Walkin’ Robin: Walk on stars with Robin Boundary conditions

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supported by:

imaging.cs.cmu.edu/walk_on_stars_robin/

\[ \Delta u = 0 \quad \text{on } \Omega \]
\[ u = g \quad \text{on } \partial \Omega_D \]
\[ \frac{\partial u}{\partial n} - \mu u = 0 \quad \text{on } \partial \Omega_R \]

Monte Carlo PDE solvers don’t require meshing

\[
\bar{u}(x) = \begin{cases} 
\rho(x, y) P(x, y) u(y) & \text{if } x \in \partial \Omega_D \\
\rho(x, y) \bar{u}(y), \text{ otherwise } & y \sim |P(x, y)|
\end{cases}
\]

at each step construct star-shaped region w/ bounded reflectance

\[
u(x) = \frac{1}{\partial \Omega_D} \int_{\partial \Omega_D} \rho(x, y) P(x, y) u(y) \, dS(x)
\]

walk on stars with Robin boundary conditions

Walk on boundary traces rays on the entire domain and has no safeguards to prevent path throughput from increasing on each step.

w/ Russian roulette proportional to reflectance

recursively estimate until walk reaches Dirichlet boundary

walk on stars is provably convergent

[Sugimoto et al. 2023] walk on stars (ours) reference

\[ R = \text{dist. to } \partial \Omega_D \text{ (multiple intersections)} \]
\[ R = \text{dist. to silhouette } (\text{single intersection}) \]
\[ R \text{ chosen s.t. } \rho_R \in [0, 1] \]

30 m, \( \varepsilon = 1e-3 \)
2h, \( \varepsilon = 2e-4 \)
8h, \( \varepsilon = 1e-4 \)

(generated with fTetWild)