *ECCV*, 2024.